Physics of the Plasma Universe
(1992) Anthony L. Peratt

Appendix A. Transmission Line Fundamentals in Space and Cosmic Plasmas

A.1 Transmission Lines

The high conductivity of cosmic plasma permits electric currents to flow that constrict the plasma to filaments. These current-carrying filaments form transmission lines which allow electric energy to be transported over large distances.

Transmission lines consist of an assemblage of two or more conducting paths. Transmission lines on earth, used for communications and the transport of electric energy, employ conductors that are usually arranged parallel to a common axis. This need not be the case in space and is often not the case in filamentary current-conducting plasma in pulsed-power generators. Nevertheless, a simplification in analysis results if we assume parallel conducting paths. The generalization to nonparallel transmission lines, such as radially converging lines, is a straightforward extension of the theory. For the case at hand, the geometric and physical parameters of the line (the nature of the conductors and of the dielectric) are assumed to be constant everywhere along the line; this is the hypothesis of homogeneity of the line. This assemblage of conductors comprises two groups of at least one conductor each, one group being the *forward conductors*, and the other the *return conductors*.

The simplified theory of lines that is to be treated here assumes that the lateral dimensions of the line are negligible, or, more precisely, that the time of propagation of the electromagnetic field between the forward and return conductors in a plane perpendicular to the axis of the line is negligible with respect to the duration of the briefest of the phenomena to be studied. This restriction leads to the second fundamental hypothesis, that of the *conservation of current* across a plane is zero, which is to say that the current through the forward conductors is equal, but in the opposite sense to the current through the return conductors.

These two fundamental hypotheses reduce the theory of transmission lines to a problem of partial differential equations in two variables (time and one space variable taken along the axis of the line). The general case would lead to partial differential equations in time and three space variables.

With no loss of generality, it can be assumed that the line is composed of only two conductors. This considerably simplifies the definition of per unit length parameters of the line. Frequently the additional hypothesis of *symmetry* of the line is made, motivated by the two-wire line having two identical cylindrical conductors. This hypothesis is by no means necessary, and most lines in space do not have this symmetry and may not even be everywhere cylindrical. Nevertheless, we will appeal to this simplification in establishing the general line equations.

A.2 Definition of the State of the Line at a Point

Let us consider an ideal two-conductor transmission line, having forward and return conductors reduced to straight lines (Figure A.1). Let *MN* be the intersection of the transmission line with a

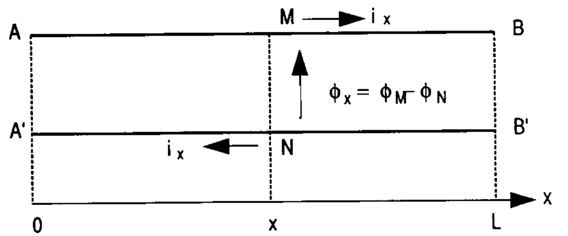

Figure A.1. A two-conductor transmission line.

plane perpendicular to the line and located a distance x along an axis Ox parallel to the line. Arbitrarily choose AB to be the forward conductor and $A'B'$ to be the return conductor.

The *current* in the line at the abscissa x is defined to be the current flowing in conductor AB at point M. It is taken to be positive if it is directed in the sense Ox, i.e., if it flows from M toward B. By the hypotheses of conservation of current, the current at N is equal and opposite to the current at M.

If ϕ_M and ϕ_N are, respectively, the potentials of points M and N with respect to some reference potential, the line voltage at the abscissa x will be

$$\phi_x = \phi_M - \phi_N$$

A.3 Primary Parameters

We will use the term *parameter* for the quantities to be defined below, rather than the term *constant* often used. The latter term arises from the fact that discussions of transmission lines on earth most often consider only sinusoidal waves of a given frequency. On the contrary, we will be especially interested in the pulse regime, which corresponds to a large *band* of frequencies. (The pulse may be measured in seconds, minutes, or hours; or in days, years, centuries, millinia, or megayears or more, but its duration is $\delta t = \delta x/c << l/c$, where l is the total length of the transmission line). At least two of the parameters of interest are strong functions of frequency: the resistance per unit length, affected by the skin depth

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$$

and the conductance per unit length, affected by the variations of dielectric losses with frequency. Nevertheless, in deriving the equations of interest, we will suppose that these parameters are constants.

Since the line is homogeneous, the total resistance of the conductors is proportional to the length of the line. A *resistance per unit length* R can thus be defined.

In the same way a *self-inductance per unit length L* can be defined, which is the result of the true self-inductance of the conductors and the mutual inductance between the two conductors.

Because of the imperfect properites of the plasma dielectric between conductors (losses in the dielectric that separates the conductors), a uniformly distributed transverse conductance appears. It is thus possible to define a *transverse conductance per unit length G*.

This is the ratio of the charge on a unit length conductor element to the voltage between the two conductors at the element considered. The *capacitance per unit length* is denoted *C*.

A.4 General Equations

A.4.1 The General Case

Consider a line element of length Δx (Figure A.2). This element can be compared to a four component discrete circuit (AA', BB') with elements $R\Delta x$, $L\Delta x$, $G\Delta x$, and $C\Delta x$.[1] The voltage rise from A to B will be

$$\Delta\phi = \phi_B - \phi_A = R\ \Delta x i - L\ \Delta x \frac{\partial i}{\partial t} \qquad (A.1)$$

The current Δi flowing into B from B' is

$$\Delta i = -G\Delta x(\phi + \Delta\phi) - C\Delta x\frac{\partial}{\partial t}(\phi + \Delta\phi)$$

$$= -G\Delta x\ \phi - C\Delta x\frac{\partial \phi}{\partial t} + G\Delta x^2\left(R i + L\frac{\partial i}{\partial t}\right) + C\Delta x^2\left(R\frac{\partial i}{\partial t} + L\frac{\partial^2 i}{\partial t^2}\right) \qquad (A.2)$$

Dividing the terms of Eq.(A.1) by Δx and letting Δx go to 0, we obtain the first basic equation:

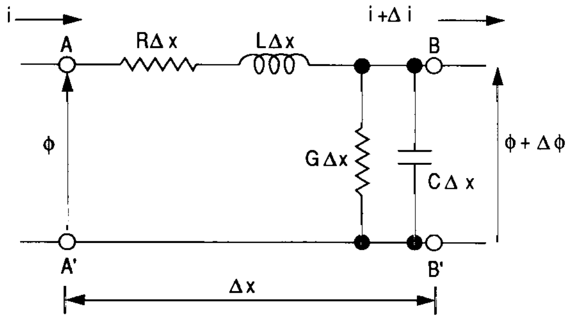

Figure A.2. An infinitesimal line element.

$$\lim_{\Delta x \to 0}\left(\frac{\Delta\phi}{\Delta x}\right)=\frac{\partial\phi}{\partial x}=-Ri-L\frac{\partial i}{\partial t} \tag{A.3}$$

In the same way, from Eq.(A.2),

$$\lim_{\Delta x \to 0}\left(\frac{\Delta i}{\Delta x}\right)=\lim_{\Delta x \to 0}\left\{-G\ \phi-C\frac{\partial\phi}{\partial t}+\Delta x\left[G\left(Ri+L\frac{\partial i}{\partial t}\right)+C\left(R\frac{\partial i}{\partial t}+L\frac{\partial^2 i}{\partial t^2}\right)\right]\right\}$$

which results in the second basic equation:

$$\lim_{\Delta x \to 0}\left(\frac{\Delta i}{\Delta x}\right)=\frac{\partial i}{\partial x}=-G\phi-C\frac{\partial\phi}{\partial t} \tag{A.4}$$

Differentiating Eq.(A.3) with respect to x yields

$$\frac{\partial^2\phi}{\partial x^2}=-\left[R\frac{\partial i}{\partial x}+L\frac{\partial}{\partial t}\left(\frac{\partial i}{\partial x}\right)\right]$$

Replacing $\partial i/\partial x$ by its value in Eq.(A.4) leads, finally, to

$$\boxed{\frac{\partial^2\phi}{\partial x^2}=RG\phi+(RC+LG)\frac{\partial\phi}{\partial t}+LC\frac{\partial^2\phi}{\partial t^2}} \tag{A.5}$$

The last relation is the *telegrapher's equation*. It can be integrated in certain special cases, e.g., if the voltage ϕ is sinusoidal or in the transient regime using operational calculus. The current equation is of the same form as Eq.(A.5) and can be obtained by differentiating Eq.(A.4) with respect to x.

A.4.2 The Special Case of the Lossless Line

If the parameters G and R can be neglected, the fundamental Eqs.(A.3) and (A.4) simplify to

$$\frac{\partial\phi}{\partial x}=-L\frac{\partial i}{\partial t} \tag{A.6}$$

$$\frac{\partial i}{\partial x}=-C\frac{\partial\phi}{\partial t} \tag{A.7}$$

These relations lead to the equation for the propagation of plane waves, which can also be obtained from Eq.(A.5) by setting $R=0$ and $G=0$:

$$\frac{\partial^2 \phi}{\partial x^2} = LC\frac{\partial^2 \phi}{\partial t^2} \tag{A.8}$$

Setting

$$\boxed{u = \frac{1}{\sqrt{LC}}} \tag{A.9}$$

in which u is the *propagation constant*, or the delay per unit length, this equation takes the form

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{1}{u^2}\frac{\partial^2 \phi}{\partial t^2}. \tag{A.10}$$

The solution of Eq.(A.10) is of the form

$$\boxed{\phi(x,t) = \phi^+(x - ut) + \phi^-(x + ut)} \tag{A.11}$$

in which ϕ^+ and ϕ^- are arbitrary functions. The current in the line can now be found from relations Eqs.(A.6), (A.7), and (A.11), which yield

$$\boxed{i(x,t) = \frac{1}{R_c}\left[\phi^+(x - ut) - \phi^-(x + ut)\right]} \tag{A.12}$$

where

$$\boxed{R_c = \sqrt{\frac{L}{C}}} \tag{A.13}$$

is by definition the *characteristic resistance* of the line.

Formulas for R_c for a number of transmission-line geometries and configurations are available in the literature [Westman 1960].

A.5 Heaviside's Operational Calculus (The Lapace Transform)

A.5.1 The Propagation Function

Consider a transmission line such as defined above, having per unit length parameters L, C, R, and G, and of length l, as shown in Figure A.3.

At the instant $t = 0$, an electromotive force $\Phi(t)$ arising from a voltage source with internal impedance Z_g, is applied to the left end, or input, of the line. The right end, or load, is terminated

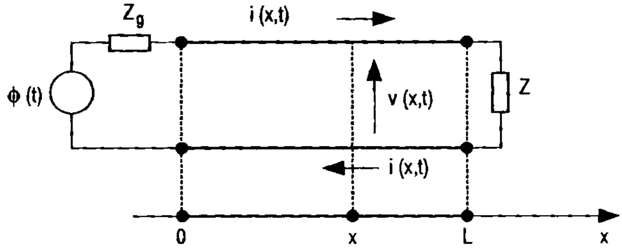

Figure A.3. Transmission line circuit under study with current i(x,t) and voltage ϕ(x,t)=V(x,t).

in an impedance Z_L. The currents and voltages at each point along the line are assumed to be zero prior to the initial time $t = 0$. The problem is to calculate the voltage $\phi(x,t)$ between the two conductors of the line, and the current $i(x,t)$ flowing in each of these conductors, at each point x and at each instant t. The distance x is taken to be positive in the direction to the left of the origin, or generator side of the line.

Equations (A.3) and (A.4) can be rewritten in terms of the Laplace transforms of the current and voltage waveforms. To that end consider

$$\Phi(x,s) \Leftrightarrow \phi(x,t); \quad I(x,s) \Leftrightarrow i(x,t)$$

Using the conditions

$$\frac{\partial \phi(x,t)}{\partial t} \Leftrightarrow s\,\Phi(x,s) - \phi(x,0^+); \quad \frac{\partial i(x,t)}{\partial t} \Leftrightarrow sI(x,s) - i(x,0^+)$$

Since the Laplace transform is defined by

$$\Phi(x,s) = \int_0^\infty e^{-st}\,\phi(x,t)\,dt$$

we have

$$\frac{\partial \Phi(x,s)}{\partial x} = \int_0^\infty e^{-st}\frac{\partial \phi(x,t)}{\partial x}dt \Leftrightarrow \frac{\partial \Phi(x,t)}{\partial x} \tag{A.14}$$

and Eqs.(A.3)–(A.4) in the transform domain become

$$\frac{\partial \Phi(x,s)}{\partial x} + (R + Ls)\Phi(x,s)=0 \tag{A.15}$$

$$\frac{\partial I(x,s)}{\partial x} + (G + Cs)I(x,s)=0 \tag{A.16}$$

Differentiating Eq.(A.15) with respect to x and using Eq.(A.16) to eliminate $\partial I(x,s)/\partial x$ from the result, we obtain

$$\frac{\partial^2 \Phi(x,s)}{\partial x^2} - \gamma^2 \Phi(x,s) = 0 \tag{A.17}$$

An analogous relation can be found for the current:

$$\frac{\partial^2 I(x,s)}{\partial x^2} - \gamma^2 I(x,s) = 0 \tag{A.18}$$

where

$$\boxed{\gamma(s) = \sqrt{(R + Ls)(G + Cs)}}$$

is called the propagation function. Note that γ is independent of x, but not of s.

A.5.2 Characteristic Impedance

The differential equation Eq.(A.17) has solutions of the form

$$\Phi(x,s) = \Phi_1(s)\, e^{-\gamma x} + \Phi_2(s)\, e^{\gamma x} \tag{A.19}$$

where $\Phi_1(s)$ and $\Phi_2(s)$ are arbitrary functions of s only, which will be simply written as Φ_1 and Φ_2.

From Eq.(A.19) there follows

$$\frac{\partial \Phi(x,s)}{\partial x} = -\gamma \left(\Phi_1\, e^{-\gamma x} - \Phi_2\, e^{\gamma x}\right)$$

which together with Eq.(A.14) yields

$$I(x,s) = \frac{\gamma}{R + Ls}\left(\Phi_1\, e^{-\gamma x} - \Phi_2\, e^{\gamma x}\right) = \left(\frac{G + Cs}{R + Ls}\right)^{1/2}\left(\Phi_1\, e^{-\gamma x} - \Phi_2\, e^{\gamma x}\right)$$

It is conventional to define

$$\boxed{Z_c(s) \equiv \sqrt{\frac{R + Ls}{G + Cs}}} \tag{A.20}$$

This last quantity, which has the dimensions of an impedance, is called the *characteristic impedance* of the line. It is related to the physical properties of the line, i.e., to its dimensions and to its conductive and dielectric properties. It is a function of s, and hence of time. We will write $Z_c(s)$ simply as Z_c.

The general solutions of Eqs.(A.17) and (A.18) are thus

$$\boxed{\begin{aligned} \Phi(x,s) &= \Phi_1\, e^{-\gamma x} + \Phi_2\, e^{\gamma x} \\ I(x,s) &= \frac{1}{Z_c}\left(\Phi_1\, e^{-\gamma x} - \Phi_2\, e^{\gamma x}\right) \end{aligned}} \tag{A.21}$$

A.5.3 Reflection Coefficients

The complete solution to Eq.(A.21) is obtained by determining the functions Φ_1 and Φ_2 using the boundary conditions at the ends of the line. In space plasmas, the "end" of a transmission line may be a planetary ionosphere or wherever else the conductivity between conducting paths becomes large. Arc discharges across dielectrics (Section 4.6.1) make excellent terminations.

Let $\Phi(s) \Leftrightarrow \phi(t)$ be the transform of the generator voltage. At the input to the line ($x = 0$)

$$\Phi(s) = Z_g\, I(0, s) + \Phi(0, s) = \frac{Z_g}{Z_c}\left(\Phi_1 - \Phi_2\right) + \Phi_1 + \Phi_2 \tag{A.22}$$

From Eqs.(A.21)–(A.22), and the definition of the voltage reflection coefficient at the input to the line

$$\boxed{\Gamma_g = \frac{Z_g - Z_c}{Z_g + Z_c}} \tag{A.23}$$

we obtain

$$\Phi_1 - \Gamma_g\, \Phi_2 = \Phi(s)\,\frac{Z_c}{Z_g + Z_c} \tag{A.24}$$

In the same way, at the output of the line ($x = l$), we find

$$\Phi_1\, e^{-\gamma l}\, \Gamma_l - \Phi_2\, e^{\gamma l} = 0 \tag{A.25}$$

where

$$\boxed{\Gamma_l = \frac{Z_l - Z_c}{Z_l + Z_c}} \tag{A.26}$$

is the *voltage reflection coefficient* at the output of the line. Solving Eqs.(A.24)–(A.25) yields

$$\Phi_1 = \Phi(s)\frac{Z_c}{Z_g + Z_c}\frac{e^{\gamma l}}{e^{\gamma l} - \Gamma_0\,\Gamma_l\,e^{-\gamma l}} = \Phi(s)\frac{Z_c}{Z_g + Z_c}\frac{1}{1 - \Gamma_g\,\Gamma_l\,e^{-2\gamma l}}$$

$$\Phi_2 = \Gamma_l\,\Phi_1\,e^{-2\gamma l} = \Phi(s)\frac{Z_c}{Z_g + Z_c}\frac{\Gamma_l\,e^{-2\gamma l}}{1 - \Gamma_g\,\Gamma_l\,e^{-2\gamma l}}$$

which, when substituted into Eq.(A.21) gives

$$\Phi(x, s) = \Phi(s)\frac{Z_c}{Z_g + Z_c}\frac{e^{-\gamma x} + \Gamma_l\,e^{-\gamma(2l - x)}}{1 - \Gamma_g\,\Gamma_l\,e^{-2\gamma l}} \tag{A.27}$$

$$I(x, s) = \Phi(s)\frac{1}{Z_g + Z_c}\frac{e^{-\gamma x} - \Gamma_l\,e^{-\gamma(2l - x)}}{1 - \Gamma_g\,\Gamma_l\,e^{-2\gamma l}} \tag{A.28}$$

With the aid of the convergent series expansion

$$\frac{1}{1 - \Gamma_g\,\Gamma_l\,e^{-2\gamma l}} = 1 + \Gamma_g\,\Gamma_l\,e^{-2\gamma l} + \ldots + \Gamma_g^n\,\Gamma_l^n\,e^{-2n\gamma l} + \ldots$$

$$= \sum_{n=0}^{\infty} \Gamma_g^n\,\Gamma_l^n\,e^{-2n\gamma l}$$

we arrive at

$$\boxed{\begin{aligned} \Phi(x, s) &= \Phi(s)\frac{Z_c}{Z_g + Z_c}\left[e^{-\gamma x} + \Gamma_l\,e^{-\gamma(2l - x)}\right]\sum_{n=0}^{\infty} \Gamma_g^n\,\Gamma_l^n\,e^{-2n\gamma l} \\ I(x, s) &= \Phi(s)\frac{1}{Z_g + Z_c}\left[e^{-\gamma x} - \Gamma_l\,e^{-\gamma(2l - x)}\right]\sum_{n=0}^{\infty} \Gamma_g^n\,\Gamma_l^n\,e^{-2n\gamma l} \end{aligned}} \tag{A.29}$$

A.6 Time-Domain Reflectometry

Consider the case of a lossless line $R = G = 0$. For this case $\gamma = \sqrt{LC}\, s = u^{-1} s$ and $Z_c = R_c$. Taking the inverse Laplace transform of Eq.(A.29) by using the translation identity

$$L^{-1}\left[\Phi(s)\, e^{-\gamma(s)}\right] = L^{-1}\left[\Phi(s)\, e^{-\frac{x}{u}s}\right] = \phi\left(t - \frac{x}{u}\right)$$

gives, as parameters for Eqs.(A.11) and (A.12)

$$\phi^{+} = \frac{R_c}{R_g + R_c} \sum_{n=0}^{\infty} \Gamma_g^n \Gamma_l^n \, \phi\left\{t - \frac{1}{u}[x + 2ln]\right\}$$

$$\phi^{-} = \frac{R_c}{R_g + R_c} \sum_{n=0}^{\infty} \Gamma_g^n \Gamma_l^{n+1} \, \phi\left\{t - \frac{1}{u}[-x + 2l(n+1)]\right\} \quad \text{(A.30)}$$

Equations (A.11), (A.12), and (A.30) give waveforms in remarkably good agreement with waveforms measured by probes placed within the transmission line. For example, very low resistances caused by dielectric surface flashover in pulsed-power transmission lines can be determined to an accuracy of less than 1% if waveforms are available for at least two spatial locations in the line. The problem reduces to iterating Eq.(A.30) in R_g and R_l until a best fit between calculated and measured waveforms is obtained. Losses in the propagating current and voltage pulses are of course determined by this procedure.

Example A.1 Cosmic transmission-line. Consider for illustration a hypothetical planetary ionosphere-magnetosphere transmission line model (Figure A.4) that might be applied to the

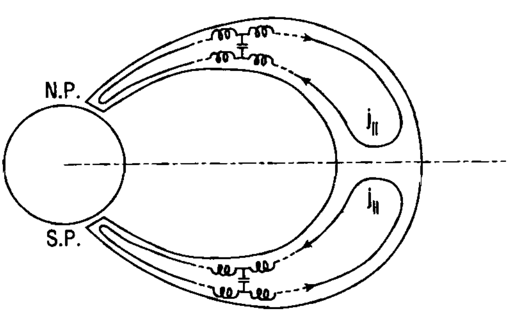

Figure A.4. An ionosphere coupling model. The north and south pole transmission lines need not be symmetric (adapted from Sato, 1978).

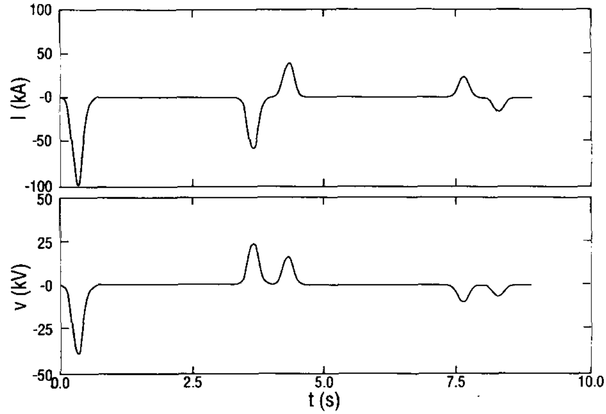

Figure A.5. Current and voltage waveforms associated with a – 40 kV and 250 ms source perturbation on a 6×10^8 m, 0.4Ω transmission line. The source impedance is 2Ω and the ionosphere (load) impedance is 0.1Ω. The probe is located 1×10^8 m from the source.

geometry of Figures 1.9 and 4.24. Assume a –40 kV and 250 ms perturbation at the source. We arbitrarily take a source impedance of 2 Ω, a characteristic transmission line impedance of 0.4 Ω, and an ionosphere (load) impedance of 0.1 Ω. The transmission line length is 6×10^8 m. With these parameter values a probe (spacecraft) located at a distance 10^8 m from the source would measure the current and voltage transients depicted in Figure A.5. After a time 3.3 s following the peak voltage spike from the perturbation, the probe would measure the first ionospheric reflection signal and, if the source impedance differs from the transmission line impedance, it would also measure a reflected signal from the source 0.7 s later. Since the amplitude of the reflections depend on the reflection coefficients Eq.(A.23) and Eq.(A.26), an accurate determination of the source and ionospheric impedance can be made. If probe measurements are available at two spatial locations, the waveforms from Eq.(A.30) are uniquely determined and can be used to ascertain the impedances and their locations. In laboratory application, precise determination of the impedances and positions of high-voltage surface flashovers (Section 4.6.1) has been achieved.

Notes

[1] In rationalized MKS units the acutal values of the line constants for a differential length of line are $L\Delta\, x$ henrys, $R\Delta\, x$ ohms, $C\Delta\, x$ farads, and $G\Delta\, x$ siemens.

Appendix B. Polarization of Electromagnetic Waves in Plasma

A wave equation is derivable from Maxwell-Hertz-Heaviside's equations Eqs.(1.1)–(1.4)

$$\nabla \times \mathbf{H} = \mathbf{j} - i\,\omega\,\varepsilon_0\,\mathbf{E} = -\,i\,\omega\,\varepsilon_0\left[\mathbf{E} + \frac{i}{\omega\,\varepsilon_0}\mathbf{j}\right] = -\,i\,\omega\,\varepsilon_0\,\mathbf{K}\cdot\mathbf{E}$$

(B.1)

where **K**, the relative dielectric tensor, is given by

$$\mathbf{K} = \begin{bmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{bmatrix} \tag{B.2}$$

and derives from the solution to the current density **j** [Stix 1962]. For cold plasma, the matrix elements are

$$S = \frac{1}{2}(R + L)$$

$$D = \frac{1}{2}(R - L)$$

$$P = 1 - \sum_k \frac{\omega_{pk}^2}{\omega^2} \tag{B.3}$$

$$R = 1 - \sum_k \frac{\omega_{pk}^2}{\omega^2}\frac{\omega}{\omega - \omega_b}$$

$$L = 1 - \sum_k \frac{\omega_{pk}^2}{\omega^2}\frac{\omega}{\omega + \omega_b}$$

The vector fields have been taken to be the sum of a zero (external field) and a first order field (e.g. $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1$, $\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_1$) where the first-order quantities vary as $e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$. Setting $\nabla \rightarrow i\,\mathbf{k}$ in Eq.(B.1) gives

$$\mathbf{k} \times \mathbf{B} = -\,\omega\,\mu_0\,\varepsilon_0\,\mathbf{K}\cdot\mathbf{E} = -\frac{\omega^2}{c^2}\mathbf{K}\cdot\mathbf{E} \tag{B.4}$$

Likewise, setting $\partial / \partial t \rightarrow -\,i\,\omega$ in Eq.(1.1) gives

$$\mathbf{k} \times \mathbf{E} = \omega \ \mathbf{B} \tag{B.5}$$

Crossing Eq.(B.5) with **k**, then substituting into Eq.(B.4) yields a wave equation

$$\mathbf{k} \times (\mathbf{k} \times \mathbf{E}) + \frac{\omega^2}{c^2} \mathbf{K} \cdot \mathbf{E} = 0 \tag{B.6}$$

At this point it is convenient to introduce the dimensionless vector **n** which has the direction of the propagation vector **n** and the magnitude of the refractive index

$$\boxed{\mathbf{n} = \mathbf{k} \frac{c}{\omega}} \tag{B.7}$$

The magnitude $n = |\mathbf{n}|$ is the ratio of light to the phase velocity. The reciprocal of n is the wave phase velocity divided by the velocity of light. The wave normal surface is the locus of the tip of the vector $\mathbf{n}^{-1} \equiv \mathbf{n} / n^2$.

Figure B.1 is a plot of phase velocity surfaces for electromagnetic waves. The ordinate is ω_b^2 / ω^2 while the abscissa is ω_p^2 / ω^2. The symbols L, R, X, and O denote left-hand circularly polarized (LHCP), right-hand circularly polarized (RHCP), extraordinary, and ordinary wave types, respectively. This figure is called a Clemmow–Mullaly–Allis or CMA diagram [Allis, Buchsbaum, and Bers 1963] and is to be interpreted as a "plasma pond" for a two-component plasma where cross-sections of the allowable wave normal surfaces are shown. The surfaces are typically in the form of spheres, ellipsoids, and wheel and dumbbell lemniscoids. The diagram is divided up into 13 regions, each of which supports two independent modes.

The significance of the different regions is due to "boundaries" where the refractive indices go either as $n^2 \to 0$ ($v_{ph} \to \infty$) called a "cutoff" condition, or $n^2 \to \infty$ ($v_{ph} \to 0$) called a "resonance" condition (Table B.1). Waves are reflected at cutoffs and absorbed at resonances. The cross-sections in Figure B.1 are not to scale, but the speed of light in relation to the velocities lies generally between the two cross-sections in each region. This divides the wave normal surfaces into "fast" and "slow" modes. The various wave types and their locations in Figure B.1 are delineated in Table B.2.

Substituting Eq.(B.7) into Eq.(B.6) gives a wave equation in terms of **n**

$$\mathbf{n} \times (\mathbf{n} \times \mathbf{E}) + \mathbf{K} \cdot \mathbf{E} = 0 \tag{B.8}$$

Table B.1 Nomenclature for cutoffs and resonances

$P = 0$	plasma cutoff
$S = 0$	plasma resonance, $\theta = \pi/2$
$L = 0$	ion cyclotron cutoff
$R = 0$	electron cyclotron cutoff
$L = \infty$	ion cyclotron resonance
$R = \infty$	electron cyclotron resonance

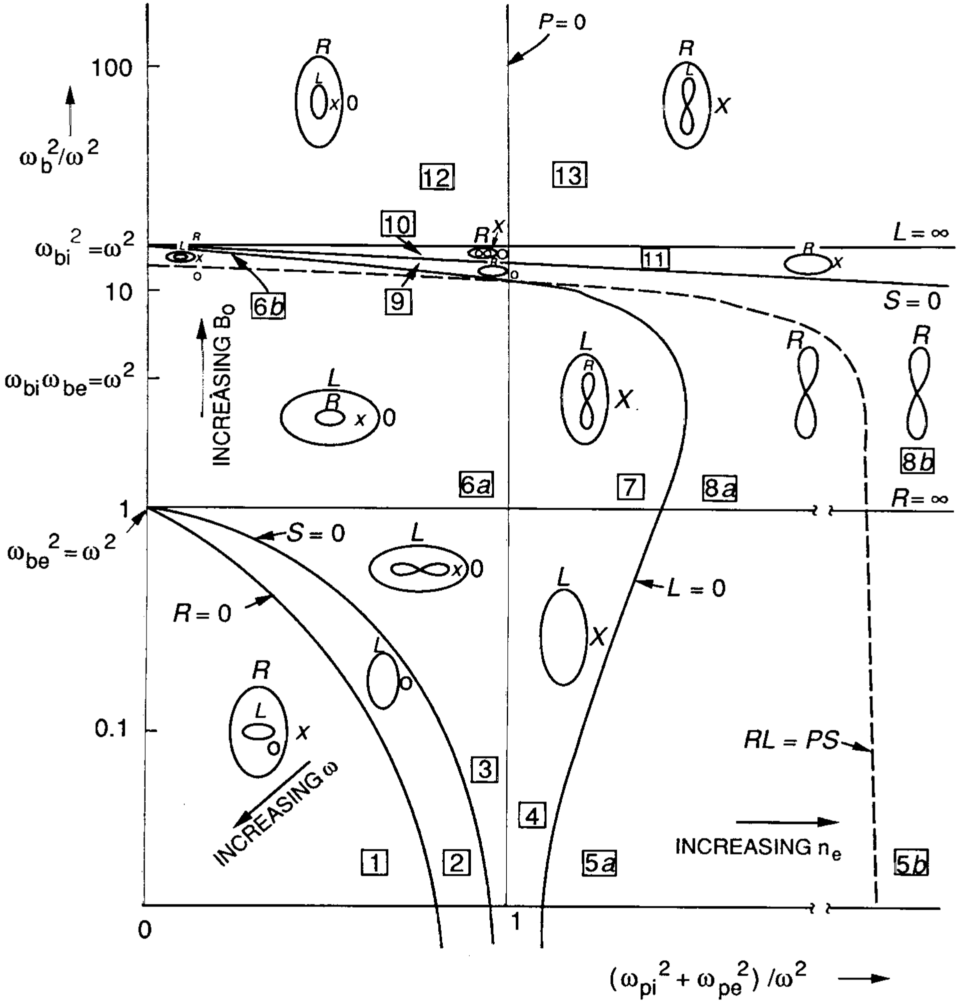

Figure B.1. CMA diagram for a two-component plasma with $m_i/m_e = 4$. Bounding surfaces appear as lines in the two-dimensional parameter space. Cross sections of wave-normal surfaces are sketched and labeled for each region. For these sketches the direction of the magnetic field is vertical.

which, when used for the orientation of the vectors shown in Figure B.2, becomes

$$\begin{pmatrix} S - n^2\cos^2\theta & -iD & n^2\cos\theta\sin\theta \\ iD & S - n^2 & 0 \\ n^2\cos\theta\sin\theta & 0 & P - n^2\sin^2\theta \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0 \tag{B.9}$$

Table B.2. Regions of plasma wave types

type	*region*
Ordinary	1,2,3,4,6,7,12,13
Extraordinary	1,2,3,4,6,7,10,11,12,13
RHCP	1,6,7,8,9,10,11,12,13
LHCP	1,2,3,4,6,7,12,13
Whistler	8
Electron Cyclotron	7,8
Quasi-Transverse Ordinary	1,2,3,6,7,8
Alfvén-Astrom waves, Ion Cyclotron waves	13

The condition for a nontrivial solution to Eq.(B.9) is that the determinant of the square matrix be zero. This condition gives the dispersion relation, or the equation for the wave normal surface

$$An^4 - Bn^2 + C = 0 \tag{B.10}$$

$$A = S\sin^2\theta + P\cos^2\theta$$
$$B = RL\sin^2\theta + PS\left(1+\cos^2\theta\right)$$

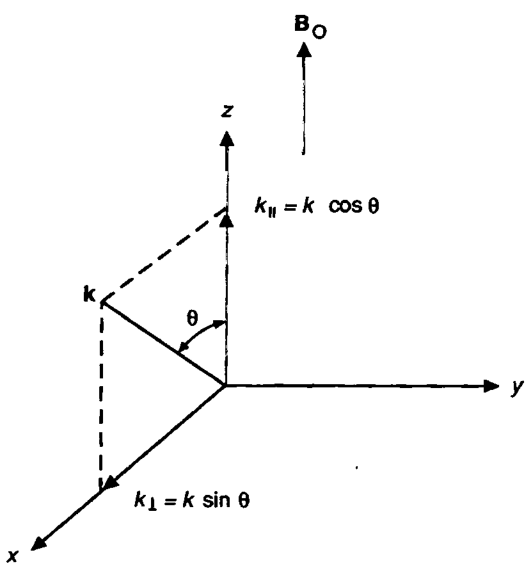

Figure B.2. The propagation vector **k** in relation to the static magnetic field $\mathbf{B}_0$.

$$C = PRL$$

The solutions of Eq.(B.10) reduce to simple expressions when $\theta = 0$ and $\theta = \pi/2$:

$$n^2 = R, L \quad \theta = 0$$
$$n^2 = RL/S, P \quad \theta = \pi/2 \tag{B.11}$$

The polarization relations between the cartesian field components of **E** follow from Eq.(B.10)

$$E_x : E_y : E_z = \left(S - n^2\right)\left(P - n^2 \sin^2\theta\right) : -iD\left(P - n^2 \sin^2\theta\right) : -\left(S - n^2\right) n^2 \cos\theta \sin\theta \tag{B.12}$$

If $\theta = 0$, Eq.(B.12) shows that $E_z = 0$ $\left(\mathbf{E} \perp \mathbf{B}_0\right)$ and, if $n^2 = R$, then $E_x/E_y = -i$. This means that E_x is 90° ahead of E_y and, by convention, the polarization is right-hand circular. If $n^2 = L$, $E_x/E_y = i$ and the polarization is left-hand circular. Figure B.3a illustrates these modes of propagation.

For the case $\theta = \pi/2$ and $n^2 = P$, $E_x = E_y = 0$ ($\mathbf{E} \parallel \mathbf{B}_0$). When $n^2 = RL/S$, $E_z = 0$ $\left(\mathbf{E} \perp \mathbf{B}_0\right)$. For this case the electric field circumscribes an ellipse in a plane of y. The modes $n^2 = P$ and $n^2 = RL/S$ are called the "ordinary" and "extraordinary" waves, respectively. These terms have been taken from crystal optics; however the terms have been interchanged in plasma physics, since the "extraordinary" mode is affected by $\mathbf{B}_0$ whereas the "ordinary" mode is not. Figure B.3b illustrates these modes of propagation.

Expressed in spherical coordinates Eq.(B.12) is

$$E_k : E_\theta : E_\phi = \left(S - n^2\right)\left(P - n^2\right) \sin\theta : -\left(S - n^2\right) P \cos\theta : -iD\left(P - n^2 \sin^2\theta\right) \tag{B.13}$$

which shows that E_k and E_θ are in phase while E_ϕ is out of phase by 90°. Figure B.4 shows the orientation of the field vectors. The vector **E** is elliptically polarized in a plane containing the y direction and the resultant of E_k and E_θ.

Example B.1 Faraday rotation. The magnitudes of the RHCP and LHCP propagation vectors are $k^R = (\omega / c)\sqrt{R}$ and $k^L = (\omega / c)\sqrt{L}$. Faraday rotation is given by

$$\tau = \frac{1}{2}\left(k^L - k^R\right)$$

At high frequencies where $\omega >> \omega_p$, ω_b, leading to the approximations,

$$k^{L,R} \approx \frac{\omega}{c}\left[1 - \frac{\omega_p^2}{2\,\omega^2}\left(1 \pm \frac{\omega_b}{\omega}\right)\right]$$

Hence, the rotation angle of the linearly polarized wave as it propagates through magnetized plasma is

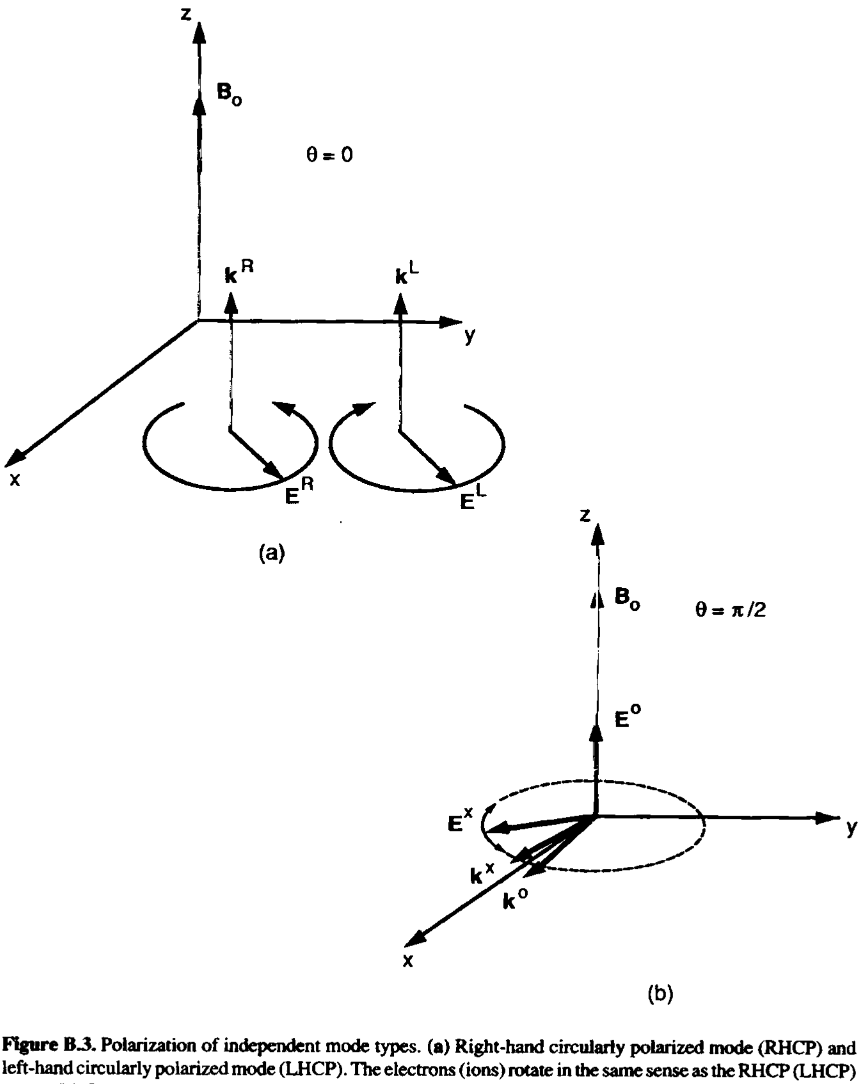

Figure B.3. Polarization of independent mode types. (**a**) Right-hand circularly polarized mode (RHCP) and left-hand circularly polarized mode (LHCP). The electrons (ions) rotate in the same sense as the RHCP (LHCP) mode. (**b**) Ordinary and extraordinary modes.

$$\Delta\chi = \tau\, l = \frac{\omega_p^2\, \omega_b}{2\, c\, \omega^2} = \frac{e^3}{2\, m^2\, c\, \varepsilon_0}\, \frac{n_e\, B_{\parallel}\, l}{\omega^2} \tag{B.14}$$

where $B_{\parallel}$ is the magnitude of **B** along the direction of wave propagation through a plasma of length l.

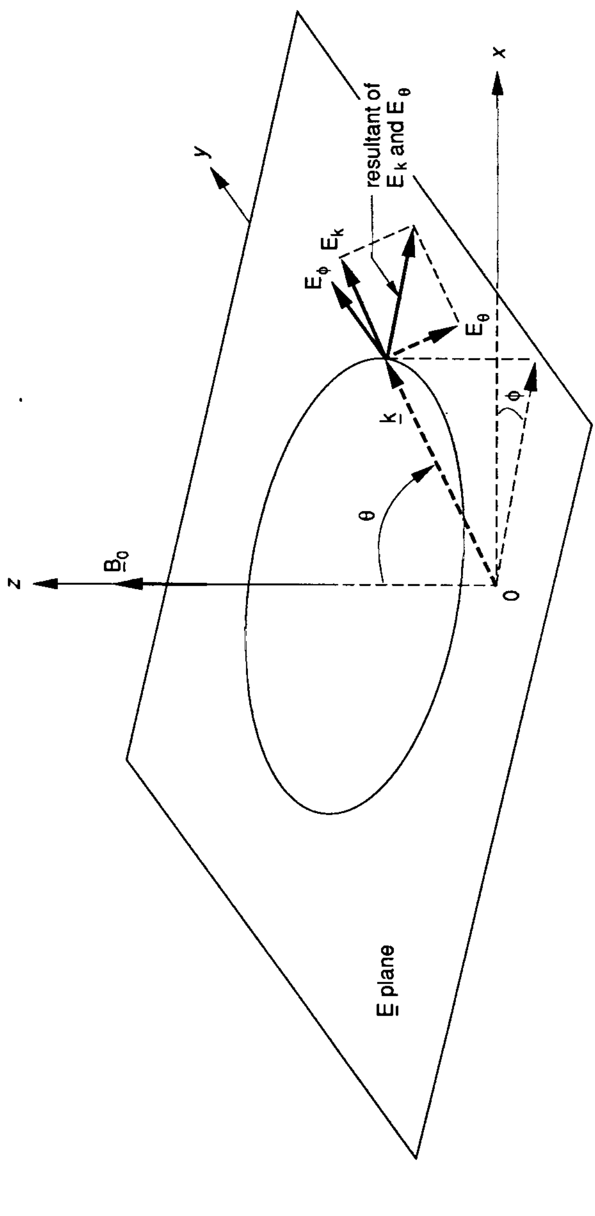

Figure B.4. Orientation of wave fields in spherical coordinates.

Appendix C. Dusty and Grain Plasmas

In general, the motion of a solid particle in a plasma obeys Eq.(2.11)

$$m\frac{d\mathbf{v}}{dt} = m\mathbf{g} + q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) - m\,v_c\ \mathbf{v} + \mathbf{f} \qquad \text{(C.1)}$$

where m and q are the mass and the electric charge of the particle, respectively, $\mathbf{g} = \hat{\mathbf{r}}G\ m(r)\,/\,r^2$ is the gravitational acceleration, $-\,m\,v_c\ \mathbf{v}$ is due to viscosity, and $\mathbf{f}$ is the sum of all other forces, including the radiation pressure.

Depending on the size of the particle, four cases are delineable:

(1) *Very small particles.* The term $q(\mathbf{E}+\mathbf{v}\times\mathbf{B})$ in Eq.(C.1) dominates over $m\mathbf{g}$ and the particle is part of a dusty plasma (Section C.1). Under cosmic conditions this is true if the size of the particle is less than 10 nm. In the case of large electric charges the limiting size may rise to 100 nm.

(2) *Small Grains.* For this case, $q/m \approx \sqrt{G}$. Plasma effects still play a major role in system dynamics (Section C.2).

(3) *Large Grains.* If the size of the particle is so large that the electromagnetic term is negligible, we have an intermediate case dominated by viscosity and gravity. The particles in this regime are referred to as grains. Their equation of motion is

$$m\,v_c\mathbf{v} = m\mathbf{g} \qquad \text{(C.2)}$$

Under conditions in interstellar clouds this may be valid for particles of the order of 10 μm.

(4) *Large solid bodies.* For "particles" of the size of kilometers or more, the inertia and gravitational terms dominate. Electromagnetic forces are negligible, and viscous forces can be considered as perturbations which may change the orbit slowly. Depending on the properties of the cosmic cloud, viscous forces become important for meter or centimeter sizes. The equation of motion Eq.(C.1) is then,

$$m\frac{d\mathbf{v}}{dt} = m\,\mathbf{g} - m\,v_c\ \mathbf{v} \qquad \text{(C.3)}$$

The transition of plasma into stars involves the formation of dusty plasma, the sedimentation of the dust into grains, the formation of stellesimals, and then the collapse into a stellar state [Alfvén and Arrhenius 1976, Alfvén and Carlqvist 1978].

C.1 Dusty Plasma

An important class of cosmic plasmas are those which are "dusty" (i.e., plasmas that contain solid matter in the form of very small dust grains). These grains are electrically charged and if q/m is large enough, the dynamics of the dust grain is controlled by electromagnetic forces [Mendis 1979, Hill and Mendis 1979, 1980, Houpis 1987, Azar and Thompson 1989, Horanyi and Goertz 1990].

The temperature of the dust in a cosmic plasma may differ by orders of magnitude from the temperature of the plasma. For example, the temperature of the dust radiating into space through a transparent plasma may be 10 K, the molecular temperature 100 K, the ion temperature 10^3 K, and the electron temperature 10^4 K.

Dusty plasma is characterized by solid particles of mass m, charge q, and a charge-to-mass ratio that is much greater than the square-root of the gravitational constant, $q/m >> \sqrt{G}$. The dust grains are charged negatively by impacts from streams of electrons. The loss of negative charge can be due to the photoeffect, field emission, and ion impacts. Normally, the potential of a dust grain may be 1–10 V, positive or negative. However, if the electron stream responsible for the charge is relativistic, the grain can charge up to several kilovolts [Deforest 1972, Reasoner 1976, Mendis 1979].

C.2 Grain Plasma

Consider a plasma whose dust may have accreted into macroscopic solid matter. For simplicity, the plasma is taken to consist of two components: *grains* with mass m_g and charge q_g, and *particles* with mass m_p and charge q_p. The grains are assumed to be weakly charged with $q_g/m_g \sim \sqrt{G}$. The particles may be electrons, ions, dust, or some other unspecified charge mass with the property $m_p << m_g$ [Wollman 1988, Gisler and Wollman 1988].

Consider a spherically symmetric gravitational condensation of the plasma, so that the parameters depend only on the distance r from the center. The dynamics of the grain fluid are specified by Eqs.(2.13) and (2.15) where $n_g(r)$ is the number density, and $N(r)$ and $M(r)$ are the total grain number and grain mass inside r, respectively. The total mass and net charge inside radius r are

$$M_T \equiv M_T(r) \equiv m_g N_g + m_p N_p$$

$$Q_T \equiv Q_T(r) \equiv q_g N_g - q_p N_p$$

If the plasma is tenous and in thermal equilibrium, $dv/dt = 0$; the electrostatic potential at q_g is $\phi = Q_T / 4\pi\varepsilon_0 r$, and the gravitational potential at m_g is $\phi_G = -G M_T / r$. Substituting these parameters into Eq.(2.13) while (momentarily) neglecting **B**, gives the following, for grains and particles, respectively:

$$0 = \frac{n_g q_g Q_T}{4\pi\varepsilon_0 r^2} - \frac{n_g m_g G M_T}{r^2} - kT_g \frac{dn_g}{dr} \tag{C.4}$$

$$0 = \frac{-n_p q_p Q_T}{4\pi\varepsilon_0 r^2} - \frac{n_p \langle\gamma\rangle m_p G M_T}{r^2} - kT_p \frac{dn_p}{dr} \tag{C.5}$$

The particles may be relativistic with mean Lorentz factor $\langle\gamma\rangle$.

Equations (C.4) and (C.5) have as solutions

$$n_g(r) = n_{0g}\left(\frac{r_0}{r}\right)^2$$

$$n_p(r) = n_{0p}\left(\frac{r_0}{r}\right)^2$$

Some insight into the meaning of Eqs.(C.4) and (C.5) is possible by considering the special case $T_g = T_p = T$, and scale lengths $n_g^{-1}\, d n_g / d r = n_p^{-1}\, d n_p / d r$, for the grain plasma and particle plasmas, respectively. For these conditions, Eqs.(C.4) and (C.5) reduce to

$$\frac{q_g\, Q_T}{4\pi\varepsilon_0} - m_g G\, M_T = -\frac{q_p\, Q_T}{4\pi\varepsilon_0} - \langle\gamma\rangle m_p G\, M_T \tag{C.6}$$

If the grain thermal speed is much less than c, then $\langle\gamma\rangle m_p << m_g$ and Eq.(C.6) may be written,

$$\frac{q_g}{\sqrt{4\pi\varepsilon_0 G}\, m_g} \frac{Q_T}{\sqrt{4\pi\varepsilon_0 G}\, M_T} = \frac{q_g}{q_g + q_p} \tag{C.7}$$

Now consider the behavior of the particle scale height relative to the grain scale height. The particle scale height parameter r_{0p} is defined by the relation

$$q_p N_p(r_{0p}) \quad = q_g N_g(r_0)$$

Now write, ignoring the particle mass with respect to the grain mass,

$$\frac{Q_T}{\sqrt{4\pi\varepsilon_0 G}\, M_T} \approx \frac{q_g\, N_g(r) - q_p\, N_p(r)}{\sqrt{4\pi\varepsilon_0 G}\, m_g\, N_g(r)} = \frac{q_g}{\sqrt{4\pi\varepsilon_0 G}\, m_g}\left(1 - \frac{q_p\, N_p(r)}{q_g\, N_g(r)}\right)$$

The ratio of particle numbers is

$$\frac{N_p(r)}{N_g(r)} = \frac{N_p(r_0)}{N_g(r_0)} = \frac{N_p(r_{0p})\, N_p(r_0)}{N_g(r_0)\, N_g(r_{0p})} = \frac{q_g\, r_0}{q_p\, r_{0p}}$$

The last identity is due to the fact that for a r^{-2} density distribution, $N(r) \propto r$. Hence,

$$\frac{Q_T}{\sqrt{4\pi\varepsilon_0 G}\, M_T} = \frac{q_g}{\sqrt{4\pi\varepsilon_0 G}\, m_g}\left(1 - \frac{r_0}{r_{0p}}\right) \tag{C.8}$$

Combining Eqs.(C.7) and (C.8) yields,

$$\frac{r_0}{r_{0p}} = 1 - \frac{4\pi\varepsilon_0 G\ m_g^2}{q_g^2}\left(\frac{q_g}{q_g + q_p}\right)$$

For an atomic plasma,

$$\frac{q_g^2}{4\pi\varepsilon_0 G\ m_g^2} >> \left(\frac{q_g}{q_g + q_p}\right)$$

and the radial electrical polarization, due to the separation $|r_0 - r_{0p}|$, is negligible. The condition for maximum polarization (i.e., when the particles are removed to infinity so that $r_{0p} \to \infty$), is

$$\frac{q_g}{\sqrt{4\pi\varepsilon_0 G}\ m_g} = \sqrt{\frac{q_g}{q_g + q_p}} \qquad \text{(C.9)}$$

or, since $q \equiv Ze$,

$$m_g = \sqrt{\frac{q_g\left(q_g + q_p\right)}{4\pi\varepsilon_0 G}} = 1.9\times10^{-9}\sqrt{Z_g\left(Z_g + Z_p\right)} \quad \text{kg}$$

The grain mass, then, is of the order of micrograms.

It is possible to state the condition for significant large-scale separation. The Jeans wave number $k_J = 2\pi/\lambda_J$ is from Eq.(2.75),

$$k_J^2 = \frac{4\pi G\ m_g \rho_m}{kT}\left(\frac{Z_p}{Z_g + Z_p}\right)$$

The particle Debye length is given by

$$\lambda_p^2 = \frac{kT_p/m_p}{n_p q_p^2/m_p \varepsilon_0} = \left[\frac{kT}{4\pi G\ m_g \rho_m}\right]\frac{4\pi\varepsilon_0 G\ m_g^2}{q_p q_g}$$

Hence, Eq.(C.9) is equivalent to

$$k_J \lambda_p = 1$$

Thus, the condition for maximum polarization can be written

$$\frac{\text{Jeans length}}{\text{Debye length}} = 2\pi \qquad \text{(C.10)}$$

This analysis is valid insofar as thermalization is efficient, so that the formation of condensation heats the plasma and $T_p = T_g$. This increases the Debye length. Then the condition for charge separation is equivalent to the condition for poor shielding of Jeans mass concentrations.

The inclusion of **B** causes a **E**×**B** drift of the low mass component around the axis at the center of the spherical condensation. If the grain plasma is in the presence of a strong magnetic field, the Jeans mass may be radically altered (Section 2.7.1).

Appendix D. Some Useful Units and Constants

Length

meter	m = 100 cm
kilometer	km = 10^5 cm = 10^3 m
millimeter	mm = 10^{-1} cm = 10^{-3} m
micron	μ = μm = 10^{-4} cm = 10^{-6} m
angstrom	Å = 10^{-8} cm = 10^{-10} m
fermi (femtometer)	fm = 10^{-13} cm = 10^{-15} m
astronomical unit (= mean sun-earth distance)	AU = 1.49598×10^{11} m
light year	ly = 9.46053×10^{15} m =63240 AU
parsec	pc = 3.08568×10^{16} m =3.26163 ly
solar radius	$R_s = 6.9599 \times 10^8$m
earth equatorial radius	$R_e = 6.3782 \times 10^6$ m = 6378 km

Volume

cubic meter	$m^3 = 10^6$ cm^3
cubic parsec	$pc^3 = 2.938 \times 10^{49}$ m^3 = 34.7 ly^3
cubic kiloparsec	$kpc^3 = 2.938 \times 10^{58}$ m^3 = 3.470×10^{10} ly^3

Time

minute	min = 60 s
hour	h = 3600 s = 60 min
day	d = 86400 s = 24 h
sidereal year	y = 365.256 d = 3.15581×10^7 s
aeon	10^9 y

Mass

solar mass	$M_s = 1.989 \times 10^{30}$ kg
earth mass	$M_e = 5.976 \times 10^{24}$ kg
atomic mass unit (^{12}C = 12 scale)	amu = 1.66056×10^{-27} kg
electron mass	$m_e = 9.10953 \times 10^{-31}$ kg = 5.4858×10^{-4} amu
proton mass	$m_p = 1.67265 \times 10^{-27}$ kg = 1.00728 amu ($m_p/m_e = 1836.15$)
mass of 1H atom	$m_H = 1.67356 \times 10^{-27}$ kg = 1.00783 amu

Energy

joule	J, 1 kg m^2 s^{-2}
erg	1 erg = 10^{-7} J
calorie	cal = 4.1868 J
electron volt	eV = 1.60219 × 10^{-19} J = 10^{-3} keV = 10^{-6} MeV = 10^{-9} GeV
mass energy of 1 amu	1.49243 × 10^{-10} J =931.502 MeV
rest mass energy of electron	$m_e c^2$ = 511.003 keV
wavelength associated with 1 eV	1.2398 μm = 1239.8 nm
frequency associated with 1 eV	2.4180 × 10^{14} Hz
temperature associated with 1 eV	11,604 K
detonation energy of 1 kiloton of high explosive	4.2 × 10^{12} J = 4.2 TJ

Power

watt	W = J s^{-1}
solar luminosity	L_s = 3.826 × 10^{26} W = 2.388 × 10^{39} MeV s^{-1}
jansky	J_y = 10^{-26} W m^{-2} Hz^{-1}

Velocity

velocity of light	c = 2.997925 × 10^8 m s^{-1} = 2.997925 × 10^4 cm μs^{-1} = 2.997925 × 10^5 km s^{-1} (10 cm μs^{-1} = 100 km s^{-1})

Pressure

pascal	Pa, kg m^{-1} s^{-2}
bar	10^5 Pa
atmosphere	atm = 1.01325 bar = 760 torr
millimeter of mercury	mm Hg =133.322 Pa =1.315 × 10^{-3} atm = 1.298 mbar

Temperature

temperature comparisons	0°C = 273.150 K 100°C = 373.150 K

Angle, Solid Angle

degree	deg = 1° = right angle/90 = 60 minutes of arc (60

	arcmin) = 3600 seconds of arc (arcsec = 3600")
radian	rad = 57°.29578
steradian	sr = 3282.8 deg^2

Angular momentum

quantum unit	$\hbar = 1.0546 \times 10^{-34}$ J s $= 6.5822 \times 10^{-16}$ eV s
Planck's constant	$h = 2\pi\hbar = 6.6262 \times 10^{-34}$ J s

Electric charge

Coulomb	$C = -6.24145 \times 10^{18}$ electrons
electron charge	$e = 1.60219 \times 10^{-19}$ C

Magnetic field

tesla	T = 10^4 gauss
gauss	G = 10^{-4} T = 1 oersted = 79.58 amp-turn m^{-1}
gamma	$\gamma = 10^{-9}$ T = 1 nT = 10^{-5} G
earth's nominal magnetic field	0.5 G = 50,000 nT

Some physical constants

Boltzmann constant	$k = 1.3807 \times 10^{-23}$ J K^{-1}
gravitational constant	$G = 6.6726 \times 10^{-11}$ m^2 s^{-2} kg^{-1}
gravitational acceleration, earth	$g = 9.8067$ m s^{-2}
permittivity of free space	$\varepsilon_0 = 8.8542 \times 10^{-12}$ F m^{-1}
permeability of free space	$\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}
Stefan-Boltzmann constant	$\sigma = 5.6705 \times 10^{-8}$ W m^{-2} K^{-4}
Avogadro number	$N_A = 6.0221 \times 10^{23}$ mol^{-1}

Appendix E. TRISTAN

```
C  TRIdimensional STANford code, TRISTAN, fully electromagnetic,
C  with full relativistic particle dynamics. Written during spring
C  1990 by OSCAR BUNEMAN, with help from TORSTEN NEUBERT and
C  KEN NISHIKAWA.
      real me,mi
      common /partls/x(8192),y(8192),z(8192),u(8192),v(8192),w(8192)
      common /fields/ex1(8192),ey1(8192),ez1(8192),bx1(8192),
     &by1(8192),bz1(8137)
     &,sm(27),q,qe,qi,qme,qmi,rs,ps,os,c
     &,ms(27),mx,my,mz,ix,iy,iz,lot,maxptl,ions,lecs,nstep
C  The sizes of the particle and field arrays must be chosen in
C  accordance with the requirements of the problem and the limitation
C  of the computer memory. The choice '8192' (with the bz1 array
C  curtailed to accommodate the remainder of the fields common) was
C  made to suit the segmented memory of a PC. Any changes in these
C  array sizes must be copied exactly into the COMMON statements of
C  the "surface", "mover" and "depsit" subroutines.
C Fields can be treated as single-indexed or triple-indexed:
C For CRAY-s, the first two field dimensions should be ODD, as here:
      dimension ex(21,19,20),ey(21,19,20),ez(21,19,20),
     &bx(21,19,20),by(21,19,20),bz(21,19,20)
      equivalence(ex1(1),ex(1,1,1)),(ey1(1),ey(1,1,1)),(ez1(1),ez(1,
     &1,1)),(bx1(1),bx(1,1,1)),(by1(1),by(1,1,1)),(bz1(1),bz(1,1,1))
      maxptl=8192
      mx=21
      my=19
      mz=20
C Strides for single-indexed field arrays:
      ix=1
      iy=mx
      iz=iy*my
      lot=iz*mz
C Miscellaneuos constants:
      qe=-.0625
      qi=.0625
      me=.0625
      mi=1.
      qme=qe/me
      qmi=qi/mi
      c=.5
C Our finite difference equations imply delta_t = delta_x =
C delta_y = delta_z = 1. So c must satisfy the Courant condition.
C The bx,by and bz arrays are really records of c*Bx, c*By,
C c*Bz: this makes for e <-----> b symmetry in Maxwell's equations.
C Otherwise, units are such that epsilon_0  is 1.0 and hence mu_0
C is 1/c**2. This means that for one electron per cell (see example
C of particle initialisation below) omega_p-squared is qe**2/me.
C For use in the boundary field calculation:
      rs=(1.-c)/(1.+c)
      tsq=.1666667
      ps=(1.-tsq)*c/(1.+c)
      os=.5*(1.+tsq)*c/(1.+c)
C Data for smoothing: the currents fed into Maxwell's equations
C are smoothed by convolving with the sequence  .25, .5, .25 in
C each dimension. Generate the 27 weights ("sm") and index
C displacements ("ms"):
      n=1
      do 1 nz=-1,1
      do 1 ny=-1,1
      do 1 nx=-1,1
```

```
      sm(n)=.015625*(2-nx*nx)*(2-ny*ny)*(2-nz*nz)
      ms(n)=ix*nx+iy*ny+iz*nz
  1   n=n+1
C In the particle arrays the ions are at the bottom, the electrons
C are stacked against the top, the total number not exceeding maxptl:
C The number of ions, "ions", need not be the same as the number of
C electrons, "lecs".
C The code treats unpaired electrons as having been initially
C dissociated from infinitely heavy ions which remain in situ.
C Initialise the particles: Place electrons in same locations as ions
C for zero initial net charge density. Keep particles 2 units away from
C the lower boundaries of the field domain, 3 units away from the upper
C boundaries. For instance, fill the interior uniformily:
            ions=0
            do 80 k=1,mz-5
            do 80 j=1,my-5
            do 80 i=1,mx-5
            ions=ions+1
            x(ions)= 2.5+i
            y(ions)= 2.5+j
 80         z(ions)= 2.5+k
C Put electrons in the same places:
      lecs=ions
      do 4 n=1,lecs
      x(n+maxptl-lecs)=x(n)
      y(n+maxptl-lecs)=y(n)
  4   z(n+maxptl-lecs)=z(n)
C Initialise velocities: these should not exceed  c  in magnitude!
C For thermal distributions, add three or four random numbers for
C each component and scale.
C  Initialise random number generator:
      lk=12345
      lp=29
      do 85 n=1,ions
      u(n)=0.046875*(rndm(lk,lp)+rndm(lk,lp)+rndm(lk,lp))
      v(n)=0.046875*(rndm(lk,lp)+rndm(lk,lp)+rndm(lk,lp))
      w(n)=0.046875*(rndm(lk,lp)+rndm(lk,lp)+rndm(lk,lp))
      u(maxptl-lecs+n)=0.1875*(rndm(lk,lp)+rndm(lk,lp)+rndm(lk,lp))
      v(maxptl-lecs+n)=0.1875*(rndm(lk,lp)+rndm(lk,lp)+rndm(lk,lp))
 85   w(maxptl-lecs+n)=0.1875*(rndm(lk,lp)+rndm(lk,lp)+rndm(lk,lp))
C Initialise the fields, typically to uniform components, such as
C just a uniform magnetic field parallel to the z-axis:
      do 5 k=1,mz
      do 5 j=1,my
      do 5 i=1,mx
      ex(i,j,k)=0.
      ey(i,j,k)=0.
      ez(i,j,k)=0.
      bx(i,j,k)=0.
      by(i,j,k)=0.
  5   bz(i,j,k)=c*1.5
C (Remember that bx,by,bz are really c*Bx, c*By, c*Bz.)
C Initial fields, both electric and magnetic, must be divergence-free.
C Part of the Earth's magnetic field would be ok. If the Earth is included
C in the field domain, its magnetic dipole field is readily established
C by maintaining a steady ring current in the Earth's core.
C     Begin time stepping
      last=32
      nstep=1
C Before moving particles, the magnetic field is Maxwell-advanced
C by half a timestep:
  6   do 7 i=1,mx-1
      do 7 j=1,my-1
      do 7 k=1,mz-1
      bx(i,j,k)=bx(i,j,k) + (.5*c) *
     &          (ey(i,j,k+1)-ey(i,j,k)-ez(i,j+1,k)+ez(i,j,k))
      by(i,j,k)=by(i,j,k) + (.5*c) *
     &          (ez(i+1,j,k)-ez(i,j,k)-ex(i,j,k+1)+ex(i,j,k))
  7   bz(i,j,k)=bz(i,j,k) + (.5*c) *
     &          (ex(i,j+1,k)-ex(i,j,k)-ey(i+1,j,k)+ey(i,j,k))
```

```
C Now move ions:
      call mover(1,ions,qmi)
C and electrons:
      call mover(maxptl-lecs+1,maxptl,qme)
C The Maxwell-advance of the fields begins with another half-step
C advance of the magnetic field since for Maxwell's equations the
C B - information and the E - information must be staggered in time.
C In space, their information is also staggered. Here we show the
C locations where field components are recorded:
C
C
C     Ex(i,j,k) = value of Ex at x=i+.5, y=j,    z=k
C     Ey(i,j,k) = value of Ey at x=i,    y=j+.5, z=k
C     Ez(i,j,k) = value of Ez at x=i,    y=j,    z=k+.5
C
C     Bx(i,j,k) = value of Bx at x=i,    y=j+.5, z=k+.5
C     By(i,j,k) = value of By at x=i+.5, y=j,    z=k+.5
C     Bz(i,j,k) = value of Bz at x=i+.5, y=j+.5, z=k
C
C     Maxwell's laws are implemented (to central difference
C     accuracy), in the form:
C
C     Change of flux of B through a cell face
C         = - circulation of E around that face
C
C     Change of flux of E through a cell face of the offset grid
C         = circulation of B around that face - current through it
C
C
C                z
C                ^
C                |
C                |
C                |
C                *---------------Ey---------------*
C               /|                                |
C              / |                                |
C             /  |                                |
C           Ex   |                                |
C           /    |                                |
C          /     |                                |
C         /      |                                |
C        *       |                                |
C        |       |                                |
C        |       |                                |
C        |       |                                |
C        |       Ez              Bx               Ez
C        |       |                                |
C        |       |                                |
C        |       |                                |
C        |   By  |                                |
C        |       |                                |
C        |       |                                |
C        |       |                                |
C        Ez      |                                |
C        |       |                                |
C        |       |                                |
C        |       |                                |
C        |       *---------------Ey---------------*-----> y
C        |      /                                /
C        |     /                                /
C        |    /                                /
C        |   Ex              Bz               Ex
C        |  /                                /
C        | /                                /
C        |/                                /
C        *---------------Ey---------------*
C       /
C      /
C     /
C    x
```

```
c
c
c Second half-advance of magnetic field:
      do 8 i=1,mx-1
      do 8 j=1,my-1
      do 8 k=1,mz-1
      bx(i,j,k)=bx(i,j,k) + (.5*c) *
     &          (ey(i,j,k+1)-ey(i,j,k)-ez(i,j+1,k)+ez(i,j,k))
      by(i,j,k)=by(i,j,k) + (.5*c) *
     &          (ez(i+1,j,k)-ez(i,j,k)-ex(i,j,k+1)+ex(i,j,k))
    8 bz(i,j,k)=bz(i,j,k) + (.5*c) *
     &          (ex(i,j+1,k)-ex(i,j,k)-ey(i+1,j,k)+ey(i,j,k))
c Front,right and top layers of B must be obtained from a special
c boundary routine based on Lindman's method:
      call surface(by,bz,bx,ey,ez,ex,iy,iz,ix,my,mz,mx,1)
      call surface(bz,bx,by,ez,ex,ey,iz,ix,iy,mz,mx,my,1)
      call surface(bx,by,bz,ex,ey,ez,ix,iy,iz,mx,my,mz,1)
      call edge(bx,ix,iy,iz,mx,my,mz,1)
      call edge(by,iy,iz,ix,my,mz,mx,1)
      call edge(bz,iz,ix,iy,mz,mx,my,1)
c Full advance of the electric field:
      do 9 i=2,mx
      do 9 j=2,my
      do 9 k=2,mz
      ex(i,j,k)=ex(i,j,k) + c *
     &          (by(i,j,k-1)-by(i,j,k)-bz(i,j-1,k)+bz(i,j,k))
      ey(i,j,k)=ey(i,j,k) + c *
     &          (bz(i-1,j,k)-bz(i,j,k)-bx(i,j,k-1)+bx(i,j,k))
    9 ez(i,j,k)=ez(i,j,k) + c *
     &          (bx(i,j-1,k)-bx(i,j,k)-by(i-1,j,k)+by(i,j,k))
c Boundary values of the E - field must be provided at rear, left
c and bottom faces of the field domain:
      call surface(ey,ez,ex,by,bz,bx,-iy,-iz,-ix,my,mz,mx,lot)
      call surface(ez,ex,ey,bz,bx,by,-iz,-ix,-iy,mz,mx,my,lot)
      call surface(ex,ey,ez,bx,by,bz,-ix,-iy,-iz,mx,my,mz,lot)
      call edge(ex,-ix,-iy,-iz,mx,my,mz,lot)
      call edge(ey,-iy,-iz,-ix,my,mz,mx,lot)
      call edge(ez,-iz,-ix,-iy,mz,mx,my,lot)
c The currents due to the movement of each charge q are applied to the
c E-M fields as decrements of E-flux through cell faces. The movement
c of particles which themselves cross cell boundaries has to be split
c into several separate moves, each only within one cell. Each of
c these moves contributes to flux across twelve faces.
c Ions and electrons are processed in two loops, changing the sign
c of the charge in-between. These loops cannot be vectorised:
c particles get processed one by one. Here is a good place to
c insert the statements for applying boundary conditions to
c the particles, such as reflection, periodicity, replacement
c by inward moving thermal or streaming particles, etc.
c Split and deposit ions currents:
      q=qi
      n1=1
      n2=ions
   52 do 53 n=n1,n2
c  Previous position:
      x0=x(n)-u(n)
      y0=y(n)-v(n)
      z0=z(n)-w(n)
c  Reflect particles at x=3, x=mx-2, y=3, y=my-2, z=3, z=mz-2:
      u(n)=u(n)*sign(1.,x(n)-3.)*sign(1.,mx-2.-x(n))
      x(n)=mx-2.- abs(mx-5.-abs(x(n)-3.))
      v(n)=v(n)*sign(1.,y(n)-3.)*sign(1.,my-2.-y(n))
      y(n)=my-2.- abs(my-5.-abs(y(n)-3.))
      w(n)=w(n)*sign(1.,z(n)-3.)*sign(1.,mz-2.-z(n))
      z(n)=mz-2.- abs(mz-5.-abs(z(n)-3.))
```

```
C  Alternatively, apply periodicity to particles:
CC     x(n)=x(n)+sign(.5*(mx-5.),mx-2.-x(n))-sign(.5*(mx-5.),x(n)-3.)
CC     y(n)=y(n)+sign(.5*(my-5.),my-2.-y(n))-sign(.5*(my-5.),y(n)-3.)
CC     z(n)=z(n)+sign(.5*(mz-5.),mz-2.-z(n))-sign(.5*(mz-5.),z(n)-3.)
  53  call xsplit(x(n),y(n),z(n),x0,y0,z0)
C  The split routines call the deposit routine.
      if(n2.eq.maxptl)go to 54
C Split and deposit electron currents:
      q=qe
      n1=maxptl-lecs+1
      n2=maxptl
      go to 52
C Countdown:
  54  nstep=nstep+1
      if (nstep.le.last) go to 6
C  The user must decide what information is to be written out at
C  each timestep, what only occasionally, and what only at the end.
C  It may be wise to write out both COMMONS before stopping: they
C  can then be read in again for a continuation of the run.
      stop
      end
C --------
      subroutine surface(bx,by,bz,ex,ey,ez,ix,iy,iz,mx,my,mz,m00)
C (Field components are treated as single-indexed in this subroutine)
      dimension bx(1),by(1),bz(1),ex(1),ey(1),ez(1)
      common /fields/ex1(8192),ey1(8192),ez1(8192),bx1(8192),
     &by1(8192),bz1(8137)
     &,sm(27),q,qe,qi,qme,qmi,rs,ps,os,c
     &,ms(38)
      m0=m00+iz*(mz-1)
      assign 5 to next
   6  m=m0
      do 2 j=1,my-1
      n=m
      do 1 i=1,mx-1
      bz(n)=bz(n)+.5*c*(ex(n+iy)-ex(n)-ey(n+ix)+ey(n))
   1  n=n+ix
   2  m=m+iy
      go to next (5,7)
   7  return
   5  m=m0+ix+iy
      do 4 j=2,my-1
      n=m
C  Directive specifically for the CRAY cft77 compiler:
cdir$ ivdep
      do 3 i=2,mx-1
      bx(n)=bx(n-iz)+rs*(bx(n)-bx(n-iz))+ps*(bz(n)-bz(n-ix))-os*(
     &ez(n+iy)-ez(n))-(os-c)*(ez(n+iy-iz)-ez(n-iz))-c*(ey(n)-ey(n-iz))
      by(n)=by(n-iz)+rs*(by(n)-by(n-iz))+ps*(bz(n)-bz(n-iy))+os*(
     &ez(n+ix)-ez(n))+(os-c)*(ez(n+ix-iz)-ez(n-iz))+c*(ex(n)-ex(n-iz))
   3  n=n+ix
   4  m=m+iy
      assign 7 to next
      go to 6
      end
C   -----------
      subroutine edge(bx,ix,iy,iz,mx,my,mz,m00)
      dimension bx(1)
      lx=ix*(mx-1)
      ly=iy*(my-1)
      lz=iz*(mz-1)
      n=m00+iy+lz
cdir$ ivdep
      do 1 j=2,my-1
      bx(n)=bx(n+ix)+bx(n-iz)-bx(n+ix-iz)
      bx(n+lx)=bx(n+lx-ix)+bx(n+lx-iz)-bx(n+lx-ix-iz)
```

```
  1   n=n+iy
      n=m00+ly+iz
cdir$ ivdep
      do 2 k=2,mz-1
      bx(n)=bx(n+ix)+bx(n-iy)-bx(n+ix-iy)
      bx(n+lx)=bx(n+lx-ix)+bx(n+lx-iy)-bx(n+lx-ix-iy)
  2   n=n+iz
      n=m00+ly+lz
cdir$ ivdep
      do 3 i=1,mx
      bx(n)=bx(n-iy)+bx(n-iz)-bx(n-iy-iz)
      bx(n-ly)=bx(n-ly+iy)+bx(n-ly-iz)-bx(n-ly+iy-iz)
      bx(n-lz)=bx(n-lz-iy)+bx(n-lz+iz)-bx(n-lz-iy+iz)
  3   n=n+ix
      return
      end
c--------
      subroutine mover(n1,n2,qm)
      common /partls/ x(8192),y(8192),z(8192),u(8192),v(8192),w(8192)
C (Field components are treated as single-indexed in this subroutine)
      common /fields/ ex(8192),ey(8192),ez(8192),
     &bx(8192),by(8192),bz(8137)
     &,sm(27),q,qe,qi,qme,qmi,rs,ps,os,c
     &,ms(27),mx,my,mz,ix,iy,iz,lot,maxptl,ions,lecs,nstep
      do 1 n=n1,n2
C Cell index & displacement in cell:
      i=x(n)
      dx=x(n)-i
      j=y(n)
      dy=y(n)-j
      k=z(n)
      dz=z(n)-k
      l=i+iy*(j-1)+iz*(k-1)
C Field interpolations are tri-linear (linear in x times linear in y
C times linear in z). This amounts to the 3-D generalisation of "area
C weighting". A modification of the simple linear interpolation formula
C          f(i+dx) = f(i) + dx * (f(i+1)-f(i))
C is needed since fields are recorded at half-integer locations in certain
C dimensions: see comments and illustration with the Maxwell part of this
C code. One then has first to interpolate from "midpoints" to "gridpoints"
C by averaging neighbors. Then one proceeds with normal interpolation.
C Combining these two steps leads to:
C   f at location i+dx  = half of f(i)+f(i-1) + dx*(f(i+1)-f(i-1))
C where now f(i) means f at location i+1/2. The halving is absorbed
C in the final scaling.
C   E-component interpolations:
      f=ex(l)+ex(l-ix)+dx*(ex(l+ix)-ex(l-ix))
      f=f+dy*(ex(l+iy)+ex(l-ix+iy)+dx*(ex(l+ix+iy)-ex(l-ix+iy))-f)
      g=ex(l+iz)+ex(l-ix+iz)+dx*(ex(l+ix+iz)-ex(l-ix+iz))
      g=g+dy*
     & (ex(l+iy+iz)+ex(l-ix+iy+iz)+dx*(ex(l+ix+iy+iz)-ex(l-ix+iy+iz))-g)
      ex0=(f+dz*(g-f))*(.25*qm)
C   ------------
      f=ey(l)+ey(l-iy)+dy*(ey(l+iy)-ey(l-iy))
      f=f+dz*(ey(l+iz)+ey(l-iy+iz)+dy*(ey(l+iy+iz)-ey(l-iy+iz))-f)
      g=ey(l+ix)+ey(l-iy+ix)+dy*(ey(l+iy+ix)-ey(l-iy+ix))
      g=g+dz*
     & (ey(l+iz+ix)+ey(l-iy+iz+ix)+dy*(ey(l+iy+iz+ix)-ey(l-iy+iz+ix))-g)
      ey0=(f+dx*(g-f))*(.25*qm)
C   ------------
      f=ez(l)+ez(l-iz)+dz*(ez(l+iz)-ez(l-iz))
      f=f+dx*(ez(l+ix)+ez(l-iz+ix)+dz*(ez(l+iz+ix)-ez(l-iz+ix))-f)
      g=ez(l+iy)+ez(l-iz+iy)+dz*(ez(l+iz+iy)-ez(l-iz+iy))
      g=g+dx*
     & (ez(l+ix+iy)+ez(l-iz+ix+iy)+dz*(ez(l+iz+ix+iy)-ez(l-iz+ix+iy))-g)
      ez0=(f+dy*(g-f))*(.25*qm)
```

```
c    ---------
c    B-component interpolations:
       f=bx(l-iy)+bx(l-iy-iz)+dz*(bx(l-iy+iz)-bx(l-iy-iz))
       f=bx(l)+bx(l-iz)+dz*(bx(l+iz)-bx(l-iz))+f+dy*
      & (bx(l+iy)+bx(l+iy-iz)+dz*(bx(l+iy+iz)-bx(l+iy-iz))-f)
       g=bx(l+ix-iy)+bx(l+ix-iy-iz)+dz*(bx(l+ix-iy+iz)-bx(l+ix-iy-iz))
       g=bx(l+ix)+bx(l+ix-iz)+dz*(bx(l+ix+iz)-bx(l+ix-iz))+g+dy*
      & (bx(l+ix+iy)+bx(l+ix+iy-iz)+dz*(bx(l+ix+iy+iz)-bx(l+ix+iy-iz))-g)
       bx0=(f+dx*(g-f))*(.125*qm/c)
c    ------------
       f=by(l-iz)+by(l-iz-ix)+dx*(by(l-iz+ix)-by(l-iz-ix))
       f=by(l)+by(l-ix)+dx*(by(l+ix)-by(l-ix))+f+dz*
      & (by(l+iz)+by(l+iz-ix)+dx*(by(l+iz+ix)-by(l+iz-ix))-f)
       g=by(l+iy-iz)+by(l+iy-iz-ix)+dx*(by(l+iy-iz+ix)-by(l+iy-iz-ix))
       g=by(l+iy)+by(l+iy-ix)+dx*(by(l+iy+ix)-by(l+iy-ix))+g+dz*
      & (by(l+iy+iz)+by(l+iy+iz-ix)+dx*(by(l+iy+iz+ix)-by(l+iy+iz-ix))-g)
       by0=(f+dy*(g-f))*(.125*qm/c)
c    ------------
       f=bz(l-ix)+bz(l-ix-iy)+dy*(bz(l-ix+iy)-bz(l-ix-iy))
       f=bz(l)+bz(l-iy)+dy*(bz(l+iy)-bz(l-iy))+f+dx*
      & (bz(l+ix)+bz(l+ix-iy)+dy*(bz(l+ix+iy)-bz(l+ix-iy))-f)
       g=bz(l+iz-ix)+bz(l+iz-ix-iy)+dy*(bz(l+iz-ix+iy)-bz(l+iz-ix-iy))
       g=bz(l+iz)+bz(l+iz-iy)+dy*(bz(l+iz+iy)-bz(l+iz-iy))+g+dx*
      & (bz(l+iz+ix)+bz(l+iz+ix-iy)+dy*(bz(l+iz+ix+iy)-bz(l+iz+ix-iy))-g)
       bz0=(f+dz*(g-f))*(.125*qm/c)
c    ---------
c    First half electric acceleration, with relativity's gamma:
       g=c/sqrt(c**2-u(n)**2-v(n)**2-w(n)**2)
       u0=g*u(n)+ex0
       v0=g*v(n)+ey0
       w0=g*w(n)+ez0
c    First half magnetic rotation, with relativity's gamma:
       g=c/sqrt(c**2+u0**2+v0**2+w0**2)
       bx0=g*bx0
       by0=g*by0
       bz0=g*bz0
       f=2./(1.+bx0*bx0+by0*by0+bz0*bz0)
       u1=(u0+v0*bz0-w0*by0)*f
       v1=(v0+w0*bx0-u0*bz0)*f
       w1=(w0+u0*by0-v0*bx0)*f
c    Second half mag. rot'n & el. acc'n:
       u0=u0+v1*bz0-w1*by0+ex0
       v0=v0+w1*bx0-u1*bz0+ey0
       w0=w0+u1*by0-v1*bx0+ez0
c    Relativity's gamma:
       g=c/sqrt(c**2+u0**2+v0**2+w0**2)
       u(n)=g*u0
       v(n)=g*v0
       w(n)=g*w0
c    Position advance:
       x(n)=x(n)+u(n)
       y(n)=y(n)+v(n)
  1    z(n)=z(n)+w(n)
       return
       end
c --------
       subroutine xsplit(x,y,z,x0,y0,z0)
       if(ifix(x).ne.ifix(x0))go to 1
       call ysplit(x,y,z,x0,y0,z0)
       return
  1    x1=.5*(1+ifix(x)+ifix(x0))
       y1=y0+(y-y0)*((x1-x0)/(x-x0))
       z1=z0+(z-z0)*((x1-x0)/(x-x0))
       call ysplit(x,y,z,x1,y1,z1)
       call ysplit(x1,y1,z1,x0,y0,z0)
       return
       end
```

```
c --------
      subroutine ysplit(x,y,z,x0,y0,z0)
      if(ifix(y).ne.ifix(y0))go to 1
      call zsplit(x,y,z,x0,y0,z0)
      return
  1   y1=.5*(1+ifix(y)+ifix(y0))
      z1=z0+(z-z0)*((y1-y0)/(y-y0))
      x1=x0+(x-x0)*((y1-y0)/(y-y0))
      call zsplit(x,y,z,x1,y1,z1)
      call zsplit(x1,y1,z1,x0,y0,z0)
      return
      end
c --------
      subroutine zsplit(x,y,z,x0,y0,z0)
      if(ifix(z).ne.ifix(z0))go to 1
      call depsit(x,y,z,x0,y0,z0)
      return
  1   z1=.5*(1+ifix(z)+ifix(z0))
      x1=x0+(x-x0)*((z1-z0)/(z-z0))
      y1=y0+(y-y0)*((z1-z0)/(z-z0))
      call depsit(x,y,z,x1,y1,z1)
      call depsit(x1,y1,z1,x0,y0,z0)
      return
      end
c --------
      subroutine depsit(x,y,z,x0,y0,z0)
c (Field components are treated as single-indexed in this subroutine)
      common /fields/ex(8192),ey(8192),ez(8192),bx(8192),
     &by(8192),bz(8137)
     &,sm(27),q,qe,qi,qme,qmi,rs,ps,os,c
     &,ms(27),mx,my,mz,ix,iy,iz,lot,maxptl,ions,lecs,nstep
c   cell indices of half-way point:
      i=.5*(x+x0)
      j=.5*(y+y0)
      k=.5*(z+z0)
c   displacements in cell of half-way point:
      dx=.5*(x+x0) - i
      dy=.5*(y+y0) - j
      dz=.5*(z+z0) - k
      l=i+iy*(j-1)+iz*(k-1)
c   current elements:
      qu=q*(x-x0)
      qv=q*(y-y0)
      qw=q*(z-z0)
      delt=.08333333*qu*(y-y0)*(z-z0)
c  Directive specifically for the CRAY cft77 compiler:
cdir$ ivdep
c  (This will make the compiler use the "gather-scatter" hardware.)
c  If one desires NO smoothing (risking the presence of alias-prone
c  high harmonics), one can replace the statement "do 1 n=1,27" by
c     n=14
c  and boost the value of  q  by a factor 8.
      do 1 n=1,27
      ex(ms(n)+l+iy+iz)=ex(ms(n)+l+iy+iz)-sm(n)*(qu*dy*dz+delt)
      ex(ms(n)+l+iz)=ex(ms(n)+l+iz)-sm(n)*(qu*(1.-dy)*dz-delt)
      ex(ms(n)+l+iy)=ex(ms(n)+l+iy)-sm(n)*(qu*dy*(1.-dz)-delt)
      ex(ms(n)+l)=ex(ms(n)+l)-sm(n)*(qu*(1.-dy)*(1.-dz)+delt)
      ey(ms(n)+l+iz+ix)=ey(ms(n)+l+iz+ix)-sm(n)*(qv*dz*dx+delt)
      ey(ms(n)+l+ix)=ey(ms(n)+l+ix)-sm(n)*(qv*(1.-dz)*dx-delt)
      ey(ms(n)+l+iz)=ey(ms(n)+l+iz)-sm(n)*(qv*dz*(1.-dx)-delt)
      ey(ms(n)+l)=ey(ms(n)+l)-sm(n)*(qv*(1.-dz)*(1.-dx)+delt)
      ez(ms(n)+l+ix+iy)=ez(ms(n)+l+ix+iy)-sm(n)*(qw*dx*dy+delt)
      ez(ms(n)+l+iy)=ez(ms(n)+l+iy)-sm(n)*(qw*(1.-dx)*dy-delt)
      ez(ms(n)+l+ix)=ez(ms(n)+l+ix)-sm(n)*(qw*dx*(1.-dy)-delt)
 1    ez(ms(n)+l)=ez(ms(n)+l)-sm(n)*(qw*(1.-dx)*(1.-dy)+delt)
      return
      end
```

```
C -------------
C  Generator of randoms uniformily distributed between -.5 and .5 :
      function rndm(lucky,leap)
C On CRAY-s, the next 5 statements should be replaced by:
C 1   lucky=and((lucky*261),32767) - and((lucky*261),32768)
C and "mini(1)" in statement labelled 2 replaced by "lucky".
      integer*2 mini(2)
      equivalence (maxy,mini(1))
  1   maxy=lucky*261
      mini(2)=0
      lucky=maxy
      leap=leap-1
      if(leap.ne.0)go to 2
      leap=37
      go to 1
  2   rndm=(1./65536.)*float(mini(1))
      return
      end
```