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Demography and the Long-Run Predictability of the Stock Market

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DEMOGRAPHY AND THE LONG-RUN PREDICTABILITY OF THE STOCK MARKET

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Abstract

Stock market price/earnings ratios should be influenced by demography. Since demography is predictable, stock returns should be as well. We provide a simple stochastic OLG model with a cyclical structure which generates cyclical P/E ratios. We calibrate the model to roughly fit the cyclical features of historical P/E ratios.

Keywords: Demography, Price earnings ratio, Returns, Efficient markets, Baby-boom, Savings

JEL Classification: E21, E32, E43, E44, G12, J11

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Demography and the Long-run Predictability of the Stock Market

1. Introduction

A striking outcome that emerges from the explicit introduction of demographic structure into a model of capital market equilibrium is that the future course of stock prices becomes to a significant degree predictable. As is commonly observed, agents have typical and distinct financial needs at different periods of their lives—borrowing when young, investing for retirement in middle age, and disinvesting in retirement. In this paper we will show that as a result security prices are to an important extent pinned down by the relative numbers of agents in the population who are at the different stages of their lifecycle. Since the relative sizes of the cohorts that will trade 20 years from now can be deduced from the age distribution of the population today, introducing demographic structure leads to a significant element of predictability for future security prices.

The idea that demographic forces have a powerful impact on economic activity more generally—on capital accumulation and output—and hence on the stock market, is far from new. It formed the basis for the classic studies of Kuznets (1958, 1961) on the influence of long swings in the growth of population on capital accumulation and the stock market in the late 19^{th} and early 20^{th} centuries. More recently the controversial paper of Mankiw and Weil (1989) studied the impact of predictable demographic change on the housing market, a problem which is close in spirit to the problem that we propose to study in this paper for the stock market. Their analysis was based on a partial equilibrium model of the demand for housing by agents at different periods of their life, and their central theoretical conclusion was that the market for housing is not efficient:

> "the fluctuations in prices caused by fluctuations in demand do not appear to be foreseen by the market even though these fluctuations in demand were foreseeable. . . ."

A similar partial equilibrium and econometric analysis was carried out by Bergantino (1997): according to his model, a significant part (about 40%) of the increase in real housing prices between 1965 and 1980 can be accounted for by the baby-boom induced growth in demand and, since peak demand for equity occurs some twenty years after the peak demand for housing, a significant part (about 30%) of the increase in stock prices after 1985 can be attributed to the same demographic phenomenon. He then notes:

> "What makes this conclusion somewhat believable is that the demographic demand variables used to generate it are derived from observed age profiles of investment in housing and stocks.

What makes this conclusion unbelievable, however, is that it implies predictability of long-run asset price cycles. . . . Many people are sure to be bothered by the thought of predictable booms and busts in stock market prices, since arbitrage by investors with rational expectations would seem to preclude such scenarios. . . . A question that does need to be answered . . . is whether or not the observed timing of movements in asset prices relative to changes in the age distribution of the population is consistent with rational expectations."

Our objective is to provide an analytical framework for studying this relation between changing demographic structure and capital market equilibrium. In the setting of a general equilibrium model which respects the tenets of rational expectations and absence of arbitrage opportunities, we study if predictable change in demographic structure can lead to predictable future change in asset prices—and how significant such prices changes can be.[1] The natural instrument for such an analysis is the *overlapping generations* (OLG) model. Up till now the study of this model has mainly focused on the special case where demographic structure is unchanged: the population is assumed to grow at a constant rate, so that the age pyramid consists of a finite number of cohorts of identical size which grow at the same rate. The relative demands for securities are thus unchanged over time. Our objective is to study how the equilibrium of the model is altered when there are systematic changes in the number of births over time which lead to systematic changes in the age pyramid.

Some recent papers have studied this problem in the context of the Diamond OLG model (1965) with production and with random birth rates. However the use of the Diamond model presents difficulties for studying the stock market, since in this model the equity price of a firm coincides with the quantity of capital it embodies. The variations in the capital stock delivered by the model are not of the order of magnitude of the variations in equity prices observed over the last century (see Brooks (2001)). Abel (2000, 2001) seeks to remedy this problem by considering a Diamond model with convex adjustment costs, in which the 'price' of capital is the reciprocal of the marginal product of investment: however since in equilibrium the value of capital is proportional to output, and since observed variations in output have not been of the same order of magnitude as variations in equity prices, this modified Diamond model suffers from the same problem. We thus resort to a

[1] Bakshi and Chen (1994) seem to have been the first to observe the striking relation between the age wave—defined as the average age of the US population over 20—and the movements in the S&P 500 Index after 1945. They attempt to construct an infinite-horizon representative-agent pricing model to account for the behavior of security prices: the representative agent is a "stand in" for all agents, having an age which is the average age of the population. One of the key assumptions is that the relative risk aversion of the representative agent is an increasing function of the average age. While their analysis is suggestive and provides insights into the behavior of the risk premium, it gives no clear insight into the way demographic forces influence the prices of securities over time.

simpler exchange framework in which equity is modeled as a purely financial security.

For the United States, the 20th century can be divided into approximately five twenty year periods of alternately high and low birth rates, generating the successive Baby Boom and Bust generations of the 1910's, 30's, 50's, and 70's and 90's—the most famous and remarkable being the post war Baby Boom generation of the 1950's. To mimic in broad outline the demographic changes that have occurred in the US while keeping the model simple, we focus on the case where the number of births is alternately high and low. The model is a basic OLG exchange economy with a single good, in which agents have a random endowment in youth and middle age, and land produces an infinite stream of random output. The securities consist of a risk free bond and equity in the land. The population structure at any date is summarized by an age pyramid giving the number of agents in each age cohort: the number of children, young, middle-aged and retired agents. Changes in the number of births overtime lead to changes in the age pyramid — and these changes in the relative sizes of the cohorts alive at any date, change the relative demands for the different securities on the financial markets. We compute the stationary markov equilibrium of the resulting economy by a method similar to that used recently by Constantinides, Donaldson, and Mehra (2000) for an OLG economy with three-period lived agents—although in their setting the structure of the population is unchanged.

The results that we obtain strongly support the view that changes in demographic structure induce significant changes in security prices—and in a way that is robust to variations in the underlying parameters. When we parametrize the model to US data, we obtain variations in the price-earnings ratios which approximate those observed in the US over the last 50 years, and in line with recent work of Campbell and Shiller (2001), the model supports the view that a substantial fall in the price-earnings ratio is likely in the next 20 years. For the 40 year cycle in population pyramids, gives rise to a 40 year cycle in equity prices—and the prices, although random, have a strong predictable component. These 40 year cycles are reminiscent, although somewhat shorter, than the 50 year cycles identified by Kondratieff (1926) for wages and interest rates in England and France during the 18^{th} and 19^{th} century.

The 'demographic' stock market equilibrium has a second striking property which is a consequence of the cyclical fluctuations in security prices: although all agents are assumed to have the same preferences, and face the same lifetime earnings and dividend processes, the size of the cohort to which an agent belongs crucially influences his utility through the profile of bond and equity prices that the agent encounters over his lifetime. Agents in small (large) cohorts face favored (unfavorable) terms of trade on the financial markets throughout their life, with the result that the agents in the favored cohorts have greater lifetime consumption—we call this the *favored cohort effect*.

In Section 2 we begin by outlining a simple demographic model in which generations are alternately large and small, and equity in land (or "trees") yields a constant stream of dividends each period forever: we choose the sizes of the generations, and the dividends and wages for young and middle-aged, in accordance with historical averages for the US. Section 3 studies the rational expectations equilibrium of this economy, and shows that the fluctuations in the demographic structure lead to cyclical fluctuations in the equity prices: the returns on equity are predictable, but consistent with equilibrium, since the interest rate also varies predictably. Since the demographic fluctuations are by themselves not sufficient to explain the observed variation in equity prices, in Section 4 we extend the model to incorporate the effect of business cycle shocks, choosing the parameters approximately in accordance with macroeconomic measures of dividends, wages, and output fluctuations, as well as plausible measures of risk aversion for the representative agent in each generation. A stationary equilibrium of this stochastic economy is more complex to compute, and its properties are studied in Section 5, where we show that the resulting long-term fluctuations in the price-earnings ratio of equity are approximately in accordance with what has been observed in postwar US data.

2. Simple Model with Demographic Fluctuation

Consider an overlapping generations exchange economy with a single good (income) in which the economic life of an agent lasts for three periods: young, middle-age and retired. All agents have the same preferences and endowments and only differ by the date at which they enter the economic scene. Their preferences over lifetime consumption streams are represented by a standard discounted sum of expected utilities

$$U(c) = E\Big(u(c^y) + \delta u(c^m) + \delta^2 u(c^r)\Big), \qquad \delta > 0 \tag{1}$$

where $c = (c^y, c^m, c^r)$ denotes the random consumption stream of an agent when young, middle-aged and retired. For the calibration, u will be taken to be a power utility function

$$u(x) = \frac{1}{1-\alpha} x^{1-\alpha}, \quad \alpha > 0$$

where α is the *coefficient of relative risk aversion.* Since a "period" in the model represents 20 years in the lifetime of an agent, we take the discount factor to be $\delta = 0.5$ (corresponding to a standard annual discount factor of 0.97).

In this Section we outline the basic features of the model and explain how we choose average values for the calibration: these average values can be taken as the characteristics of a deterministic economy whose equilibrium is easy to compute, and provides a first approximation for the effect of demographic fluctuations on the stock market.

Each agent has an endowment $w = (w^y, w^m, 0)$ which can be interpreted as the agent's labor income when young and middle-aged, and income in retirement is zero. There are two financial instruments—a riskless bond and an equity contract—which agents can trade to redistribute income over time (and, in the stochastic version of the model, to alter their exposure to risk). The (real) bond pays one unit of income (for sure) next period and is in zero net supply; the equity contract is an infinite-lived security in positive supply (normalized to 1), which pays a dividend each period. Agents own the financial instruments only by virtue of having bought them in the past: they are not initially in any agent's endowment. In this section the dividends and wage income are nonstochastic, so that the bond and equity are perfect substitutes: in Section 4 where we introduce random shocks to both dividends and wages, bond and equity cease to be perfect substitutes.

To simplify the calculation of the stationary equilibrium we assume that the model has been "detrended" so that the systematic sources of growth of dividends and wages arising from population growth, capital accumulation and technical progress are factored out. The sole source of variation in total output comes from the cyclical change in the demographic structure, to which we now turn, and from the random "business cycle" shocks introduced in Section 4.

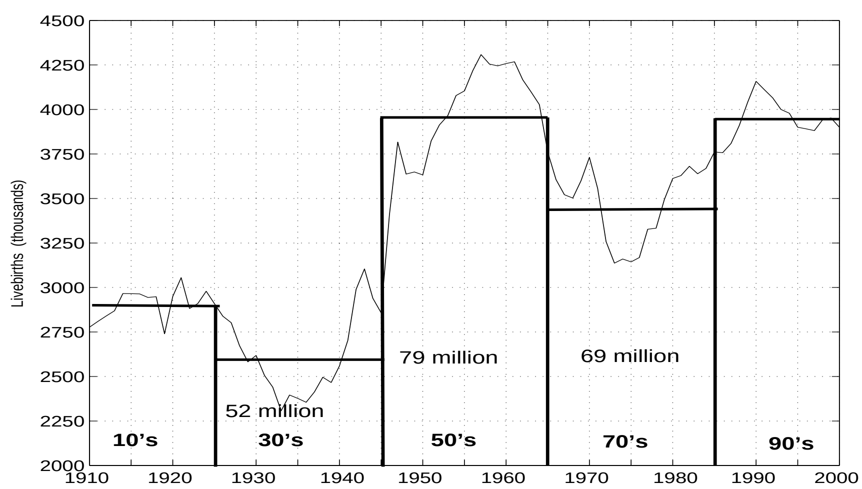


Figure 1: *US livebirths and the 5 cohorts of the 20^{th} century*

Demographic Structure. Livebirths induce the subsequent age structure of the population: the annual livebirths for the US during the 20th century[2] are shown in Figure 1. If the livebirths for a sequence of twenty adjoining years are grouped together into a cohort, then the number of births can be approximated by five twenty-year periods which create the alternatively large and small cohorts known as the 10's, 30's, 50's, 70's and 90's generations. We seek the simplest way of modeling this alternating sequence of generation sizes: time is divided into a sequence of 20 year periods and we let

$$\Delta_t = (\Delta_t^y, \Delta_t^m, \Delta_t^r)$$

denote the *age pyramid* at date t, where Δ_t^y denotes the number of young, Δ_t^m the number of middle-aged, and Δ_t^r the number of retired agents.[3] We assume that in each odd period a large cohort (N) enters the economic scene as young, while in each even period a small cohort (n) enters: thus the age pyramid is $\Delta_t = (N, n, N)$ in every odd period, and $\Delta_t = (n, N, n)$ in every even period. We let $\Delta_1 = (N, n, N)$ and $\Delta_2 = (n, N, n)$, and call $\Delta = (\Delta_1, \Delta_2)$ the set of pyramid states.

Because the typical lifetime income of an individual is small in youth, high in middle age and small or nonexistent in retirement, agents typically seek to borrow in youth, invest in equity and bonds in middle age, and live off this middle-age investment in their retirement. As we shall see, this lifecycle portfolio behavior implies that the relative size of the middle and young cohorts, which can be summarized in the *medium-young cohort ratio* $\delta_t = \frac{\Delta_t^m}{\Delta_t^y}$, plays an important role in determining the behavior of the equilibrium prices on the bond and equity markets. For the above alternating cohort structure, the medium-young cohort ratio (MY, for short) alternates between $\delta_1 = \frac{n}{N} < 1$ in odd periods and $\delta_2 = \frac{N}{n} > 1$ in even periods: since $\delta_2 = \frac{1}{\delta_1}$, it is convenient to refer to δ_2 (namely, the ratio in pyramid Δ_2) as the MY cohort ratio of this alternating pyramid structure.

The demographic structure shown in Figure 1 is not perfectly stationary. There were 52 million live births in the Great Depression generation from 1925-1944, and 79 million born in the Baby Boom from 1945-1964; these two generations traded as medium and young in the period 1965 - 1984. In the Baby-Bust (Xer) generation between 1965 and 1984 births fell, but only to 69 million; the Baby Boom and Xer generations have traded with each other from 1985-2004. The Echo Baby Boom generation born between 1985 and the present seems headed for the same order of magnitude

[2] Historical Statistics of the United States, Series B1, and Bureau of the Census

[3] An individual's "biological life" is divided into 4 periods, 0-19 (child), 20-39 (young) 40-59 (middle age), 60 -79 (retired); the agent's "economic life" (earning income, trading on financial markets) begins when the agent is young. For simplicity we include only the "economic agents" in the age pyramid: the Δ_t^y young were the children born at date $t-1$.

as the Baby Boom generation; the Echo Baby Boom generation and the Xer generation will trade with each other from 2005-2024.

In order to mimic the actual history with a stationary economy, we are thus led to study two cases: in the first $n = 52$ and $N = 79$. The equilibrium for this case gives an idea of the potential change in equity prices in the transition from a pyramid state Δ_1 in which the large generation is young, to a pyramid state Δ_2 in which the large generation is middle aged, such as happened between 1965-84 and 1985-2004. Given that the Baby Bust (Xer) generation born 1965-84 was larger (69 million) than the Great Depression generation, this equilibrium will somewhat exaggerate the increase in price that a calibrated model predicts for the period 1985-2004, and substantially overestimate the predicted decline in prices when the Baby Boomers retire. To correct for this, we compute the equilibrium for a second case in which N is kept at 79, and set $n = 69$. This case in turn will underestimate the increase in prices for the period 1985-2004, but should give a fair estimate of the decline in prices predicted when the Baby Boomers retire. In the analysis that follows we will refer to these two cases as the 'high" and the "low" MY ratio cases.

Wage Income and Dividends. The exchange economy is viewed as an economy with "fixed production plans". Equity in land or trees yields a steady stream of dividends D each period, and each young and middle-aged worker produces output w^y and w^m respectively. To calibrate the relative shares of wage income going to young and middle-aged agents, we draw on data from the Bureau of the Census shown in Figure 2: the maximum ratio of the average annual real income of agents in the age-groups 45-54 and 25-34 is 1.54: we round this to 1.5 and calibrate the model on the basis of a wage income of $w^y = 2$ for each young agent and $w^m = 3$ for each middle-aged agent. Since the agents have homothetic (CES) preferences, the absolute levels of endowments and dividends do not influence the relative prices or relative consumption levels, which will be the primary focus of the study.

Since the wage income of middle-aged agents is greater than that of the young, a change in the age pyramid leads to a change in the total wages in the economy: the total wage is greater when the middle-aged generation is large (pyramid Δ_2) than when the young generation is large (pyramid Δ_1). Since the active population is constant, this increase in wages has to be interpreted as coming from an increase in the average productivity of labor: implicitly the model presumes that middle-aged agents are more experienced and productive than the young since they are paid higher wages.

In the high MY ratio economy, in which $(N, n) = (79, 52)$, the total wages alternate between

$$341 = 79 \times 3 + 52 \times 2 \quad \text{and} \quad 314 = 79 \times 2 + 52 \times 3$$

When the demographic structure is less skewed, as in the low MY economy with $(N, n) = (79, 69)$, the total wage income alternates between

$$375 = 79 \times 3 + 69 \times 2 \quad \text{and} \quad 365 = 79 \times 2 + 69 \times 3$$

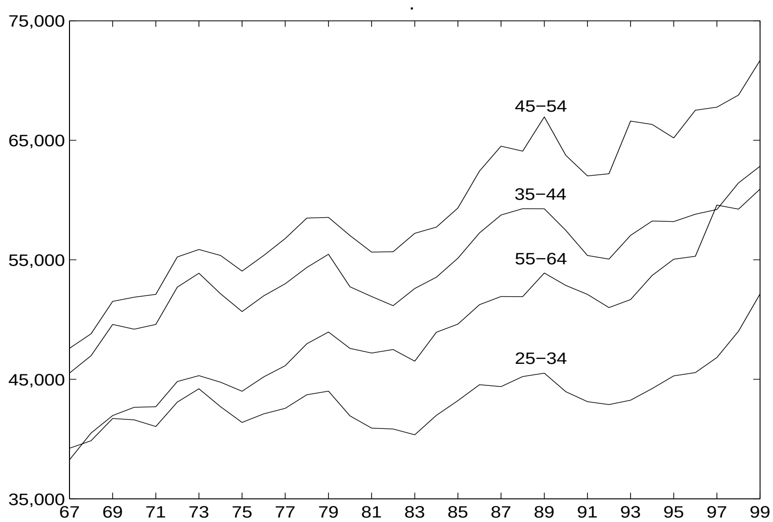


Figure 2: *Real wage income of different age cohorts over time (1999 dollars)*

Land produces output which is distributed as dividends to the equity holders. We take the ratio of the dividends to wages to be of the same order of magnitude as the ratio of stock market dividends to wages in the US economy: from the macro literature we take the standard share of output going to labor to be 70% and to capital 30%. Of the 30% going to capital we take the postwar historical average of 50% to be distributed as dividends[4], the other half being kept by the firms as retained earnings. We incorporate this into the model by assuming that the wages of the young and medium agents represent on average 70% of output, while 30% is produced by land. Of the 30%—which we call the "earnings"—half is distributed as dividends to the equity holders, and the other half is used to maintain the land. Thus the ratio of dividends to average wages is 15/70: in the low MY economy, we take $D = \frac{15}{70}(341 + 314)/2 = 70.18$, and in the high MY economy we take $D = \frac{15}{70}(375 + 365)/2 = 79.29$.

[4]For the period 1951-2000 the average payout ratio for the firms in the S&P500 Index was 0.51.

For the demographic structure $(N, n) = (79, 52)$ in which there is a large variation in the cohort ratio, total income (wages plus dividends) is on average 7.0% higher with Δ_2 than with Δ_1: for the case $(N, n) = (79, 69)$ with its smaller variation in the cohort ratio, the output difference between Δ_2 and Δ_1 is 2.3%.

3. Pure Demographic Equilibrium

Equilibrium Equations. When the only source of change in the economy comes from fluctuations in the demographic structure, it is straightforward to describe and solve for the stationary equilibrium. Let q_t^b be the price of the bond at time t, that is, the amount of good that is required in period t to buy one unit of output in the next period. Since a period is twenty years, $q_t^b = 1/(1 + r_t^{an})^{20}$, where r_t^{an} is the annualized interest rate for borrowing during period t. It is easy to prove that there must be an equilibrium in which $q_t^b = q_1$ whenever t is odd, and $q_t^b = q_2$ whenever t is even. Moreover, all the N agents who are young in an odd period choose the same consumption stream $(c^y(1), c^m(1), c^r(1))$, and every agent young in an even period makes the choice $(c^y(2), c^m(2), c^r(2))$. In equilibrium we must have

$$\begin{aligned} Nc^y(1) + nc^m(2) + Nc^r(1) &= N \times 2 + n \times 3 + D \\ nc^y(2) + Nc^m(1) + nc^r(2) &= n \times 2 + N \times 3 + D \end{aligned}$$

Since there is no uncertainty the bond and equity must be perfect substitutes in each period. Since any agent can roll over the bond, or sell his equity as he gets older, it follows that agents who become young in odd periods face the present-value budget constraint

$$c^y(1) + q_1 c^m(1) + q_1 q_2 c^r(1) = w^y + q_1 w^m + q_1 q_2 w^r = 2 + q_1 3$$

while agents who are young in even periods face the budget constraint

$$c^y(2) + q_2 c^m(2) + q_1 q_2 c^r(2) = w^y + q_2 w^m + q_1 q_2 w^r = 2 + q_2 3$$

From the no-arbitrage property of equilibrium, if bond prices alternate between q_1 and q_2, then the price of equity must alternate between q_1^e and q_2^e, where

$$\begin{aligned} q_1^e &= Dq_1 + Dq_1q_2 + Dq_1q_2q_1 + Dq_1q_2q_1q_2 + ... \\ q_2^e &= Dq_2 + Dq_2q_1 + Dq_2q_1q_2 + Dq_2q_1q_2q_1 + ... \end{aligned}$$

Since bonds and equity are perfect substitutes, $q_1 < q_2$ if and only if $q_1^e < q_2^e$; and it is easy to show that

$$q_1^e/D = (q_1q_2 + q_1)/(1 - q_1q_2) \quad \text{and} \quad q_2^e/D = (q_1q_2 + q_2)/(1 - q_1q_2)$$

Since dividends are half of earnings, the two PE ratios (equity price to annual earnings) are given by

$$PE(1) = q_1^e/(2D/20) = 10q_1^e/D \quad \text{and} \quad PE(2) = 10q_2^e/D$$

Properties of Equilibrium. If the bond prices were to coincide with the consumer discount rate, $q_1 = q_2 = 0.5$, then individuals would attempt to completely smooth their consumption, demanding the stream $(c^y, c^m, c^r) = (2, 2, 2)$. But then, in the case where the population structure is $(N, n) = (79, 52)$, in odd periods the aggregate excess demand for consumption would be $79(2-2)+52(2-3)+79(2-0)-70.18 = 33.82$, while in even periods it would be $52(0)+79(-1)+52(2)-70.18 = -45.18$. Thus in odd periods there is excess demand for consumption as the relatively poor young and retired consume beyond their income, while in even periods when the middle-aged cohort is large, there is excess demand for savings as those households seek to invest for their retirement. To clear markets, the interest rates must adjust, discouraging consumption (stimulating savings) in odd periods, and discouraging saving (stimulating consumption) in even periods: as a result equilibrium bond prices must be below 0.5 in odd periods, and above 0.5 in even periods.

But, if $q_1 < q_2$, then budget constraints of agents in the odd and even cohorts are not the same: the difference is the price of middle-aged consumption. Let $U(1)$ (resp. $U(2)$) be the maximum utility that can be obtained facing the first (resp. second) budget constraint. Since agents are always saving (for their retirement years) when middle-aged, then $q_1 < q_2$ implies that $U(1) < U(2)$, so that agents born in small cohorts are favored relative to those born in large cohorts. We call this the *favored cohort effect*.

Calculating the stationary equilibrium for the economy with $(N, n) = (79, 52)$, and utility function parameter $\alpha = 2$, gives equity prices, annual interest rates and PE ratios

$$(q_1^e, q_2^e,\ r_1^{an}, r_2^{an},\ PE(1), PE(2)) = (55, 92,\ 5.6\,\%, 1.5\,\%, 7.8, 13.1)$$

the consumption streams $c(i) = (c^y(i), c^m(i), c^r(i)),\ i = 1, 2$, and utilities

$$(c(1), c(2),\ U(1), U(2)) = ((1.81, 2.21, 1.82), (2.27, 1.87, 2.28),\ (-.91, -.82))$$

As expected, when the large cohort is young and the small cohort middle-aged, the interest rate is high (5.6 %) and the equity price is low (55), with a low price-earnings ratio (7.8): when the large cohort moves into middle-age and seeks to save for retirement, the interest rate falls to 1.5 %, while the equity price is about 70 % higher ($q_2^e/q_1^e = 1.7$), the PE ratio increasing to 13.1. As predicted by the favored cohort effect, the smaller generation is better off ($-.82 > -.91$.).

When the demographic structure (N, n) gets less skewed, the disequilibrium implied by the bond prices $q_1 = q_2$ is less pronounced, so that bond and equity prices do not need to fluctuate as much to establish equilibrium. Thus with the same utility parameter $\alpha = 2$, if $(N, n) = (79, 69)$ then the equilibrium prices are

$$(q_1^e, q_2^e,\ r_1^{an}, r_2^{an},\ PE(1), PE(2)) = (69, 82,\ 4.4\,\%, 3\,\%, 8.7, 10.3)$$

and the consumption and utility levels are

$$(c(1), c(2),\ U(1), U(2)) = ((1.92, 2.08, 1.98), (2.06, 1.97, 2.13),\ -.89, -.85)$$

For a given demographic structure, the price change must become more volatile, the higher the aversion to consumption variability. Thus when $(N, n) = (79, 52)$, and $\alpha = 4$, equilibrium macro values are

$$(q_1^e, q_2^e,\ r_1^{an}, r_2^{an},\ PE(1), PE(2)) = (49, 118,\ 6.9\,\%, 0.06\,\%, 7.1, 16.9)$$

and the individual values are

$$(c(1), c(2),\ U(1), U(2)) = ((1.78, 2.09, 1.76), (2.38, 2.00, 2.35),\ -0.09, -0.05)$$

Effect of Bequests. The model presented above assumes that agents fully deplete their wealth in the last period of their life. In practice people end up with wealth at the time of their death both because they hold precautionary balances against the uncertain time of death, and because they derive a direct utility from the bequest they leave to their children[5]. Poterba (2000) has argued that the presence of such bequests will attenuate—if not cancel—the decrease in security prices that is expected when the Baby Boomers go into retirement, since they will not attempt to sell all their securities. However if all generations transfer bequests, it still implies that a large generation will need to sell the share of its wealth that it needs as retirement income to a smaller generation of middle-aged savers. Abel (2001) has shown that in his model with production and two-period-lived agents, the presence of bequests does not change the equilibrium. In our model adding a bequest motive does affect the equilibrium, but does not significantly alter the ratio of equity prices q_2^e/q_1^e: the main effect is to decrease interest rates and thus to increase equity prices and price-earnings ratios in both pyramid states.

Suppose that the utility function of the representative agent is modified to incorporate the bequest motive

$$U(c) = \frac{1}{1-\alpha}\Big((c^y)^{1-\alpha} + \delta(c^m)^{1-\alpha} + \delta^2((c^r)^{1-\beta}b^\beta)^{1-\alpha}\Big)$$

[5] See Modigliani (1986) for a discussion and estimation of the proportion of wealth transferred through bequests.

where b is the bequest, and suppose that the retired agents of date t transfer their bequests to the agents who are middle-aged at date $t+1$: since the number of these agents is the same, we can assume that each retired agent at date t saves b_t from his available income in the form of bond or equity holding, and each middle-aged agent receives $b_t(1+r_t) = b_t/q_t$ at the beginning of date $t+1$. The model has a stationary equilibrium with bond prices (q_1, q_2), the budget constraint of an agent born in an odd generation (Δ_1) being

$$c^y(1) + q_1 c^m(1) + q_1 q_2 (c^r(1) + b(1)) = w^y + q_1(w^m + b(1)/q_1)$$

where the term $b(1)$ on the leftside is the agent's choice of bequest, while the term $b(1)$ on the rightside is exogenous since it is the bequest received from an old agent born in pyramid Δ_1. A similar budget constraint holds for the consumption and bequest $(c(2), b(2))$ of an agent born in an even generation with q_1 being replaced by q_2, and the market clearing equations are unchanged. With $(N, n) = (79, 52)$, $\alpha = 2$, and $\beta = 0.1$, the equilibrium prices are

$$(q_1^e, q_2^e,\ r_1^{an}, r_2^{an},\ PE(1), PE(2)) = (74, 121,\ 4.9\,\%, 0.9\,\%, 10.5, 17.3)$$

while the consumption streams $(c(1)) = (1.89, 2.15, 1.76)$, and $c(2) = (2.40, 1.85, 2.24)$ for the two generations are similar to those without bequest. Since the middle-age agents have more income and thus a higher propensity to save, the main change in the equilibrium is decrease in the rate of interest in both pyramid states. This decrease in interest rates favors the young agents who consume slightly more than in the equilibrium without bequests. The decrease in interest rates translates into higher equity prices and price-earnings ratios, but the ratio of equity prices $q_2^e/q_1^e = 1.64$ is of the same order of magnitude as the ratio 1.7 found in the equilibrium without bequests. With the same parameters, except for greater utility of bequests $\beta = 0.2$ the ratio is 1.6; with $\alpha = 4$ the ratio is 2.4 without bequests, 2.2 with $\beta = 0.1$, and 2.1 with $\beta = 0.2$.

4. Introducing Business Cycle Shocks

The demographic model studied above is unable to explain the full increase in equity prices observed over the last twenty years. Using the cohort sizes $(N, n) = (79, 52)$ and the transition from the population pyramid $\Delta_1 = (N, n, N)$ in which the Baby Boom generation was young, to the pyramid $\Delta_2 = (n, N, n)$ in which they became middle aged, the equilibrium in the previous section gave the ratio of equity prices $q_2^e/q_1^e = 1.7$ with $\alpha = 2$, implying a 70% increase in prices, and the ratio $q_2^e/q_1^e = 2.4$ with $\alpha = 4$, implying that equity prices would have somewhat more than doubled. In fact the real S&P500 increased by a factor of more than 7 from its low of around 190 in 1982 to a high of around 1400 in 2000 (see Figure 4). The rise in stock prices over this period coincided not only with a change in demographic structure but also with the passage from a period in which aggregate shocks were mainly negative—the oil shocks of the 70's, the bursts of high inflation followed by restrictive monetary policy, leading to unemployment and low productivity—to the period of the 90's in which aggregate shocks were mainly positive—low inflation and energy prices, rapid technical change resulting in low unemployment and high productivity. We thus add to the demographic model of the previous section the possibility of random shocks to output to study the combined effect for security prices of demographic and business cycle fluctuations. For simplicity we calculate the equilibrium for the case where agents do not leave bequests.

Risk Structure. We model the risk structure of the economy by assuming that the wages and dividends on equity are subject to shocks. At each date there are four possible states of nature (shocks), s_1=(high wage, high dividend), s_2=(high wage, low dividend), s_3=(low wage, high dividend), s_4=(low wage, low dividend). Given the nature of the risks and the very extended length of time represented by a period (20 years), we have chosen not to a invoke a markov structure, but rather to assume that the shocks are i.i.d. To reflect the fact that aggregate income and dividends are positively correlated we assume that s_1 and s_4 are more likely (probability 0.4 each), than s_2 and s_3, and this gives rise to a correlation between dividends and wages of 0.6.

Figure 2 shows that the maximum variability of the real annual wage income of the 45-54 cohort is about 4%: in the recession of 1990-91 the mean wage of this cohort went from 65 to 60 (thousands of 1999 dollars), a variability of (2.5/62.5)=0.04; the variability of the wage income of the 25-34 cohort is somewhat lower. To take into account the fact that negative output shocks result in unemployment and that some periods, like the period 1970-1983, experienced a sequence of negative shocks, in the calibration we increase the coefficient of variation (CV) of the wage income of the middle-aged to 16.7% and that of the young to 10% . Corresponding to the two demographic cases of a 'high" and a "low" MY cohort ratio, the CV of dividends is taken to be 21.4% and 19% respectively: this leads to a CV for aggregate output of about 15%. In the appendix we show the impact on the equilibrium of decreasing the income risks of the agents: we give (less detailed) results for the "low risk case" in which the CV of the wage income is 5% for the young and 6.7% for the middle-aged, the CV of dividends is 14% leading to a CV of 7.5% for total output.

Stationary Equilibrium. Since the economy $\mathcal{E}(u, w, d, \Delta)$ has a stationary structure, it is natural to look for a stationary equilibrium. Unlike the deterministic case of the previous section, the equilibrium cannot be solved in terms of the consumption variables with a single present-value budget constraint. For each date-event can be followed by 4 possible states of nature and agents only trade 2 independent securities—the equity in land and the bond[6]—so that markets are incomplete. We thus need to change the focus to portfolio optimization and market clearing asset prices. Since agents' (economic) lives span 3 periods, it can be shown that a markov equilibrium which depends on the exogenous states—the pyramid and shock states—does not exist. What is needed is an endogenous variable which summarizes the dependence of the equilibrium on the past—the income which the middle-aged agents inherit from their portfolio decision in their youth. Thus we will study equilibria with a state space $\Xi = G \times K \times S$ where G is a compact subset of $\mathbb{R}_+$, $K = \{1, 2\}$ is the set of pyramid states (indexed by $k \in \{1, 2\}$) and $S = \{s_1, s_2, s_3, s_4\}$ is the set of shock states: we let $\xi = (\gamma, k, s)$ denote a typical element of the state space Ξ, γ denoting the portfolio income inherited by the middle-aged agents from their youth. The pyramid state k determines the age pyramid $\Delta_k = \left(\Delta_k^y, \Delta_k^m, \Delta_k^r\right)$. If k is the population state at date t, let k^+ (resp. k^-) denote the pyramid state at date $t+1$ (resp. $t-1$). Since the pyramid states alternate, if $k = 1$ then $k^+ = k^- = 2$. The output shock $s \in S$ determines the incomes $w^y = (w_s^y, s \in S)$ and $w^m = (w_s^m, s \in S)$ of the young and middle-aged agents, as well as the dividend $D = (D_s, s \in S)$ on the equity contract.

To find a markov equilibrium, we note that the security prices only need to make the portfolio trades of the young and middle-aged agents compatible: the retired agents have no portfolio decision to make—they collect the dividends and sell their equity holdings. Thus we are led to study the portfolio problems of the young and the middle-aged agents, the latter inheriting the income γ, and to look for security prices which clear the markets. This problem can be reduced to the study of a family of two-period portfolio problems in which middle-aged agents anticipate the consequences of their decisions for their retirement—they need to anticipate the next period equity price Q^e—and young agents anticipate the portfolio income they will transfer into middle age (which also depends on Q^e) and the saving decision F that they will make next period to provide income for their retirement. A correct expectations equilibrium then has the property that the agents' expectations are fulfilled in the next period. Given that an equilibrium will involve both current and anticipated variables we introduce the convention that current variables are denoted by lower case letters, while anticipated variables are denoted by capitals. A stationary markov equilibrium will be a function $\Phi : \Xi \longrightarrow \mathbb{R}^4 \times \mathbb{R}_+^2 \times \mathbb{R}_+^8$ with $\Phi = (z, q, Q^e, F)$, where $z = (z^y, z^m) = (z_b^y, z_e^y, z_b^m, z_e^m)$ is the vector of bond and equity holdings of the young and middle-aged agents respectively, $q = (q^b, q^e)$ is the vector of *current* prices for bond and equity, $Q^e = (Q_s^e, s \in S)$ is the vector of *anticipated*

[6] They are now no longer perfect substitutes, since the payoff of equity is risky.

next period equity prices and $F = (F_s, s \in S)$ is the vector of *anticipated* next period savings of the young. To express the condition on correct expectations we need the following notation: if in state ξ young agents choose a portfolio $z^y(\xi)$ and anticipate equity prices $Q^e(\xi)$, then the income $\Gamma(\xi) = \Big(\Gamma_s(\xi), s \in S\Big)$ which they *anticipate* transferring into middle age is given by

$$\Gamma(\xi) = V(\xi) z^y(\xi), \quad \xi \in \Xi$$

where $V(\xi) = (\mathbf{1}, D + Q^e(\xi))$, $\mathbf{1} = (1, \ldots, 1) \in \mathbb{R}^4$ denoting the sure payoff on the bond and $D = (D_s, s \in S)$ the random dividend on equity. We let $f(\xi)$ denote the actual savings chosen by middle-aged agents when the state is ξ, thus

$$f(\xi) = q(\xi) z^m(\xi), \quad \xi \in \Xi$$

Definition. A function $\Phi = (z, q, Q^e, F) : \Xi \longrightarrow \mathbb{R}^4 \times \mathbb{R}^2_+ \times \mathbb{R}^8_+$ is a *stationary (markov) equilibrium* of the economy $\mathcal{E}(u, w, D, \Delta)$ if, $\forall\ \xi = (\gamma, k, s) \in \Xi$,

$$\text{(i)}\ \ z^y(\xi) = \arg\max_{z^y \in \mathbb{R}^2} \left\{ u(c^y) + \delta \sum_{s' \in S} \rho_{s'} u(C^m_{s'}) \ \middle|\ \begin{array}{l} c^y = w^y_s - q(\xi) z^y \\ \\ C^m = w^m + V(\xi) z^y - F(\xi) \end{array} \right\}$$

$$\text{(ii)}\ \ z^m(\xi) = \arg\max_{z^m \in \mathbb{R}^2} \left\{ u(c^m) + \delta \sum_{s' \in S} \rho_{s'} u(C^r_{s'}) \ \middle|\ \begin{array}{l} c^m = w^m_s + \gamma - q(\xi) z^m \\ \\ C^r = V(\xi) z^m \end{array} \right\}$$

$$\text{(iii)}\ \ \Delta^y_k\, z^y_b(\xi) + \Delta^m_k\, z^m_b(\xi) = 0, \quad \Delta^y_k\, z^y_e(\xi) + \Delta^m_k\, z^m_e(\xi) = 1$$

$$\text{(iv)}\ \ Q^e_{s'}(\xi) = q^e\Big(\Gamma_{s'}(\xi), k^+, s'\Big),\ \forall\ s' \in S, \qquad F_{s'}(\xi) = f\Big(\Gamma_{s'}(\xi), k^+, s'\Big),\ \forall\ s' \in S$$

(i) and (ii) are the conditions requiring maximizing behavior on the part of young and middle-aged agents who anticipate the equity prices $Q^e(\xi)$ and, in the case of the young agents, anticipate the savings $F(\xi)$. Note that the vector of consumption $C^m \in \mathbb{R}^4_+$ which a young agent anticipates for middle age (hence the capital letter) must be distinguished from $c^m(\xi) \in \mathbb{R}$ which is the current consumption of a middle-aged agent. (iii) requires that the aggregate demands of the two cohorts for the bond and equity clear the markets. (iv) is the condition requiring the agents' expectations be correct. In choosing their portfolio $z^y(\xi)$ in state ξ, young agents anticipate transferring the income $\Gamma(\xi) = V(\xi) z^y(\xi)$ to the next period—where $V(\xi)$ is the anticipated payoff of the securities depending on $Q^e(\xi)$. In order that $Q^e_{s'}(\xi)$ be a correct expectation, it must coincide with the price

$q^e\Big(\Gamma_{s'}(\xi), k^+, s'\Big)$ which is realized in output state s' when middle-aged agents receive the portfolio income $\gamma' = \Gamma_{s'}(\xi)$ and the pyramid state is k^+; in the same way the saving $F_{s'}(\xi)$ that the young anticipate doing in their middle age must coincide with the actual savings of a middle-aged agent with asset income $\gamma' = \Gamma_{s'}(\xi)$.

To derive the equilibrium of the calibrated model we take a grid over the asset income carried over by the middle-aged representative agent, and by a sequential procedure compute an approximation to the equilibrium (see the Appendix for further explanation of the procedure for computing equilibrium).

5. Calibration Results

We study the equilibrium trajectories of an economy $\mathcal{E}(u, w, D, \Delta)$ with alternating pyramid structures $(\Delta_t)_{t\geq 1}$, where $\Delta_t = \Delta_1 = (N, n, N)$ if t is odd, and $\Delta_t = \Delta_2 = (n, N, n)$ if t is even. Table 1 gives the statistics for the prices on a typical trajectory for the *Baby-Boom/Depression* (high) cohort ratio $\delta_2 = 79/52 = 1.5$, while Table 2 shows the price statistics for the *Baby-Boom/Xer* (low) cohort ratio $\delta_2 = 79/69 = 1.14$. In each case the endowment and dividend processes (w, D) satisfy the assumptions given earlier, and agents' preferences are characterized by the relative risk aversion parameter α=2, 4, or 6.

An interesting feature of the equilibrium trajectories shown in Tables 1 and 2 is that, while a state $\xi = (\gamma, k, s)$ must involve the (endogenous) portfolio income γ inherited by the middle-aged agents, the *resulting stationary equilibrium depends essentially only on the exogenous states* (k, s): the standard deviation of prices (the numbers between parentheses) about their means (the numbers without parentheses) are small for all pyramid-shock states (k, s). Thus the average values of the equity price, price-earning ratio and interest rate in the different states (k, s) give a rather precise description of the trajectories.

A new variable which arises when uncertainty is introduced is the equity premium—namely the amount by which the expected return on equity exceeds the return on bonds. The annual equity premium is defined by

$$rp^{an} = \text{average}\left(\left[\left(\frac{q^e_{t+1} + D_{t+1}}{q^e_t}\right)^{\frac{1}{20}} - 1\right] - r^{an}\right)$$

This is the only variable which has a high variance even for a given pyramid-shock state (k, s): this is natural since the realized equity premium is greater than average when a favorable state follows state s, while it is lower than average when an unfavorable state follows state s.

Table 1: High Cohort Ratio

$N = 79,\ n = 52,\ w^y = (2.2, 2.2, 1.8, 1.8),\ w^m = (3.5, 3.5, 2.5, 2.5),\ D = (85, 55, 85, 55)$

		pyramid 1: MY ratio=0.66				**pyramid 2: MY ratio=1.5**			
		q^e	q^e/E	r^{an}	rp^{an}	q^e	q^e/E	r^{an}	rp^{an}
$\boldsymbol{\alpha = 2}$	s_1	71 (.8)	8.3 (.09)	3.9 % (.01)	0.2 % (1.3)	126 (4.5)	15 (.5)	-0.3 % (.03)	0.25 % (1)
	s_2	70 (.6)	13 (.1)	4 % (0)	0.2 % (1.2)	118 (3.7)	21.5 (.7)	0.07 % (.03)	0.2 % (1)
	s_3	45 (.5)	5.3 (.06)	6.5 % (.01)	0.25 % (1.3)	76 (3)	8.9 (.3)	2.3 % (.04)	0.25 % (1)
	s_4	44 (.4)	8 (.07)	6.7 % (0)	0.22 % (1.3)	69 (2.3)	12.5 (.4)	2.8% (.03)	0.22 % (1)
	Average	57 (13.3)	8.3 (1.7)	5.3 % (1.4)	0.22 % (1.3)	97 (28)	14 % (3)	1.3 % (1.5)	0.23 % (1)
		ratio of av. prices: 1.7 [0.6], *peak / trough*: 2.9, *trough / peak* 0.35							
$\boldsymbol{\alpha = 4}$	s_1	85 (1.4)	10 (.2)	3.4 % (.01)	0.8 % (2.3)	247 (25)	29 (2.9)	-3.9 % (.2)	0.7 % (1.4)
	s_2	79 (1)	14.4 (.2)	3.9 % (.01)	0.7 % (2.3)	202 (19)	37 (3.4)	-3 % (.2)	0.7 % (1.3)
	s_3	40 (0.6)	4.7 (.07)	7.8 % (.02)	0.7 % (2.5)	92 (9)	10.8 (1.1)	0.9 % (.25)	0.7 % (1.4)
	s_4	37 (.4)	6.7 (.08)	8.5 % (.01)	0.8 % (2.5)	68 (6)	12.3 (1.1)	2.4 % (.2)	0.7 % (1.4)
	Average	60 (23)	8.6 (2.7)	5.9 % (2.5)	0.8 % (2.4)	154 (85)	21 (9.8)	-0.7 % (3)	0.7 % (1.4)
		ratio of av. prices: 2.5 [0.4], *peak / trough*: 6.7, *trough / peak* 0.15							
$\boldsymbol{\alpha = 6}$	s_1	106 (6)	12.5 (.7)	2.4 % (.2)	1.5 % (3.5)	485 (93)	57 (11)	-7.2 % (.8)	1.2 % (1.6)
	s_2	94 (4)	17.2 (.8)	3.3 % (.1)	1.5 % (3.6)	352 (63)	64 (12)	-5.8 % (.7)	1.1 % (1.6)
	s_3	41 (1.7)	4.8 (.2)	8.3 % (.2)	1.4 % (4)	110 (21)	13 (2.4)	-0.3 % (.8)	1.1 % (1.7)
	s_4	36 (1)	6.5 (.2)	9.7 % (.1)	1.5 % (4)	64 (11)	11.5 (2)	2.5 % (.8)	1.2 % (1.7)
	Average	70 (34)	9.8 (4)	6 % (3.5)	1.5 % (3.9)	260 (206)	34 (25)	-2.3 % (4.6)	1.2 % (1.7)
		ratio of av. prices: 3.7 [0.3], *peak / trough*: 13.5, *trough / peak* 0.07							

Table 2. Low Cohort Ratio

$N = 79,\ n = 69,\ w^y = (2.2, 2.2, 1.8, 1.8),\ w^m = (3.5, 3.5, 2.5, 2.5),\ D = (95, 65, 95, 65)$

		pyramid 1: MY ratio=0.87				**pyramid 2: MY ratio=1.14**			
		q^e	q^e/E	r^{an}	rp^{an}	q^e	q^e/E	r^{an}	rp^{an}
$\boldsymbol{\alpha = 2}$	s_1	90 (1.6)	9.5 (.2)	2.7 % (.01)	0.2 % (1.1)	110 (3)	11.5 (.3)	1.3 % (.02)	0.2 % (1)
	s_2	88 (1.4)	13.6 (.2)	2.8 % (.01)	0.3 % (1)	105 (2.4)	16.2 (.4)	1.6 % (.02)	0.2 % (1)
	s_3	56 (1)	5.9 (.1)	5.3 % (.02)	0.2 % (1.1)	67 (2)	7 (.2)	3.9 % (.03)	0.2 % (1)
	s_4	54 (.9)	8.3 (.2)	5.5 % (.01)	0.2 % (1.1)	63 (1.5)	9.7 (.2)	4.2 % (.02)	0.2 % (1)
	Average	72 (18)	9 (2)	4.1 % (1.4)	0.2 % (1.1)	86 (23)	10.8 (2.3)	2.8 % (1.4)	0.2 % (1)
		ratio of av. prices: 1.2 [0.83], *peak / trough*: 2, *trough / peak* : 0.5							
$\boldsymbol{\alpha = 4}$	s_1	117 (3)	12.3 (.3)	1.3 % (.02)	0.7 % (1.8)	168 (10)	18 (1.1)	-1.2 % (.1)	0.7 % (1.5)
	s_2	106 (2)	16.4 (.4)	1.8 % (.02)	0.7 % (1.8)	146 (8)	22.5 (1.2)	-0.4 % (.08)	0.7 % (1.5)
	s_3	49 (1.2)	5.2 (.1)	5.9 % (4)	0.7 % (1.9)	66 (3)	6.9 (.4)	3.6 % (.1)	0.7 % (1.6)
	s_4	44 (.8)	6.7 (.1)	6.7 % (.02)	0.7 % (2)	54 (2.6)	8.3 (.4)	4.7 % (.08)	0.7 % (1.6)
	Average	80 (35)	9.8 (3.6)	4 % (2.6)	0.7 % (1.9)	109 (55)	13.2 (5.6)	1.8 % (2.8)	0.7 % (1.6)
		ratio of av. prices: 1.35 [0.74], *peak / trough*: 3.8, *trough / peak* : 0.26							
$\boldsymbol{\alpha = 6}$	s_1	165 (5)	17.3 (.6)	-0.6 % (.03)	1.4 % (2.7)	278 (30)	29 (3)	-3.8 % (.3)	1.3 % (2)
	s_2	139 (4)	21.3 (.6)	0.4 % (.02)	1.5 % (2.7)	219 (21)	34 (3.3)	-2.6 % (.2)	1.2 % (2)
	s_3	50 (1.3)	5.3 (.2)	5.7 % (.03)	1.5 % (2.9)	71 (7)	7.4 (.7)	2.9 % (.3)	1.4 % (2)
	s_4	39 (.8)	6 (.1)	7.5 % (.02)	1.6 % (3)	47 (4)	7.3 (.6)	5.1 % (.2)	1.4 % (2.2)
	Average	100 (60)	12 (6)	3.4 % (3.8)	1.5 % (2.9)	157 (110)	19 (12)	0.6 % (4.2)	1.3 % (2.1)
		ratio of av. prices: 1.6 [0.63], *peak / trough*: 7.1, *trough / peak* : 0.14							

Cyclical Fluctuations of Security Prices. In the model with both demographic and business cycle shocks, agents must decide how to allocate their savings between the two securities—equity and bonds. However, the fundamental principle that underlies the certainty model—namely that with unchanged prices total savings would be too large in even periods and too small in odd periods, carries over to the economy with uncertainty. To understand the pattern of the equilibrium prices, consider pyramid Δ_2: the large cohort of middle-aged agents seeking to invest for retirement creates a high demand for equity and the bond, and since the young cohort is small there is a small supply of the bond, creating a high bond price (a low interest rate). The price on the equity market must also be high to prevent arbitrage: for the low interest rate encourages the young to borrow—not only to increase consumption in youth but also to invest in equity—and this demand, when added to the equity demand of the middle-aged, pushes up the price on the equity market. In Δ_1 the cohort sizes are reversed: a small middle-aged cohort creates a small demand for equity and the bond, while the large cohort of young creates a high supply of the bond, leading to a low bond price: the resulting high interest rate discourages the young from borrowing to invest on the equity market, and when added to the low demand of the middle aged, leads to a low equity price.

This intuitive explanation is very stylized—in particular it leaves aside agents' anticipations, the fact that each agent examines not only his lifetime needs but also the course of equity and bond prices over the three periods of his life—in short, no simple verbal explanation can substitute for the subtle balance of forces leading to a stationary equilibrium. The outcome however is striking: *in the economy with both demographic and business cycle shocks, the stochastic sequence of equilibrium security prices* (q_t^e, q_t^b) *co-moves with the cohort ratio* δ_t*, being higher (lower) than average when the* MY *cohort ratio is high (low).* Thus long-run fluctuations in demographic structure lead to long-run cyclical fluctuations in security prices over time. The order of magnitude of the demographic effect is indicated in Tables 1 and 2 by the *ratio of the average prices* in the two pyramid states—the first number giving the average price increase when the economy moves from Δ_1 to Δ_2, the second [in square brackets] the average price decrease in the move from Δ_2 to Δ_1.

Why does this predictable long-run rise and fall of equity prices not give rise to arbitrage opportunities? The answer, of course, lies in the fact that this is a simultaneous equilibrium on the bond and equity markets, and it is the variation in the interest rate which prevents arbitrage opportunities: when equity prices are sure to rise "on average", the interest rate is sufficiently high to ensure that a strategy of borrowing to buy equity will give rise to a negative payoff in the unfavorable states (3 and 4); when equity prices are expected to fall, short-selling equity and buying bonds with the proceeds, gives rise to a negative payoff in the favorable states (1 and 2).

Adding shocks to output opens up the possibility of greater variations in equity prices, the greatest increase occurring when the economy moves from (Δ_1, s_4) to (Δ_2, s_1), namely from a large young cohort with an unfavorable output shock to a large middle-aged cohort with a favorable shock. The ratio of these prices is given in Tables 1 and 2 by the peak to trough ratio (and its reciprocal, the trough to peak ratio). It is of some interest to ask what proportion of this overall increase in security prices is accounted for by the demographic effect. If we take the ratio of the average prices to represent the demographic effect d, then the rest of the increase is due to the business cycle effect b. In Table 1, for each value of α, these two effects account for the same share of the increase in equity prices: for with $\alpha = 2$, the peak/trough ratio of 2.9 can be written as the product db with $(d, b) = (1.7, 1.7)$, with $\alpha = 4$, $(d, b) = (2.5, 2.7)$ and with $\alpha = 6$, $(d, b) = (3.7, 3.7)$. In Table 2 where the difference between the cohort sizes has been reduced, as would be expected, the demographic effect is smaller than the business cycle effect. If we use this table to study the decline in prices, we can take d, the demographic effect, to be the ratio of the average price in Δ_1 to the average price in Δ_2: the demographic effect implies an average decrease of 17% for $\alpha = 2$, 26% for $\alpha = 4$, and 37% for $\alpha = 6$. The trough to peak ratio can be decomposed as the product db where b is the business cycle effect, with $(d, b) = (0.83, 0.59)$ for $\alpha = 2$, $(d, b) = (0.74, 0.35)$ for $\alpha = 4$, $(d, b) = (0.63, 0.22)$ for $\alpha = 6$. With the smaller difference between the cohort sizes, the share of the decline in prices due to the demographic effect is now approximately one half that due to the business cycle effect.

Favored Cohort Effect. As in the model of Section 3, the long-run cyclical fluctuations in the demographic structure give rise to a second striking property of the capital market equilibrium, dual to the cyclical fluctuations in security prices: *agents in small cohorts receive more favorable equilibrium lifetime consumption streams than agents in large cohorts.* The lifetime equilibrium consumption streams of agents born into the small (respectively, large) cohorts is shown in Table 3—they have been multiplied by 10,000 to make the comparison of the consumption streams more intuitive. Even though all agents begin with the same average lifetime wage income (20,000, 30,000, 0), the average lifetime consumption stream of an agent born into a small cohort is significantly greater than that of an agent in a large cohort. This difference arises from the cyclical fluctuations in the security prices: the column below the average consumption streams shows the average prices (the equity price and the interest rate) that the corresponding agent faces during his lifetime. In their youth, agents in the small cohort—the Xer's—can (on average) borrow at a lower interest rate than young agents in the large cohort; furthermore, in their middle age, when they seek to invest for their retirement, the interest rate is high, and (on average) prices on the equity market are

Table 3: Lifetime Consumption and the Favored Cohort Effect

$N = 79,\ n = 52,\ w^y = (2.2, 2.2, 1.8, 1.8),\ w^m = (3.5, 3.5, 2.5, 2.5),\ D = (85, 55, 85, 55)$

		$\boldsymbol{\alpha = 2}$		$\boldsymbol{\alpha = 4}$		$\boldsymbol{\alpha = 6}$	
		large cohort	*small cohort*	*large cohort*	*small cohort*	*large cohort*	*small cohort*
Average consumption	*young*	18,100 (1,900)	22,700 (3,000)	17,600 (1,600)	23,800 (3,800)	17,000 (1,300)	24,600 (4,500)
	middle	22,000 (3,500)	18,800 (3,200)	20,500 (3,000)	20,400 (3,500)	19,300 (2,600)	21,900 (3,700)
	retired	18,000 (2,700)	22,700 (3,700)	17,500 (2,800)	23,600 (3,700)	17,200 (2,900)	24,700 (3,800)

		q^e	r	q^e	r	q^e	r	q^e	r	q^e	r	q^e	r
Average market prices	*young*	57 (13)	5.3 % (1.4)	97 (28)	1.3 % (1.5)	60 (23)	5.9% (2.5)	154 (85)	-0.7 % (3)	70 (34)	6 % (3.5)	260 (206)	-2.3% (4.6)
	middle	97 (28)	1.3 % (1.5)	57 (13)	5.3 % (1.4)	154 (85)	-0.7 % (3)	60 (23)	5.9% (2.5)	260 (206)	-2.3% (4.6)	70 (34)	6 % (3.5)
	retired	57 (13)		97 (28)		60 (23)		154 (85)		70 (34)		260 (206)	

$N = 79,\ n = 69,\ w^y = (2.2, 2.2, 1.8, 1.8),\ w^m = (3.5, 3.5, 2.5, 2.5),\ D = (95, 65, 95, 65)$

		$\boldsymbol{\alpha = 2}$		$\boldsymbol{\alpha = 4}$		$\boldsymbol{\alpha = 6}$	
		large cohort	*small cohort*	*large cohort*	*small cohort*	*large cohort*	*small cohort*
Average consumption	*young*	19,200 (2,100)	20,500 (2,500)	19,100 (2,000)	21,000 (2,700)	18,900 (1,900)	21,300 (2,900)
	middle	20,800 (3,300)	19,800 (3,200)	20,400 (3,000)	20,200 (3,200)	19,900 (2,800)	20,700 (3,200)
	retired	19,800 (3,000)	21,200 (3,200)	19,500 (3,000)	21,500 (3,200)	19,400 (3,000)	21,700 (3,200)

		q^e	r	q^e	r	q^e	r	q^e	r	q^e	r	q^e	r
Average market prices	*young*	72 (18)	4.1 % (1.4)	86 (23)	2.8 % (1.4)	80 (35)	4 % (2.6)	109 (55)	1.8 % (2.8)	100 (60)	3.4 % (3.8)	157 (110)	0.6 % (4.2)
	middle	86 (23)	2.8 % (1.4)	72 (18)	4.1 % (1.4)	109 (55)	1.8 % (2.8)	80 (35)	4 % (2.6)	157 (110)	0.6 % (4.2)	100 (60)	3.4 % (3.8)
	retired	72 (18)		86 (23)		80 (35)		109 (55)		100 (60)		157 (110)	

sure to rise. Their counterparts in the large cohort—the Baby Boomers—face exactly the opposite conditions: in youth it is expensive to borrow, in middle age not only does the bond market give a miserable return, but equity is expensive to buy and is virtually certain to decline in value when they come to sell in retirement. The alternating cohort sizes create fluctuations in security prices, and agents in small cohorts are always *in phase* with these fluctuations, boosting their consumption streams, while agents in large cohorts are always *out of phase*, thereby shrinking their lifetime consumption. The extent to which the small cohort is favored depends on the magnitude of the fluctuations in security prices: *the greater the difference in the cohort sizes, the greater the degree of relative risk aversion, or the greater the variability of agents' endowment streams*[7] *the greater the fluctuations in security prices, and the greater the extent to which capital markets favor the small cohort.*

Other authors, in particular Easterlin (1987), have pointed out that the Baby Boomers, being a large cohort, face more competition on the labor market and thus receive lower wages than the small cohort which preceded them: this labor-market cohort effect, which has been documented by Welch (1979), is absent from our model since we assume that agents have the same lifetime wage profile in both cohorts. Our model shows however that large cohorts face a second curse from the financial markets: by being so numerous they drive the terms of trade against themselves, favoring the small cohort on the other side of the market which follows or precedes them.

Differential Participation. How sensitive is the equilibrium of the model to restricted participation? After all, even though the proportion of US household investing on the stock market has increased significantly over the last fifty years[8], it still remains less than 50%. To take this effect into account, we solved for the equilibrium under the assumption that half the agents in any cohort do not have access to the stock market and must restrict their financial transactions to the bond market. The resulting equilibrium for the case $\alpha = 4$ is shown in Table 5 in the Appendix: compared with the equilibrium with the same parameters in Table 1, the fluctuation of the equity prices is essentially unchanged: however given that it shifts half the middle-aged agents off the equity market and onto the bond market, and reduces the demand for loans by the young who would otherwise borrow to invest in equity—the rate of interest falls, and since equity prices are only

[7]Comparing Table 1 with Table 4, in the Appendix, shows the effect of decreasing the coefficient of variation of the wage income of the young and middle aged from 10% to 5%, and 16.7% to 6.7% respectively, while decreasing the variability of dividends: the demographic effect remains unchanged, but the variability of prices due to the business cycle effect is reduced.

[8]Vissing-Jorgensen(1999) estimates the participation rate in the stock market at around 6% in the early 1950's and around 40% in 1995.

marginally increased, it leads to an increase in the equity premium[9] which however remains lower than the historical premium in the US. Note that the historical average of the equity premium taken over the last 100 or 200 years (see Goetzmann-Jorion (1996) and Siegel (1994)) is a short piece of a trajectory when taken from the perspective of our model. With an average of about 1.7% (resp. 1.1%) and a standard deviation of 2.2% (resp. 1.4%) in pyramid Δ_1 (resp. Δ_2), the equilibrium of Table 5 is able to generate short-run averages of the order of 3% on sections of trajectories which have a succession of positive shocks: this is less than the average of approximately 4.5% reported by Goetzmann-Jorion and Siegel, but not entirely out of bounds.

Comparing calibration with observations. One of the predictions of the model is that there should be a close correlation between the middle-young (MY) cohort ratio and equity prices. To investigate this we present three graphs: first, the ratio of the size of the cohort 40-49 to the size of the cohort 20-29 in the US population[10] (Figure 3), which we take as our proxy[11] for the middle-young (MY) cohort ratio; second, the real Standard and Poors (S&P) index expressed in dollars of 2000 (Figure 4); third, the price earnings ratio for the S&P[12](Figure 5).

Up to the late 1940's there were no significant variations in the MY ratio, and this corresponds roughly with the lack of systematic long-run movement in the S&P index around its trend over this period. To be sure there were 10 year fluctuations in the 30's and 40's, and the 10 year boom of the Roaring 20's, which we think of as shorter run business cycle fluctuations. Starting in the late 40s and continuing all through the 50's and early 60's the ratio of middle-aged to young agents was rising: the middle-aged agents were born at the turn of the century, a period of relatively high birth rates (see Figure 1) and immigration, while the young were the small generation born during the Great Depression. During this same period equity prices were steadily rising. Stock market prices declined in real terms during the 60's and the 70's, during which the MY cohort ratio also declined significantly. (The small Great Depression cohort became the middle-aged, but it was the continual and rapid increase in the denominator, reflecting the Baby Boom surge in births between 1945-1960, which accounted for most of the temporal movement in the MY ratio during this period.) In the early 80's equity prices began their remarkable ascent to their current level,

[9] This is consistent with the findings of Heaton and Lucas (1999) who explore—in an OLG model with two-period lived agents—the idea of using restricted participation as a way of increasing the equity premium: however in our model participation has a bigger impact on the premium.

[10] Derived from Series A 33-35 in *Historical Statistics of the US* and Bureau of the Census data.

[11] Among the many ratios of cohort sizes which can be chosen to represent the ratio of middle-aged to young, this is the one which best corresponds to the moves in bond and equity prices: most major borrowing by the young (mortgages for house purchase) is done between 20-29, and the major part of investment for retirement is set aside between 40-49.

[12] We are grateful to Robert Shiller for making the data set for the Standard and Poors index available.

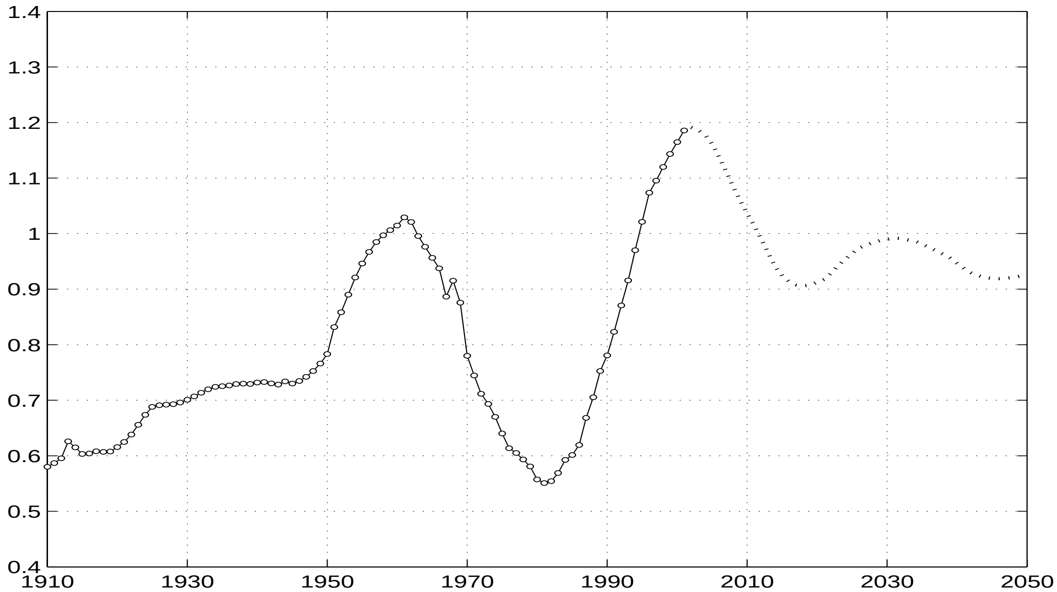


Figure 3: *The middle-young (MY) cohort ratio.*

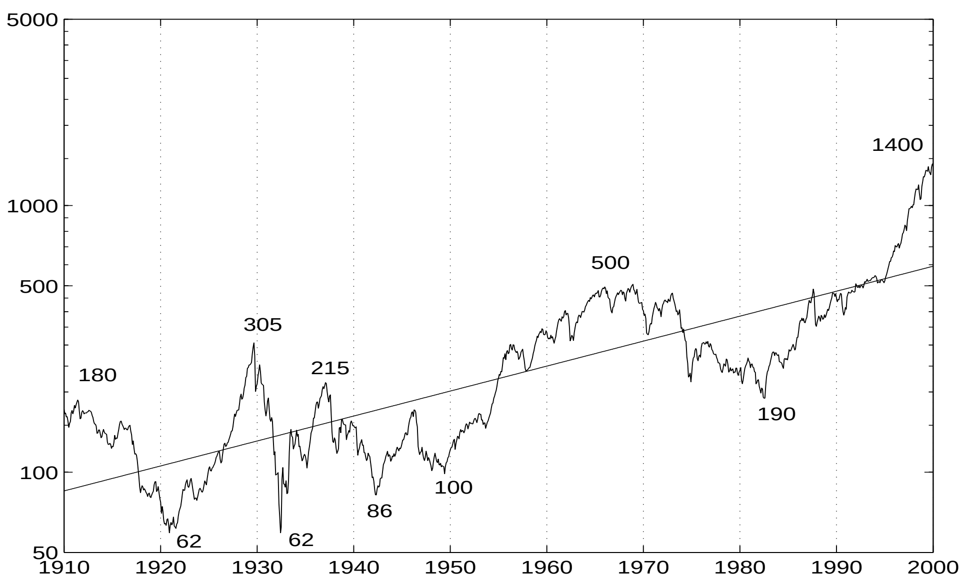


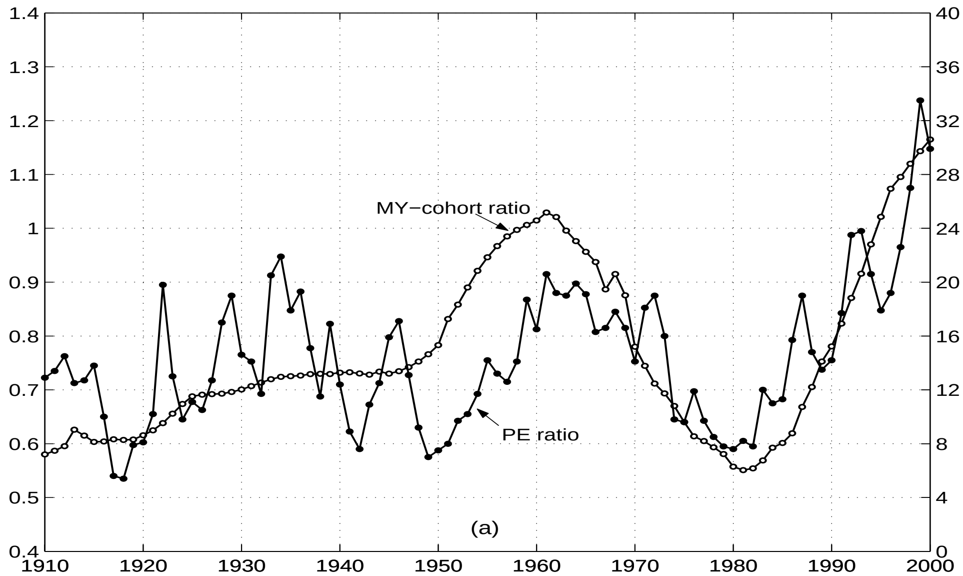
Figure 4: *Real Standard and Poors Index of Common Stock Prices 1910-2000.*

and it was during this period that the plentiful Baby Boomers moved into middle-age, while the small cohort born in the 70's entered their economic life, creating the equally dramatic surge in the MY ratio.

Figure 4 shows the peak and trough values of the real S&P index. Since the 50's and 60's were a period of prosperity, the 70's and early 80's troubled economic times with the successive oil shocks and bursts of inflation, while the late 80's and 90's were a boom period, only interrupted by the Gulf War recession, the numbers can be compared with the peak and trough values of Tables 1 and 2. However the comparison is not exact since the S&P index increases with the overall growth of the economy, while our "detrended" model factors out growth. Between the peak of 1965 and the trough of 1982, the S&P index lost over 60% of its value in real terms—which is in line with the trough to peak numbers found in Tables 1 and 2 . For the "reasonable" cases $\alpha = 2$ and $\alpha = 4$, the ratio of the equity price in the worst state (s_4) of pyramid Δ_1 to the price in the good state (s_1) of pyramid Δ_2 is between 0.5 and 0.15 in Tables 1 and 2—representing a loss of between 50 and 85 percent. During the big rise in equity prices from 1982 to 2000, the real S&P index increased by a factor of 7.4, which is more than predicted by the cases $\alpha = 2$ and $\alpha = 4$ of Table 1 with peak to trough measures between 2.9 and 6.7—however growth is absent from our model.

The price-earnings ratio is a normalized measure of price which has the advantage of factoring out growth and is thus more directly comparable with the results of our model. As Figure 5(a) shows, the PE ratio follows roughly the same pattern as the real S&P index and corresponds well with the long-run fluctuations in the MY cohort ratio. The PE ratio increases from a low of 8 to around 20 in the 60's, decreases in the seventies and early 80's to around 8, after which it increases to around 30 in 2000. These numbers correspond well with the predictions of the model for the case $\alpha = 4$: in Table 2, the PE ratio increases from 6.7 to 18 when going from s_4 in pyramid Δ_1 to s_1 in pyramid Δ_2, and in Table 1, the PE ratio moves between 6.7 and 29.

As can be seen from Figure 5(a), never before during this century has the PE ratio (of the S&P index) attained its current high level of 30 or more. This has given rise to considerable controversy. Are there new forces at work on the stock market which make this phenomenon seem natural, or is it, as Shiller (2000) has argued, a bubble created by the "irrational exuberance" of investors? Some authors have argued that we have entered a New Economy in which conventional concepts of the underlying economy have to be radically altered. (See Hall (2001), McGrattan and Prescott (2001)). Others (like Heaton and Lucas (1999)) have sought an explanation for the high PE ratio in a decrease of the equity premium due to a combination of factors: a decrease in the risk aversion of investors, an increased participation in the stock market, and a decrease in the risk of agents'



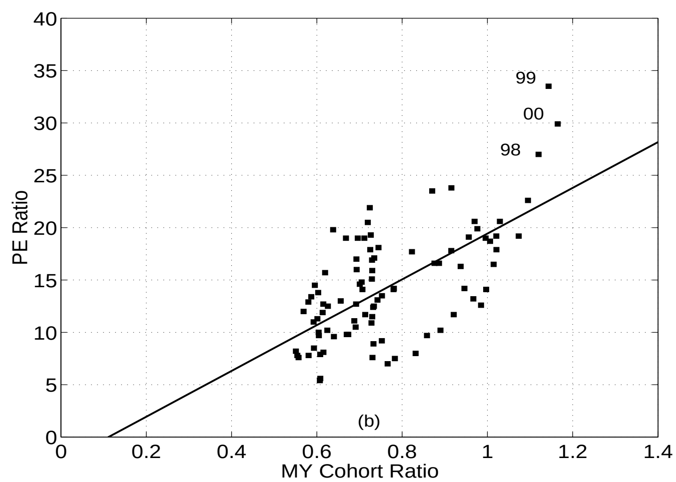


Figure 5: *(a) Real S&P Price-Earnings ratio and MY cohort ratio; (b) Regression of PE ratio on MY cohort ratio.*

portfolios due to easier access to diversification. Our analysis suggests an alternative explanation based on demography and the differing asset demands of agents at different points of their life cycle: given that the ratio of middle-aged agents to young agents is at an all time high, low interest rates and high equity prices are to be expected. Note however that if the historical data shown in Figure 5(b) are used to predict the PE ratio given the cohort ratio, then the most recent points corresponding to 1998, 2000, and especially 1999, seem unusually far from the regression line.

Long-run predictability. In a stylized way the stock market can be viewed as playing two important roles: the first is to provide an instrument for diversifying the production risks of the economy, the second is to provide an instrument for the transfer of ownership of firms from one generation to the next. Much of the finance literature focuses on the first role, and with it comes naturally an emphasis on the short-run unpredictability of the market. The latter role, when coupled with the explicit demographic structure of our model, leads to a quite different perspective on the predictability of the market. For the cohort of children today will enter their economic life as young adults in twenty years, when the young of today will move into their middle age: thus the ratio of middled-aged to young is predictable with a high degree of confidence over the next twenty years or so. Since in our model this ratio determines (modulo constancy of other parameters) an interval within which the PE ratio can be expected to lie, the demographic approach changes the focus from short-run unpredictability to long-run predictability of the stock market. Calculations from the annual population tables of the Bureau of the Census shows that this ratio will decline from its current value to around 0.9 in 2018 (see Figure 3). Using Table 2 , which has a high cohort ratio close to 0.9, with $\alpha = 2$ or $\alpha = 4$, our model predicts a decline in the PE ratio from around 30 to a ratio between 5 and 16 in the next twenty years. This is somewhat lower than the interval [12.5, 22.5] predicted by the historical regression of the PE ratio on the MY cohort ratio[13] or the interval [10, 20] around the historical mean of the PE ratio ($\mu_{PE} = 15$) which is essentially the prediction of Campbell and Shiller (2001) based on mean-reversion of the PE process.

Appendix

Computation of Recursive Equilibrium. For given anticipation functions

$$(Q^e, F) : \Xi \to \mathbb{R}^4_+ \times \mathbb{R}^4_+$$

[13]Using one standard deviation ($\sigma_{PE} = 5$) around the conditional mean.

(i), (ii), and (iii) in the Definition of a stationary equilibrium in Section 4, define a family of two-period equilibria indexed by $\xi = (\gamma, k, s) \in \Xi$. Assuming uniqueness of the equilibria, let

$$\left(z_{(Q^e,F)}(\xi), q_{(Q^e,F)}(\xi), \Gamma_{(Q^e,F)}(\xi), f_{(Q^e,F)}(\xi)\right)$$

denote the equilibrium portfolios, prices, anticipated income transfers by the young, and the actual savings of the middle aged, for each $\xi \in \Xi$. Finding a recursive equilibrium amounts to finding functions (Q^e, F) such that (iv) is satisfied i.e.

$$\text{(E)} \qquad \begin{bmatrix} Q^e_{s'}(\xi) \\ F_{s'}(\xi) \end{bmatrix} = \begin{bmatrix} q^e_{(Q^e,F)}(\Gamma_{(Q^e,F)s'}(\xi), k^+, s') \\ f_{(Q^e,F)}(\Gamma_{(Q^e,F)s'}(\xi), k^+, s') \end{bmatrix} \quad \forall s' \in S,\ \forall \xi = (\gamma, k, s) \in \Xi$$

Assuming that the anticipation functions as well as the equilibrium functions are continuous, an equilibrium is a fixed point on the space of continuous functions $\mathcal{C}(\Xi, \mathbb{R}^8_+)$ of the form $(Q^e, F) = \psi(Q^e, F)$ where $\psi(Q^e, F)$ is defined by the RHS of (E). We look for an approximate equilibrium in the space of piecewise linear functions on $G \times K \times S$, calculating "as if" ψ was a contraction.

We begin by choosing an interval $G = [\underline{\gamma}, \bar{\gamma}]$ and a grid $G_m = \{g_1 \ldots, g_m\}$ on this interval, and then choose arbitrary initial anticipation functions $(Q^{e,0}, F^0)$ on $G_m \times K \times S$. By solving a sequence of two-period equilibrium problems we can then compute the family of associated two-period equilibria $(z^0(\xi), q^0(\xi), \Gamma^0(\xi), f^0(\xi),\ \xi \in G_m \times K \times S)$, possibly modifying the interval G so that $\Gamma^0_s(\xi) \in G$ for all s and all $\xi \in G_m \times K \times S$. Then by recursion we define for $n \geq 1$ the anticipation functions $(Q^{e,n}, F^n)$ by

$$\begin{bmatrix} Q^{e,n}_{s'}(\xi) \\ F^n_{s'}(\xi) \end{bmatrix} = \text{Lin} \begin{bmatrix} q^{e,n-1}(\Gamma^{n-1}_{s'}(\xi), k_+, s') \\ f^{n-1}(\Gamma^{n-1}_{s'}(\xi), k_+, s') \end{bmatrix} \quad \forall s' \in S,\ \forall \xi \in G_m \times K \times S$$

where $(z^{n-1}, q^{n-1}, \Gamma^{n-1}, f^{n-1})$ is the family of two-period equilibria associated with $(Q^{e,n-1}, F^{n-1})$, and Lin denotes the linear interpolation

$$\text{Lin}\, q^{e,n-1}(\Gamma^{n-1}_{s'}(\xi), k_+, s') = \lambda q^{e,n-1}(g_j, k^+, s') + (1-\lambda) q^{e,n-1}(g_{j+1}, k^+, s')$$

if $\Gamma^{n-1}_{s'}(\xi) = \lambda g_j + (1-\lambda) g_{j+1}$. At each step we modify G if necessary so that $\Gamma^n_s(\xi) \in G$ for all s and all $\xi \in G_m \times K \times S$. Although it seems difficult to prove formally that the properties of uniqueness and continuity of the two-period equilibria are satisfied, and that ψ is a contraction, in practice these properties hold and the algorithm converges in less than 100 iterations.

Table 4: Low Income Risk

$N = 79,\ n = 52,\ w^y = (2.1, 2.1, 1.9, 1.9),\ w^m = (3.2, 3.2, 2.8, 2.8),\ D = (80, 60, 80, 60),\ \alpha = 4$

	pyramid 1: MY ratio=0.66				**pyramid 2: MY ratio=1.5**			
	q^e	q^e/E	r^{an}	rp^{an}	q^e	q^e/E	r^{an}	rp^{an}
s_1	63 (.6)	7.9 (.07)	5.4 % (.01)	0.18 % (1.2)	165 (8)	20.5 (1)	-1.7 % (.06)	0.2 % (.8)
s_2	60 (.5)	10 (.08)	5.8 % (.01)	0.15 % (1.2)	141 (6)	23.5 (1)	-0.9 % (.2)	0.2 % (.8)
s_3	44 (.4)	5.5 (.05)	7.4 % (.01)	0.15 % (1.2)	106 (5)	13.3 (.6)	0.4 % (.06)	0.2 % (.8)
s_4	42 (.3)	6.9 (.05)	7.9 % (.01)	0.15 % (1.2)	88 (4)	14.7 (.7)	1.4 % (.05)	0.18 % (.8)
Average	52 (10)	7.5 (1.1)	6.6 % (1.2)	0.17 % (1.2)	125 (36)	17.7 (3.6)	-0.16 % (1.4)	0.19 % (.8)
	ratio of av. prices: 2.4 [0.42], *peak / trough*: 3.9, *trough / peak*: 0.25							

Table 5: 50% Participation Rate in the Equity Market

$N = 79,\ n = 52,\ w^y = (2.2, 2.2, 1.8, 1.8),\ w^m = (3.5, 3.5, 2.5, 2.5),\ D = (85, 55, 85, 55),\ \alpha = 4$

	pyramid 1: MY ratio=0.66				**pyramid 2: MY ratio=1.5**			
	q^e	q^e/E	r^{an}	rp^{an}	q^e	q^e/E	r^{an}	rp^{an}
s_1	94 (1.6)	11.1 (.2)	2.2 % (.1)	1.6 % (2.1)	250 (30)	30 (3.6)	-4 % (.6)	1 % (1.3)
s_2	88 (.8)	16 (.1)	2.7 % (.06)	1.7 % (2.1)	212 (23)	39 (4.2)	-3.3 % (.5)	1.1 % (1.3)
s_3	45 (.4)	5.3 (.05)	6.4 % (.08)	1.7% (2.3)	102 (12)	12 (1.4)	0.3 % (.6)	1.2 % (1.4)
s_4	41 (.5)	7.5 (.09)	7.1 % (.1)	1.9 % (2.4)	79 (8)	14.4 (1.4)	1.5 % (.5)	1.2 % (1.5)
Average	68 (26)	9.6 (3)	4.6 % (2.3)	1.7 % (2.2)	161 (83)	22.5 (9.5)	-1.3% (2.7)	1.1 % (1.4)
	ratio of av. prices: 2.4 [0.42], *peak / trough*: 6.1, *trough / peak*: 0.16							

References

Abel, A.B. (2000), "The Effects of a Baby Boom on Stock Prices and Capital Accumulation in the Presence of Social Security", mimeo, The Wharton School, University of Pennsylvania.

Abel, A.B. (2001), "Will Bequests Attenuate The Predicted Meltdown in Stock Prices When Baby Boomers Retire?" NBER Discussion Paper 8131.

Bakshi, G.S. and Z. Chen (1994), "Baby Boom, Population Aging and Capital Markets", *Journal of Business*, 67, 165-202.

Bergantino, S.M. (1997), "Booms, Busts and Babies: The Effect of Demographics on Housing and Stock Prices", MIT Discussion Paper.

Brooks, R. (2001) "Asset Markets Effects of the Baby Boom and Social Security Reform," forthcoming in *American Economic Review, Papers and Proceedings.*

Campbell, J and R.J. Shiller (2001), "Valuation Ratios and the Long-Run Stock Market Outlook: An Update", NBER Working Paper 8221.

Constantinides, G.M., Donaldson J.B. and R. Mehra (2000) "Junior Can't Borrow: A New Perspective on the Equity Premium Puzzle", forthcoming, *Quarterly Journal of Economics.*

Easterlin, R.A. (1987), *Birth and Fortune*, University of Chicago Press.

Hall, R.E. (2000), "E-Capital: TheLink between the Stock Market and the Labor Market in the 1990's,"*Brookings Papers on Economic Activity*, 73-118.

Heaton, J. and D. Lucas (1999), "Stock Prices and Fundamentals," in B.S. Bernanke and J.J. Rotemberg, eds., *NBER Macroeconomic Annual,* 213-241.

Jorion, P. and W.N. Goetzmann (1999), "Global Stock Markets of the Twentieth Century", *Journal of Finance*, 54, 953-980.

Kondratieff, N.D. (1935), "The Long Waves in Economic Life," *Review of Economic Statistics*,17,105-115; translation of "Die langen Wellen der Konjunktur," *Archiv fű́r Sozialwissenschaften und Sozialpolitik*, 56, 573-609.

Kuznets, S. (1958), "Long Swings in the Growth of Population and in Related Economic Variables," *Proceedings of the American Philosophical Society*, 102, 25-52.

Kuznets, S. (1961), *Capital and the American Economy: its Formation and Financing*, Princeton University Press.

Mankiw, N. and D. Weil (1989), "The Baby Boom, the Baby Bust and the Housing Market," *Regional Science and Urban Economics,* 19, 235-258.

McGrattan, E.R. and E.C. Prescott (2001), "Is the Stock Market Overvalued," NBER Working Paper.

Modligiani, F. (1986), "Life Cycle, Individual Thrift, and the Wealth of Nations," *American Economic Review*, 76, 297-313.

Poterba, J.M. (2000), "Demographic Structure and Asset Returns," mimeo, Massachusetts Institute of Technology.

Shiller, R.J. (2000), *Irrational Exuberance*, Princeton University Press.

Siegel, J. (1994), *Stocks for the Long Term*, New York: R.D. Irwin.

Vissing-Jorgenson, A. (1999),"Comment" (on Heaton and Lucas's paper), in B.S. Bernanke and J.J. Rotemberg, eds., *NBER Macroeconomic Annual,* 242-253.

Welch, F. (1979), "The Effects of Cohort Size on Earnings: the Baby Boom Babies' Financial Bust," *Journal of Political Economy*, 87, 65-97.