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Lyapunov–Krasovskii approach to the robust stability analysis of time-delay systems☆

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Abstract

In this paper, a procedure for construction of quadratic Lyapunov–Krasovskii functionals for linear time-delay systems is proposed. It is shown that these functionals admit a quadratic low bound. The functionals are used to derive robust stability conditions.

Keywords: Time-delay system; Robust stability

1. Introduction

It is well known that there are several difficult aspects in the application of the Lyapunov–Krasovskii approach to the stability analysis of uncertain time-delay systems. One of them is a lack of efficient algorithms for constructing the corresponding Lyapunov–Krasovskii functionals. The common practice consists in exploiting various reduced type functionals, see review papers, Dugard and Verriest (1997) and Kharitonov (1999). However, there is no guarantee that any of such reduced type functionals is really suitable for a particular time delay system under consideration.

An interesting numerical scheme for the construction of full size Lyapunov–Krasovskii functionals has been proposed in Gu (1997). The scheme is based on the LMI technics.

Some general expressions for quadratic functionals for linear systems with one delay have been proposed in Repin (1965), Datko (1971) and Infante and Castelan (1978).

In Juang (1989) some of the results have been extended to the case of general linear time-delay systems. It has been shown there that the functionals admit a local cubic bound. The functionals are quite appropriate for the stability analysis of a given time-delay system, but they become inadequate when one is addressing to the stability analysis of uncertain time-delay systems. This is mainly because of the fact that the functionals are such that their time derivative includes only terms which depend on the present state of the system. The problem is that for the robust stability analysis one needs functionals for which the time derivative includes also terms which depend on past states of the system.

In this paper, we propose a procedure for construction of quadratic functionals, whose time derivative includes terms which depend on the past states of the system and show how one can use such functionals for the robust stability analysis of time-delay systems.

It is also shown in the paper that these functionals admit a quadratic low bound in contrast with the functionals proposed in Juang (1989), for which only a local cubic bound has been derived.

2. Basic facts

In this section, some useful basic results are given. We consider the time-delay system with concentrated delays

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = A_0 x(t) + \sum_{j=1}^{m} A_j x(t-h_j). \tag{1}$$

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Here $A_0, A_1, \ldots, A_m$ are constant $n \times n$ matrices and $0 < h_1 < \cdots < h_m = h$ are positive delays.

Definition 1. System (1) is said to be exponentially stable if there exist $\alpha > 0$ and $\gamma \geqslant 1$ such that for every solution $x(t,\phi)$ of the system with initial function $\phi(\theta)$, $\theta \in [-h, 0]$, the following condition holds:

$$\|x(t,\phi)\| \leqslant \gamma \|\phi\|_h e^{-\alpha t}, \quad \forall t \geqslant 0.$$

Here $\|\phi\|_h = \max_{\theta\in[-h,0]} \|\phi(\theta)\|$. For simplicity we will call an exponentially stable system just stable.

The following equivalent of the Cauchy formula for solutions of system (1) will be used later on.

Theorem 1 (Bellman & Cooke, 1963). *Let $n \times n$ matrix function $K(t)$ satisfy the equation*

$$\frac{d}{dt}K(t) = A_0K(t) + \sum_{j=1}^{m} A_jK(t-h_j), \quad t \geqslant 0$$

with the initial condition: $K(0) = E$, and $K(t) = 0$ for $t < 0$ (E is the identity matrix). Then for $t \geqslant 0$

$$x(t,\phi) = K(t)\phi(0) + \sum_{j=1}^{m} \int_{-h_j}^{0} K(t-h_j-\theta)A_j\phi(\theta)\,d\theta. \tag{2}$$

Corollary 2. *Matrix $K(t)$ satisfies also the equation*

$$\frac{d}{dt}K(t) = K(t)A_0 + \sum_{j=1}^{m} K(t-h_j)A_j, \quad t \geqslant 0. \tag{3}$$

Matrix $K(t)$ is known as the *fundamental matrix* of system (1). It follows from the definition that every column of $K(t)$ is a solution of system (1), so if the system is stable then the matrix also satisfies the inequality

$$\|K(t)\| \leqslant \gamma e^{-\alpha t} \quad \text{for all } t \geqslant 0. \tag{4}$$

Lemma 3. *Let system (1) be stable, then for every $n \times n$ constant matrix W the matrix*

$$U(\tau) = \int_0^{\infty} K^{\top}(t)WK(t+\tau)\,dt \tag{5}$$

is well defined for all $\tau \in R$.

Now, we introduce some useful properties of $U(\tau)$.

Remark 1. If W is a symmetric matrix then $U(-\tau) = U^{\top}(\tau)$, for all $\tau \geqslant 0$.

Proof. By definition

$$U(-\tau) = \int_0^{\infty} K^{\top}(t)WK(t-\tau)\,dt.$$

Taking into account the fact that $K(t-\tau) = 0$, for $t \in [0,\tau)$, we obtain that

$$U(-\tau) = \int_{\tau}^{\infty} K^{\top}(t)WK(t-\tau)\,dt = \left(\int_0^{\infty} K^{\top}(\xi)WK(\xi+\tau)\,d\xi\right)^{\top} = U^{\top}(\tau).$$

In particular, $U(0)$ is a symmetric matrix. □

Remark 2. Matrix $U(\tau)$ is such that

$$-W = U(0)A_0 + A_0^{\top}U(0) + \sum_{j=1}^{m}[U^{\top}(h_j)A_j + A_j^{\top}U(h_j)].$$

Proof. From (3)

$$\frac{d}{dt}(K^{\top}(t)WK(t)) = K^{\top}(t)WK(t)A_0 + A_0^{\top}K^{\top}(t)WK(t) + \sum_{j=1}^{m}[K^{\top}(t)WK(t-h_j)A_j + A_j^{\top}K^{\top}(t-h_j)WK(t)].$$

Integrating both sides of the equality from 0 to ∞ one concludes that

$$-W = U(0)A_0 + A_0^{\top}U(0) + \sum_{k=1}^{m}[U^{\top}(h_k)A_k + A_k^{\top}U(h_k)]. \quad \square$$

Remark 3. Matrix $U(\tau)$ satisfies the equation

$$\frac{d}{d\tau}U(\tau) = U(\tau)A_0 + \sum_{k=1}^{m} U(\tau-h_k)A_k, \quad \tau \in [0,h].$$

Proof. The statement can be proved by direct calculations. □

3. Lyapunov–Krasovskii functionals

Given definite positive $n \times n$ matrices $W_0, W_1, \ldots, W_m$, $R_1, R_2, \ldots, R_m$. Let $\varphi(\theta)$ be a continuous initial function on $[-h, 0]$. Consider the functional

$$w(\phi(\cdot)) = \phi^{\top}(0)W_0\phi(0) + \sum_{k=1}^{m} \phi^{\top}(-h_k)W_k\phi(-h_k) + \sum_{k=1}^{m}\int_{-h_k}^{0} \phi^{\top}(\theta)R_k\phi(\theta)\,d\theta. \tag{6}$$

We are looking for the Lyapunov–Krasovskii functional $v(\phi(\cdot))$ such that

$$\left.\frac{\mathrm{d}v(x(t+\cdot,\varphi))}{\mathrm{d}t}\right|_{(1)} = -w(x(t+\cdot,\varphi)).$$

If system (1) is stable then the functional really exists and can be written as

$$v(\varphi(\cdot)) = \int_0^\infty w(x(t+\cdot,\varphi))\,\mathrm{d}t.$$

Many papers are addressed for the construction of Lyapunov–Krasovskii functionals for (1), but usually only special reduced type functionals have been studied. In this section, it is shown how one may find full size functionals and how these functionals can be used for the stability analysis of uncertain time-delay systems.

Functional $v(\phi(\cdot))$ may be decomposed into $2m+1$ components. Each of them corresponds to one of the $2m+1$ summands on the right-hand side of (6).

We start with the summand

$$w_0(\phi(\cdot),W_0) = \phi^\top(0)W_0\phi(0).$$

The corresponding component of $v(\phi(\cdot))$ can be defined as

$$v_0(\phi(\cdot),W_0) = \int_0^\infty w_0(x(t+\cdot,\phi),W_0)\,\mathrm{d}t = \int_0^\infty x^\top(t,\phi)W_0x(t,\phi)\,\mathrm{d}t.$$

Using expression (2) one can write the component as

$$\begin{aligned} v_0(\phi(\cdot),W_0) &= \phi^\top(0)U_0(0)\phi(0) \\ &+ \sum_{k=1}^m 2\phi^\top(0)\int_{-h_k}^0 U_0(-h_k-\theta)A_k\phi(\theta)\,\mathrm{d}\theta \\ &+ \sum_{k=1}^m\sum_{j=1}^m \int_{-h_k}^0 \phi^\top(\theta_2)A_k^\top \\ &\times\left[\int_{-h_j}^0 U_0(\theta_2-\theta_1+h_k-h_j)A_j\phi(\theta_1)\,\mathrm{d}\theta_1\right]\mathrm{d}\theta_2. \end{aligned}$$

Here matrix $U_0(\tau)$ is defined as

$$U_0(\tau) = \int_0^\infty K^\top(t)W_0K(t+\tau)\,\mathrm{d}t.$$

In order to compute $v_0(\phi(\cdot),W_0)$ one has to know the matrix function $U_0(\tau)$ for $\tau\in[-h,h]$.

We now address the component of $v(\phi(\cdot))$ which corresponds to the integral term

$$w_k(\phi(\cdot),R_k) = \int_{-h_k}^0 \phi^\top(\theta)R_k\phi(\theta)\,\mathrm{d}\theta.$$

The component can be expressed as

$$\begin{aligned} v_k(\phi(\cdot),R_k) &= \int_0^\infty w_k(x(t+\cdot,\phi),R_k)\,\mathrm{d}t \\ &= \int_0^\infty\left(\int_{-h_k}^0 x^\top(t+\theta,\phi)R_kx(t+\theta,\phi)\,\mathrm{d}\theta\right)\mathrm{d}t. \end{aligned}$$

Changing the integration order in the double integral one comes to the following expression

$$\begin{aligned} v_k(\phi(\cdot),R_k) = \int_{-h_k}^0 &\left[\int_\theta^0 \phi^\top(s)R_k\phi(s)\,\mathrm{d}s\right. \\ &\left.+\int_0^\infty x^\top(s,\phi)R_kx(s,\phi)\,\mathrm{d}s\right]\mathrm{d}\theta \end{aligned}$$

and therefore

$$\begin{aligned} v_k(\phi(\cdot),R_k) &= h_kv_0(\phi(\cdot),R_k) \\ &+\int_{-h_k}^0 (h_k+\theta)\phi^\top(\theta)R_k\phi(\theta)\,\mathrm{d}\theta. \end{aligned}$$

The component of $v(\phi(\cdot))$, which corresponds to the delay term

$$w_{m+j}(\phi(\cdot),W_j) = \phi^\top(-h_j)W_j\phi(-h_j)$$

is

$$v_{m+j}(\phi(\cdot),W_j) = \int_0^\infty x^\top(t-h_j,\phi)W_jx(t-h_j,\phi)\,\mathrm{d}t.$$

This equality allows us to rewrite $v_{m+j}(\phi(\cdot),W_j)$ as

$$v_{m+j}(\phi(\cdot),W_j) = v_0(\phi(\cdot),W_j) + \int_{-h_j}^0 \phi^\top(\theta)W_j\phi(\theta)\,\mathrm{d}\theta.$$

Now, gathering all components together, we obtain that

$$\begin{aligned} v(\phi(\cdot)) &= v_0\left(\phi(\cdot),W_0+\sum_{k=1}^m(W_k+h_kR_k)\right) \\ &+\sum_{k=1}^m\int_{-h_k}^0 \phi^\top(\theta)[W_k+(h_k+\theta)R_k]\phi(\theta)\,\mathrm{d}\theta. \end{aligned}$$

The value of the functional for the trajectory segment is

$$\begin{aligned} v(x(t+\cdot)) &= x^\top(t)U(0)x(t) \\ &+\sum_{k=1}^m 2x^\top(t)\int_{-h_k}^0 U(-h_k-\theta)A_kx(t+\theta)\,\mathrm{d}\theta \\ &+\sum_{k=1}^m\sum_{j=1}^m\int_{-h_k}^0 x^\top(t+\theta_2)A_k^\top \end{aligned}$$

$$\times \int_{-h_j}^{0} U(\theta_1-\theta_2+h_k-h_j)$$
$$\times A_j x(t+\theta_1)\,\mathrm{d}\theta_1\,\mathrm{d}\theta_2$$
$$+\sum_{k=1}^{m}\int_{-h_k}^{0} x^{\top}(t+\theta)[(h_k+\theta)R_k+W_k]$$
$$\times x(t+\theta)\,\mathrm{d}\theta, \tag{7}$$

where

$$U(\tau)=\int_0^{\infty} K^{\top}(t)\left[W_0+\sum_{k=1}^{m}(W_k+h_kR_k)\right]K(t+\tau)\,\mathrm{d}t.$$

Theorem 4. *Let system (1) be stable. Given definite positive $n\times n$ matrices W_0, W_k, R_k, $k=1,2,\ldots,m$, the functional (7) satisfies the condition*

$$\left.\frac{\mathrm{d}}{\mathrm{d}t}v(x(t+\cdot,\varphi))\right|_{(1)}=-w(x(t+\cdot,\varphi)).$$

Proof. The statement may be checked by direct calculations. □

Theorem 5. *For some $\varepsilon>0$ functional (7) admits the quadratic low bound*

$$\varepsilon\|\varphi(0)\|^2\leqslant v(\varphi(\cdot)).$$

Proof. Let us define the functional

$$v^{(\varepsilon)}(\varphi(\cdot))=v(\varphi(\cdot))-\varepsilon\|\varphi(0)\|^2.$$

Then

$$\frac{\mathrm{d}}{\mathrm{d}t}v^{(\varepsilon)}(x(t+\cdot,\varphi))=-w^{(\varepsilon)}(x(t+\cdot,\varphi))$$
$$=-w(x(t+\cdot,\varphi))-2\varepsilon x^{\mathrm{T}}(t,\varphi)$$
$$\times\left[A_0x(t,\varphi)+\sum_{k=1}^{m}A_kx(t-h_k,\varphi)\right].$$

It follows from (6) and our assumption that all matrices W_k and R_k are positive definite that, for a sufficiently small $\varepsilon>0$,

$$w^{(\varepsilon)}(x(t+\cdot,\varphi))\geqslant 0 \quad \text{for all } t\geqslant 0.$$

So

$$v^{(\varepsilon)}(\varphi(\cdot))=\int_0^{\infty} w^{(\varepsilon)}(x(t+\cdot,\varphi))\,\mathrm{d}t\geqslant 0$$

and therefore $v(\varphi(\cdot))\geqslant\varepsilon\|\varphi(0)\|^2$. □

4. Robust stability

Assume now that matrices A_0, A_k are perturbed as follows:

$$A_k \rightsquigarrow A_k+\Delta_k, \quad k=0,1,\ldots,m$$

and consider the perturbed system

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t}=(A_0+\Delta_0)y(t)+\sum_{k=1}^{m}(A_k+\Delta_k)y(t-h_k). \tag{8}$$

Let the original system (1) be stable. Now our aim is to derive conditions under which the perturbed system remains stable. To this end we will use functional (7) constructed for the original system.

The derivative of $v(\phi(\cdot))$ along trajectories of system (8) is

$$\left.\frac{\mathrm{d}}{\mathrm{d}t}v(y(t+\cdot))\right|_{(8)}$$
$$=-w(y(t+\cdot))$$
$$+2\left[\Delta_0y(t)+\sum_{k=1}^{m}\Delta_ky(t-h_k)\right]^{\top}$$
$$\times\left[U(0)y(t)+\sum_{j=1}^{m}\int_{-h_j}^{0}U^{\top}(h_j+\theta)A_jy(t+\theta)\,\mathrm{d}\theta\right].$$

Let the only information available about matrices Δ_0 and Δ_k be that they are constant and satisfy the condition

$$\Delta_k^{\top}H_k\Delta_k\leqslant\rho_kE, \quad k=0,1,\ldots,m, \tag{9}$$

where H_k are definite positive matrices and ρ_k are given positive numbers. Then, using the inequality

$$2\mathbf{a}^{\top}\mathbf{b}\leqslant\mathbf{a}^{\top}H\mathbf{a}+\mathbf{b}^{\top}H^{-1}\mathbf{b}$$

which holds for arbitrary definite positive matrix H, one can easily arrive at the following upper bound for the derivative

$$\left.\frac{\mathrm{d}}{\mathrm{d}t}v(y(t+\cdot))\right|_{(8)}$$
$$\leqslant -y^{\top}(t)\left[W_0-\frac{\rho_0}{\mu}\left(1+\sum_{k=1}^{m}h_k\right)E\right.$$
$$\left.-\mu U(0)\left(\sum_{k=0}^{m}H_k^{-1}\right)U(0)\right]y(t)$$
$$-\sum_{j=1}^{m}\sum_{k=0}^{m}\int_{-h_j}^{0}y^{\top}(t+\theta)[R_j-\mu A_j^TU(h_j+\theta)$$
$$\times H_k^{-1}U^{\top}(h_j+\theta)A_j]y(t+\theta)\,\mathrm{d}\theta$$
$$-\sum_{k=1}^{m}y^{\top}(t-h_k)\left[W_k-\frac{\rho_k}{\mu}\left(1+\sum_{j=1}^{m}h_j\right)E\right]$$
$$\times y(t-h_k).$$

Here it is assumed that $H=(1/\mu)H_j$, $j=0,1,\ldots,m$.

The next theorem follows directly from this inequality.

Theorem 6. *Let system (1) be stable. Then system (8) remains stable for all perturbations satisfying (9) if there* *exist definite positive matrices* W_0, W_k, R_k, $k = 1, 2, \ldots, m$, *and a positive value* μ, *such that*

- $W_0 > \mu \sum_{k=0}^{m} U(0) H_k^{-1} U(0) + \rho_0 \mu^{-1}(1 + \sum_{j=1}^{m} h_j)E$,
- $R_k > \mu A_k^T U(h_k + \theta)(\sum_{j=0}^{m} H_j^{-1}) U^\top(h_k + \theta) A_k$, *for all* $\theta \in [-h, 0]$; $k = 1, 2, \ldots, m$,
- $W_k > \rho_k \mu^{-1}(1 + \sum_{j=1}^{m} h_j)E$, $k = 1, 2, \ldots, m$.

Remark 4. If the theorem conditions are satisfied, then matrices Δ_k, $k = 0, 1, \ldots, m$, may be time varying and even may depend on $y(t)$, $y(t - h_j)$, $j = 1, 2, \ldots, m$. The only assumption one does really need is that they are continuous with respect to these arguments and satisfy to (9) for all values of the arguments.

Proof. In fact, for some $\varepsilon > 0$ and $\gamma > 0$ functional $v(\varphi)$ satisfies the inequalities

$$\varepsilon \|\varphi(0)\|^2 \leqslant v(\varphi) \leqslant \gamma \|\varphi\|_h^2. \tag{10}$$

Along the trajectories of the perturbed system (8) the derivative of the functional is

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t} v(y_t) &= -\tilde{w}(y_t) = -w(y_t) \\ &+ 2\left[\Delta_0 y(t) + \sum_{k=1}^{m} \Delta_k y(t - h_k)\right]^\top \\ &\times \left[U(0) y(t) + \sum_{j=1}^{m} \int_{-h_j}^{0} U^\top(h_j + \theta) A_j y(t + \theta)\, \mathrm{d}\theta\right].\end{aligned}$$

If matrices Δ_k, $k = 0, 1, \ldots, m$, depend on t and (or) on $y(t - h_k)$, $k = 0, 1, \ldots, m$, but satisfy inequalities (9), then there exists $\epsilon > 0$ such that

$$\tilde{w}(\varphi) \geqslant \epsilon w(\varphi). \tag{11}$$

Conditions (10) and (11) imply that the zero solution of the perturbed system (8) is globally asymptotically stable. □

Example 1. Consider the system

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix} x(t) + \begin{pmatrix} 0 & 0 \\ -1 & 1 \end{pmatrix} x(t - 1).$$

The system is stable because all zeros of the characteristic quasipolynomial of the system have negative real parts. For $W_0 = W_1 = R_1 = E$ the components of $U(\tau)$, $\tau \in [0, 1]$, are given in Fig. 1.

If we assume that $H_0 = H_1 = E$ then direct calculations show that all conditions of Theorem 6 are fulfilled when

$$\rho_0 < 1.3 \times 10^{-4} \quad \text{and} \quad \rho_1 < 1.5 \times 10^{-4}.$$

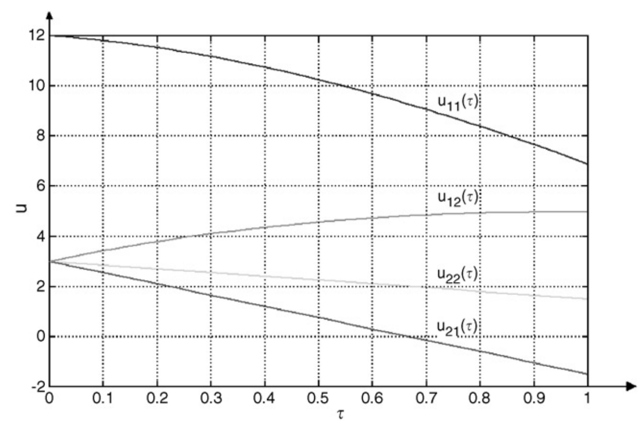

Fig. 1. Matrix U components.

5. Case *m* = 1

Some specific peculiarities of the case of systems with one delay

$$\dot{x}(t) = A_0 x(t) + A_1 x(t - h), \tag{12}$$

will be discussed in the section.

The matrix $U(\tau)$ satisfies now the equation

$$U'(\tau) = U(\tau) A_0 + U(\tau - h) A_1, \quad \text{for } \tau \geqslant 0 \tag{13}$$

and the additional condition

$$W + U(0) A_0 + A_0^\top U(0) + U^\top(h) A_1 + A_1^\top U(h) = 0.$$

In order to define a particular solution of (13) one needs to know the corresponding initial matrix function $\Phi(\theta)$, $\theta \in [-h, 0]$. Such initial matrix function for $U(\tau)$ is not given explicitly, but Remark 1 provides sufficient information for construction of $U(\tau)$. In fact, the remark shows that the required solution of (13) possesses some symmetry property with respect to the point $\tau = 0$. Using this property one can replace the delay term in the right-hand side of (13) as follows:

$$U'(\tau) = U(\tau) A_0 + U^\top(h - \tau) A_1, \quad \text{for } \tau \in [0, h].$$

Differentiating the last equation one arrives at the equation

$$\begin{aligned}U''(\tau) &= [U(\tau) A_0 + U^\top(h - \tau) A_1] A_0 \\ &\quad - [U(\tau) A_0 + U^\top(h - \tau) A_1]^\top A_1.\end{aligned}$$

Substituting here the delay term $U^\top(h - \tau) A_1$ by the corresponding expression from (13), and collecting similar terms, one obtains the second-order ordinary differential matrix equation (see, Juang, 1989)

$$\begin{aligned}U''(\tau) &= U'(\tau) A_0 - A_0^\top U'(\tau) \\ &\quad + A_0^\top U(\tau) A_0 - A_1^\top U(\tau) A_1.\end{aligned} \tag{14}$$

The following theorem shows that the construction of matrix $U(\tau)$ is reduced to the search of a particular solution of Eq. (14).

Theorem 7. *Matrix $U(\tau)$ is a solution of the second-order ordinary differential equation (14) which satisfies the following additional conditions*

- $U'(0) + [U'(0)]^{\mathrm{T}} = -W$,
- $U'(0) = U(0)A_0 + U^{\top}(h)A_1$.

Proof. The proof follows directly from the previous manipulations. □

It is well known that the general solution of (14) depends on two free matrices which define initial conditions $C_0 = U(0)$ and $C_1 = U'(0)$. Matrix C_0 should be symmetric because of Remark 1. Therefore, it contains $n(n+1)/2$ free components, and additionally there are n^2 free components of matrix C_1. In total there exist $N = n^2 + (n(n+1)/2)$ free scalar parameters. For these parameters the theorem defines the same number of scalar linear relations, $n(n+1)/2$ of the relations are given in the first condition, and the second condition provides the remaining n^2 relations. So, in general, there exists only one solution of (14) which satisfies these conditions. This solution is the required matrix $U(\tau)$.

Remark 5. It is interesting to mention the fact that the spectrum set of equation (14) is symmetric with respect to the imaginary axis.

Remark 6. Matrix function $U(\tau)$ which satisfies the theorem conditions may exist even if system (12) is not stable.

6. Conclusions

In this paper, some explicit expressions for the full size Lyapunov–Krasovskii functionals are obtained along with some robust stability results based on the use of the functionals.

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