Refutation of the logical syntax of IT architectures

Abstract: We evaluate two conditions in the antecedent of a definition, both separately and as respective negations, then proceed to a definition of the consequent. The conjecture to define the symmetric binary relations of an n-tier hierarchy is *not* tautologous. The conjecture with substitutions in a subsequent lemma is also denied. This refutes the conjecture of the logical syntax of IT architecture. These results form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬; + Or, ∨, ∪, ⊔; - Not Or; & And, ∧, ∩, ⊓, ·, ∘, ⊗; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡,:=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ∃!, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Strnadl, C.F. (2020). The logical syntax of IT architectures. arxiv.org/pdf/2004.03719.pdf

2 Theory Let us directly start with the definition of an architecture. Our notation follows standard model theory ..

Definition 9 (elementary n-tier architecture). *Let* $T_n = \{1, 2, ..., n\}$ *and define the symmetric binary relation* T^L *("linked to") over* T_n^2 *as follows:*

$$|i-j| \leq 1 \Rightarrow T^L(i,j),$$
$$|i-j| > 1 \Rightarrow \neg T^L(i,j)$$

Then the B&C architecture

$$\mathscr{T}_n = \langle T_n, \{T^L\} \rangle$$

is called the **elementary n-tier architecture**.

(2.9.1.1), (2.9.2.1)

LET p, q, r: i, j, T^L(i,j).

~((%s>#s)<(p-q))>r ; **F**NNN TTTT **F**NNN TTTT (2.9.1.2)
((p-q)>(%s>#s))>~r ; TTTT C**FFF** TTTT C**FFF** (2.9.2.2)

Remark 2.9.1.2, ..2.2: Eqs. 2.9.1.2 and 2.9.2.2 as rendered are *not* tautologous, hence refuting the definition of the symmetric binary relation and subsequent conjectures. If the definitions are respective negations, then the conjunction should be contrary.

(2.9.3.1)

(~((%s>#s)<(p-q))>r)&(((p-q)>(%s>#s))>~r) ;
FNNN C**FFF** **F**NNN C**FFF** (2.9.3.2)

Remark 2.9.3.2: Eq. 2.9.3.2 is *not* contrary, hence refuting the antecedent definitions as respective negations.

We attempt to resuscitate Def. 9 by completing the argument according to the apparent intention. The comma in the text separating Eqs. 2.9.1.1 and 2.9.2.1 taken to mean "And" should read "Or" as 2.9.1.1 or 2.9.2.1. (2.9.4.1)

```
(~((%s>#s)<(p-q))>r)+(((p-q)>(%s>#s))>~r) ;
                              TTTT TTTT TTTT TTTT          (2.9.4.2)
```

The goal was apparently to express th4e antecedent in Def. 9 as a tautology.

The argument in Def. 9 then becomes the antecedent as true to imply the consequent of B&C Architecture (B&C) such that T implies (B&C). For this to hold, the goal is for T to imply T and not for the disallowed T to imply **F**.

B&C is defined in Def. 5 below:

Definition 5 (boxes & connectors (B&C) architecture)**.** *An architecture* $\mathscr{A} = \langle A, \mathbf{R} \subseteq \mathcal{P}(A^2), \emptyset\rangle$ *is called a* **boxes & connectors (B&C) architecture**.

A B&C architecture is called **connected** *if* $\forall a \in A : \exists R \in \mathbf{R}, \exists a^\star \in A : R(a, a^\star) \vee R(a^\star, a) \wedge a \neq a^\star$*. Otherwise it is called* **disconnected**.

We shall often write $\mathscr{A} = \langle A, \mathbf{R}\rangle$ *for a B&C architecture in the following.*

Architects frequently used so-called n-tier[7] architectures which can be rigorously defined as followed.

(2.5.1)

LET t, u, v, w, x: a, a*, A, R, **R**.

```
((#t<v)>((%w<x)&(%u<v)))>((w&(t&u)) +((w&(t&u))&(t@u))) ;
                              FFFF FFFF FFFF FFFF(  1)}2}8
                              NNNN NNNN NNNN NNNN(  1)}  }
                              FFFF FFFF FFFF FFFF(  5)}1}
                              NNNN NNNN NNNN NNNN(  1)}  }
                              FFFF FFFF FFFF FFFF(  1)}  }
                              TTTT TTTT TTTT TTTT(  1)}  }
                              FFFF FFFF FFFF FFFF(  3)}  }
                              TTTT TTTT TTTT TTTT(  1)}  }   (2.5.2)
```

Remark 2.5.2: Eq 2.5.2 is *not* tautologous as required to confirm Def. 9. In fact, completing the argument for Def. 9 with the Eq. 2.9.4.1 as antecedent and 2.5.1 as consequent produces the same result as 2.5.2. This means Def. 9 is *not* tautologous using Def. 5 which is also *not* tautologous, hence refuting the claimed conjecture of a syntax for IT architecture. We include a subsequent Lemma 1 which by substitutions in its proof is similarly denied for the same reasons.

3.2 Theorems

Here we demonstrate that our theory of (IT) architectures also provides a mathematical framework for reasoning (and proving) facts about architectures.

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Lemma 1. *Every n-tier B&C architecture $\mathscr{A}$ is homomorphic to the elementary n-tier architecture $\mathscr{T}_n$, that is there exists a homomorphism $h : \mathscr{A} \mapsto \mathscr{T}_n$.*

Proof. Let $\mathscr{A} = \langle A, \mathbf{R} \rangle$ be a n-tier B&C architecture. Then, by definition, every element $a \in A$ belongs to exactly one tier C_i with $1 \leq i \leq n$. We also have $R \subseteq A^2$ because of the "boxes & connectors" property. Then define the map $h : \mathscr{A} \mapsto \mathscr{T}_n$ as follows:

$$h(\cdot) : \begin{cases} a \in C_i & \mapsto \; i \in T_n, \\ R \in \mathbf{R} & \mapsto \; T^L \end{cases}$$

Assume $R(c_i, c_j)$ with $c_i \in C_i$ and $c_j \in C_j$. Then $h(c_i) = i$ and $h(c_j) = j$. Because $\mathscr{A}$ is an n-tier architecture, we have

$$|i - j| \leq 1 \Rightarrow |\, h(c_i) - h(c_j)\,| \leq 1 = T^L(h(c_i), h(c_j)) = h(R)(h(c_i), h(c_j)).$$

Therefore h is a homomorphism. □