Consequences of pacing the Pleistocene 100 kyr ice ages by nonlinear phase locking to Milankovitch forcing

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[1] The consequences of the hypothesis that Milankovitch forcing affects the phase (e.g., termination times) of the 100 kyr glacial cycles via a mechanism known as "nonlinear phase locking" are examined. Phase locking provides a mechanism by which Milankovitch forcing can act as the "pacemaker" of the glacial cycles. Nonlinear phase locking can determine the timing of the major deglaciations, nearly independently of the specific mechanism or model that is responsible for these cycles as long as this mechanism is suitably nonlinear. A consequence of this is that the fit of a certain model output to the observed ice volume record cannot be used as an indication that the glacial mechanism in this model is necessarily correct. Phase locking to obliquity and possibly precession variations is distinct from mechanisms relying on a linear or nonlinear amplification of the eccentricity forcing. Nonlinear phase locking may determine the phase of the glacial cycles even in the presence of noise in the climate system and can be effective at setting glacial termination times even when the precession and obliquity bands account only for a small portion of the total power of an ice volume record. Nonlinear phase locking can also result in the observed "quantization" of the glacial period into multiples of the obliquity or precession periods.

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1. Introduction

[2] *Hays et al.* [1976] established that Milankovitch forcing (i.e., variations in orbital parameters and their effect on the insolation at the top of the atmosphere) plays a role in glacial cycle dynamics. However, precisely what that role is, and what is meant by "Milankovitch theories" remains unclear despite decades of work on the subject [e.g., *Wunsch*, 2004; *Rial and Anaclerio*, 2000]. Current views vary from the inference that Milankovitch variations in insolation drives the glacial cycle (i.e., the cycles would not exist without Milankovitch variations), to the Milankovitch forcing causing only weak climate perturbations superimposed on the glacial cycles. A further possibility is that the primary influence of the Milankovitch forcing is to set the frequency and phase of the cycles (e.g., controlling the timing of glacial terminations or of glacial inceptions). In the latter case, glacial cycles would exist even in the absence of the insolation changes, but with different timing.

[3] Here we consider the possibility that Milankovitch forcing could affect the phase of the glacial cycles via a mechanism known as "nonlinear phase locking," following the pioneering work on the role of phase locking to Milankovitch forcing by Saltzman and collaborators [e.g., *Saltzman et al.*, 1984] and as more specifically proposed by *Gildor and Tziperman* [2000]. We describe the nonlinear phase locking mechanism and show how it could permit Milankovitch forcing to "pace" the glacial cycles. These ideas are demonstrated using a simple model of the glacial cycles.

[4] Our main message, however, is that the timing of the major deglaciations can be set by Milankovitch forcing nearly independent of the specific mechanism (or model) that is responsible for these cycles. Thus distinguishing between different models by their fit to the ice volume proxy record is therefore difficult.

[5] "Nonlinear phase locking" may be briefly summarized with the help of Figure 1b which shows time series of land ice volume from two model runs starting from two different initial conditions (thin color lines). In the presence of Milankovitch forcing (affecting ablation in the model), the glacial cycle phase (e.g., termination times) is independent of the initial conditions of the climate system after some initial adjustment period.

[6] The purpose of this paper is to examine the case for nonlinear phase locking setting the phase of the glacial cycles and to study some of the consequences of this hypothesis. A condition for the phase locking mechanism to work is that the 100 kyr cycles are the result of some nonlinear process of very general type. We will show that the phase locking mechanism can explain why very differ-

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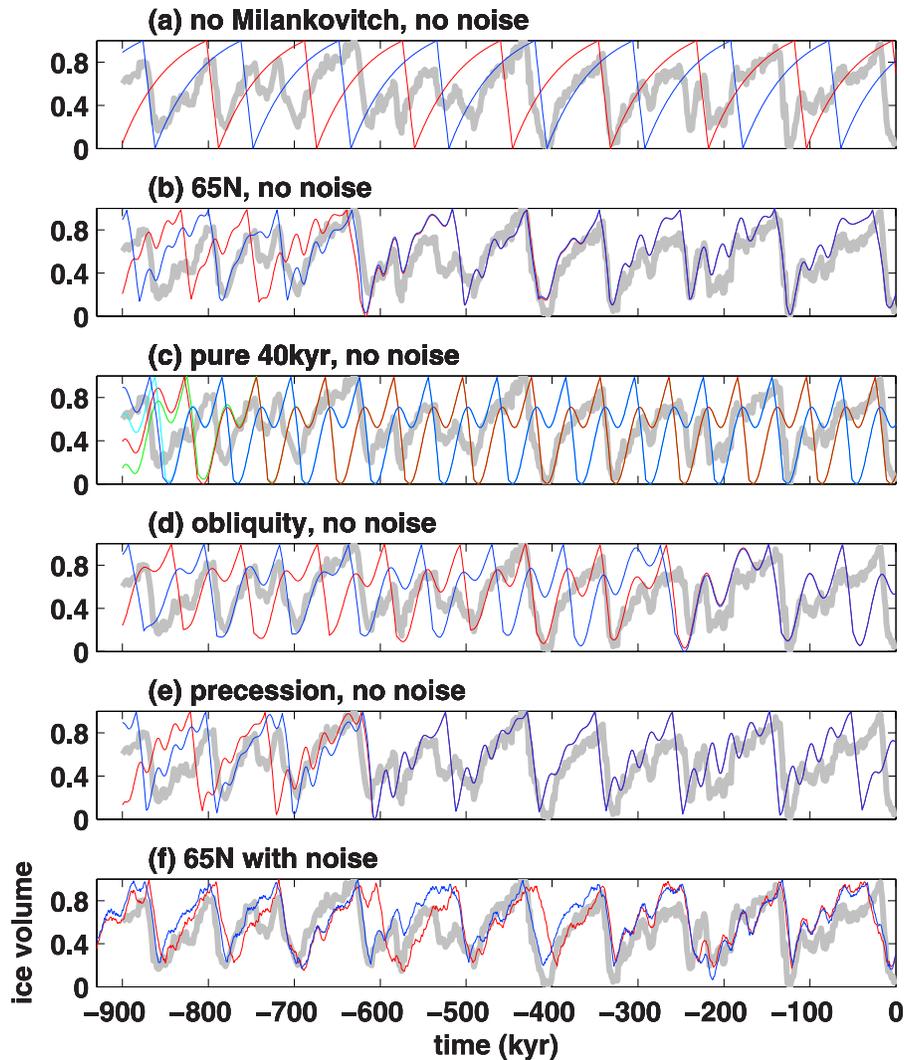
Figure 1. Phase locking to Milankovitch forcing. The gray curve is the proxy $\delta^{18}O$ compilation of *Huybers and Wunsch* [2004]; the thin color curves are ice volume time series from different model runs using different initial conditions. The model description is given in Appendix A. (a) A model run with no Milankovitch forcing. (b) Model forced by and phase locked to Milankovitch forcing (65N summer insolation). (c) Model run forced by and phase locked to a periodic 40 kyr forcing (i.e., not the obliquity time series but a sine wave with a 40 kyr period). (d) Model forced by obliquity variations only. (e) Model forced by precession only. (f) Same as Figure 1b but in the presence of noise.

ent physical mechanisms and models [e.g., *Pollard*, 1982; *Berger and Loutre*, 1996; *Saltzman et al.*, 1984; *Hyde and Peltier*, 1987; *Maasch and Saltzman*, 1990; *Saltzman and Verbitsky*, 1994; *Paillard*, 1998; *Gildor and Tziperman*, 2000; *Ashkenazy and Tziperman*, 2004; *Huybers and Wunsch*, 2005] are all able to produce a good fit to the observed ice volume record. A consequence of the efficiency of nonlinear phase locking in many different models is that it is too simple to obtain a good fit to SPECMAP-like records [*Imbrie et al.*, 1984] by including Milankovitch forcing in a given model. We therefore emphasize that obtaining such a fit by a given model (including the highly idealized and unrealistic model used here) is not necessarily

an indication that the glacial mechanism represented by this model is correct.

[7] *Wunsch* [2004] showed that the spectral power within the precession and obliquity Milankovitch bands is only a small fraction of the total power in the observed proxy time series. Does this mean that Milankovitch forcing does not play a significant role in glacial dynamics? We try to address this question by showing that in many models, even when the Milankovitch forcing completely determines the model termination times of the ice ages via nonlinear phase locking, the Milankovitch period bands of 19, 23 and 41 kyr are still responsible for only a small fraction of the spectral power, as observed.

[8] The climate system is characterized by processes that operate on multiple timescales. The shorter-timescale variability, often represented and thought of as a stochastic (noise) process, almost certainly affects the glacial cycle dynamics, as well as the proxy records that are used to study glacial cycles. *Wunsch* [2004] has shown that glacial cycles may be consistently fit by a "linear" random process (defined in section 3.2, equation (3)), except for short segments associated with the glacial terminations. This raises the important question of whether one may rule out a deterministic "nonlinear" mechanism for the glacial cycles. We demonstrate that the very fact that the observed record may be fit with a linear stochastic process does not necessarily imply that the underlying mechanism is linear or stochastic. We also show that the phase locking mechanism can set the phase of the ice ages even in the presence of noise.

[9] Many investigators have looked for a consistent and uniform phase relation between Milankovitch forcing and termination times in proxy climate records [e.g., *Imbrie et al.*, 1992; *Raymo*, 1997]. Major obstacles are the time-scale uncertainties and the potentially nonlinear relationship between Milankovitch forcing and the glacial response. Recently, *Huybers and Wunsch* [2005] found a consistent phase relation between terminations and obliquity, but age uncertainties prevented them from determining whether precession also helps phase lock the glacial cycles.

[10] Should one expect an exact relationship between Milankovitch forcing and each termination [*Huybers and Wunsch*, 2005] if termination times are set by nonlinear phase locking? We show below that in one simple model used here, terminations may initiate at minimum high-latitude summer insolation forcing at one time, and at nonminimum insolation phase at other times. This leaves open the possibility that nonlinear phase locking to Milankovitch forcing is responsible for the pacing of the glacial cycles even if a variable phase relation between them is observed.

[11] A related issue concerns determining which of the Milankovitch frequencies acts as the pacemaker: For example, is it obliquity or precession that is more critical? We show that the fact that the Milankovitch forcing is not a pure sine wave with a single frequency of 41 kyr or 23 kyr, and the strong amplitude modulation of the orbital forcing may determine the particular observed ice age termination times. Thus a unique phase locking probably requires the broadband nature of the Milankovitch forcing. The nonperiodic nature of the obliquity and precession parameters may be critical, although with the available data we cannot determine if either of these two orbital parameters dominates the phase locking.

[12] In the following sections we explain the phase locking mechanism and its implications (section 2), consider the effects of noise on the dynamics of the phase-locked glacial cycles (section 3, which is more technical and may perhaps be skipped on first reading), interpret the results of other studies from the perspective of phase locking (section 4), and conclude in section 5.

The highly simplified model used here is described in the appendix.

2. Phase Locking: A Candidate for the Glacial Cycle Pacing Mechanism

2.1. Mechanism of Phase Locking, Linear Versus Nonlinear Oscillations

[13] The term "phase locking" has been used in many different and perhaps even ambiguous ways in past works on glacial cycles, and it is worthwhile explaining what precisely is meant here by this term. The phenomenon of nonlinear phase locking was discovered by the Dutch physicist Huygens [*Huygens*, 1669] in the 17th century. He noted that two pendulum clocks that are suspended from the same wooden frame (such that minute vibrations can pass from one to the other) became synchronous, although the time shown by the two drifted apart when the clocks were separated. In the context of glacial cycles and Milankovitch forcing, one could think of the Milankovitch forcing (specifically, the precession and obliquity variations and their effects on the insolation) as being one clock. The second clock would be the glacial cycles, and the interaction between the two is one-sided with Milankovitch forcing being an "external clock" affecting the glacial cycles but not being affected by them [*Gildor and Tziperman*, 2000] (although see *Bills* [1994]). Moreover, because the frequencies of precession and obliquity are significantly different from that of the 100 kyr ice volume cycles, the phase locking cannot take the form of synchronous oscillations, but a somewhat different form as we explain shortly.

[14] Figure 1 demonstrates how phase locking to Milankovitch forcing affects glacial cycles in an idealized model. Figure 1a shows glacial oscillations in this model in the absence of any Milankovitch forcing. The different time series shown by the thin color lines correspond to model runs using different initial conditions for the ice volume, while the thicker gray curve is the $\delta^{18}O$ proxy data compilation of *Huybers and Wunsch* [2004] (similar results are obtained when comparing to the compilation of *Lisiecki and Raymo* [2005] or to SPECMAP). Figure 1b shows two ice volume time series from the same model, with the only difference being that Milankovitch forcing is now included in the model dynamics (in the representation of the ablation; see Appendix A). Note that in this case the initially different model solutions all converge to a single time series within a few hundred thousands years (see merging of thin colored lines in Figure 1b). This results from the model glacial cycles becoming "phase locked" to the external Milankovitch "clock." Of interest, of course, is that this phase locked model time series also roughly fits the observed proxy ice volume record (thick gray line) quite well for such a simple model (although note the misfit at the last termination).

[15] Phase locking is abundant in nature, resulting, for example, in the synchronization of planetary motion. For example, the moon spins exactly once around itself for every rotation around the Earth, which is a 1:1 phase locking, also termed 1:1 "nonlinear resonance" (which is synonymous with "nonlinear phase locking"); the planet Mercury turns three times for every two orbits around the Sun, which is a

2:3 nonlinear resonance. In general, if two oscillators are nonlinearly phase locked, their frequencies ω_1, ω_2 must be related as the ratio of some two integers p and q,

$$\omega_1/\omega_2 = p/q. \qquad (1)$$

Similarly, the oscillation and forcing periods are related as the inverse of equation (1). In the case of a linear resonance, the forcing frequency is equal to that of the oscillator, $\omega = \omega_f$, similarly to the case of a nonlinear resonance with $p/q = 1/1$, although the two cases are still distinct. Certain fireflies manage to flash exactly synchronously via a nonlinear phase locking; and there are numerous other examples varying from the atomic scale to planetary motion, to biological examples, to engineering examples [*Strogatz*, 2003]. A more technical description is given by *Strogatz* [1994], *Schuster* [1989] and *Pikovsky et al.* [2001]; *Tziperman et al.* [1995] discuss this issue in the context of ENSO and *Kaspi et al.* [2004] discuss the issue in the context of Heinrich events. Unfortunately, the term "phase locking" appears in the literature with many different meanings, not necessarily equivalent to the above. It is important to understand that when we use the term nonlinear phase locking we refer to a very specific mathematical phenomenon whose properties and implications are explored below.

[16] What are the conditions for nonlinear phase locking to occur? The most important one is that the oscillation mechanism must be nonlinear. An example of a "linear" oscillator is a simple pendulum undergoing small oscillations, described by the equation $d^2x/dt^2 = \ddot{x} = -\omega^2 x$, which is completely linear in the model variable $x(t)$. The solutions of this equation are oscillations of the form $x(t) = A \cos(\omega t + \phi)$, with the amplitude, A, and phase, ϕ, determined by the initial conditions (e.g., the strength of the initial push used to get it started). Note, in contrast, that the solution of the phase-locked nonlinear model shown in Figure 1b evolves to independence of the initial conditions. More important, the frequency of a linear oscillation (ω in the above example) does not depend on the amplitude of the oscillation.

[17] A nonlinear oscillator, on the other hand, is one satisfying an equation that is nonlinear in the variable $x(t)$, such as $\ddot{x} = -\omega^2 x - \varepsilon(x2 - 1)\dot{x}$ (the Van der Pol equation), or $\ddot{x} = -(g/\ell) \sin x$ (a pendulum under gravity g, composed of a rigid rod of length ℓ with a rotation axis at one end and a mass attached to the other). These equations are both nonlinear in the variable x. The important property of such nonlinear oscillations for the purpose of the phase locking mechanism, is that they may change their period as function of their amplitude. In the case of the pendulum, for example, the period of the oscillation is longer when the oscillator makes larger swings.

[18] Phase locking also requires some friction or dissipation of which there is no lack in the climate system, from the friction associated with glaciers sliding over their bed to the viscosity affecting ocean currents etc. Dissipation "erases" the memory of the initial conditions and therefore allows the oscillations to phase lock as demonstrated by the two converging time series in Figure 1b. Our model is clearly too simple to be able to quantitatively estimate the amount of dissipation, and therefor cannot be used to estimate the time it would require for the glacial cycles to phase lock to the Milankovitch starting from some arbitrary initial conditions.

[19] An historically important example of a nonlinear oscillator is the model of *Imbrie and Imbrie* [1980] which may be written as

$$dV/dt = (i(t) - V)/\tau(V), \qquad (2)$$

where V is the land ice volume, $i(t)$ is related to the Milankovitch summer insolation at 65N, and $\tau(V)$ is a specified timescale for the growth and withdrawal of the land ice sheets, which is a function of the ice volume V and switches values depending on whether ice volume is increasing or decreasing. The right-hand side of (2) is nonlinear because $\tau(V)$ switches values as function of the ice volume V. (Although such a model is sometimes referred to as "piecewise linear," it is actually one of a class of nonlinear models.) The model of *Paillard* [1998] is also nonlinear for a similar reason.

[20] We have some idea of the nonlinearities in the equations governing the oceans, atmosphere, ice sheets and even vegetation dynamics [*Claussen and Gayler*, 1997]. However, we cannot determine which, if any, of these nonlinearities is the critical one. Nor can we determine if the nonlinearity must be that of the Van der Pol oscillator above, or the very different "threshold" nonlinearity of the *Imbrie and Imbrie* [1980] model, or the threshold nonlinearity of the stochastic model of *Wunsch* [2003], or of another form. In particular, we cannot differentiate between mechanisms which involve a nonlinearity throughout the glacial cycle, or mechanisms which invoke it only in the transition from accumulation to deglaciation phase.

2.2. Role of the Nonperiodic Nature of Milankovitch Forcing and Glacial Period "Quantization"

[21] Milankovitch forcing, as manifested for example by the 65N summer insolation, is not a simple periodic signal [*Melice et al.*, 2001], and this has interesting consequences for the phase locking mechanism. Figure 1c and Figure 1d contrast the model results when forced by a perfectly periodic 40 kyr forcing (an idealization of obliquity forcing) and when forced by the 65N summer insolation. There are two lessons to be learned from this comparison. The first is related to the observation that the phase-locked ice volume time series in this case (Figure 1c) has a period of 80 kyr. This is a special case of the nonlinear resonance condition (1). The phase-locked system (ice volume time series in our case) has a frequency ω that is related to that of the forcing, $\omega_f = 2\pi/40$ kyr, as the ratio of two integers, $\omega/\omega_f = p/q$. In the case of Figure 1c the period of the glacial oscillation is 80 kyr, which is precisely twice that of the specified forcing in that model run.

[22] Without the external periodic forcing, a nonlinear oscillator such as our model is characterized by a period ω that does not satisfy (1) for nearly all parameter choices. The external forcing acts to change the amplitude of the oscillator, and because of its nonlinearity, the frequency is affected as well, such that the nonlinear resonance condition (1) is

satisfied. This result would not be possible in a linear oscillator forced by external periodic forcing.

[23] *Raymo* [1997] and *Ridgwell et al.* [1999] [see also *Huybers and Wunsch*, 2005] suggested that the glacial period is "quantized" into multiples of the precession or obliquity periods. The phase locking mechanism indeed provides an explanation for such a quantization. This result arises from the phase locking condition (1), which states precisely that the glacial oscillation period is quantized by that of the Milankovitch forcing. Had the relevant insolation period been exactly 20 kyr, for example, the glacial period could have been 80 kyr ($p/q = 1/4$), 100 kyr (1/5) or 120 kyr (1/6). If the relevant insolation period is that of obliquity, 41 kyr, some of the possible phase locked glacial periods are 82 kyr ($p/q = 1/2$), 102.5 kyr ($p/q = 2/5$) and 123 kyr ($p/q = 1/3$).

[24] As the Milankovitch forcing is not perfectly periodic but contains several amplitude modulated frequencies, it is not clear whether the forcing frequency ω_f to be used in (1) should only be that of obliquity, precession, or must retain all the frequencies present in the Milankovitch forcing. The amplitude modulations of the Milankovitch forcing can cause the p/q ratio to change from cycle to cycle. Note that in our model the period is constant in the absence of external Milankovitch forcing (Figure 1a). Adding a perfectly periodic forcing with a period of 40 kyr changes the glacial period to a constant 80 kyr. The phase locking to the nonperiodic Milankovitch 65N summer insolation (Figure 1b) results in the nonuniform glacial period (nonconstant p/q) as observed. We conclude that the variations in the length of individual glacial cycles, and their seeming variable quantization by the precession and obliquity frequencies, may be due to the nonuniform character of the Milankovitch forcing. (These variations may also be due to stochastic forcing as discussed below.) *Rial* [1999] attributed the variations in the length of the glacial cycles to frequency modulation by the lowest frequency 1/413 kyr component of the eccentricity variations, but a physical component of the climate system capable of undergoing such frequency modulation remains elusive.

[25] The second important lesson to be learned from the model run forced by a pure sine wave (Figure 1c) is that the nonsinusoidal nature of the Milankovitch insolation time series is of major importance for the uniqueness of the phase locking. Note first that phase locking to a pure sine wave is not unique, unlike the locking to the modulated Milankovitch forcing. That is, when the model is forced by the pure sine wave, the different initial conditions for the ice volume converge to two different groups rather than just one. The reason is that the model forced by the pure sine wave is phase locked at $p/q = 1/2$ in this case, but the model time series may adjust such that terminations happen at years T_0, $T_0 + 80$, $T_0 + 160, \ldots$ or at years $T_0 + 40$, $T_0 + 120$, $T_0 + 200, \ldots$ etc, where T_0 is the relevant initial time. In other words, the nonuniqueness of the phase locking seen in Figure 1c occurs because shifting the forcing time series by 40 kyr results in exactly the same forcing. The nonperiodic nature of the Milankovitch forcing eliminates this nonuniqueness because shifting the Milankovitch forcing by 20 or 40 kyr results in a different time series. As a result, all initial conditions converge to the same model time series,

and termination times are uniquely determined by the Milankovitch forcing. The roughly 100 kyr glacial period in Figure 1 is intrinsic to the climate system, is not a response to direct eccentricity forcing, and is phase locked by the precession and obliquity periods.

2.3. Phase Between Termination Times and Milankovitch Forcing

[26] *Raymo* [1997], *Ridgwell et al.* [1999] and others have suggested that termination times are linked to times of increase in Northern Hemisphere high-latitude summer insolation. Figure 2a shows all the terminations in our simple model during the past 800 kyr, superimposed (from the run shown in Figure 1b). Also shown is the Milankovitch forcing during each of these model terminations. The phase of the Milankovitch forcing is *not* the same during all terminations, and spans an approximately 90° interval. This is a result of the nonperiodic nature (amplitude modulation) of the Milankovitch forcing. When the insolation forcing is set to be periodic, there is a unique phase relation between the termination time and the Milankovitch forcing (Figure 2b).

[27] It is evident from the above discussion that it is not only the value of Milankovitch forcing prior to a termination that sets the time of the termination. If the maximum insolation is the only factor that sets the phase of the glacial cycles, we would expect a consistent phase between Milankovitch forcing and termination times. Instead, Milankovitch forcing seems to slowly nudge the model ice sheets (via the accumulation and ablation terms) throughout the cycle, and this way affect the glacial cycle amplitude. Because our model is nonlinear, its period is a function of the amplitude. The changes to the amplitude therefore result in changes to the length of individual glacial cycles in a way that causes different initial conditions to all phase lock to Milankovitch forcing. Different variations in the Milankovitch forcing during the different glacial cycles may result in varying phase relationships between Milankovitch forcing and ice volume during the different terminations.

2.4. Which Orbital Parameter Is Responsible for the Phase Locking?

[28] So far we have used the high-latitude 65N summer insolation to represent the Milankovitch forcing. However, which part of the Milankovitch forcing is in fact critical: precession, obliquity [*Huybers and Wunsch*, 2005], or both? Is it the summer insolation, winter insolation, the insolation gradient with latitude, or the value at some specific latitude? We have run our simple model with precession and obliquity time series as the Milankovitch forcing instead of the full 65N summer insolation, and the results are shown in Figures 1d and 1e. By slightly tuning the model coefficients (most variables are changed by less than 10%), the results of either an obliquity forced or precession forced model run can be made to resemble the observed record. As with the forcing by 65N insolation, the oscillation is again not periodic because of the irregular character of the precession and obliquity parameters.

[29] We conclude that our model is too simple to deduce which element of the Milankovitch forcing is responsible for the phase locking. In any case, it is not clear that any
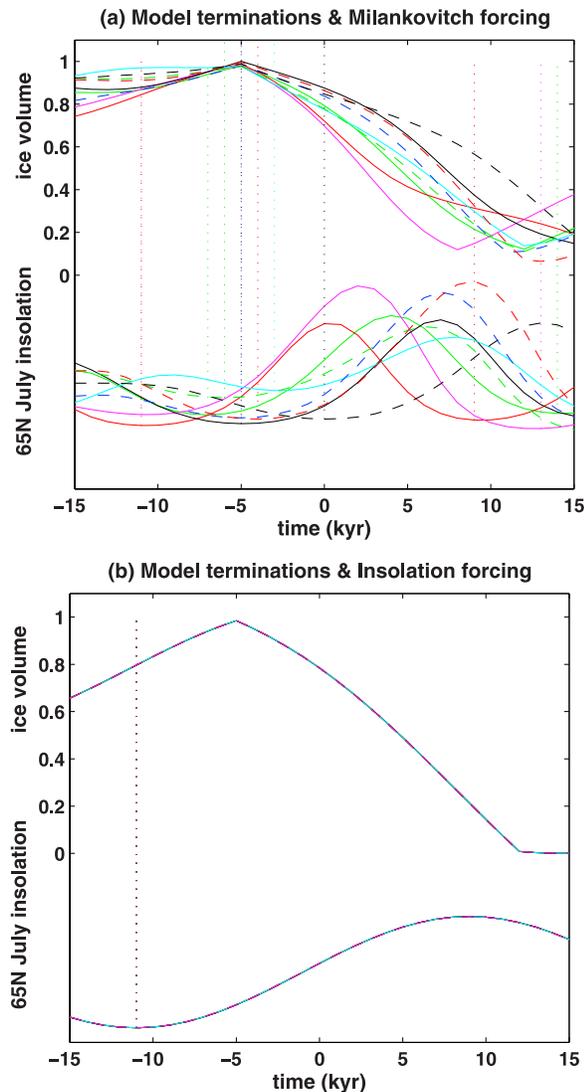
Figure 2. Phase of Milankovitch forcing during glacial terminations. Shown are superimposed model time series of ice volume versus time during model glacial terminations (top curves), shown with the Milankovitch forcing used to obtain these time series (bottom curves). The vertical dotted lines indicate the time of minimum insolation. (a) Terminations when the model is forced by Milankovitch summer radiation at 65N (Figure 1b). (b) Model run forced by a perfectly periodic 41 kyr signal (Figure 1c). The ice volume for the model forced by a purely periodic forcing is always in phase with the insolation forcing, while the nonperiodic Milankovitch 65N summer radiation has different phases during different model terminations.

single Milankovitch component is mainly responsible for the locking.

3. Phase Locking in the Presence of Noise

[30] The climate system contains numerous processes that produce variability on all timescales and the shorter-time-scale processes may perhaps best be treated as "noise" or stochastic forcing. For the phase locking mechanism to be relevant, it must be robust to a reasonable amplitude of noise (section 3.1). Milankovitch forcing is next shown to be able to phase lock the model glacial cycles even when the Milankovitch bands compose only a small portion of the total spectral power. Finally, we show that even when glacial cycle time series are well fit by a linear random process, we cannot rule out the possibility that the cycles are nonlinear and phase locked to Milankovitch forcing (section 3.2).

3.1. Can Milankovitch Forcing Phase Lock the Glacial Cycles in Spite of its Small Contribution to the Total Spectral Power?

[31] A possible challenge to the phase locking mechanism comes from the observation that in the late Pleistocene the total power at the Milankovitch bands of precession and obliquity is actually quite low [*Wunsch*, 2004] and does not exceed 15% or so. Is this observation consistent with the suggestion that Milankovitch forcing can phase lock and therefore pace the glacial cycles?

[32] Figure 1f shows several time series of model runs with different realizations of noise and different initial conditions. The times series are still phase locked to each other and to the proxy record, although the presence of noise causes them to occasionally diverge from each other, unlike those in Figure 1b. (Additional model time series with different initial conditions and different noise realizations make Figure 1f less legible, but demonstrate the same result.)

[33] Next, we consider the spectrum of a time series from such a model run (Figure 3). Following the same procedure used by *Wunsch* [2004], we find that the precession and Milankovitch bands account for only 18% of the total power, similar to the results found by Wunsch in an actual proxy record. (We use a somewhat more generous definition of the Milankovitch bands than used by *Wunsch* [2004], taking periods from 17 to 25 kyr for the precession, and 37 kyr to 45 kyr for obliquity.) This result indicates that Milankovitch forcing could be setting the phase of the glacial cycles via nonlinear phase locking even in the presence of the abundant noise in the climate system and even though Milankovitch bands account for only a small portion of the total spectrum.

[34] We cannot accurately calculate the amplitude of the noise that should be added to our simple model, because of the uncertainty in what this noise represents, how large it was during the glacial cycles, and because of the extreme simplicity of our model. The noise chosen is of a plausible amplitude, based on the observation that the model spectrum seems to have roughly the same power distribution between the different periods as observed for the proxies (compare Figures 3a and 3b). A detailed description of the noise form and amplitude may be found in Appendix A.

3.2. Can We Distinguish Between a Linearly Driven Random Process and a Nonlinear Phase-Locked Scenario Using Proxy Time Series?

[35] Another observation, which may seem contradictory to the nonlinear phase locking mechanism, is that proxy records of the glacial cycles can be well fit by a linear Markov (autoregressive, AR) random process [*Wunsch*,

2004]. An order-two AR process for the ice volume $V(t)$, for example, is defined to be the outcome of the equation

$$V(t) = a_1 V(t - \Delta t) + a_2 V(t - 2\Delta t) + \theta(t) \qquad (3)$$

where a_1 and a_2 are coefficients determined from fitting this equation to the proxy record for land ice volume, and $\theta(t)$ is a white noise (random) forcing term (see Appendix A). This equation can be stepped forward in time steps of $\Delta t = 1$ kyr to calculate the ice volume at a time t from that at the two previous steps.

[36] That such a linear equation provides a good fit to the glacial cycles indicates that the proxy records of the glacial cycles are consistent with the assumption that these cycles are a random linear process driven by noise. One wonders whether this implies that phase locking, a deterministic and nonlinear mechanism, is inconsistent with the observations.

[37] Figure 4a shows the fit of an AR(2) process to a model time series such as those in Figure 1d; much of the structure remains in the disturbance series $\theta(t)$. While the innovations (residuals) from the fit (Figure 4b) whose histogram is shown in Figure 4c do not satisfy the formal requirements of a white noise process (the Akaike criterion suggests that a 10th-order AR process would be required), they are reasonably close. It seems therefore that a phase locked model solution that is deterministic and self-sustained, but with some additive noise, can pass the tests applied by *Wunsch* [2004] to the proxy time series. We conclude that these tests may not be able to rule out the possibility that the observed ice volume proxy record is phase locked to Milankovitch forcing.

4. Discussion

[38] Because of the robustness of the phase locking mechanism, the underlying mechanism responsible for the existence of the glacial cycles is not well constrained by the observed ice volume record. The proxy record can be fit using glacial cycle models that are based on jumps between multiple equilibria [*Paillard*, 1998], but explanations such as explored here, not based on such jumps, produce an equally good fit. Nonlinear phase locking is effective within mechanisms in which CO_2 variations drive the cycles [*Saltzman*, 2001] but also when CO_2 variations only amplify the glacial amplitude [*Gildor et al.*, 2002]. The phase locking mechanism can work with a variety of nonlinear mechanisms for glacial terminations [e.g., *Yoshimori et al.*, 2001] and for the ice ages. These very different scenarios and many others all result in a good fit to ice volume records. The edifice of particular physical mechanisms, while interesting, obscure the simpler point that the models actually fit the proxy record because they are phase locked to the Milankovitch forcing. Figuring out the correct glacial mechanism will require more than just a good fit to ice volume record.

[39] That our model displays ~100 kyr glacial oscillations even in the absence of Milankovitch forcing (i.e., our oscillations are "self-sustained"), is not essential for the nonlinear phase locking mechanism. The phase locked models of *Imbrie and Imbrie* [1980] and *Paillard* [1998],

(a) model: prec=5%; obliq=8% 100kyr=58%

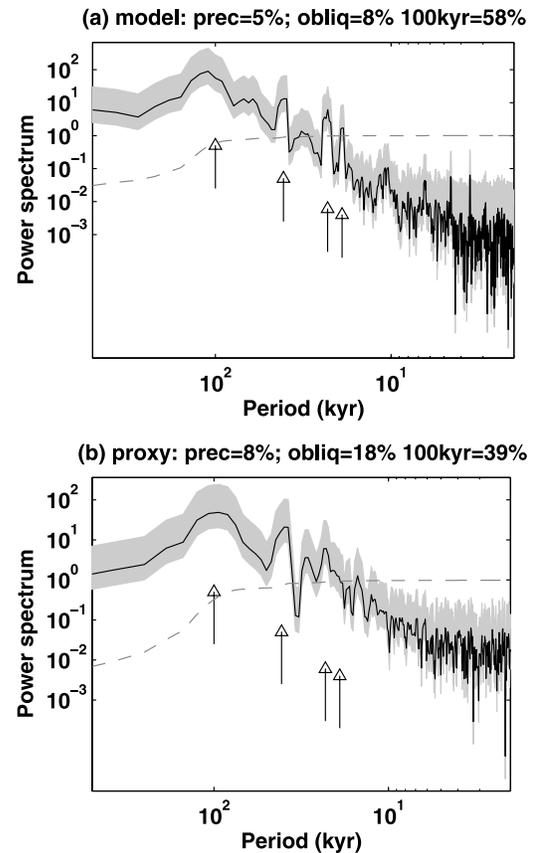**(b)** proxy: prec=8%; obliq=18% 100kyr=39%

Figure 3. Spectrum of ice volume time series for (a) model driven by 65N insolation and noise and (b) proxy record (see text for details). The 100, 41, 23, and 19 kyr periods are indicated by the vertical arrows. Note that the total power explained by the Milankovitch bands is small, yet in this model the Milankovitch forcing still sets the phase of the cycles (i.e., the time of the terminations).

for example, would display no variability in the absence of Milankovitch forcing. The models of *Saltzman* [2001] and *Huybers and Wunsch* [2005] were also self-sustained and phase locked to Milankovitch forcing, hence their good fit to the ice volume proxy record. Phase locking may have played a role in the 41 kyr glacial oscillation prior to the mid-Pleistocene transition [*Ashkenazy and Tziperman*, 2004], although dating difficulties make it more difficult to investigate the relative timing of terminations and Milankovitch forcing then.

[40] That glacial cycles are phase locked to Milankovitch forcing, and that this enables a good fit to the observed ice volume record, was already realized by *Saltzman et al.* [1984, p. 3387] who asked:

[41] "How does small amplitude periodic forcing control phase in a complex nonlinear oscillatory system, and is there a good physical interpretation for this phase locking phenomenon?"

[42] These authors recognized the important role played by Milankovitch forcing as elaborated by *Gildor and Tziperman* [2000] and here. The model of *Saltzman and Verbitsky* [1994] was nonlinearly phase locked, yet they write (p. 774)
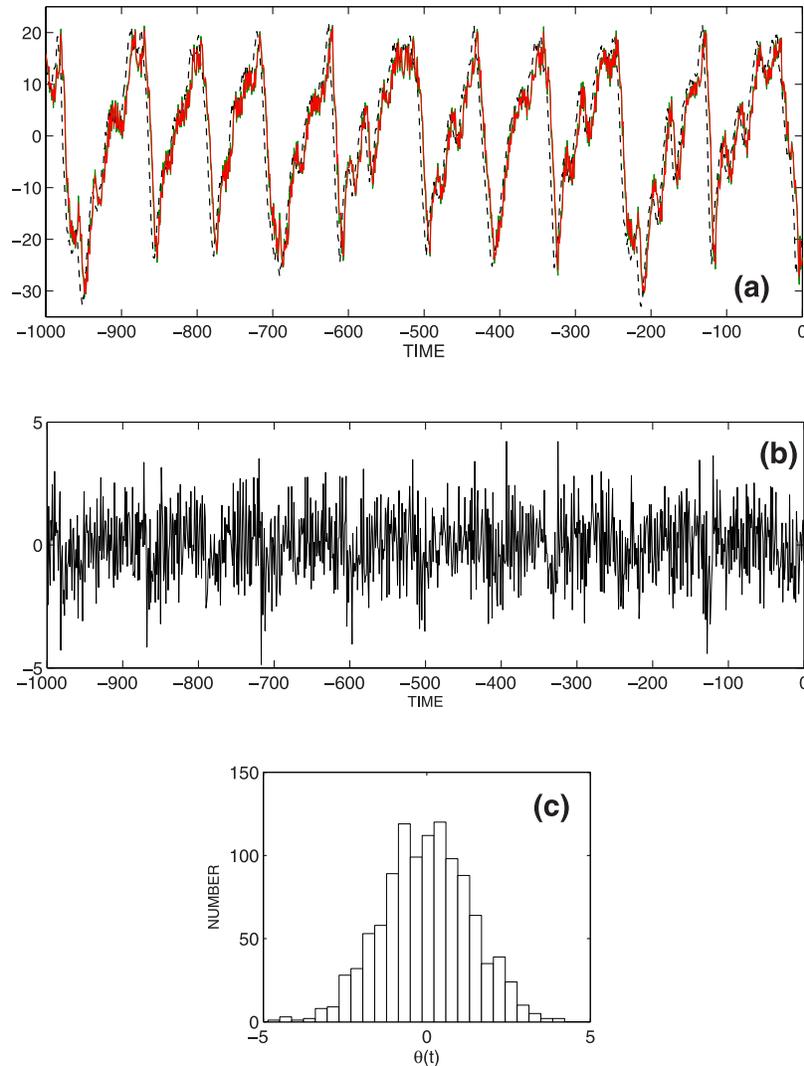
Figure 4. A stochastic model fit to the phase-locked result of the model. (a) Autoregressive (AR)(2) fit (solid green) to the model time series (black dash). (b) Innovations (residuals) of the fit ice volume model time series. (c) Histogram of the innovations showing that they are nearly Gaussian. One may conclude that a good fit of proxy ice volume time series by a random process does not necessarily rule out that the underlying process is not self-sustained and phase locked.

[43] "The chronology for this 100 kyr period cyclic behavior is set by the Milankovitch forcing which imposes a "phase lock" on the cycle by constructive interference of the maximum (and minima) in high-latitude incoming radiation of near 20- and 40-kyr period with maximum (and minima) of CO_2 of near 100-kyr period."

[44] This explanation of the phase locking is different from the mechanism presented here. An "interference" is normally a linear phenomenon in which two time series are superimposed without affecting each other. This is in contrast to the nonlinear phase locking mechanism in which the external Milankovitch forcing affects the amplitude and thus the period of the glacial oscillations to satisfy the nonlinear resonance condition (1), rather than just super-imposing a 20 and 41 kyr signal on the glacial oscillations.

[45] *Hyde and Peltier* [1985, 1987] have also realized many of the characteristics of nonlinear phase locking, including insensitivity to initial conditions. They suggest, though, that the glacial period must be a multiple of the forcing period, rather than the more accurate and general condition of equation (1). More importantly, like many other studies mentioned above, they proceed to suggest that the fit to the ice volume record serves to validate their model mechanism, while we conclude that this, unfortu-nately, is not the case.

[46] Understanding how Milankovitch forcing sets the phase of the glacial cycles via the nonlinear phase locking mecha-nism contributes significantly to the understanding of glacial cycle dynamics. It can provide the "pacemaker" mechanism via the dependence of the period of a nonlinear oscillator on its

amplitude; it predicts the "quantization" of the glacial period via the nonlinear resonance condition (equation (1)); and it reveals why very different glacial mechanisms and models are all able to fit the ice volume record.

[47] Nonlinearity in proxy glacial time series has been identified in many previous studies of both the 100 kyr cycles and the 41 kyr cycles [*Hagelberg et al.*, 1991, 1994; *Rial*, 1999, 2004; *Rial and Anaclerio*, 2000; *Huybers and Wunsch*, 2004; *Ashkenazy and Tziperman*, 2004]. See also the reviews by *Hinnov* [2000] and *Elkibbi and Rial* [2001] and references therein, including a discussion of the glacial cycle mechanism relying on the nonlinear production of combination tones of the precession orbital forcing [*Le-Treut and Ghil*, 1983]. Note that the glacial mechanisms suggested by these papers are different from, although sometimes related to, the phase locking mechanism examined here. *Hagelberg et al.* [1994], for example, suggested that the 100 kyr cycles may be explained as a linear resonance of the climate system with eccentricity forcing. Such a resonance was also proposed by *Imbrie et al.* [1993] to support the hypothesis that the 100 kyr cycles are a response to eccentricity forcing, and in order to explain the absence of 400 kyr variability in the climate record. We emphasize that the phase locking mechanism discussed in this paper for the pacing of the glacial cycles is not based on the amplification of the 100 kyr eccentricity forcing, but rather relies on the obliquity and precession Milankovitch forcing.

[48] One element of the late Pleistocene $\delta^{18}O$ record we do not discuss is the nature of the continuum away from the Milankovitch bands [*Huybers and Curry*, 2006]. While the source of the observed continuum of spectral climate response is beyond the scope of the present paper, it is useful to keep in mind that its presence would need to be explained as part of a satisfactory glacial theory.

5. Conclusions

[49] We have investigated some implications of the phase locking mechanism as a possible pace maker for the glacial cycles [*Gildor and Tziperman*, 2000]. The model we have used is not realistic and is not meant to accurately represent the glacial cycle dynamics. Rather, our objective was to demonstrate the plausibility of Milankovitch insolation variations setting the phase (e.g., termination times) of the glacial cycles via the mechanism of nonlinear phase locking. This mechanism is effective regardless of the specifics of the model [e.g., *Pollard*, 1982; *Berger and Loutre*, 1996; *Saltzman et al.*, 1984; *Hyde and Peltier*, 1987; *Maasch and Saltzman*, 1990; *Saltzman and Verbitsky*, 1994; *Paillard*, 1998; *Gildor and Tziperman*, 2000; *Ashkenazy and Tziperman*, 2004; *Huybers and Wunsch*, 2005] or of the glacial mechanism itself. The only required condition is that the glacial mechanism is nonlinear, as explained above. A good fit to the ice volume record should therefore not be taken as an indication that the glacial mechanism used in any of these models is necessarily correct, but only that the model is nonlinearly phase locked to Milankovitch forcing. This also suggests that the actual glacial cycles may also be similarly nonlinearly phase locked to the Milankovitch forcing.

[50] We found that the phase locking mechanism has several interesting additional implications:

[51] 1. Phase locking may be effective despite the presence of abundant noise in the climate system.

[52] 2. Phase locking can occur because of precession and obliquity variations. Eccentricity may be important only indirectly, in modulating the higher frequency Milankovitch forcing.

[53] 3. Phase locking tends to result in a quantization of the glacial period into multiples of the precession or obliquity frequencies [*Raymo*, 1997; *Huybers and Wunsch*, 2005] via the nonlinear resonance condition of equation (1).

[54] 4. The nonsinusoidal nature of the Milankovitch forcing (that is, the modulation of the amplitude of the precession and obliquity parameters) can account for varying glacial periods, and may uniquely determine the timing of glacial terminations.

[55] These findings suggest that the ice ages problem is effectively divided into two separate sub problems: the first is explaining the phase or timing of the cycles, and the second is finding the physical mechanism that gives rise to these cycles. We believe that nonlinear phase locking provides a good framework for understanding the first problem, even if the second is still far from being resolved.

[56] Identifying nonlinear phase locking dynamics merely by analyzing proxy ice volume time series is difficult. With only eight 100 kyr cycles, making a statistical differentiation between linear, stochastically driven cycles, and nonlinear phase locked cycles, is a challenging task. It is possible that more realistic, three-dimensional, models with less scope for tuning, in conjunction with many more, geographically distributed, proxy records will eventually permit the rejection of different mechanisms and models. It is simple, though, to see that nonlinear phase locking is active in a given model: the nonlinear resonance condition (1) when the model is driven with some pure frequency, and the convergence of different initial conditions to a single time series are all that is needed to verify that the model is phase locked by this mechanism.

[57] Nonlinear phase locking provides a simple and robust description of how Milankovitch forcing could pace the glacial cycles. In this light, the "pacemaker" analogy used by *Hays et al.* [1976] seems exceptionally insightful.

Appendix A: Model Description

[58] The simple model used in this paper is that of *Ashkenazy and Tziperman* [2004]. The model ice volume freely oscillates when not forced by Milankovitch forcing, at a period that is just over 100 kyr. The mechanism of the oscillation is the sea ice switch mechanism of *Gildor and Tziperman* [2000]. The specifics of the mechanism as well as the fact that it is self-sustained are not relevant, as the main message of this paper is that the timing of the terminations is set by nonlinear phase locking regardless of the glacial cycle mechanism.

[59] Assume changes to Northern Hemisphere ice volume V to be due to the difference between net snow precipitation over land P and total ablation S (melting, ice sheet surges,

wind erosion etc.). The ablation is assumed to be composed of a constant factor S_0 plus terms that depend on Milankovitch forcing and possibly on some random noise process $\nu(t)$,

$$S = S_0(1 + \nu(t)) + S_M\,I(t), \qquad (A1)$$

where $I(t)$ is the July insolation at 65°N [*Berger and Loutre*, 1991] (or the time series of obliquity or precession parameters, or a purely periodic 40 kyr signal in some of our model runs) normalized to zero mean and unit variance (and filtered using the same procedure used by *Paillard* [1998] for runs in Figures 1b and 1f); S_M is a constant.

[60] We also assume that accumulation rate P is small for large ice volume and large for small ice volume because of the temperature precipitation feedback [*Källén et al.*, 1979; *Ghil*, 1994]. A simple formulation of accumulation reducing with ice volume is of the form P(no sea ice) $= p_0 - kV$, where p_0 and k are constants: p_0 is the precipitation rate when the ice sheets are completely melted and k is the growth rate constant of the ice sheet.

[61] Following *Gildor and Tziperman* [2000] we assume that when the ice volume reaches a certain specified maximal ice volume V_{max}, the atmospheric temperature becomes sufficiently low such that a significant sea ice cover rapidly forms and atmospheric temperature drop. Precipitation over land ice then reduces very sharply, and thus accumulation in the presence of sea ice may be written as P(with sea ice) $= (p_0 - kV)(1 - a_{si-on})$. This reduced accumulation in the presence of sea ice results in land ice withdrawing. When land ice volume drops below a certain minimal ice volume V_{min} (resulting in warming due to ice albedo feedback) the sea ice melts rapidly and precipitation returns to its original rate without sea ice.

[62] Finally, combining the above expressions for the ablation and accumulation, the ice volume mass balance may be written as

$$\frac{dV}{dt} = [(p_0 - kV)(1 - a_{si})] - [S(1 + \nu(t)) + S_M\,I(t)], \qquad (A2)$$

where the first term in square brackets on the right-hand side is the accumulation, the second is the ablation; a_{si} is the relative area of the sea ice ($a_{si} = a_{si-on} > 0$ when sea ice is "on" and $a_{si} = 0$ when sea ice is "off").

[63] The model is nonlinear because of the dependence of the sea ice area a_{si} on the ice volume at the two threshold points $V_{max} = 45e6 * km3$ and $V_{min} = 3e6 * km3$. The other model parameters are $a_{si-on} = 0.46$, $p_0 = 0.26\ Sv$, $k = 0.70/(40 * kyr)$, $S = 0.23\ Sv$, and $S_M = 0.03\ Sv$. When noise is included in the model $\nu(t) = \nu(n\Delta t)$ is set to a simple first-order Markov process with an amplitude of 0.04 and a correlation time of 3 kyr. That is,

$$\nu_n = R * \nu_{n-1} + \sqrt{1 - R2} * \theta_n; \qquad (A3)$$

where $R = \exp(-\Delta t/3\ kyr)$ and θ_n is a white noise with zero mean, such that $\langle\theta_n\theta_m\rangle = 0.042\delta_{nm}$ and δ_{nm} is the Kronecker delta. For the run shown in Figure 1b the ratio of the Milankovitch forcing variance to the noise variance is near four to one. Additionally, a second similar noise term is added to model time series once the integration is complete, with an amplitude of $0.05V_{max}$. This second noise term represents measurement noise and other non-ice-volume effects on the proxy time series.

[64] The parameters for the model run in Figure 1d are $V_{max} = 49.5e6 * km3$, $V_{min} = 3.00e6 * km3$, $a_{si-on} = 0.34$, $p_0 = 0.22\ Sv$, $k = .91/(40 * kyr)$, $S = 0.17\ Sv$, and $S_M = 0.023\ Sv$. Parameters for Figure 1e are $V_{max} = 33.3e6 * km3$, $V_{min} = 3.00e6 * km3$, $a_{si-on} = 0.16$, $p_0 = 0.28\ Sv$, $k = 1.3/(40 * kyr)$, $S = 0.24\ Sv$, and $S_M = 0.02\ Sv$.

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