The Obama-Tribe "Curvature of Constitutional Space" Paper is Crackpot Physics

F. J. Tipler
Department of Mathematics and Department of Physics, Tulane University, New Orleans, LA 70118
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The Harvard Law School professor Laurence Tribe published a paper entitled "The Curvature of Constitutional Space," wherein he argued that the strict constructionist interpretations of the U.S. Constitution were obsolete, being based on a Newtonian world-view, and need to be replaced by a more modern relativistic and quantum mechanical world-view. I shall show on the contrary that in using general relativity and quantum mechanics, we have never left the Newtonian world-view. It was shown in 1923 by the greatest geometer of the twentieth century, Elie Cartan, that in Newtonian theory, gravity is curvature just as it is in general relativity. The greatest twentieth century theoretical physicist in Poland, Andrzej Trautman, showed in 1966 that the equations of general relativity are mathematically equivalent to Newtonian gravitational field equations interacting with the luminiferous æther. Physics Nobel Prize winner Lev Landau showed in the 1930's that the Schrödinger equation, the basic equation of quantum mechanics, is a special case of the Hamilton-Jacobi equation, proven in 1837 to be the most powerful formulation of Newtonian mechanics. Erwin Schrödinger himself proved that his equation had nothing to do with probabilities or fundamental uncertainties. Since it was demonstrated mathematically decades ago that twentieth century physics is Newtonian mechanics, then by Laurence Tribe's own argument, it follows that all objections to strict constructionism are without merit. Tribe's physics is not post-Newtonian but pre-Newtonian, the physics of Aristotle, in which the arbitrary will of the powerful is the dominant influence in reality. Tribe's politics is, like his physics, profoundly reactionary, replacing unalterable law with the ever changing personal preferences of judges. As I shall demonstrate, the recent *Boumedienne vs. Bush* decision is a particularly egregious example of such replacement. Furthermore, Tribe's main books on Constitutional law are adversely influenced by his bad physics.

The key thesis of the paper "The Curvature of Constitutional Space," written by Laurence Tribe with the assistance of the then editor in chief of the Harvard Law Review B. Obama, is contained in the opening three sentences of its abstract: "Twentieth-century physics revolutionized our understanding of the physical world. Relativity theory replaced a view of the universe as made up of isolated objects acting upon one another at a distance with a model in which space itself was curved and changed by the presence and movement of objects. Quantum physics undermined the confidence of scientists in their ability to observe and understand a phenomenon without fundamentally altering it in the process [1]."

All three of these sentences express complete nonsense. In Newtonian theory, gravity is space-time curvature just as it is in general relativity. In fact, Einstein's general relativity is just a special case of Newtonian gravity theory incorporating the æther. Quantum physics is also just a special case of Newtonian mechanics in its wave-particle formulation (called Hamilton-Jacobi theory) incorporating the very modest requirement that this formulation be mathematically consistent. There was absolutely nothing revolutionary about twentieth century physics. There has been no "paradigm shift" in physics. The magnificent intellectual edifice created by Isaac Newton stands unshaken. The center holds [54].

In other words, I shall demonstrate that [1] is a crackpot paper. Understanding exactly why it is a crackpot paper is important for experts in constitutional law, because, as Obama and Tribe themselves emphasize (in one passage with which I heartily agree) "How we think about these institutions [e.g., the court system, and constitutional law] has been fundamentally influenced by new insights into the operation of the physical world ([1], p. 2)." But if these "new insights" are in complete error, then it is exceedingly likely that "how we think about these institutions" is also likely to be in complete error.

Obama and Tribe assert: "To search the sciences for authoritative answers to legal questions, *or any questions for that matter* [my emphasis], is misguided. The formalist philosophy which views science as a 'collection' of the 'proven' or even of the 'provable' is based upon an inappropriate reification. The better vision of science is as a continual and, above all, critical exploration of fruitful insights; the better metaphor is that of a journey. Science is not so much about proving as it is about *im*proving. [Obama-Tribe's emphasis]. To look to the natural sciences for authority — that is, for certainty — is to look for what is not there ([1], p. 2)."

In this passage, and in the references they cite to support it, Obama and Tribe reveal a philosophical dependence on the view of science due to the philosophers David Hume, Karl Popper, and Thomas Kuhn. Their entire paper is permeated with this view. To cite just one example, on page 10 we find the expression "Within the majority's stilted pre-modern paradigm ..." and this word "paradigm" is used in the sense introduced by Thomas Kuhn. I shall therefore devote an entire section of this paper to refuting this view in detail. In particular, I shall demonstrate that Newtonian mechanics, although indeed not absolutely certain, is so nearly certain that its truth is a practical certainty. Newton got it right, and he got it right more than three centuries ago.

Obama and Tribe point out, correctly, that "Early in our nation's history it was commonplace, for example, to say that the 1787 Constitution was Newtonian in design, with its carefully counterpoised forces and counterforces, its checks and balances, structured like a 'machine that would go of itself' to meet the crises of the future ([1], p. 3)."

I shall demonstrate that to the extent that this is true — that the Constitution is indeed a Newtonian machine that would go of itself — then the Founders constructed well. Judges should not tamper with the Constitutional machine, especially if they no longer understand how it works, or understand how it was intended to work.

If you don't understand it, don't mess with it.

To demonstrate a paper to be "crackpot," I shall need a precise definition of "crackpot." I shall say that a paper that purports to use physics is a "crackpot" paper if it makes claims about physics, claims that are central to its thesis, that have been known to be false many years before the paper is written. Furthermore, I shall require that the erroneous claims have been pointed out to be erroneous in at least one textbook written by physicists universally recognized as experts in the field.

This definition is important because the unfortunate fact is, many professors of physics at Harvard have also written, and are continuing to write, crackpot papers on general relativity. The reason is, that most professors of physics at Harvard have never taken a course in general relativity, which is Einstein's theory of gravity, either as an undergraduate student or as a graduate student. Many professors of physics at Harvard, in other words, do not understand general relativity, even though they think they do, and are considered by non-physicists to understand general relativity. But alas, they do not.

I do understand general relativity because I was fortunate to have taken a course in general relativity taught by the great physicist Steven Weinberg while I was an undergraduate at M.I.T. in the late 1960's and another course in general relativity taught out of the textbook *Gravitation*, written by the three greatest experts in general relativity, Charles W. Misner, Kip S. Thorne, and John Archibald Wheeler. Misner and Thorne were themselves the students of Wheeler, and I myself later became a post-doctoral student of Wheeler. All the false claims in the Obama-Tribe paper are carefully pointed out to be false in *Gravitation,* published in 1973, many years before the Obama-Tribe paper was written. The claims were actually known to be false many decades before the Obama-Tribe paper was written, but they did not make many textbooks in general relativity because, once again, most textbooks were and are written by physicists who do not understand general relativity.

Notice that I am making an outrageous claim: most Harvard professors of physics are crackpots in physics. Far more likely, it would seem, that it is Frank Tipler, and not the Harvard professors, who is the crackpot. After all, I am only a professor of mathematical physics at Tulane University, and not a professor of physics at Harvard, the greatest and most famous university in the world. Furthermore, I have written several books which have been considered (by the Harvard physics professors) to make crackpot claims. My claims are not crackpot, but they are based on general relativity, more specifically on a subfield of general relativity, global general relativity, which very few physicists are taught.

In such a case, you should check my claims for yourself. Do not take anyone's word for anything. Fortunately, all my claims are easily checked, at least if you know high school mathematics.

Demonstrating that the Obama-Tribe paper is a crackpot paper necessarily requires mathematics. But as I said, the reader will need only what is now high school mathematics: you will need to know what a "partial derivative" is. In symbols, you will need to know what $\partial f/\partial x$ means. Einstein once said that to understand physics, all you really need to understand is partial derivatives, which is what these symbols mean. I shall show that Einstein was right. The supposedly advanced mathematical concept of "curvature," which is the central concept in Newton's theory of gravity, in Einstein's theory of gravity, and in the Obama-Tribe paper, is really just a way of organizing a vast number of partial derivatives. That is, of arranging partial derivatives in such a way that the limited human mind can grasp them.

In this paper, I shall state in words without mathematics roughly why the Obama-Tribe claims about the relationship between Newtonian and Einsteinian physics are wrong, and were known to be wrong before Obama was born. There is no excuse for either Obama or Tribe to be ignorant of these mathematical facts. The central results were in the textbooks before the Tribe paper was written. However, the reader should not take my word or anyone else's word for my claims. The reader should confirm my claims personally by going through the mathematics. I shall insert sections entitled "mathematical interlude" wherein the mathematics is presented. For reasons given above, the reader should not trust any authority about the validity of my claims. If you, the reader of this article, are unable to confirm or refute my claims by yourself, you are uneducated. It matters not how many degrees you have — you may have several Ph.D.'s — but you are uneducated if you cannot follow the very elementary mathematical arguments I shall give in this paper. If you cannot follow the simple mathematics in this paper, you cannot follow any intellectual argument. Mathematics is central to understanding the world we live in.

The opinions of those who can't count, don't count.

This has has always been true. Basic mathematical knowledge has always been considered a requirement for any serious reflection on any human endeavor. The greatest minds in history have expressed the same opinion. For examples:

"Let no one ignorant of Mathematics enter here." Plato, the greatest of the philosophers, is said to have inscribed these words above the entrance to his school, *Academe.*

The ancient Roman philosopher Boethius (480–525

C.E.) says in the Second Prologue to his book *Arithmetic*, "If an inquirer lacks the four parts of mathematics, he has very little ability to discover truth" [2]

The Medieval philosopher Roger Bacon (1214–1294 C.E.) wrote: "Of [all the] sciences the gate and key is mathematics ... Neglect of this branch now for thirty or forty years has destroyed the whole system of the Latins. Since he who is ignorant of this cannot know the other sciences nor the affairs of this world ..." [3], and also wrote: "Wherefore it is evident that if in other sciences we should arrive at certainty without doubt and truth without error, it behooves us to place the foundations of knowledge in mathematics ... [4]."

In his only book on the philosophy of science, *The Assayer,* the great Italian physicist Galileo wrote: "Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these, one wanders about in a dark labyrinth [5]."

The greatest mathematician of the ancient world, Euclid, also believed that a knowledge of mathematics was essential for political leadership, even if learning mathematics requires a great deal of effort. In his *Eudemiarz* Summary, Proclus (410-485 C.E.) tells us that Ptolemy Soter, the first King of Egypt and the founder of the Alexandrian Museum, patronized the Museum by studying geometry there under Euclid. He found the subject difficult and one day asked his teacher if there weren't some easier way to learn the material. To this Euclid replied, "Oh King, in the real world there are two kinds of roads, roads for the common people to travel upon and roads reserved for the King to travel upon. In geometry there is no royal road." Similar anecdotes about the importance of mathematical knowledge to kingship appear elsewhere in the surviving literature from Greece. Stobaeus has narrated it in connection with Menaechmus when serving as instructor to Alexander the Great [17].

It was the surprising lack of mathematical knowledge in "Curvature of Constitutional Space" that leads me to attribute the paper to both Tribe and Obama. Indeed, Tribe is listed as the sole author, and Obama is listed only on page 1, in a footnote as one of five who provided "analytic and research assistance." Laurence Tribe has been caught [7] plagiarizing other sources in key parts of his famous book *God Save This Honorable Court: How the Choice of Supreme Court Justices Shapes Our History* [6]. Tribe's admission of guilt, when combined with his earlier remarks (in the context of his discussion of plagiarism charges made against a colleague), makes it clear that the plagiarized passages were not Tribe's own contribution, but those of his research assistants. Tribe, like many Harvard professors these days [9], just puts his name on the papers which his assistants have researched and written. Tribe's paper demonstrates an appalling ignorance of elementary mathematics, which would be surprising if he himself had written the paper (or had read it) because Tribe had graduated *summa cum laude* in mathematics from Harvard in 1962, a time when Harvard provided a very good undergraduate education in mathematics. There is no way Tribe could be ignorant of partial derivatives, which is all the mathematics required to understand the arguments in this paper. I highlight Obama as the main culprit, because Tribe has publicly called Obama "the best student I ever had," thus presumably the most capable of the five research assistants, and, in the footnote, Tribe thanks the assistants for "analytic assistance." In other words, the assistants provided, by Tribe's own admission, "analysis", which is to say, some of the central ideas. Tribe was more explicit about Obama's contribution to his article when campaigning for him in 2007. According to the *Concord Monitor*, "Tribe called Obama the 'best student I ever had' and the 'most exciting research assistant.' He recalled Obama's ability to turn an abstract theoretical paper into language lawyers typically use 'so people don't think you're a pointy-headed conehead' [10]." The paper "Curvature of Constitutional Space" was the only paper Tribe published in the *Harvard Law Review* during Obama's tenure as Law Review president, so presumably it is this paper he was referring to when he effectively claimed Obama wrote the paper.

As regards people thinking "you're a pointy-headed conehead," I myself would rather be right than popular. One of my personal heros, the Nobel prize winning physicist Richard Feynman, expressed my attitude in the title [11] of one of his books: *What Do YOU Care What Other People Think?*. Mathematics is essential in order to think correctly about any important issue, even if knowledge of mathematics at the high school levels causes most law professors to think that you are "a pointy-headed conehead."

As regards plagiarism, I think the charge against Tribe was ridiculous [8], even it he did admit it, and I mention it only because it bears on who wrote the article I am discussing. What is important is not whether a passage is copied from someone else, but whether the passage is RIGHT. One of the central points in this paper is that virtually nothing I write in this paper is original. Almost every thing I assert here has been known before Obama was born. If anyone can find in this paper a passage that I have plagiarized, it will only strengthen my argument.

Let us consider two central passages in the Obama-Tribe paper where they describe the physics as they understand it, and where they get the physics wrong.

Obama and Tribe assert: "... the general theory of relativity has demonstrated, among other things, that the universe, as seen through a telescope, can be explained only by realizing that objects like stars and planets *change* the space around them — they literally 'warp' it — so that their effect is both complex and interactive ([1], p. 4)."

I emphasize once again that I shall demonstrate, on the contrary, that in Newtonian gravity theory exactly the same is true. Stars and planets in this theory also change the space around them. That is, in contradiction to the title of their paper, in Newtonian theory, gravity, like in general relativity theory, is curvature. I shall also demonstrate that the general theory of relativity is just a special case of Newtonian gravity theory. I shall show explicitly how one can rigorously derive the Einstein field equations of general relativity as a special case of the Newton gravity equations.

Obama and Tribe assert: "A second advance over Newtonian physics — quantum theory — also offers significant heuristic insights for legal analysis. One of the familiar postulates of quantum theory is the Heisenberg Uncertainty Principle, which exploded the assumption that, by taking enough care and remaining sufficiently uncoupled from the system, one could detect, with any desired degree of precision the behavior of all objects in the universe ([1], p. 17)."

I shall demonstrate that quantum mechanics was NOT an advance over Newtonian mechanics. Quantum mechanics is just a special case of Newtonian mechanics. It follows from this that any property of quantum mechanics, however counter-intuitive like the Uncertainty Principle, must be also a property of classical mechanics. But I shall also demonstrate that the Uncertainty Principle, though indeed a property of quantum mechanics, and hence necessarily a property of Newtonian mechanics, has nothing to do with humans interfering with the observed by the process of observing the observed. I shall demonstrate this by actually deriving the Uncertainty Principle from the Hamilton-Jacobi equation, constrained to be mathematically consistent. Interactions by the measuring apparatus never appear in the derivation. The quantum mechanical system, and any human who observes it, are totally deterministic Newtonian machines. Of course, if one omits from consideration some of the Newtonian machines which are coupled to the machine one is observing, then the future behavior of the observed machine is not determined by this less than total data. There is a limit to how much data one can obtain, but this limitation is due to mathematical consistency, and not due to human interaction with a quantum mechanical system.

Eighteenth century physicists were well aware that mathematical consistency would limit data collection, and hence predictability, even though the total Newtonian machine was deterministic. To predict the entire future of the universe, one would need, not only to collect the data, but also to locate this data inside a computer inside the universe itself. This tiny subset of the universe, in other words, would have to be exactly as complex as the entire universe itself. No one three centuries ago believed this. We should not believe it now.

Nevertheless, it is possible to reduce outside interference in a measurement to a minimum, so that fundamental properties of system can be measured to arbitrary precision. It is possible to measure the energies of the ground state of an atom with arbitrary precision, for example. The experimenter only has to decide what sort of interference with the observed he wishes to minimize.

The Founders framed the Constitution with the intent of minimizing interference from elite aristocrats who think that they know more than the people, who alone have sovereignty. For this reason, most power was placed in the political branches of government, precisely so that power would be under the control of the people. Even the power of the people was minimized, by making it exceedingly difficult to amend the Constitution. Amending the Constitution by a 5-4 vote of the Supreme Court does not minimize interference from elite aristocrats who are called "judges."

Obama and Tribe, in their analysis of quantum mechanics, claim in effect that in quantum mechanics, in contrast to classical Newtonian mechanics, a particle is both a wave and a particle. It is certainly true that in quantum mechanics a particle, a photon, the particle of light, is both a wave and a particle. But none other than Sir Isaac Newton himself, in his book *Opticks*, argued for the theory that light was simultaneously a wave phenomenon and a particle phenomenon. For instance in Query 17, Newton conjectured that a ray of a light particle was accompanied by a wave that guides the particle: ". . . and are not these Vibrations [waves] propagated from the point of Incidence to great distances? And do they not overtake the Rays of light, and by overtaking them successively, do they not put them into the Fits of easy Reflexion and easy Transmission described above? For if the Rays endeavor to recede from the densest part of the Vibration, they may be alternatively accelerated and retarded by the Vibrations overtaking them [15]."

Newton asserted this as a query (conjecture), because he did not know how to formulate mechanics in a way that would make light simultaneously a particle and a wave. This was, however, achieved [13] by the great Irish mathematical physicist Sir William Rowan Hamilton in 1834, and simplified by the great German mathematician Karl Gustav Jacob Jacobi in 1837 [14]. Their theory is appropriately called "Hamilton-Jacobi theory," and has been in all the textbooks of classical Newtonian mechanics (e.g. [12], p. 147) for nearly two centuries now. I shall show below that the central equation of quantum mechanics, the Schrödinger equation, is not a generalization of the Hamilton-Jacobi equation, but instead a *specialization.* In other words, all the mathematics of quantum mechanics is already present in this ancient equation: quantum mechanics is a special case of Newtonian mechanics.

Thomas Young, the Englishman who, along with the Frenchman Augustin-Jean Fresnel, established that light was a wave phenomenon in the early part of the nineteenth century, devoted the first few pages of his first paper [16] on the wave theory of light to a series of quotes from Isaac Newton's works, showing that Newton believed that light was both a particle and a wave, and that Newton also believed in the æther: " A luminiferous Ether pervades the Universe, rare and elastic in a high degree ([16], p. 14) ... All bodies have an attraction for the Ethereal Medium ([16], p. 21." Both Young and Newton thought that light could be both a particle and a wave, though lesser minds thought these properties contradictory. Newton and Young are correct, and understanding how all objects are both particles and waves is the key to understanding why we have never left Newtonian mechanics.

All physics is nothing but a series of footnotes to Newton.

The reader will need to understand a central fact that should be understood by professional mathematicians and physicists, even though most mathematicians and physicists do not: if two theories are mathematically equivalent, then they are the same theory. Only the language used to express the theories is different.

To take a very simple example, when I say $2 + 2 = 4$, I am saying exactly the same thing as when I say "two plus two equals four." In the former case, I am expressing a mathematical identity in Arabic numerals, in the latter case, I am expressing the same mathematical identity in standard English. I could have also have written "zwei und zwei ist vier." Once again, I have expressed the same mathematical identity, but this time in German. I also could write II + II = IV using Roman numerals.

Mathematical identities can be quite different in appearance. For example, the repeated decimal number $0.99999999\ldots$, where the 9's go on forever (this is the meaning of $\ldots$), is actually another way to write the number 1. To prove this, let us use a little algebra. Set $x = 0.99999999\ldots$, and then multiply both sides of this equation by 10, obtaining

$$10x = 9.99999999\ldots$$

Now subtract the original equation $x = 0.99999999\ldots$ from the above equation, obtaining

$$9x = 9$$

Dividing both sides of this new equation by 9 gives us $x = 1$, or

$$1 = 0.99999999\ldots$$

That wasn't hard, was it? You now see that your elementary knowledge of algebra is sufficient to enable you to prove something counter-intuitive [18].

For most people, it indeed seems counter-intuitive that $0.99999999\ldots$ could equal one. Until we think about it deeply — as we have just done — we tend to think that $0.99999999\ldots$ must really be smaller than 1. The reason for this is our intuition misleads us. We don't *really* believe that the 9's in $0.99999999\ldots$ go on forever, even if we claim that it does. In the back of our minds, if not actually expressed, is the idea that nothing can go on forever, and indeed, if the number of 9's in $0.99999999\ldots$ were finite, the number would in fact be less than 1.

The expression of a number in decimal notion has an important feature: it allows one to multiply two numbers together very easily, using the procedure we were taught in grammar school. In fact, this procedure is called the "grammar school algorithm" (an "algorithm" is the term mathematicians use for an effective procedure.) The way that everyone has been taught to multiply two numbers together was invented by Hindu mathematicians in the 5th century A.D. (approximately), and is a *much* easier way to multiply two numbers than trying to multiply using Roman numerals (try it!). However, computer scientists have discovered another way to multiply two numbers together, a way that is even faster if the two numbers are really big. This way involves fast Fourier transforms. (Don't ask; you'll never need to know the details. If you need to multiply two big numbers together, use a standard computer program like Mathematica or Maple. These programs have the fast Fourier transform technique already coded in.) However, the fast Fourier transform technique usually works better if the numbers are written in binary rather than decimal notation.

The point is, different ways of formulating a theory can yield different ways of thinking about reality, even if the two formulations are mathematically equivalent. For some types of problems, one way will be the most efficient technique, for another type, another technique will be more efficient.

The reason for the importance of this fact, the identity of two physics theories if they are mathematically equivalent, is that I am going to prove all my claims by proving equivalence. First, I shall show that in Newton's theory, gravity is curvature just as it is in Einstein's theory by re-expressing Newton's theory in the language of curvature. Second, I shall show that quantum mechanics is just a special case of Newton's mechanics in its most advanced mathematical formulation, a formulation which Newton himself, as I pointed out above, believed in but was unable to discover. Third, I shall show that the full theory of general relativity, Einstein's theory of gravity, is mathematically equivalent to a Newtonian gravity theory in which the matter is coupled in a special way to an æther that permeates all of space. As is well-known, nineteenth century physicists believed in the luminiferous æther.

The bottom line: contrary to what Obama and Tribe claim, we've never left Newtonian mechanics. There never was a revolution in physics in the twentieth century.

Mathematical Interlude

Proof that Newtonian Gravity is also Curvature

In Einstein's theory and also in Newton's theory, gravity is curvature. In both theories, curvature comes from an *affine connection*, generally written in terms of its coefficients $\Gamma^{\alpha}{}_{\beta\gamma}$, where both the superscripts and the subscripts take on the values t, x, y, or z, which are time, and the three spatial dimensions respectively. We say that Γ^{t}_{xy} is one component of the connection. There are 40 distinct such components, because the connection coefficients are symmetric in the two subscripts: $\Gamma^{\alpha}{}_{\beta\gamma} = \Gamma^{\alpha}{}_{\gamma\beta}$. Sometimes we will want to consider only the spatial components, and when we do, we shall so indicate by using a Latin letter rather than a Greek letter. Thus, $\Gamma^{\alpha}{}_{\beta i}$ will denote one of three possibilities: $\Gamma^{\alpha}{}_{\beta x}$, $\Gamma^{\alpha}{}_{\beta y}$, or $\Gamma^{\alpha}{}_{\beta z}$. It is also convenient to use the Einstein summation convention, which says that whenever you see the same letter repeated (generally once in a superscript and once in a subscript), this means that a summation of terms is assumed. For example,

$$\Gamma^{\mu}{}_{\beta\mu} \equiv \Gamma^{t}{}_{\beta t} + \Gamma^{x}{}_{\beta x} + \Gamma^{y}{}_{\beta y} + \Gamma^{z}{}_{\beta z} \tag{1}$$

where the symbol $\equiv$ means "equivalent to'." If it is Latin letters that are repeated in the superscript and subscript, then only the spatial components are summed over:

$$\Gamma^{i}{}_{\beta i} \equiv \Gamma^{x}{}_{\beta x} + \Gamma^{y}{}_{\beta y} + \Gamma^{z}{}_{\beta z} \tag{2}$$

Once we know the connection coefficients, we know the curvature, which is given by the *Riemann curvature tensor*:

$$R^{\alpha}{}_{\beta\gamma\delta} = \frac{\partial \Gamma^{\alpha}{}_{\beta\delta}}{\partial x^{\gamma}} - \frac{\partial \Gamma^{\alpha}{}_{\beta\gamma}}{\partial x^{\delta}} + \Gamma^{\alpha}{}_{\mu\gamma}\Gamma^{\mu}{}_{\beta\delta} - \Gamma^{\alpha}{}_{\mu\delta}\Gamma^{\mu}{}_{\beta\gamma} \tag{3}$$

Let us now recall the basic equation of Newtonian mechanics, Newton's Second Law of motion:

$$F^{i} = ma^{i} = m\frac{d^2x^i}{dt^2} \tag{4}$$

where F^i are the components of the force and a^i are the components of the acceleration, as are $\frac{d^2x^i}{dt^2}$. That is, I have written the components of the vector $\vec{a} = (a^x, a^y, a^z)$ as a^i where the superscript i can take on the values x, y, or z, for the three dimensions x, y, and z. If the force F^i is gravity, then if this force of gravity acting on a particle of mass m is due to a point particle of mass M located at the origin of coordinates,

$$F^{i} = m\Big[\frac{GM}{x^2+y^2+z^2}\Big]\Big[\frac{-x^i}{(x^2+y^2+z^2)^{1/2}}\Big] \tag{5}$$

where G is the gravitational constant. Equation (5) can be written

$$F^{i} = m\frac{\partial}{\partial x^i}\Big[\frac{GM}{(x^2+y^2+z^2)^{1/2}}\Big] \tag{6}$$

The quantity in brackets is the negative of what is called the *gravitational potential* Φ, because it is the gravitational potential energy, normalized to zero at spatial infinity. For Newtonian gravity generated by a distribution of matter, with a spatial density ρ which is a function of spatial position (that is, we have $\rho(x,y,z)$), the potential satisfies the Poisson equation:

$$\nabla^2\Phi \equiv \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2} = 4\pi G\rho \tag{7}$$

If we know the potential Φ, then we can combine the Second Law equation (4) with equation (6) to get

$$m\frac{d^2x^i}{dt^2} = -m\frac{\partial\Phi}{\partial x^i} \tag{8}$$

The crucial step that will allow us to show that Newtonian gravity is also curvature is to cancel out the mass m of the particle on both sides of the equation (8) which becomes

$$\frac{d^2x^i}{dt^2} + \frac{\partial\Phi}{\partial x^i}\Big(\frac{dt}{dt}\Big)^2 = 0 \tag{9}$$

where I have used the trivial identity $dt/dt = 1$ to write (9) in a very suggestive way.

Now Newton's Second Law is invariant under a linear re-scaling of the time, i..e, $\tau = at + b$, where a and b are constants, so in terms of this new time variable τ we have

$$\frac{d^2t}{d\tau^2} = 0 \tag{10}$$

$$\frac{d^2x^i}{d\tau^2} + \frac{\partial\Phi}{\partial x^i}\Big(\frac{dt}{d\tau}\Big)^2 = 0 \tag{11}$$

If we compare these two equations to the geodesic equation

$$\frac{d^2x^\alpha}{d\tau^2} + \Gamma^{\alpha}_{\beta\gamma}\frac{dx^\beta}{d\tau}\frac{dx^\gamma}{d\tau} = 0 \tag{12}$$

we see that the factor $\partial\Phi/\partial x^i$ is really just an affine connection coefficient for curvature:

$$\Gamma^{i}_{tt} = \frac{\partial\Phi}{\partial x^i} \tag{13}$$

and all other affine connection coefficients are zero.

Now the geodesic equation is the equation obeyed by curves of extremal length. It is a postulate of Einstein gravity theory that particles travel along geodesics, which is to say, the paths of particles obey the geodesic equation. We have just shown that exactly the same is true in Newtonian gravity: particles travel along geodesics.

Inserting (13) into the standard formula for the Riemann curvature tensor $R^\alpha{}_{\beta\gamma\delta}$ gives

$$R^i{}_{tjt} = -R^i{}_{ttj} = \frac{\partial^2 \Phi}{\partial x^i \partial x^j} \tag{14}$$

and all other components $R^\alpha{}_{\beta\gamma\delta}$ are equal to zero.

We first contract the Riemann tensor to get the Ricci curvature tensor ("contracting" just means sum over some of the super and subscripts):

$$R_{\alpha\beta} \equiv R^\mu{}_{\alpha\mu\beta} \tag{15}$$

which gives for the only non-zero component of the Ricci tensor (remember the Einstein summation convention, so that we sum over repeated indices):

$$R_{tt} = \frac{\partial^2 \Phi}{\partial x^i \partial x^i} \equiv \nabla^2 \Phi \tag{16}$$

which in turn means that the Poisson equation for the gravitation potential can be written as an equation for the Ricci curvature:

$$R_{tt} = 4\pi G \rho \tag{17}$$

In Newtonian gravity, the affine connection is fundamental, and it is determined by the distribution of matter. In Einsteinian gravity, the affine connection is of a special type, an affine connection that arises from a metric. Thus Einstein gravity theory is a specialization of Newtonian gravity. Both theories assert that particles travel along geodesics, and both theories assert that the affine connection that defines the geodesics is determined by the distribution of matter.

End Mathematical Interlude

I have not defined "curvature" or "affine connection," because all textbooks in general relativity give perfectly good definitions, and it is not necessary for my purpose here. In fact, all you need to know for the present purpose is that these concepts serve to organize partial derivatives. In Einstein's gravity theory, the connection coefficients are defined in terms of a collection of partial derivatives. And what a complicated collection it is! In the general case, with none of the 10 $g_{\mu\nu}$'s non-zero, there would be 40 connection coefficients, each consisting of a sum of 30 partial derivatives, since there would be 10 of the $g^{\mu\nu}$'s, each of which would multiply 3 partial derivatives. And to calculate the Riemann curvature tensor, 4 partial derivatives of each of these 40 connection coefficients would have to be written down, and added to products of connection coefficients. Instead, all of this complexity can be neatly written down in three lines using the concepts of connection and curvature. But still, as Einstein said, all one really needs to understand is partial derivatives.

The huge number of partial derivatives that appear in Einstein's theory, as opposed to the relatively tiny number that appear in Newton's theory (three partial derivatives in the only non-vanishing affine connection, and three second order partial derivatives in the curvature) also shows us why the concept of curvature is never introduced in Newton's gravity theory: the concept is not necessary to organize the partial derivatives. Instead, the concept of gravity as a force works quite nicely for most problems.

It is often asserted that Einstein proved that gravity is the same thing as being in an accelerated frame of reference. This is complete nonsense. Einstein did no such thing. In both Newtonian theory and Einsteinian theory, gravity is curvature. If the Riemann curvature tensor is zero, then there is no gravity, whether or not the observer is accelerated.

It is also often asserted that Einstein proved that "all things are relative". Actually, the theory of relativity, in both its special and general forms, asserts the exact opposite: physics is concerned with the absolute. The laws of physics do *not* depend of the observer. Prior to Einstein, standard formulations of electromagnetism did depend on the observer, and the elimination of this observer dependence was the chief contribution and intended goal of Einstein. The term "relativity theory" was introduced in 1907, not by Albert Einstein, but by Max Planck. Einstein himself used, up to 1911, the expression "so-called relativity theory" to refer to special relativity in his published papers, and used the word *Invariantentheorie* — theory of invariance — in his private letters on the subject. Many scholars, like the great mathematician Felix Klein, aware of the general public's misunderstanding of the meaning of the phrase "theory of relativity", urged Einstein to adopt his original term *Invariantentheorie,* but Einstein wrote in response: "Now to the name relativity theory. I admit that it is unfortunate, and has given occasion to philosophical misunderstandings ... The description you proposed would perhaps be better, but I believe it would cause confusion to change the generally accepted name after all this time ([23], p. xv)."

There is, however, an important qualification to my statement that theories which are mathematically equivalent are the same theory. Two theories which are mathematically equivalent may not be *psychologically* equivalent, and they may not be *computationally* equivalent.

Let me illustrate these distinctions by examples. To illustrate "psychological" inequivalence, let me tell you about a problem I set for my students whenever I teach a course in general relativity. I point out to my students that in general relativity, all coordinate systems are equal. Therefore, it would appear that Copernicus was wrong to claim that the Sun, not the Earth was the center of the Solar System. For, according to general relativity, all coordinate systems are equal — this is what "general relativity" means, that the laws of the physics do not depend on the particular coordinate system we may choose to use — so we could just as well use a coordinate system whose origin of coordinates is the Earth rather than the Sun, and in such a coordinate system, the Earth is indeed the center of the universe. I ask my students to show that it still makes sense, even in general relativity, to say that it is the Sun, and not the Earth, that is the center of the Solar System.

The answer to this final exam problem is that, although in general relativity all coordinate systems are equal, some coordinate systems are more equal than others. The fact of the matter is, studying the Solar System in a coordinate system which has the Sun as the center is much easier than trying it in a geocentric coordinate system which assumes the Earth to be immovable. Consider the effect of just the rotation of the Earth. If we use a coordinate system in which the Earth is stationary, we must consider the stars and planets to be in motion, revolving around our fixed Earth. That is, rather than picturing the Earth to rotate on its axis every 24 hours, we must instead regard the stars and planets as revolving around the Earth every 24 hours.

But this means that the further an object is from the Earth, the faster it must move. An object a fixed distance R away from the Earth must revolve around a circle of radius R every 24 hours. That is, the object must move around the circumference of a circle of radius R every 24 hours. The circumference of a circle is $2\pi R$, so the object's speed must be $2\pi R$ divided by 24 hours, since speed is defined to be the distance traveled divided by the time needed to traverse that distance. Thus there will be a distance from the Earth at which all objects at that distance or larger will be moving at a speed greater than the speed of light. This distance is easily computed to be 28 Astronomical Units, the term astronomers use for the distance from the Earth to the Sun. Since the planets Neptune and Pluto are at a distance from the Sun of 30 A.U. and 40 A.U. respectively, they would apparently be moving faster than the speed of light, as would the stars, the nearest of which, Alpha Centauri, is four and one half light years away, or more than a quarter of a million Astronomical Units, away from the Earth.

However, it is a fundamental principle of relativity theory that nothing can move faster than the speed of light, so what is happening here? What is happening is a breakdown in the unmoving geocentric coordinate system if you go beyond 28 astronomical units. The stars are *not* moving faster than light. The equations describing the coordinate system themselves assert that they cannot be used beyond 28 astronomical units. To describe the nearest star one would have to use another coordinate system, for instance one in which the Sun is not moving. We cannot describe mathematically the stars in the unmoving geocentric coordinate system even though we can see the stars at night. It is far better to use a coordinate system in which the stars can actually exist. So although the geocentric and heliocentric coordinate systems are equal, the heliocentric coordinate system is more equal than the geocentric coordinate system.

The idea of "psychological inequivalence" is closely related to "computational inequivalence." What humans can see as obviously true in one language can be exceedingly difficult to see in another language. And a "language change" can be as simple as replacing a single word by a pair of words.

Let me take an example, the law on abortion, discussed by Tribe at length in the very paper being analyzed here. Tribe uses words that make it appear that the only issue is whether women have the right to control their own bodies. But suppose that we change the word used to describe the entity inside a pregnant woman's womb from "fetus" to "unborn child." Then it becomes obvious that in an abortion decision, there are the rights of two people that must be weighed against each other. We now have to consider whether the right of the mother to control her own body outweighs the right to life of the unborn child. Furthermore, if "unborn child" is used rather than "fetus," most who make the language change think that the word "murder" is more accurate description of the doctor's action than "abortion." This little change of language leads to a profound psychological change. These two languages are psychologically inequivalent, indeed profoundly so.

So which of the two languages is more appropriate in the fetus/unborn child legal situation? In physics, this sort of question would be decided by experiment. Or rather, it should be decided by experiment. As we shall see shortly, the physics departments at the elite universities like Harvard are increasingly falling under the control of people who believe that the laws of physics are determined, not by experiment, but by a "consensus" of people who hold physics professorships at the elite universities. These people — I am reluctant to call them "physicists" — are at bottom motivated by the same lack of understanding of quantum mechanics and general relativity which has led to Tribe's view that constitutional law should reflect, not unchanging natural or constitutional law, but rather a "consensus" of Supreme Court justices and law professors at elite universities. This is the way physical law was determined prior to Galileo: the professors of physics at the elite universities decided among themselves what the laws of physics were. Galileo argued against this, saying that if elite opinion disagreed with experiment, then elite opinion was wrong. Furthermore, Galileo insisted on simple experiments, experiments that anyone could carry out. Galileo's stress on simple experiments was once again based on his deep suspicion of authority: can the elite professors be trusted not to fake the data if the evidence goes against them? Data faking is not possible if anyone can do the experiment.

In Galileo's case, the physics professors — they were called "professors of philosophy" in Galileo's time — used the power of the state to force Galileo to shut up. Subsequent professors have re-written history to make it appear that Galileo was sentenced to perpetual house arrest because modern science was inconsistent with Christian dogma, but the world's leading Galileo scholar, Stillman Drake has demonstrated [33] conclusively that it was the physics professors that were behind Galileo's trial, and that the issue before the Holy Inquisition was physics and not theology. Before he became a professor himself as a result of his important work on Galileo, Drake was a stock broker, and not part of the professor's guild, so he was not motivated to conceal the fact that physics professors used the Church to silence Galileo to protect elite opinion. Drake showed, in fact, that the Inquisition was able to silence Galileo only because Galileo was what we would now call a "fundamentalist" Catholic who considered himself as subject to the Inquisition. Had Galileo wished, he could have moved to Venice where the Inquisition was not allowed.

As I said earlier, truth can be understood by anyone with a high school level of knowledge, and can be verified by anyone with this knowledge, a principle advanced by Galileo against the elite professors of his day. In Galileo's day, elite professors would write only for other elitists, in Latin, so that the common people would not understand. Galileo instead wrote his books in the Italian language, precisely to enable the common people to understand. Galileo was denounced by the seventeenth century professors for doing this, just as today professors say an argument cannot be believed until it has been "peer reviewed" — that is, to say, found to agree with the biases of the elite — and published in an elite journal in an arcane language understood only by the elite.

Galileo thought this was nonsense, and so do I, which is why I am writing this paper in elementary high school mathematics: so that I can bypass elite opinion in my day, for the same reason Galileo bypassed elite opinion in his day. As Galileo exhorted his readers, so I exhort you: go through my mathematics yourself, and don't take the word of anyone — including me! — as to its validity. If you must take the word of someone else, that someone is your master. You are nothing but his intellectual serf.

Tribe wishes us to return to the pre-Galileo, pre-Newton world in which the elite professors, and not experiment, determine truth, which can therefore change whenever elite opinion changes.

As it has changed in the question of whether homosexual behavior is to be lawfully permitted, a question which Tribe has argued before the Supreme court. The U.S. Supreme Court ruled in 1986 (Bowers v. Hardwick [34] against Tribe's argument that it is to be allowed. The United Supreme Court ruled in 2003 [35] in favor of allowing homosexual activity. Elite opinion on the matter changed. Or more precisely, the makeup of the Supreme Court changed. In 1986, White, Burger, Powell, Rehnquist, and O'Connor upheld the law banning homosexual activity, while Blackmun, Brennan, Marshall, and Stevens dissented. In 2003, Kennedy, Stevens, Souter, Ginsburg, Breyer, and O'Connor overturned the law banning homosexual behavior, while Scalia, Rehnquist, and Thomas dissented. O'Connor changed her mind, but given a 6 to 3 decision, her change of heart did not change the outcome. Since Tribe does not discuss homosexuality in his paper on the Curvature of Constitutional Space, I shall not discuss the question here. My only concern is to point out that the only thing that has changed is elite opinion.

Tribe's views are profoundly reactionary.

In the abortion case, Tribe wishes to return to the ancient Roman world in which abortion was allowed. In which anything was allowed that did not oppose elite opinion [25].

Tribe has written a book *Abortion: the Clash of Absolutes.* Like Einstein, I love absolutes, I insist on absolutes. I demand theories that do NOT depend on the observer or any human opinion. Clearly, we need to test the abortion question by experiment to determine which absolute is correct.

New experimental evidence has appeared with the development of ultrasound technology that allows anyone to see inside a woman's womb, and observe the fetus/unborn baby. Most who observe the fetus/unborn baby, at least after the sixth month, believe that it is an unborn baby and not a fetus. If informed consent is to be required by law before a surgical procedure is to be performed, then letting a pregnant woman observe what she intends to destroy should be required by law. But there have been attempts to ban such ultrasound requirements. No experimental challenge to elite opinion is to be permitted [26]

Tribe describes an exchange between Justice O'Connor and Charles Fried, who was trying to persuade the Supreme Court to overturn *Roe v. Wade:*

Justice O'Connor: "Do you think that the state has the right to, if ... we had a serious population problem, ... require women to have abortions after so many children?

Charles Fried: "I surely do not. That would be quite a different matter."

Justice O'Connor: "What do you rest that on?"

Charles Fried: "Because unlike abortion ... that would involve not preventing an operation but violently taking hands on a woman and submitting her to an operation.(quoted from [1], p. 14, with his ellipsis).

A better reply would have been to apply the "unborn baby" language. If babies are people, then we can no more solve the population problem by abortion than we can solve it by "aborting" (killing) the elderly. In fact, killing the elderly would be even more in the interests of the state, because such abortion would not only solve the population problem, it would solve the social security and government workers pension underfunding problems! Justice O'Connor, being closer to retirement than to birth at the time she questioned Fried, might have been more able to see the force of the argument against elderly abortion to solve the population problem.

What persuades everyone that the elderly, even those who have had strokes and can no longer talk, must not be aborted, is that they look like people! (Huge surprise.) This is also what changes the minds of those who view a fetus via ultrasound. They don't look like "fetuses," they look like "unborn babies". I have provided a computer theory argument elsewhere [30] that indeed life begins at conception. Experiment should always trump theory, but many are unwilling to believe an experiment until it is confirmed by theory.

The Scots philosopher David Hume has given advice on what to do with the Obama-Tribe paper:

"If we take in our hand any volume, ... let us ask *Does it contain any abstract reasoning concerning quantity or number?* No. *Does it contain any experimental reasoning concerning matter of fact and existence?* No."

Hume's emphasis in both of the above sentences. In my paper — which you have in your hands — you will find plenty of mathematics (abstract reasoning concerning quantity or number), and plenty of physics, with extensive references to the actual experiments that back up my mathematical physics. You will find neither in the Obama-Tribe paper. What then does Hume suggest we do with the Obama-Tribe paper?

"Commit it then to the flames: For it can contain nothing but sophistry and illusion [66]."

As we shall now see, Hume's advice applies to Hume's own book. In fact, the above passage is one of the few correct remarks Hume that ever wrote.

Thomas S. Kuhn and His Damn Fool Idea of "Paradigm"

Obama and Tribe repeatedly refer to "paradigm" and "paradigm change," referring of course to the famous book by Thomas S. Kuhn *The Structure of Scientific Revolutions.* They advocate a change from a "Newtonian" to a "post-Newtonian" paradigm. Since I have demonstrated that physics never underwent such a paradigm change, they have no justification for such a change in constitutional law. Nevertheless, since Kuhn's theses loom large in their paper, I shall devote this section to pointing out just what caused Kuhn and others to be misled into thinking that there were scientific revolutions in the early twentieth century, and what the intellectual implications of this serious mistake were.

As has been pointed out by the Australian philosopher David Stove, in his important book *Scientific Irrationalism: Origins of a Postmodern Cult* [56], Kuhn built his own theories upon the attack on science by the Austrian philosopher Karl Popper, who in turn built upon the attack on science by the Scots philosopher David Hume.

The negative effects of David Hume's attack on all reason — for this is just what it was — can scarcely be understated. As the British philosopher Bertrand Russell put it in the early twentieth century: "The growth of unreason throughout the nineteenth and what has passed of the twentieth century is a natural sequel of Hume's destruction of empiricism. ... Hume's scepticism rests entirely upon his rejection of the principle of induction" ([73], p. 673). ... if the first half of Hume's doctrine is admitted, the rejection of induction makes all expectation as to the future irrational, even the expectation that we shall continue to feel expectations. I do not mean merely that our expectation *may* be mistaken; that in any case must be admitted. I mean that, taking even our firmest expectation, such as that the Sun will rise tomorrow, there is not a shadow of a reason for supposing them more likely to be verified than not" ([73], p. 668).

The principle of induction roughly states that if we observe nature, and from these observations, form a theory — such as the theory that the Sun will rise once a day — then repeated observations confirming the theory — such as indeed the Sun has risen once a day throughout our lifetimes — increases our confidence that the theory — the Sun will rise once a day — is true. Hume claimed that this belief is irrational, because it tacitly assumes the principle of uniformity of nature. That is, Hume claims we are assuming that what has happened in the past will be likely to happen in the future. Since we have no reason to believe this except from our observations of nature, we can prove the uniformity of nature principle only by using induction, which itself assumes the uniformity of nature principle. Thus any expectation of the future is based on circular reasoning, and hence it is irrational. Hume really believed this nonsense. As he wrote in the end of *An Enquiry Concerning Human Understanding:* "... we cannot give a satisfactory reason, why we believe, after a thousand experiments, that a stone will fall, or fire burn ..." [65].

Obviously, it is David Hume who is irrational, and not us when we daily reason inductively. We are sane, while David Hume is quite mad. But let me prove it.

David Hume's attack on induction can be refuted by an elementary algebra calculation. Anyone who has knowledge of junior high school algebra can follow the argument. The only algebra that you need to know is that if a, b, c, and d are any real numbers and b and d are not equal to zero, and also

$$ab = cd \tag{18}$$

where ab just means that the numbers a and b are multiplied together, then by dividing both sides of equation (18) by the product bd gives the equation

$$\frac{a}{d} = \frac{c}{b} \tag{19}$$

where $\frac{a}{d}$ just means that the number a is divided by the number d. If you followed the above, you can follow the proof of the Induction Principle. But be warned: the proof will require very subtle reasoning about certain particular numbers a, b, c, and d. Which means that you will have to think hard at certain steps in the argument. Mathematics is just a language, so anyone who is capable of learning to speak any human language — this means anyone who is reading this paper — is capable of learning mathematics. Because mathematics is much more precise and concise than any natural human language, it allows us to reason more deeply than would be possible in any natural human language. But to reason deeply requires more mental effort. Most people, claiming that they are not mathematically talented, will not make the effort. It's not that you are mathematically untalented; it's just that you are lazy.

The elementary proof of the Principle of Induction is not original with me but was discovered in 1961 by Richard Cox, a professor of physics (naturally!) at Johns Hopkins University (and later Dean of Arts and Sciences there). All we need are the central equations of probability theory, which are merely a precise mathematical formulation of the degree of human ignorance. Being a precise measure of human ignorance, a probability is necessarily conditional, which is to say, the likelihood that a claim is true is dependent on all our knowledge, or at least the part of part of our knowledge that is relevant to the claim. Let us call the claim whose probability we are investigating A, and for our knowledge relevant to the truth of A, we will use the symbol B. Then we will represent the probability that A is true given B as:

$$p(\mathrm{A}|\mathrm{B}) \tag{20}$$

The two central algebra equations of probability theory are:

$$p(\mathrm{AB}|\mathrm{C}) = p(\mathrm{A}|\mathrm{C})p(\mathrm{B}|\mathrm{AC}) = p(\mathrm{B}|\mathrm{C})p(\mathrm{A}|\mathrm{BC}) \tag{21}$$

which is called the *product rule*, and

$$p(\mathrm{A}|\mathrm{B}) + p(\overline{\mathrm{A}}|\mathrm{B}) = 1 \tag{22}$$

which is called the *sum rule*.

A few points on the notation. In the expression for the product rule, I have written $p(\mathrm{AB}|\mathrm{C})$. This just means the probability that two claims, namely A and B, are both true given our knowledge C. In the expression for the sum rule, I have written $p(\overline{\mathrm{A}}|\mathrm{B})$. This just means the probability that the claim A is false, given our knowledge C. I shall not prove these two central theorems of probability theory. (A proof can be found in [60] and in [59].) What I want to discuss is the reasons why these equations are necessarily true.

The first important assumption in probability is that we want probabilities to be real numbers, so that we can use the natural ordering on the real numbers to express the idea that, given my knowledge, some claims are more likely than other claims. For example, I am far more confident of the truth of the claim "the sky is blue" than I am of the truth of the claim "fairies are now dancing outside my window." In fact, I am virtually certain that there are no fairies at all, but perhaps I just cannot see them, being a total skeptic. We can express this mathematically as

$$p(\mathrm{B}|\mathrm{T}) > p(\mathrm{F}|\mathrm{T}) \tag{23}$$

where I have used the symbol T for my knowledge, B for the claim that the sky is blue, and F for the claim that the fairies are dancing outside my window.

Although I am virtually certain that fairies do not exist, I am not absolutely certain. But there are certain claims which I am absolutely certain are true: the claim that $2 + 2 = 4$, for example. Being absolutely certain means that no additional knowledge will change my mind on the truth of the claim that $2 + 2 = 4$. So the theory of probability has to express this fact, namely, the probability that this absolutely certain claim is true is not changed by any additional knowledge. Similarly, there are claims that I am absolutely certain are false, for example the claim that $2+2 = 5$. No additional knowledge will change my mind that this statement is false.

In summary, the theory of probability must express the idea that there are two extreme types of claims, ones we are absolutely certain are true and ones which we are absolutely certain are false. Additional knowledge cannot change the probability that the former claims are true, and the latter claims are false.

Look now at the product rule, which expresses what we must know in order for two claims to be simultaneously true. It says that this probability is the product of two other probabilities. If we let C be our initial knowledge and A, say, is a claim which we know to be absolutely certain, then we must have

$$p(\mathrm{AB}|\mathrm{C}) = p(\mathrm{B}|\mathrm{AC}) \tag{24}$$

since the probability of B cannot be changed by any consideration of whether A is true, because we know for certain that it is. By comparing equality (24) with the product rule, we see that

$$p(\mathrm{A}|\mathrm{C}) = 1 \tag{25}$$

if we know that the claim A is certain. Which means that probability of a certain statement is one. A similar argument will show that if a claim A is known with certainty to be false, then the probability that A is true is

$$p(\mathrm{A}|\mathrm{C}) = 0 \tag{26}$$

The argument leading to equations (25) and (26) also shows that any probability p must lie somewhere between zero and one; that is, we must have $0 \leq p \leq 1$. The numbers zero and one have one unique property among the real numbers: if they are multiplied by themselves, the value is unchanged, that is, $0 \times 0 = 0$ and $1 \times 1 = 1$. If the product rule is to hold, then zero must correspond to knowing for certain that a claim is false, and one must correspond to knowing for certain that a claim is true. Every other claim, claims we do not know for certain, must lie somewhere between these two extreme values.

Now we are in a position to understand the sum rule. It is a particular case of knowing something that is certain. For example, if we let A be the claim that "the sky is blue", then $\overline{\mathrm{A}}$ is necessarily the statement "the sky is NOT blue." (Sometimes negation is expressed in more stilted language as "it is not the case that the sky is blue." Here I will consider these two statements to be equivalent.) Now we know for certain that the sky is either blue or it is not. If the sky were not colored at all — if the sky were such that the color concept did not apply to it – then the sky would definitely not be blue, since blue is a color. The sum rule is essential when one tries to actually compute the actual numerical values of the probabilities, and we will use it later to see how probabilities arise in quantum mechanics.

But for now, let us use these basic equations of probability to prove the principle of induction, which claims that testing a theory over and over again, always with a confirmation, increases the probability that it is true.

By the second equality of the product rule, we have by elementary algebra:

$$\frac{p(\mathrm{B|AC})}{p(\mathrm{A|BC})} = \frac{p(\mathrm{B|C})}{p(\mathrm{A|C})} \tag{27}$$

Now let us suppose that B is theory which we are testing, and suppose that A and C are two claims which we can deduce from the theory. Think of A and C as two distinct claims which we want to verify experimentally to test the theory B. For example, from Newtonian gravity theory we can deduce that the orbits of the planets are ellipses with the Sun in one focus (Kepler's First Law). Let A be the claim that the planet Mars orbits the Sun in an ellipse, and C the claim that the planet Jupiter orbits the Sun in an ellipse.

Since both A and C are both necessary claims, given the truth of the theory B, we must have $p(\mathrm{A|BC}) = 1$, because A is already implied by the theory B; the truth of C does not increase the probability of the theory B since is is already included in the theory. Thus equation (27) becomes

$$p(\mathrm{B|AC}) = \frac{p(\mathrm{B|C})}{p(\mathrm{A|C})} \tag{28}$$

Now it cannot be the case that $p(\mathrm{B|C}) = 0$, because this would mean that the theory would be determined to be false by a confirmation of its own implication. For example, this would be saying that Newtonian gravity theory is refuted by the observation that the planet Jupiter in fact moves around the Sun in an ellipse with the Sun in one focus. Also, it cannot be the case that $p(\mathrm{A|C}) = 1$ since this would mean that there is no information in A that is already contained in C alone. For example, it does not follow with absolute certainty that knowing Jupiter moves around the Sun in an ellipse that Mars does also. It is logically possible that Jupiter obeys Newtonian gravity theory, but Mars moves according to the whims of those fairies that I don't believe in.

Thus we must have

$$p(\mathrm{B|AC}) > p(\mathrm{B|C}) \tag{29}$$

That is, experimentally confirming that both A and C are true increases the probability that the theory B is true. For example, verifying that Mars and Jupiter both orbit the Sun in ellipses increases the probability that Newtonian gravity is true. To put it another way, the probability that Newtonian gravity theory is true is greater if we know both Mars and Jupiter orbit the Sun in ellipses than it is if we know only that Jupiter orbits the Sun in an ellipse, and know nothing about the orbit of Mars.

Equation (29) is the Principle of Induction, which I have deduced using nothing but elementary algebra from the fundamental equations of probability theory. I have made no assumptions about the "uniformity of nature," even though philosophers, following Hume, have claimed that I must. The formulation of probability theory that I have used to prove the Principle of Induction was developed [61] by the great French mathematical physicist Pierre Simon de Laplace in the late eighteenth century, and independently discovered earlier by the (obscure) English mathematician Thomas Bayes, who derived it in 1748 in a successful attempt to refute Hume's 1748 attack on the method of inductive reasoning [63]. The product rule, when unfortunately written in a restricted form omitting the crucial claim C (thereby making it useless for proving the Induction Principle) is called "Bayes' Theorem." This usual restriction is a great pity, because when Bayes' Theorem is expressed in full generality as the product rule, using it to prove the Induction Principle requires only a knowledge of elementary algebra. Bayes refuted Hume in the same year that Hume's attack on induction became generally known. Subsequent writers on induction, especially Popper and Kuhn, have no excuse for their ignorance especially since Cox's clear derivation [58] was published in 1961. The basic probability theory was discovered, not only before either Popper or Kuhn were born, but even before the United States was born. The United States Constitution was adopted on September 17, 1787, the new government began operations on March 4, 1789, and the last of the thirteen states ratified the Constitution on May 29, 1790. Your choice for the "birthday" of the United States.

We can also prove, again using simple algebra, that the confirmation of an unexpected and counter-intuitive implication of a theory enormously increases the probability that a theory is true. This follows if we assume that $p(\mathrm{A|C})$ in equation (28) is very small. This factor is called the "prior probability of A," because it quantifies, *before* we test the claim A, the likelihood that A is true, given only our knowledge that the claim C is true. For example, knowing only that the planets move (now, to a high degree of approximation, but not exactly) in ellipses, does not suggest that one can calculate the location of a planet no one has ever seen. Thus $p(\mathrm{A}|\mathrm{C})$ for these two claims was very low in the years prior to 1846. Yet two astronomers Urbain Jean Joseph Le Verrier and John Couch Adams, using B, the claim that Newtonian gravity theory is true, were able to compute the location of an unknown planet, which was discovered on September 23, 1846, very close to where Adams and Le Verrier predicted that it would be. This new planet, which we now call "Neptune," created a sensation when its discovery, due to the efforts of Le Verrier to persuade astronomers to look for the undiscovered planet at the place he calculated, was announced by Johann Galle, the man who first observed it. The reason it created a sensation all over the world was precisely because $p(\mathrm{A}|\mathrm{C})$ was so small a number. If A had been the claim "The Sun will rise tomorrow," and C had been "the Sun has risen once a day for as long as the human race has existed," then in this case, $p(\mathrm{A}|\mathrm{C})$ is very close to one, and no sensation is created when the Sun actually rises tomorrow. (A sensation would be created if those fairies I don't believe in suspended the laws of physics and prevented the Sun from rising tomorrow.)

If $p(\mathrm{A}|\mathrm{C})$ is indeed very small, then by equation (28) tells us that

$$\frac{p(\mathrm{B}|\mathrm{AC})}{p(\mathrm{B}|\mathrm{C})} = \frac{1}{p(\mathrm{A}|\mathrm{C})} \tag{30}$$

and so the ratio on the left hand side of equation (30), which is the amount by which the verification of A increases the probability that the theory B is true, is very large. For example, suppose that $p(\mathrm{A}|\mathrm{C}) = 1/1,000$. Then $1/p(\mathrm{A}|\mathrm{C}) = 1/(1/1,000) = 1,000$ by elementary algebra. As I said, believing that the observation of Neptune enormously increased the probability that Newtonian gravity theory is true was exactly the reasoning of the scientists at the time of the discovery of Neptune. I have now proven that their reasoning was valid according to rigorous probability theory. The history of physics is filled with such examples [67].

Thus, since the general theory of relativity and quantum mechanics are valid implications of Newtonian mechanics, their confirmation should have increased physicists' confidence in the validity of Newtonian theory. Instead, confirmation of these two true theories convinced physicists that Newtonian mechanics was false! It was as if the Sun rising once again today, and no other observation, had convinced scientists that the Sun would not rise tomorrow! What caused this extraordinary reaction?

The reaction was due to the fact that, for political reasons, general relativity and quantum mechanics were falsely presented as theories opposed to Newtonian mechanics. Furthermore, the political beliefs that caused this false representation were very close to the political beliefs of Tribe and Obama.

On November 6, 1919, Sir Arthur Eddington, the Plumian Professor of Astronomy at Cambridge University, and Frank Dyson, the Astronomer Royal of England, announced to a joint meeting of the Royal Society of London and the Royal Astronomical Society, that Newtonian gravity theory – the Newtonian Empire — had been overthrown by Einstein. However, physicists have known for decades that the observational data presented at this meeting was inadequate to justify distinguishing between Newtonian theory without æther effects (what Eddington termed "Newtonian theory without qualification, though, as we shall see, he knew better), and Newtonian theory including æther effects (also known as Einstein's general theory of relativity). What Eddington claimed was that starlight had been bent by gravity as it passed by the limb of the Sun during an eclipse. But this is a very small effect, and it was not until the 1970's that technology improved enough to allow this effect to be measured accurately, thereby confirming Einstein for real. So why did Eddington and Dyson claim that Einstein had overthrown Newton?

For political purposes. The First World War had ended the year before, and had left bad feelings between scientists of the opposing sides. Scientists of the allied nations refused to allow scientists of the Central Powers, such a Germany and Austria, to attend meetings in their countries. Eddington was a Quaker and pacifist, and was justly appalled by this attitude. He wanted a reconciliation. By announcing that a Briton (himself) had made an enormous effort to confirm the theory of a German (Einstein), he showed to the allied scientists that they could not afford to ignore the work done by the German scientists. Indeed true, but Eddington decided to generate vast publicity for "a Briton confirms a German theory," by further claiming that the Briton Newton was overthrown. Eddington fudged the data to make it appear that Einstein had overthown Newton [85].

The announcement that Einstein had overthrown Newton was front page news all over the world, and its intellectual impact was tremendous. Karl Popper tells us that it was in the "autumn of 1919 [i.e., at the exact time of the Eddington announcement] when I first began to grapple with the problem, *When should a theory be ranked as scientific?* [Popper's emphasis] ([75], p.33) ... We all — the small circle of students to which I belonged — were thrilled with the result of Eddington's eclipse observations which in 1919 brought the first important confirmation of Einstein's theory of gravitation. It was a great experience for us, and one which had a lasting influence on my intellectual development ([75], p.34)." Popper contrasted the situation of Newton against Einstein (as it was presented in the newspapers) with the psychological theories of Alfred Adler, which Popper claimed could explain any observation as consistent with the theory: "With Einstein's theory the situation was strikingly different. Take one typical instance — Einstein's prediction, just then confirmed by the findings of Eddington's expedition. Einstein's gravitational theory had led to the result that light must be attracted by heavy bodies (such as the Sun), precisely as material bodies were attracted. As a consequence it could be calculated that light from a distant fixed star whose apparent position was close to the Sun would reach the Earth from such a direction that the star would seem to be slightly shifted away from the Sun; or, in other words, that stars close to the Sun would look as if they had moved a little away from the Sun, and from one another. This is a thing which cannot normally be observed since such stars are rendered invisible in daytime by the Sun's overwhelming brightness; but during an eclipse it is possible to take photographs of them. If the same constellation is photographed at night one can measure the distances on the two photographs, and check the predicted effect.

"Now the impressive thing about this case is the risk involved in a prediction of this kind. If observation shows that the predicted effect is definitely absent, then the theory is simply refuted. The theory is incompatible with certain possible results of observation — in fact with results which everybody before Einstein would have expected. This is quite different from the situation I have previously described [e.g. Adler's theories], when it turned out that the theories in question were compatible with the most divergent human behavior, so that it was practically impossible to describe any human behavior that might not be claimed to be a verification of these theories.

"These considerations led me in the winter of 1919-20 to conclusions which I may now reformulate as follows. . . . (4) *A theory which is not refutable by any conceivable event is nonscientific. Irrefutability is not a virtue of theory (as people often think) but a vice* [my emphasis]" ([75], p.35–36).

The last sentence is an extraordinary claim. The mathematical identity $2+2=4$ is not refutable by any conceivable event; it is absolutely true. Nevertheless it, and the rest of mathematics, is extremely useful, precisely because it is not refutable. Also, a physical theory could be not refutable because it is in fact true. We should search for irrefutable physical theories because if we found one, we could trust it just as we can trust $2+2=4$.

In fact, since Newtonian theory has been confirmed, not refuted, by general relativity and quantum mechanics, we have no evidence whatsoever that it is wrong, and overwhelming evidence that it is correct. It is thus a candidate for an irrefutable theory. It is a practical certainty that Newtonian theory is an irrefutable universal theory, in the sense that the probability of its being true is very close to one.

Popper himself gives, without realizing that he had done so, the real reason why Adlerian psychology was probably false: bad observations, bad data. The same error that persuaded the world that Einstein had overthrown Newton. As Popper records: "As for Adler, I was much impressed by a personal experience. Once, in 1919, I reported to him a case which to me did not seem particularly Adlerian, but which he found no difficulty in analysing in terms of his theory of inferiority feelings, although he had not even seen the child. Slightly shocked, I asked him how he could be so sure. 'Because of my thousandfold experience,' he replied; whereupon I could not help saying: 'And with this new case, I suppose, your experience has become thousand-and-one-fold.'

"What I had in mind was that his previous observations may not have been much sounder than this new one; that each in its turn had been interpreted in the light of 'previous experience,' and at the same time counted as additional confirmation" ([75], p.35).

I would simply state the obvious: bad observations cannot confirm any theory. But good observations can. How can we tell the difference between the two? By whether the observations themsleves have been confirmed, and whether they can be easily confirmed by anyone. In 1846, anyone with a decent telescope could see Neptune for himself, that is, see Neptune's disk, and anyone with a pair of field glasses could confirm that the point of light, which the people with a decent telescope claimed was a planet, was in fact a planet. In contrast, with Eddington and Dyson, one had to trust the elite observers. We now know that this trust was misplaced. With Adler, Popper himself records that the trust was misplaced. The probability that a theory is true is increased dramatically if the theory makes a counterintuitive prediction that is easy to confirm by anyone, and is confirmed.

However, Popper ignores the obvious, and replaces the Induction Principle, which I proved above, with the Falsification Principle: "(5) Every genuine test of a theory is an attempt to falsify it, or to refute it. Testability is falsifiability ([75], p.36). Since Popper rejects the Induction Principle, he necessarily regards all theories, including Newtonian mechanics, general relativity, and quantum mechanics, as mere guesses ([75], p. 115; [76], p.9)). Of course; if one cannot use observations to induce a theory, all of these theories can be nothing else than mere guesses.

But Popper himself induced from his perception that Newtonian mechanics had been overthrown to conclude ". . . all universal theories [theories like Newtonian mechanics, general relativity, or quantum mechanics, which claim validity over the entire universe, spatially and temporally] whatever their content, have zero probability" ([74], p.373). Why bother to test any universal theory then, if one knows that it is certainly false? According to Popper, the whole point of testing is to determine if the theory is false, and we know before we begin the test that it is false! Furthermore, if one rejects the Induction Principle, knowing that the theory is false will provide no knowledge about the nature of a better theory.

Quantum mechanics was developed in the mid 1920's, once again just after the First World War. The central mathematical object in quantum mechanics is the wave function, represented by the Greek letter ψ. The word "function just means that its value depends on its spatial position x, and the time t, so we write this sort of dependence as $\psi(x,t)$. The phrase "wave function" just means that $\psi(x,t)$ depends continuously on spatial position and the time, and satisfies an equation that resembles the equation for wave motion. This equation is deterministic, which means that if the wave function exists at one instant of time t_0 — notice that I don't say that we know what it is, just that ψ exists — then this value of the wave function at the particular instant t_0 determines the wave function at all other times. The particular instant t_0 is entirely arbitrary.

If the wave function $\psi(x,t)$ is the central entity in physics, then it is obviously of central importance to know what it means. As we shall see, Erwin Schrödinger, the discoverer of the wave equation for the wave function — this wave equation is now called the "Schrödinger equation" — came very close to discovering its true meaning. Schrödinger showed in 1926 that $|\psi|^2$ was proportional to a density of some sort. We shall see that the discovery of just what sort of density $|\psi|^2$ is had to wait until 1957. But once it is known what $|\psi|^2$ is, it is easy to show that it has nothing to do with uncertainty. Reality is, as Einstein always insisted, completely deterministic.

However, Paul Forman, the world's leading historian of physics for the years 1910–1930, has shown [68] that the political climate of the 1920's made a deterministic meaning for $|\psi|^2$ unacceptable to German physicists of the time. Forman pointed out that prior to the First World War, indeed prior to the sudden German defeat in 1918, German physicists were very happy with the idea of determinism, for obviously (to the Germans), it was determined that German culture and (after the war began) Germany herself, would dominate and eventually rule the entire world. A German defeat in 1918 obviously meant (to the Germans) that the world must be fundamentally indeterministic. Determinism would have necessarily given a German victory. So by the 1920's, German physicists were searching desperately for a way to prove that physics, in spite of its deterministic equations, was nevertheless indeterministic.

The German physicist Max Born achieved the political goal of German physicists. As I shall show later, mathematical consistency of Newtonian mechanics requires that, whatever the wave function means physically, multiplying the wave function by an arbitrary constant cannot change this physical meaning, whatever it is. Born seized upon this fact to argue that $|\psi|^2$ was not a density of something real, but instead a *probability* density. That is, we can always choose the arbitrary constant so that summing all the values of $|\psi|^2$ over all space will give 1. This statement is just a variant of the sum rule for probabilities. Also, as I shall show below, the Schrödinger equation itself implies that, in many cases, in particular all the cases that the physicists of the 1920's considered, $|\psi|^2$ will approach, in the limit of an infinite number of measurements, a "relative frequency." In treatises on probability theory — for instance [60] — it is proven that if the probability of a coin coming up heads is 1/2, then the relative number of times we will observe heads is 1/2. That is, if we toss the coin a very large number of times, the best estimate of the relative number of times it comes up heads is 1/2.

Born made a common error and identified the probability, which is a measure of human ignorance, with the relative frequency. In most (but not all) cases that physicists considered then and now, it can be proven that the relative frequency will approach the probability in the limit of an infinite number of measurements, so unfortunately very few physicists, in the 1920's and since, are aware that a probability is NOT a relative frequency. So Born was able to convince physicists all over the world — most German physicists needed very little convincing — that $|\psi|^2$ was a "probability density" hence ψ, the ultimate physical entity, was intrinsically "probabilistic." Thus, since a probability is a measure of ignorance, it must mean that Nature herself is "ignorant," which is to say, indeterministic.

The political goal was achieved! But having probability — ignorance — as a physical ultimate means that irrationality is also an ultimate. This gave additional support to the irrationality of Hume and Popper.

Thomas Kuhn developed [71] Hume's and Popper's scientific irrationalism into a general theory of "Scientific Revolutions," of which his main examples were the non-revolutions of general relativity and quantum mechanics. Kuhn's theory has been wittily described by David Stove: "Kuhn claims to have detected a certain cycle in the history of science. First there is a 'pre-paradigm' stage, when a chaos of facts overwhelms all students, theories proliferate but no theory wins, one man's solution is another man's problem, and so on. Then a *paradigm* [my emphasis], some acknowledged model of how things should be done, emerges. ... The paradigm imposes an intelligible order on the welter of known facts, solves some problems decisively, and indicates the lines along which many others will be able to be solved. Guided by the paradigm, scientists mop up the problem areas: this is the period Kuhn calls 'normal science.' Sooner or later, however, difficulties accumulate on the successful paradigm, like barnacles and weed on a ship's hull: 'anomalies' multiply. When this process has reached a sufficiently serious point, then if, and only if, a new paradigm offers itself, you have a period of 'paradigm-shift' and 'revolutionary science'. Young scientists desert the old paradigm for the new, like rats deserting a ship which, though it is not yet actually sinking, they somehow know is doomed. The new paradigm triumphs, a new period of normal science begins, and ... away we go again" ([57], p. 9).

Kuhn always denied that his theory of paradigm change was anti-scientific and irrational. He claimed that "Some sense of my surprise and chagrin over this and related ways of reading my book [*The Structure of Scientific Revolutions*] may be generated by the following anecdote. During a meeting I was talking to a usually far-distant friend and colleague whom I knew, from a published review, to be enthusiastic about my book. She turned to me and said, 'Well, Tom, it seems to me that your biggest problem now is showing in what sense science can be empirical," My jaw dropped and still sags slightly. I have total visual recall of that scene and of no other since de Gaulle's entry into Paris on 1944" ([70], footnote on page 263).

Kuhn's colleague was absolutely correct. Kuhn avoids coming to terms with essential questions like the problem posed by his colleague by systematically misstating the questions, even after he is told the correct way to formulate them. For example, in answer to the claim by Karl Popper that theory replacement in physics entails an advance in knowledge, Kuhn wrote: "To say of a field theory that it 'approach[es] more closely to the truth' than an older matter-and-force theory should mean, unless words are being oddly used, that the ultimate constituents of nature are more like fields than like matter and force. But in this ontological context it is far from clear how the phrase 'more like' is to be applied. Comparison of historical theories gives no sense that their ontologies are approaching a limit: in some fundamental ways, Einstein's general relativity resembles Aristotle's physics more than Newton's" ((([70] p. 265).

This last sentence is complete nonsense. I myself am an expert in general relativity. I obtained my Ph.D. in 1976 in the field of general relativity, after which I became the post doctoral student, first of John A. Wheeler (the man who named the black hole, and who founded the most important school of general relativity in the United States), and then the postdoctoral student of Dennis Sciama (the founder of the most important school of general relativity in Great Britain; Sciama's most famous student was Stephen Hawking, probably the most famous relativist since Einstein himself). The key difference between Einstein and Newton on the one hand, and Aristotle on the other, is fact that the theories of Einstein and Newton are *mathematical*, whereas Aristotle repeatedly insisted that it was impossible to express physics in mathematical terms. Further, as I shall demonstrate — mathematically — below, general relativity *is* Newtonian gravitational theory coupled to the luminiferous æther. More precisely, the full theory of general relativity is a special case of the classical mechanics of Newton as developed in the 19th century. But I have demonstrated above that the idea of gravity being curvature rather than a force is already present in the etherless version of gravity developed by Newton himself. Worse of all, for Kuhn, this fact of Newtonian gravity was established by the greatest geometer of the twentieth century, the French mathematician Elie Cartan, in the same year (1922) that Kuhn was born (though I will grant that Cartan's papers establishing this were not published until Kuhn was one and two years of age). Kuhn, writing the above words in 1969, nearly fifty years after the publication of the theorems of Cartan, has no excuse for his ignorance. It is true that Cartan's papers were published in French, a major language of physics in the 1920's, but the physicist Ludwik Silberstein published, also in 1923, a paper [21] in the world's leading science journal *Nature* containing a proof in the English language that a particle trajectory in Newtonian gravity can be considered a geodesic. (Silberstein did not show, as did Cartan, that the Poisson equation was an equation for the curvature, nor did he regard the affine connection as fundamental.)

Furthermore, the dichotomy between fields on the one hand and matter and force on the other, is a false dichotomy, as was pointed out by none other that Kuhn's most famous opponent, Karl Popper, in the very paper to which the above quoted passage of Kuhn's was a response. Popper had pointed out the true debate among physicists was not a over a dichotomy but over a trichotomy: "In connection with the problem of matter, we have had at least three dominant theories competing since antiquity: the continuity theories, the atomic theories, and those theories which tried to combine the two. ([72], pp. 54–55)." The last of the three possibilities is the actual one and always has been, because Newtonian mechanics has always had both particles and continua as fundamental, as I showed above when I pointed out that Newton himself regarded light as having both particle and wave (continua) aspects. The only question has always been, what exactly is the continuum and how do particles interact with it? The atomic nature of atoms, which is due to the quantization of energy and angular momentum, is due to boundary conditions on a continuum equation, Schrödinger's equation, itself a special case of a continuum equation, the Hamilton-Jacobi equation. In the case of Schrödinger's equation or equivalently, the Hamilton-Jacobi equation, the answer to the "what is the continuum" question is "the multiverse." In the case of general relativity, the answer is "the æther." In the case of quantized general relativity, the answer is "a multiverse of an æther continua." Obviously the correct answer. What else could it be? Such continua will necessarily have particle aspects, and physicists have known this for decades.

Once again, we have an example of equivalent languages. Mathematicians have known for at least a century that in the case of the mathematics of the real line, one can regard the real line — the continuum — as fundamental, and consider the integers (whole numbers) as just special real numbers. Or one can regard the integers as fundamental, and derive the real line in steps, first by defining the rational numbers — ratios of integers – and then by defining the irrational numbers — real numbers which cannot be written as ratios of integers — as sets containing infinite numbers of rational numbers. In my book *The Physics of Christianity*, I have described on a popular level this latter definition ([30], chapter 4). So neither the continuum nor atoms are fundamental, or both are, since they can be defined in terms of each other. I myself, like most physicists, mostly use continuum language as fundamental, since in current mathematics, problems are generally easer to solve in continuum language. But not always.

It is always possible to state a problem in a language, or in such a way, that makes the problem impossible to solve, and in essence, that is exactly what Hume, Popper and Kuhn have done. After solving the Problem of Induction using elementary algebra above — see what the appropriate language can do! — I described some of the misuse of terminology by Hume, Popper, and Kuhn. The philosopher David Stove has given ([56], [57]) an even more extensive list of misuse of language by these three "great" philosophers, leading one to appreciate the great mathematician John von Neumann's definition of philosophy as "the systematic misuse of words designed for that purpose."

I am far more concerned with the effects of this madness on law professors like Obama and Tribe, and on physicists. Unfortunately, madness can be a contagious disease, and as the Obama-Tribe paper makes clear, philosophical insanity has spread outside of philosophy departments.

Hume, Popper, and Kuhn were monsters of philosophical depravity. They have much to answer for. They are responsible for the phenomenon of multiculturalism which is afflicting all university departments. Just as there is one reality, there is one physics, and there is only one way to approach this physics, and that is via the culture of technological civilization. This one physics can be expressed in many equivalent languages, but these must be mathematically equivalent if they correctly express the same one reality. The notion of "incommensurable paradigm" is nonsense, because there is only one paradigm, the paradigm of Newtonian mechanics.

I shall prove the claim that multiculturalism is founded on the ideas of Hume, Popper, and Kuhn in a book which I am now writing, *The Physics of Multiculturalism.* Here I shall limit myself to pointing out a few examples showing that the ideas Popper and Kuhn have begun to undermine even physics departments. The disease of multiculturalism is not limited to humanities departments. Philosophy of Science and the History of Science are the bridges between the humanities and the hard sciences. If these two fields become diseased, the contagion travels across the bridge.

Popper's most important book *The Logic of Scientific Discovery* was translated into English in 1959. Kuhn's main book *The Structure of Scientific Revolutions* first appeared in 1962. Cox's book *The Algebra of Probable Inference*, which contained the elementary proof of the Induction Principle I gave above and thereby refuted the central idea of both Popper and Kuhn, was published, in 1961, between the two books of Popper and Kuhn. Cox's contribution was not to prove the Induction Principle — it is a very simple implication of the multiplication rule, which was known to Bayes and Laplace in the eighteenth century — but rather to derive the multiplication rule from the most general requirements of rational thought in the circumstances of incomplete knowledge.

Cox's work was not mentioned in any of the philosophy of physics courses that I took as an undergraduate at MIT in the late 1960's. But Popper and Kuhn were studied in depth. In 1979, Kuhn became the highly paid Laurance S. Rockefeller Professor of Philosophy at MIT. Thus I and many other physics majors at MIT were taught Popperian and Kuhnian irrationality. All physicists of my age and younger came to accept this irrationality, because we were unaware of the rational alternative. I began a course in probability theory at MIT, but dropped it because the material seemed irrelevant to real world problems.

The Bayes-Laplace-Cox approach to probability theory is even today not mentioned at all, or is dismissed out of hand by the overwhelming majority of philosophers and mathematicians. In fact, this approach to probability theory was given the name "Bayesian probability theory," in the 1930's by the British statistician Ronald Fisher as a term of derision.

One way used to dismiss Bayesian probability is to claim that probabilities cannot apply to claims about anything that happens in the real world. This is obvious nonsense. For example, it is obvious that it made sense to say, in the Fall of 2007, that "Obama is unlikely to win the Democratic Party nomination for President." Notice that I am not saying that this statement is true, I am only saying that it makes sense. If it makes sense, then what we were doing was placing a probability, in the Fall of 2007, on the future claim "Obama was the 2008 Democratic Presidential Nominee." As I write these words, the claim is now known to be true for certain, because the claim has been verified. Also as I write these words, it is still uncertain whether the claim "Obama was elected President of the United States in 2008." is true. Thus, at the present time, all we can do is place a probability on the truth of this claim. Hopefully, by November 5, 2008, we will know with certainty whether the claim is true. Hopefully we will not have a repeat of the 2000 election. But we can place a probability on whether 2008 was a repeat of 2000.

It is thus obvious that all the time we place probabilities on claims. In fact, we can scarcely carry out any act without doing so, since we almost never have complete information about anything. What Cox did was to state, in a mathematically precise form, the various rules of thumb we necessarily use all the time, and show that mathematical consistency between these rules of thumb implied the multiplication rule and the sum rule. Hence mathematical consistency implies the Induction Principle.

Remember those fairies I don't believe in? Sir Arthur Conan Doyle, the creator of Sherlock Holmes, did believe in them [78]. He believed that they had been photographed. His book [77] defending his belief in fairies is appalling to read, because the photographs, which he reprints in his book, are obvious fakes, and it is painful to watch an intelligent man go through such contortions to avoid the obvious. Reading philosophers of the past two centuries, in particular, Hume, Popper, and Kuhn, is equally painful, for exactly the same reason. It is appalling to watch intelligent men take leave of their senses.

These men, having gone mad like Doyle, go to enormous lengths to defend their madness. As I have repeatedly said, the proof of the Principle of Induction is not original with me. It originated in Hume's own lifetime. Men like Hume, Popper, and Kuhn — and Obama and Tribe — remain willfully ignorant of the elementary proof.

Their willful ignorance takes many forms. One way that has been used to avoid the truth is to avoid studying the elementary mathematics, justifying this behavior by saying a "consensus of experts" has established that the mathematics is nonsense. It is not nonsense. Check it out for yourself above. It is as certain as $2 + 2 = 4$.

I've had people — some of them professors of mathematics, extraordinary as it may seem to those unacquainted with modern university mathematics departments — reply to the last statement by denying that $2 + 2 = 4$!

Their argument is as follows. No measurement is absolutely precise (true.) Therefore we cannot know for certain that a measurement of anything is precisely two (also true). Therefore we cannot be certain that adding two gallons of gasoline to two gallons of gasoline will result in four gallons of gasoline.

This argument confuses a measurement with the logical analysis of the measurement. If a measurement gives a value a and we are uncertain of its actual value by an amount Δa, where Δa can be positive or negative, then we manipulate the measurement by writing the value as $a + \Delta a$, and treating this quantity as absolutely exact in the analysis of the data. We will then know, at the end of the manipulation, by how much we are uncertain of the actual value of the final calculated value.

But if we ignored this obvious procedure, and insisted on confusing measurement with logical analysis, then — well, let's see what can happen.

Since you don't believe that $2+2 = 4$, then by the same argument there is no reason to believe that $2 - 2 = 0$. Consider the identity $2^2 - 2^2 = 2^2 - 2^2$. Factor the right hand side of this identity into $2^2 - 2^2 = (2 + 2)(2 - 2)$, and the left hand side into $2^2 - 2^2 = 2(2 - 2)$. So we have $(2 + 2)(2 - 2) = 2(2 - 2)$. Now divide both sides by $2 - 2$, which we have, by assumption, no reason to believe is equal to zero. The result is, since the factor $(2 - 2)$ cancels from both sides, is $2 + 2 = 2$. Subtracting 2 from both sides of this gives $2 = 0$. Adding one to this gives $3 = 1$. Adding 1 to this gives $4 = 2$, which is also, we have shown is also equal to 0. Continuing in this way, we can show that all whole numbers are equal to zero.

This nonsense is generated, of course, by the assumption $2 - 2$ is not equal to zero, whereas obviously it is. The above "calculation" shows why division by zero is not allowed. It also shows why we must use rigorous mathematics in logical analysis, and distinguish sharply between the concepts of measurements, and the concept of logical analysis.

I have gone on at length about this crucial distinction, because ignoring this crucial distinction is the central error of Popper and Kuhn. These two, and all of their followers, confuse the history of the theories of physics with the logical analysis of the theories of physics. Popper and Kuhn, and their followers such as Obama and Tribe, assume that because most physicists of the twentieth century — a consensus of twentieth physicists — believed that relativity and quantum mechanics were revolutionary theories that were generalizations of, and not special cases of, Newtonian mechanics, then this statement is a mathematical fact. It is not a mathematical fact. Most twentieth century physicists have made and are making a mathematical error. A mathematical error cannot be made into a mathematical fact by a majority vote by anyone or any group whatsoever.

If a majority of the Harvard Mathematics faculty were to vote that $\pi = 3$ exactly, this would not establish that π, which is in fact an irrational number, is exactly equal to 3. It would establish only that the Harvard Mathematics faculty had taken leave of their senses.

Scientists are still unaware of the fact that the Induction Principle has been proven, having been kept in the dark by their equally ignorant teachers for centuries. I myself learned about Bayesian probability theory by reading a novel [79] on global warming!

The theory that humans are responsible for global warming is an example of the corrosive influence of Popper and Kuhn on physics ([80]). Global warming theory is a branch of meteorology, itself a branch of the physics of the atmosphere. Kuhn claims that a paradigm change is due to a change in the consensus of opinion among the experts in a branch of physics, not by the evidence accumulated. Notice that we are told that we must believe human activity is causing the warming of the Earth, because a "consensus of experts" believes it. This argument is pure Kuhn. Before Kuhn, the public would be told that we must believe a theory because new observations show the theory to be true. This was the story in the world press in 1919, as they reported on the results of the solar eclipse expedition that apparently confirmed Einstein's theory of general relativity. The observations were actually inconclusive, but at least people were not told in 1919 that we must accept Einstein because a "consensus of experts" believe in relativity theory.

In reality, we simply do not know if humans are responsible for global warming. Everybody is aware from their own experience that weather predictions are notoriously poor. And predictions of large-scale weather patterns are also very poor. After a season of a very large number of hurricanes in 2005 — as a resident of New Orleans, I was painfully aware of Hurricane Katrina — the 2006 and 2007 hurricane seasons were very quiet, in spite of predictions by the leading hurricane experts that these years would be years of above normal hurricane activity. The computer programs that predict that global warming is caused by humans are simplified, stripped down versions of the computer programs that fail to predict our daily weather. Why should we believe that these programs can predict what the weather will be like 50 years from now?

We are told that a "consensus of experts" believes that the computer programs are reliable over 50 year intervals even though they are not reliable over periods which we ourselves can check. This may be the case, but how do we know that it is the case? The "experts" never even bother to offer a prediction we ourselves can check, a prediction that, given the present knowledge of all of us, we would judge to be highly unlikely. Remember the theorem on induction I proved above. If theory makes an unlikely claim, and the claim is verified, then that theory becomes much more probable.

Instead, we are told, a la Kuhn building on Popper, that we must accept a "consensus of experts." Who are you going to believe, me or your lying eyes?

Induction theory is indeed just common sense made mathematically precise. The irrationalism of Hume, Popper and Kuhn have undermined common sense in the field of weather physics.

It has also undermined common sense in high energy physics. Today, this area of physics is almost totally dominated by "string theory," and has been for over a quarter of a century. Yet in contrast to Newtonian mechanics, Maxwellian electromagnetic theory, general relativity, and quantum mechanics, string theory has not made *any* counter-intuitive prediction, whose confirmation, in accordance with the Induction Principle, has convinced string theorists, other theoretical physicists, and the population as a whole, that string theory is true. Instead, once again, we are told [82] we must follow Kuhn and accept the "consensus of the experts." This is even worse than global warming. String theory is supposed to be a branch of mathematical physics, and thus prove its mathematical claims rigorously. But most of the central claims in string theory are held to be true [83] merely because they are "believed by the experts." The theoretical physicist Lee Smolin [81] has divided up the past two centuries into 25-year intervals, the length of time that string theory has ruled the physics departments at the elite universities. Smolin lists the great breakthroughs in physics that occurred during these eight intervals. There is one interval in which there have been no breakthroughs at all, namely the period of string theory dominance. This period is continuing, and unfortunately may be indefinitely prolonged.

As long as the Kuhnian concepts rule physics departments, the disasters that are global warming theory and string theory will continue to generate nonsense in physics departments as well as humanities departments.

The notion that a "consensus of experts," rather than experiment, determines truth, is turning physics into a religion. In orthodox Christianity, the hallmark of truth is *quod ubique, quod semper, quod ad omnibus.* That is, believed everywhere, believed always, and believed by everybody. That is, believed by a consensus. This is the canon of St. Vincent, (Vincentius, fl. 435 C.E.) and I never thought to see it invoked in physics. In science, construction of theories must be guided by experiment, and then, after construction, the theories must be confirmed by experiment. The opinion of "experts" is totally irrelevant. Acceptance of the Kuhn-St. Vincent rule in physics means that physics is no longer a science. Experiment, and only experiment, determines truth. The fact that many, if not most, contemporary physicists reject the rule of experiment in favor of a consensus of "experts" is why I have insisted that you, the reader, go through my mathematics yourself, and confirm for yourself that it is correct. I would bet my bottom dollar that a consensus of experts will deny the validity of my mathematics. Or they will deny the truth of quantum mechanics and relativity. I have listened to chaired professors of physics at the elite universities deny quantum mechanics and relativity for years. I've listened long enough. Now I am following Galileo, and taking the science, the real physics of quantum mechanics and relativity, to you the people. I am also following the great physicist Richard Feynman, who wrote in his popular book *Surely You're Joking Mr. Feynman!*: "... when I became interested in beta-decay, I read all these reports by the 'beta-decay experts' ... I only read those reports, like a dope. Had I been a *good* physicist ... I would have immediately looked up [the original data] — that would have been the sensible thing to do. I would have recognized right away [that the experts were wrong]. Since then I never pay any attention to anything by 'experts.' I calculate everything myself [84]."

The concept of a "paradigm," the idea that the truth of this paradigm is to be decided by appointed "experts" rather than by obtaining experimental evidence, accessible and understandable to anyone, for a theory, will lead to equally bad results in constitutional law, where "consensus of the experts" means a "consensus of Supreme Court justices." Instead, basic constitutional law should be Newtonian, true and unalterable, just as the Founders intended. Changes should be made only by constitutional amendment, exceedingly difficult *only* because such amendment would require convincing, not just the elite judges and the elite law school professors, but instead the vast majority of the American people: an amendment must be approved first by a two-thirds vote in both houses of Congress, and then approved by majorities in the legislatures in three-fourths of the states, or the legislatures of two-thirds of the states can call a Constitutional Convention to consider one or more amendments

Mathematical Interlude

Proof that the Schrödinger Equation Is a Hamilton-Jacobi Equation Special Case

To see that quantum mechanics is a special case of Newtonian mechanics, let me show that the basic equation of quantum mechanics, the Schrödinger equation, is a special case of the most general form of Newtonian mechanics, the Hamilton-Jacobi equation. You can verify, by checking out any textbook on quantum mechanics, that the Schrödinger equation for spinless particles with potential is

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V(\vec{x})\psi \tag{31}$$

Bohm [36], [37] and Landau ([38], p. 51–52) pointed out that the substitution

$$\psi = \mathcal{R}\exp(i\varphi/h) \tag{32}$$

for two real functions $\mathcal{R} = \mathcal{R}(\vec{x},t)$ and $\varphi = \varphi(\vec{x},t)$ yields the two equations

$$\frac{\partial\varphi}{\partial t} = -\frac{(\vec{\nabla}\varphi)^2}{2m} - V + \left(\frac{\hbar^2}{2m}\right)\frac{\nabla^2\mathcal{R}}{\mathcal{R}} \tag{33}$$

$$\frac{\partial\mathcal{R}^2}{\partial t} + \vec{\nabla}\cdot\left(\mathcal{R}^2\frac{\vec{\nabla}\varphi}{m}\right) = 0 \tag{34}$$

Equation (33) is recognized (e.g. [12], p. 147) as the classical Hamilton-Jacobi equation for a single particle moving in the modied potential [41]:

$$U = V - \left(\frac{\hbar^2}{2m}\right)\frac{\nabla^2\mathcal{R}}{\mathcal{R}} \tag{35}$$

This modified potential is required by global determinism: without it, a general nonsingular potential $V(\vec{x})$ will cause caustic singularities to develop in the wave φ after a finite time. As an example of this, consider a spherically symmetric attractive potential that drops off as r^{-1} far from the center of the potential and a wave which is plane far from this center. The normals to the wave fronts (the tangents to the particle trajectories) will be focussed a finite distance behind the center, and this focal point will be the cusp singularity where the normal to the wave ceases to be defined. With the modified potential (35) this will never happen, because the added term forces the trajectories apart (Schrödinger's equation is linear, and hence cannot develop caustics). The universes of the multiverse governed by the modified classical Hamilton-Jacobi equation are conserved.

The Hamilton-Jacobi equation was recognized both in the nineteenth century and today as the most powerful mathematical expression of classical mechanics. But it is clear that the Hamilton-Jacobi equation is a multiverse expression of classical mechanics. In the nineteenth century and often even today, this multiverse nature was not taken seriously — both then and now physicists have difficulty taking the fundamental equations of physics seriously; they cannot accept that their equations may be in one-to-one correspondence with reality — and so only one trajectory of the Hamilton-Jacobi equation was believed to actually exist. But the other worlds of the multiverse really do exist even in classical mechanics: it is the collision of the worlds that yield the caustics. Both classical mechanics and quantum mechanics are multiverse theories. Quantum mechanics is nothing but classical mechanics made globally deterministic by the addition of a term to the potential.

Equation (34) is the conservation equation for the universes, and it is expressed in standard form for a conservation equation, which therefore allows us to recognize that $\mathcal{R}^2$ is the density of universes. Thus the total number of what I shall term "effectively distinguishable" universes is the space integral of $\mathcal{R}^2$, and this integral may be infinite. The total number of universes is necessarily uncountably infinite if $\mathcal{R}^2$ varies continuously with space and time, as it does in the Schrödinger equation. However, the expression $\mathcal{R}^2 d^3x$ can still be considered as describing a single universe if d^3x is an infinitesimal volume element. Astrophysicists (see any elementary astrophysics text book, e.g., [42], p. 82), correctly term $L_\lambda d\lambda = d\lambda(4\pi A\hbar c^2)/(\lambda^5[e^{2\pi\hbar c/\lambda kT)} - 1])$ the "monochromatic luminosity" of a star — i.e., literally, the luminosity of a star at a *single* wavelength — with surface area A, with $d\lambda$ an infinitesimal bandwidth, even though there are literally an uncountable infinity of wavelengths present in any finite bandwidth. We see that Schrödinger's equation does not require the integral of $\mathcal{R}^2$ to be finite, and there will be many cases of physical interest in which it is not. The plane waves are one important and indispensable example, and physicists use various delta function normalizations in this case. An infinite integral of $\mathcal{R}^2$ for the wave function of the multiverse has been shown [43] to provide a natural and purely kinematic explanation for the observed flatness of the universe.

However, in most cases of physical interest, the integral of $\mathcal{R}^2$ will be finite, and if we pose questions that involve the ratio of the number of "effectively distinguishable" worlds to the total number of "effectively distinguishable" worlds, it is convenient to normalize the spatial integral of $\mathcal{R}^2$ to be 1. With this normalization, $\mathcal{R}^2 d^3X$ is then the ratio of the number of "effectively distinguishable" universes in the region d^3x to the total number of universes.

Erwin Schrödinger came very close to the interpretation described here. In his English language summary of his new theory, published in 1926, Schrödinger wrote that "... the quantity $\psi\psi^*$ [is] a sort of weight function in the configuration space ([40], p. 1068)." In other words, he discovered his famous Schrödinger's equation in 1925, and within a year he knew that the wave function was a wave, not in ordinary space of three dimensions, but in the $3N$ dimensional configuration space of the N particles. Operationally, Schrödinger treated $\psi\psi^* = \mathcal{R}^2$ as proportional to a density of the N particles, not in ordinary three-dimensional space, but in the $3N$ dimensional configuration space of these particles. If one assumes only a single universe exists, this makes no sense, since there are only three spatial dimensions, but the base space of the multiverse — the space upon which the wave function is defined — has the dimensionality of the total number of degrees of freedom of the system.

The Heisenberg Uncertainty Principle does NOT measure any fundamental limitation on our ability to measure a physical quantity, or any limitation from physics to determinism — Einstein was absolutely correct in claiming that God does not play dice with the universe — but rather it is a reflection that after we carry out a measurement, we cannot be sure which universe we are in. There are an infinity of universes, and an infinity of universes which at any instant are identical to each other.

To see this I am going to give an elementary derivation of the Uncertainty Principle, a derivation due to the great German mathematician Hermann Weyl in the 1930's, a derivation which is much simpler than one can find in most textbooks on quantum mechanics,

The usual derivation of the uncertainty principle concludes the following. Define the "expectation value" of an operator A as

$$\langle A\rangle \equiv \int_{-\infty}^{+\infty} \psi^* A\psi\, dx \tag{36}$$

Now define the "variance ΔA of an operator A as

$$\Delta A \equiv \sqrt{\langle A^2\rangle - \langle A\rangle^2} \tag{37}$$

Then the usual expression of the "uncertainty principle" is

$$\Delta A \Delta B \geq \frac{\hbar}{2} \tag{38}$$

where B is some other operator. Inequality (38) will not apply to all operators A and B, just some pairs of operators. In other words, for some pairs of operators, there is no constraint on the product of the "variance" of the operators, and hence, even if we adopted the usual interpretation of the uncertainty principle as a limitation on measurement, there would, in the case of such a pair, be no limitation on the precision of measurement, even according to the indeterminist view.

A pair of operators to which the uncertainty principle does apply is the "operator" of the position of a particle, and the "operator" of the momentum of a particle. It is this example which is most often given when the "limitation of measurement" claim is made, so I shall focus on this case. The Weyl proof, which is described in ([38], p. 48) deals with this case, and restricts itself to just one spatial dimension, chosen to be the x location of the particle. For simplicity, Weyl also assumes that $\langle x\rangle = 0$, and $\langle p\rangle = 0$, where p is the momentum of the particle in the x direction, so we only have to show that if

$$\langle x^2\rangle \equiv \int_{-\infty}^{+\infty} \psi^* x^2\psi\, dx \tag{39}$$

and

$$\langle p^2\rangle \equiv \int_{-\infty}^{+\infty} \psi^* p^2\psi\, dx \tag{40}$$

then

$$(\Delta x)^2(\Delta p)^2 \geq \left(\frac{\hbar}{2}\right)^2 \tag{41}$$

Weyl's proof begins with the inequality whose truth is obvious:

$$\int_{-\infty}^{+\infty} \left|\alpha x\psi + \frac{d\psi}{dx}\right|^2 dx \geq 0 \tag{42}$$

where the absolute value signs in the integral just means $|Q|^2 \equiv Q^*Q$ and Q^* just means the complex conjugate of the function Q. The number α is assumed to be any real number.

When expanded out algebraically, the integral becomes the sum of three integrals, one of which can be evaluated as

$$\int_{-\infty}^{+\infty} \left(x\frac{d\psi^*}{dx}\psi + x\psi^*\frac{d\psi}{dx}\right) dx = \int_{-\infty}^{+\infty} x\frac{d|\psi|^2}{dx}\, dx \tag{43}$$

Integration by parts of the last integral, and combined with the assumed vanishing of $|\psi|^2$ at infinity (otherwise the integral of $|\psi|^2$ over all space would not be finite gives

$$\int_{-\infty}^{+\infty} \left(x\frac{\psi^*}{dx}\psi + x\psi^*\frac{d\psi}{dx}\right) dx = -\int_{-\infty}^{+\infty} |\psi|^2\, dx = -1 \tag{44}$$

Another integral obtained is

$$\int_{-\infty}^{+\infty} \frac{d\psi^*}{dx}\frac{d\psi}{dx}\, dx = -\int_{-\infty}^{+\infty} \psi^*\frac{d^2\psi}{dx^2}\, dx \tag{45}$$

where once again integration by parts has been used.

Now the "operators" for x^2 and p^2 are assumed to be x^2 and $(-\hbar^2)d^2/dx^2$ respectively. Thus we have

$$(\Delta x)^2 = \int_{-\infty}^{+\infty} x^2\psi^*\psi\, dx \equiv \int_{-\infty}^{+\infty} x^2|\psi|^2\, dx \tag{46}$$

and

$$(\Delta p)^2 = \int_{-\infty}^{+\infty} \psi^* p^2\psi\, dx = -\hbar^2\int_{-\infty}^{+\infty} \psi^*\frac{d^2\psi}{dx^2}\, dx \tag{47}$$

The last integral in (47) is just the last integral in (45), multiplied by $\hbar^2$. Thus we can write the "'obvious inequality" (42) as

$$\alpha^2(\Delta x)^2 - \alpha + \frac{1}{\hbar^2}(\Delta p)^2 \geq 0 \tag{48}$$

Think of this inequality as an inequality in the real number α. If the equality were to hold, then it would be a quadratic equation in α, a special case of the general quadratic equation:

$$a\alpha^2 + b\alpha + c = 0 \tag{49}$$

which has general solution

$$\alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{50}$$

If α is to be a real number, it must be the case that the expression under the square root sign must be non-negative, or $-4ac \leq b^2$, which means that, in the case of (48), $b = -1$, so $-4ac \leq 1$, or $4ac \geq 1$. Putting in $a = (\Delta x)^2$ and $c = (1/\hbar^2)(\Delta p)^2$, we get

$$4(\Delta x)^2(1/\hbar^2)(\Delta p)^2 \geq 1 \tag{51}$$

which can be written as

$$(\Delta x)^2(\Delta p)^2 \geq \left(\frac{\hbar}{2}\right)^2 \tag{52}$$

which is just the uncertainty relation (41).

Notice that the Weyl derivation of the uncertainty principle makes no reference to any measurement. No rigorous derivation does. Therefore, the uncertainty principle cannot be a limitation on measurement.

Furthermore, in terminology, the standard expression of the uncertainty principle (38) makes explicit reference to "expectation value" and "variance," concepts which are statistical in nature, and thus not concerned with an individual case. Thus the uncertainty principle (38) can have nothing whatsoever to do with the measurement of an individual particle's position and momentum. The hand-waving non-proofs one finds in many textbooks trying to convince the reader that the Uncertainty Principle is concerned with a limitation on measurement are all invalid. And it is obvious that they are invalid, because if there was a valid, rigorous proof, it would have been discovered by now, three quarters of a century after the discovery of Schrödinger's equation, and the discovery of the above, completely rigorous proof.

The cause of the uncertainty principle is not a limitation on measurement, but rather an interaction of the other universes of the multiverse with our universe. To see this, solve the Schrödinger equation (31) for the second derivative of ψ, and substitute this into the last integral of (45). Then calculate $\partial\psi/\partial t$ by using the expression (32) for ψ. The result is a sum of three integrals, one of which must vanish, since otherwise it would give an imaginary contribution to (45), whereas we know this expression is purely real. Then equation (45) is also equal to

$$\int_{-\infty}^{+\infty} |\psi|^2 \left(\frac{1}{\hbar^2}\left(\frac{\partial\varphi}{\partial x}\right)^2 + \frac{\nabla^2\mathcal{R}}{\mathcal{R}}\right) dx \tag{53}$$

Thus we can write the "variances" of both position and momentum in the same form for comparison:

$$(\Delta p)^2 = \int_{-\infty}^{+\infty} |\psi|^2 \left(\left(\frac{\partial\varphi}{\partial x}\right)^2 + \hbar^2\frac{\nabla^2\mathcal{R}}{\mathcal{R}}\right) dx \tag{54}$$

and

$$(\Delta x)^2 = \int_{-\infty}^{+\infty} |\psi|^2 x^2 \, dx \tag{55}$$

This allows us to see the origin of the uncertainty principle. In nineteenth century Hamilon-Jacobi theory, which did not consider the quantum potential term, the momentum of a particle was just $p = \partial\varphi/\partial x$. The ancients also assumed that only one particle existed, which would mean that there was only one universe. Mathematically, this says that the density of universes is zero except for a single point. This is usually expressed by saying that $|\psi|^2$ is a delta function. A delta function $\delta(x - x_0)$ is defined by

$$\int_{-\infty}^{+\infty} \delta(x - x_0) f(x) \, dx = f(x_0) \tag{56}$$

where x_0 is a constant, and $f(x)$ is any function. Since in the above derivation of the uncertainty principle we have assumed to simplify the mathematics that the expectation values of both the position and the momentum are both zero, which means that we must set $x_0 = 0$. and also $p = \partial\varphi/\partial x = 0$ at $x = 0$. This gives $(\Delta x)^2 = (\Delta p)^2 = 0$, which is to say we have both variances equal to zero simultaneously.

But mathematical consistency requires that the quantum potential be non-zero. And this means that we can no longer set $|\psi|^2 = \delta(x - x_0)$, because if we did, the quantum potential, which is proportional to the second derivative of $|\psi| = \mathcal{R}$, would make the integral in (54) infinite. Now since $|\psi|^2$ is proportional to the density of universes in the multiverse, this means that the ultimate reason for the uncertainty principle is not a limitation on our ability to measure position and momentum simultaneously, but rather that any attempt to measure these quantities with absolute precision in one universe would increase the interference with our measurements from the other universes to infinity.

Remember the reason we have to add the quantum potential: to insure mathematical consistency for the ultimate expression of Newtonian mechanics, the Hamilton-Jacobi equation.

End Mathematical Interlude

All of reality, which is to say, the multiverse, is a totally deterministic Newtonian machine. The equation for the time evolution of the multiverse, Schrödinger's equation, is completely deterministic: given the wave function at one time, the wave function at all other times are completely determined. Einstein was exactly correct: God does indeed not play dice with the multiverse. We humans, however, do not exist over the entire multiverse but only in a limited subset of the multiverse. If we leave out the effects of the rest of reality when we are trying to predict the future, then of course we cannot expect our predictions to be valid.

This failure to be able to predict is not a failure of determinism, it is only a failure to be able to obtain all the necessary data from all the universes, and bring this information back to this single universe of ours. No nineteenth century physicist would be surprised at such a limitation in our human ability to predict. No nineteenth century physicist would consider this inability to predict to be a breakdown in determinism.

It is the height of human arrogance to assume that a limitation on human ability means a limitation on reality itself.

Let me now prove, using Schrödinger's insight that $|\psi|^2$ is proportional to the density of universes in the multiverse, that, in the limit of a very large number of trials, the measured relative frequencies will approach the probabilities — the measure of human ignorance of the other universes of the multiverse. The proof depends crucially on the indistinguishability of the initial states of identical observers in the many worlds of the multiverse.

In the eighteenth century, Laplace showed how to use the sum rule to assign probabilities. Let us consider a fair coin. Let A be the claim that the coin will land up heads, and thus $\overline{\mathrm{A}}$ is the opposite claim, that the coin will not land up heads. I will assume that we can ignore the possibility that the coin does not land at all (say someone catches it) and the possibility that it lands, not only one side or the other, but on its edge. I shall assume in other words, that $\overline{\mathrm{A}}$ is just another expression for the claim that the coin comes up tails.

Since by assumption I have fair coin, the probabilities of coming up heads must equal the probability of coming up tails — this is a definition of what I mean by a fair coin. Thus we must have

$$p(\mathrm{A}|\mathrm{B}) = p(\overline{\mathrm{A}}|\mathrm{B}) \tag{57}$$

But since the sum rule tell us that the two probabilities must sum to one, we must have

$$p(\mathrm{A}|\mathrm{B}) = p(\overline{\mathrm{A}}|\mathrm{B}) = \frac{1}{2} \tag{58}$$

We have just derived, using rigorous mathematics, the fact that for a fair coin, the probability that it comes up heads is 1/2. We shall derive the probability that we shall see an event in a quantum mechanical universe in exactly the same way.

But before we do, let us think a bit more deeply about why we assigned equal probabilities to the two sides of the coin. We said a "fair coin" requires both sides are equally likely, but *why* do we think this?

We think that both sides of a coin are equally likely because in the coin toss, the shapes on the two sides have no significant effect on whether the coin will come up head or tails. In other words, although we can ourselves easily see the difference between head and tails, the coin toss procedure cannot "see" the difference. From the point of view of toss, the two sides are *indistinguishable*. This is the crucial word, so I've emphasized it. All assignments of specific real numbers to probabilities require this basic concept of indistinguishability.

A singe die (the singular of the plural word "dice") has six sides, so the sum rule, with a little effort, gives us a sum of six probabilities. Since there is no reason to prefer one side over another — for die tossing, as in coin tossing, the procedure cannot distinguish any of the six sides — these probabilities must be equal, and hence equal to 1/6. Of course, we ourselves can see the different labels on the different sides.

But suppose there were no way, by any logically possible type of experiment, to distinguish between several physical states. Then by the same logic that forced us to assign the same probabilities to each of the possible outcomes of the coin toss and the die toss, we would have to assign identical probabilities to each of these truly indistinguishable physical states.

Such an assignment of probabilities to truly indistinguishable physical states was first made in the the late nineteenth century by the American physicist J. Willard Gibbs of Yale University, who pointed out that a problem in the physical calculation of the entropy of a gas could be solved if we assumed that interchanging two molecules of the same chemical element had no effect whatsoever. In other words, he assumed that two molecules of the same chemical were absolutely indistinguishable.

Gibbs is given credit for introducing the concept of indistinguishability into physics, but the basic idea is much older, going back at least to the seventeenth century and the German philosopher Leibniz — the same Liebniz of the Liebniz product rule in calculus — who insisted on the "identity of indiscernibles."

The probabilities of quantum mechanics arise from this fundamental, irremovable sort of indistinquishability.

In the Hamilton-Jacobi equation without the quantum potential, we can assume that only one trajectory actually exists. But we can no longer make this assumption if the quantum potential is non-zero, because these other trajectories are clearly having a physical effect on the particular trajectory we have singled out. Thus we are forced to assume that all the trajectories exist simultaneously. But if we generalize the equation to include all particles, then each trajectory corresponds to an entire universe. So we are forced to assume that reality consists not of a single universe, but a multiverse of universes.

In particular, anytime any physicist carries out any measurement, there must exist an infinity of identical analogues of the physicist carrying out the identical experiment. Now when I write “identical” I mean precisely that: the physicist and the analogues are “indistinguishable” in the sense of Gibbs.

It is this indistinguishability of the physicists doing a measurement that gives rise to probabilities in quantum mechanics, and I shall show if a physicist and his analogues carry out a series of measurements — say toss a fair coin a very large number of times – the relative number of times heads will come up will approach the probability that heads will come up, namely 1/2.

I shall now prove this for a a typical series of measurements of a quantum mechanical variable, rather than a coin toss.

Mathematical Interlude

Proof that the Relative Frequencies Approach the Indistinguishability Probabilities

For simplicity I shall prove that the observed relative frequencies will approach the probabilities for the wave function for the spin of an electron, or more generally the spins of a pair of electrons which are together in the singlet state, which means that the two–electron system has zero total spin angular momentum. The proof can be generalized to any physical system, but the mathematics will be more complex. I shall end the derivation with a statement from the general case that is easily testable.

First, let me review the standard notation for the electron spin states. An electron naturally spins like a top, but its spin cannot be zero. Rather, it must necessarily spin in one of two directions, and these directions are necessarily opposite to each other. The particular pair of directions is determined, not by the electron alone, but by the observers choice of which pair of directions to measure. The pair of directions could be up-down, left-right, north-south, east- west, or any such opposite pair. The standard notation for up-down is $|\uparrow>$ for the state of an electron with spin up, and $|\downarrow>$ for the state of an electron with spin down. I shall use a subscript to denote which electron of a pair I am referring to. Thus, the state of electron number 1 is $|\uparrow>_1$ or $|\downarrow>_1$.

If we decide to measure the particle spins in the up-down direction, the wave function of a two-electron singlet state is

$$|\Psi>=\frac{|\uparrow>_1|\downarrow>_2-|\downarrow>_1|\uparrow>_2}{\sqrt{2}} \tag{59}$$

where, as mentioned above, the direction of the arrow denotes the direction of spin, and the subscript identifies the particle. To see that in quantum mechanics, as in standard probability theory, probabilities are a measure of human ignorance rather than an intrinsic property of nature, we must apply quantum mechanics to both the observer and the observed, that is, we must assume that the Hamilton-Jacobi equation applies both to the observer and to the observed. Or equivalently the Schrödinger equation applies to both the observer and the observed. Probabilities are a precise measure of human ignorance of the alternative versions of themselves in the various worlds of the multiverse.

The importance of applying quantum mechanics to the observer is immediately seen in the fact that the two spin 1/2 particle singlet state can also be written as

$$|\Psi>=\frac{|\leftarrow>_1|\rightarrow>_2-|\rightarrow>_1|\leftarrow>_2}{\sqrt{2}} \tag{60}$$

Which is correct? The answer, of course, is that both are correct. But equation (59) is more appropriate if the measuring apparatus is set to measure the spin as spin up or down — in the vertical direction — and equation (60) is more appropriate if the measuring apparatus is set to measure the spin as left or right — in the horizontal direction.

Let $M_i(...)$ be the state of the electron spin detection device, or a record book, before the spin is recorded) of the ith electron. To measure the spin of two electrons, we will need two detectors, whose interaction with the ith electron we will denote by the operator $\mathcal{U}_i$, whose effect is completely described by its effects on the basis states. For two vertically oriented electron spin detectors, it is

$$\begin{aligned}\mathcal{U}_1M_1(...)|\uparrow>_1 &= M_1(\uparrow)|\uparrow>_1\\ \mathcal{U}_1M_1(...)|\downarrow>_1 &= M_1(\downarrow)|\downarrow>_1 \qquad (61)\\ \mathcal{U}_2M_2(...)|\uparrow>_2 &= M_2(\uparrow)|\uparrow>_2\\ \mathcal{U}_2M_2(...)|\downarrow>_2 &= M_2(\downarrow)|\downarrow>_2 \qquad (62)\end{aligned}$$

where the combined state of the electron spin and the atomic center of mass (or the electron spin and the notebook) is represented by the product of the wave functions of the two objects. In particular, if particle 1 is in an eigenstate of spin up, and particle 2 is in an eigenstate of spin down, then the effect of the $\mathcal{U}_i$'s together is

$$\mathcal{U}_2\mathcal{U}_1M_1(...)M_2(...)|\uparrow>_1|\downarrow>_2=M_1(\uparrow)M_2(\downarrow)|\uparrow>_1|\downarrow>_2 \tag{63}$$

Then the effect of the two vertically oriented detection devices, both measuring the spins of the electrons is

$$\mathcal{U}_2\mathcal{U}_1 M_2(...)M_1(...)\left[\frac{|\uparrow>_1|\downarrow>_2 - |\downarrow>_1|\uparrow>_2}{\sqrt{2}}\right] =$$
$$\mathcal{U}_2 M_2(...)\left[\frac{M_1(\uparrow)|\uparrow>_1|\downarrow>_2}{\sqrt{2}} - \frac{M_1(\downarrow)|\downarrow>_1|\uparrow>_2}{\sqrt{2}}\right] =$$
$$\frac{M_2(\downarrow)M_1(\uparrow)|\uparrow>_1|\downarrow>_2}{\sqrt{2}} - \frac{M_2(\uparrow)M_1(\downarrow)|\downarrow>_1|\uparrow>_2}{\sqrt{2}} \quad (64)$$

The last line of (64) is in the older literature on the Many-Worlds said to indicate that the universe has "split" into two universes, in one of which electron 1 has spin up and electron 2 has spin down, and in the other universe, electron 1 has spin down, and electron 2 has spin up. However, since in the state space of quantum mechanics, multiplication distributes over addition:

$$M_2(...)M_1(...)\left[\frac{|\uparrow>_1|\downarrow>_2 - |\downarrow>_1|\uparrow>_2}{\sqrt{2}}\right] =$$
$$\frac{M_2)(...)M_1(...)\,[|\uparrow>_1|\downarrow>_2]}{\sqrt{2}} -$$
$$\frac{M_2)(...)M_1(...)\,[|\downarrow>_1|\uparrow>_2]}{\sqrt{2}} \quad (65)$$

equation (64) could have been written:

$$\mathcal{U}_2\mathcal{U}_1 M_2(...)M_1(...)\left[\frac{|\uparrow>_1|\downarrow>_2 - |\downarrow>_1|\uparrow>_2}{\sqrt{2}}\right] =$$
$$\frac{\mathcal{U}_2\mathcal{U}_1 M_2)(...)M_1(...)\,[|\uparrow>_1|\downarrow>_2]}{\sqrt{2}} -$$
$$\frac{\mathcal{U}_2\mathcal{U}_1 M_2)(...)M_1(...)\,[|\downarrow>_1|\uparrow>_2]}{\sqrt{2}}$$
$$\frac{M_2(\downarrow)M_1(\uparrow)|\uparrow>_1|\downarrow>_2}{\sqrt{2}} - \frac{M_2(\uparrow)M_1(\downarrow)|\downarrow>_1|\uparrow>_2}{\sqrt{2}} \quad (66)$$

The only difference between (64) and (66) is the middle line. But the middle line in (66) indicates that the universe is "split" *before* the measurement is carried out, or rather, that the multiverse consists of the distinct universes before the measurement is carried out, the measurement having the effect of causing the universes to differentiate. Since the two descriptions are mathematically equivalent, they are the same physically, and I shall use the two languages of "splitting universes" and "differentiating universes" interchangeably. But how the universes split or differentiate, into universes in which the electron spin is spin up and down, or left or right, is determined by the specific interaction that is used in the experiment. The interaction is not chosen by us on the fundamental level; this "choice" is determined like everything else in physical reality. I shall in what follows not explicitly write in the measurement states, but they are physically present, and they, together with the measurement interaction, determine the state of the multiverse, and the probabilities we assign to the outcome of the measurements.

To show how probability comes about by measurements splitting (differentiating) the universe into distinct worlds, I follow standard notation and write the singlet state (59) with respect to a basis in a more general direction $\hat{\mathbf{n}}_1$ as

$$|\Psi> = (1/\sqrt{2})(|\hat{\mathbf{n}}_1,\uparrow>_1|\hat{\mathbf{n}}_1,\downarrow>_2 - |\hat{\mathbf{n}}_1,\downarrow>_1|\hat{\mathbf{n}}_1,\uparrow>_2) \quad (67)$$

Let another direction $\hat{\mathbf{n}}_2$ be the polar axis, with θ the polar angle of $\hat{\mathbf{n}}_1$ relative to $\hat{\mathbf{n}}_2$. Without loss of generality, we can choose the other coordinates so that the azimuthal angle of $\hat{\mathbf{n}}_1$ is zero. Standard rotation operators for spinor states then give [38]:

$$|\hat{\mathbf{n}}_1,\uparrow>_2 = (\cos\theta/2)|\hat{\mathbf{n}}_2,\uparrow>_2 + (\sin\theta/2)|\hat{\mathbf{n}}_2,\downarrow>_2$$
$$|\hat{\mathbf{n}}_1,\downarrow>_2 = -(\sin\theta/2)|\hat{\mathbf{n}}_2,\uparrow>_2 + (\cos\theta/2)|\hat{\mathbf{n}}_2,\downarrow>_2$$

which yields

$$\begin{aligned}|\Psi> = (1/\sqrt{2})[&-(\sin\theta/2)|\hat{\mathbf{n}}_1,\uparrow>_1|\hat{\mathbf{n}}_2,\uparrow>_2 \\ &+(\cos\theta/2)|\hat{\mathbf{n}}_1,\uparrow>_1|\hat{\mathbf{n}}_2,\downarrow>_2 \\ &-(\cos\theta/2)|\hat{\mathbf{n}}_1,\downarrow>_1|\hat{\mathbf{n}}_2,\uparrow>_2 \\ &-(\sin\theta/2)|\hat{\mathbf{n}}_1,\downarrow>_1|\hat{\mathbf{n}}_2,\downarrow>_2] \quad (68)\end{aligned}$$

In other words, if two different devices measure the spins of the two electrons in two distinct arbitrary directions, there will be a split into *four* worlds, one for each possible permutation of the electron spins. Just as in the case with $\hat{\mathbf{n}}_1 = \hat{\mathbf{n}}_2$, normalization of the devices on eigenstates plus linearity forces the devices to split into all of these four worlds, which are the only possible worlds, since each observer must measure the electron to have spin $+1$ or -1.

The fact that the splits are determined by the nature of the measurement apparatus is the key to deriving the Born Interpretation (BI) wherein the squares of the coefficients in (68) are the "probabilities" of an observed occurrence of the four respective outcomes in (68). Note that all two or four outcomes actually happen: the sums in (68) (or (59) are in 1 to 1 correspondence with *real* universes. Since the observers are unaware of the other versions of themselves after a measurement, ignoring the existence of the other versions necessarily means a loss of information available to one observer, and it is this loss of information that results in probabilities. The information is still in the collection of observers. but it is now divided between the four versions, who are now mutually incommunicado.

Consider a measurement of (59) or (68) with $\theta = \pi/2$. In either case, the initial state of the observer is the same, and there is no way even in principle of distinguishing the two or four final states of (59) or (68) respectively. Since there is no difference between the initial state observer, there is no difference in the terms of the expression except for the labels I have given them, and the labels can be interchanged leaving the physics invariant. This interchange of labels forms a group, and shows that the probabilities assigned to each state must be the same. This transformation group argument for assigning a probability distribution is originally due to the great French mathematical physicist Henri Poincaré [39]; see [60] for a modern discussion. Thus the invariance of the physics under the relabeling of the states yields the "Principle of Indifference": we must assign equal probabilities to each of two or four states respectively, and so the probabilities must be 1/2 or 1/4 respectively. These are seen to be the relative numbers of distinguishable universes in these states. In summary, it is the indistinguishability of the initial state of the observer in all two or four final states that forces us to equate the probabilities with the relative number of distinguishable universes in the final state. The same argument gives the same equation of the probability of the general orientation state in (68) with arbitrary θ with the squares of the coefficients of the states in (68) with the relative number of effectively distinguishable universes in the final states.

It is very important to note that I have not assigned probabilities to the electron states. Rather, I have assigned probabilities to the observers in the universes of the multiverse who will measure the electron states. Differentiating language is better for seeing how to assign probabilities. Refer again to equation (66). Notice that just before the measurement the two observers are identical. The two (or four) observers differ, just before the measurement, only in the label we give them. They are physically equivalent. There is no way, even in principle, to distinguish between them. The situation is just like the coin or die examples, except that here there is no difference between the observers before the measurement, even in principle. The electron spins provide the label, but also the weight. If there are more universes with spin up than down, there will be more labels with spin up than spin down. Thus we must assign probabilities according to the relative size of $\mathcal{R}^2$.

Notice that this does not give the Born Interpretation in the usual sense of "probabilities mean relative frequencies as the number of observations approaches infinity." In Laplacean (Bayesian) probability theory, the relative frequency is a parameter to be estimated from a probability, not a probability itself (see [60] for a detailed discussion of this point). However, the most probable value of the relative frequency has been shown ([60],pp. 336–339, 367-368, 393-394, 576–578) to be equal in classical physics to the probability (in the Laplacean sense) that the event will occur.

I shall assume that the spins of a series of electrons are measured, and that the spins of all the measured electrons are spin up before the measurement. I shall also assume that the measuring apparatus is at an arbitrary angle θ with respect to the vertical in all the universes. In this case the Laplacean probabilities for measuring spin up along the axis of the apparatus is $p_{\uparrow,\,\theta} = \cos^2(\theta/2) \equiv p$ and for measuring spin as anti-aligned with the axis is $p_{\downarrow,\,\theta} = \sin^2(\theta/2) \equiv q$, respectively, for $0 \leq \theta \leq \pi/2$. The probability $\mathrm{prob}(r|N)$ that an observer in a particular universe will, after N measurements of N different electrons but with all in the spin up state, see the electron as having spin aligned with the apparatus r times, is

$$\mathrm{prob}(r|N) = \sum_k \mathrm{prob}(r, S_k|N)$$
$$= \sum_k \mathrm{prob}(r|S_k,\, N) \times \mathrm{prob}(S_k|N) \qquad (69)$$

where the summation is over all the 2^N sequences of outcomes S_k, each of which actually occurs in some universe of the multiverse, after N measurements in each of these now 2^N distinct universes. The first term in the second line of (69) will equal one if S_k records exactly r measurements of the spin in the θ direction, and will be zero otherwise. Since the N electrons are independent, the probability of getting any particular sequence S_k depends only on the number of electrons with spins measured to be in the θ direction, and on the number with spins measured in the opposite direction. In particular, since the only sequences that contribute to (69) are those with r spins measured to be in the θ direction and those with $N-r$ spins to be in the opposite direction, we have

$$\mathrm{prob}(S_k|N) = p^r q^{N-r} \qquad (70)$$

However, the order in which the r aligned spins and the $N-r$ anti-aligned spins are obtained are irrelevant, so the number of times (70) appears in the sum (69) will be C^N_r, the number of combinations. Thus the sum (69) is

$$\mathrm{prob}(r\,|\,N) = \frac{N!}{r!(N-r)!} p^r q^{n_r} \qquad (71)$$

The relative number of universes in which we would expect to measure aligned spin r times —that is to say, the expected value of the frequency with which we would measure the electron spin to be aligned with the axis of the measuring apparatus — is

$$\langle f \rangle = \left\langle \frac{r}{N} \right\rangle = \sum_{r=0}^{N} \left(\frac{r}{N}\right) \mathrm{prob}(r\,|\,N)$$

$$= \sum_{r=1}^{N} \frac{(N-1)!}{(r-1)!(N-r)!} p^r q^{N-r} = p(p+q)^{N-1} = p \quad (72)$$

where the lower limit has been replaced by one, since the value of the $r=0$ term is zero.

The sum in the second line of (72) has been evaluated by differentiating the generating function of the binomial series $\sum_{r=0}^{N} C^N_r p^r q^{N-r} = (p+q)^N$. That is, we have

$\langle r^m \rangle = (p[d/dp])^m (p+q)^N$, where q is regarded as a constant in the differentiation, setting $p+q=1$ at the end. This trick also allows us to show that the variance of the difference between the frequency $f = r/N$ and the probability p vanishes as $N \to \infty$, since we have

$$\left\langle \left(\frac{r}{N} - p \right)^2 \right\rangle = \frac{pq}{N} \tag{73}$$

In fact, all moments of the difference between f and p vanish as $N \to \infty$, since a calculation using the above generating function gives

$$\left\langle \left(\frac{r}{N} - p \right)^m \right\rangle \sim \frac{1}{N} + \text{higher order terms in } \frac{1}{N} \tag{74}$$

So we have

$$\lim_{N\to\infty} \left(\frac{r}{N} \right) = p \tag{75}$$

in the sense that all the moments vanish as $1/N$ as $N \to \infty$. This law of large numbers explains why it has been possible to believe, incorrectly, that probabilities are frequencies. Not so, as Laplace emphasized over two hundred years ago. It is, instead, that the quantum property of indistinguishability, applied to the observers, forces the measured frequencies to approach the probabilities.

This proof then justifies applying the standard uncertainty principle to large numbers of observations. Indeed the product of the variance of the position and momentum operators must satisfy the uncertainty relation. But the uncertainty is not due to our inability to measure the position and the momentum of a particle. Because whenever we carry out a measurement, the identical analogues of ourselves out in the multiverse are carrying out the measurement at the same time, and because we and our analogues are truly indistinguishable, the uncertainty is a reflection of us not being able to tell which universe we are in.

And this mulitverse approach to the uncertainty principle and quantum mechanics is testable experimentally. Notice that the above calculation shows that it is only in the limit of a large number of observations that the observed frequencies will approach the probabilities. If we make only one or 2 measurements, it is obvious that we need not expect to see the probabilities identical to the observed frequencies. If we toss a coin twice, it is not guaranteed that we will get one head and one tail. We might very well get two heads, or two tails.

But knowing that the observed approach to the quantum mechanical frequencies is caused by the existence of the other versions of ourselves in the other universes of the multiverse we can derive a simple formula for exactly how rapidly this approach will be.

For simplicity, I will assume that we are measuring the frequency distribution of photons or electrons incident on a screen after passing through a single (or double) slit. The distribution will depend on one variable, the distance along the screen, call it x, with $x = 0$ the location of the central peak, or equivalently, the location on the screen exactly opposite to the center of the single slit, or midway between the two slits. The variable $x \in (-\infty, +\infty)$, so divide up this region into $M + 2$ bins, one of size $(-\infty, m_-)$ one of size $(m_+, +\infty)$, and M bins of equal size $\Delta\ell$. The numbers m_- and m_+ are determined by the condition that there are no observed particles in either region $(-\infty, m_-)$ or in region $m_+, +\infty)$.

Let N be the total number of particles observed, and let N_i be the number of particles observed to be in the ith bin. We observe a pattern on the screen after N particle observations,. This pattern will be approaching the ratio

$$\frac{\int_{-\infty}^{x} \psi^*(t)\psi(t)\,dt}{\int_{-\infty}^{+\infty} \psi^*(t)\psi(t)\,dt} \tag{76}$$

This is the general case, but in what follows I shall simplify by using the standard normalization, which I am allowed to to by the linearity of the Schrödinger's equation:

$$\int_{-\infty}^{+\infty} \psi^*(t)\psi(t)\,dt = 1 \tag{77}$$

The observed pattern on the screen is measured by the ratio

$$\frac{\sum_{i=1}^{j(x)} N_i \Delta\ell}{\sum_{i=1}^{M} N_i \Delta\ell} \tag{78}$$

where $j(x)$ is the jth bin, chosen so that the upper end of the jth bin is in position x on the real line. The factor $\Delta\ell$ cancels out, so we can state the **EASILY TESTABLE FORMULA** for the Many-Worlds prediction for how rapidly the observed frequencies will approach the quantum relative frequencies as

$$\left| \frac{\sum_{i=1}^{j(x)} N_i}{N} - \int_{-\infty}^{x} \psi^*(t)\psi(t)\,dt \right| \leq \frac{C(M,e)}{N} \tag{79}$$

where $C(M.e)$ is a constant that will depend on the number of bins M, and on the detector efficiency e. The important fact is the left-hand side (LHS) of (79) will be independent of x, and the approach of the two terms in (79) to each other will be $\sim 1/N$.The experimental strategy will be to record the locations of the particles as they are detected one by one, and after all the data is taken, chose the bin size, the number of bins, and the numbers m_- and m_+. Then see if there is a constant such that (79) holds as N is increased.

It is obvious that (79) holds in two extreme cases. If $N = 1$, (79) holds for any $j(x)$ and x, with $C(M,e) = 1$.

If $x = +\infty$, so that $\sum_{i=1}^{j(+\infty)} N_i = N$, the LHS of (79) is $|1-1| = 0$, so (79) holds with any $N \neq 0$ and any $C(M, e) > 0$. If $N = 0$, both sides of (79) are indeterminate.

End Mathematical Interlude

The formula (79) counts as counter-intuitive for two reasons. First, the "political interpretation" of the wave function, due to Born and unfortunately still found in most textbooks, gives no way to calculate the actual approach of the observed frequencies to the probabilities required by the indistinguishability of the observers in the different universes of the multiverse. Second, an analogy with a standard formula in statistics, called the Central Limit Theorem, would lead us to believe that the approach would be not $\sim 1/N$, but rather $\sim 1/\sqrt{N}$. This sort of standard approach is called the Berry-Esseen Theorem, but discussing it would lead us far beyond high school calculus. If you are interested, look it up on Wikipedia.

So you so-called physicists who want to defend Obama and Tribe, I DARE YOU to test formula (79)!

Now that I have shown where the relative frequencies in quantum mechanics come from, it is a simple mathematical calculation to show that a series of measurements on a series of particles each with the same wave function will satisfy the standard Uncertainty Principle (38). But the variance in the measurements of position and momentum (say) will not be the result of any disturbance due to the observation, but rather to the fact that, because of indistinguishability, we cannot tell which universe we are in at any point in the series of measurements. In fact, it is possible that we will find ourselves in a universe in which the variances are in wide disagreement with (38)! But such universes are in the extremely small minority.

So it is possible – though very unlikely (!) — that I will lose the challenge I just made to physicists.

Physicists have not generally appreciated this connection between the Schrödinger equation and the ultimate equation of classical mechanics, the Hamilton-Jacobi equation, because they have formulated the Hamilton-Jacobi equation in a form — the eikonal equation — that obscures the relation between the two equations. There are plenty of examples of this sort of error in the history of physics. For example, the chairman of the Nobel Committee for Physics in the 1920's Allvar Gullstrand, made just such an error in analyzing Einstein's general theory of relativity. The most important early prediction of Einstein's theory was that the perihelion of Mercury should precess at 43 seconds of arc per century. This is exactly what is observed. But Gullstrand, believed that this prediction was an artifact of the coordinate system that Einstein used [85]

The view of the æther that I shall need was expressed by one of the first English-speaking authors of a treatise on special relativity, published in 1914 before Einstein even discovered the general theory of relativity. His name was Ebenezer Cunningham, and he wrote the following words in 1921, in a paper on relativity for the British journal *Nature:*

"The æther builders succeeded too well, and constructed, not one but an infinity of æthers, any one having a uniform translatory motion relative to any other. ... With the lack of determinateness in the æther goes a similar ambiguity in the measures of time and space. Each of these æthers has its proper scale of time and space. Events which are simultaneous in one æther are not simultaneous in another, and, since none can tell which is the true æther [I would say that none is the true æther; they are all equally true æthers, and this is Cunningham's point], none can tell whether two events are simultaneous or not. This is where the theories of Lorentz and Larmor lead us ... ([22], p. 785)." The idea that Cunningham is expressing is that rather than a single Newtonian time, the æther gives us an infinity of possible time directions, each depending on the velocity of the observer. Each velocity defines a different time direction, and this is the idea that Trautman used to extend Newtonian curvature of time into the Einsteinian curvature of time and space.

It is worth quoting at length from a letter which none other than Sir Arthur Eddington himself published in the world's leading science journal *Nature* on April 14, 1921, since the leter expresses exactly the same view as regards relativity and the æther theory that is developed in this paper that you are now reading,

"Your readers are indebted to Mr. Bonacina's letter in *Nature* of April 7 for a very clear statement of a fundamental point in the relativity controversy, and it is important that the views held with regard to it should be clearly understood. The issue is stated concisely in the sentence 'the relativists seem now to indicate that space, instead of being conditioned by matter, is itself the foundation of matter and physical forces'. Now it seems clear that if any relativist expresses himself in terms like these he cannot be regarding space as mere emptiness or as the arbitrary co-ordinate system of the pure mathematician; for him it is the substitution of matter, light, and electric force — that is to say, it is the thing which most of us call æther. Since it is not matter, it has not (and we ought not to expect it to have) the material properties of density, elasticity, or even velocity; but it has other dynamical attributes, measured by tensor-expressions, which stand in much the same relation towards it that mass and strain do towards matter. It is, in short, a physical medium. It is sometimes stated that the relativity theory does away with the æther; the defence of this statement must be left to those who make it; I do not think it is the view of Prof. Einstein. It seems more reasonable to say that relativity has added to the importance of the æther by enlarging its functions. ... The statement that the phenomena of mechanics are the outcome of the geometry of the world implies the complementary statement that the phenomena of experimental geometry are the outcome of the mechanics of the world. Either form expresses central truth of the generalized relativity theory ... Since it is the medium the condition of which determines light and electromagnetic force, we may call it *æther*; since it is the subject-matter of the science of geometry, we can call it *space*; sometimes, in order to avoid giving preference to either aspect, it is called by Minkowski's term *world* [87]."

In other words, æther language and geometry language are just two mathematically equivalent expressions for the same theory. Which is the central point of this paper.

Einstein agreed with Eddington and myself. In a lecture which Einstein delivered at the University of Leyden on May 5, 1920, entitled "Æther and the Theory of Relativity," Einstein said:

"More careful reflection teaches us, however, that the special theory of relativity does not compel us to deny æther. We may assume the existence of an æther; only we must give up ascribing a definite state of motion to it, i.e. we must by abstraction take from it the last mechanical characteristic which Lorentz had still left it. We shall see later that this point of view, the conceivability of which I shall at once endeavour to make more intelligible by a somewhat halting comparison, is justified by the results of the general theory of relativity ([88], p. 13). . . . What is fundamentally new in the æther of the general theory of relativity as opposed to the æther of Lorentz consists in this, that the state of the former is at every place determined by connections with the matter and the state of the æther in neighbouring places, which are amenable to law in the form of differential equations; whereas the state of the Lorentzian æther in the absence of electromagnetic fields is conditioned by nothing outside itself, and is everywhere the same. The æther of the general theory of relativity is transmuted conceptually into the æther of Lorentz if we substitute constants for the functions of space which describe the former, disregarding the causes which condition its state. Thus we may also say, I think, that the æther of the general theory of relativity is the outcome of the Lorentzian æther, through relativation ([88], pp. 19–20). . . . Recapitulating, we may say that according to the general theory of relativity space is endowed with physical qualities; in this sense, therefore, there exists an æther. According to the general theory of relativity space without æther is unthinkable; for in such space there not only would be no propagation of light, but also no possibility of existence for standards of space and time (measuring-rods and clocks), nor therefore any space-time intervals in the physical sense. But this æther may not be thought of as endowed with the quality characteristic of ponderable media, as consisting of parts which may be tracked through time. The idea of motion may not be applied to it" ([88], p 23–24).

In the second edition off his great work on the history of the æther, Sir Edmund Whittaker expressed the view that what has been called the zero point energy of the vacuum should more properly thought of as the æther: "As everyone knows, the æther played a great part in the physics of the nineteenth century; but in the first decade of the twentieth, chiefly as a result of the failure of attempts to observe the earth's motion relative to the æther, and the acceptance of the principle that such attempts must always fail, the word 'æther' fell out of favour, and it became customary to refer to the interplanetary spaces as 'vacuous'; the vacuum being conceived as mere emptiness, having no properties except that of propagating electromagnetic waves. But with the development of quantum electrodynamics, the vacuum has come to be regarded as the seat of the 'zero-point' oscillations of the electromagnetic field, of the 'zero-point' fluctuations of electric charge and current, and of a 'polarisation' corresponding to a dielectric constant different from unity. It seems absurd to retain the name 'vacuum' for an entity so rich in physical properties, and the historical word 'æther' may fitly be retained" ([89], p. v [Preface to the Second Edition]).

I agree with Whittaker. In fact, in the last ten years we are seeing the gravitational effect of the æther directly, as the cause of the acceleration of the universe as a whole.

One of the questions which laymen often pose to experts on relativity such as myself, is, what is so important about the speed of light? Why not some other speed? The reason is that the particles of light have no mass; any particle that has no mass would travel at this speed. But we were unaware of other particles with zero mass until the twentieth century. Since light was discovered first, we naturally called this special speed, "the speed of light."

In fact, we now know that *all* fundamental particles — electrons, quarks, neutrinos, gluons, for examples — have zero mass. The reason that these zero mass particles are measured to have non-zero mass is due to their interaction with another æther, called the Higgs field. We now know that there are ten æthers (four æthers that are analogues of the luminiferous æther — one of these is the gluon æther — and six æthers corresponding to particles like the electrons, quarks and neutrinos, coupled together in a way that is far beyond the scope of this paper to describe. but gravitationally, their effect is described by the Einstein equations for general relativity. These ten æthers and their mutual interactions are collectively called the Standard Model. The bottom line is that twentieth century physics has never left the fundamental ideas of Newton; there was no "scientific revolution" in the twentieth century.

Let me now give a mathematical proof that in using general relativity, we are just using æther mechanics.

Mathematical Interlude

Proof that the Einstein Gravity Equations are a Special Case of the Newtonian Gravity Equations Coupled to a Luminiferous Æther

Both Newtonian physics and the physics of Einstein are based on the idea that gravity is curvature. However, for Einstein the curvature is determined not only by the density of matter, but also by the density of the luminiferous æther. The original proof of this appeared in 1966 in a paper by the great Polish theoretical physicist Andrzej Trautman [46], but I shall instead use a more elegant derivation discovered by the Tulane University mathematician Maurice Dupre [48], who has pointed out to me that his derivation is very similar to that given by none other than Albert Einstein, in his important 1916 paper "The Foundation of the General Theory of Relativity" [51], given on pages 148–149 of [50]. I shall still use Trautman's definition of the æther, however. I used Trautman's approach in my undergraduate senior thesis at M.I.T., which was carried out under the direction of the great astrophysicist, the late Institute Professor Philip Morrison.

The equations of the luminiferous æther are the four Maxwell equations, which in terms of the electric vector $\vec{\mathrm{E}}$ and the magnetic vector $\vec{\mathrm{B}}$ are

$$\nabla \cdot \vec{\mathrm{E}} = 4\pi\rho \tag{80}$$

$$\nabla \times \vec{\mathrm{B}} - \frac{1}{\mathrm{c}}\frac{\partial \vec{\mathrm{E}}}{\partial t} = \frac{4\pi}{\mathrm{c}}\vec{\mathrm{J}} \tag{81}$$

$$\nabla \times \vec{\mathrm{E}} + \frac{1}{\mathrm{c}}\frac{\partial \vec{\mathrm{B}}}{\partial t} = 0 \tag{82}$$

$$\nabla \cdot \vec{\mathrm{B}} = 0 \tag{83}$$

where ρ is the density of the electric charge, and the vector $\vec{\mathrm{J}}$ is the electric current density. For our purposes, the most important thing to notice is the constant c, which as you correctly guessed, is the speed of light. It is important because this speed does not make any reference to any reference frame. So one would expect that the equations must specify a preferred frame of reference, the frame in which medium that carries the electromagnetic field is not moving. This medium that carries the electromagnetic field is, by definition, the luminiferous æther. This æther would then define a frame of rest over the entire universe. Or to put it another way, there would then be a vector field $\vec{v}$ defined everywhere else in the universe, and this velocity measures the speed at which every other body in the universe moves with respect to the æther. But as was first noticed by Lorentz, if one moves with respect to the æther, the time coordinate is also effected by the motion. This means that motion with respect to the æther cannot be detected, and hence the speed of light will be measured to be the same by every observer.

The Maxwell equations for the the æther are naturally written in 4-dimensional spacetime notation as follows. First, we combine the electric field vector $\vec{E}$ and the magnetic field vector $\vec{B}$ into a 4×4 array called a *tensor*, which is called the Faraday tensor $F_{\alpha\beta}$:

$$F_{\alpha\beta} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{pmatrix} \tag{84}$$

where E_x, E_y, E_z and B_x, B_y, B_z are respectively the vector components of the electric and magnetic vectors $\vec{E}$ and $\vec{B}$. In the Maxwell equations, notice that the partial derivatives with respect to time is always multiplied by the speed of light c. If we write $T = ct$, then T has the dimensions of space just like x, y, z, and we can treat time as a fourth spatial dimension, and thus the indices α, β take on the values T, x, y, z. (This change is what allows $\vec{E}$ and $\vec{B}$ to have the same units.) We can now write the last two Maxwell equations (those which have no reference to charges or currents) in the same notation that we used for the Newtonian equation for gravity, except that we now have four dimensions:

$$\frac{\partial F_{\alpha\beta}}{\partial x^\gamma} + \frac{\partial F_{\beta\gamma}}{\partial x^\alpha} + \frac{\partial F_{\gamma\alpha}}{\partial x^\beta} = 0 \tag{85}$$

since this equation reduces to $\nabla \cdot \vec{\mathrm{B}} = 0$ whenever one takes $\alpha = x$, $\beta = y$, and $\gamma = z$, whereas it reduces to $\nabla \times \vec{\mathrm{E}} + \frac{1}{\mathrm{c}}\frac{\partial \vec{\mathrm{B}}}{\partial t} = 0$ if one sets any index equal to T.

To write the other two Maxwell equations in four-dimensional form, we are going to have to have a way of raising indices, because the other two equations require the Einstein summation, and this requires one of the indices we are going to sum over to be raised, and the other index summed over to be lowered. This raising of an index requires the existence of a four-dimensional metric to do the raising, and providing such a metric is the essential role of the æther.

As was first pointed out by great Dutch physicist Anton Lorentz in 1904, what the æther does is define a different time scale at each point in space and at each time, and this allows us to define a 4-dimensional metric. The first step is to think of the direction of time as a 4-dimensional vector. We do this by writing the time vector as

$$t_\mu \equiv \left(\frac{\partial t}{\partial (ct)},\, 0,\, 0,\, 0\right) \tag{86}$$

where the zero entries are the spatial components of the vector. Thus the time vector points entirely in the time direction, which is what we want.

Now imagine that at each point in space and time, there exists another 4-dimensional vector u_μ, but with

$$u_\mu \equiv (u_t,\, u_x,\, u_y,\, u_z) \tag{87}$$

where now the spatial components are non-zero, but constant over all space. The vector u_μ is called a *rigging* of space and time. The vector u_μ is given the dimension of velocity. It is called the "four-velocity" of the æther with respect to the Newtonian rest frame. Now we can define a 4-dimensional metric as follows:

$$\eta_{\mu\nu} \equiv h_{\mu\nu} - \frac{u_\mu u_\nu}{c^2} \tag{88}$$

where $h_{\mu\nu}$ is a 4×4 symmetric matrix, with components

$$\begin{pmatrix} h_{tt} & h_{tx} & h_{ty} & h_{tz} \\ h_{xt} & h_{xx} & h_{xy} & h_{xz} \\ h_{yt} & h_{yz} & h_{yy} & h_{yz} \\ h_{zt} & h_{zx} & h_{zy} & h_{zz} \end{pmatrix}$$

with all components having a subscript t being zero. In other words, the matrix $h_{\mu\nu}$, which is a tensor in the four dimensions of space and time, is physically just the usual metric on 3-dimensional Euclidean space written in four-dimensional form.

The tensor $\eta^{\mu\nu}$ is defined to be the inverse of the matrix $\eta_{\mu\nu}$, and we can treat the matrix $\eta^{\mu\nu}$ as a 4-dimensional metric to raise and lower indices in space and time, which have now been unified into what is now called *spacetime*. The vector u_μ is usually normalized via

$$u^\mu t_\mu \equiv \eta^{\mu\nu} u_\mu t_\nu = -1 \tag{89}$$

where the Einstein summation convention is in operation, only now in four dimensions. Thus with the standard Cartesian coordinates x, y, z, we have

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{90}$$

Now that we have a 4-dimensional metric, we can raise the indices of the tensor $F_{\alpha\beta}$ as follows:

$$F^{\alpha\beta} \equiv \eta^{\alpha\mu} \eta^{\beta\nu} F_{\mu\nu} \tag{91}$$

where once again, the Einstein summation convention is in operation, now over both the μ index, and also over the ν index.

If we write the combination of charge density ρ and current density $\vec{J}$ as a 4-vector $J^\alpha = (\rho, \vec{J})$, then

$$\frac{\partial F^{\alpha\beta}}{\partial x^\beta} = 4\pi J^\alpha \tag{92}$$

with the Einstein summation over the index β, then equations (92) are the Maxwell equations (80) and (81).

I have ascribed a velocity u_μ to the æther. However, there is no way to measure this velocity using the electromagnetic field, as Lorentz and Einstein pointed out in 1904 and 1905 respectively, because Maxwell's equations are invariant under Lorentz transformations between any two "inertial frames," that is, between any two systems moving with constant velocity with respect to one another. If x^μ is the coordinate position in space and time in the original frame, and $x^{\mu'}$ is the coordinate position in space and time in frame moving at constant speed v with respect to the original frame, then the Lorentz transformations are defined as follows. Let the velocity $\vec{v}$ of the moving frame be in the direction $\vec{n}$ according to the original frame, where $\vec{n} \equiv (n_x, n_y, n_z)$ are the usual direction cosines, with $n^2 = n_x^2 + n_y^2 + n_z^2 = 1$. Then we can write the transformation from one set of coordinates to another as

$$x^{\mu'} = \Lambda^{\mu'}{}_{\nu} x^\nu \tag{93}$$

where the Lorentz transformation matrix is given by

$$\Lambda^{t;}{}_{t} = \gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{94}$$

$$\Lambda^{t'}{}_{i} = -\frac{v}{c} \gamma n^i \tag{95}$$

$$\Lambda^{j'}{}_{i} = (\gamma - 1) n^j n^i + \delta^{ij} \tag{96}$$

If the electromagnetic field tensor is also transformed along with the coordinates using the above matrix as follows:

$$F^{\alpha'\beta'} = \Lambda^{\alpha'}{}_{\alpha} \Lambda^{\beta'}{}_{\beta} F^{\alpha\beta} \tag{97}$$

then Maxwell's equations are unchanged. So, for that matter, is the 4-dimensional metric $\eta_{\mu\nu}$.

Thus there are an infinite number of possible velocities we could ascribe to the æther. This fact is crucial to understanding how to make the Newtonian theory of gravity consistent with the existence of the luminiferous æther. For Einstein's most famous formula, $E = mc^2$, was originally derived for the case in which the energy was that of an electromagnetic field. Thus, in the Newtonian gravity equation $R_{tt} = 4\pi G\rho$, some of the mass density ρ will be contributed by the electromagnetic field. Since this density can be measured in any inertial frame defined by the Lorentz transformations, we must have at least

$$R_{tt'} = 4\pi G \rho' \tag{98}$$

where ρ' is some density appropriate to this frame ([44]). But equation (98) is really a tensor equation of the form

$$R_{\mu\nu} = 4\pi G S_{\mu\nu} \tag{99}$$

because the LHS of (98) are components of a tensor, and further, if two symmetric tensors agree for all possible t' time coordinates, they are the same tensor in all space and time dimensions. (A symmetric tensor is a tensor for which $S_{\mu\nu} = S_{\nu\mu}$; that is, it is unchanged if the indices are interchanged. We must have symmetry because $R_{tt'} = R_{t't}$; that is, we can interchange the order of frames without effecting the physics. Also, the symmetry of the Ricci tensor $R_{\mu\nu}$ follows from the symmetry of the affine connection.

Let me prove this statement. I will actually give two proofs of this statement. Both proofs will follow the proof given in ([45] , p. 72, Proposition 3.3.4), but the first proof will actually be less complicated and more general, and will be based on the fact that Newtonian gravity theory is more general than Einsteinian gravity theory: Newtonian gravity theory allows the density ρ' in principle to depend on two, not one æther directions, as is clear from equation (98). I am giving two proofs, for two reasons. First, I want to emphasize the greater generality of Newtonian gravity theory, and second because familiarity with the simpler first proof will allow the reader to follow the second with ease.

A sufficiently general timelike vector T^μ (recall that a timelike vector is one for which $\eta^{\mu\nu}T_\mu T_\nu < 0$; a spacelike vector s_μ is one for which $\eta^{\mu\nu}s_\mu s_\nu > 0$; "unit" just means $\eta^{\mu\nu}T_\mu T_\nu = -1$ and $\eta^{\mu\nu}s_\mu s_\nu = 1$ respectively); can be written as

$$T^\mu = t^\mu + us^\mu \tag{100}$$

for $u \in [0, b)$ for some real number b, where t^μ is a fixed timelike vector, and s^μ is a unit spacelike vector. The key idea is to realize that, if u is sufficiently small, then the vector T^μ will be still be timelike. This is what I meant by saying that T^μ is "sufficiently general." Let $S_{\mu\nu}$ and $S'_{\mu\nu}$ be two tensors which agree for all possible time directions at a given spacetime point. That is,

$$S_{\mu\nu}T^\mu\overline{T}^\nu = S'_{\mu\nu}T^\mu\overline{T}^\nu \tag{101}$$

Now substitute $T^\mu = t^\mu + us^\mu$ and $\overline{T}^\mu = t^\mu + uw^\mu$ into $S_{\mu\nu}T^\mu W^\nu$, where $\overline{T}^\mu$ and w^μ are two new unit timelike and spacelike vectors respectively, obtaining

$$S_{\mu\nu}T^\mu\overline{T}^\nu = S_{\mu\nu}t^\mu t^\nu + uS_{\mu\nu}t^\mu s^\nu + uS_{\mu\nu}w^\mu t^\nu + u^2 S_{\mu\nu}s^\mu w^\nu \tag{102}$$

Differentiating (102) with respect to u gives

$$\frac{d}{du}S_{\mu\nu}T^\mu\overline{T}^\nu = S_{\mu\nu}t^\mu s^\nu + S_{\mu\nu}w^\mu t^\nu + 2uS_{\mu\nu}s^\mu w^\nu \tag{103}$$

and taking the second derivative gives

$$\frac{d^2}{du^2}S_{\mu\nu}T^\mu\overline{T}^\nu = 2S_{\mu\nu}s^\mu w^\nu \tag{104}$$

Thus taking the second derivative with respect to u of equation (101) gives

$$S_{\mu\nu}s^\mu w^\nu = S'_{\mu\nu}s^\mu w^\nu \tag{105}$$

which is to say, that these two tensors, which agree for all timelike components, also agree in all spacelike components.

We only have to show that the two tensors agree also for mixed time and space indices. This is easy. Take the first derivative of equation (101), set $u = 0$, giving

$$S_{\mu\nu}t^\mu s^\nu + S_{\mu\nu}w^\mu t^\nu = S'_{\mu\nu}t^\mu s^\nu + S'_{\mu\nu}w^\mu t^\nu \tag{106}$$

Now use the assumed symmetry of the two tensors, and set $s^\mu = w^\mu$ obtaining

$$S_{\mu\nu}t^\mu s^\nu = S'_{\mu\nu}t^\mu s^\nu \tag{107}$$

which completes the proof, showing that $S_{\mu\nu} = S'_{\mu\nu}$, since they agree in all their components. Hence, equation (98) is equivalent to equation (99) and the problem is to determine the tensor $S_{\mu\nu}$ in equation (99).

I assumed in the above that I would use two timelike vectors $T^\mu = t^\mu + us^\mu$ and $\overline{T}^\mu = t^\mu + uw^\mu$. I did this because in the LHS of (17), we have the component R_{tt}, which in the notion of Einstein summation, is $R_{\mu\nu}T^\mu T^\nu$, where T^μ is the four-vector pointing in the direction of Newtonian absolute time. But we now have an infinity of time directions, all physically equivalent, so that we might expect to see$R_{\mu\nu}T^\mu T^\nu$ replaced by $R_{\mu\nu}T^\mu\overline{T}^\nu$, which is to say, by two distinct time directions.

Now let me give a proof that if

$$S_{\mu\nu}T^\mu T^\nu = S'_{\mu\nu}T^\mu T^\nu \tag{108}$$

for any timelike T^μ, then the primed and unprimed tensors are equal. The difference between assumption (101) and assumption (108) is that in the former, I assumed there were two timelike vectors, T^μ and $\overline{T}^\nu$ and now I am assuming only one timelike vector, namely T^μ.

But now define T^μ as

$$T^\mu = t^\mu + us^\mu + vw^\mu \tag{109}$$

for $u \in [0, b)$ for some real number b, $v \in [0, c)$ for some real number c, where as before t^μ is a fixed timelike vector, s^μ is a unit spacelike vector and w^μ is a unit spacelike vector . I also assume that the parameters u and v are independent, so we can take the standard partial derivatives with respect to either of these variables. As in the first proof, we substitute (109) into $S_{\mu\nu}T^\mu T^\nu$, and expand this out algebraically as in the first proof. Since you are now familiar with the procedure, I shall not give the algebra, only the results.

In particular, we get, if we set $u = 0$ and $v = 0$ after taking the first partial derivative $S_{\mu\nu}T^\mu T^\nu$:

$$\frac{\partial}{\partial u} S_{\mu\nu}T^\mu T^\nu = S_{\mu\nu}(t^\mu s^\nu + s^\mu t^\nu) = 2S_{\mu\nu}t^\mu s^\nu \qquad (110)$$

where I have used the symmetry of $S_{\mu\nu}$ in the last step. (It may be helpful to remember that μ and ν are dummy indices, so that $S_{\mu\nu}t^\mu s^\nu = S_{\nu\mu}t^\nu s^\mu$. The word "dummy" just means that they are just labels for summing over, and I can change what I call the labels. Once again, we see using equivalent languages allows a simplification.)

Furthermore, we obtain for the second mixed patial derivative of $S_{\mu\nu}T^\mu T^\nu$:

$$\frac{\partial^2}{\partial u \partial v} S_{\mu\nu}T^\mu T^\nu = S_{\mu\nu}(w^\mu s^\nu + s^\mu w^\nu) = 2S_{\mu\nu}w^\mu s^\nu \qquad (111)$$

where one again I have used the symmetry of $S_{\mu\nu}$ in the last step. Doing the same steps on $S'_{\mu\nu}T^\mu T^\nu$, which is, by assumption equal to $S_{\mu\nu}T^\mu T^\nu$, gives two equations, which are

$$S_{\mu\nu}s^\mu w^\nu = S'_{\mu\nu}s^\mu w^\nu \qquad (112)$$

and

$$S_{\mu\nu}t^\mu s^\nu = S'_{\mu\nu}t^\mu s^\nu \qquad (113)$$

which are the same as equations (105) and (113). Thus all the components of the two tensors are equal and thus the two tensors are equal. This completes the second proof.

Instead of a single Ricci tensor component R_{tt}, determined by a three component connection coefficient Γ^i_{tt}, we have the full symmetric Ricci tensor $R_{\mu\nu}$ with 10 components, which geometry tells us is determined by a connection coefficient $\Gamma^\alpha{}_{\beta\gamma}$, with as many as 40 components, if (as we shall see is the case) the connection coefficients are symmetric in the two lower indices. Where do these components come from? The answer is, from the structure of the æther.

So far, I have assumed that the rigging vector which defines the æther is a constant for all space and time. But of course, there is no reason to make this assumption. Let us suppose in fact that the 4-vector u^μ is a function of both space and time:

$$u_\mu(t,\vec{x}) = (u_t(t,\vec{x}),\, u_x(t,\vec{x}),\, u_y(t,\vec{x}),\, u_z(t,\vec{x})) \qquad (114)$$

In this case, the 4-dimensional tensor η is itself no longer a constant but varies over all of space and time. We call this more general raising and lowering tensor $g_{\mu\nu}$:

$$g_{\mu\nu}(t,\vec{x}) \equiv h_{\mu\nu} - \frac{u_\mu(t,\vec{x})u_\nu(t,\vec{x})}{c^2} \qquad (115)$$

where the Euclidean metric tensor $h_{\mu\nu}$ is still as in Euclidean space, and a constant if we use Cartesian coordinates. But the tensor $g_{\mu\nu}$ is now itself a metric for 4-dimensional spacetime.

We would expect this allowed variation of the æther from point to point would have the effect of an energy density, so let us write equation (99) as

$$R_{\mu\nu} = 4\pi G S_{\mu\nu} = 4\pi G \left(T_{\mu\nu} + T^{\text{aether}}_{\mu\nu}\right) \qquad (116)$$

where $T_{\mu\nu}$ is the tensor for the energy density of ordinary ponderable matter, and $T^{\text{aether}}_{\mu\nu}$ is the energy density of the æther. In his 1916 paper, Einstein called $T^{\text{aether}}_{\mu\nu}$ "the gravitational energy" (which he represented by the expression $t_{\mu\nu}$, which, as Einstein realized, is not a tensor) and emphasized that "... if we consider a complete system (e.g. the solar system), the total mass of the system, and therefore its total gravitating action as well, will depend on the total energy of the system, and therefore on the ponderable energy together with the gravitational energy. This will allow itself to be expressed by introducing into [the vacuum equations $R_{\mu\nu} = 0$], in place of the energy components of the gravitational field alone, the sums $t_{\mu\nu} + T_{\mu\nu}$ of the energy components of matter of of the gravitational field ([50], p. 148)." As we shall see later, Einstein considered the terms "energy of the æther" and "energy of the gravitational field" to be interchangeable. The derivation I shall give here is superior to Einstein's original derivation, because my analogue of $t_{\mu\nu}$ is a true tensor, whereas $t_{\mu\nu}$ is not a tensor, as Einstein realized.

In relativity, mass is energy, so energy can generate curvature. it turns out that this ability of the energy of particles to generate curvature can be taken into account by the distribution of pressure, if the pressure in the ith direction is represented by p_i, and momentum flow density in the ith direction is represented by P_i then we can form another 4×4 symmetric matrix $T_{\mu\nu}$, called the *stress-energy tensor*, which can be locally diagonalized spatially into the following form:

$$\begin{pmatrix} \rho & P_x & P_y & P_z \\ P_x & p_x/c^2 & 0 & 0 \\ P_y & 0 & p_y/c^2 & 0 \\ P_z & 0 & 0 & p_z/c^2 \end{pmatrix}$$

where, as always in relativity, the letter c denotes the speed of light. Without the local spatial diagonalization, the symmetric tensor $T_{\mu\nu}$ would be written in matrix form as

$$\begin{pmatrix} T_{tt} & T_{tx} & T_{ty} & T_{tz} \\ T_{xt} & T_{xx} & T_{xy} & T_{xz} \\ T_{yt} & T_{yz} & T_{yy} & T_{yz} \\ T_{zt} & T_{zx} & T_{zy} & T_{zz} \end{pmatrix}$$

so that in the particular case of the diagonal matrix, we have $T_{tt} = \rho$, $T_{xx} = p_x/c^2$, $T_{yy} = p_y/c^2$, $T_{zz} = p_z/c^2$, and all other components are zero. As pointed out above, the tensor $T_{\mu\nu}$ can be spatially diagonalized by a suitable choice of coordinate system. In fact, for almost all types of matter, the matrix $T_{\mu\nu}$ can be completely diagonalized (Hawking and Ellis ([47], p. 89) call such matter Type I matter). The only exception to complete diagonalization is the case of matter consisting entirely of electromagnetic radiation moving inonly one direction, Type II matter in the terminology of Hawking and Ellis.

I shall show that $T_{\mu\nu}^{\text{aether}} = T_{\mu\nu} - g_{\mu\nu}g^{\alpha\beta}T_{\alpha\beta}$, where $g^{\alpha\beta}$ is the matrix inverse to $g_{\alpha\beta}$, and once again the Einstein summation convention is applying to the indices α and β. Thus Newton's theory of gravity, including not only ponderable matter but also the gravitating æther, is (writing $T \equiv g^{\alpha\beta}T_{\alpha\beta}$):

$$R_{\mu\nu} = 8\pi G\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right) \tag{117}$$

These equations are called the Einstein field equations, since they were discovered by Einstein in 1916. Thus the Einstein equations ARE the Newton equations. only including what prior to Einstein had been omitted, the gravitational effect of the æther. Nineteenth century physicists knew that the gravitational effect of the æther had to be included in the gravitational equations, but they did not know how to do this. Einstein discovered how, and this was his greatest achievement.

These field equations (117) — there are 10 equations here, since the tensors $T_{\mu\nu}$ and $g_{\mu\nu}$ are symmetric — reduce to the single Poisson equation (7) in the case that the pressures divided by the speed of light squared are small in comparison to the mass density, and all curvatures are close to zero. Equivalently, we can say that the equations (117) reduce to (7) when the various effect of the æther can be neglected.

My proof that that $T_{\mu\nu}^{\text{aether}} = T_{\mu\nu} - g_{\mu\nu}g^{\alpha\beta}T_{\alpha\beta}$ is a modification of an alternative way of deriving the Einstein equations due to my Tulane University colleague, Professor of Mathematics Maurice Dupre [48]. Professor Dupre attributes the added term to the gravitational field, whereas I attribute it to the æther. As we shall see, Einstein himself regarded these as mathematically equivalent.

We start with the result, originally derived by Maxwell in 1873 ([32], section 792; p. 391 of volume II), and now in any standard physics textbook, that for an electromagnetic wave traveling in the ith direction, where i is either x, y or z in the æther, then $\rho = p_i/c^2$ where ρ is the "mass" density of the electromagnetic radiation, and the p_i are the pressures in the ith direction. With this relation between the mass density and the pressure, we can compute the speed of sound in the æther medium which carries a light wave in the ith direction using the standard Laplace (adiabatic) formula for the speed of sound v_i in the i direction, where i is x, y, or z. This formula for the sound speed is $v_i^2 = \partial p_i/\partial\rho$. so we obtain for the sound speed:

$$v_i^2 = \frac{\partial p_i^{\text{EM}}}{\partial \rho_{\text{EM}}} = c^2 \tag{118}$$

For an electromagnetic wave traveling in an arbitrary direction, we would have for the relation between the density and the pressure:

$$\rho_{\text{EM}} = (p_x^{\text{EM}} + p_y^{\text{EM}} + p_z^{\text{EM}})/c^2 \tag{119}$$

which also will give the speed of sound as c in an electromagnetic wave.

However, the physicists of the nineteenth century wanted the sound speed to be the æther sound speed, which requires

$$\rho_{\text{aether}} = (p_x^{\text{aether}} + p_y^{\text{aether}} + p_z^{\text{aether}})/c^2 \tag{120}$$

Equation (120) is the central equation of nineteenth century æther physics. The whole point of introducing the æther in the first place was to have a medium that would carry the light as a "sound" wave in that medium.. Given equation (120), we can compute the speed of sound in the æther medium using once again the Laplace formula for the speed of sound in the æther:

$$v_i^2 = \frac{\partial p_i^{\text{aether}}}{\partial \rho_{\text{aether}}} = c^2 \tag{121}$$

Notice, however that we now have two, not one, equations for the sound speed: The speed of sound in an electromagnetic wave, and the speed of sound in the æther. The nineteenth century physicists would consider these two equations one and the same, although strictly speaking they are not. The nineteenth century physicists would have no hesitation in identifying the electromagnetic wave pressures as æther pressures; they would regard the pressure exerted by electromagnetic wave as due to the æther. In other words, they would equate the pressures:

$$(p_x^{\text{aether}} + p_y^{\text{aether}} + p_z^{\text{aether}}) = (p_x^{\text{EM}} + p_y^{\text{EM}} + p_z^{\text{EM}}) \tag{122}$$

Combining (122) and (120), we obtain a relation between the electromagnetic wave pressures and the æther density:

$$\rho_{\text{aether}} = (p_x^{\text{EM}} + p_y^{\text{EM}} + p_z^{\text{EM}})/c^2 \tag{123}$$

As I showed earlier, we have $T_{\mu\nu}^{\text{aether}}$ if we know $T_{\mu\nu}^{\text{aether}}t^\mu t^\nu$ for all timelike vectors t^μ. In order to compute $T_{\mu\nu}^{\text{aether}}t^\mu t^\nu \equiv \rho_{\text{aether}}$, in the general case, where we have any sort of matter passing through the æther, the natural assumption (and notice here I am making an assumption) is that equation (123) applies whatever the material passing through the æther [49]. That is, the natural assumption is obtain ρ_{aether} in the general case by simply dropping the superscript EM in equation (123):

$$\rho_{\text{aether}} = (p_x + p_y + p_z)/c^2 \tag{124}$$

This equation implies $T^{\text{aether}}_{\mu\nu} = T_{\mu\nu} - g_{\mu\nu}g^{\alpha\beta}T_{\alpha\beta}$. To see this, let me use the mathematical fact that in a sufficiently small region, one can choose coordinates to that $g_{\alpha\beta} = \eta_{\alpha\beta}$, so that in particular $g_{tt} = -1$, and thus

$$\begin{aligned}\rho^{aether} \equiv T^{aether}_{tt} &= (p_x + p_y + p_z)/c^2 \\ &= T_{tt} + [-T_{tt} + (p_x + p_y + p_z)/c^2] \\ &= T_{tt} - g_{tt}[-T_{tt} + (p_x + p_y + p_z)/c^2] \\ &= T_{tt} - g_{tt}g^{\alpha\beta}T_{\alpha\beta} = T_{tt} - g_{tt}T \end{aligned}\tag{125}$$

where I have written the density of ordinary matter as T_{tt}. In other words, if there are no labels to the tensor T it is the tensor with only non- æther material. Thus we have

$$T^{\text{aether}}_{\mu\nu} t^\mu t^\nu = T_{tt} - g_{tt}T = (T_{\mu\nu} - g_{\mu\nu}T)t^\mu t^\nu \tag{126}$$

Hence, we have an equality between two tensors for all timelike unit vectors t^μ (recall that "unit timelike vector" means, by definition, that $(g_{\mu\nu}t^\mu t^\nu) = -1$. But I showed earlier that this equality implies the equality of the tensors themselves:

$$T^{\text{aether}}_{\mu\nu} = T_{\mu\nu} - Tg_{\mu\nu} \tag{127}$$

This completes the derivation of the energy tensor for the æther, and thus derives the Einstein field equations as the Newtonian equations for gravity in which the gravitating æther is included.

Equation (127), also give the æther pressures in terms of the normal matter density and the normal matter pressures, since $p^{aether}_i \equiv T^{\text{aether}}_{ii}$. Hence, we have for p^{aether}_x, the component of the pressure in the x direction:

$$p^{aether}_x = \rho c^2 - p_y - p_z \tag{128}$$

where, as above, the quantities without the æther super or subscript are normal matter quantities.

Alternatively, we can express the normal matter density in terms of the normal pressures and the æther pressure in any of the following three ways:

$$\rho = (p^{aether}_x + p_y + p_z)/c^2 \tag{129}$$

$$\rho = (p_x + p^{aether}_y + p_z)/c^2 \tag{130}$$

$$\rho = (p_x + p_y + p^{aether}_z)/c^2 \tag{131}$$

In particular, if the normal matter pressure divided by c^2 is essentially zero — as it is in laboratory conditions on Earth, — then all the æther pressures are equal to ρc^2, which is an enormous number. For ρ equal to the density of water, the æther pressures are all equal (isotropic pressure) and equal to 8×10^{14} atmospheres, or 14 thousand trillion pounds per square inch. But if the normal matter pressures are zero, then equation (124) gives $\rho^{aether} = 0$. So in the presence of normal matter only, the æther would have no density but an enormous pressure. This is what Isaac Newton meant when he wrote that the æther is enormously rarified, but enormously rigid.

In the twentieth century, physicists have had no difficulty accepting such numbers. According to the Standard Model of particle physics — confirmed by all experiments to date — the entire universe is filled by the Higgs energy field, with pressures a factor of 10^{26} larger than these, and with a density even lower than zero. (The Higgs field has a negative mass density, that is, $\rho_{Higgs} = -1.0 \times 10^{26}(m_H/246\text{GeV})^2\text{gm/cm}^3$, where m_H is the mass of the Higgs boson, known to be larger than 100 GeV. I predicted years ago that $m_H = 220 \pm 20$ GeV.) If twentieth century physicists can accept such huge numbers, then it is plausible that nineteenth century physicists would accept lesser numbers for the æther.

In his 1873 book, cited above, Maxwell actually proved that the *energy* density ρ_E was equal to the sum of the pressures: $\rho_E = p_x + p_y + p_z$. I have used the Einstein equation $E = mc^2$ to convert energy density to mass density. This was derived by Einstein in a 1905 paper [93], but according to the historian of the æther Edmund Whittaker, it was originally derived for the electromagnetic field by the great French mathematical physicist Henri Poincare in 1900 ([90], p.51). Priority for discovering that $E = mc^2$ is of no interest here, but the fact was definitely known before relativity was considered a "revolutionary theory." The equality of mass and energy (up to the factor c^2) is important, since if energy had no mass, it presumably would not generate gravity.

In the absence of ordinary matter, equation (127) asserts that there would be no æther either, so the æther is intimately bound up with ordinary matter. The equations in the absence of matter are thus

$$R_{\mu\nu} = 0 \tag{132}$$

In the absence of pressure in ordinary matter, the Newton æther gravitational equations become

$$R_{tt} = 4\pi G\rho \tag{133}$$

which is exactly the same as the Newton gravity equation (17), and

$$R_{\alpha\beta} = 0 \tag{134}$$

for all $\alpha \neq \beta$. If $\alpha = \beta = i$, then

$$R_{ii} = 4\pi G\rho \tag{135}$$

since $T_{\mu\nu} - (1/2)Tg_{\mu\nu} = (\rho/2)I$, where I is the identity matrix.

Equations (134) and (135) are additional constraints on the affine connection, a constraint not present in Newtonian gravity in the absence of the æther. Once again this constraint signals that Einstein gravity theory is constrained Newtonian theory, the latter being more general. Equations (132), (133), (134) and (135) imply a remarkable fact: the æther has an effect, even when its gravitational effects are not present! This is due to the fact that the rigging which defines the æther is still present even when the æther exerts no gravitational effect. To put it another way, In Newton-æther gravity theory, the affine connection must satisfy an additional constraint: the connection is forced to arise from the rigging, or in equivalent language, from the metric $g_{\mu\nu}$. I can show this as follows.

The metric $g_{\mu\nu}$ is a 4×4 symmetric matrix, with inverse $g^{\mu\nu}$. The existence of the metric allows us to define a Ricci scalar R as

$$R \equiv g^{\mu\nu} R_{\mu\nu} \tag{136}$$

where once again we recall that repeated superscripts and subscripts mean summation. The fundamental assumption of Einstein, in his discovery of general relativity, is that he assumed that the affine connection comes from a metric. Then geometry tells us that the connection coefficients are given by the metric by the equation

$$\Gamma^{\alpha}{}_{\beta\gamma} = \frac{g^{\alpha\mu}}{2}\left(\frac{\partial g_{\mu\beta}}{\partial x^{\gamma}} + \frac{\partial g_{\mu\gamma}}{\partial x^{\beta}} - \frac{\partial g_{\beta\gamma}}{\partial x^{\mu}}\right) \tag{137}$$

Let me give an argument (not a proof) that the existence of the æther requires that the affine connecttion must arise from the metric $g_{\mu\nu}$. That is, I shall argue that (137) holds if the Newtonian geodesic equation holds in the presence of an æther defined by equation (115).

It will simplify the calculation if I first define the notion of *covariant derivative* for a vector $t_\alpha \equiv t^\nu g_{\nu\alpha}$ and a tensor $S_{\alpha\beta}$ respectively as:

$$t_{\alpha;\mu} \equiv \frac{\partial t_\alpha}{\partial x^\mu} - \Gamma^{\nu}_{\alpha\mu} t_\nu \tag{138}$$

$$S_{\alpha\beta;\mu} \equiv \frac{\partial S_{\alpha\beta}}{\partial x^\mu} - \Gamma^{\nu}_{\alpha\mu} S_{\nu\beta} - \Gamma^{\nu}_{\beta\mu} S_{\alpha\nu} \tag{139}$$

Then in terms of the covariant derivative, the geodesic equation (12) can be written as (where we set $t^\alpha = dx^\alpha/d\tau$):

$$\frac{d^2x^\alpha}{d\tau^2} + \Gamma^{\alpha}_{\beta\gamma}\frac{dx^\beta}{d\tau}\frac{dx^\gamma}{d\tau} = t^\mu t^\alpha{}_{;\mu} = 0 \tag{140}$$

This equation expresses the assumption that a particle acting only under the force of Newtonian gravity must move along a geodesic, with tangent vector t^μ. But in the presence of the æther, we have no reason to prefer Newtonian absolute time, so the direction of the æther rigging vector u^α ought to be a direction for a geodesic also. This gives

$$u^\mu u^\alpha{}_{;\mu} = 0 \tag{141}$$

Now assume that we can project the covariant derivative of the æther rigging vector u^α into any time direction, not just the direction of Newtonian absolute time t, so that

$$s^\mu u^\alpha{}_{;\mu} = 0 \tag{142}$$

for any timelike vector $s^\mu = dx'^\mu/d\tau$. I proved earlier that if two tensors are equal when Einstein summed over all possible timelike vectors, then they are equal. Thus we have

$$u^\alpha{}_{;\mu} = 0 \tag{143}$$

Notice that I have not proven (143). The founder of Newtonian-æther gravity theory, A. Trautman, simply assumed (143). I have in the above calculation only derived (143) from slightly weaker assumptions. Now (143) implies that

$$g_{\alpha\beta;\mu} = 0 \tag{144}$$

A little algebra, with extensive use of $g_{\alpha\beta}$ and $g^{\alpha\beta}$ to raise and lower indices, will show that equation (144) is equivalent to equation (137). Most textbooks give the proof of this statement —for example a proof is worked out in detail in Exercise 8.15 on page 216 of the textbook by Misner, Thorne, and Wheeler [31] — so I shall not give the detailed algebra here.

So the gravitational force is ultimately due to a spacetime metric, which is itself a manifestation of the luminiferous æther.

As in the Newtonian case, the motion of particles is determined by the geodesic equation, although the time parameter is no longer Newtonian time, but any function that allows the characteristic form of the geodesic equation. This "time" is called an "affine parameter," since it is determined by the affine connection.

Notice that my development of general relativity has made no use of the fact that $g_{\mu\nu}$ is a *metric*, which means that $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$, with $dx^t = d(ct)$, measures length in spacetime. Instead, my entire derivation merely used $g_{\mu\nu}$ as a device to raise and lower indices, so that the Einstein summation convention could be used, and also to distinguish between timelke and spacelike directions. The standard approaches to general relativity usually begin with the metric nature of $g_{\mu\nu}$. The word "metric" has the same meaning that it does in high school calculus of several variables, with one caveat: since ds^2 will be negative if a curve has a timelike tangent vector, the distance along such a curve is obtained by integrating $ds = \sqrt{-g_{\mu\nu}dx^\mu dx^\nu}$ along the curve. For curves whose tangent vectors are everywhere spacelike, the distance is obtained as usual by integrating $ds = \sqrt{g_{\mu\nu}dx^\mu dx^\nu}$. There is a third possibility: the tangent vector could satisfy $t^\mu t_\mu = 0$ with $t^\mu \neq 0$. Such curves are called "null curves" and they represent the paths in spacetime of light rays. The spacetime length of such paths are indeed zero.

It has been known for many years that if we did not have charged particles like electrons to worry about – that is, if we could assume that the charge density ρ and the current density $\vec{J}$ were zero — then we would have an already unified theory of electromagnetism and gravity. Rainich in 1925 [52] and independently Misner and Wheeler in 1957 [53] proved that this already unified theory followed from the following conditions on the Ricci tensor:

$$R^\mu{}_\mu = 0 \tag{145}$$

$$R_\mu{}^\alpha R_\alpha{}^\nu = \delta_\mu{}^\nu (R_{\alpha\beta}R^{\alpha\beta}/4) \tag{146}$$

In the absence of electric charge, these conditions, called the Rainch conditions, determine the electric and magnetic fields up to an arbitrary angle α called the "complexion:"

$$\vec{B}_{\rm new} = \vec{B}\cos\alpha + \vec{E}\sin\alpha \tag{147}$$

$$\vec{E}_{\rm new} = -\vec{B}\sin\alpha + \vec{E}\cos\alpha \tag{148}$$

in a local coordinate system where it can be be assumed that $g_{\mu\nu} = \eta_{\mu\nu}$. (The reason that the fields are determined only up to the complexion is due to the fact that electric charges break the symmetry between electric and magnetic fields. Electrons are electric monopoles, but there are no magnetic monopoles.)

Thus in the absence of electric charges, we have an "already unified theory" of electromagnetism and gravitation. In such a theory, the magnetic and electric fields can be expressed in term of the components of the Ricci tensor. I will not bother the reader with the detailed algorithm for doing so; the algorithm is given in the technical papers cited above.

Since electrons exist, and since we have discovered other vector fields like the electromagnetic field (these are gauge fields) and finally a scalar field called the Higgs field, we have in effect several æthers. The interesting question is, can all of these æthers be incorporated into the gravitational field like the electromagnetic field can. The answer is, we don't know. We do know that the answer is no on the classical level. But in this paper, I have treated quantum mechanics and general relativity separately, because this was what was done by the scholars, Obama and Tribe, whom I am criticizing. However, when gravity and quantum mechanics are combined, one obtains a far more complex equation than the Einstein equation which I derived above. Whether there are conditions analogous to the Rainich conditions on these more complicated equations that will incorporate all the non-gravitational forces into a pure æther field, we simply do not know.

Many physicists have believed that incorporating the fact that electrons have spin 1/2 into Newtonian mechanics is impossible. (The expression "spin 1/2" just means that the electron's intrinsic angular momentum has magnitude $\hbar/2$ whereas the spin of the photon is 1, (meaning that the photon's intrinsic angular momentum is $\hbar$ and the graviton, the particle that is the multiverse manifestation of the gravitational field, has spin 2.) But this is false, and this error is ultimately due to the fact that most physicists still do not accept the fact that the wave function in quantum mechanics is a measure of the density of universes in the multiverse, a fact recognized by Schrödinger, the creator of quantum mechanics.

More precisely, it is not the wave function itself but the quantity $\mathcal{R}^2$ that is proportional to the density of universes in the multiverse, and this has a profound implication: there are two possible types of statistics allowed in mathematically consistent Newtonian mechanics. We call these two types of statistics Bose-Einstein statistics and Fermi-Dirac statistics.

Indistinguishability is ordinarily applied by writing the wave function as a function of the $3N$ variables of configuration space and satisfying Schrödinger's equation (31) in these $3N$ variables. It is easily verified that the transformation (32) with the functions $\mathcal{R}$ and φ also in $3N$ variables, yields the Hamilton-Jacobi equation (33) and equation (34), both in $3N$ variables. Now we have another symmetry: these two equations are invariant under particle exchange, provided that (as usual) the potential is invariant under the exchange, and that the function $\mathcal{R}$ satisfies either

$$\mathcal{R}(\vec{x}_1, \ldots \vec{x}_i \ldots \vec{x}_j \ldots \vec{x}_n) = \mathcal{R}(\vec{x}_1, \ldots \vec{x}_j \ldots \vec{x}_i \ldots \vec{x}_n) \tag{149}$$

or

$$\mathcal{R}(\vec{x}_1, \ldots \vec{x}_i \ldots \vec{x}_j \ldots \vec{x}_n) = -\mathcal{R}(\vec{x}_1, \ldots \vec{x}_j \ldots \vec{x}_i \ldots \vec{x}_n) \tag{150}$$

since the function $\mathcal{R}$ appears only as a square in equation (34) and in the combination $\nabla^2\mathcal{R}/\mathcal{R}$ in equation (33).

End Mathematical Interlude

As any physics textbook will inform you, the equation (149) gives Bose-Einstein statistics, and the equation (150) gives Fermi-Dirac statistics. In other words, these two distinct types of statistics are already present in Newtonian mechanics, or more exactly, in the Hamilton-Jacobi form of Newtonian mechanics. The connection between the spin and the statistics comes from considering how the æther interacts with particles. A nice elementary proof of the necessary connection between the spin and the statistics, considering the effect of the æther, was published, two years before the publication of the Tribe paper, by the great physicist Richard Feynman [86]. Feynman held the opinion, which I have assumed throughout this paper, that any fundamental idea in physics can be understood by anyone who has a high school knowledge of calculus. Feynman expressed this by saying that if a physicist cannot explain any idea in physics in a single lecture to an audience of freshman physics students, he does not understand the idea himself. The proof by Feynman [86] of what is called the Spin-Statistics Theorem, is indeed understandable by anyone with a knowledge of high school calculus, so I shall not repeat it here. I shall only need the conclusion: any particle that obeys Fermi-Dirac statistics necessarily has half integral spin (that is, its intrinsic spin is $\hbar/2$, or $3\hbar/2$), and any particle that obeys Bose-Einstein statistics necessarily has integral spin (that is, its intrinsic spin is eithert 0, $\hbar$, or $2\hbar$. Feynman prefers the standard language of special relativity rather than the language of æther mechanics to derive the Spin-Statistics Theorem, and it may be that this theorem is easier to prove in this traditional language.

So once again, we are still using Newtonian mechanics when we analyze the implications of the electron's intrinsic spin. Most physicists have not realized this because they have been incorrectly taught that it is the electron's spin that is fundamental, whereas it is actually the statistics that the electron obeys that is fundamental, and the value of the spin follows from combining the statistics with æther theory (special relativity).

The *Boumedienne v. Bush* Decision: An Example of Tribe and Obama's Bad Physics

To demonstrate that *Boumedienne v. Bush* is fundamentally based on the "metaphors" taken from the bad physics of Obama and Tribe, all I really have to do is quote the dissenting opinions of Roberts and Scalia. Obama himself said that he voted against confirming Chief Justice Roberts because he (Obama) rejected the idea of the Constitution being a self-correcting Newtonian machine, which would operate best without continual interference from the personal values of Supreme Court Justices:

"... I have not only argued cases before appellate courts but for 10 years was a member of the University of Chicago Law School faculty and taught courses in constitutional law. ... It is absolutely clear to me that Judge Roberts truly loves the law. He couldn't have achieved his excellent record as an advocate before the Supreme Court without that passion for the law, and it became apparent to me in our conversation that he does, in fact, deeply respect the basic precepts that go into deciding 95 percent of the cases that come before the Federal court — adherence to precedence, a certain modesty in reading statutes and constitutional text, a respect for procedural regularity, and an impartiality in presiding over the adversarial system. ... adherence to legal precedent and rules of statutory or constitutional construction will dispose of 95 percent of the cases that come before a court, so that both a Scalia and a Ginsburg will arrive at the same place most of the time on those 95 percent of the cases – what matters on the Supreme Court is those 5 percent of cases that are truly difficult. ... In those 5 percent of hard cases, the constitutional text will not be directly on point. The language of the statute will not be perfectly clear. Legal process alone will not lead you to a rule of decision. In those circumstances, your decisions about whether affirmative action is an appropriate response to the history of discrimination in this country or whether a general right of privacy encompasses a more specific right of women to control their reproductive decisions or whether the commerce clause empowers Congress to speak on those issues of broad national concern that may be only tangentially related to what is easily defined as interstate commerce, whether a person who is disabled has the right to be accommodated so they can work alongside those who are nondisabled – in those difficult cases, the critical ingredient is supplied by what is in the judge's heart [94]."

Needless to say, all important cases are in "those 5 percent of hard cases." In these, Obama wishes decisions to be based on "what is in the judge's heart ." Arbitrary human will, rather than unalterable law, is held to control the universe. I disagree.

This passage provides evidence of Obama's innumeracy, even at the elementary school level. In the extended quote from Senator Obama's press release, we have a definition of "hard case" — those in which Justice Ginsburg and Justice Scalia will find themselves on opposite sides. Yet, in the 2006–2007 Supreme Court session, according to Laurence Tribe [10], who does understand mathematics — at least at the elementary school level today, and forty years ago, at a much higher level — "24 out of 72 cases were decided by a 5-4 vote, with Justice Anthony Kennedy as the swing vote." In $24/72 = 1/3$ of the cases, at a bare minimum, Ginsburg and Scalia were on the opposite sides, hence at least one third of the cases that were decided by the Supreme Court in the 2006–2007 session were "hard cases." It should be obvious that $1/3$ is not equal to $5/100 = 1/20$. It should be equally obvious that one third is substantially greater than one-twentieth. Furthermore, it is likely that Justice Ginsburg and Justice Scalia were on opposite sides in some decisions that were decided by a majority greater than 5-4.

The passage also provides evidence of Obama's lack of heart. He wants "women to control their reproductive decisions." In particular, he has repeatedly supported partial birth abortion, a horror large majorities of both the House and Senate (but not Obama) voted to ban, and the Supreme Court (by a 5-4 vote) upheld. Obama wants judges who will allow partial birth abortion.

Partial birth abortion involves using a vacuum device to suck out the brains of a full term baby while it is still in its mother's womb. Films of this monstrous act are available, but banned from the mass media, and from Youtube. Any sane person witnessing this outrage would know that it should be banned, and the perpetrator sent to jail for life, or until full and convincing repentance. Although films of the actual act are banned, you can watch a fictionalization of the sucking out of brains in the movie *Starship Troopers.* At least in the movie, an insect-like alien from another planet did the brain sucking, and not a human "doctor." (I put "doctor" in quotes, because performing abortions is strictly prohibited by the Hippocratic Oath.) At least in the movie, an adult human soldier's brains are sucked out, and not a baby's. In both the movie and in a partial birth abortion, the human being screams as his brains are sucked out. Which is why "experts" do not want you to see a real partial birth abortion. The average person is not mad, and would immediately realize the meaning of this experiment: it is a form of death by torture. Another reason why professors at the elite universities want to redefine science from "confirmation by experiment," to "consensus by experts."

To demonstrate that *Boumediene v. Bush* is an example of the Obama-Tribe thesis that the arbitrary will of judges should replace unalterable law requires nothing but a few quotes from Associate Justice Scalia's dissent, beginning with the last clause in his first paragraph: "... the Court's intervention in this military matter is entirely *ultra vires.*" On the second page of his dissent, Justice Scalia complains about the abandonment of precedent: "But it is this Court's blatant *abandonment* of such a principle [precedent] that produces the decision today. The President relied on our settled precedent in *Johnson v. Eisentrager*, 339 U.S. 763 (1950), when he established the prison at Guantanamo Bay for enemy aliens."

As Justice Scalia pointed out, two centuries of precedent established that treatment of enemy soldiers was determined by the President, in his military role of Commander-in- Chief, and Congress. Now, "As THE CHIEF JUSTICE's dissent makes clear, we have no idea what those procedual and evidentiary rules are, but they will be determined by civil court ... (p. 4). In other words, determined by the arbitrary will of the civil court judges.

And a majority of the current Supreme Court Justices feel free to set aside at any moment their own earlier decisions: "And today it is not just the military that the Court elbows aside. A mere two Terms ago in *Hamdan v. Rumsfeld*, 548 U.S. 557 (2006), when the Court held (quite amazingly) that the Detainee Treatment Act of 2005 had not stripped habeas jurisdiction over Guantanamo petitioners' claims, four Members of today's five-Justice Majority joined an opinion saying the following: 'Nothing prevents the President from returning to Congress to seek the authority ... he believes necessary. ... The Constitution places its faith in those democratic means.' Turns out they [the earlier majority] were just kidding. ... What the Court apparently means is that the political branches [Congress and the President] can debate, after which the Third Branch will decide" (p. 5).

Justice Scalia summarizes that the Court claims the power "to say what the law is" (p. 17).

Is it just a matter of time before the Supreme Court declares what the laws of physics are?

This is not at all a rhetorical question. The Federal courts have been involved over the past few decades in the question of the mechanism of evolution. All mechanisms are ultimately physical mechanisms, since, as I have demonstrated, we live in a Newtonian mechanical universe, a deterministic mechanical universe. Darwinism, in contrast, insists on a fundamental randomness in nature. Darwinian theory claims that there is no cosmic teleology, no ultimate goal for reality. But the universe (actually the multiverse) is deterministic, and this determinism works both ways: past to future determinism is mathematically equivalent to future to past determinism [95]. In other words, to say that we humans evolved because the physical multiverse was in a certain state 10 billion years ago is mathematically completely equivalent to saying that we humans evolved because our actions today are necessary in order for the multiverse to achieve a certain goal in the far future. Just because we do not know what that goal is does not mean that there is no goal. Unless, once again, one is arrogant enough to assume that just because we humans are unaware of a goal means that indeed there is no goal to cosmic, universal history. I find it extraordinary that there are people in important positions who think they can dictate to the entire universe what it must be like.

The first Supreme Court decision on the evolution question was *Epperson v. Arkansas*, 393 U.S. 97 (1968), which invalidated an Arkansas state constitutional provision forbidding the teaching of evolution in public schools. The state of Arkansas eventually tried to reintroduce the prohibition by requiring "equal time" for "creation science" if evolution was taught. This act was invalided by Federal District Judge William R. Overton, in *McLean v. Arkansas Board of Education* (529 F. Supp. 1255, 1258-1264 (ED Ark. 1982)). In his decision, Judge Overton himself forbade any discussion in public schools of any theory of the universe that had a beginning in time (Overton used the expression "sudden creation of the universe ... from nothing" [96]), on the grounds that such a theory — professional cosmologists call this theory the "Big Bang" theory — is inherently religious and unscientific. I had, at the time Judge Overton handed down his decision, just completed post-doctoral work researching the Big Bang theory at a public school, the University of Texas at Austin. I wrote to my teacher, John Wheeler, pointing out that he was now forbidden to continue his work on Big Bang cosmology. John Wheeler wrote back that he would be unable to obey the law of the land.

So Federal judges are indeed now arrogant enough to decree what the laws of physics must entail. It is this sad state to which the carckpot physics of Tribe and Obama have led us.

[1] Laurence H. Tribe "The Curvature of Constitutional Space: What Lawyers Can Learn from Modern Physics," *Harvard Law Review* **103**, 1–39.

[2] Quoted by Roger Bacon, in Chapter 2 of the First Distinction of Part IV of *Opus Majus* (p. 117 of Volume 1 of the Robert Belle Burke translation (University of Pennsylvania Press, 1928).

[3] Roger Bacon, *Opus Majus*, part IV First Distinction, Chapter 1 (Rome, 1267) p. 116 of Volume 1 of the Robert Belle Burke translation (University of Pennsylvania Press, 1928).

[4] Roger Bacon, Chapter 3 of the First Distinction of Part IV of *Opus Majus* (p. 120 of Volume 1 of the Robert Belle Burke translation (University of Pennsylvania Press, 1928).

[5] Galileo Galilei, *Il Saggiatore (The Assayer)* Rome, October 1623, translated by Stillman Drake Discoveries and Opinions of Galileo (Doubleday, New York 1957) pp. 238-239.

[6] Laurence H. Tribe *God Save This Honorable Court: How the Choice of Supreme Court Justices Shapes Our History* (New York, Random House,1985).

[7] Joseph Bottum "The Big Mahatma: Laurence Tribe and Problem of Borrowed Scholarship," *The Weekly Standard* **10** (October 4, 2004). available online.

[8] Peter W. Morgan and Glenn H. Reynolds *The Appearance of Impropriety: How the Ethics Wars Have Undermined American Government, Business, and Society* (New York, Free Press,1997). See especially chapter five, "A Plague of Originality," which is available online.

[9] Jacob Hale Russell "A Million Little Writers," *02138 Magazine* (November/December 2007), page 78. Available online at www.02138mag.com.

[10] Shira Schoenberg "Law expert: Obama will preserve Constitution," *Concord Monitor* November 14, 2007. Available online at http://www.cmonitor.com

[11] Richard P. Feynman *What Do YOU Care What Other People Think?* (New York, Norton, 1988).

[12] Lev D. Landau and E. M. Lifschitz *Mechanics* (New York, Addison Wesley, Reading [MA], 1960).

[13] Michiyo Nakane and Craig G. Fraser, "The Early History of Hamilton-Jacobi Dynamics 1834–1837," *Centaurus* **44** (2002), 161–227.

[14] Thomas L. Hankins *Sir William Rowan Hamilton* (Baltimore, Johns Hopkins University Press, 1980), p. 196. Hamilton obtained two equations, one in the initial data and one in the final data, but Jacobi pointed out that only one of these equations was necessary. As to why quantum mechanics was not developed early in the nineteenth century, Hankins has a few suggestions. in chapter 14, particularly pages 202–207. I would add to Hankins' opinion the fact that physicists then and now do not take their equations seriously. Physics Nobel Laureate Steven Weinberg in his *The First Three Minutes* (Collins, Glasgow, 1977) remarks on page 128: "This is often the way it is in physics — our mistake is not that we take our theories too seriously, but that we do not take them seriously enough. It is always hard to realize these numbers and equations we play with at our desks have something to do with the real world." In the Hamitlon-Jacobi case, no mathematician ever asked what is required in order for the equation to apply for all times and all places. Insisting that the H-J equation apply always and everywhere leads to quantum mechanics of mathematical necessity, as I shall show in the Mathematical Interlude on quantum mechanics.

[15] Isaac Newton, *Opticks* (Dover, New York, 1952), p. 348.

[16] Thomas Young "The Bakerian Lecture: On the Theory of Light and Colours," *Philosophical Transactions of the Royal Society of London* **92** (1802), 12–48.

[17] The entire paragraphs have been taken word for word from http://library.thinkquest.org/22494/stories/Euclid.htm)

[18] An alternative proof that $1 = 0.99999999\ldots$, apparently more rigorous, but actually equivalent to the proof in the text is as follows. The number $0.99999999\ldots$ is really the infinite geometric series $9/10 + 9/10^2 + 9/10^3 + 9/10^4 + \ldots$, which is proven in any textbook on infinite series to converge to $9/(1 - \frac{1}{10}) - 9 = 9/(9/10) - 9 = 10 - 9 = 1$.

[19] Élie Cartan "Sur les variétés à connexion affine et la théorie de la relativité generalisée (premiére partie)," *Ann. École Norm. Sup.* **40** (1923), 325–412.

[20] Élie Cartan "Sur les variétés à connexion affine et la théorie de la relativité generalisée (suite)," *Ann. École Norm. Sup.* **41** (1924), 1–25.

[21] Ludwik Silberstein "The True Relation of Einstein's to Newton's Equations of Motion," *Nature* **112**, 778–779.

[22] Ebenezer Cunningham "Relativity: The Growth of an Idea" *Nature* **106**, 784–786.

[23] Gerald Holton and Yehuda Elkana, editors 1982 *Albert Einstein: Historical and Cultural Perspectives* (Princeton University Press, Princeton.

[24] Bernard N. Nathanson *The Hand of God* (Regnery, Washington, 1996) documents the effect ultrasound technology had in transforming him from an abortionist into a pro-life medical doctor. He also gives examples of how attempts have been made to ban ultrasound movies of abortions, the first of which *Silent Scream*, was made by Dr. Nathanson. Fortunately, such efforts have not yet succeeded, and such movies are available on the internet, for examples: YouTube and at http://www.silentscream.org/. More than 45 million abortions have been performed in the United States since *Roe vs. Wade*. See also Ramesh Ponnuru *The Party of Death* (Regnery, Washington, 2006).

[25] Michael Gorman *Abortion and the Early Church: Christian, Jewish and Pagan Attitudes in the Greco-Roman World* (Eugene (OR), Wipf and Stock, 1998).

[26] The Christian Church has *always* opposed abortion. The historical evidence is overwhelming, despite attempts of professors at elite universities to falsify the historical record. House Speaker Nancy Pelosi's spokesman, for example, falsely cited St. Augustine in support of her prochoice position. In fact, St. Augustine followed the standard Christian opposition to abortion, opposition which was already ancient in the 5th century of St. Augustine. A summary of St. Augustine's position on abortion can be found in [29], which I now quote:

"**Abortion** Augustine, in common with most other ecclesiastical writers of his period, vigorously condemned the practice of induced abortion. Procreation was one of

the goods of marriage; abortion figured as a means, along with drugs which cause sterility, of frustrating this good. It lay along a continuum which included infanticide as an instance of "lustful cruelty" or "cruel lust" (nupt. et conc. 1.15.17). Augustine called the use of means to avoid the birth of a child an "evil work": a reference to either abortion or contraception or both (b. conjug. 5.5).

"Augustine accepted the distinction between "formed" and "unformed" fetuses found in the Septuagint version of Exodus 21:22-23. While the Hebrew text provided for compensation in the case of a man striking a woman so as to cause a miscarriage, and for the penalty to be exacted if further harm were done, the Septuagint translated the word "harm" as "form," introducing a distinction between a "formed" and an "unformed" fetus. The mistranslation was rooted in an Aristotelian distinction between the fetus before and after its supposed "vivification" (at forty days for males, ninety days for females). According to the Septuagint, the miscarriage of an unvivified fetus were vivified, the punishment wa a capital one.

"Augustine disapproved of the abortion of both the vivified and unvivified fetus, but distinguished between the two. The unvivified fetus died before it lived, while the vivified fetus died before it was born (nupt. et con. 1.15.17). In referring back to Exodus 21:22-23, he observed that the abortion of an unformed fetus was not considered murder, since it could not be said whether the soul was yet present (qu. 2.80).

"The question of the resurrection of the fetus also exercised Augustine, and sheds some light on his views on abortion. Here again he referred to the distinction between the formed and unformed fetus. Though he acknowledged that it was possible that the unformed fetus might perish like a seed, it was also possible that, in the resurrection, God would supply all that was lacking in the unformed fetus, just as he would renew all that was defective in an adult. This notion, Augustine remarked, few would dare to deny, though few would venture to affirm it (ench. 33.85). At another point Augustine would neither affirm nor deny whether the aborted fetus would rise again, though if it should be excluded from the number of the dead, he did not see how it could be excluded from the resurrection (civ. Dei 22.13)."

The Christian Church dropped the distinction between vivified and univvfied fetus in the nineteenth century, once scientific embryology research made it clear that this was a false distinction, and that the fetus was ensouled from the moment of conception. Pelosi, like Tribe and Obama, insist on sticking to 5th century science, which held that the fetus was formed from a woman's menstrual blood and that this blood was not vivified (ensouled) until 40 days after conception for males, and 90 days for females. Once again, the views of Obama, Pelosi, and Tribe are demonstrated to be profoundly reactionary.

But the error on ensoulment never influenced the Christian Church's opposition to abortion. In the *Didache,* (Greek for "Teaching") thought to be the first Catechism, or summary of basic Church teaching, and dated by most modern scholars to the late first or early second century (80 A.D.— 120 A.D.) we find: "You shall not kill the embryo by abortion and shall not cause the newborn to perish." (*Didache 2,2* translation from passage 2271 of *Catechism of the Catholic Church*, on page 606 of [28]. Additional valid (needless to say, not from professors at elite universities), evidence that the Church has from its very beginnings been opposed to abortion can be found in [27] and [28]. The executive summary of the historical evidence has been given in the Catholic Catechism: "Since the first century the Church has affirmed the moral evil of every procured abortion. This teaching has not changed and remains unchangeable."

Notice that the Church's view of the moral law is exactly the same as my view of Newtonian mechanics, as developed in this paper: "unchanged and unchangeable." The basic law is correct, just a few minor adjustments have to make it completely consistent. The Church had to abandon the false distinction between vivified and unvivfied fetus, a distinction which can be used to undermine its always correct moral position on abortion, as the citations by pro-choice politicians to the works of Augustine and Aquinas have made clear. Similarly, physicists have to abandon the mathematically inconsistent version of the Hamilton-Jacobi theory — thereby getting Schrödinger's equation — and their insistence that Newtonian mechanics be restricted to non-æther mechanics — thereby getting general relativity. Quantum mechanics and general relativity are correct physics — and they are both Newtonian mechanics.

[27] John R. Connery *Abortion, the Development of the Roman Catholic Perspective* (Chicago, Loyola Univeristy Press, 1977).

[28] Joseph Ratzinger *Catechism of the Catholic Church* (New York, Doubleday, 1995). The passages 2270–2275 on abortion are on pages 608–609 of this translation.

[29] Allan D. Fitzgerald, general editor *Augustine Through the Ages : an Encyclopedia* (Grand Rapids, (Mich). ; Cambridge, (U.K.), William B. Eerdmans, 1999). The quoted passage on "abortion" was written by John C. Bauerschmidt, and this passage is available online at amazon.com.

[30] Frank J. Tipler *The Physics of Christianity* (New York, Doubleday, 2007).

[31] Charles W. Misner, Kip S. Thorne, and John A. Wheeler 1973 *Gravitation* (Freeman, San Francisco).

[32] J. C. Maxwell, *A Treatise on Electricity and Magnetism* (Clarendon Press, Oxford, 1873), in two volumes. Ernst Mach's personal copy is available online.

[33] Stillman Drake *Galileo: Oxford Past Masters Series* (New York: Hill and Wang, 1980).

[34] Bowers v. Hardwick, 478 U.S. 186 (1986).

[35] Lawrence v. Texas, 539 U.S. 558 (2003).

[36] David Bohm "Suggested Interpretation of the Quantum Theory in Terms of Hidden Variables: 1" *Phys Rev* **85** (1952), 166–179.

[37] David Bohm "Suggested Interpretation of the Quantum Theory in Terms of Hidden Variables: 2" *Phys Rev* **85** (1952) 180–193.

[38] Lev D Landau and E M Lifshitz 1977 *QuantumMechanics: Non-relativistic Theory, Third Edition* (Oxford: Pergamon Press). The first edition appeared in 1958, before Obama was born.

[39] Henri Poincaré *Calcul des Probabilités*, second edition, (Gauthier-Villars, Paris, 1912).

[40] Erwin Schrödinger "An Undulatory Theory of the Mechanics of Atoms and Molecules," *Physical Review* **28** (1926), 1049–1070.

[41] Max Jammer 1974 *The Philosophy of Quantum Mechan-*

ics (New York: Wiley).

[42] B. W. Carroll and D. A. Ostlie *An Introduction to Modern Astrophysics* (Addison-Wesley, New York, 1996).

[43] F. J. Tipler *Rep. Prog. Phys.* **68** (2005), 897–964.

[44] The Newtonian gravitational equation $R_{tt} = 4\pi G\rho$, which, recall, is Poisson's equation $\nabla^2\Phi = 4\pi G\rho$ in geometrical language, should have been regarded as too restrictive by nineteenth century physicists for reasons independent of the existence of the æther. Georg Friedrich Bernhard Riemann gave a famous lecture, "On the Hypotheses Which Lie a the Bases of Geometry," where he proposed that the universe might be a spatially a three-sphere rather than Euclidean three-space. It is well-known that Karl Friedrich Gauss was in the audience, and was enthusiastic about the lecture — in fact, Gauss himself chose the topic of Riemann's lecture. (A translation of Riemann's lecture was made for *Nature* (**8** (1873) 14–17, 46–37) by William Kingdon Clifford). So Riemann, Gauss, and Clifford — three of the greatest mathematicians of the nineteenth century — believed that a three-sphere universe was a real possibility. However, if mass is positive, then Poisson's equation does not allow a three-sphere universe. An elementary mathematics proof of this is as follows. Integrate $\nabla^2\Phi = 4\pi G\rho$ over the entire three-sphere, getting $(1/(4\pi G))\int \rho\sqrt{g}d^3x = \int \nabla^2\Phi\sqrt{g}d^3x = \int div\, grad\Phi\sqrt{g}d^3x = \int_S grad\Phi\, dS = 0$, the last two steps using the Gauss Divergence theorem and the fact that the last integral is over the boundary S of the three-sphere, and thus this integral is zero since the three-sphere has no boundary. But $\int \rho\sqrt{g}d^3x = 0$ is impossible if $\rho \geq 0$, unless $\rho = 0$ everywhere in the universe. This elementary proof could have been discovered by any of these great nineteenth century mathematicians. A more general textbook proof, based on differential forms, is as follows. First, it is easily shown (pp. 221–222 of *Foundations of Differentiable Manifolds and Lie Groups* by Frank W. Warner [Scott, Foresman, London 1971]) that the only harmonic functions f (solutions of $\nabla^2 f = 0$) on a compact, connected, oriented Riemannian manifold are the constant functions. In particular, the three-sphere is a compact, connected, oriented Riemannian manifold. Then by the Hodge Decomposition Theorem (p. 233 of Warner), applied to functions (which are 0-forms in the language of the Hodge Theorem), the equation $\nabla^2\Phi = 4\pi G\rho$ will have a solution only if ρ is orthogonal (defined by equation (5) on page 221 of Warner) to the harmonic functions. This means that ρ must satisfy $\int f\rho\sqrt{g}d^3x = 0$, where the integral is over the entire compact manifold, f is any constant, and $\sqrt{g}d^3x$ is the volume element of the Riemannian manifold. This is impossible, as before, if $\rho \geq 0$ (or if $\rho \leq 0$). In fact, it is impossible unless the average value of the positive mass is equal to the average value of the negative mass. Thus, if the universe is a three-sphere and Poisson's equation holds, then not only must matter with negative mass exist, but this negative mass matter must make up half of the matter in the universe! Clearly, Poisson's equation is too restrictive for the world-view of the great nineteenth century mathematical physicists. A three-sphere universe, however, is allowed if the æther's gravitational effects are taken into account, as I show in the text of this paper.

[45] Rainer Kurt Sachs and Hung-Hsi Wu *General Relativity for Mathematicians* (Springer-Verlag, Heidelberg, 1977). I am grateful to Professor M. Dupre for pointing out this theorem to me. I am embarrassed that I was not aware of it, since I was Professor Sachs' postdoc in 1979.

[46] Andrzej Trautman "Comparison of Newtonian and Relativistic Theories of Space-Time," in *Perspectives in Geometry and Relativity: Essays in Honor of Václav Hlavatý*, edited by Banesh Hoffmann (Indiana University Press, Bloomington, 1966), pp. 413–425.

[47] S. W. Hawking and G. F. R. Ellis *The Large Scale Structure of Space-Time* (Cambridge (UK), Cambridge University Press, 1973).

[48] Maurice Dupre, "The Einstein Equation and the Energy Density of the Gravitational field," arXive:0803. 1684v1[math-ph] 11Mar2008.

[49] One might be tempted to generalize from the electromagnetic wave by trying $\rho_{aether} = \rho$, the density of normal matter. But if this held for all possible time directions, then it would imply $T^{aether}_{\mu\nu} = T_{\mu\nu}$; that is, we would have the æther tensor equal to the normal matter tensor. This would eliminate the gravitational effect of the æther altogether, since such an equality would just mean an effective doubling of the gravitating matter in a region. To make this consistent with experiment, the gravitational constant G would have to reduced by a factor of 1/2. However, the gravity equations would become $R_{\mu\nu} = 4\pi G T_{\mu\nu}$, which would violate conservation of energy and momentum, since $R^{\mu\nu}{}_{;\nu} \neq 0$. This conservation law violation would be drastic, and show up even at speeds small compared with light speed. It is well-known that the standard rocket exhaust equations would be grossly violated, contrary to experiment. In an earlier version of this paper, I attempted a third approach, setting the æther pressures equal to the matter pressures. This would work only if $T = 0$, which is too restrictive. I thank Professor Dupre for pointing out this error.

[50] Albert Einstein, H. A. Lorentz, Hermann Weyl, Hermann Minkowski, Arnold Sommerfeld *The Principle of Relativity* (Dover Publicaitions, New York, 1923).

[51] Albert Einstein"Die Grundlage der allgemeinen Relativitätstheorie," *Annalen der Physik* **49** (1919) translated into English as "The Foundations of the General Theory," in [50], pp. 109–164.

[52] G. Y. Rainich, *Trans. American Mathematical Society* **27** (1925) 106.

[53] John A. Wheeler *Geometrodynamcis* (Academic Press, New York, 1962), pp. 227–228, and 239. The reader is warned that there pages missing from this collection of papers; to see the complete details, the reader should go to the original technical articles.

[54] Peter Novick, That Noble Dream: The "Objectivity Question" and the American Historical Profession (Cambridge: Cambridge University Press, 1988). Novick argues that objectivity has been abandoned in the history profession, and in part he justifies this abandonment, by claiming, like Obama and Tribe, that physics has abandoned objectivity. Novick gave the title "The Center Does Not Hold," to his chapter on physics. I agree with Novick (and Obama and Tribe) that physics is the center of intellectual life. I disagree that this center has abandoned either objectivity or Newtonian mechanics. The Center Holds! Novick, like myself, is taking the phrase "the center does not hold,' from a famous poem by Yeats. I shall not generally give references to passages which I have

taken form other authors; I shall feel free to plagiarize at will. As I have emphasized in the body of this paper, my argument becomes all the stronger if my paper can be demonstrated to be *entirely* plagiarized. The more plagiarism the better.

[55] James Lindgren "Review: Fall from Grace: Arming America and the Bellesiles Scandal," *The Yale Law Journal* **111** (2002), 2195–2249. This article summaries a scandal in the history profession, exposed in large part by professors of law. In contrast, a preliminary version of the book *Arming America*, was awarded a prize by the Organization of American Historians for the best article to appear in 1996 in the *Journal of American History*. In April 2001, Columbia University awarded the Bancroft Prize for history to *Arming America* (after the book was exposed as a fraud, the Bancroft Prize was rescinded). When the president of the National Rifle Association, Charlton Heston was critical of the methodology of the book, the book's author responded "When Professor Heston gets his Ph.D. and does the research, I might be open to persuasion." In other words, professors who teach at elite universities know better than the average person. Bellesiles was supported by other history professors. Heston was right; the professors at the elite universities were wrong.

[56] David Stove *Scientific Irrationalism: Origins of a Postmodern Cult* (London, Transaction Publishers, 2007).

[57] David Stove "Cole Porter and Karl Popper: The Jazz Age in the Philosophy of Science," originally published in *Encounter* (June 1985). Reprinted in David Stove *The Plato Cult and other Philosophical Folllies* (Oxford, Basil Blackwell, 1991), pp. 1–26.

[58] Richard T. Cox *The Algebra of Probable Inference* (Baltimore, Johns Hopkins University Press, 1961), p. 91.

[59] Many mathematicians, recognizing that the multiplication rule and the sum rule are central to probability theory, have given alternative derivations from other postulates. Maurice J. Dupre and I have provided a general list of these alternative derivations in "The Cox Theorem: Unknowns and Plausible Value," available on the arXiv at math.PR/0611795.

[60] E. T. Jaynes *Probability Theory: The Logic of Science* (Cambridge: Cambridge University Press, 2003), p. 33.

[61] Marquis de Laplace *A philosophical Essay on Probabilities* (New York, Dover Publications, 1995). Translation of the the 1814 book in French.

[62] Harold Jefferys *Theory of Probability* (Clarendon Press, Oxford 1939)

[63] T. Bayes "An Essay Toward Solving a Problem in the Doctrine of Chances," *Philosophical Transactions of the Royal Society of London* **53** (1773), 370–418. The entire article is available online via Wikikpedia (click on "Thomas Bayes "). The evidence that Bayes derived the product rule in 1748 to refute Hume's attack on inducitve reasoning can be found in S.M. Stigler, "Who Discovered Bayes' Theorem?" *Am. Stat* **37** (1983), 290–296. and in S.L. Zabell, "The Rule of Succession, " *Erkenntnis* **31** (1989), 283–321. See also the discussion in S. M. Stigler *The History of Statistics* (Cambridge (UK), Harvard University Press, 1986).

[64] David Hume *An Enquiry concerning Human Understanding* in *Essential Works of David Hume* edited by Ralph Cohen (New York, Bantam Books, 1965). As the editor points out (p. 31), the first edition of this work appeared in 1748, under the title *Philosophical Essays Concerning Human Understanding*. The second edition, with the now standard title appeared in 1758. The final edition, rewritten by Hume, was published in 1777, after his death the preceding year. The book was largely a rewriting of Book I of his *Treatise of Human Nature*, published in 1738. Indeed, Bayes could have been aware of Hume's attack on the Principle of Induction.

[65] This quote can be found on page 165 of [64]. It can also be found in any edition of *An Enquiry concerning Human Understanding*, in the latter part of Section XII, Part III of this book. Read the context yourself, and you will see that I have *not* taken this passage out of context. Also, although Hume claimed to have himself solved the Problem of Induction (a problem he himself created), few philosophers, in his time or later, believe that he did. Hume wasn't stupid, just crazy. So it is unlikely he believed his own solution.

[66] This quote can be found on page 167 of [64]. These words are the very last paragraph in Hume's *An Enquiry concerning Human Understanding*.

[67] Richard Cox ([58], p. 92) discusses what convinced physicists that the Young-Fresnel wave theory of light was true. The great French mathematician Poisson pointed out to Fresnel his equations implied that if a circular disk were placed in front of a beam of light, there would have to be a bright spot of light in the center of the shadow behind the disk. This had never been seen, and everyone, including Fresnel, believed the existence of such a bright spot to be very improbable. However, Fresnel could find nothing wrong with Poisson's calculation, so he went to his laboratory to look for it. He found it. As the theory of probability requires, this confirmation of a vary improbable implication of a theory vastly increases the probability that the theory is true. See [91], p. 115 for a further discussion of the reaction to the discovery of what is today called the Fresnel Bright Spot.

[68] Paul Forman *Historical Studies in the Exact Sciences* **3** (1971) 1–115.

[69] S. G. Brush *Social Studies of Science* **10** (1978), 393–447.

[70] Thomas S. Kuhn "Reflections on My Critics, in [72].

[71] Thomas S. Kuhn *The Structure of Scientific Revolutions, Second Edition* (Chicago, University of Chicago Press, 1970).

[72] Imre Lakatos and Alan Musgrave, editors *Criticism and the Growth of Knowledge* (Cambridge (UK), Cmmbridge University Press, 1970).

[73] Bertrand Russell *A History of Western Philosophy* (New York, Simon and Schuster, 1963).

[74] Karl R. Popper *The Logic of Scientific Discovery* (Harper and Row, New York, 1965).

[75] Karl R. Popper *Conjectures and Refutations* (Harper and Row, New York, 1968).

[76] Karl R. Popper *Objective Knowledge* (Oxford (UK), Oxford University Press, 1972).

[77] Sir Arthur Conan Doyle *The Coming of the Fairies* (Lincoln (NE), University of Nebraska Press, 2006). Originally published in 1922 by Hodder and Stoughton, London.

[78] James Randi *Flim-Flam!* (Buffalo, New York: Prometheus Books,1982). See Randi's online discussion of this appalling example of madness at http://www.randi.org/library/cottingley/movie.html.

[79] Michael Crichton *State of Fear: a Novel* (New York,

Harper-Collins, 2004). This novel has an exceedingly interesting appendix, with references to the technical literature supporting Crichton's thesis, namely that we do not know if human activity is responsible for the apparently observed temperature increase of the Earth over the last century. I wrote "apparently observed" because we now know there is considerable doubt as to the reliability of the temperature observations. I myself looked at the recording thermometer in New Orleans, and it is now located next to a gravel boat loading area, after having been moved from a grassy area in a park. Can we trust that the corrections that must be made because the thermometer is now next to a heat absorber, have been made correctly? Also, any measurement of a global temperature increase must depend on observations made in foreign countries, many with past and present dictatorships. Can we trust the data from these countries if the tyrants in control want the people of the US to believe in global warming, because this will lead the US government to control carbon dioxide emission, thereby weakening the US economy, thereby weakening US military power? Don't believe a claim unless you yourself have checked it out, or you are sure of the motivations of the people making the observations. Crichton referenced an online paper by a physicist named Matthews, who presented enough of Bayesian probability theory to convince me that it was correct, and referred me to E. T. Jaynes's magnificent book [60].

[80] For those interested in reading criticisms of the idea of human-caused global warming by first class climatologists, I can recommend two books. The first is Roy Spencer's *Climate Confusion: How Global Warming Hysteria Leads to Bad Science, Pandering Politicians and Misguided Policies that Hurt the Poor* (New York, Encounter Books, 2008). As the title of his book suggests, Spencer also believes that the physicists who work on global warming have become corrupted. The second is S. Fred Singer's *Unstoppable Global Warming: Every 1,500 Years* (London and Lanham (MD), Rowman and Littlefield, 2008). These two books present experimental evidence for their conclusions, rather than appeal to a "consensus of experts." Though they could have appealed to a "consensus of experts." There is a petition that has been signed by more than 31,000 "experts" in the physical sciences expressing doubt about human-caused global warming. But I warn you: don't believe in any "expert," or group of "experts." Trust only your own observations. Which is better, picking a mechanic to fix your car on the basis of the number of degrees he has from elite universities, or on the basis of the fact that he has previously fixed your car, quickly and cheaply? It is better to use common sense in physics, too. The best short discussion of why global warming is probably not due to human activity is by a journalist, Lord Monckton of Brenchley. See his open letter to Senator John McCain on this subject, available on the internet at http://www.americanthinker.com/2008/10.

[81] Lee Smolin, *The Trouble with Physics : the Rise of String Theory, the Fall of a Science, and What Comes Next* (Boston, Houghton Mifflin, 2006)

[82] Peter Woit *Not Even Wrong : the Failure of String Theory and the Search for Unity in Physical Law* (New York, Basic Books, 2006).

[83] James Holt "Unstrung" *The New Yorker* (October 2, 2006). available online.

[84] Richard P. Feynman *Surely You're Joking Mr. Feynman!* (Bantam Books, New York, 1985), p. 233 (in the chapter entitled "The 7 Percent Solution," in other editions).

[85] John Earman and Michel Janssen 1993 "Einstein's Explanation of the Motion of Mercury's Perihelion," in *The Attraction of Gravitation: New Sutdies in General Relativity* (Birkhäuser, Boston, 1993).

[86] Richard P. Feynman "The Reason for Antiparticles," in Richard P. Feynman and Steven Weinberg, *Elementary Particles and the Laws of Physics: the 1986 Dirac Memorial Lectures* (Cambridge (U.K.), Cambridge University Press, 1987).

[87] Arthur S. Eddington, " 'Space' or 'Æther'?" *Nature* **107** (1921), 201–201. (A 21st century physicist like myself is astonished by the speed of publication of Eddington's letter, the week immediately following the publication of the paper upon which he is commenting. A 21st century "rapid publication journal" — for example, *Nature* itself, would take at least 6 months to publish such a reply. That's "progress" typical of science over the past few decades.)

[88] Albert Einstein *Sidelights on Relativity* (Dover Publications, New York, 1983) reprinting"Ether and Relativity Theory (original in German, with the title "Äther und Relativitäts-Theorie: Rede Gehalten am 5.Mai 1920 an der Reichs-Universität zu Leiden") *Sidelights on Relativity* was originally published by E.P. Dutton, New York, 1923. This book is available on Google Books for limited viewing.

[89] Edmund T. Whittaker, *A History of the Theories of Æther and Electricity: The Classical Theories* (Thomas Nelson and Sons, London, 1951).

[90] Edmund T. Whittaker, *A History of the Theories of Æther and Electricity: The Modern Theories* (Philosophical Library, New York, 1954).

[91] Edmund T. Whittaker, *A History of the Theories of Æther and Electricity* (Longmand, Green, and Company, London, 1910). This book is the first edition of [89] and [90].

[92] Harold Jefferys "The Physical Status of 'Space' " *Nature* **107** (1921), 394–394.

[93] Albert Einstein "Zur Elektrodynamik bewegter Körper," *Annalen der Physik* P**17**, (1905) 891– 921.

[94] Barack H. Obama "Remarks of Senator Barack Obama on the Confirmation of Judge John Roberts, Thursday, September 22, 2005," Senate Press release, available online at http://obama.senate.gov/press/050922-remarks_of_sena/.

[95] In physics, this two-way determinism — cosmic teleology — is given the technical name "unitarity." A mathematical proof that unitarity means two way determinism can be found on pages 145–146 of the well-known textbook by James D. Bjorken and Sidney D. Drell *Relativistic Quantum Fields* (New York, McGraw-Hill, 1965). Unitarity is a central postulate of quantum mechanics (it immediately follows from Schrödinger's equation, but it also applies in the fully relativistic case). Unitarity has all sorts of important consequences, for instance the CPT theorem. ALL of its consequences have been verified experimentally. One important thing to note about the proof of Bjorken and Drell. It actually establishes far more than the authors realized. No fault of theirs; they were writ-

ing 40 years ago, when many physicists still believed that quantum mechanics was about probabilities, and not about a deterministic multiverse. Bjorken and Drell thought they had merely proved that probabilities satisfied two-way determinism. They actually proved that the multiverse itself was two-way deterministic. Check their proof out for yourself, keeping in mind my proof, given in the body of this paper, of how "probabilities" in the sense of relative frequencies, arise in real world experiments.

[96] *McLean v. Arkansas Board of Education*, 529 F. Supp. 1255, 1258-1264 (ED Ark. 1982).In section III of his Memorandum Opinion. Judge Overton quoted Section 4 of the Act being overturned: "... Creation-science includes the scientific evidences and related inferences that indicate: (1) Sudden creation of the universe, energy, and life from nothing ... Judge Overton ruled that this definition made "creation science"; inherently religious: "The argument that creation from nothing in 4(a)(1) does not involve a supernatural deity has no evidentiary or rational support. To the contrary, "creation out of nothing" is a concept unique to Western religions. In traditional Western religious thought, the conception of a creator of the world is a conception of God. Indeed, creation of the world "out of nothing" is the ultimate religious statement because God is the only actor. ...." Cosmologists routinely use the expression "creation of the universe" to refer to the beginning of time in Big Bang Cosmology. No reference to a "creator" is implied when cosmologists use this expression. The Overton decision was not appealed, so it had an effect only in Arkansas, but it had a significant influence on the most recent Supreme Court decision on the scientific status of cosmology, *Edward v. Aguillard* (482 U.S. 578 (1987)). If the original Arkansas statue had been upheld in *Epperson v. Arkansas*, on the grounds that a State can forbid the teaching, in public schools, of any subject whatsoever, the Court would have avoided deciding, in effect, what the laws of physics are. The political branches surely should have the power to decide how scarce resources for public education are to be allocated. The legislature should, for example, have the power to decide that Spanish and German only are to be taught in public schools, and not Latin and Greek. Somebody has to decide which foreign languages are to be taught, since there are dozens of major languages (defined as those languages which have more than 100 million native speakers), and public schools cannot offer all of them. Why should Spanish or German be taught, and not Chinese or Hindi? I myself think that a physics course should be a high school requirement — necessitating more hires of physics teachers — but in most states, the legislatures disagree, insisting that physics be an elective that most students elect not to take. Why should not the legislatures have the power to disagree with me, and the Supreme Court, on allocation of scare resources? Instead, the Court decided they themselves can, and must, decree where the truth in science lies. I happen to agree with the Court that creation science is not science, but this is a scientific judgment I am making *qua* scientist. Deciding what "science" means is necessarily a scientific judgment, calling for extensive experience in scientific research. The Supreme Court is demonstrating supreme arrogance by asserting that they have the right to make scientific judgments, instead of leaving these judgments to professional scientists such as myself. Justice Scalia made this very point in his dissent (joined by Chief Justice Rehnquist) in *Edward v. Aguillard.* I have given reasons in the body of this paper why I think we are headed for a *Newton and Einstein v. Darwin* fight in science, and I do not think J.D.'s without even a high school physics course are competent to decide such issues. I will go further: any Federal judge who thinks he is the intellectual equal of Albert Einstein is completely wrong.