SOLAR MAGNETIC ACTIVITY AND TOTAL IRRADIANCE SINCE THE MAUNDER MINIMUM

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Abstract.
We develop a model for estimating solar total irradiance since 1600 AD using the sunspot number record as input, since this is the only intrinsic record of solar activity extending back far enough in time. Sunspot number is strongly-correlated, albeit non-linearly with the 10.7 cm radio flux ($F_{10.7}$), which forms a continuous record back to 1947. This enables the non-linear relationship to be estimated with usable accuracy and shows that relationship to be consistent over multiple solar activity cycles. From the sunspot number record we estimate $F_{10.7}$ values back to 1600 AD. $F_{10.7}$ is linearly correlated with the total amount of magnetic flux in active regions, and we use it as input to a simple cascade model for the other magnetic flux components. The irradiance record is estimated using these magnetic flux components plus a very rudimentary model for the modulation of energy flow to the photosphere by the sub-photospheric magnetic flux reservoir feeding the photospheric magnetic structures. Including a Monte Carlo analysis of the consequences of measurement and fitting errors, the model indicates the mean irradiance during the Maunder Minimum was about 1 ± 0.4 W m^{-2} lower than the mean irradiance over the last solar activity cycle.

Keywords: Maunder Minimum, solar variability, irradiance

1. Introduction

Understanding the variability in solar irradiance and its role in global climate change requires as long as possible a record of historical irradiance variations. In the absence of observations currently extending back in the past beyond about three solar activity cycles, this can only be obtained through proxy quantities and modelling. The modelling process is extremely complicated, entailing physics that is poorly understood in many cases, and the use of proxies with connections with the values being represented weak to doubtful. In addition, the required precision is not available through calculations, so fitting the model to irradiance data is usually needed. Finally, deriving the required historical record requires extrapolation of the model well outside the parameter space used to derive it.

Over the last 2000 years there have been instances where solar activity fell to a low level for some decades. These have become known as the Wolf (1280-1350 AD), Spörer (1420-1540 AD), Maunder (1645-1715 AD) and Dalton (1795-1825 AD) minima. Connections between these minima and periods of anomalous global cooling were first pointed out by Eddy (1976 *a*, *b*, 1977, 1979, 1980). The period between 1700 AD and the present is important in that there is a continuous record of sunspot number, which is a directly measured index of solar activity, of known pedigree with established relationships with other activity indices, and which antedates the rapid increase in anthropogenic greenhouse gases that began with the industrial revolution. That there is a connection between solar variability and terrestrial environmental change is well-documented, and evident in both recent, mediaeval and geological records (see for example, Anderson, 1991).

The currently available database of total irradiance measurements spans only about three solar activity cycles, and is a composite of contributions from several instruments. The integration of these measurements was very difficult, involving significant issues with calibration. The data, together with the integration and calibration processes are described in Pap and Fröhlich (1999) and Fröhlich (2004).

The importance of the historical irradiance issue has let to intense study and development of models. However, since they all involve extensive use of proxies and other empirical approaches, it has proved difficult to isolate the key processes driving long-term irradiance change. For a comprehensive discussion of modelling approaches together with the underlying physics and assumptions, see Lockwood (2004). Most models are based upon the assumption of two irradiance contributions, both of which vary with time. Following the terminology used by Solanki and Krivova (2004), these are a *cyclic* component, following the rhythm of the solar activity cycle as exemplified by activity indices such as sunspot number or 10.7 cm solar radio flux, and a separate, *secular* irradiance component, with characteristic variational timescales much longer than the solar activity cycle. One of the reasons for the desire to extend models and estimates back into the Maunder Minimum is that during the minimum the cyclic component was small or absent, and that the only variable irradiance contribution would be from the secular component. There is evidence that the Maunder Minimum did not mark a complete cessation of the activity cycle, and that flux emergence and submergence continued throughout (Beer *et al.*, 1990), so it is likely that the Maunder Minimum did not mark a discontinuous change in solar behaviour that models might not bridge.

It is well established that the shorter-term irradiance variations follow changes in solar magnetic activity (Willson and Hudson, 1991; Fröhlich, 1994; Kuhn, 1996). and that changes in solar irradiance correlate with bright faculae and magnetic network (Foukal and Lean, 1988). However, the rhythm of the solar activity cycle so deeply modulates the various aspects of solar activity that correlations might not indicate a causal relationship. This contributed to two broad schools of thought regarding the causes of irradiance change. One invokes purely the photospheric phenomena associated with solar activity, such as sunspots, faculae and active network. Examples included Foukal and Lean (1988), Schatten and Orosz (1990), Zhang *et al.* (1994), Lean, Beer and Bradley (1995). Lean *et al.* (1998), Solanki and Fligge (1998 and 1999), Lockwood and Stamper (1999), Lean (2000), Fligge, Solanki and Unruh (2000), Foster (2004) Solanki and Krivova (2004) and Krivova, Balmaceda and Solanki (2007).

The other school of thought proposes that to adequately model irradiance variations, some additional phenomena associated with more global solar processes need to be included. Suggestions include variations in the solar radius (Delache, Lacrare and Sadsaout, 1986; Hoyt and Schatten; 1993, Ulrich and Bertello, 1995; Antia, 2003), large convective cells and changes in internal structure (Ribes, Mein and Mangeney, 1985; Fox and Sofia, 1994; Sofia, 2004; Sofia and Li, 2004). Interesting insights into the relationship between these schools of thought are given in Foukal and Spruit (2004). Considering the diverse modelling approaches, it is perhaps not surprising there is considerable spread in the results, with the irradiance during the Maunder Minimum being estimated to be between 0.1 and 0.4% - a factor of about 4, where the cycle-related variability is smoothed out.

A discussion of the magnetic interpretation of solar activity is given by Parker (1994 *a* and *b*). Its visible manifestations take place primarily in *complexes of activity*, that may contain several active regions at various stages in their evolution, together with their decay products. These complexes persist for up to a dozen solar rotations. A detailed study of these complexes and their evolution is given by Gaizauskas *et al.* (1983). Photospheric magnetic activity comprises three main processes: the emergence of new magnetic flux at the the photosphere and its subsequent submergence; the fragmentation of a smaller part of the concentrations of magnetic flux in areas of high magnetic field strength into flux fragments of lower magnetic field strength, and their diffusion away across the photosphere (mainly poleward). Some signs of magnetic activity are always present, even at solar activity minima. There are ephemeral regions (short-lived magnetic bipoles) forming and dissipating, and large areas of weaker magnetic flux remaining from the decay of old active regions (see the discussion in Zwaan and Harvey, 1994). Sunspots might be a good indicator of magnetic activity when present, but they are not useful when activity is low. When examining solar activity during a sustained change in the solar activity cycle, one needs to examine two issues: firstly, does the nature of the process by which magnetic flux is processed change, and secondly, what is the solar activity machine below the photosphere doing?

There have been previous attempts at modelling the history of solar magnetic activity. The use of solar magnetic fields as a basis for irradiance modelling was examined by Harvey (1994), and more recently by Fox (2004). In work conducted since, particularly noteworthy are models by Solanki, Schlüssler and Fligge (2000, 2002), which are based upon the processing of magnetic flux over the solar cycle. In the case of this investigation, the input to the model has to be sunspot number, which is the only direct index of solar activity available. In this paper we develop a model for the processing of solar magnetic flux and use it to model the historical record of total irradiance. In the discussions here we use sunspot number and the 10.7 cm solar radio flux as starting points. Here we use $F_{10.7}$ to estimate the total amount of magnetic flux in active regions, use this as input to a magnetic flux processing model, and thence estimate irradiance contributions. Using $F_{10.7}$ we estimate solar total irradiance back to 1947, when $F_{10.7}$ record begins, and then, using sunspot number to estimate the 10.7 cm solar radio flux, we extend the estimates back to 1700 AD and the Maunder Minimum.

2. Input Data

2.1. Sunspot Number

Sunspot number is the oldest directly-measured index of solar activity, and now forms a consistent, continuous record covering more than 300 years. The main database of sunspot numbers is maintained by the Solar Influences Data Centre (SIDC) in Belgium, but available through a number of sources worldwide. The sunspot number data used in this paper, covering the period 1700 to the present, were obtained from the National Geophysical Data Center, Boulder, Colorado.The sunspot number index (more correctly referred to as the Zürich Sunspot Number) has proved itself to be a very useful indicator of the level of solar activity. Its disadvantage is that it is empirical and cannot be physically related to solar and active region parameters, and the slope of plots of sunspot number against some other indices changes at the low-activity end, making extrapolation uncertain. However, operating the model requires sunspot data antedating the Maunder Minimum. Thomson (private communication), using research by Hoyt and Schatten (1995*a* and *b*, 1996 and 1998*a* and *b*) made estimates of sunspot number from 1610 onwards.

2.2. 10.7 cm Solar Radio Flux

The 10.7 cm solar radio flux values are measurements of the intensity, at 10.7 cm wavelength, of the slowly-varying component (S-component) of solar radio emission. The S-component can be observed at wavelengths from about 50 to 1 cm, but is brightest at wavelengths close to 10 cm, hence the value of measurements at 10.7 cm wavelength. It is thermal in origin, and comes from coronal plasma trapped in the magnetic fields overlying active regions.

In active regions where the ambient magnetic fields supporting the plasma are low enough for the electron gyrofrequency (roughly $f_B = 2.8 \times 10^6 B$ Hz, where B is in Gauss) is less than about a third of the observing frequency, the main contributor to the total absorption coefficient is free-free interactions between thermal electrons and ions. In places where the magnetic fields are stronger, for example over sunspots, thermal gyroresonance can produce much brighter emission due to the higher absorption coefficient. A more detailed discussion of the S-component and its origins are given in monographs by Kundu (1965) and Krüger (1979), and in Tapping and Harvey (1994). The intensity of the S-component at 10.7 cm wavelength, which is known as the 10.7 cm solar radio flux, or $F_{10.7}$, is a well-established index of solar activity. A specific discussion of the 10.7 cm solar radio flux is given in Tapping (1987). The S-component is a composite of contributions from all the active regions on the solar disc plus emissions originating outside active regions. Studies of active region radio sources as contributors to the S-component have been made by Tapping and Zwaan (2002) and Tapping, Cameron and Willis (2003). The 10.7 cm solar radio flux is given in solar flux units (1 solar flux unit (sfu) $\equiv 10^{-22}\mathrm{Wm}^{-2}\mathrm{Hz}^{-1}$). In this discussion it is the enhancement of the emission over the Quiet Sun base level that is important, so we define the *10.7 cm S-component flux*, $S_{10.7}$, which is the strength of the slowly-varying component at 10.7 cm wavelength, which is $F_{10.7}$ with the Quiet Sun flux density (assumed here to be 68 solar flux units) subtracted. The old assumed value for the quiet sun solar flux of 64 sfu was associated with a linear extrapolation of a plot of $F_{10.7}$ to zero sunspot number. However, taking into account the slight nonlinearity of the plot at the low activity end, the flux density at zero activity is estimated at 68 sfu.

2.3. Magnetic Flux Measurements

For more than two decades, the magnetograph at the National Solar Observatory at Kitt Peak has been used to measure the distribution of magnetic flux at the photosphere with near-arc-second resolution. The instrument is essentially a high-angular resolution spectrometer that can be used to measure the Zeeman splitting of selected photospheric spectral lines). Harvey (1992) analyzed magnetograms over more than a solar activity cycle to estimate the contributions to the total magnetic flux coming from the various types of magnetic structure, including sunspots, faculae, the active network, the intra-network fields and possibly a background component. She found that to a very large degree, the magnetic elements measured by the magnetograph fall into two distinct classes (see also Zwaan and Harvey, 1994): *strong-field elements* having average magnetic field strengths higher than a certain threshold value, and *weak-field elements*, located outside active regions, with average magnetic field strengths smaller than this value. Moreover, she demonstrated that this threshold magnetic field strength could be varied over the range 25-40 Gauss without significantly affecting the relative contributions to the total magnetic flux, so clearly different populations are identified. Elements having intermediate values for magnetic field strength exist only transiently during region fragmentation. Following the practice used by Harvey, all the magnetic flux values used here, unless it is stated otherwise, are given in *magnetic flux units* (1 magnetic flux unit (mfu) $\equiv 10^{22}$ Maxwells).

The strong-field elements lie in active regions and in elements of the active network. Their intrinsic field strengths range from about 1,000 Gauss (in network elements) to 3,000 Gauss (in sunspots). Since these elements often lie below the resolution of the magnetograms, the field strengths measured are averages over the element and consequently smaller. Ephemeral regions are considered to be simply the small-scale end of the size distribution of active regions. Setting them aside as a separate category of active structures may well be an artifact of previous lines of research (Zwaan and Harvey, 1994). The weak-field elements include the *intra-network field* (INF), which is probably a product of collapse of field lines in network elements, that due to their weakness, are trapped between the canopy of strong field lines connecting the network elements, and which repeatedly emerge and submerge above and below the photosphere between their end-points. The INF occurs all over the photosphere.

The weak-field elements are scattered over the rest of the solar disc. The calibration of the strong-field magnetic flux (Φ_S) values is taken as correct, but it has been pointed out there is a question about the values of the weaker-field components. Fortunately, for more than three decades, the National Solar Observatory has provided values of the average photospheric magnetic field strength (B_{av}), from which we can calculate the total photospheric magnetic flux ($\Phi_T = 4\pi R_\odot^2 B_{av}$), where $R_\odot$ is the photospheric radius, from which the weaker-field magnetic flux (Φ_W) may be obtained by difference: $\Phi_W = \Phi_T - \Phi_S$. The values of Φ_S and Φ_W over one solar activity cycle are shown in Figure 1.

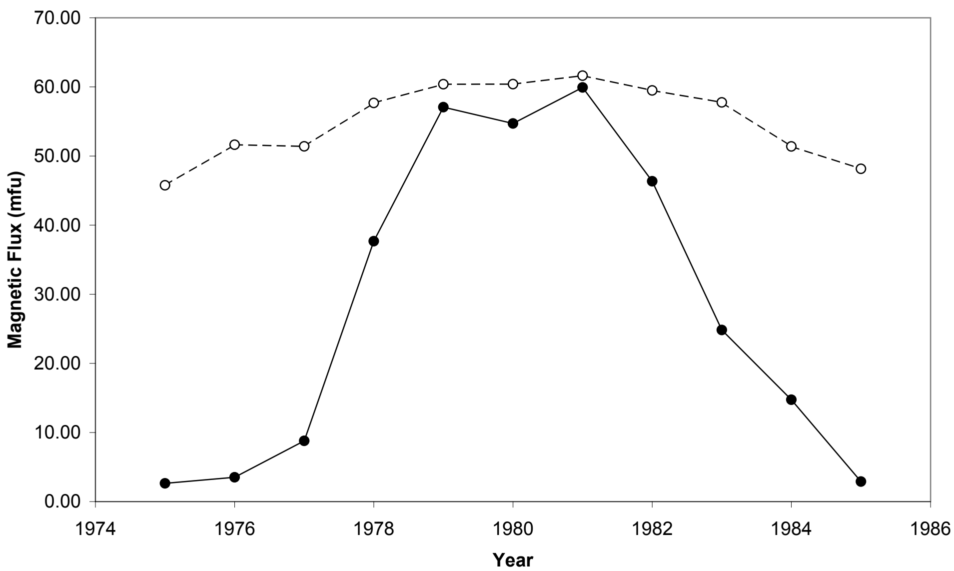

Figure 1. Annually-averaged values of total strong (solid circles/solid lines) and weak-field (open circles/dashed lines) magnetic fluxes over a solar activity cycle.

2.4. Irradiance

Through a succession of instruments, irradiance measurements have been made over almost three solar activity cycles so far, from 1976 to the present. A review of the measurements, measurement methods and instrumentation is given by Fröhlich *et al.* (1991). Integration of these separate data sets into a single, consistent and calibrated record is described by Pap and Fröhlich (1999) and Fröhlich (2004), and references therein (see also Fröhlich and Lean 1998*a* and b). The uncertainties and problems in this integration process are matters of continuing discussion. This integrated irradiance database was obtained from PMOD/WRC, Davos, Switzerland. The irradiance values used in this paper are annual averages.

3. The Model

In this model, the cyclic component of irradiance variability is assumed mainly to be due to the modulation of irradiance by photospheric magnetic structures associated with solar activity (active regions and their decay products). The secular component is assumed due to a reservoir of sub-photospheric magnetic flux, which modulates energy flow to the photosphere, causing small temperature variations and consequent changes in irradiance.

In stars like the Sun, magnetic flux is generated mainly at the tachocline. It then moves into the convection zone, forming a reservoir of magnetic flux. Some emerges through the photosphere to maintain the photosphere, chromosphere and corona, and to drive the phenomena of solar activity. However, most never emerges above the photosphere. Although some variation in total magnetic flux in the convection zone would occur over the solar activity cycle, the stability in the structure of the photosphere, chromosphere and corona, and of large-scale patterns of photospheric activity suggests the total amount of sub-photospheric magnetic flux exceeds significantly the amount of magnetic flux appearing and disappearing at and above the photosphere over the solar activity cycle. The magnetic flux in the reservoir would be expected to vary slightly over the solar activity cycle and over longer timescales. The cycle-related component would contribute to the cyclic component of irradiance variation, and the slower changes would drive the secular variation component.

Sunspot number is used as a proxy for historical values of the 10.7 cm solar radio flux, which in turn are used to estimate the total active region magnetic flux. The other magnetic flux components are obtained used a simple cascade model. These are then used to estimate the cyclic component of the irradiance variability. A smoothed total photospheric magnetic flux value is used as an estimate of the sub-photospheric magnetic flux and thence to obtain the secular irradiance component.

3.1. $F_{10.7}$ AND $S_{10.7}$ FROM N_s

The 10.7 cm solar radio flux $F_{10.7}$ and the 10.7 cm S-component flux $S_{10.7}$ ($S_{10.7}$ is simply $F_{10.7}$ with the quiet sun level subtracted) are measures of the total amount of strong-field magnetic flux. The fluxes are highly correlated, but the relationship between them is non-linear. Figure 2 shows annually-averaged $F_{10.7}$ values plotted against sunspot number (N_s).

An empirical relationship that relates the two quantities is:

$$F_{10.7} = \frac{N_s}{2} \left(2 - \exp(-0.01 N_s)\right) + 68 \qquad (1)$$

and $S_{10.7} = F_{10.7} - 68$.

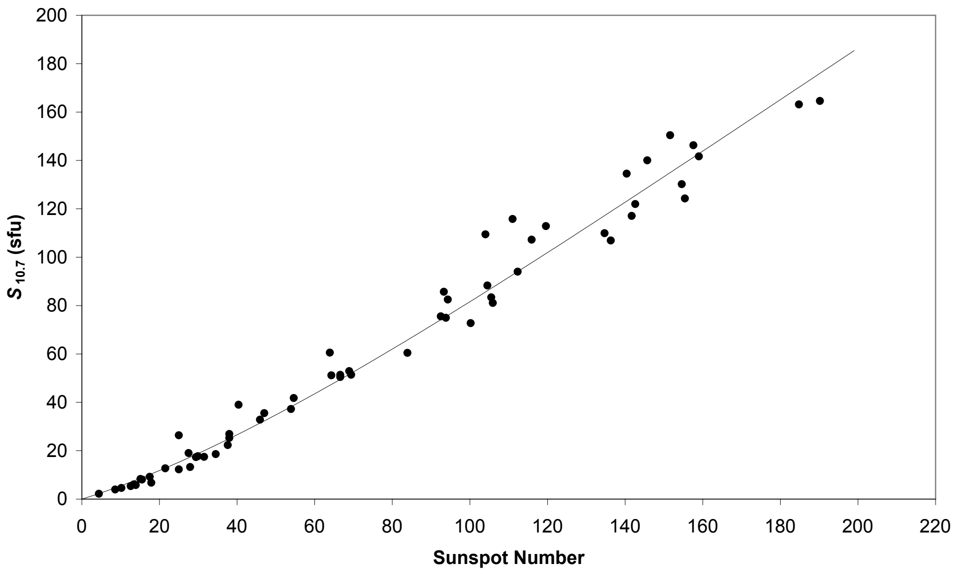

Figure 2. Annually-averaged values of the 10.7 cm solar radio flux ($F_{10.7}$) since 1947 plotted against sunspot number.

3.2. Magnetic Flux

Figure 3 shows the strong-field magnetic flux over a solar activity cycle plotted against the S-component flux.The relationship between the two quantities is linear ($\Phi_S = \psi S_{10.7}$, where $\psi = 0.446$), and with a high correlation coefficient ($R^2 = 0.99$).

The rate of change of the weak-field magnetic flux can be described by an equation of the form:

$$\frac{\partial \Phi_W}{\partial t} = E_W - S_W + I_W - L_W, \tag{2}$$

where E_W and S_W are rates of emergence and submergence of magnetic flux into that structure class, I_W is the input from the decay of strong-field elements, and L_W is the loss by dissipation (such as cascading to ever-smaller scales and eventual reconnection). A truly representative model would involve establishing all four functions individually. However, it is not really practicable in this case, and simplifications are needed. Assuming all

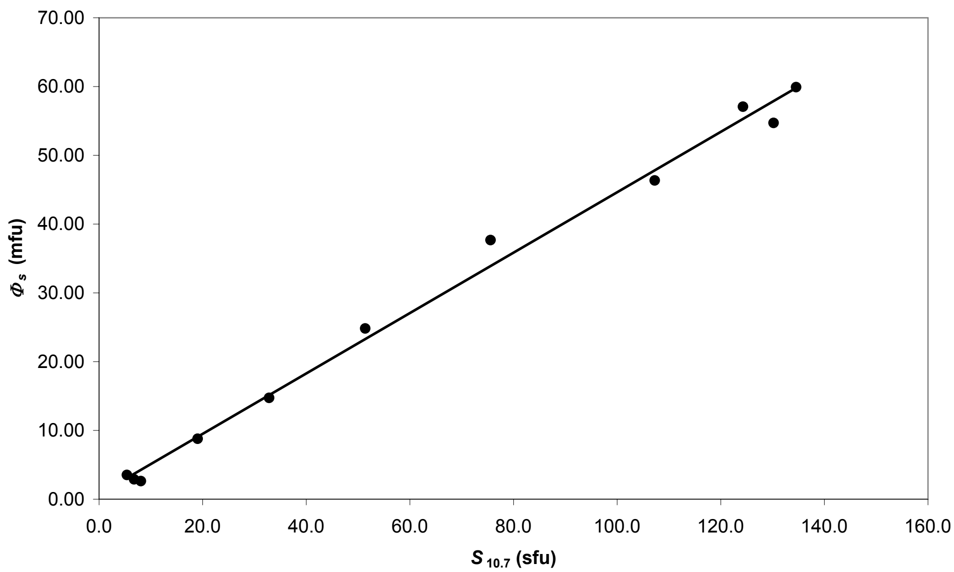

Figure 3. Annually-averaged values of the strong-field magnetic flux plotted against similarly-averaged values of the 10.7-cm S-component flux, together with a fitted line.

the magnetic flux that emerges as weak-field flux elements submerges again, and all the weak-field flux that comes from fragmentation continues the process until it is dissipated, the weak-field magnetic flux can be separated into two distinct components $\Phi_{W,1}$ and $\Phi_{W,2}$. The absence of a phase shift suggests that $E_W - S_W$ can be approximated using the time-differential of a standard solar activity index, so $\Phi_{W,1} = \alpha\Phi_S$, where α is a constant.

If active regions fragment mainly around the periphery, the rate of fragmentation will be related to the perimeter distance, which would in turn be related to $A^{1/2}$, and $\Phi_S \propto A_S$. If the dissipation is exponential:

$$\frac{\partial\Phi_{W,2}}{\partial t} = a\Phi_S^{1/2} - \beta\Phi_{W,2}. \tag{3}$$

The evolutionary timescales of active regions are typically one or two solar rotations. Over a year the time-differential would be small, so $\Phi_{W,2} = (a/\beta)\Phi_S^{1/2}$, so combining the two contributions:

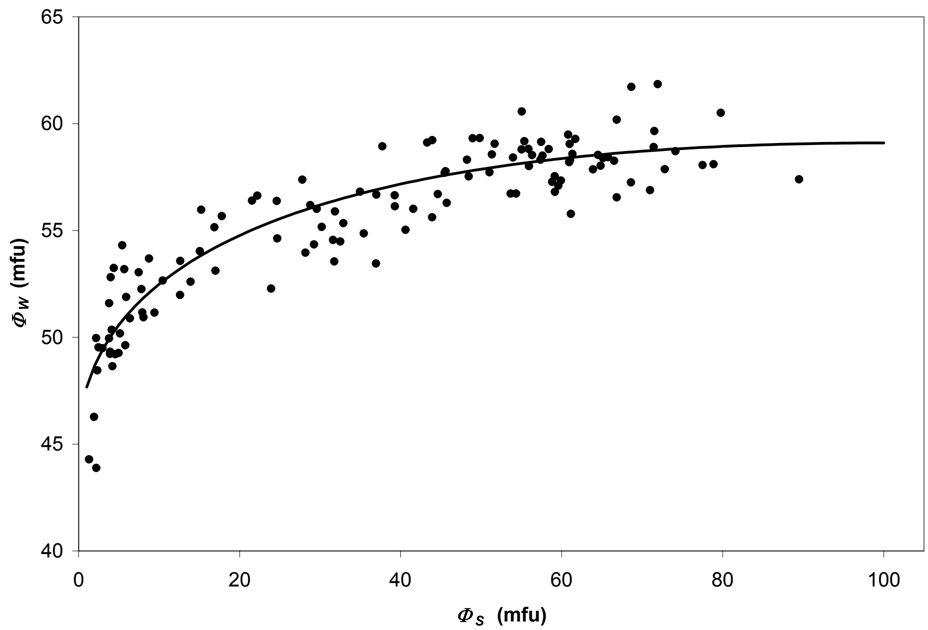

Figure 4. Rotationally-averaged values of the weak-field magnetic flux plotted against the strong-field magnetic flux, with the fitted model. The assumed constant background of 45 mfu is included in the weak-field flux.

$$\Phi_W = \alpha \Phi_S + \frac{a}{\beta} \Phi_S^{1/2} + \Phi_0. \tag{4}$$

The background magnetic flux Φ_0 is a constant of integration and represents a component of the weak-field magnetic flux that can be assumed constant over the calculation timescale. Estimated by inspection, $\Phi_0 \approx 45$ mfu.

A fit to the observed data yields $a/\beta = 2.81$ and $\alpha = -0.14$. The fitted model and rotationally-averaged weak and strong magnetic fluxes are shown in Figure 4.

Figure 5 shows the estimated strong and weak-field magnetic fluxes since 1600 derived from the sunspot number data. To simplify the plot, the constant background of about 45 mfu has not been included.

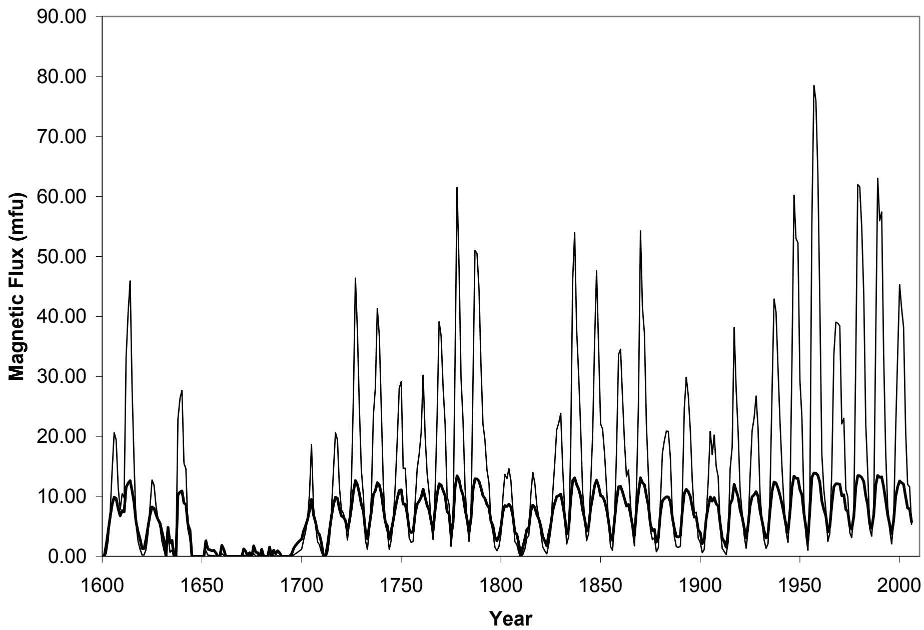

Figure 5. Estimated strong and weak-field magnetic fluxes since 1600, derived from the sunspot number record. The thin line showing the higher amplitude variation represents the strong-field magnetic flux, and the thick line showing a lower amplitude variation represents the weak-field magnetic flux. The assumed constant background flux of 45 mfu is not included in the plot.

3.3. Irradiance

Figure 6 shows annually-averaged total irradiance and $F_{10.7}$ plotted against time. Two points stand out: firstly there is the variation in total irradiance over the solar activity cycle, as exemplified by $F_{10.7}$, and there is a downward trend in the irradiance minima of about 0.006 W m^{-2}.year^{-1}.

This can be described in terms of a two-component model: $I(t) = I_a(t) + I_\odot(t)$, where $I_a(t)$ the irradiance contributions from active region structures and $I_\odot(t)$ comes from a generally-varying brightness of the photosphere outside the active regions. This "disc" irradiance is the equivalent of the secular irradiance component discussed elsewhere, but since the disc irradiance is associated with a specific model approach, whereas the secular irradiance

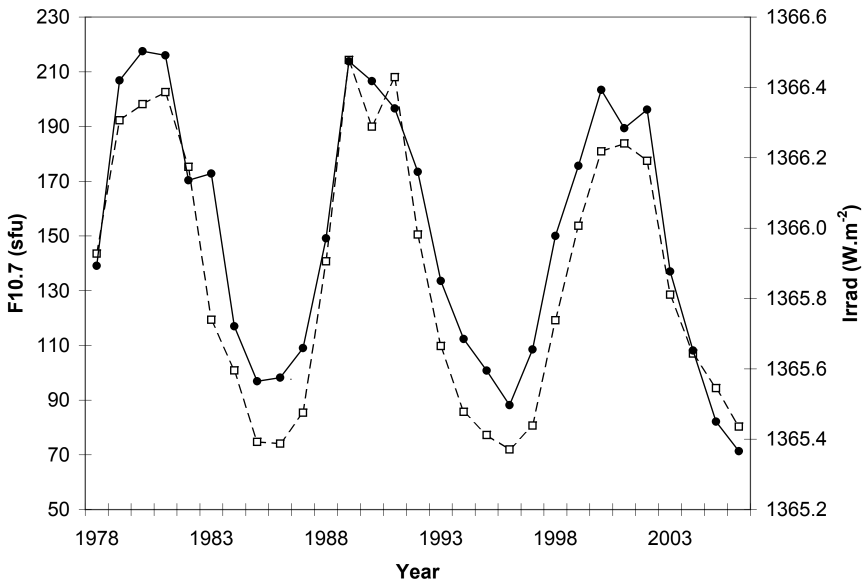

Figure 6. Annually-averaged values of the 10.7 cm solar radio flux (dashed line and open squares) and total irradiance (solid line and circles) plotted against time.

component is defined independent of models, we use the disc irradiance here. The variation of irradiance following the rhythm of the solar activity cycle is assumed to be related to the formation and decay of active regions. Since the variations between the irradiance and the 10.7 cm solar radio flux are in-phase, the background cannot be due to decay of active regions, because the observed variation showed by a line through the irradiance minima in the plot would require a decay time constant big enough to generate an immediately-visible phase delay between irradiance and $F_{10.7}$.

With the high correlation between the activity index and irradiance, together with the absence of any phase delay between them comparable with or larger than the one-year averaging, the baseline slope is assumed to be another phenomenon, so that the irradiance model would be of the form:

$$I(t) = I_a(t) + I_{\odot}(t). \tag{5}$$

In terms of the areas of active structure classes:

$$I(t) = \sum_i (\epsilon_i - \epsilon_\odot) A_i(t) + I_\odot(t), \tag{6}$$

where ϵ_i is the emissivity of the ith class of active structure and $\epsilon_\odot$ is the emissivity of the surrounding photosphere. The area of each class of active structure is proportional to the total magnetic flux, so $A_i \approx \Phi_i/\bar{B}_i$, where $\bar{B}_i$ is the average magnetic field strength in that active region. So we can define a new variable combining these parameters, which describes the increase in irradiance from the active region compared with the disc background:

$$\eta_i = \frac{\epsilon_i - \epsilon_\odot}{\bar{B}_i}. \tag{7}$$

The model then becomes:

$$I(t) = \sum_i \eta_i \Phi_i(t) + I_\odot(t). \tag{8}$$

In the case of there being two classes of magnetic structure, strong-field and weak-field structure:

$$I(t) = \eta_S \Phi_S(t) + \eta_W \Phi_W(t) + I_\odot(t). \tag{9}$$

3.3.1. $I_\odot(t)$

The formation and decay of active regions marks the emergence of magnetic flux from and submergence back into a reservoir of sub-photospheric magnetic flux. This reservoir varies over much longer timescales than the evolution of active regions, and can modulate the transfer of heat to the photosphere.The energy that is radiated into space from photospheric magnetic structures has first to find its way from the Sun's core to the photosphere, through the reservoir of sub-photospheric magnetic flux feeding the emergence and absorbing the submergence of photospheric magnetic flux. One possible modulation mechanism could be the role this magnetic flux would have in the pressure balance below the photosphere, causing small changes in structure of the Sun and small changes in photospheric temperature. Spruit (2000) showed the thermal time constant increases with depth below the photosphere, so there could be considerable smoothing of the variations in the rate of radial heat outflow. As a rough approximation, we assume the total amount of magnetic flux in this "reservoir" would be related to one of the standard solar activity indices that has been low-pass filtered to remove the strong modulation by the solar activity cycle. This was done by applying a three-point running average 50 times in succession to the modelled $S_{10.7}$ record. The resulting filtered record is shown in Figure 7.

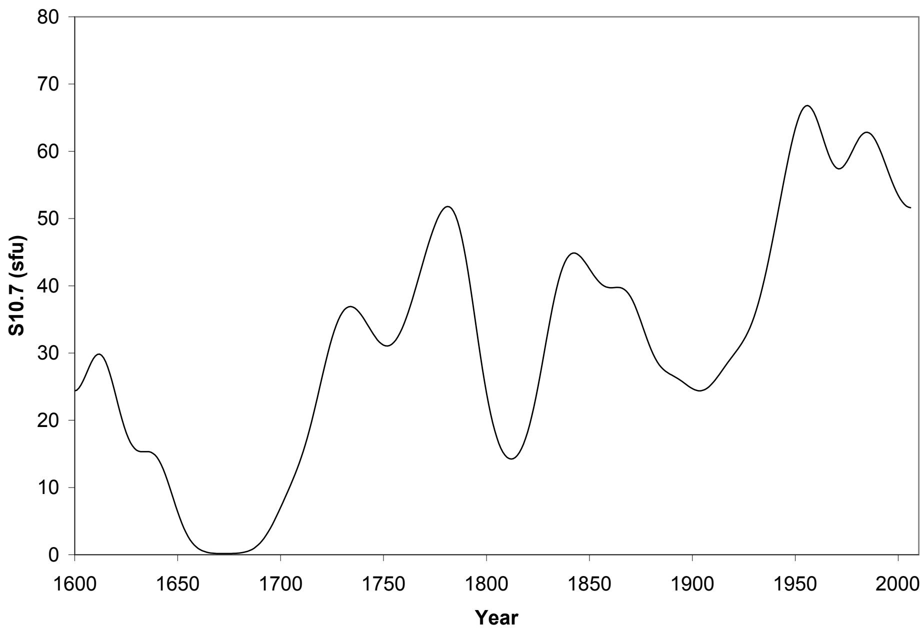

Figure 7. The variation in modelled $S_{10.7}$ since 1600 after filtering to remove the modulation by the solar activity cycle.

For small degrees of modulation, and considering that we do not have what can be considered a real *model*, we assume a linear model for $I_{\odot}$ in terms of the smoothed activity variation:

$$I_{\odot}(t) = I_0 + \xi \mathcal{S}_{10.7}, \qquad (10)$$

where I_0 is some constant "base" irradiance contribution, $\mathcal{S}_{10.7}$ is the smoothed version of $S_{10.7}$ and ξ is a constant parameter.A regression analysis gives $\xi = 0.017$ W m^{-2}.sfu^{-1}, and $I_0 = 1364.5$ W m^{-2}. Figure 8 shows the total irradiance measurements plus the estimated disc irradiance, which forms a baseline.

We can use this equation to remove the effect of the variation in $I_{\odot}$ from $I(t)$, leaving $I_a(t)$, the variation due to active regions.

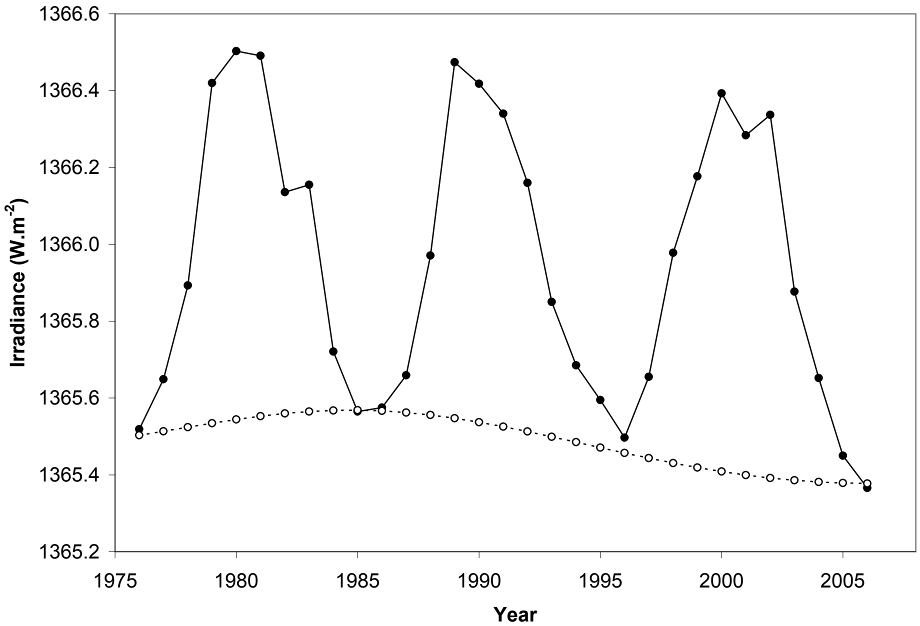

Figure 8. Total irradiance measurements plus estimated disc irradiance since 1976.

3.3.2. *Total Irradiance*

The irradiance contribution by the active regions, I_a, is given by subtracting the disc irradiance $I_\odot$ from the observed irradiance I. Fitting the magnetic flux model, we obtain $\eta_S = 0.015$ and $\eta_W = 0.003$ W m^{-2}.mfu^{-1}, and the total irradiance model is:

$$I_a(t) = 0.015\Phi_S(t) + 0.003\Phi_W(t) \tag{11}$$

and

$$I_\odot = 0.017\mathcal{S}_{10.7} + 1364.5. \tag{12}$$

The total irradiance $I = I_a + I_\odot$. Using this model, the estimated values of $I_\odot$ and $I = I_a + I_\odot$ since 1600 are shown in Figure 9.

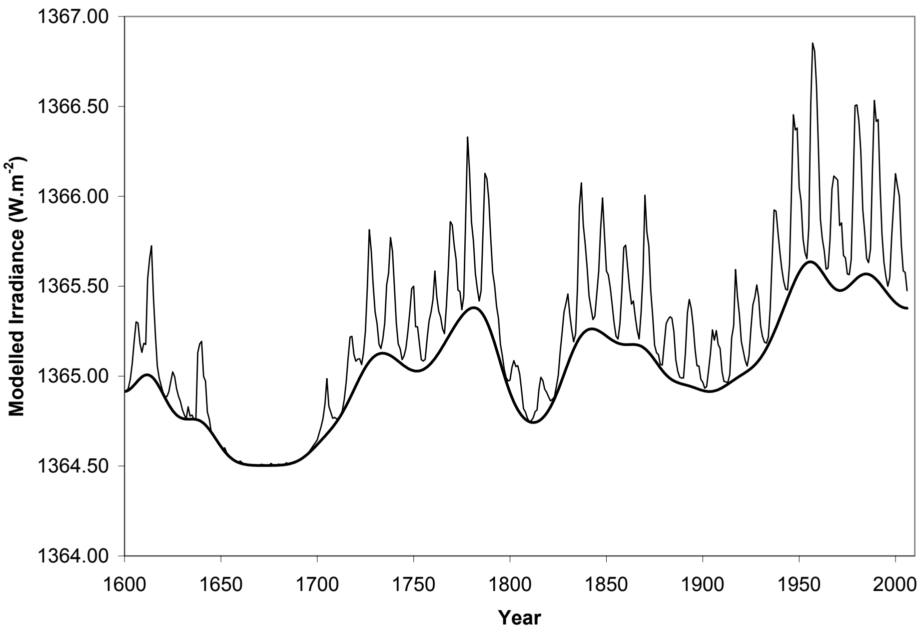

Figure 9. Modelled total irradiance and disc irradiance since 1600.

Using the model estimation process described above, the total irradiance during the Maunder Minimum is estimated to have fallen to 1364.5 $\pm$ 0.4 W m^{-2}.

4. Estimation of Errors

A highly-empirical process of estimation like that described here, depending heavily upon proxies, fits and extrapolation, present many opportunities for errors to be made. However, systematic errors will tend to vanish by being incorporated into fits, leaving the errors arising being mostly in the fitting of coefficients. The errors were estimated by regression analysis. In cases where a simple linear model between two parameters was applied, the regression line fit could be applied directly. In cases where the relationship involved two or three independent parameters, the errors of which could not easily be determined independently (*e.g.* the summation of irradiance contributions, or where the relationship is non-linear *e.g.* $\Phi_W = \alpha\Phi_S + \frac{a}{\beta}\Phi_S^{1/2} + \Phi_0$), the error was estimated by plotting the modelled parameter against the observed one, which should always produce a linear relationship. In all cases the percentages represent the 90% confidence range. The error for I_0 was estimated as about 0.05%, and is treated as negligible compared with the other errors.

The estimated errors are listed in the table below.

Parameter	Quantity	Expected Value	Error
$\Phi_S = \psi S_{10.7}$	ψ	0.446	5%
$\Phi_W - \Phi_0 = a_W(\alpha\Phi_s + a/\beta\Phi_S^{1/2})$	a_W	1	20%
$I_a = a_I(\eta_S\Phi_S + \eta_W\Phi_W)$	a_I	1	20%
$I_\odot = a_\mathcal{S}\xi\mathcal{S}_{10.7} + a_0 I_0$	$a_\mathcal{S}$	1	35%
	a_0	1	small

The coefficients are assumed to have errors that have Gaussian distributions with standard deviations (as percentages of the mean value) equal to the values listed. The coefficients listed above have errors added that are sampled from these Gaussians. A constant is then added to bring the plot through the reference point at 2000 AD. The model was run to produce a list of irradiance values since 1600, and the process was repeated 10^5 times and a standard deviation calculated for the error scattered values for each year. Figure 10 shows a central plot showing values where no error is assumed, and the lines one standard deviation above and below, outlining the estimated error band.

5. Discussion and Conclusions

According to the model discussed here, the total irradiance during the Maunder Minimum was about 1364.5 ± about 0.4 W m^{-2}. Compared with the average measured irradiance over the period 1996-2006 (inclusive), this is a decrease of about 0.1%. There have been many other attempts to model historical irradiance, using a wide variety of models. These, together with some of the underlying physics, are discussed by Lockwood (2004). Using the same definition as above, these indicate irradiance reductions during the Maunder Minimum ranging from 0.1 to 0.4%. The results of the model described here fall into the range embracing the other models.

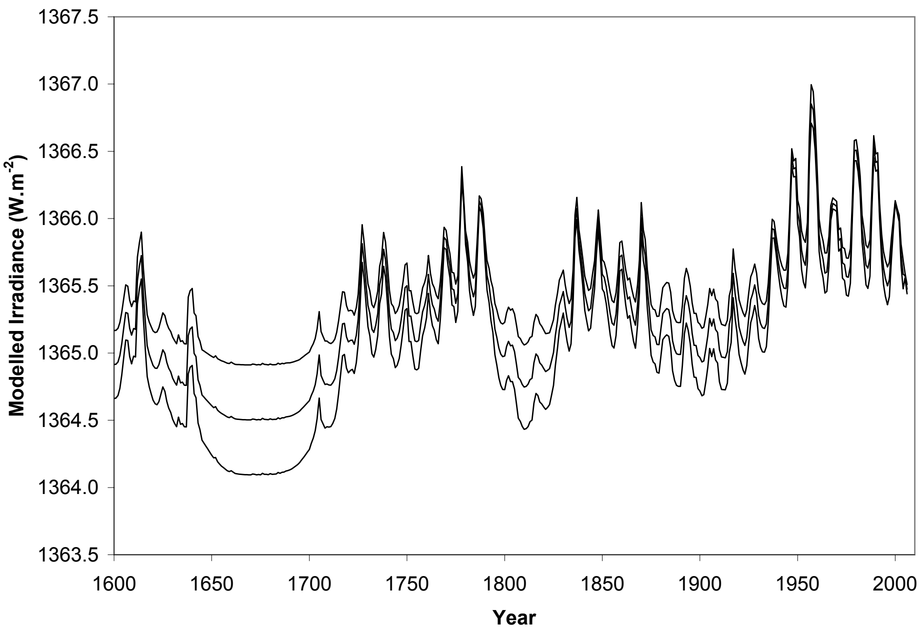

Figure 10. The central (inner) line shows the modelled irradiance values and the thin lines above and below lie one standard deviation from the expected values.

Following the terminology used by Solanki and Krivova (2004), the varying total irradiance can be decomposed into two components: a *cyclic component*, describing modulation of irradiance by the solar activity cycle, and a *secular component*, describing longer term (over time scales longer than a solar activity cycle). The characteristic timescale of the secular is sufficiently longer than the solar activity cycle, so plotting total irradiance against activity indices such as sunspot number yields a noisy but linear relationship, without signs of loops due to time-delayed components. The disc irradiance component used in this model would be the equivalent of the secular component. However the secular component, as defined, is not associated with any particular modelling approach.

Although the cyclic and secular components are generally modelled separately, they are often modelled from the same roots, as is the case in this paper and in calculations done by for example Lean *et al.*, 1998, and Lean

(2000), where the cyclic component was estimated using sunspot number as a proxy, and the secular component estimated from smoothed sunspot number.

There are two main schools in irradiance modelling. One school assumes irradiance variability is a photospheric process, involving the changing emissivities of the various magnetic structures associated with solar activity, such as faculae, sunspots and elements of the active network (*e.g.* Foukal and Lean, 1990; Lean *et al,* (1998, 1999 and 2000). Others examine more global variability, due to very small changes in the photospheric temperature outside active regions. This might involve small changes in the photospheric radius (Delache, Lacrare and Sadsaout, 1986; Ulrich and Bertello, 1995; Antia, 2003), large convective cells and modulation of radial energy transfer (Ribes *et al,*, 1985; Fox and Sofia, 1994; Sofia, 2004; Sofia and Li, 2004). The model discussed in this paper is a hybrid, in that the cyclic component comes from active region structures and the secular component from modulation of radial energy transfer due to a reservoir of sub-photospheric magnetic flux.

The presence of the photosphere, chromosphere and corona is due to magnetic flux. That this structural regime remains stable over solar activity cycles suggests the total amount of magnetic flux below the photosphere is significantly larger than the amounts of photospheric magnetic flux we observe over the solar activity cycle. This, together with the observed stability of large-scale activity patterns and complexes of activity (Gaizauskas *et al.*, 1983) and "sunspot nests" (Castenmiller, Zwaan and van der Zalm, 1986) suggests they are firmly anchored in a reservoir of sub-photospheric flux containing significantly more magnetic flux than we see at and above the photosphere.

In the case of solar-like stars, magnetic flux is generated mainly at the tachocline, rather than in the overlying convection zone (see discussion in Giampapa, 2004). The magnetic flux reservoir in the convection zone receives new magnetic flux rising from the tachocline, and loses it through emergence above the photosphere and possibly through reconnection. It also receives input through submergence of magnetic flux from above the photosphere. The total amount of magnetic flux in the tachocline and in the overlying convection zone is suggested to be much higher than the total amount of magnetic flux visible at the photosphere. For example, a magnetic field of 10^4 Gauss pervading the equatorial half of a tachocline alone with a thickness of $0.05\,R_\odot$ contains up to 400 mfu of unsigned magnetic flux. Estimating the total amount of magnetic flux in the convection zone is more difficult, due to the limiting effects of buoyant and other losses. Using calculations by Parker (1994*a* and *b*) the average magnetic field strength in the convection zone is very roughly estimated to range between 10 and 100 Gauss. This would place between 10^3 and 10^4 mfu of unsigned magnetic flux in the convection zone. In contrast, the amount of magnetic flux emerging during the rising phase of an activity cycle is of the order of 60 mfu (c.f. Figure 1 herein).

The cyclic component of irradiance variability is due to modulation by the phenomena of the solar activity cycle, and the secular component is suggested to be due to modulation of energy flow by the slower variations in the total amount of sub-photospheric magnetic flux, which would cause very small changes in the temperature of the photosphere. The temperature changes producing significant changes in the disc irradiance would be too small to detect in indices such as the 10.7 cm solar radio flux. For example, an increase in disc temperature of about 10 Kelvins would add about 0.9 W m^{-2} to the irradiance, but less than 0.02 solar flux units to the 10.7 cm solar radio flux. This is about 0.03% of the quiet sun flux value, and far too small to measure.

The only direct index of solar activity extending back to the Maunder Minimum and possibly beyond is sunspot number. The 10.7 cm solar radio flux record began in 1947. This 60-year overlap can be used to address a slight non-linearity in the relationship between sunspot number and magnetic flux components. Consequently, many models (*e.g.* Lean *et al.*, 1998, 1999 and 2000) use an index of solar activity, such as sunspot number or 10.7 cm solar radio flux as a proxy for the cyclic component of irradiance variation and a smoothed version (using a running mean) of the record of that index as a proxy for the secular component *i.e.*:

$$I = I_{cyclic} + I_{secular} = f(x) + \int_{t-\tau/2}^{t+\tau/2} g(x).dt \qquad (13)$$

where x is an index of solar activity. With the fitting, largely empirical approaches, relatively crude physical models and many uncertainties, the functions $f(x)$ and $g(x)$ are relatively simple, either linear or simple polynomials, regardless of the underlying model, it is possible the differences in irradiance during the Maunder Minimum predicted by the various models are at least largely due to the difficulty in dealing with the secular component rather than fundamental differences between the models.

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