Theorem prover Meth8 applies four valued Boolean logic for modal interpretation

Point of contact:

Colin James III, Director
Ersatz Systems Machine Cognition, LLC
Colorado Springs, CO 80904-5220 USA

Email: info@cec-services.com
Mobile: +011 (719) 210 – 9534

Websites: ersatz-systems.com; cec-services.com

Purpose, approach, and motivation

- Meth8 is a theorem prover to account for alethic logic as the most conservative extension of classic logic
- The logic system adopted is bivalent multi-valued logic as a quaternary system based on the 2-tuple.
- The current need is to upgrade devices for situational awareness in real time scenarios for command and control.
- The problem is that many theorems implemented in modal logic parts are flawed, and hence decision making is degraded.
- This deficiency is overcome with a variant of the $Ł_4$ system that corrects these mistakes.

Overview

- Theorem prover for modal logic : the *2-tuple*
- Jan Łukasiewicz : $Ł_4$
- Garry Goodwin : $Ł_4$ Variants
- Test results : "A as true does not imply necessarily A."

Overview: modal logic theorem prover

- 2-tuple as 2-sides to map alternative quaternary, 4-valued logics:
  - *left bit is sinister* **false 0**;
  - *right bit is dexter* **true 1**
- Bivalent and decidable:
  - not vector space;
  - not undecidable
- Scalable and expandable
  - 8-bit version has 256 connectives each as its own truth table for *non* quantified expressions.
  - 16-bit version has 65,536 connectives each as its own truth table for quantified expressions.

Overview: Jan Łukasiewicz

- Untenable $Ł_4$:
  - Béziau : ◊A & ◊B → ◊(A & B)

    If possibly Booth killed Lincoln and possibly Booth never killed anyone, then it is possible that Booth both killed Lincoln and never killed anyone.
  - Font, Hájek □A → (◊B → □B)

    Necessarily every coin has two sides implies that if possibly the next flip of the coin lands heads, then necessarily the coin lands heads.
- Tenable $Ł_4$: Łukasiewicz left a door open in two matrices M9 and M13 that prove the same system when kept apart, not combined.
- Prefix or Polish notation [+ 3 4] versus:
  - Infix notation [(3)+(4)]
  - Postfix or Reverse Polish notation (fewer parens) [3 4 +]

Overview: Garry Goodwin

semantic-qube.com.uk

- Multi-valued logic
  - Cartesian graphing in N-dimensions
  - Color mapping in RGB
  - Quantified axiom mappings in the tesseract
- Modal logic in plausible variants of $Ł_4$ as matrix $Ł_{4.M9}$, $Ł_{4.M13}$
  - 3 variants with 3 options
  - With and without *R* reference for worlds and frames
  - Multi-valued Boolean logic leads to incompleteness
  - "Some arguments which are never false fail to be theorems."
- Postfix notation: no ambiguity of ◊□A in i order as $A◊^{ii}□^{i}$

Acronym

- Meth8: **Me**chanical **Th**eorem in **8**-bits
  - Based on the 256-connectives of **8**-bits:

    Connective number below is 0011 0011 [conditional] 0101 0101.

    ```
    & AND:  17 0001 0001: 00 00 00 00  00 01 00 01  00 00 10 10  00 01 10 11
    > IMP: 221 1101 1101: 11 11 11 11  10 11 10 11  01 01 11 11  00 01 10 11
    ```
  - Distinct from 65536-connectives of 16-bits:

    For proofs with existential ∃ and universal ∀ quantifiers

    ```
    & AND: 4369: 00010001 00010001: 00 00 00 00  00 00 00 00..11 00 11 01  11 10 11 11
    > IMP:56797: 11011101 11011101: 11 11 11 11  11 11 11 11..11 00 11 01  11 10 11 11
    ```
- *Demo* Meth8 is free with literals scaled down.
  - Two literals for propositions: p, q
  - Two literals for theorems: A, B

Meth8 logical expressions

- Antecedent and Consequent
  - Literal type:

    13 theorems: A, B, C, D, E, F, G, H, I, J, K, L, M

    13 propositions: n, o, p, q, r, s, t, u, v, w, x, y, z
  - Modal operators: # necessarily □ ; % possibly ◊ ; ~ not ¬

    3 affirmed: [none], #, %

    3 dis-affirmed: ~ , ~#, ~%
- Conditional: AND & ; OR + ; IMP > ; EQV =
  - 4 affirmed: &, +, >, =
  - 4 dis-affirmed: ~&, ~+, ~>, ~=

Example: Meth8 data structure

```
LET antecedent  = 13          ! A, … , M; or n , … , z
LET consequent = 13           ! A, … , M; or n , … , z
LET modifier    =  6          ! [none], ~, # , % , ~#, ~%
LET conditional  =  8         ! &, +, >, =, ~&, ~+, ~>, ~=
LET model       =  3          ! based on 3 variants
LET rows         =  4         ! rows per truth table
DIM expr$( 0, 0, 0, 0, 0, 0, 0)  !  2-bits * 4 values = 1 byte
MAT REDIM expr$( 6, 13, 8, 6, 13, 3, 4)      ! 7-dimensions
```

Example: Meth8 memory footprint

- Literal expressions: 13 theorems; 13 propositions
- Combination of antecedent, consequent, conditional

Antecedent (6 modifiers * 13 literals)

* Consequent (6 modifiers * 13 literals)

* Conditional (8 connectives) = 48,672 expressions

- Combination of literal segments and number of modes

* 3 models = 146,016 combinations

- Memory size (8-bits/row * 4 rows/table = 32-bits = 4-bytes)

* 4-bytes per expression = 584, 064 bytes ≈ 571 KB (< 1 MB part)

Meth8 paren parsing : Sliding Window

1. Map L/R parens
2. Slide window L
3. Tag first adjacent L/R pair [*04*, *09*]; then next [12,17]
4. Write stack list [*04*, *09*], [12, 17]
5. Match remaining parens [**02**, **18**]

			01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	
1.			□	(	□	(	◊	A	→	B	)	↔	□	(	A	→	□	B	)	)	S5:58.
				L		L					R			L					R	R	
				02		04					09			12					17	18	
2.	<<	01	02		04					09			12					17	18		<<
			L		L					R			L					R	R		
3.				**L**		*L*					R			L					R	R	
				02		*04*					09			12					17	18	
	<<	01	02		04					*09*			12					17	**18**		<<
			L		L					*R*			L					R	**R**		
4.	L			**02**		*04*								12							
	R					*09*								17					**18**		
5.	L			**02**		*04*								12							
	R			**18**		*09*								17							

Meth*N* scalability

- Meth8 [2^3] uses 8-bit connectives (256) in truth tables of 32-bits (of 8-bits / row).
- Meth16 [2^4] uses 16-bit connectives (65,536) in truth tables of 1024-bits (of 64-bits / row).

But do we need a logic with more than 65,536 connectives?

Maybe to map the human brain using the *Method and system for Kanban cell neuron network,* U.S. Patent (to issue 12/2015).

- Meth*N* [2^(log*N*/log2)] uses *N*-bit connectives (2^*N*) in truth tables of (2^*N)/N*-bits (of (2^*N)/N/4*-bits / row).

What Meth*N* is NOT

- Lattice logic of informational states as{ T, F, TF, 0}
  - “The cat is on the mat.” {TF}
  - “Tabby the cat is on the mat outside.” {T}
  - “Tabby and Kitty are on the mat inside.” {F}
  - “Kitty is the name of a stuffed animal.” {0}
- Paraconsistent logic (“beside the consistent”)
  - Bit-*in*consistent truth tables
  - *Non*-classical connectives or rules of inference
- Quantum logic (no law of distribution)
- Probabilistic logic (Bayesian analysis)

Variants of $Ł_4$: the Option Approach

- The 2-tuple follows $Ł_4$: { 00, 10, 01, 11 }, 11 as designated.
  - For { Contradiction, False, True, Tautology } : Boolean B_4
- Variant 1: { F, C, N, T } with designated value as T
  - For { False, Contingent, Non-Contingent, True }
- Variant 2: { U, I, P, E } with designated value as E
  - For { Un-Evaluated, Improper, Proper, Evaluated }
- Option 1, 2, 3:
  - Strategies disambiguate the middle rows of truth tables where two unary propositions fall within the scope of the same modal operator.

Variants 1, 2a, 2b

B_4	V1 ~	**V1**	V1 []	V1 <>	V1 ~[]	V1 ~<>	V2a ~	**V2a**	V2a []	V2a <>	V2a ~[]	V2a ~<>	V2b ~	**V2b**	V2b []	V2b <>	V2b ~[]	V2b ~<>	**B_4**
00	*T*	**F**	F	C	T	N	*E*	**U**	U	I	E	P	*E*	**U**	U	P	E	I	**00**
01	*C*	**N**	N	T	C	F	*I*	**P**	P	E	I	U	*P*	**I**	I	E	P	U	**01**
10	*N*	**C**	F	C	T	N	*P*	**I**	U	I	E	P	*I*	**P**	U	P	E	I	**10**
11	*F*	**T**	N	T	C	F	*U*	**E**	P	E	I	U	*U*	**E**	I	E	P	U	**11**

The interpretation of truth tables is non standard: truth possibilities are (T, N, C) and (E, P, I); but proof is (T) and (E).

Truth possibilities are bivalent in V2a as 'is the case' or in V2b as 'is not the case'. Use of 'the case' disambiguates truth possibility from truth evaluations. Truth values are truth evaluations.

Middle rows in V2b 'not the case' contain a mix of the same values, making it unclear which matrix (V2a or V2b) to apply to the same modal operator. Hence these rows are not applicable.

Variant 2 modal operations for mixed truth combinations

	□	◊
Option 1	x **E**	+ **U**
Option 2	x **U**	+ **E**
Option 3	x **P**, x **I**	+ **P**, + **I**

Options 1, 2, and 3 cover permutations of modal operators.

Option 1 is for “won't decide”, leaving the rows of mixed truth combinations unchanged.

Option 2 is for “can't decide”, applying to Variants 2a and 2b.

Option 3 applies Variants 2a and 2b separately and records the different results.

The user may invoke any or all options.

Meta-variables {A, B} of Variants 1 and 2

	Variant 1		Variant 2		
	◊A	□A	◊A	□A	◊AA
Top Row	A+M1	A*M1	A+M2a	A*M2a	AA+M2a
Middle Rows	A+M1	A*M1	n/a	n/a	AA
Bottom Row	A+M1	A*M1	A+M2b	A*M2b	AA+M2b

Variant 1 shows every truth function for an expression in that form.

Variant 2 shows unary propositions that do not mix truth combinations.

The doubled meta-variable {AA, BB} shows complex expressions of two or more propositions that mix truth possibilities.

For example, ◊AA may be ◊(p & q) or ◊(p & □q).

When testing an expression we apply matrices by this table.

Two examples disprove the implausible theorems of $Ł_4$

$\Diamond A \,\&\, \Diamond B \rightarrow \Diamond(A \,\&\, B)$

If possibly Booth killed Lincoln and possibly Booth never killed anyone, then it is possible that Booth both killed Lincoln and never killed anyone.

$\Box A \rightarrow (\Diamond B \rightarrow \Box B)$

Necessarily every coin has two sides implies that if possibly the next flip of the coin lands heads, then necessarily the coin lands heads.

Meta-data of all permutations in six models for (◊A & ◊B) → ◊(A & B)

	(◊A	&	◊B)	→	◊	(A	&	B)
1.0	A_1+M1	(A_1+M1)*(B_1+M1)	B_1+M1		(A_1*B_1) +M1	A_1	A_1*B_1	B
2.0	A_2+M2a	(A_2+M2a)*(B_2+M2a)	B_2+M2a		A_2*B_2	A_2	A_2*B_2	B_2
2.1	A_2+M2a	(A_2+M2a)*(B_2+M2b)	B_2+M2b		A_2*B_2	A_2	A_2*B_2	B_2
2.2	A_2+M2a	(A_2+M2a)*BB_2	BB_2		A_2*BB_2	A_2	A_2*BB_2	BB_2
2.3	AA_2	AA_2*(B_2+M2a)	B_2+M2a		AA_2*B_2	AA_2	AA_2*B_2	B_2
2.4	AA_2	AA_2*BB_2	BB_2		AA_2*BB_2	AA_2	AA_2*BB_2	BB_2

Removing one model based solely on V2b which represents a model based solely on V2a reduces redundancy.

Models that also mix V2a and V2b are removed.

There are five V2 models and one V1 model for six models.

This becomes its own proof method, where each row is treated as its own model.

Non redundant models for (◊A&◊B)→◊(A&B)

This accounts for all truth tables in unique models.

Underlined consequents are un-evaluated results where the antecedent has an evaluation.

This means some models have the antecedent as un-evaluated where the consequent is true.

This disproves the Ł_4 theorem.

	(◊A	&	◊B)	→	◊	(A	&	B)
1.0	CCTT	CCTT	TTTT	TTTT	CCTT	FCNT	FCNT	TTTT
	CCTT	CCTT	TTTT	TTTT	CCTT	FCNT	FFNN	NNNN
	CCTT	CCCC	CCCC	TTTT	CCCC	FCNT	FCFC	CCCC
	CCTT	CCCC	CCCC	TTTT	CCCC	FCNT	FFFF	FFFF
2.0	IIEE	IIEE	EEEE	PEPE	UIPE	UIPE	UIPE	EEEE
	IIEE	IIEE	EEEE	PPPP	UUPP	UIPE	UUPP	PPPP
	IIEE	IIII	IIII	PEPE	UIUI	UIPE	UIUI	IIII
	IIEE	IIII	IIII	PPPP	UUUU	UIPE	UUUU	UUUU
2.1	IIEE	IIEE	EEEE	PEPE	UIPE	UIPE	UIPE	EEEE
	IIEE	UUPP	PPPP	EEEE	UUPP	UIPE	UUPP	PPPP
	IIEE	IIEE	EEEE	PEPE	UIUI	UIPE	UIUI	IIII
	IIEE	UUPP	PPPP	EEPP	UUUU	UIPE	UUUU	UUUU
2.2	IIEE	IIEE	EEEE	PEPE	UIPE	UIPE	UIPE	EEEE
	IIEE	UUPP	PPPP	EEEE	UUPP	UIPE	UUPP	PPPP
	IIEE	IIII	IIII	PEPE	UIUI	UIPE	UIUI	IIII
	IIEE	UUUU	UUUU	EEEE	UUUU	UIPE	UUUU	UUUU
2.3	UIPE	UIPE	EEEE	EEEE	UIPE	UIPE	UIPE	EEEE
	UIPE	UIPE	EEEE	EPEP	UUPP	UIPE	UUPP	PPPP
	UIPE	UIUI	IIII	EEEE	UIUI	UIPE	UIUI	IIII
	UIPE	UIUI	IIII	EPEP	UUUU	UIPE	UUUU	UUUU
2.4	UIPE	UIPE	EEEE	EEEE	UIPE	UIPE	UIPE	EEEE
	UIPE	UUPP	PPPP	EEEE	UUPP	UIPE	UUPP	PPPP
	UIPE	UIUI	IIII	EEEE	UIUI	UIPE	UIUI	IIII
	UIPE	UUUU	UUUU	EEEE	UUUU	UIPE	UUUU	UUUU

Meta-data of all permutations in five models for $\Box A \rightarrow \Diamond(B \rightarrow \Box B)$

	$(\Box A$	$\rightarrow$	$(\Diamond B$	$\rightarrow$	$\Box B)$
1.0	A_1*M1		B_1+M1		B_1*M1
2.0	A_2*M2a		B_2+M2b		B_2*M2b
2.1	A_2*M2a		BB_2		BB_2
2.2	AA_2		B_2+M2b		B_2+M2b
2.3	AA_2		BB_2		BB_2

There are five non redundant models of the generalized theorem.

Non redundant models for $\Box A \rightarrow \Diamond(B \rightarrow \Box B)$

This shows the $Ł_4$ theorem is not a theorem in this variant.

It is also worth noting that

$$\Box A \rightarrow (B \rightarrow \Box B)$$

is also not a theorem.

	(□A	→	(◊B	→	□B)
1.0	FFNN FFNN FFNN FFNN	TTTT TTTT TTTT TTTT	TTTT TTTT CCCC CCCC	NNNN NNNN NNNN NNNN	NNNN NNNN FFFF FFFF
2.0	UUPP UUPP UUPP UUPP	EEII EEII EEII EEII	EEEE PPPP EEEE PPPP	IIII IIII IIII IIII	IIII UUUU IIII UUUU
2.1	UUPP UUPP UUPP UUPP	EEEE EEEE EEEE EEEE	EEEE PPPP IIII UUUU	EEEE EEEE EEEE EEEE	EEEE PPPP IIII UUUU
2.2	UIPE UIPE UIPE UIPE	EEII EEII EEII EEII	EEEE PPPP EEEE PPPP	IIII IIII IIII IIII	IIII UUUU IIII UUUU
2.3	UIPE UIPE UIPE UIPE	EEEE EEEE EEEE EEEE	EEEE PPPP IIII UUUU	EEEE EEEE EEEE EEEE	EEEE PPPP IIII UUUU

Some general test results

- Not general theorems
  - $S5 \vdash \Box A \rightarrow \Box A$
- Not validated, but the consequent is never false
  - $\Diamond(A \& B) \rightarrow \Diamond A \& \Diamond B$
  - $\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$
  - $\Box A \rightarrow (\Diamond B \rightarrow \Box B)$
  - $\Box A \rightarrow (B \rightarrow \Box B)$
- Limited validity
  - Axiom K
  - Axiom B: $A \rightarrow \Box\Diamond A$
  - Axiom 5: $\Diamond A \rightarrow \Box\Diamond A$

Not general theorems, and with semantic consequence

- K axiom $(A \to B) \to (\Box A \to \Box B)$

  08 $\neg\Diamond(A \,\&\, B) \leftrightarrow \Box(A \to \neg B)$

  15 $\Box(A \vee B) \to (\Box A \vee \Diamond B)$

- D axiom

  33 $(\Diamond\neg A \vee \Diamond\neg B) \vee \Diamond(A \,\&\, B)$

- B axiom

  40 $\neg\Diamond(\Diamond\Box\Diamond A \,\&\, \Box\neg A)$

- S5 axiom

  57 $\Box(\Box A \to \Box B) \vee \Box(\Box B \to \Box A)$

Not general theorems, but with*out* semantic consequence

- K axiom

  19 $(\Box A \,\&\, \Diamond B) \rightarrow \Diamond(A \,\&\, B)$

  21 $(\Box A \rightarrow \Diamond B) \rightarrow \Diamond(A \rightarrow B)$

- S4 axiom

  42 $(\Diamond A \,\&\, \Box B) \rightarrow \Diamond(A \,\&\, \Box B)$

- S5 axiom

  54 $(\Box A \vee \Diamond B) \rightarrow \Box(A \vee \Diamond B)$

  55 $(\Diamond A \,\&\, \Diamond B) \rightarrow \Diamond(A \,\&\, \Diamond B)$

  56 $(\Diamond A \,\&\, \Box B) \rightarrow \Diamond(A \,\&\, \Box B)$, also $(A \,\&\, \Box B) \rightarrow (\Diamond A \,\&\, \Box B)$

Point of contact:

Colin James III, Director
Ersatz Systems Machine Cognition, LLC
Colorado Springs, CO 80904-5220 USA

Email: info@cec-services.com
Mobile: +011 (719) 210 – 9534

Websites: ersatz-systems.com; cec-services.com