Report on the Evaluation of Appendix IV

# Analytical Equations to Generate the Free Electron Current-Vector Field and the Angular-Momentum-Density Function $\mathbf{Y}_{0}{ }^{\mathbf{o}}(\boldsymbol{\theta}, \boldsymbol{\phi})$ in <br> "The Grand Unified Theory of <br> Classical Physics" by Dr. Randell L. Mills 

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## Executive Summary

In my analysis, I verified calculations and equations involving the calculation and generation of the BECVF and OCVF current vector fields (CVFs) found in Appendix IV of the book "The Grand Unified Theory of Classical Physics" (January 2020 edition) by Dr. Randell L. Mills.

There is a remarkable agreement between the equations found in the chapter and the equations I get from my calculations. I verified that all the equations found in the chapter from Equation (1) through Equation (32) were in fact true, and reproducible.

## Purpose

In Appendix IV, putting the electron current in the counter-clockwise direction, the Larmor precession of the angular momentum vector of the free electron is seen to be about two axes simultaneously: the ( $\mathbf{i}_{x}, 0 \mathbf{i}_{\mathbf{y}}, \mathbf{i}_{\mathbf{z}}$ )-axis and the lab-frame z -axis (which is defined by the direction of an applied magnetic field). The motion generates CVFs (current vector fields), with the first motion sweeping out a BECVF and the rotation about the z-axis sweeping out an OCVF. The combined motion is a convolution of the BECVF with the OCVF.

One motion is a $2 \pi$ rotation about the ( $\mathbf{i}_{\mathrm{x}}, 0 \mathbf{i}_{\mathrm{y}}, \mathbf{i}_{\mathrm{z}}$ )-axis whereby the angular momentum vector of the free electron sweeps out a cone about the ( $\mathbf{i}_{x}, 0 \mathbf{i}_{y}, \mathbf{i}_{z}$ )-axis. The rotational matrix about the ( $\mathbf{i}_{x}, 0 \mathbf{i}_{y}, \mathbf{i}_{z}$ )-axis is given in this appendix, then evaluated based on the rotational matrices in Equations (1.81-1.82). Next, the BECVF convolution is given. From this, the integral form of the convolution is given, along with the infinite sum of great circles that make up the BECVF. Figure IV. 1 shows the current pattern generated.

The rotation of the free-electron disc formed by the variation of $\rho$ in a continuous manner forms two conical surfaces that join at the origin and face in opposite directions along the ( $\mathbf{i}_{x}, 0 \mathbf{i}_{y}, \mathbf{i}_{z}$ )-axis, as shown in Figure IV.2.

Next is considered the Larmor precession of the free electron about the z-axis by a rotation of $2 \pi$ about the $\left(-\mathbf{i}_{x}, 0 \mathbf{i}_{y}, \mathbf{i}_{z}\right)$-axis. The rotational matrix about the ( $-\mathbf{i}_{x}, 0 \mathbf{i}_{y}, \mathbf{i}_{z}$ )-axis is given in this appendix, then is evaluated based on the rotational matrices in Equations (1.81-1.82). Next the BECVF convolution is given. And from this, the integral form of the convolution is given, along with the infinite sum of great circles that make up the BECVF. Figure IV. 3 shows the current pattern thus generated.

The rotation of the free-electron disc formed by the variation of $\rho$ in a continuous manner forms two conical surfaces that join at the origin and face in opposite directions along the $\left(-\mathbf{i}_{x}, 0 \mathbf{i}_{y}, \mathbf{i}_{z}\right)$-axis, as shown in Figure IV.4.

Next, the momentum-density function $\mathrm{Y}^{\circ}{ }^{\circ}(\theta, \phi)$ is obtained by convolving the BECVF with the OCVF for the first case of the $\left(\mathbf{i}_{\mathbf{x}}, 0 \mathbf{i}_{y}, \mathbf{i}_{z}\right)$-axis. This operation is the same as incrementally rotating the BECVF about the z -axis by $2 \pi$.

Similarly we can also repeat this same procedure for the ( $-\mathbf{i}_{\mathrm{x}}, 0 \mathbf{i}_{\mathrm{y}}, \mathbf{i}_{z}$ )-axis we considered that is, the second case we considered above.

Next are considered matrices to visualize the momentum-density $\mathrm{Y}_{0}{ }^{\circ}(\theta, \phi)$ for the combined precession motion of the free electron about the ( $\mathbf{i}_{x}, 0 \mathbf{i}_{y}, \mathbf{i}_{z}$ )-axis and the z-axis. The coordinates of the great circle basis element to generate the OCVF are given in this appendix. The OCVF is generated by rotating this basis element great circle about the zaxis using $\mathrm{R}_{z}(\theta)$ over the range of $2 \pi$. The OCVF matrices are included in this appendix. And using them, the infinite sum of great circles representation of the OCVF is given. Figure IV. 5 shows the current pattern generated.

The great-circle distribution $\mathrm{Y}_{\mathrm{o}}{ }^{\circ}(\theta, \phi)$ is then generated by the convolution of either BECVF we generated with its corresponding OCVF over the range of $2 \pi$. The corresponding BECVF replaces the great circle basis element now to form $\mathrm{Y}_{0}{ }^{\circ}(\theta, \phi)$. The result is a $\mathrm{Y}_{0}{ }^{\circ}(\theta, \phi)$ that is azimuthally symmetric about the $z$-axis. This azimuthally symmetric property is proven explicitly in the next-to-last section of this appendix - the section on the Azimuthal Uniformity Proof of $\mathrm{Y}_{\mathrm{o}}{ }^{\circ}(\theta, \phi)$.

The $\mathrm{Y}_{\mathrm{o}}{ }^{\circ}(\theta, \phi)$ is formed by convolving the OCVF with the BECVF. The matrix representation of the convolution (involving a series of delta functions) is given next in this appendix. Then the integral form of the same convolution is given. Lastly, the integration results in the infinite double sum of great circles that yields $\mathrm{Y}_{0}{ }^{\circ}(\theta, \phi)$. This produces a continuous distribution.

Next a discrete representation of $\mathrm{Y}_{0}{ }^{\circ}(\theta, \phi)$ can be generated. This representation is shown in Equation (18) and the corresponding momentum density $\mathrm{Y}_{0}{ }^{\circ}(\theta, \phi)$ is shown in Figures IV. 6 and IV.7. Computer modeling to generate the free electron CVFs and the azimuthally-symmetric $\mathrm{Y}_{0}{ }^{\circ}(\theta, \phi)$ are available at the Brilliantlightpower.com website [Reference 3 for this Appendix.]

Next are considered matrices to visualize the momentum-density $\mathrm{Y}_{0}{ }^{\circ}(\theta, \phi)$ for the combined precession motion of the free electron about the ( $-\mathbf{i}_{\mathrm{x}}, 0 \mathbf{i}_{\mathrm{y}}, \mathbf{i}_{z}$ )-axis and the z -axis. The coordinates of the great circle basis element to generate the OCVF are given in this appendix. The OCVF is generated by rotating this basis element great circle about the zaxis using $\mathrm{R}_{z}(\theta)$ over the range of $2 \pi$. The OCVF matrices are included in this appendix. And using them, the infinite $\mathrm{Y}_{0}{ }^{\circ}(\theta, \phi)$ sum of great circles representation of the OCVF is given. Figure IV. 8 shows the current pattern generated.

The great-circle distribution $\mathrm{Y}_{\mathrm{o}}{ }^{\circ}(\theta, \phi)$ is then generated by the convolution of the BECVF we generated with its corresponding OCVF over the range of $2 \pi$. The corresponding BECVF replaces the great circle basis element now to form $\mathrm{Y}_{\mathrm{o}}{ }^{\circ}(\theta, \phi)$ that is azimuthally symmetric about the z-axis. This azimuthally symmetric property is proven explicitly in
the next-to-last section of this appendix - the section on the Azimuthal Uniformity Proof of $\mathrm{Y}_{\mathrm{o}}{ }^{\circ}(\theta, \phi)$.

The $\mathrm{Y}_{\mathrm{o}}{ }^{\circ}(\theta, \phi)$ is formed by convolving the OCVF with the BECVF. The result is the infinite double sum of great circles that yields $\mathrm{Y}_{\mathrm{o}}{ }^{\circ}(\theta, \phi)$. This produces a continuous distribution.

Next a discrete representation of $\mathrm{Y}_{0}{ }^{\circ}(\theta, \phi)$ can be generated. This representation is shown in Equation (23) and the corresponding momentum density $\mathrm{Y}_{\mathrm{o}}{ }^{\circ}(\theta, \phi)$ are equivalent to those shown in Figures IV. 6 and IV.7.

Next this appendix includes a section on a proof of the azimuthal symmetry of $\mathrm{Y}_{0}{ }^{\circ}(\theta, \phi)$.
This appendix ends with a discussion of spin-flip transitions. The electron can flip between two spin states - one having the magnetic moment parallel to the z -axis and the other having the magnetic moment antiparallel to the z-axis. The BECVFs, OCVF, and $\mathrm{Y}_{0}{ }^{\circ}(\theta, \phi)$ developed earlier in this appendix apply to both states. The transition corresponds to a $\pm \pi$ rotation of the $\mathrm{Y}_{0}{ }^{\circ}(\theta, \phi)$ distribution about the x -axis using $\mathrm{R}_{\mathrm{x}}(\theta)$ given by Equation (1.80). The infinite double sum of great circles for this process is found in this appendix, based on the methods developed earlier in this appendix. The first representation describes a continuous distribution. However, a discrete representation can also be found, and is included at the end of this appendix. The representation of the current pattern generated are equivalent to Figures IV. 6 and IV.7, but with the current direction reversed.

## Calculations

I have verified that Equations (1)-(32) are in fact correct as listed in the GUTCP book.

## Conclusion

I was able to verify the results of Appendix IV in excellent agreement with my own calculations and derivations of equations. I successfully reproduced all of the equations and derivations found in Appendix IV, up through Equation (32).

This appendix concerned itself with the calculation and generation of the BECVF and OCVF current vector fields (CVFs). I find my results and calculations to be confirmation that the derivations and equations of Appendix IV are indeed valid, reproducible, and accurate.

