

Recent advances in the modal model checker Meth8 and VL4 universal logic

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In applied and theoretical mathematics, assertions are categorized in alphabetical order as: axiom; conjecture; definition, entry; equation; expression; formula; functor; hypothesis; inequality; metatheorem; paradox; problem; proof; schema; system; theorem; and thesis. We evaluate 635 artifacts in 3304 assertions to confirm 548 as tautology and 2756 as *not* (83.4%). We use Meth8, a modal logic checker in five models.

The semantic content or predicate basis of some expressions on their face does not disqualify them from evaluation by Meth8 in classical modal logic. However, the rules of classical logic, as based on the corrected Square of Opposition by Meth8, apply to virtually any logic system. Consequently some numerical equations are mapped to classical logic as Meth8 scripts.

The rationale for mapping quantifiers as modal operators is based on satisfiability and reproducibility of validation of the 24-syllogisms from the corrected Square of Opposition.

Test results are refuted as *not* tautologous, confirmed as tautologous, or neither. For a paradox, *not* tautologous means it is not a paradox, but not necessarily a contradiction either.

The experimental tests used variables for 4 propositions, 4 theorems, and 11 propositions. The size of truth tables are respectively for 16-, 256-, and 2048- truth values, using recent advances in look up table indexing.

The Meth8 modal theorem prover implements the logic system variant VL4 which corrects the quaternary Ł4 of Łukasiewicz. There are two sets of truth values on the 2-tuple {00, 10, 01, 11} as respectively {False for contradiction; Contingent for falsity; Non contingent for truthity; Tautology for proof} and {Unevaluated; Improper; Proper; Evaluated}. The designated *proof* value is T for tautology and E for evaluated. The model checker contains recent advances in parsing technology named sliding window.

The mapping of formulas in Meth8 script was performed by hand, checked, and tested for accuracy of intent. The Meth8 script uses literals and connectives in one-character. Propositions are p-z, and theorems are A-B. The connectives for {and, or, imply, equivalent} are {&, +, >, =}. The negated connectives for {nand; nor; not imply; exclusive-or} are {\, -, <, @}. The operators for {not; possibility $\diamond\exists$; necessity $\square\forall$ } are {~, %, #}. Expressions are adopted for clarity as: (p=p) for tautologous; (p@p) for contradiction; and (x<y) for $x \in y$. The expression $x \leq y$ as "x less than or equal to y" is rendered in the negative as $\sim(y < x)$ or as $(\sim x > \sim y)$.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Binary ordinal
1	p=p	T	tautology	proof	11	3
2	p@p	F	contradiction	absurdum	00	0
3	%p>#p	N	non-contingency	truthity	01	1
4	%p<#p	C	contingency	falsity	10	2

Note the meaning of (%p>#p): a possibility of p implies the necessity of p; and some p implies all p. In other words, if a possibility of p then the necessity of p; and if some p then all p.

This shows equivalence of respective modal operators and quantified operators as in Appendix.

For Meth8 an immediate further application is mapping sentences of natural language into logical formulas, so a semi-automation of that linguistic process is near completion.

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