The Universal Electrodynamic Force

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A new classical electrodynamic force law for real finite-size elastic charged particles is derived in a proper axiomatic fashion by solving simultaneously the fundamental empirical laws of classical electrodynamics, i.e. Gauss's laws, Ampere's generalized law, Faraday's law, and Lenz's law assuming Galilean invariance. This derived version of the electromagnetic force law incorporates the effects of the self fields of real finite-size elastic particles as observed in particle scattering experiments. It satisfies Newton's Third Law, conservation of energy and momentum, conservation of charge, and Mach's Principle. Hooper's experiments showing that the fields of a moving charge move with the charge require that the electrodynamic force be a contact force based on field extensions of the charge instead of action-at-a-distance. These field extensions of the charges in the universe physically define the fabric of space or the ether of Einstein's General Relativity Theory and Newton's Universal Gravitation Force. A proof is given that the Lorentz force is derived from Galilean invariance instead of Lorentz invariance. The derived force law appears to be a universal force law, because it can account for all electrodynamics including radiation and radiation reaction plus gravity, inertia, quantum, and relativistic effects. The derived force law has a combination of spherical and chiral symmetry which is experimentally observed in the structure of matter on all size scales including elementary particles, nuclei, atoms, molecules, crystals, solar system, galaxies, and the structure of the observable universe as a whole. This universal electrodynamic force law appears to be superior (theoretically and experimentally) to previous force laws, i.e. relativistic quantum electrodynamic, gravitational, inertial, strong interaction and weak interaction.

1. Introduction

Henri Poincare was a famous philosopher and mathematician. He and Einstein were the authors of Special Relativity Theory. He also formulated a logical criterion[1] for fundamental theories in science. According to this criterion no two fundamental scientific theories may use the same fundamental constant. Poincare noticed that four "so-called" fundamental theories used the same fundamental constant c for the velocity of light, i.e. electrodynamics, special relativity, quantum mechanics, and general relativity. According to his logical criterion only one of these four theories could possibly be fundamental. Poincare proposed that the only fundamental theory in the group was electrodynamics, and it should explain what all the others were invented to explain.

Stephen W. Hawking in his book **A Brief History of Time: From the Big Bang to Black Holes** notes that **"scientists today describe the universe in terms of two basic partial theories—the general theory of relativity and quantum mechanics. The general theory of relativity describes the force of gravity and the large-scale structure of the universe... Quantum mechanics, on the other hand, deals with extremely small scale phenomena such as elementary particles, atoms, nuclei. Unfortunately, these two theories are known to be inconsistent with each other—they cannot both be correct."**[2]

The purpose of this paper is to derive a better electrodynamic force law that appears to be the universal electrodynamic force law capable of replacing both quantum mechanics and relativity theory plus many other theories in modern science.

2. A Proper Axiomatic Foundation

Euclid laid the foundation for the development of natural philosophy when he developed geometry on an axiomatic basis. Tradition tells us that Plato had the inscription engraved above the door of his Academy of natural philosophy in Athens, ***"Let No One Ignorant of Geometry Enter Here"***. This was in recognition of Euclid's development of the axiomatic method of proofs in geometry, one of the first properly developed areas of natural philosophy. History has agreed with Plato. Euclid's book **Elements (of Geometry)** has been published in more editions and translated into more languages than any other science or math book in the history of the world.

According to Euclid's axiomatic process for the development of theories in natural philosophy, one starts with the most minimal set of true postulates based on observation and then develops through the rigorous application of logic various theorems or theories. In developing a universal force law, it is instructive to examine the postulates of some of the principal theories that are being replaced by this proper theory of electrodynamics in order to see the faulty postulates that contributed to their failures. (The faulty postulates are colored red).

Postulates of Newton's Mechanics and Gravity

1. Newton's 1st Law Conservation of Momentum
2. Newton's 2nd Law Inertial Force F=ma is fundamental
3. Newton's 3rd Law $F_{Reaction}=-F_{Action}$
4. Conservation of energy
5. Galilean invariance
6. Absolute time

7. Force of Gravity $F=GM_1M_2/R_{12}^2$ is fundamental
8. Instantaneous action-at-a-distance gravitational force
9. Mass is fundamental property of matter
10. Point-particle idealization for finite-size bodies
11. Linear Superposition Principle for forces
12. Force of gravity independent of motion

Postulates of Maxwell's Electrodynamics

1. Coulomb's Law for force between static charges
2. Ampere's Law for force between current elements
3. Faraday's Law of electromagnetic induction
4. Lorentz Force Law
5. Constant forces - no radiation or energy loss
6. Newton's 3rd Law
7. Conservation of charge
8. Conservation of energy
9. Conservation of momentum of fields and particles
10. Point-like charge particle idealization
11. No deformation or elasticity of charges
12. Action-at-a-distance forces
13. Galilean invariance
14. Linear Superposition Principle for forces and fields
15. Mach's Principle need not be satisfied
16. Absolute time

Postulates of Einstein's Special Relativity Theory

1. Maxwell electrodynamics
2. Lorentz Force Law
3. Action-at-a-distance forces
4. Point-like particle idealization
5. No deformation or elasticity of charges or particles
6. Space is homogeneous and isotropic
7. Nothing can move faster than speed of light c
8. Invariance of the velocity of light c
9. Classical addition of velocities law invalid for light
10. Quantum effects do not exist
11. Non-Galilean relativity
12. Linear Superposition Principle
13. Mach's Principle need not be satisfied
14. Euclidean Geometry
15. Inertial frames of reference are unique
16. Equivalence of physical laws in inertial frames
17. Rest mass is a form of energy $E=mc^2$
18. Preferred state of motion defined by inertial frames
19. State of a system determined by particles in inertial frame
20. Inertial frames move at constant velocity a=da/dt=0

Postulates of Einstein's General Relativity Theory

1. Point-like particle idealization
2. No deformation or elasticity of particle masses
3. Quantum effects do not exist
4. Non-Galilean relativity
5. Linear Superposition Principle
6. Matter only is responsible for space-time curvature
7. Non-Euclidean Geometry

Postulates of Dirac's Quantum Mechanics

1. Point-like particle idealization
2. No deformation or elasticity of particle masses
3. Linear Superposition Principle
4. $\Psi^*\Psi$ Statistical interpretation of nature
5. No Law of Cause and Effect
6. State of a system determined by $\Psi(x,t)$ waves
7. Uncertainty Principle

Postulates for Universal Electrodynamic Force

1. Coulomb's Law for force between static charges
2. Ampere's Law for force between current elements
3. Faraday's Law of electromagnetic induction
4. Lenz's Law for non-conservative non-linear forces
5. Newton's 3rd Law
6. Conservation of charge
7. Conservation of energy
8. Conservation of momentum of fields and particles
9. Finite-size and elasticity of charged particles
10. Fields of charges remain attached when charges move
11. Electromagnetic fields have tensile strength
12. Law of Cause and Effect
13. Mach's Principle
14. Galilean Invariance
15. Absolute time
16. Euclidean Geometry
17. Electrical charge systems are non-linear

According to Einstein, **"A theory is the more impressive the greater the simplicity of its premises is, the more different kinds of things it relates, and the more extended is its area of applicability".**[3] In this paper a proper axiomatic derivation/proof of an improved classical electrodynamic force law will be obtained which appears to be a good candidate for the universal force law, because from it better force laws for gravity and inertia can be derived and better theories of elementary particles, nuclei, atoms and molecules can be built.

3. Derivation of Improved Classical Electrodynamic Force Law for Finite-Size Elastic Charged Particles Moving at Constant Velocity

In the derivation that follows it is assumed that the electrodynamic force law at constant velocity is based on five empirical laws of electrodynamics, i.e.

Grassmann form of Ampere's Law-constant velocity current loops

$$\vec{B}_i(\vec{r},\vec{v},t) = q\frac{\frac{\vec{v}}{c}\times\vec{r}'}{|\vec{r}'|^3} = \frac{\vec{v}}{c}\times\vec{E}(\vec{r}',t') \quad (1)$$

Faraday's Law of induction - constant velocity currents

$$\int \vec{E}(\vec{r}',t')d\vec{l}' = -\frac{1}{c}dt\int \vec{B}(\vec{r},\vec{v},t)\cdot\hat{n}da \quad (2)$$

Gauss Electrostatic Law - zero velocity

$$\int \vec{E}(\vec{r}',\vec{v}',t')\cdot\hat{n}da' = 4\delta q \quad (3)$$

Gauss Magnetostatic Law - zero velocity

$$\nabla\cdot\vec{B}(\vec{r},\vec{v},t) = 0 \quad (4)$$

Lenz's Law for non-conservative forces - constant velocity

$$\vec{E}(\vec{r},\vec{v},t) \propto -\vec{E}_0(\vec{r},t) \text{ where } \vec{E}(\vec{r},t) = \vec{E}_0(\vec{r},t) + \vec{E}_i(\vec{r},\vec{v},t) \quad (5)$$

Note that the relativistic type notation is used, and that both Ampere's Law and Faraday's Law involve the observer's reference frame and a moving frame of reference that are related by the Galilean transformation. See Figure 1.

Galilean Transformation

$$\vec{r}\,' = \vec{r} - \vec{v}t \text{ and } t' = t \text{ and } c' \neq c \quad (6)$$

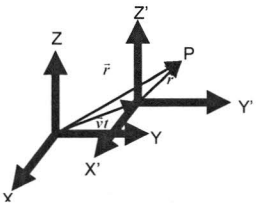

Figure 1 Coordinates of Charged Particle at Point P

Previous papers[4-18] deriving the improved electrodynamic force law have included a derivation of the Biot-Savart Law and the Grassmann law from the generalized Ampere Law. The derivation shows that both the Biot-Savart Law and the Grassmann Law satisfy Newton's third Law, when one notes that the additional term in the generalized Ampere Law, which is missing in the Grassmann form of Ampere's Law, gives no contribution for closed current loops. **Grassmann's form of Ampere's Law is the best form, when one notes that Ampere's Law only applies to closed current loops!!**

Note that equation (1) for a point element of charge gives the transformation of the static electric field $\mathbf{E_0(r',t')}$ in the moving frame of reference to the induced magnetic field $\mathbf{B_i(r,t)}$ in the observer's frame of reference. This is assumed valid for all velocities, whether constant in time or changing, and will be used to obtain the fields of a charged particle with internal charge distribution. Note that if Ampere's law, as represented by equation (1), is cast into its usual Maxwell equation form

$$\nabla \times \vec{B}(\vec{r},t) = \frac{4\delta}{c}\vec{J}(\vec{r},t) \quad (7)$$

the reference frame transformation information of equation (1) is lost. Jackson [19 p. 138 or 20 p. 174 or 21 p.179] points out that

$$\nabla \times \vec{B}(\vec{r},t) = \frac{4\delta}{c}\vec{J}(\vec{r},t) + \frac{1}{c}\nabla \int \frac{\nabla' \cdot \vec{J}(\vec{r}',t')}{|\vec{r}-\vec{r}'|} d^3r' \quad (8)$$

The second term on the right is not zero for finite size particles and induced self-field effects. For these reasons it appears that the covariant form of electrodynamics, based on Maxwell's equations, is technically incorrect being based on significant approximations. It is certainly not as fundamental as the empirical laws upon which Maxwell's equations are based.

Note that classical electrodynamics was originally based on Galilean relativity of equation (6) which assumed instant action-at-a-distance for longitudinal forces due to the Coulomb gauge potential[19 pp. 181-183, 20 pp. 220-223, 21 pp. 240-242] and absolute time such that the retarded time $\tau = t' - t = 0$. **Thus it was logically inconsistent to introduce retardation effects in classical electrodynamics!!**

Also note from Ampere's Law that $\mathbf{B_i(r,t)}$ will be time varying. According to Faraday's law [19 pp. 170-173 or 20 pp. 210-213 or 21 pp. 208-211] $\mathbf{dB_i\ (r,t)/dt}$ introduces an additional electric field $\mathbf{E_i(r,t)}$. The distribution of charge within the finite-size elastic elementary charged particle rearranges itself to be consistent with the induced fields. This rearrangement causes a change in the internal binding energy of the particle and the kinetic energy stored in the particle's external electromagnetic fields. Using the fundamental empirical equations of electrodynamics one can calculate the induced fields in order to obtain the total fields of the moving charged particle, i.e.

$$\vec{E}(\vec{r},\vec{v},t) = \vec{E}_0(\vec{r},t) + \vec{E}_i(\vec{r},\vec{v},t) \quad (9)$$
$$\vec{B}(\vec{r},\vec{v},t) = \vec{B}_0(\vec{r},t) + \vec{B}_i(\vec{r},\vec{v},t)$$

where $\mathbf{E_0(r,t)}$ is the electrostatic field and $\mathbf{E_i(r,v,t)}$ is the induced field due to motion **v**.

In 1957 Cullwick[22] published results of his research indicating that the magnetic flux loops discovered by Oersted[23] were actually in motion along the linear conductor in the direction of the electron current giving rise to it. The loops moved with the electron drift velocity. This result was confirmed experimentally by Hooper.[24] This means that when motion is involved $\mathbf{E(r,t)}$ must be written in terms of the coordinates of its rest frame, i.e. $\mathbf{E(r,t)}$ becomes $\mathbf{E(r',t') = E(r-vt,t)}$. Faraday believed that the fields were an extension of the charge to infinity, that the field lines had tensile strength to attract and repel as shown in Figures 2a, 2b and 2c, and that light was merely a ripple of angular momentum in the fields of the charge.[25] This experimental result explains why the laws of physics are only the same in inertial frames of reference in Special Relativity Theory.

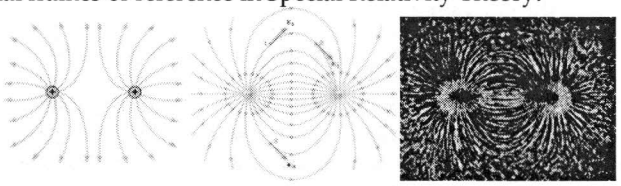

Figure 2a Repulsion of Positive charges

Figure 2b Attraction of Opposite Charges

Figure 2c Iron Filings Showing Field Lines

Experiments[24] show that induced fields are not electrostatic in nature, c.g. electrostatic fields can not be shielded but induced fields can be shielded. According to Culwick[22], Hooper[24], and Moon and Spencer[26], this means that the superposition principle as applied to electrical fields does not hold for the **B** x **v**/c generated electric fields. Thus in electrodynamics one must explicitly keep track of both electrostatic fields and the induced fields. Using the basic equations of electrodynamics one must calculate explicitly the induced fields in order to obtain the total fields of the moving charged particle. (Note that the covariant form of electrodynamics based upon Maxwell's equations assumes that the superposition principle holds and does not treat electrostatic and induced fields separately in disagreement with the experimental results cited above.)

From the discussion above and equations (1) and (9), the magnetic field for a charged particle with finite size and internal charge structure with total charge q must be written in terms of the position of the charge in its current inertial rest frame (See Figure 1 and note origin of B in primed coordinate system)

$$\boldsymbol{B(\vec{r}',t') = \vec{B}_i(\vec{r}-\vec{v}t,t) = \frac{\vec{v}}{c} \times \left(\vec{E}_0(\vec{r}',t') + \vec{E}_i(\vec{r}',t')\right)} \quad \mathbf{(10)}$$

Note that $\mathbf{Bi(r',t')}$ is not a function of $\mathbf{(r,t)}$ but of $\mathbf{(r',t')=(r-vt,t)}$ due to the fact that the induced field remains attached to the charge and reflects all aspects of the motion of the charge. Assuming that a particle with charge q is moving with relative ve-

locity v in the z-direction and that from symmetry the induced electric field Ei is in the radial direction, one obtains

$$\vec{B}_i(\vec{r}-\vec{v}t,t)=\frac{\vec{v}\times\vec{r}'}{c}\frac{q}{|\vec{r}'|^3}+\frac{\vec{v}}{c}\times\vec{E}_i(\vec{r}',t') \quad (11)$$

Using the Galilean transformation of Equation (6) for constant relative velocity one obtains

$$\vec{B}_i(\vec{r}-\vec{v}t,t)=\frac{vr\sin\grave{e}}{c}\frac{q\hat{o}'}{|\vec{r}-\vec{v}t|^3}+\frac{vsin\grave{e}}{c}E_i(\vec{r}-\vec{v}t,t)\hat{o}' \quad (12)$$

where spherical coordinates are used with

$$\frac{\partial\vec{B}_i(\vec{r}-\vec{v}t,t)}{c\partial t}=\frac{vr\sin\grave{e}\hat{o}'}{c}\left[\frac{3qv(rcos\grave{e}-vt)}{c|r-vt|^5}+\frac{\partial}{rc\partial t}\vec{E}_i(\vec{r}-\vec{v}t(13)\right.$$

where

$$|\vec{r}'|=|\vec{r}-\vec{v}t|=\sqrt{r^2sin^2\grave{e}+(rcos\grave{e}-vt)^2}$$

$$\vec{v}\times\vec{r}'=\vec{v}\times(\vec{r}-\vec{v}t)=\vec{v}\times\vec{r}=vr\sin\grave{e}\hat{o}' \quad (14)$$

Now the induced time varying magnetic field Bi(r-vt,t) causes an electric field Ei(r-vt,t) to be induced. According to Faraday's law[19 p. 173 or 20 p. 213 or 21 p. 211] the changing magnetic flux linked by a circuit is proportional to the induced electric field around the circuit, i.e.

$$\int_C\vec{E}'_i(\vec{r}',t')\cdot d\vec{l}'=-\frac{1}{c}\frac{d}{dt}\int_S\vec{B}_i(\vec{r}-\vec{v}t,t)\cdot\hat{n}\,da' \quad (15)$$

where the circuits and fields are defined in Figure 3. Note that Faraday's law gives the relationship between E'i(r',t') in the moving frame to Bi(r-vt,t) in the observer's frame. This relationship is lost if one casts Faraday's law into its usual covariant Maxwell equation form as shown below.

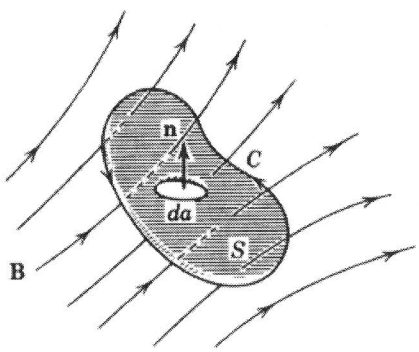

Figure 3 Definition of Fields for Faraday's law

Faraday's law can be used as a second equation to go with equation (10) to solve for the induced Bi and Ei fields. **Eventually a complete set of all the fundamental equations of electrodynamics will be required to obtain the complete solution.** Faraday's law can be rewritten using Stokes theorem as

$$\int_C\left(\vec{E}'_i(\vec{r}',t')-\frac{1}{c}(\vec{v}\times\vec{B}_i(\vec{r}-\vec{v}t,t))\right)\cdot d\vec{l}'=-\frac{1}{c}\int_S\frac{\partial\vec{B}_i(\vec{r}-\vec{v}t,t)}{\partial t}\cdot\hat{n}da' \quad (16)$$

where use of the convective derivative

$$\frac{d}{dt}=\frac{\partial}{\partial t}+\vec{v}\cdot\nabla' \quad (17)$$

gives

$$\frac{d\vec{B}_i(\vec{r}-\vec{v}t,t)}{dt}=\frac{\partial\vec{B}_i(\vec{r}-\vec{v}t,t)}{\partial t}+(\vec{v}\cdot\nabla')\vec{B}_i(\vec{r}-\vec{v}t,t)$$

$$=\frac{\partial\vec{B}_i(\vec{r}-\vec{v}t,t)}{\partial t}+\nabla'\times\left[\vec{B}_i(\vec{r}-\vec{v}t,t)\times\vec{v}\right]+\vec{v}\left[\nabla'\cdot\vec{B}_i(\vec{r}-\vec{v}t,t)\right]$$

$$=\frac{\partial\vec{B}_i(\vec{r}-\vec{v}t,t)}{\partial t}+\nabla'\times\left[\vec{B}_i(\vec{r}-\vec{v}t,t)\times\vec{v}\right]\ since\ \nabla'\cdot\vec{B}=0 \quad (18)$$

This is a statement of Faraday's law applied to a moving circuit C. We could also apply Faraday's law to a circuit C instantaneously at rest with v=0. In this case one obtains

$$\oint_C\vec{E}_i(\vec{r}-\vec{v}t,t)\cdot d\vec{l}'=-\frac{1}{c}\oint_S\frac{\partial\vec{B}_i(\vec{r}-\vec{v}t,t)}{\partial t}\cdot\hat{n}'da' \quad (19)$$

Galilean invariance requires that in this case E'=E and F'=F. Thus the left-hand sides of equations (16) and (19) are equal giving

$$\vec{E}'(\vec{r}',t')=\vec{E}_i(\vec{r}-\vec{v}t,t)+\frac{1}{c}\left[\vec{v}\times\vec{B}_i(\vec{r}-\vec{v}t,t)\right] \quad (20)$$

or

$$\vec{F}'(\vec{r}',t')=\vec{F}_0'(\vec{r}',t')+\vec{F}_i'(\vec{r}',t')=q\vec{E}_0'(\vec{r}',t')+q\vec{E}_i'(\vec{r}',t')$$

$$=q\vec{E}_0(\vec{r}-\vec{v}t,t)+q\vec{E}_i(\vec{r}-\vec{v}t,t)+\frac{q}{c}\left[\vec{v}\times\vec{B}_i(\vec{r}-\vec{v}t,t)\right] \quad (21)$$

Thus we obtain the Lorentz Force

$$\vec{F}(\vec{r},t)=q\vec{E}(\vec{r}-\vec{v}t,t)+\frac{q}{c}\left[\vec{v}\times\vec{B}_i(\vec{r}-\vec{v}t,t)\right] \quad (22)$$

where static fields are identical in all frames of reference. **Note that the Lorentz force law has been derived from Galilean invariance plus the experimental fact that fields are a physical extension of the charge making the electromagnetic force a contact type force.**

Faraday's law of equation (2) can be put into differential form by use of Stoke's Theorem and equation (17). The transformation of the line integral for the electric field into a surface integral leads to

$$\oint_S\left[\nabla'\times\vec{E}_i(\vec{r}-\vec{v}t,t)+\frac{1}{c}\frac{\partial}{\partial t}\vec{B}_i(\vec{r}-\vec{v}t,t)\right]\cdot\hat{n}'da' \quad (23)$$

Since the circuit C and the bounding surface S are arbitrary, the integrand must vanish at all points in space. Thus the differential form of Faraday's law for the special case v=0 is

$$\nabla'\times\vec{E}_i(\vec{r}-\vec{v}t,t)=-\frac{1}{c}\frac{\partial}{\partial t}\vec{B}_i(\vec{r}-\vec{v}t,t) \quad (24)$$

Note that for self-consistency the charge distribution within the finite-size elementary particle has been altered to produce the field E(r-vt,t)=Eo(r-vt,t)+Ei(r-vt,t), because the charge distribution is assumed to be in equilibrium with all forces or fields. Using equation (13) one may write Faraday's law in spherical coordinates as

$$\nabla'\times\vec{E}_i(\vec{r}-\vec{v}t,t)=\frac{\hat{r}'}{rsin\grave{e}}\left[\frac{\partial}{\partial\grave{e}}\left(sin\grave{e}\vec{E}_i(\vec{r}',t)\cdot\hat{o}'\right)-\frac{\partial}{\partial\ddot{o}}\left(\vec{E}_i(\vec{r}',t)\cdot\hat{e}'\right)\right]+\hat{e}'\left[\frac{1}{rsin\grave{e}}\frac{\partial}{\partial\ddot{o}}\left(\vec{E}_i(\vec{r}',t)\cdot\hat{r}'\right)-\frac{1}{r}\frac{\partial}{\partial r}\left(r\vec{E}_i(\vec{r}',t)\cdot\hat{e}'\right)\right]+\frac{\hat{o}'}{r}\left[\frac{\partial}{\partial r}\left(r\vec{E}_i(\vec{r}',t)\cdot\hat{e}'\right)-\frac{\partial}{\partial\grave{e}}\left(\vec{E}_i(\vec{r}',t)\cdot\hat{r}'\right)\right] \quad (25)$$

$$=-\frac{1}{c}\frac{\partial}{\partial t}\vec{B}_i(\vec{r}',t)=-\frac{v}{c}rsin\grave{e}\left[\frac{3qv(rcos\grave{e}-vt)}{c|r-vt|^5}+\frac{1}{rc}\frac{\partial}{\partial t}E_i(\vec{r}-\vec{v}t,t)\right]\hat{o}'$$

The symmetry of

$$\frac{\partial}{\partial t}\vec{B}_i(\vec{r}-\vec{v}t,t)\propto\hat{o}'\ and\ \vec{E}_i(\vec{r}-\vec{v}t,t)\propto\hat{r}'$$

allows equation (25) to be reduced to

$$-\frac{1}{r}\frac{\partial}{\partial\grave{e}'}E_i(r-vt,t)\hat{o}'=-\frac{v}{c}rsin\grave{e}\left[\frac{3vq(rcos\grave{e}-vt)}{c|r-vt|^5}+\frac{1}{rc}\frac{\partial}{\partial t}E_i(\vec{r}-\vec{v}t,t)\right]\hat{o}'$$

and

$$\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta'}E_i(\vec{r}-\vec{v}t,t)\hat{\theta}'=0 \quad (26)$$

Or rearranging equation (26) and integrating over θ obtain

$$\int_0^{\theta'} dE_i(\vec{r}-\vec{v}t,t)=E_i(\vec{r}-\vec{v}t,t)-E_i(\vec{r}-\vec{v}t,t)|_{\theta'=0}$$
$$=\frac{3v^2r^2q}{c^2}\int_0^{\theta'}\frac{(r\cos\theta'-vt)}{|\vec{r}-\vec{v}t|^5}\sin\theta'd\theta'$$
$$+\frac{vr^2}{c}\int_0^{\theta'}\frac{1}{rc}\frac{\partial}{\partial t}E_i(\vec{r}-\vec{v}t,t)\sin\theta'd\theta' \quad (27)$$

where the constant Ei(r-vt,t) diminishes the original electrostatic field. From Lenz's law and symmetry of local forces, Bi(r-vt,t) | θ=0 should oppose the induced field Ei(r-vt,t)r′, which is proportional to the moving static field Eo(r-vt,t)r′, i.e.

$$E_i(r-vt,t)|_{\theta'=0}\hat{r}'=-\theta E_0(\vec{r}-\vec{v}t,t)\hat{r}' \quad (28)$$

The use of Lenz's law allows one to satisfy Mach's principle.[27] According to Mach any correct physical theory must take into account all the charges and masses in the universe in a consistent way, because the electromagnetic and gravitational forces have infinite range. Since all massive particles seem to have charge structure, the use of Lenz's law appears to satisfy Mach's principle by taking into account the electromagnetic interaction of all other particles in the universe. The covariant form of electrodynamics based on Maxwell's equations fails to satisfy Mach's principle, because it is not based on Lenz's Law. Einstein's Special Relativity Theory (SRT) incorporating the Lorentz transformation does not satisfy Mach's principle, because it is based on Maxwell electrodynamics.

Equation (27) can be solved by iterative substitution for Ei(r-vt,t) where each successive iteration takes into account the next order of the induced field effect. In order to obtain the results observed in the laboratory frame, it is necessary to evaluate Bi(r-vt,t) at t = 0 when the moving frame coincides with the laboratory frame after all terms for a given iteration have been evaluated. In order to simplify the iteration of successive terms the r′ = r - vt terms are left in place in order to keep track of the correct power of r′ for the derivative in the iterative term. The vt(cosθ-1) terms are explicitly dropped for t=0. Substituting equation (28) into equation (27) and using β = v/c obtain

$$E_i(\vec{r}-\vec{v}t,t)|_{t=0}+\theta E_0(\vec{r}-\vec{v}t,t)|_{t=0}$$
$$=3r^2\beta^2\left[\frac{q}{|\vec{r}-\vec{v}t|^5}\left(\frac{r}{2}\sin^2\theta'+vt(\cos\theta'-1)\right)\right]_{t=0}$$
$$+\beta r^2\int_0^{\theta'}\frac{1}{rc}\frac{\partial}{\partial t}E_i(\vec{r}-\vec{v}t,t)\sin\theta'd\theta'|_{t=0} \quad (29)$$

Iterating equation (29) obtain

$$E_i(\vec{r}-\vec{v}t,t)|_{t=0}=\frac{3}{2}\beta^2r^3\frac{q}{|\vec{r}-\vec{v}t|^5}\sin^2\theta|_{t=0}-\theta r\frac{q}{|\vec{r}-\vec{v}t|^3}|_{t=0}$$
$$+\beta r^2\int_0^{\theta}\frac{1}{rc}\frac{\partial}{\partial t}\left[\frac{3}{2}\beta^2r^3\frac{q}{|\vec{r}-\vec{v}t|^5}\sin^2\theta'\right.$$
$$-\frac{\theta rq}{|\vec{r}-\vec{v}t|^3}$$
$$+\beta r^2\int_0^{\theta'}\frac{1}{rc}\frac{\partial}{\partial t}E_i(\vec{r}$$
$$\left.-\vec{v}t,t)\sin\theta'd\theta'\right]\sin\theta d\theta|_{t=0}$$

$$E_i(\vec{r}-\vec{v}t,t)|_{t=0}=\frac{3}{2}\beta^2r^3\frac{q}{|\vec{r}-\vec{v}t|^5}\sin^2\theta|_{t=0}-\theta r\frac{q}{|\vec{r}-\vec{v}t|^3}|_{t=0}$$
$$+\beta r^2\int_0^{\theta}\left[\frac{3}{2}\beta^2r^3\frac{q}{|\vec{r}-\vec{v}t|^7}5\beta r\cos\theta'\sin^2\theta'\right.$$
$$\left.-\frac{\theta rq}{|\vec{r}-\vec{v}t|^3}3\beta r\cos\theta'\right]\sin\theta'd\theta'|_{t=0}$$
$$+\beta^2r^4\int_0^{\theta'}\left[\frac{1}{rc}\frac{\partial}{\partial t}\int_0^{\theta'}\frac{1}{rc}\frac{\partial}{\partial t}E_i(\vec{r}-\vec{v}t,t)\sin\theta d\theta\right]\sin\theta'd\theta'|_{t=0}$$

In order to obtain the terms from the first iteration, one can take the limit t=0 and θ′=θ to those terms to obtain

$$E_i(r)=\frac{q}{r^2}\frac{3\beta^2}{2}\sin^2\theta-\frac{\theta q}{r^2}+\frac{q}{r^2}\frac{15\beta^4}{2}\int_0^{\theta}\sin^3\theta d\sin\theta-\frac{\theta q}{r^2}3\beta^2\int_0^{\theta}\sin\theta d\sin\theta$$
$$=\frac{q}{r^2}\frac{3\beta^2}{2}\sin^2\theta-\frac{\theta q}{r^2}+\frac{q}{r^2}\frac{15\beta^4}{8}\sin^4\theta-\frac{\theta q}{r^2}\frac{3\beta^2}{2}\sin^2\theta$$
$$=\frac{q}{r^2}\left[\frac{3\beta^2}{2}\sin^2\theta+\frac{15\beta^4}{8}\sin^4\theta\right]-\frac{\theta q}{r^2}\left[1+\frac{3\beta^2}{2}\sin^2\theta\right]$$
$$=E_0(r)\left[\frac{3\beta^2}{2}\sin^2\theta+\frac{15\beta^4}{8}\sin^4\theta\right]$$
$$-\theta E_0(r)\left[1+\frac{3\beta^2}{2}\sin^2\theta\right] \quad (30)$$

Solving for E the total field equal to Eo + Ei one obtains the terms from the first iteration

$$E(r)=E_0(r)+E_i(r)$$
$$=E_0(r)\left[1+\frac{3\beta^2}{2}\sin^2\theta+\frac{15\beta^4}{8}\sin^4\theta\right]$$
$$-\theta E_0(r)\left[1+\frac{3\beta^2}{2}\sin^2\theta\right] \quad (31)$$

From the definition of the binomial expansion

$$(1-\beta^2\sin^2\theta)^{-3/2}=1+\frac{3\beta^2}{2}\sin^2\theta+\frac{15\beta^4}{8}\sin^4\theta+\cdots \quad (32)$$

one can see that the first iteration has given the first few terms of the series. Calculations of the second and third iterative terms confirms that the additional terms are part of this binomial series. Using the binomial expansion to sum the infinite series of iteration terms gives

$$E(r)=E_0(r)+E_i(r)=E_0(r)\frac{(1-\theta)}{(1-\beta^2\sin^2\theta)^{3/2}} \quad (33)$$

Equation (33) gives the resultant self-consistent electric field of an elastic finite-size moving charged particle with total charge q as seen by an observer in his frame of reference.

In order to evaluate the constant λ one uses Gauss's law for electric charge [19 p.4 or 20 p.30 or 21 p.27] where

$$\oint_S\vec{E}(r)\cdot\hat{n}da=4\pi q \quad (34)$$

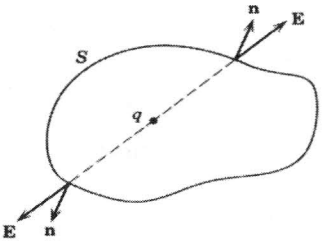

Figure 3 Geometry for Gauss's Law

Using a spherical surface centered about the charge distribution q with spherical coordinates and noting that E and n will then be in the same direction, equation (34) becomes

$$4\pi q=\int_0^{2\pi}\int_0^{\pi}E_0(r)\frac{(1-\theta)}{(1-\beta^2\sin^2\theta)^{3/2}}r^2\sin\theta d\theta d\phi$$
$$=4\pi q\frac{1-\theta}{1-\beta^2} \quad (35)$$

where

$$\int_0^{\pi}(1-\beta^2\sin^2\theta)^{-3/2}\sin\theta d\theta=\frac{2}{1-\beta^2}\ and\ E_0(r)=\frac{q}{r^2} \quad (36)$$

So λ must be equal to β^2 in order for the total flux of **E** to be conserved. Thus the expression for the self-consistent fields may be written as

$$\vec{E}(\vec{r},\vec{v}) = \frac{q(1-\hat{a}^2)\vec{r}}{r^3(1-\hat{a}^2 sin^2\grave{e})^{3/2}} = \frac{q(1-\vec{\hat{a}}^2)\vec{r}}{\left[\vec{r}^2 - \frac{\{\vec{r}\times(\vec{r}\times\vec{\hat{a}})\}}{\vec{r}^2}\right]^{3/2}} \quad (37a)$$

$$\vec{B}_i(\vec{r},\vec{v}) = \frac{\vec{v}}{c}\times\vec{E}(\vec{r},\vec{v}) = \frac{q(1-\vec{\hat{a}}^2)\vec{\hat{a}}\times\vec{r}}{\left[\vec{r}^2 - \frac{\{\vec{r}\times(\vec{r}\times\vec{\hat{a}})\}}{\vec{r}^2}\right]^{3/2}} \quad (37b)$$

$$\vec{F}(\vec{r},\vec{v}) = q'\left[\vec{E}(\vec{r},\vec{v}) + \frac{\vec{v}}{c}\times\vec{B}_i(\vec{r},\vec{v})\right] \quad (37c)$$

Note that the $\beta^2\sin^2\theta$ term in Equation (37a) is written as the triple vector product **r**x(**r**x**β**) instead of **r**x**β** for consistency with the triple vector product of the Lorentz Force Law of equation (37c). This requirement will become more obvious when the potential U is defined and the force is obtained from $dU/dt = -\mathbf{v}\cdot\mathbf{F}$. Equations (37a, b) are the derived version of the **E** and $\mathbf{B}_i$ fields in the lab frame to order v in the Galilean transformation. These equations are precisely the same as one would obtain from Special Relativity Theory using the Lorentz transformation for the electric and magnetic fields observed in the observer's frame of reference for a point charge q passing by with relative uniform velocity v. [20 p. 555 or 21 p. 560]

Although the mathematical expressions are the same for constant velocity v, this axiomatic derivation is superior, because it is based upon causality and all the fundamental empirical laws of electrodynamics, including Lenz's Law, uses real elastic particles with finite size and an internal charge structure, and contact forces satisfying Mach's principle. **Perhaps the most significant observation of all is that the scheme of Special Relativity Theory only imitates the self-field effects to time derivative order v. The terms involving acceleration, reaction, and other higher order time derivatives are missing!**

Now the total electromagnetic force **F** exerted by the moving charge distribution on a test charge q' is

$$\vec{F}(\vec{r},\vec{v}) = q'\left[\vec{E}(\vec{r},\vec{v}) + \frac{\vec{v}}{c}\times\vec{B}_i(\vec{r},\vec{v})\right]$$

$$= qq'(1-\vec{\hat{a}}^2)\frac{\vec{r}+\vec{\hat{a}}\times(\vec{\hat{a}}\times\vec{r})}{\left[\vec{r}^2 - \frac{\{\vec{r}\times(\vec{r}\times\vec{\hat{a}})\}^2}{\vec{r}^2}\right]^{3/2}} \quad (38)$$

Using the vector identities below the triple cross product **β** x (**β** x **r**) may be rewritten

$$\vec{A}\times(\vec{B}\times\vec{C}) = (\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C} \quad (39)$$

$$\vec{r}\times(\vec{r}\times\vec{\hat{a}}) = (\vec{r}\cdot\vec{\hat{a}})\vec{r} - (\vec{r}\cdot\vec{r})\vec{\hat{a}} \;\; or \;\; \hat{a} = \frac{(\vec{r}\cdot\vec{\hat{a}})\vec{r} - \vec{r}\times(\vec{r}\times\vec{\hat{a}})}{\vec{r}^2}$$

$$\vec{\hat{a}}\times(\vec{\hat{a}}\times\vec{r}) = (\vec{\hat{a}}\cdot\vec{r})\vec{\hat{a}} - (\vec{\hat{a}}\cdot\vec{\hat{a}})\vec{r} = (\vec{\hat{a}}\cdot\vec{r})\frac{(\vec{r}\cdot\vec{\hat{a}})\vec{r} - \vec{r}\times(\vec{r}\times\vec{\hat{a}})}{\vec{r}^2} - \vec{\hat{a}}^2\vec{r}$$

Thus

$$\vec{F}(\vec{r},\vec{v}) = q'\left[\vec{E}(\vec{r},\vec{v}) + \frac{\vec{v}}{c}\times\vec{B}_i(\vec{r},\vec{v})\right]$$

$$= qq'(1-\vec{\hat{a}}^2)\frac{\vec{r}+\vec{r}\times(\vec{\hat{a}}\times\vec{r})}{\left[\vec{r}^2 - \frac{\{\vec{r}\times(\vec{r}\times\vec{\hat{a}})\}^2}{\vec{r}^2}\right]^{3/2}}$$

$$= \frac{qq'}{\vec{r}^2}\frac{(1-\vec{\hat{a}}^2)\vec{r}}{\left[\vec{r}^2 - \frac{\{\vec{r}\times(\vec{r}\times\vec{\hat{a}})\}^2}{\vec{r}^2}\right]^{1/2}} - \frac{qq'}{\vec{r}^2}\frac{(1-\vec{\hat{a}}^2)(\vec{\hat{a}}\cdot\vec{r})\vec{r}\times(\vec{r}\times\vec{\hat{a}})}{\left[\vec{r}^2 - \frac{\{\vec{r}\times(\vec{r}\times\vec{\hat{a}})\}^2}{\vec{r}^2}\right]^{3/2}} \quad (40)$$

Equation (40) is a derived version of the electromagnetic force for an elastic finite-size charged particle moving at constant velocity. Note that in the limit $\beta\to 0$ the force approaches the static Coulomb force. Also the second term produces a spiraling or helix type motion that is commonly observed in electric plasma spheres, accelerators, and fluorescent lights.

The electromagnetic potential U corresponding to equation (40) that is accurate to order v in the derivatives of the relative separation distance is

$$U(\vec{r},\vec{v}) = \frac{qq'}{r}\frac{\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{v^2}{c^2}sin^2\grave{e}\right)^{1/2}} = \frac{qq'\left(1-\frac{\vec{v}^2}{c^2}\right)}{\left[\vec{r}^2 - \frac{\{\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\}^2}{\vec{r}^2}\right]^{1/2}} \quad (41)$$

From conservation of energy one has the relationship for the electromagnetic potential U

$$\frac{dU(\vec{r},\vec{v})}{dt} = -\vec{F}(\vec{r},\vec{v},\vec{a})\cdot\vec{v} \quad (42)$$

where **v** is the relative velocity between the moving charges. Assuming that the electromagnetic potential is a regular function and well-behaved, one can obtain some of the acceleration terms of the force by allowing the velocity to be a function of time.

$$\vec{v}\cdot\vec{F}(\vec{r},\vec{v},\vec{a}) = -\frac{dU(\vec{r},\vec{v})}{dt} = -\frac{d}{dt}\frac{qq'\left(1-\frac{\vec{v}^2}{c^2}\right)}{\left[\vec{r}^2 - \frac{\left\{\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\right\}^2}{\vec{r}^2}\right]^{\frac{1}{2}}}$$

$$= \frac{-qq'\left(\frac{-2\vec{v}\cdot\vec{a}}{c^2}\right)}{\left[\vec{r}^2 - \frac{\{\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\}^2}{\vec{r}^2}\right]^{1/2}} - qq'(1-\vec{\hat{a}}^2)\frac{\left(\frac{-1}{2}\right)\frac{d}{dt}\left[\vec{r}^2 - \frac{\{\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\}^2}{\vec{r}^2}\right]}{\left[\vec{r}^2 - \frac{\{\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\}^2}{\vec{r}^2}\right]^{3/2}}$$

$$= qq'\frac{\frac{2\vec{v}\cdot\vec{a}}{c^2}}{\left[\vec{r}^2 - \frac{\{\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\}^2}{\vec{r}^2}\right]^{1/2}}$$

$$+ \frac{qq'(1-\vec{\hat{a}}^2)}{2}\frac{2\vec{r}\cdot\vec{v} + 2\frac{\{\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\}^2}{\vec{r}^4}\vec{r}\cdot\vec{v}}{\left[\vec{r}^2 - \frac{\{\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\}^2}{\vec{r}^2}\right]^{3/2}}$$

$$+ \frac{\frac{qq'(1-\vec{\hat{a}}^2)\left\{\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\right\}}{\vec{r}^2}\cdot\left\{\vec{v}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right) + \vec{r}\times\left(\vec{v}\times\frac{\vec{v}}{c}\right) + \vec{r}\times(\vec{r}\times\frac{\vec{a}}{c})\right\}}{\left[\vec{r}^2 - \frac{\{\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\}^2}{\vec{r}^2}\right]^{3/2}}$$

$$= qq' \frac{\frac{2\vec{v}\cdot\vec{a}}{c^2}}{\left[\vec{r}^2 - \frac{\left\{\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\right\}^2}{\vec{r}^2}\right]^{\frac{1}{2}}} \quad (43)$$

$$-qq'(1-\vec{\hat{a}}^2)\frac{(\vec{r}\cdot\vec{v})\left[1-\frac{\{\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\}^2}{\vec{r}^4}\right]}{\left[\vec{r}^2 - \frac{\{\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\}^2}{\vec{r}^2}\right]^{3/2}}$$

$$-qq'(1-\vec{\hat{a}}^2)\frac{\left\{\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\right\}\cdot\left\{\vec{v}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)+\vec{r}\times\left(\vec{v}\times\frac{\vec{v}}{c}\right)+\vec{r}\times\left(\vec{r}\times\frac{\vec{a}}{c}\right)\right\}}{\vec{r}^2\left[\vec{r}^2 - \frac{\left\{\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\right\}^2}{\vec{r}^2}\right]^{\frac{3}{2}}}$$

Thus

$$\vec{v}\cdot\vec{F}(\vec{r},\vec{v},\vec{a}) = qq'\frac{\frac{2\vec{r}^2(\vec{v}\cdot\vec{a})}{c^2}+(1-\vec{\hat{a}}^2)(\vec{r}\cdot\vec{v})}{\vec{r}^2\left[\vec{r}^2 - \frac{\left\{\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\right\}^2}{\vec{r}^2}\right]^{\frac{1}{2}}} \quad (44)$$

$$-\frac{qq'\left(1-\vec{\hat{a}}^2\right)\left\{\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\right\}\cdot\left\{\vec{v}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\right\}}{\vec{r}^2\left[\vec{r}^2 - \frac{\left\{\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\right\}^2}{\vec{r}^2}\right]^{\frac{3}{2}}}$$

$$-\frac{qq'(1-\vec{\hat{a}}^2)\left\{\left(\vec{r}\cdot\frac{\vec{v}}{c}\right)\vec{r}-(\vec{r}\cdot\vec{r})\frac{\vec{v}}{c}\right\}\cdot\left\{\vec{r}\times\left(\vec{r}\times\frac{\vec{a}}{c}\right)\right\}}{\vec{r}^2\left[\vec{r}^2 - \frac{\left\{\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\right\}^2}{\vec{r}^2}\right]^{\frac{3}{2}}}$$

where

$$\vec{r}\times\left(\vec{v}\times\frac{\vec{v}}{c}\right) = \mathbf{0}\ \textit{and}\ \vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right) = \left(\vec{r}\cdot\frac{\vec{v}}{c}\right)\vec{r}-(\vec{r}\cdot\vec{r})\frac{\vec{v}}{c}$$

Using

$$\vec{r}\cdot\vec{r}\times\left(\vec{r}\times\frac{\vec{a}}{c}\right) = \mathbf{0}$$

$$\vec{v}\cdot\vec{F}(\vec{r},\vec{v},\vec{a}) = qq'\frac{\frac{2\vec{r}^2(\vec{v}\cdot\vec{a})}{c^2}+(1-\vec{\hat{a}}^2)(\vec{r}\cdot\vec{v})}{\vec{r}^2\left[\vec{r}^2 - \frac{\left\{\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\right\}^2}{\vec{r}^2}\right]^{\frac{1}{2}}} \quad (45)$$

$$-qq'(1-\vec{\hat{a}}^2)\frac{\left\{\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\right\}\cdot\left\{\vec{v}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\right\}+\left\{(\vec{r}\cdot\vec{r})\frac{\vec{v}}{c}\right\}\cdot\left\{\vec{r}\times\left(\vec{r}\times\frac{\vec{a}}{c}\right)\right\}}{\vec{r}^2\left[\vec{r}^2 - \frac{\left\{\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\right\}^2}{\vec{r}^2}\right]^{\frac{3}{2}}}$$

Using

$$\vec{v}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right) = \left(\vec{v}\cdot\frac{\vec{v}}{c}\right)\vec{r}-(\vec{v}\cdot\vec{r})\frac{\vec{v}}{c}\ \textit{and}\ \mathbf{r}\cdot\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right) = \mathbf{0}$$

$$\vec{v}\cdot\vec{F}(\vec{r},\vec{v},\vec{a}) = qq'\frac{\frac{2\vec{r}^2(\vec{v}\cdot\vec{a})}{c^2}+(1-\vec{\hat{a}}^2)(\vec{r}\cdot\vec{v})}{\vec{r}^2\left[\vec{r}^2 - \frac{\left\{\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\right\}^2}{\vec{r}^2}\right]^{\frac{1}{2}}} \quad (46)$$

$$-qq'(1-\vec{\hat{a}}^2)\frac{\left\{\left(\vec{r}\cdot\frac{\vec{v}}{c}\right)\vec{v}\cdot\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)-(\vec{r}\cdot\vec{r})\frac{\vec{v}}{c}\cdot\vec{r}\times\left(\vec{r}\times\frac{\vec{a}}{c}\right)\right\}}{\vec{r}^2\left[\vec{r}^2 - \frac{\left\{\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\right\}^2}{\vec{r}^2}\right]^{\frac{3}{2}}}$$

Thus

$$\vec{F}(\vec{r},\vec{v},\vec{a}) = \frac{qq'}{\vec{r}^2}\frac{(1-\vec{\hat{a}}^2)\vec{r}+\frac{2\vec{r}^2\vec{a}}{c^2}}{\left[\vec{r}^2 - \frac{\{\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\}^2}{\vec{r}^2}\right]^{1/2}} \quad (47)$$

$$-\frac{qq'(1-\vec{\hat{a}}^2)}{\vec{r}^2}\frac{\left\{\left(\vec{r}\cdot\frac{\vec{v}}{c}\right)\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)-(\vec{r}\cdot\vec{r})\vec{r}\times\left(\vec{r}\times\frac{\vec{a}}{c^2}\right)\right\}}{\left[\vec{r}^2 - \frac{\left\{\vec{r}\times\left(\vec{r}\times\frac{\vec{v}}{c}\right)\right\}^2}{\vec{r}^2}\right]^{\frac{3}{2}}}$$

This force in the limit of constant velocity, i.e. **a**=0, corresponds to the covariant relativistic electrodynamic force of equation (37) based on Maxwell's equations. [20 p. 555 or 21 p. 560] Also note that this derived electrodynamic force law has chiral symmetry due to the **r** x (**r** x **v**) and **r** x (**r** x **a**) terms. This is very significant, since all matter on all size scales is observed to have chiral symmetry, i.e. elementary particles, nuclei, atoms, molecules, crystals, plant leaf patterns, flower petal patterns, and seed pod patterns, solar system, galaxies, and universe as a whole.[7,9,12-18]

Note that the second group of terms in the equation for the force is responsible for the curling corkscrew type of motion of moving charges as seen in plasma currents in fluorescent lights and plasma lightning balls sold at many gift shops. Figure 5 shows the curling corkscrew motion of charge due to the electrodynamic force for constant velocity. In general there is a second smaller scale curling motion superimposed upon the first due to acceleration.

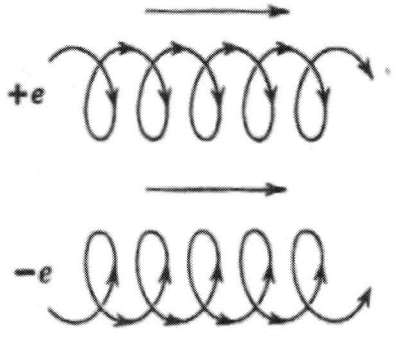

Figure 4 Corkscrew Motion of Charge Due to Electromagnetic Force

According to Ampere [28] the effect of a current flowing in a circuit twisted into small sinuosities is the same as if the circuit

were smoothed out. For $v^2/c^2 << 1$ and $aR/c^2 << 1$ this is true, but not for relativistic velocities $v/c \sim 1$. **Thus the force between two moving charges is increased due to the reduction of the effective distance R between the charges due to the spiraling motion of each charge according to the R x (R x V/c) term in the denominator of the electromagnetic force.**

4. Comparison of Derived Universal Force with Other Work

In section 3 of this paper the universal force law for constant velocity was derived in equation (40). Then this result for the force was extended to cover acceleration terms by assuming that the force was continuous and well-behaved to obtain the more general equation (47) .

This force, in the limit of constant velocity with acceleration **a** = 0, corresponds to the covariant relativistic electrodynamic force based on Maxwell's equations. [20 p. 555 or 21 p. 560]

$$\vec{F}(\vec{r},\vec{v},\vec{a})_{\vec{a}=0} = \frac{qq'}{\vec{r}^2}\frac{\left[1-\frac{v^2}{c^2}\right]\hat{r}}{\left[1-\frac{v^2}{c^2}sin^2\grave{e}\right]^{1/2}} + \frac{qq'}{\vec{r}^2}\frac{\left[1-\frac{v^2}{c^2}\right]\left(\hat{r}\cdot\frac{\vec{v}}{c}\right)\left\{\hat{r}\times\left(\hat{r}\times\frac{\vec{v}}{c}\right)\right\}}{\left[1-\frac{v^2}{c^2}sin^2\grave{e}\right]^{3/2}} \quad (48)$$

This universal force appears to be fully "relativistic" without any reference to Einstein's Special Relativity Theory or retarded fields or their unique assumptions. It can account for relativistic radiation and radiation reaction as will be shown in the next section of the paper.

There are other versions of the electrodynamic force in the scientific literature. This derived version of the electromagnetic force law is equivalent to Phipps's[29] and Wesley's[30] proposed relativistic version in the limiting case of transverse or circular motion sin θ=1 with **a** = 0 to order $\beta^2=v^2/c^2$

$$\vec{F}(\vec{r},\vec{v},\vec{a})_{\vec{r}\perp\vec{v},\vec{a}=0} = \frac{qq'\hat{r}}{\vec{r}^2}\left[\frac{(1-\hat{a}^2)}{(1-\hat{a}^2sin^2\grave{e})^{\frac{1}{2}}}\right]_{\grave{e}=\frac{\eth}{2}} \quad (49)$$

$$= \frac{qq'\hat{r}}{\vec{r}^2}[1-\hat{a}^2]^{\frac{1}{2}} \approx \vec{F}_{Phipps}$$

$$= \frac{qq'\hat{r}}{\vec{r}^2}[1-\hat{a}^2]^{\frac{1}{2}} \approx \vec{F}_{Wesley}$$

It is also equivalent to Weber's [31] version of the force law for linear motion, i.e. $\mathbf{v} \| \mathbf{r} \| \mathbf{a}$ such that sinθ=0.

$$\vec{F}(\vec{r},\vec{v},\vec{a})_{\vec{r}\|\vec{v}\|\vec{a}} = \frac{qq'}{|\vec{r}|^3}\left[(1-\hat{a}^2)\vec{r} + \frac{2\vec{r}^2}{c^2}\vec{a} + \cdots\right] \approx \vec{F}_{Weber} \quad (50)$$

5. ACCELERATION FIELDS AND RADIATION

In order to determine the radiation fields due to acceleration, it is convenient to define the electromagnetic fields and force **F** in terms of their electric and magnetic components.

$$\vec{F}_E = q\vec{E}(\vec{r}) = q\left[\vec{E}_0(\vec{r}) + \vec{E}_i(\vec{r})\right] \quad (51)$$

$$\vec{F}_M = \frac{q\vec{v}}{c}\times\left[\vec{B}_0(\vec{r}) + \vec{B}_i(\vec{r})\right]$$

The induced fields may be defined in terms of the type of motion that produced them, i.e.

$$\vec{E}_i(\vec{r}) = \vec{E}_V(\vec{r}) + \vec{E}_A(\vec{r}) + \vec{E}_{Rad}(\vec{r}) + \cdots$$

$$\vec{B}_i(\vec{r}) = \vec{B}_V(\vec{r}) + \vec{B}_A(\vec{r}) + \vec{B}_{Rad}(\vec{r}) + \cdots \quad (52)$$

Thus

$$\vec{F} = \vec{F}_{Eo} + \vec{F}_{Ev} + \vec{F}_{Ea} + \cdots \vec{F}_{Bo} + \vec{F}_{Bv} + \vec{F}_{Ba} + \cdots + \vec{F}_{Rad} \quad (53)$$

Since the velocity term in the induced fields is known experimentally to not obey the superposition principle, it is assumed that the acceleration and higher order time derivative terms also do not obey the superposition principle. Thus each of these fields should have unique properties and may correspond to the unique static, velocity and acceleration electric and magnetic fields reported by Hooper[24, pp. 9-15].

Comparing equation (53) with equation (48) one obtains

$$\vec{F}_{Eo} + \vec{F}_{Ev} = \frac{qq'\vec{r}}{|\vec{r}|^3}\frac{(1-\hat{a}^2)}{(1-\hat{a}^2sin^2\grave{e})^{3/2}} = q'(\vec{E}_0 + \vec{E}_V)$$

$$\vec{F}_{Bo} + \vec{F}_{Bv} = \frac{qq'\vec{r}}{|\vec{r}|^3}\frac{(1-\hat{a}^2)\hat{a}^2sin^2\grave{e}}{(1-\hat{a}^2sin^2\grave{e})^{\frac{3}{2}}} + \frac{qq'\vec{r}}{|\vec{r}|^4}\frac{(1-\hat{a}^2)(\vec{a}\cdot\vec{r})\vec{r}\times(\vec{r}\times\vec{a})}{(1-\hat{a}^2sin^2\grave{e})^{\frac{3}{2}}} \quad (54)$$

$$\vec{F}_{Ea} = \frac{qq'}{|\vec{r}|^3}\frac{\frac{2\vec{r}^2}{c^2}\vec{a}}{(1-\hat{a}^2sin^2\grave{e})^{3/2}}$$

$$\vec{F}_{Rad} = \frac{qq'}{c^3|\vec{r}|^3}\frac{(1-\hat{a}^2)\vec{r}\times(\vec{r}\times\vec{a})}{(1-\hat{a}^2sin^2\grave{e})^{3/2}} = q'\vec{E}_{Rad}$$

where the explicit vector dependence has been removed from the denominator.

6. RADIATION FROM REAL FINITE-SIZE PARTICLES

If a charged particle is accelerated, it is observed to give off radiation. This radiation is due to the curling motion of the electric and magnetic fields produced by the acceleration. The instantaneous energy flux of the radiation is given by the Poynting vector S [21, p. 665].

$$\vec{S} = \frac{c}{4\eth}\vec{E}_{Rad}x\vec{B}_{Rad} = \frac{c}{4\eth}|\vec{E}_{Rad}|^2\hat{r} = \frac{q^2}{4\eth c^3\vec{r}^2}\frac{(1-\hat{a}^2)^2|\hat{r}\times(\hat{r}\times\vec{a})|^2}{(1-\hat{a}^2sin^2\grave{e})^3}\hat{R} \quad (55)$$

where θ is the angle between **r** and **v**, and all observed radiation is due to acceleration .

In the non-relativistic limit $v/c << 1$

$$\frac{dP}{d\grave{U}} = \frac{q^2}{4\eth c^3}|\vec{a}|^2sin^2\grave{e}' \quad (56)$$

where θ′ is the angle between **r** and **a**. This is the standard Larmor formula for total power radiated from a non-relativistic accelerated charge. [19 p. 469 or 20 p. 659 or 21, p. 665] The total instantaneous power radiated is obtained by integrating $dP/d\Omega$ over all solid angles.

For the relativistic case where **v** is perpendicular to **r** such that sin θ = 1 or **r**·**v** = 0

$$\frac{dP}{d\grave{U}} = \frac{q^2}{4\eth c^3}\frac{(1-\hat{a}^2)^2}{(1-\hat{a}^2sin^2\grave{e})^3}\vec{a}^2sin^2\grave{e}$$

$$P = \frac{2q^2}{3c^3}\tilde{a}^6\vec{a}^2(1-\hat{a}^2) = \frac{2q^2}{3c^3}\tilde{a}^6[\vec{a}^2 - (\vec{v}\times\vec{a})^2] \quad (57)$$

where

$$\vec{a}^2(1-\hat{a}^2) = \vec{a}^2 - \left(\frac{\vec{v}}{c}\times\vec{a}\right)^2 \; since\; \vec{v}\cdot\vec{a} = 0 \quad (58)$$

This is the fully relativistic Lienard result first obtained in 1898. [19 p. 470, 20 p. 660, 21 p. 666] Note that there is a relationship between θ and θ′, i.e.

$$\sin\grave{e} \to 1 \quad \sin\grave{e}' \to 0 \qquad (59)$$

This causes all the radiation to be emitted at very small angles θ′ producing the search light effect observed in particle accelerators and shown in Figure 6.

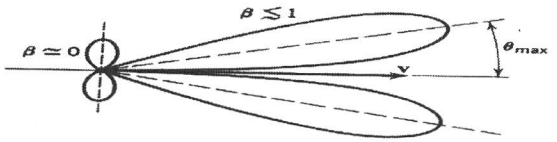

Figure 6 Searchlight Effect in Radiation Pattern of Charges Accelerated in Direction of Velocity

7. RADIATION REACTION FORCE

If we assume the motion of a charged particle is periodic or the charge in a charged particle flows in closed loops[6,7,13], then the average work done on the particle by the electrodynamic force in emitting radiation is the negative of the power integrated over one period from τ_1 to τ_2

$$\int_{\hat{o}_1}^{\hat{o}_2} \vec{F}_{Rad} \cdot \vec{v} dt = \int_{\hat{o}_1}^{\hat{o}_2} -P dt = -\int_{\hat{o}_1}^{\hat{o}_2} \frac{\grave{i}_0 q^2 \vec{a}^2}{6\eth c} dt$$
$$= -\int_{\hat{o}_1}^{\hat{o}_2} \frac{\grave{i}_0 q^2}{6\eth c} \frac{d\vec{v}}{dt} \cdot \frac{d\vec{v}}{dt} dt \qquad (60)$$

Notice that we can integrate the above expression by parts. If we assume that there is periodic motion, the boundary term in the integral by parts disappears to give

$$\int_{\hat{o}_1}^{\hat{o}_2} \vec{F}_{Rad} \cdot \vec{v} dt = -\frac{\grave{i}_0 q^2}{6\eth c} \frac{d\vec{v}}{dt} \cdot \vec{v}|_{\hat{o}_2} + \frac{\grave{i}_0 q^2}{6\eth c} \frac{d\vec{v}}{dt} \cdot \vec{v}|_{\hat{o}_1}$$
$$+ \int_{\hat{o}_1}^{\hat{o}_2} \frac{\grave{i}_0 q^2}{6\eth c} \frac{d^2\vec{v}}{dt^2} \cdot \vec{v} dt$$
$$= 0 + \int_{\hat{o}_1}^{\hat{o}_2} \frac{\grave{i}_0 q^2}{6\eth c} \frac{d\vec{a}}{dt} \cdot \vec{v} dt \qquad (61)$$

Thus we can identify the radiation reaction force [21, p. 748] as

$$\vec{F}_{Rad} = \frac{\grave{i}_0 q^2}{6\eth c} \frac{d\vec{a}}{dt} \quad (62)$$

8. NATURE OF LIGHT AND ITS CIRCUIT

The electrodynamic force is a local contact type of force in which the fields are an extension of the charge. The fields have tensile strength and extend to great distances. Light is a ripple of angular momentum in the fields of the particle. All field lines start on a charge and end on a charge as shown in Figures 2b and 2c.

Since most charges appear in neutral pairs throughout the universe, most of the field lines return to the nearby charge of the opposite sign. Thus there is a circuit of some sort that brings light back to its original source, since it travels on the fields of the particle that emitted it. This notion is similar to some ideas in General Relativity Theory. **These field extensions of the charge define the fabric of space.** They physically define the ether of General Relativity Theory which Einstein was never able to do. He was only able to describe it mathematically.

9. SUMMARY OF RESULTS

An electrodynamic force law for elastic finite-size charged particles was derived by simultaneously solving, by substitution, the fundamental empirical laws of classical electrodynamics, i.e. Gauss's laws, Ampere's law, Faraday's law, and Lenz's law assuming Galilean invariance. The mathematics of the derivation and Hooper's experimental results[24] require that the electrodynamic force be a local contact force based on the electromagnetic fields being an extension of the charges.

This derivation of the electrodynamic force law is a **proof** that "relativistic effects" are due entirely to the self-fields of finite size elastic charged particles and Lenz's Law without any reference to Einstein's Special Relativity Theory or retarded fields. It accounts for relativistic effects everywhere in electrodynamics including radiation and radiation reaction. It explains the physical basis for Special Relativity Theory's use of the inertial frame as being due to the electromagnetic fields remaining attached to the physical charges at rest in the inertial frame. This derivation of the electrodynamic force law is also a **proof** that the Lorentz force law is a consequence of Galilean invariance instead of Lorentz invariance. **By Einstein's own criteria given in his Nobel prize address[3], this work is more impressive than his work in relativity theory, because of the simplicity of its assumptions and its ability to explain relativistic effects without a separate theory.**

This proper axiomatic approach deals with a number of loose ends in electrodynamics. It deals with real elastic particles with finite size and self fields. It does not use fictitious action-at-a-distance forces but uses real contact forces based on the physical extension of charges via the charge's fields. It satisfies Mach's principle, which Einstein was never able to do with Special Relativity, by using Lenz's law. It uses the Galilean transformation based upon causality. It conserves energy and momentum 100% of the time compared to quantum electrodynamics which depends on the uncertainty principle to allow it to escape these conservation laws for brief periods of time. It only depends on the relative distance r, the relative velocity v, the relative acceleration a, and the reaction jerk da/dt between the interacting charges implying that the force has the same value for all observers irrespective of their states of motion or of their inertial frame of reference. It is a proper relativistic theory according to the classical definition, because it only depends on relative coordinates instead of relative reference frames. The field extensions of the charges in the universe physically define the fabric of space or the ether of General Relativity Theory and Newton's Universal Gravitation Force.

Finally the derived universal electrodynamic force has chiral symmetry. This symmetry is missing in covariant Maxwell electrodynamics, quantum mechanics, and relativity theory, but is observed in elementary particles, nuclei, atoms, molecules, crystals, plant leaf patterns, flower petal patterns, and seed pod patterns, the solar system, galaxies, and the structure of the observable universe.[7,9,12-18]

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