Recent advances in the modal model checker Meth8 and VŁ4 universal logic

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In applied and theoretical mathematics, assertions are categorized in alphabetical order as: axiom; conjecture; definition, entry; equation; expression; formula; functor; hypothesis; inequality; metatheorem; paradox; problem; proof; schema; system; theorem; and thesis. We evaluate 635 artifacts in 3304 assertions to confirm 548 as tautology and 2756 as *not* (83.4%). We use Meth8, a modal logic checker in five models.

The semantic content or predicate basis of some expressions on their face does not disqualify them from evaluation by Meth8 in classical modal logic. However, the rules of classical logic, as based on the corrected Square of Opposition by Meth8, apply to virtually any logic system. Consequently some numerical equations are mapped to classical logic as Meth8 scripts.

The rationale for mapping quantifiers as modal operators is based on satisfiability and reproducibility of validation of the 24-syllogisms from the corrected Square of Opposition.

Test results are refuted as *not* tautologous, confirmed as tautologous, or neither. For a paradox, *not* tautologous means it is not a paradox, but not necessarily a contradiction either.

The experimental tests used variables for 4 propositions, 4 theorems, and 11 propositions. The size of truth tables are respectively for 16-, 256-, and 2048- truth values, using recent advances in look up table indexing.

The Meth8 modal theorem prover implements the logic system variant VŁ4 which corrects the quaternary Ł4 of Łukasiewicz. There are two sets of truth values on the 2-tuple {00, 10, 01, 11} as respectively {False for contradiction; Contingent for falsity; Non contingent for truthity; Tautology for proof} and {Unevaluated; Improper; Proper; Evaluated}. The designated *proof* value is T for tautology and E for evaluated. The model checker contains recent advances in parsing technology named sliding window.

The mapping of formulas in Meth8 script was performed by hand, checked, and tested for accuracy of intent. The Meth8 script uses literals and connectives in one-character. Propositions are p-z, and theorems are A-B. The connectives for {and, or, imply, equivalent} are {&, +, >, =}. The negated connectives for {nand; nor; not imply; exclusive-or} are {\, -, <, @}. The operators for {not; possibility ◊∃; necessity □∀} are {~, %, #}. Expressions are adopted for clarity as: (p=p) for tautologous; (p@p) for contradiction; and (x<y) for x∈y. The expression x≤y as "x less than or equal to y" is rendered in the negative as ~(y<x) or as (~x>~y).

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Binary ordinal
1	p=p	T	tautology	proof	11	3
2	p@p	F	contradiction	absurdum	00	0
3	%p>#p	N	non-contingency	truthity	01	1
4	%p<#p	C	contingency	falsity	10	2

Note the meaning of (%p>#p): a possibility of p implies the necessity of p; and some p implies all p. In other words, if a possibility of p then the necessity of p; and if some p then all p. This shows equivalence of respective modal operators and quantified operators as in Appendix.

For Meth8 an immediate further application is mapping sentences of natural language into logical formulas, so a semi-automation of that linguistic process is near completion.

References

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No.	Name of artifact	Type of artifact	Non Taut.	Tautology
1	ABC	Conjecture	1	
2	Shevenyonov’s proof	Conjecture	6	
3	Abductive reasoning	System	1	
4	Abduction, induction, deduction: Peirce	Inference	2	1
5	Abstract theory of segments using Prolog	Proofs	15	3
6	Adaptive algorithm for molecular simulation	Conjecture	5	
7	AGM / Levi and Harper bridging principles	Postulates	7	3
8	AGM: remainder sets, paraconsistent revisions	Operators	5	
9	Agnostic hypothesis testing	Hexagon	9	2
10	Agnosticism as subset of atheism from no-belief	Conjecture	4	2
11	AI: divide the dollar competition	Experiment	1	
12	AI: reduction for scalable deep learning	Algorithm	3	
13	Ackermann’s approach to quantifier reduction	System	20	2
14	Alcoholics Anonymous BB: We agnostics, p 53	Conjecture	5	
15	Aphorism acceptance inverse expectation	Proportion	1	
16	Contradictory sayings	Paradoxes	4	
17	Alexandroff correspondence	Conditional	3	
18	Alternating Turing machines (ATMs)	Problem	3	
19	Analysis as both correct and informative	Paradox	5	
20	Analytic principles of choice and dependent choice	Axioms	3	
21	Anderson division by zero as nullity	Theorem	3	
22	Approximations of theories	System	1	
23	Arrow’s impossibility	Theorem	7	
24	Athanasian creed (Holy Trinity)	Credo	1	2
25	Axiomatizing category theory in free logic	Axioms	5	5
26	Axiomatizing fuzzy logic with graded modalities	Conjecture	1	
27	Banach order space, generating positive cone	Definition	2	1
28	Banach-Tarski, shorter resuscitation attempt	Paradox	2	
29	Crucial claim in Step 3 of proof, shortest	Paradox	1	
30	Bar recursion	Mapping	2	
31	Barcan	Formula	37	32
32	Barwise compactness	Theorem	2	
33	Bayes rule	Rule	11	11
34	BCI / BCK algebra	System	5	3

No.	Name of artifact	Type of artifact	Non Taut.	Tautology
35	Bell / CHSH / Spekken toy model	Inequalities	7	
36	A shorter refutation	Theorem	2	
37	A simpler refutation	Theorem	2	
38	Bell's inequality by axiom of empty set	Theorem	2	
39	Bogus Bellian logic (BBL)	Theorem	5	
40	Coercive proof	Theorem	2	
41	Coin toss proof	Conjectures	2	
42	Original inequality with assumption	Conjecture	4	
43	Original inequality with assumptions	Conjectures	5	
44	Original inequality and CHSH	Conjectures	6	
45	Positive reasons proof	Theorem	6	3
46	Temporal logic	Theorem	1	
47	Tropical sum	Method	1	
48	Bellman's Lost in the forest problem, solution	Theorem		4
49	Berkeley	Paradox	1	
50	Bernstein-Vazirani	Algorithm	4	1
51	No-cloning theorem	Theorem	2	
52	Bertrand-Chebyshev theorem / postulate	Theorem	2	
53	Betweenness theory	Axioms	9	7
54	BF calculus, Spencer-Brown Primitive arithmetic	System	8	
55	Biscuit conditionals	System	13	
56	Bisimulation, coinduction, Howes' congruence	Proof method	5	
57	Bitstring and question-answer	Semantics	18	
58	Blameworthiness degrees	Coalition	3	5
59	Block argumentation	Method	15	
60	Blok-Esakia	Theorems	5	
61	Bogdanov map, 2D conjugate of Hénon map	Formula	1	
62	Boolean polynomials	Conjecture	3	
63	Boone-Rogers, uniform word problem	Theorem	2	1
64	Borel base and hull	Conjectures	2	
65	Born rule	Theorem	6	
66	Exclusivity rule as basis	Theorem	5	
67	Borsuk-Ulam theorem (BUT)	Conjecture	1	
68	Bounded and Σ_1 formulas in PA	Conjecture	1	

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69	Bounding modal logics on transitive frames	Conjecture	6	
70	Bourbaki-Moroianu-Tarski fixed point theorem	Theorems	10	
71	Boyer’s question	Paradox	3	
72	Branching quantifier (Hintikka)	System	3	
73	Brouwer fixed point theorem (BFPT)	Conjecture	3	2
74	Browder's theorem	Conjecture	1	
75	Buddhist logic	Tetralemma	12	
76	Buridan's Ass	Paradox	11	
77	Buss’s bounded arithmetics	Hierachy	6	
78	Cabannas objectivity	Theory	1	1
79	Hypothesis recast in equational arithmetic	Conjecture	13	
80	Cantor's continuum conjecture, binary injection	Hypothesis	2	
81	Continuum conjecture, interpreted	Hypothesis	3	
82	Diagonal argument	Proof	3	
83	Pairing	Functor	2	
84	Carroll's tortoise and Achilles	Paradox	3	2
85	Caswell's significant curriculum issues (1952)	Theorem	5	1
86	Causality	Axiom	3	2
87	Category composition of morphisms	Definition	1	
88	Category Tannakian via Iwasawa embedding	Axioms	1	
89	Category theory by lattice identity / partitions	Conjectures	5	1
90	CC conjecture of Lin Fan Mao	Conjecture	2	
91	Chaitin incompleteness and L constant	Theorem	3	1
92	Chinese room argument: Brain Simulator Reply	Conjecture		3
93	CHSH inequality	Conjecture	1	
94	Dual reality	Conjecture	1	
95	Church	Thesis	2	
96	Church-Rosser	Theorem	2	
97	Clausius-Clapeyron on spacial dimension	Equation	1	
98	Clifford tori 2D / Kanban cell neuron	Definition	2	3
99	Coherence in modal logic	Refutation	1	
100	Collatz (briefest known confirmation)	Conjecture	4	1
101	Collection theory as set of all closure sentences	Schema	1	
102	Commutative short circuit on propositional logic	System	1	

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103	Comorphism of sites for Grothendieck toposes	Theory	1	
104	Complementarity inequality	Refutation	2	1
105	Completeness: inclusion/equivalence of universality	Conjecture	1	
106	Complex (ℂ), imaginary number rendering	Method	1	1
107	Computer simulation model theory (CSMT)	Method	2	
108	Conceivable statement of Perez confirmed	Definition	18	1
109	Conceptivistic / containment logic: proscription	Systems	3	
110	Conditional logic: improved Adams	Hypothesis	7	
111	Events in QL	Conjectures	8	1
112	Conditional necessitarianism	Conjectures	6	
113	Confluence in rewrite systems	Property	3	
114	Connexive logic: Wansing's nightmare	System	2	
115	Aristotle, Boethius justifying connexive	Theses	4	
116	De Finettian logic conditionals	Indicatives	10	
117	Constructive math: Ishihara tricks onLPO/WLPO	Principles	3	
118	Constructivistic logic	System	14	2
119	Contingent, necessitarian, and internal, external	Conjectures	5	
120	Continuum hypothesis	Conjecture	2	1
121	Cook-Reckhow	Definition	6	
122	Coq proof assistant	Prover	1	
123	Counterfactual analysis of causation	Reversal	1	
124	Counterpart theory on intensionality logic	Theory	7	3
125	Craig interpolation, constructive Fefferman	Theorem	4	2
126	Constructive using Maehara's technique	Theorem	7	
127	Creative theories in degrees of unsolvability	Theorem	1	
128	Curry-Howard correspondence	Conjecture	4	
129	D Ultrafilter contra continuum problem	Equation	1	
130	De Morgan algebras, existentailly closed	System	2	
131	Decoherence (pre-measurement) quantum mechanics	Formula	1	
132	Dedekind lattice identity	Axiom		1
133	Dempster-Shafer belief and plausibility	Theory	3	
134	Density of all Turing and truth table degrees	Formula	2	
135	Dependent choices (DC) on supercompactness	Axiom	2	
136	DC with axiom of determinacy (AC) on mice	Axioms	2	

No.	Name of artifact	Type of artifact	Non Taut.	Tautology
137	Description logic	System	2	
138	Power set POW on of any Ω-model	Operator	4	
139	Unions in descriptively near sets	Operator	6	2
140	Dialetheism	System	4	
141	Dialetheism: inconsistent	System	2	
142	Dichotomy of selection	System	1	
143	Differential reasoning	Logic	1	
144	Distributive bilattices	Variety	1	
145	Disturbance	Feature	4	
146	Diverse double compiling	Schema	1	
147	Domain theory of Dana Scott	System	6	
148	Poset: reflexive, antisymmetric, transitive	Definitions	2	1
149	Doxastic logic	System	8	13
150	Drinker's paradox	Paradox	1	
151	Duality corrected for weak by refuting strong	Theorem	7	2
152	Dyatic semantics on paraconsistent logic C_1	Approach	4	
153	E=mc^2	Theorem	3	1
154	EF-axiom: topology and near sets	Axiom	1	
155	Ehrenfeucht-Mostowski indiscernables	Theorem	1	
156	Einstein–Podolsky–Rosen (EPR)	Paradox	1	
157	Elementary constructive set theory (ECST)/CZF	Definitions	6	
158	Entanglement vs untanglement: no-no go-go	Conjecture	28	
159	Enumeration reducibility, degrees, topology	Method	1	
160	Epicurus invoked by Epictetus	Paradox	1	
161	Epimenides the Cretan	Paradox	1	
162	Epistemic coalition	Perfect recall	4	3
163	Epistemic dynamic reasoning	System	2	
164	Epistemic Hilbert substructure	System	5	
165	Epistemic for knowledge and probability	System	2	
166	Epistemic navigation	System	8	
167	Epistemic quantifiers over agents	Conjecture	8	12
168	Erdös-Strauss	Conjecture	1	
169	Ethical reasoning and HOL proof assistant	System	1	
170	Euathlus paradox	Paradox	9	

No.	Name of artifact	Type of artifact	Non Taut.	Tautology
171	Euclidean geometry embedded in non Euclidean	Conjecture	3	
172	"Ex nihilo": some thing from nothing	Conjecture	4	4
173	Fictional logic based on Levi-identity / AGM	System	5	4
174	Finitary, algebraizable logic undecidable in Hilbert	System	2	
175	Finite direct powers of ω for modal model	System	1	
176	First-order proofs without syntax	Method	7	
177	Fitch's knowability	Paradox	4	
178	Fixedpoint operator mapping mu-calculus	Conjecture	2	
179	FOL disjunctive normal forms (DNF): minimize	FOL Optimizer	2	1
180	FOL continuous induction on real closed fields	System	11	1
181	Force speed greater than light speed	Conjecture		3
182	FOT first order team logic	System	1	
183	Fodor weaker/stronger, class	Principle	5	1
184	Forcing method	Paradoxes	2	
185	Forcing to change large cardinal strength	Theorems	3	
186	Formalization of AC as equivalents using Coq	Claims	6	
187	Frauchiger-Renner conjunctives	Paradox	6	
188	Frauchiger-Renner thought experiment	Paradox	5	
189	Frauchiger-Renner thought quantum model	Paradox	14	2
190	Fredkin	Paradox	1	
191	Free choice permission (FCP) in deontic logic	Paradox	1	
192	Free logic (presupposition) for FOP (implication)	Conjecture	12	
193	Free will argument of Maimonides	Paradox	1	
194	Free will non-existence	Refutation	8	2
195	Free will theorem: Clifton-Kochen-Specker	Theorem	1	
196	Free will theorem: FIN axiom	Hypothesis	2	2
197	Free will theorem: MIN axiom	Hypothesis	1	
198	Frequency dependence of mass	Theorem	3	1
199	Functions as injective, surjective, bijective	Theorems		4
200	Gentzen proof of sequent System G-M	System	6	2
201	Gettier (justified belief)	Problem	6	
202	GHZ (Greenberger-Horne-Zeilinger)	Experiments	1	
203	Gleason	Theorem	1	
204	Gobbay separation	Theorem	17	

No.	Name of artifact	Type of artifact	Non Taut.	Tautology
205	Gödel class with identity un-solvable	Conjecture	10	1
206	Gödel compactness	Theorem	6	2
207	Gödel completeness, short	Theorem	2	
208	Gödel completeness, shorter	Theorem	4	
209	Gödel first incompleteness	Theorem	4	
210	Gödel incompleteness	Equations	14	1
211	Gödel incompleteness FOL	Contradictions	14	1
212	Gödel incompleteness theorem	Assistant tools	2	2
213	Gödel incompleteness theorem, shortest	Refutation	7	
214	Gödel incompleteness via Löb counter-examples	Refutation	9	2
215	Gödel-justification logic	Refutation	7	2
216	Gödel-justification subsets	Models	2	
217	Gödel-Löb	Axiom	1	
218	Gödel-McKinsey-Tarski translation of IPC	Theorem	1	
219	Gödel pairing function	Axiom	3	1
220	Gödel recursion	Theorem		1
221	Gödel-Scott on God	Theorem schema	5	2
222	Goldbach's conjectures	Conjectures	8	2
223	Strong conjecture	Conjecture	5	1
224	Goldblatt-Thomason with back/forth duality	Theorem	3	
225	Goodstein (with Ackermannian)	Theorem	3	
226	Graded modal logic	Frame classes	4	2
227	Grassmannian discovery	Paradox	4	
228	Hadamard gate	Theory	3	1
229	Hahn-Banach	Theorem	5	
230	Hall	Effect	1	
231	Hamiltonian quaternion	Definition	3	
232	Hamkins' embeddings in sets	Theorem	1	
233	Hardy's generalized paradox	Conjecture	6	
234	Hegel's dialectic	Method	6	3
235	Heider inspired international relations theory	Theory	5	1
236	Heisenberg uncertainty principle	Axiom	2	
237	Conjecture	Principle	1	
238	Replacement theorem	Axiom	2	3

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239	Take a picture of an electron	Principle	6	2
240	Hempel’s raven	Paradox	6	
241	Henkin cyclic algebra and FOL applications	Axioms	8	6
242	Permutation model nonrepresentable	Assertion	1	
243	Herbrand semantics	System	6	
244	Heyting algebra	Algebra	21	
245	Heyting-Brouwer intuitionistic logic	Systems	9	1
246	Heyting distributive lattices / binary operator	Conjecture	1	
247	Heyting logic	Idempotency	2	1
248	Hoop and pocrim with Prover9/Mace8	Theorems	14	
249	Hexagons of opposition for statistical modalities	Conjectures	9	1
250	Higher-order mathematical induction	Principle	2	1
251	Hilbert H10 undecidable: extended to Q in R	Problem	6	4
252	Hilbert generalization	System	1	
253	Hilbert grand hotel	Paradox	1	
254	Hilbert / Kolmogorov intuitionistic logic	Systems	9	
255	Hilbert’s first epsilon theorem quantifier shift	Logics	7	
256	HOL rejection of Lowe’s modal ontology	Argument	16	4
257	Horty's puzzles in stit logic	Corollaries	3	3
258	Hrushovski (*now* confirms Lachlan and Zil’ber)	Construction	2	
259	Huemer’s confirmation theory for induction	Proposal	3	2
260	Huhn 2-distributive lattice identity	Formula	1	
261	Hybrid systems via predicate transform semantics	Assistant	5	
262	Hydraulic forgiveness (confirmed)	Theorem	6	2
263	Hydraulic corollary of peace	Corollary		1
264	Hypersequent calculus for modal logic S5	System	2	
265	Ideals	Definition	4	
266	Ignorance of first choice	System	3	
267	Imaginary numbers	Definition	2	
268	Imperative logic	System	3	4
269	Implication combination forms from (p>r)>r	Theorems	3	16
270	Implicit logic (IL) to explicit logic (EL)	Translation	7	
271	Impossible worlds: none necessary; all possible	Definitions	4	4
272	Inclusion, dependence, independence logics	Theorem	1	

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273	Inconsistent theory	Theorem	6	2
274	Extending the monad to a triad	Formulas	10	2
275	Kunen inconsistency	Theorem	1	1
276	Independence-friendly logic (Kreiselization)	System	2	
277	Indicative conditionals	Encyclopedia entry	5	1
278	Indiscernibles in saturated free algebras	Theory	2	
279	Induction: Black raven (swan); Kripkenstein	System	3	
280	Induction: intuitionistic logic Martin-Löf type theory	Axiom	4	
281	Induction: New riddle	Paradox	2	
282	Induction, coinduction: standard and mutual	System	4	
283	Induction in Elem. Arith. for reduction property	Formulas	2	
284	Inequality: 'arbitrarily' vs 'sufficiently large	Conjecture	2	1
285	Infinite set theory	Theorem	2	
286	Information theory: mutual information	Definition	4	
287	Innovation contest in two sequential stages	Definition	2	
288	Inquisitive modal logic via flatness grade	System	2	
289	Internal logic of extensive restriction categories	Duality	3	
290	Interpretability logic	System	3	
291	Logics ILM, TOL, Vaught and adjunctive sets	Systems	10	
292	Interval logic for model checking	System	3	
293	Intuitionistic fuzzy decisions in Dempster-Shafer	Comparison	2	
294	Intuitionistic Zermelo-Fraenkel set theory (IZF)	Axioms	10	
295	Isabelle/HOLinteractive tools	Prover	14	3
296	Isabelle/HOL prover assistant	System	1	
297	Jaccard index	Statistic	3	
298	Jensen polynomial partition roots for Riemann	Hypothesis	1	
299	Join-prime in lattice theory	Definition		1
300	Jonsson positive logic: retromorphism	System	3	
301	*k*-triangular set function logic	Definition		1
302	Kanban cell neuron maps whole brain	Model	242	14
303	Kant: falsity of syllogistic figures	Theorems	8	2
304	Karpenko, S.A.	System K-Ł4	6	8
305	Keisler measure in NIP theory	Formula	1	
306	Keisler's ultraproduct	Construction	7	1

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307	Kent algebras on rough set concept analysis	Algebra	5	
308	Kleene algebra, free with domain (binary relations)	Conjecture	2	
309	Kleene lattices, reversible	Axiomization	3	1
310	Knowledge representation	Refutations	15	
311	Kramers-Kronig	Relation	1	
312	Kripke frames via BAOs with ◊⊥=⊥	Theory	10	
313	Kripke-Platek and CZF	Set Theory	1	
314	Kuratowski–Zorn lemma (Zorn's lemma)	Lemma	1	
315	Lachlan problem solution	Problem	4	
316	Lambda λ-calculus and LISP	System	7	1
317	Lattice effect algebra	Conjecture	1	
318	Lean prover from Microsoft	System	3	
319	Lebesgue spaces	Conjecture	1	
320	Leibniz' identity of indiscernibles	Theorem	1	
321	Leibniz' ontological proof	Proof	1	1
322	Briefest known ontological proof of God	Proof		2
323	Lemmon D	Axiom	1	
324	van Leunen deformation field	Conjecture	4	
325	van Leunen lattice logic, weak modular	Theorem	2	
326	Liar	Paradox	5	
327	Liar's antimony	Paradox	7	
328	Prior rendition	Paradox	4	
329	Saul Kripke rendition	Paradox	1	
330	Line through a circle	Conjecture		1
331	Linear algebra	Theorems	4	
332	Linear programming: Kharun-Kush-Tucker	Conditions	4	
333	Linear temporal logic	System	2	
334	Liouville	Theorem	2	1
335	Lipschitz horizontal vector fields	Conjecture	4	
336	Löb original, corrected	Theorem	1	1
337	Löb with Gödel incompleteness	Theorems	3	
338	Lobachevskii non Euclidean geometry	Consistency	1	1
339	Lonely runner	Conjecture	8	1
340	Refutation of and strong Löwenheim–Skolem	Theorem	4	

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341	Löwenheim–Skolem, Hilbert style	Metatheorem	4	
342	Luce model (general)	Definitions/axioms		5
343	Łukasiewicz nightmare: (◊p&◊q)→◊(p&q)	Allegation	4	3
344	Łukasiewicz modal Ł4	Objections	8	1
345	Lusin’s separation	Theorem	2	
346	Lyndon interpolation and GL	Interpolation	8	1
347	Majorana's 'root'	Equations	9	
348	Malice and Alice	Puzzle	7	
349	Matita prover superposition	Algorithms	3	
350	Mereology, behavioral	System	2	
351	Mereotopology, distributive	Relations	9	
352	Metaphysics: "Something rather than nothing"	Problem		2
353	Meth8/VŁ4 self-proof in one variable	Theorem		8
354	Meth8 versus Prover9 via Lifshitz	Problem		1
355	Minimalist foundation (MF) via Church’s thesis	System	1	
356	Minkowski plane, classical set of points/cycles	Theorems	2	1
357	Modal aleatoric calculus	System	6	
358	Modal coalgebraic geometric logic	System	2	
359	Modal logics: 2; 3; 4; B; D; E; K; M; T; and W	Systems	2	8
360	Modal GL_2	Logic	8	1
361	Modal logic for supervised learning	System	11	
362	Modal operators on rings of continuous functions	Conjecture	6	
363	Model theory = univ. algebra + mathematical logic	Language	1	
364	Modern modal logics (2): JYB4 and AR4	Axioms	12	17
365	Modified divine command	Theory	4	1
366	Modus ponens consequent as conditional	Rule	3	1
367	Molyneux's problem	Problem	1	
368	Moore's paradox	Paradox	2	1
369	Moral absolutism impossible	Conjecture	3	1
370	Relativity of moral absolutism	Conjecture	2	1
371	Most	Quantifier	2	
372	Naive scale invariance for ’t Hooft world hologram	Conjecture	1	
373	Necessitation: K,T,4,B,D,5; *D*,*M*,*S4*,*B*,*S5*	Axiom	10	7
374	Leonard Nelson's criticism of epistemology	System	2	2

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375	von Neuman-Bernays-Gödel [NBG]	Theory	2	3
376	Neutrosophic logic	Theorems	5	
377	Dezert-Smarandache	Theory	2	
378	Generalized Hegel's dialectics	Method	4	
379	Generalized intuitionistic, fuzzy logic	System	2	
380	Geometry of Smarandache / Lin Fan Mao	Definition	2	
381	MANET attack genetics	Algorithm	1	
382	Negated adjectival phrases	Lattices	2	
383	Neutrosophic logic: theory of everything	Theorem	2	
384	Neutrosophic sets	Properties	3	
385	Neutrosophy with dependent probability	Definitions	3	2
386	Pseudo-trinitarian human consciousness	Model	2	
387	Quinary	System	2	
388	Retract crisp set topology	Propositions	5	
389	Smarandache multi-space theory	System	5	
390	Soft lattice theory	Theorem	2	
391	Unification of other logics	Axioms / Rules	2	
392	Values as 1, 0, between 0 and 1	Designated logic	2	
393	Vector space on {-0, 0, 0<p<1, 1, 1+}	Multi-valued logic	3	
394	Newcomb's game	Paradox	2	
395	Noncontingency variants	Operator	5	
396	Non-deterministic logic	Completeness	8	
397	Nothing does not imply a non existent null set	Definition	7	
398	Nucleosynthesis	Definitions	4	
399	Ontology engineering for complete-debug problems	Conjecture	6	
400	Open universe causal reasoning	Conjecture	2	
401	Optimization for complex number programming	Conjecture	1	
402	Ordinal notation via simultaneous definition	Conjecture	5	
403	Ordinal Turing machine (OTM) on set theory	Conjectures	3	
404	Orthomodular law	Theorem	1	
405	Overlap algebra on constructive Boolean algebra	Conjecture	4	
406	P=NP; 3-SAT	Conjectures	5	
407	Satisfiability via claimed Boolean rules	Conjectures	4	
408	Paraconsistency, machine-assisted view	Axioms	2	3

No.	Name of artifact	Type of artifact	Non Taut.	Tautology
409	Paraconsistent contradiction	Conext	4	
410	Paraconsistent logic on one conjecture	Theorem	1	
411	Paradox refutation reduced to one variable	Method	3	9
412	Parikh’s logic G and completeness of system Par	Axiomatization	3	
413	Partial awareness	System	8	1
414	Pascal’s wager thought experiment	Theorem	4	1
415	Pauli exclusion	Principle	4	
416	Peano arithmetic 9, 1-8	Axioms	1	8
417	Extended truth definitions	Theorems	3	
418	PŁ4	System	10	
419	Playfair axiom	Axiom		1
420	Płonka sums of inclusive consequence relations	Inclusion	1	
421	Poincaré recurrence theorem	Theorem	2	
422	Poison modal logic (PML)	System	4	
423	Karl Popper on God	Proof	4	14
424	Positive modal logic	Variety	4	
425	PowerEpsilon mathematical induction	Axiom		1
426	Pratt-Floyd-Hoare logic correctness	System	5	
427	Predicative collapse, arithmetical comprehension	Conjectures	5	
428	Preference profiles	Algorithm	6	
429	Prenex normal format	Rules	11	3
430	Shortest refutation based on implication	Rules	2	2
431	Pre-orderable groups	Conjecture	3	
432	Presburger arithmetic	System	3	
433	Presupposition different from entailment	Conjecture	3	
434	Prevarieties and quasivarieties of logic	Monoids	4	
435	Prisoner's paradox	Paradox	3	
436	Probabilistic approximate logic (PALO)	System	1	
437	Program verification	Reduction	1	
438	Provability logic GL, Japaridze polymodal GLP	Logics	8	
439	Prover9 vs Meth8 differences	System	6	
440	Pure alethic modal logic	System	1	1
441	Quantified modal logic theorem proof (QMLTP)	Library	5	
442	Quantifiers and operators	Connectives	12	

No.	Name of artifact	Type of artifact	Non Taut.	Tautology
443	Quantum arithmetic for repeat-until-success	Circuits	2	
444	Block chain	Cryptography	1	
445	Classical logic as completion of QL	System	16	6
446	Control by observation	System	1	
447	Entanglement	Conjecture	5	
448	Gates correspond to logical operators	Theorems	4	1
449	Gates refuted	Theorems	8	1
450	Gedanken experiment	Conjectures	8	
451	Logic	System	17	2
452	Masking	Conjectures	5	2
453	Probability	Axiom	1	
454	Probability	Rule	2	
455	Simulation of Hamiltonian spectra	Operator	1	
456	Spin-statistics	Theorem	4	4
457	Superposition as the red herring	Conjecture	3	
458	Superposition disproves Schrödinger cat	Conjecture	5	2
459	Temporal logic based on Löwner order	System	4	
460	Ternary probability of qutrit	Hypothesis	3	
461	Three lights experiment of qutrit	Conjecture	1	
462	Questions and answers	Problem	4	1
463	Ramsey's theorem / Pythagorean triple of integers	Theorem	8	
464	Ranjan, A.	Problem		2
465	Rational emotive behavior therapy (REBT/LBT)	Systems	16	
466	Rauszer ‘concrete’ Boolean algebra by preorder	Systems	13	
467	Realizability semantics for QML	Theorems	3	
468	Reichenbach common cause / event-splitting	Principle	6	3
469	Relativization (structural induction) in weighting	Method	1	
470	Relevance logic	System	7	
471	Definition in R	Model	2	
472	Resolution-based decision procedure, 2-vars equality	Methodology	9	
473	Reverse mathematics: recursive comprehension	Principle	10	2
474	Measurability and computability	Theory	1	2
475	Nets	Conjectures	4	
476	Rewriting logic for compositional specification	Conjectures	4	

No.	Name of artifact	Type of artifact	Non Taut.	Tautology
477	Riemann: only zeroes at 0, 1/2	Hypothesis	1	
478	Excluded middle	Conjecture	4	
479	Extended complex numbers on sphere	Operators	3	2
480	Zeta function, Caceres 6	Proposition	4	
481	Zeta function, properties	Axioms	7	
482	Roman Catholic Church (RCC) Canon law	Theorem	4	
483	Erasmus contra Luther	Controversy		1
484	Infallibility and the Historic Church	Pius IX	2	
485	Invalid epiclesis with silent Holy Ghost	Canons	4	1
486	Magisterium	Paul VI	1	
487	Primacy of the Roman See	Pius XI	1	
488	Sacred heart of Jesus	Pius XI	3	
489	Tradition above scripture	Pius IX	4	
490	Twelfth promise of St Alacoque	Vision	3	
491	Rosser's theorem on consistency	Theorem	6	1
492	Rota lattice theory, distributive	Axiom	1	
493	Russell	Paradox	4	
494	Russell-Prawitz embedding	Conjectures	7	
495	S5Π+ propositional quantification	System	1	
496	Sabotage modal logic (revisited)	System	6	
497	Sacchetti's modal logics of provability	Definition	1	
498	Sahlqvist modal / quantified correspondence	Theory	7	
499	Sahlqvist non formulas as tautologous	Theory	1	
500	Schaeffer for graphs, P, NP, NPC, NPH, NPI	Theorem	10	
501	Schematic refutations of formula schemata	Method	2	
502	Schrödinger's cat	Paradox	2	
503	Scott’s existence axiom in scheaf theory	Conjecture	5	
504	Scott’s topology, temporal types, landscapes	Theory	4	
505	Search fund study	Model	2	
506	Security barrier	Model	5	2
507	Self-equilibrium	Law / Paradox	1	
508	Self-proving fixed-point predicate modal logic on GL	Definition / Theorem	4	
509	Self-verifying axioms for grounding functions	Conjecture	7	
510	Shallow embedding for rewriting equality in MLTL	Rule	7	

No.	Name of artifact	Type of artifact	Non Taut.	Tautology
511	Shevenyonov extension nary antropic to PL	System	16	2
512	Simulation argument and incompleteness	Conjecture	1	
513	Mistakes in rebuttal of refutation above	Conjectures	6	2
514	Imply operator injected for Bayes' pipe	Conjecture	1	
515	Skolem form	Axiom	1	
516	Sliding scale in law	Theorem	3	
517	Solovay arithmetical / semantical completeness	Theorem	10	2
518	Sorites	Paradox		1
519	Special theory of relativity: Crothers refutation	Confirmation	1	4
520	Square (Łukasiewicz) and Cube (Seuren)	Systems	27	15
521	Square of Opposition Meth8 Corrected	System		6
522	Square of Opposition Modern Revised	System	2	
523	Square of Opposition	Proportions	3	
524	Stable set lattices	Modal operators	2	
525	Stit logic (see to it that)	System	13	
526	Stone space type lattice logic model	Theory	2	
527	Stone-Wales rotation transform reversibility	Theorem	2	1
528	Strong jump inversion, decided saturated model	Logic	1	
529	Student quiz conjecture	Paradox	1	
530	Subdirect products on bounded homomorphisms	Lattices	2	
531	Superposition of states	Principle	6	
532	Supply and demand	Conjecture	6	
533	Surveillance objectives	Subgames	2	
534	Suzko's minimal truth values for universal logic	Problem	3	
535	Symmetry breaking, skeleton, ensemble, SMT solver	Methodology	3	
536	Symplectic vector space	Theorem	2	
537	Tarski's geometric axioms and betweenness	System	8	5
538	Tarski's undefinability of truth	Theorem	2	
539	Tarski–Grothendieck set theory	Axiom	2	
540	Tarski–Grothendieck Metamath ax-groth	Axiom	7	
541	Tarski's planar Euclidean R-geometry	System	8	2
542	Temporal logic properties	System	7	
543	Temporal logic over infinite intervals, completeness	Conjecture	6	
544	Term rewriting for automated theorem proving	Method	2	

No.	Name of artifact	Type of artifact	Non Taut.	Tautology
545	Theory of logic for glutty, gappy relations	System	16	
546	Three-valued, bivalent logic VŁ3	System		10
547	Time algorithm	Conjecture	3	
548	Time as God	Conjecture		2
549	Time as past, present, future coexisting states	Conjecture	3	
550	Time as tense via converse implication (EQT)	Conjecture		1
551	Topological manifold transition	Function	1	
552	Topological T_0-space	Differ modality	3	
553	Totherian sets	Theory	5	
554	Translation invariance of superposition calculus	Property	2	
555	Traveling salesman complexity problem	Conjecture	1	1
556	Triangle inequality	Conjecture		1
557	Trivial proofs for a troll	Conjectures		3
558	Trolley ethical thought experiment	Problem	3	
559	Turing halting problem	Problem	2	1
560	Turing's halting problem as logically unsolvable	Problem	2	
561	Twin paradox	Paradox	2	
562	Two-sided page	Paradox	5	1
563	Type theory	Conjecture	2	
564	Ultrapower of universe, projective determinancy	Hypotheses	2	
565	Unanswered logic questions	Conjectures	2	1
566	Unfalsifiability	Conjecture	2	2
567	Unification nets (canonical proof net quantifiers)	Conjecture	9	
568	Unification of simple symmetrical modal logics	Type	2	
569	Universal finite set	Theorem	2	
570	Universal network	Operator	5	
571	Universal logic VŁ4	Logic		1
572	Veblen (corrected)	Axiom	1	1
573	Vector conjecture for two points implying a third	Conjecture		1
574	Verilog / VHDL hardware logic	Connectives	36	
575	Veronoï regions (with "nonempty sets")	Definition	3	
576	Vickrey auction	Theorem	10	2
577	Vietoris space modal operators	Definitions	2	
578	Visibility graph / Kanban cell resuscitation	Algorithm	1	1

No.	Name of artifact	Type of artifact	Non Taut.	Tautology
579	VŁ4 completeness confirmed	Theorem	1	1
580	VŁ4 soundness confirmed	Theorem	2	1
581	VŁ4 theorems for S5	Theorems	8	
582	Vongehr's shift rendering QM "natural"	Paradigm	2	3
583	W (K4W)	Theorem	1	
584	Wadge order (and on Scott domain)	Theorem	4	
585	Weak set H to prove its own consistency	Theory	3	
586	Well ordering property	Axiom	1	
587	Wellfoundedness of multiset order	Theorem	2	
588	White's model of creation	System	7	1
589	Wittgenstein's ab-notation	System	3	
590	X-homology on manifolds in topology	Axiom	1	
591	Yalcin logic	Axioms	2	
592	Yinyang	System	1	3
593	Zadeh first operators on fuzzy logic	System	5	
594	Historical assumptions	Axioms	4	1
595	Perfect / strong functions, residuated operator	Conjectures	3	
596	Resolution and symmetry in Z-numbers	Definitions	3	
597	Swedes and Italians logic challenge	Problem	3	
598	Zariski topology affine varieties, scheme theory	Definitions	3	
599	Zermelo-Fraenkel constructive (CZF)	Axioms	9	1
600	Zermelo-Fraenkel (ZF) shortest contra 9 axioms	Axioms	10	
601	ZFC Empty set	Axiom	2	
602	Shortest refutation	Axiom	3	
603	ZFC Extensionality	Axiom	1	1
604	ZFC Extensionality (another rendition)	Axiom	1	
605	Other refutations of ZFC axioms (10)	Axioms	10	1
606	ZF Law of excluded middle: infinite set	Axiom	2	
607	ZF supremum and infimum	Definitions	3	
608	ZFC^0: schemas of comprehension, replacement	Axioms	2	
609	Zero and three in arithmetic	Theorems		9
610	Zero knowledge proof	Theorem	1	
611	Zeroth law of thermodynamics	Theorem	3	2
612	Rendering quantifiers as modal operators	Appendix		

No.	Name of artifact	Type of artifact	Non Taut.	Tautology
613	Meth8 on Modus Cesare and Modus Camestros	Appendix		
614	Availability of Meth8/VŁ4	Marketing collateral		
615	Availability of Meth8/VŁ4 demo for 2-variables (p,q)	Marketing collateral		
616	Scalability of Meth8/VŁ4	Marketing collateral		
		635 artifacts = 3304	2756	548

Table of Contents

Refutation of the ABC conjecture

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p,q,s integers; r relatively prime; ~ Not; + Or; & And; > Imply; = Equivalent.

The ABC conjecture is described at wiki. Basically the sentence reads:

"If p or q is equivalent to s and p,q,s are relatively prime, then p or q is tautologous". (1.0)

If the conjecture is confirmed, then it can be used as the proof for a multitude of other unrefuted conjectures.

"If p+q=s and p,q,s are relatively prime, then p+q is tautologous." (1.1)

(((p+q)=s)&(((p&q)&s)=r))>(p+q) ; **F**TTT TTTT TTTT TTTT (1.2)

Eq. 1.2 as rendered is *not* tautologous, and deviates by one value in bold. This refutes the ABC conjecture.

Denial of A.V. Shevenyonov's proof for the ABC conjecture

Abstract: The six seminal equations evaluated are *not* tautologous, refuting the subsequent claimed proof of the ABC conjecture, and forming a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Shevenyonov, A.V. (2017). Taken ABaCk by conjecturing out-of-Box. vixra.org/pdf/1712.0669v1.pdf

Remark 0: We present only script mappings with table value results, as keyed to the text, because the author has no published email address, and we decline to correspond via the Disqus forum.

Sketching the grand problem—or Is it but a very special case?

LET p, q, r: a, b, rad

((1.1.1.1)=(1.1.2.1))=(1.1.3.1) (1.1.4.1)

((r&((p&q)&(p+q)))=((r&(p&q))&(r&(p+q))))=(((r&p)&(r&q))&(r&(p+q))) ;
FFFF FFFT **FFFF FFF**T (1.1.4.2)

Remark 1.1: What the author means to say when invoking "[b]y straightforward induction" is the equation:
(((1.1.1.1)=(1.1.2.1)) and ((1.1.2.1)=(1.1.3.1))) and ((1.1.1.1)=(1.1.3.1)). (1.1.5.1)

Heuristic Support: The above result could be seconded from a number of alternative standpoints. First, the ABC conjecture appears to pass the dimensionality check: (a+b)≥rad(ab[a+b]). (1.2.1)

~((r&((p&q)&(p+q)))>(p+q)) = (p=p) ;
FFFF FFFF FFFF FFFF (1.2.2)

Remark 1.2: The inequality of Eq. 1.2.1 is *not* tautologous, contradictory, and refutes the claim.

Twin & twixt multiplicity versus additivity: 2.1.1, 2.2.1, 2.3.1

(((p-(%s>#s))\(q-(%s>#s)))&((r&q)=((q-p)\(q-(%s>#s)))))=(r&p) ;
TTTT T**FF**T TTTT T**FF**T TTTT (2.1.2)

(r&(p+q))=(((p\(p-(%s>#s)))&((r&p)-(%s>#s)))+(r&s)) ;
NNNN N**F**C**F** NNNN **F**TTT (2.2.2)

(((r&(p+q))-(r&p)) \ ((r&(p+q))-(r&q)))=(q\p) ;
FFFT **F**TT**F** **FFF**T **F**TT**F** (2.3.2)

(r&(s@s))=(((%s>#s)\(%s<#s))&((r&(%s>#s))+(r&~(%s>#s)))) ;
TTTT **FFFF** TTTT **FFF**F (2.10.2)

Remark 2: The four Eqs. 2.1.2, 2.2.2, 2.3.2, and 2.10.2 are not tautologous and hence not "consistent with orduale residuality as well as the i*nherently Diophantine* nature of primes".

Six seminal equations are *not* tautologous, refuting the subsequent claimed proof of the ABC conjecture.

Refutation of abductive reasoning

Abductive reasoning is defined from C.S. Peirce as:

> The universal fact p is a truthity. But if q was a tautology, then p would necessarily follow. Therefore possibly q exists as a truthity. (1.1)

Using Meth8/VŁ4,

> LET # necessity, for all; % possibility, for one or some; > Imply; \ Not And
> `T` is the designated proof value; `N` is truthity as (%p>#p); `C` is falsity as (%p<#p).

((#p=(%p>#p))\((q=(q=q))>#p)) > (%q=(%p>#p)) ;

`CTNN CTNN CTNN CTNN` (1.2)

Eq 1.2 as rendered is *not* tautologous. This means abductive reasoning is refuted.

Refutation of Peirce's abduction and induction, and confirmation of deduction

Abstract: We evaluate definitions of C.S. Peirce for abduction, induction and deduction: all are inversions of the same sentence. However, when the connectives are changed to implication, abduction and induction are not tautologous, leaving deduction as the only form of tautologous inference in logic.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) `C` non-contingency, ∇, ordinal 2; (%z>#z) `N` as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: iep.utm.edu/peir-log/

C.S. Peirce originally defined the three forms of inference in logic as:

Abduction: (Q is S) and (Q is P) imply (S is P) (1.1.1)

LET p, q, s: P, Q, S.

((q=s)&(q=p))>(s=p) ; `TTTT TTTT TTTT TTTT` (1.1.2)

Induction: (S is Q) and (P is Q) imply (S is P) (2.1.1)

((s=q)&(p=q))>(s=p) ; `TTTT TTTT TTTT TTTT` (2.1.2)

Deduction: (S is Q) and (Q is P) imply (S is P) (3.1.1)

((s=q)&(q=p))>(s=p) ; `TTTT TTTT TTTT TTTT` (3.1.2)

Peirce described Eqs. 1-3 as inversions of the same.

Remark: If the word "is" is taken to mean the word "implies" then the connective = is replaced with the connective > below.

Abduction: (Q implies S) and (Q implies P) imply (S implies P) (1.2.1)

((q>s)&(q>p))>(s>p) ; `TTTT TTTT` **F**`TTT` **F**`TTT` (1.2.2)

Induction: (S implies Q) and (P implies Q) imply (S implies P) (2.2.1)

((s>q)&(p>q))>(s>p) ; `TTTT TTTT TT`**`F`**`T TT`**`F`**`T` (2.2.2)

Deduction: (S implies Q) and (Q implies P) imply (S implies P) (3.2.1)

((s>q)&(q>p))>(s>p) ; `TTTT TTTT TTTT TTTT` (3.2.2)

Eqs. 1.2.2-2.2.2 as rendered for abduction and induction are *not* tautologous, but Eq. 3.2.2 is tautologous. This means that abduction and induction are not inversions of deduction, leaving deduction as the only form of tautologous inference in logic.

On the computer proof of a result in the abstract theory of segments

Abstract: We evaluate the computer proof of a result in the abstract theory of segments. Out of 18 equations, three were trivial and tautologous as expected, and 15 were *not* tautologous. The refutes the approach of using the computer programming language Prolog for such mappings.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r, s, u, v, w, z, y, z: f, q, r, s, u, v, w, z, y, z;
~ Not, ¬ ; + Or, ∨ ; - Not Or; & And, ∧; \ Not And;
> Imply, greater than, →; < Not Imply, less than, ∈;
= Equivalent, ≡; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊; # necessity, for every or all, ∀, □;
~(y < x) (x ≤ y), (x ⊆ y); (p=p) Tautology.

From: Skordev, D. (2015).
On the computer proof of a result in the abstract theory of segments.
store.fmi.uni-sofia.bg/fmi/logic/skordev/proofsrc.pdf dvak@fmi.uni-sofia.bg

Remark 0: For clarity, we distribute the quantifiers to each instance of a variable.

Problem 1 *Consider next four sentences:*

(symm) ∀x∀y∀z(r(x,y,z)→r(y,x,z)) (1.1.1)

(#r&((#x&#y)&#z))>(#r&((#y&#x)&#z)) ;

```
TTTT TTTT TTTT TTTT(16)
```
(1.1.2)

(assoc1) ∀x∀y∀u∀v∀z(r(x,y,z)∧r(z,v,u)→∃w(r(y,v,w)∧r(x,w,u))) (1.2.1)

((#r&((#x&#y)&#z))&(#r&(#z&(#v&#u))))>((#r&((#y&#v)&%w))&(#r&(#x&(%w&#u)))) ;

```
TTTT TTTT TTTT TTTT( 6)
TTTT CCCC TTTT CCCC( 2),
TTTT TTTT TTTT TTTT( 8)
```
(1.2.2)

(assoc2) ∀x∀y∀u∀v∀z(r(x,z,y)∧r(u,z,v)→∃w(r(x,v,w)∧r(u,y,w))) (1.3.1)

((#r&((#x&#y)&#z))&(#r&(#u&(#z&#v))))>((#r&((#x&#v)&%w))&
(#r&(#u&(#y&%w)))) ;

```
TTTT TTTT TTTT TTTT( 6)
TTTT CCCC TTTT CCCC( 2),
TTTT TTTT TTTT TTTT( 8)
```
(1.3.2)

(monot) ∀x∀y∀u∀z(r(x,z,u)∧r(y,y,z) →r(x,y,u)) (1.4.1)

((#r&((#x&#z)&#u))&(#r&((#y&#y)&#z)))>(#r&((#x&#y)&#u)) ;
TTTT TTTT TTTT TTTT(16) (1.4.2)

Remark 1.4: We see, as rendered, Eqs. 1.1.2 and 1.4.2 have the same truth table result as tautology, and Eqs. 1.2.2 and 1.3.2 have the same truth table result as not tautologous, but without the contradiction value **F**.

Show that their conjunction (1.5.1)

((((#r&((#x&#y)&#z))>(#r&((#y&#x)&#z)))&(((#r&((#x&#y)&#z))&
(#r&(#z&(#v&#u))))>((#r&((#y&#v)&%w))&(#r&(#x&(%w&#u))))))&
((((#r&((#x&#y)&#z))&(#r&(#u&(#z&#v))))>((#r&((#x&#v)&%w))&
(#r&(#u&(#y&%w)))))&(((#r&((#x&#z)&#u))&(#r&((#y&#y)&#z)))>
(#r&((#x&#y)&#u)))))=(p=p) ; TTTT TTTT TTTT TTTT(118),
TTTT CCCC TTTT CCCC(2),
TTTT TTTT TTTT TTTT(8) (1.5.2)

implies the sentence

$\forall x\forall y\forall z(r(z,z,x)\land r(z,z,y)\to\exists w(((w=x)\lor r(x,x,w))\land((w=y)\lor r(y,y,w))))$ (1.6.1)

((%w=#x)+(#r&((#x&#x)&%w)))>((%w=#y)+(#r&((#y&#y)&%w))) ;
TTTT TTTT TTTT TTTT(24),
CCCC CCCC CCCC CCCC(16),
TTTT TTTT TTTT TTTT(24) (1.6.2)

Conjunction of Eqs. (1.1.2 & 1.2.2 & 1.3.2 & 1.4.2) as (1.5.2) implies (1.6.2) (1.7.1)

((((#r&((#x&#y)&#z))>(#r&((#y&#x)&#z)))&(((#r&((#x&#y)&#z))&
(#r&(#z&(#v&#u))))>((#r&((#y&#v)&%w))&(#r&(#x&(%w&#u))))))&
((((#r&((#x&#y)&#z))&(#r&(#u&(#z&#v))))>((#r&((#x&#v)&%w))&
(#r&(#u&(#y&%w)))))&(((#r&((#x&#z)&#u))&(#r&((#y&#y)&#z)))>
(#r&((#x&#y)&#u)))))>
(((%w=#x)+(#r&((#x&#x)&%w)))>((%w=#y)+(#r&((#y&#y)&%w)))) ;
TTTT TTTT TTTT TTTT(24),
CCCC CCCC CCCC CCCC(16),
TTTT TTTT TTTT TTTT(24)
(solution) (1.7.2)

Remark 1.6: The consequent in Eq. 1.6.2 has the same truth table result as the implied solution in 1.7.2. The expected solution to Problem 1 is *no*t confirmed by Eq. 1.7.2 which is not tautologous, hence refuting it. However, there is no contradiction value **F**, but rather the falsity value C for contingency.

Problem 2 *Show that the conjunction of the same four sentences* **(symm)**, **(assoc1)**, **(assoc2)** *and* **(monot)** *implies the sentence* **(meet)** $\forall x\forall y\forall z(r(z,z,x)\land r(z,z,y)\to\exists w(r(x,x,w)\land r(y,y,w)))$
(2.1)

((#r&((#z&#z)&#x))&(#r&((#z&#z)&#y)))>
((#r&((#x&#x)&%w))&(#r&((#y&#y)&%w))) ;

TTTT TTTT TTTT TTTT(112),
TTTT CCCC TTTT CCCC(8),
TTTT TTTT TTTT TTTT(8) (2.2)

((((#r&((#x&#y)&#z))>(#r&((#y&#x)&#z)))&(((#r&((#x&#y)&#z))&
(#r&(#z&(#v&#u))))>((#r&((#y&#v)&%w))&(#r&(#x&(%w&#u))))))&
((((#r&((#x&#y)&#z))&(#r&(#u&(#z&#v))))>((#r&((#x&#v)&%w))&
(#r&(#u&(#y&%w)))))&(((#r&((#x&#z)&#u))&(#r&((#y&#y)&#z)))>
(#r&((#x&#y)&#u)))))
\>
(((#r&((#z&#z)&#x))&(#r&((#z&#z)&#y)))>
((#r&((#x&#x)&%w))&(#r&((#y&#y)&%w)))) ;
TTTT TTTT TTTT TTTT(112),
TTTT CCCC TTTT CCCC(6),
TTTT TTTT TTTT TTTT(10) (2.3)

Remark 2.3: The solution to Problem 2 as Eq. 2.3 is *not* tautologous.

Problem 3 (quite easy) *Show that the conjunction of* **(assoc1)** *and* **(monot)** (3.1.1)

(((#r&((#x&#y)&#z))&(#r&(#z&(#v&#u))))>((#r&((#y&#v)&%w))
&(#r&(#x&(%w&#u)))))&(((#r&((#x&#z)&#u))&(#r&((#y&#y)&#z)))>
(#r&((#x&#y)&#u))) ; TTTT TTTT TTTT TTTT(118),
TTTT CCCC TTTT CCCC(2),
TTTT TTTT TTTT TTTT(8) (3.1.2)

implies the sentence

(transit) ∀u∀x∀y∀z (r(x,u,y)∧r(y,u,z)→r(x,u,z)) (3.2.1)

(#r&((#x&#u)&#y))>(#r&((#x&#u)&#z)) ;
TTTT TTTT TTTT TTTT(50),
TTTT CCCC TTTT CCCC(2),
TTTT TTTT TTTT TTTT(2),
TTTT CCCC TTTT CCCC(2),
TTTT TTTT TTTT TTTT(2),
TTTT CCCC TTTT CCCC(2),
TTTT TTTT TTTT TTTT(2),
TTTT CCCC TTTT CCCC(2),
TTTT TTTT TTTT TTTT(64) (3.2.2)

Eqs. 3.1.2 implies 3.2.2 (3.3.1)

((((#r&((#x&#y)&#z))&(#r&(#z&(#v&#u))))>((#r&((#y&#v)&%w))
&(#r&(#x&(%w&#u)))))&(((#r&((#x&#z)&#u))&(#r&((#y&#y)&#z)))>
(#r&((#x&#y)&#u))))>
((#r&((#x&#u)&#y))>(#r&((#x&#u)&#z))) ;
TTTT TTTT TTTT TTTT(50),
TTTT CCCC TTTT CCCC(2),
TTTT TTTT TTTT TTTT(2),
TTTT CCCC TTTT CCCC(2),
TTTT TTTT TTTT TTTT(2),

```
TTTT CCCC TTTT CCCC( 2),
TTTT TTTT TTTT TTTT( 2),
TTTT CCCC TTTT CCCC( 2),
TTTT TTTT TTTT TTTT(64)
```
(3.3.2)

Remark 3.3.2: Eq. 3.3.2 is not tautologous and hence refutes the proposed solution in Problem 3.

Problem 4 (hard enough) *Show that the conjunction of* **(symm)***,* **(assoc2)** *and* **(transit)** *implies* **(meet)**.

We write this as Eqs. 1.1.1 & 1.3.1 & 3.2.1 implies 2.1. (4.1)

```
(((#r&((#x&#y)&#z))>(#r&((#y&#x)&#z)))&((((#r&((#x&#y)&#z))&
(#r&(#u&(#z&#v))))>((#r&((#x&#v)&%w))&(#r&(#u&(#y&%w)))))&
((#r&((#x&#u)&#y))>(#r&((#x&#u)&#z)))))>
(((#r&((#z&#z)&#x))&(#r&((#z&#z)&#y)))>
((#r&((#x&#x)&%w))&(#r&((#y&#y)&%w)))) ;
TTTT TTTT TTTT TTTT(112),
TTTT CCCC TTTT CCCC( 10),
TTTT TTTT TTTT TTTT(  6)
```
(4.2)

Remark 4.2: Eq. 4.2 is *not* tautologous and hence refutes the solution for Problem 4.

Problem 5 *Consider next sentence:*

(assoc2') ∀x∀y∀u∀v∀z (r(x,z,y)∧r(u,z,v)→∃w(r(v,x,w)∧ r(y,u,w))) (5.1.1)

```
((#r&((#x&#z)&#y))&(#r&((#u&#z)&#v)))>
((#r&((#v&#x)&%w))&(#r&((#y&#u)&%w))) ;
TTTT TTTT TTTT TTTT(118),
TTTT CCCC TTTT CCCC(  2),
TTTT TTTT TTTT TTTT(  8)
```
(5.1.2)

Show that the conjunction of **(assoc2')** *and* **(transit)** *implies* **(meet)**.

We write this as Eqs. 5.1.1 & 3.2.1 implies 2.1. (5.2.1)

```
((((#r&((#x&#z)&#y))&(#r&((#u&#z)&#v)))>((#r&((#v&#x)&%w))&
(#r&((#y&#u)&%w))))&((#r&((#x&#u)&#y))>(#r&((#x&#u)&#z))))>
(((#r&((#z&#z)&#x))&(#r&((#z&#z)&#y)))>
((#r&((#x&#x)&%w))&(#r&((#y&#y)&%w)))) ;
TTTT TTTT TTTT TTTT(112),
TTTT CCCC TTTT CCCC(  6),
TTTT TTTT TTTT TTTT( 10)
```
(5.2.2)

Remark 5.2.2: Eq. 5.2.s is *not* tautologous and hence refutes the solution for Problem 5.

Obviously the conjunction of **(symm)** and **(assoc2)** implies **(assoc2')**.

We rewrite this as Eqs. 1.1.1 and 1.3.1 implies 5.1.1. (5.3.1)

(((#r&((#x&#y)&#z))>(#r&((#y&#x)&#z)))&(((#r&((#x&#y)&#z))&
(#r&(#u&(#z&#v))))>((#r&((#x&#v)&%w))&(#r&(#u&(#y&
%w))))))>(((#r&((#x&#z)&#y))&(#r&((#u&#z)&#v)))>
((#r&((#v&#x)&%w))&(#r&((#y&#u)&%w)))) ;

```
TTTT TTTT TTTT TTTT( 16)
```
(5.3.2)

The following formula is a convenient prenex normal form of **(assoc2')**:

$\forall x \forall y \forall u \forall v \exists w \forall z(r(x,z,y) \wedge r(u,z,v) \rightarrow r(v,x,w) \wedge r(y,u,w))$ (5.4.1)

((#r&(#x&#y))&(#r&((#u&#z)&#v)))>
((#r&((#v&#x)&%w))&(#r&((#y&#u)&%w))) ;

```
TTTT TTTT TTTT TTTT(118),
TTTT CCCC TTTT CCCC(  2),
TTTT TTTT TTTT TTTT(  8)
```
(5.4.2)

A corresponding Skolem normal form is

$\forall x \forall y \forall u \forall v \forall z(r(x,z,y) \wedge r(u,z,v) \rightarrow r(v,x,f(x,y,u,v) \wedge r(y,u,f(x,y,u,v)))$ (5.5.1)

((#r&((#x&#z)&#y))&(#r&((#u&#z)&#v)))>
((#r&(#v&(p&((#x&#y)&(#u&#v)))))&
(#r&((#y&#u)&(p&((#x&#y)&(#u&#v)))))) ;

```
TTTT TTTT TTTT TTTT(118),
TTTT CTCT TTTT CTCT(  2),
TTTT TTTT TTTT TTTT(  6),
TTTT CTCT TTTT CTCT(  2)
```
(5.5.2)

Remark 5.3/5.4: Eqs. 5.3.2 and 5.5.2 are *not* tautologous, hence refuting the prenex normal formal of **(assoc2')** and corresponding Skolem normal form.

Eqs. 1.1.2, 1.4.2, and 5.3.2 as trivial are expected as tautologous. The other 15 equations are *not* tautologous..We evaluate the computer proof of a result in the abstract theory of segments. The refutes the approach of using the computer programming language Prolog for such mappings.

Refutation of an adaptive algorithm for molecular simulation on quantum computers

Abstract: We evaluate "a unitary variant of coupled cluster theory (UCCSD) [as] defined by replacing the excitation operators with an anti-Hermitian sum of excitation and de-excitation operators". It is *not* tautologous, hence refuting the subsequent sections for the ADAPT-VQE algorithm, molecular dissociation simulation results, and dependence of convergence on operator ordering. The conjecture of the paper is refuted from its abstract as: "an arbitrarily accurate variational algorithm that instead of fixing an ansatz upfront, … grows it systematically one operator at a time in a way dictated by the molecule being simulated … [to] highlight the potential of our adaptive algorithm for exact simulations with present-day and near-term quantum hardware." These equations form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Grimsley, H.R.; Economou, S.E.; Barnes, E.; Mayhall, N.J. (2019). An adaptive variational algorithm for exact molecular simulations on a quantum computer. arxiv.org/pdf/1812.11173.pdf

II. Results
A. Specification of the adopted notation

… In this context, a unitary variant of coupled cluster theory (UCCSD) was defined by replacing the excitation operators with an anti-Hermitian sum of excitation and de-excitation operators:

$$\hat{t}_{ij}^{ab} \rightarrow \hat{t}_{ij}^{ab} - \hat{t}_{ab}^{ij} = \hat{\tau}_{ij}^{ab}. \quad (4)$$

We write the snippet (4) above as (4.1) below:

tˆab_ij → tˆab_ij − tˆ ij_ab = ˆτ ab_ij . (4.1)

LET p, q, r, s: tˆ, ˆτ, ab_ij, ij_ab.

(p&r)>(((p&r)-(p&s))=(q&r)) ;
`TTTT TTT`**`F`** `TTTT TTT`**`F`** (4.2)

Remark 4.2: Eq. 4.2 as rendered is *not* tautologous, hence refuting the subsequent sections for the ADAPT-VQE algorithm, molecular dissociation simulation results, and dependence of convergence on operator ordering.

The conjecture of the paper is refuted from its abstract as: "an arbitrarily accurate variational algorithm that instead of fixing an ansatz upfront, … grows it systematically one operator at a time in a way dictated by the molecule being simulated … [to] highlight the potential of our adaptive algorithm for exact simulations with present-day and near-term quantum hardware."

Refutation of AGM postulates and Levi and Harper bridging principles

Abstract: We evaluate the AGM logic system in eight postulates and two bridging principles. The postulates named success, inclusion, vacuity, and inconsistency and the Levi and Harper bridging principles are *not* tautologous, hence refuting the AGM logic system.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET: ~ Not, ¬ ; + Or, ∨ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →; < Not Imply, less than, ∈;
= Equivalent, ≡; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊; # necessity, for every or all, ∀, □;
~(y < x) (x ≤ y), (x ⊆ y); (p=p) Tautology.

From: en.wikipedia.org/wiki/Belief_revision; philarchive.org/archive/LINEDD cite

The AGM postulates (named after the names of their proponents, Alchourrón, Gärdenfors, and Makinson) are properties that an operator that performs revision should satisfy in order for that operator to be considered rational. The considered setting is that of revision, that is, different pieces of information referring to the same situation. Three operations are considered: expansion (addition of a belief without a consistency check), revision (addition of a belief while maintaining consistency), and contraction (removal of a belief).

The first six postulates are called "the basic AGM postulates". In the settings considered by Alchourrón, Gärdenfors, and Makinson, the current set of beliefs is represented by a deductively closed set of logical formulae K called belief base, the new piece of information is a logical formula P, and revision is performed by a binary operator ∗ that takes as its operands the current beliefs and the new information and produces as a result a belief base representing the result of the revision. The + operator denoted expansion: K+P is the deductive closure of K∪{P}. The AGM postulates for revision are:

LET: p, q, r, s: P, Q, K, consistent

Closure: K∗P is a belief base (i.e., a deductively closed set of formulae) (1.1)

(r&p)>(r+p) ; TTTT TTTT TTTT TTTT (1.2)

Success: P∈ K∗P (2.1)

p<(r&p) ; **F**T**F**T **FFFF** **F**T**F**T **FFFF** (2.2)

Inclusion: K∗P ⊆ K+P (3.1)

~((r+p)<(r&p)) = (p=p) ; T**FTF** **F**T**F**T T**FTF** **F**T**F**T (3.2)

Vacuity: If (¬ P) ∉ K , then K∗P =K+P (4.1)

~(~p<r)>((r&p=(r+p)) ; T**F**T**F** **F**T**F**T T**F**T**F** **F**T**F**T (4.2)

Inconsistence: K∗P is inconsistent only if P is inconsistent or K is inconsistent (5.1)

((p>~s)+(q>~s))>((r&p)>~s) ;TTTT TTTT TTTT T**F**TT (5.2)

Extensionality: If P and Q are logically equivalent, then K∗P = K∗Q (6.1)

(p=q)>((r&p)=(r&q)) ; TTTT TTTT TTTT TTTT (6.2)

K∗(P∧Q) ⊆ (K∗P)+Q (7.1)

~(((r&p)+q)<(r&(p&q))) = (p=p) ;
TT**FF** T**FF**T TT**FF** T**FF**T (7.2)

If (¬ Q)∉ K∗P then (K∗P) + Q ⊆ K∗(P∧Q) (8.1)

(~q<(r&p))>~((r&(p&q))<((r&p)+q)) ;
TTTT TTTT TTTT TTTT (8.2)

From: philarchive.org/archive/LINEDD

AGM also contains ... the following bridging principles:

LET: p, q: G, α

Levi identity: (G∗α)=(G–¬α)+α (10.1)

(p&q)=((p-~q)+q) ; TT**F**T TT**F**T TT**F**T TT**F**T (10.2)

The Levi identity says that the result of revising the belief set G by the sentence α equals the result of first making room for α by (if necessary) contracting G with ¬α and then expanding the result with α.

Harper identity: (G–α)=(G∗α)∩(G∗¬α) (11.1)

(p-q)=((p&q)&(p&~q)) ; **F**TTT **F**TTT **F**TTT **F**TTT (11.2)

The Harper identity says that the result of contracting α from G is the common part of G revised with α and G revised with ¬α.

Eqs. for postulates named success, inclusion, vacuity, and inconsistency and the Levi and Harper bridging principles are *not* tautologous, hence refuting the AGM logic system.

Refutation of remainder sets for paraconsistent revisions

Abstract: Two definitions for expansion, remainder, and selection of K functions are *not* tautologous. Two definitions implication and paraconsistent/weak negation operators are *not* tautologous. These refute remainder sets and paraconsistent valuations of logic **mbC**, an extension of **CPL+**. Therefore these conjectures are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1;
(%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Resta, R.; Fermé, E.; Garapa, M.; Reis M. (2018).
How to construct remainder sets for paraconsistent revisions: preliminary report.
no emails proffered.
academia.edu/attachments/58978670/download_file?st=MTU1NjM2NDU2OCw3NS43MS4xNjEuMTQ2LDc2MDk1MzU4&s=swp-toolbar&ct=MTU1NjM2NDU2OSwxNTU2MzY0NTg2LDc2MDk1MzU4

Remark 0: The AGM model of belief systems is named for Alchourrón, Gärdenfors, and Makinson (1985). Axioms are supposed to be a subset of classical logic CPL+.

Formally we have the following:

Definition 1. *The expansion of K by α (K + α) is given by K + α = Cn*(*K*∪{*α*}) (D1.1)

LET p, q, r, s: *α* , *C*, *K*, *n*.

(r+p)=((q&s)&(r&p)) ; `T`**`F`**`T`**`F`** **`FFFF`** `T`**`F`**`T`**`F`** **`FFF`**`T` (D1.2)

Definition 2 (Remainder)**.** *The set of all the maximal subsets of K that do not entail α is called the* remainder set *of K by α and is denoted by K⊥α, that is, K′ ∈ K⊥α iff:*

(i) *K′ ⊆ K.*
(ii) *α ∉ Cn*(*K′*).
(iii) *If K′ ⊂ K″ ⊆ K then α ∈ Cn*(*K″*). (D2.1)

LET p, q, r, s, t, u: α ,C, K, n, K' , K'' .

((~(u<t)&~(p<((q&r)&t)))&(~(r<(t<u))>(p<((q&s)&u))))>(t<(r@p)) ;

TTTT T**F**T**F** TTTT T**F**T**F**(1), TTTT TTTT TTTT TTTT(2),
TTTT T**F**T**F** TTTT T**F**T**F**(2), TTTT TTTT TTTT TTTT(2),
TTTT T**F**T**F** TTTT T**F**T**F**(1) (D2.2)

Definition 3 (selection function)**.** *A selection function for K is a function γ such that, for every α:*

1. $\gamma(K\perp\alpha) \subseteq K\perp\alpha$ *if* $K\perp \neq \emptyset$.
2. $\gamma(K\perp\alpha) = \{K\}$ *otherwise*. (D3.1)

LET p, q, r: α, γ, K.

(((r@#p)@(s&s))>~((r@#p)<(q&(r@#p))))+((q&(r@#p))=r) ;
TTTT **F**NTT TTTT TTTT (D3.2)

Definition 13 (Valuations for **mbC** (Carnielli and Coniglio 2016))**.** *A function* $v : \mathbb{L} \rightarrow \{0,1\}$ *is a* valuation for **mbC** *if it satisfies the following clauses*:

($v \rightarrow$) $v(\alpha \rightarrow \beta) = 1 \Leftrightarrow v(\alpha) = 0$ *or* $v(\beta) = 1$ (Implication) (D13.3.1)

LET p, q, r: α , β , v .

((r&(p>q))=(s=s))=(((r&p)=(s@s))+((r&q)=(s=s))) ;
FFFF TTTT **FFFF** TTTT (D13.3.2)

($v\neg$) $v(\neg\alpha) = 0 \Rightarrow v(\alpha) = 1$ (Paraconsistent/Weak negation) (D13.4.1)

((r&~p)=(s@s))>((r&p)=(s=s)) ; **FFFF** TTTT **FFFF** TTTT (D13.4.2)

Remark 13: Defs. 13.3.2 and 13.4.2 are equivalent.

The Defs. D1.2-3.2 as rendered are *not* tautologous for expansion, remainder, and selection of K functions. The Defs. 13.3.2-13.4.2 are *not* tautologous for implication and paraconsistent/weak negation operators. These refute remainder sets and paraconsistent valuations of logic **mbC**, an extension of **CPL+**.

Refutation of logically-consistent hypothesis testing and the hexagon of oppositions

Abstract: Definitions for □H and ¬ ◊H are supposed to be equivalent for a classical mapping of agnostic hypothesis tests. While each definition reduces to a theorem in the conjecture, they are *not* tautologous. This refutes that agnostic hypothesis tests are proved to be logically consistent. Hence the characterization of credal modalities in agnostic hypothesis tests cannot be mapped to the hexagon of oppositions to explain the logical relations between these modalities. Therefore the 11 definitions tested form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Esteves, L.G.; Izbicki, R.; Stern, J.M.; Stern, R.B. (2019).
Logically-consistent hypothesis testing and the hexagon of oppositions.
arxiv.org/pdf/1905.07662.pdf rbstern@gmail.com

Abstract: Although logical consistency is desirable in scientific research, standard statistical hypothesis tests are typically logically inconsistent. In order to address this issue, previous work introduced agnostic hypothesis tests and proved that they can be logically consistent while retaining statistical optimality properties. This paper characterizes the credal modalities in agnostic hypothesis tests and uses the hexagon of oppositions to explain the logical relations between these modalities.

Table 1: Modalities of agnostic hypothesis tests

Remark 1: We evaluate Tab. 1 beginning with Eq. 3.1 because it is the only atomic definition without the delta or nabla injections.

LET p, H; delta Δ; nabla ∇.

Modality	**Name**	**Equivalence**	**Interpretation**	
□H	Necessity (A)	ΔH∧◊H	H is accepted.	(1.1)

(#p+~(%p&~#p))&%p ; **F**N**F**N **F**N**F**N **F**N**F**N **F**N**F**N (1.2)

#p=((#p+~(%p&~#p))&%p) ; TTTT TTTT TTTT TTTT (1.3)

¬ ◊H Impossibility (E) ΔH∧¬ □H H is rejected. (2.1)

(#p+~(%p&~#p))&~#p) ; N**F**N**F** N**F**N**F** N**F**N**F** N**F**N**F** (2.2)
~%p=((#p+~(%p&~#p))&~#p) ; TTTT TTTT TTTT TTTT (2.3)

∇H Contingency (Y) ◊H∧¬ □H H is not decided. (3.1)

∇H: %p&~#p ; CCCC CCCC CCCC CCCC (3.2)

◊H Possibility (I) □H∨∇H H is not rejected. (4.1)

#p&(%p&~#p) ; **FFFF FFFF FFFF FFFF** (4.2)
%p=(#p&(%p&~#p)) ; N**F**N**F** N**F**N**F** N**F**N**F** N**F**N**F** (4.3)

¬ □H Non-necessity (O) ¬ ◊H∨∇H H is not accepted. (5.1)

~%p&(%p&~#p) ; **FFFF FFFF FFFF FFFF** (5.2)
~#p=(~%p&(%p&~#p)) ; **F**N**F**N **F**N**F**N **F**N**F**N **F**N**F**N (5.3)

ΔH Non-contingency (U) □H∨¬ ∇H H is decided. (6.1)

ΔH: #p+~(%p&~#p) ; NNNN NNNN NNNN NNNN (6.2)

Remark 1-2: Eqs. 1.3 and 2.3 as rendered result in theorems, so we test the modalities as equivalences: □H=¬ ◊H. (7.1)

#p=~%p ; CCCC CCCC CCCC CCCC (7.2)

Eq. 7.2 is *not* tautologous. This refutes that agnostic hypothesis tests are proved as logically consistent. Therefore the characterization of credal modalities in agnostic hypothesis tests cannot be mapped to the hexagon of oppositions to explain the logical relations between these modalities.

Proof of agnosticism as a subset of atheism because both lead to non-belief

Abstract: We define belief as trust in the unseen to evaluate the belief relationship of agnosticism and atheism. Atheism asserts there is evidence not to believe God exists. Agnosticism asserts that there is no evidence neither to believe nor not to believe God exists. We simplify these definitions by removing God from the mix as the object of belief. The conjectures to test are: Does both atheism and agnosticism imply or lead to non-belief; and Does both atheism and agnosticism imply agnosticism is a subset of atheism. We prove these as theorems. The contra-arguments are found to be *not* tautologous.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET q, r: belief, evidence (knowledge)
~ Not; + Or; - Not Or; > Imply, greater than; = Equivalent.

We define belief as trust in the unseen to evaluate the belief relationship of agnosticism and atheism.

Atheism asserts there is evidence *not* to believe God exists. (1.0)

Agnosticism asserts that there is no evidence neither to believe nor not to believe God exists. (2.0)

Remark: The two definitions of Eqs. 1.0 and 2.0 are simplified by removing God from the mix as the object of belief.

Atheism asserts there is evidence *not* to believe. (1.1)

r>~q ; TTTT TT**FF** TTTT TT**FF** (1.2)

Agnosticism asserts that there is no evidence neither to believe nor not to believe. (2.1)

~r>(q-~q) ; **FFFF** TTTT **FFFF** TTTT (2.2)

The conjecture to test is if atheism and agnosticism *both* imply or lead to *non-belief*. (3.0)

Eq. 3.0 is rewritten to use the if-then construct, that is, the implication operator.

If evidence, then no belief and if no evidence then neither belief nor no belief implies no belief (3.1)

((r>~q)&(~r>(q-~q)))>~q ; TTTT TTTT TTTT TTTT (3.2)

Remark: If evidence, then no belief and if no evidence then neither belief nor no belief implies belief. (4.1) ((r>~q)&(~r>(q-~q)))> q ;   TTTT **FF**TT TTTT **FF**TT   (4.2)

We ask, Does both atheism and agnosticism imply agnosticism is a subset of atheism. (5.1)

((r>~q)&(~r>(q-~q)))>((r>~q)>(~r>(q-~q))) ;
TTTT TTTT TTTT TTTT   (5.2)

Remark: Does both atheism and agnosticism imply agnosticism is *not* a subset of atheism. (6.1)

(((r>~q)&(~r>(q-~q)))>((r>~q)<(~r>(q-~q))) ;
TTTT **FF**TT TTTT **FF**TT   (6.2)

Eqs. 3.2 and 5.2 as rendered are tautologous, and the respective contra Eqs. 4.2 and 6.2 are *not* tautologous.

Hence the two theorems in Eqs. 3.1 and 5.2 can be restated to mean:

Both atheism and agnosticism imply *no* belief.   (3.1)

Both atheism and agnosticism imply agnosticism is a *subset* of atheism.   (5.1)

Refutation of three phase, all reduce algorithm across processing units for scalable deep learning

Abstract: A three-phase algorithm to do an all-reduce across all GPUs is not tautologous and refuted.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

LET p, q, r, s; ~ Not; & And; > Imply, greater than.

From: Jia, X.; Song, S.; Shi, S.; et al. (2018). Highly scalable deep learning training system with mixed-precision: training imagenet in four minutes. arXiv:1807.11205
jiaxianyan123@126.com, csshshi@comp.hkbu.edu.hk

[G]roup *k* GPUs together, then use a three-phase algorithm to do the all-reduce across all GPUs ... Figure 5: ...

1. reduce within GPUs of the same group, (1.1)
2. store the partial results to a master GPU in each group, then ... (2.1)
3. launch Ring all-reduce across *p*/*k* groups: after each master GPU gets the final result, propagate the final result [back] to every GPU. (3.1)

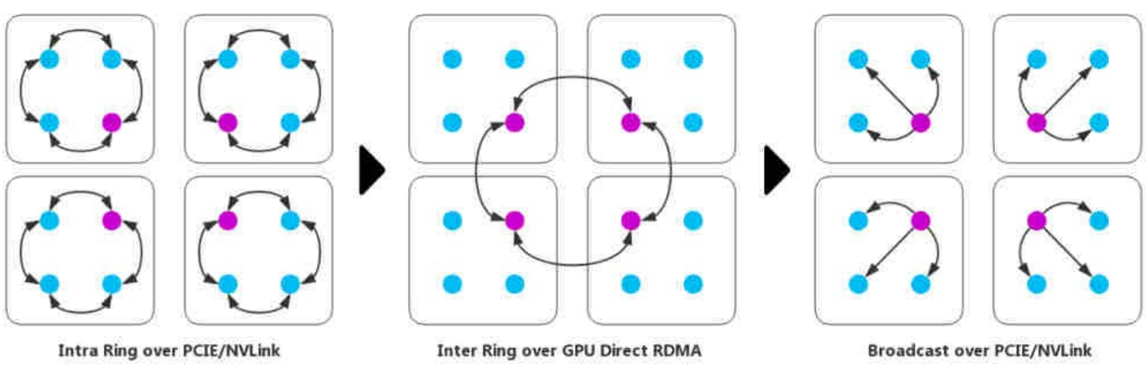

Fig 5: Three phase, all reduce algorithm for GPU aggregation.

We ignore the intra ring of phase one as trivial, and assign logic values to the four inter rings as row-major to map the data flow in both directions.

((s>r)>(p>q))&((s>q)>(p>r)) ; T**F**T**F** T**F**TT TTT**F** T**F**TT (2.2)

We map the discrete broadcast phase as:

(((s>p)&((s>q)&(s>r)))&((r>p)&((r>q)&(r>s))))&
(((q>r)&((q>s)&(q>p)))&((p>q)&((p>r)&(p>s)))) ;
T**FFF** **FFFF** **FFFF** **FFF**T (3.2)

Remark: Eq. 2.1 contains the "then" word as a connective meaning the implication operator applies to Eqs. 2.1 as implying 3.1. In other words, if Eq. 2.1, then Eq. 3.1. (4.1)

(((s>r)>(p>q))&((s>q)>(p>r))) >
((((s>p)&((s>q)&(s>r)))&((r>p)&((r>q)&(r>s))))&
(((q>r)&((q>s)&(q>p)))&((p>q)&((p>r)&(p>s))))) ;
TT**F**T **F**T**FF** **FFF**T **F**T**F**T (4.2)

Eqs. 2.2, 3.2, and 4.2 as rendered are *not* tautologous. This means the three-phase algorithm to do the all-reduce across all GPUs is refuted.

Refutation of the AI experiment for a divide the dollar competition

Abstract: We evaluate the AI experiment for a divide the dollar competition at the June CEC2019 IEEE conference in New Zealand. The apparatus definition is *not* tautologous, hence refuting the experiment.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r, s, u, v: x, y, R, s, u (contestant 1), v (contestant 2);
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⊢; < Not Imply, less than, ∈;
= Equivalent, ≡, ⊨; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(s=s) T as tautology; s@s) **F** as contradiction;
(%s<#s) C non-contingency, ∇, ordinal 2; (%s>#s) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y).

From: Ashlock, D.; Greenwood, G. (2019). Divide-the-dollar competition. CEC-C1. cec2019.org/programs/competitions.html#cec-12. dashlock@uoguelph.ca

The conventional divide-the-dollar game is a two player game where the players simultaneously bid on how to divide a dollar. If the bids sum to a dollar or less each player receives their bid, otherwise they receive nothing. ... In this game, instead of dividing a dollar, a scoring set, S ⊂ RN is used. Each player bids a point coordinate and, if the resulting point is in the scoring set, then the players receive their bid, otherwise nothing. ... The contest will use sets not seen by the players before and will be restricted to the two-player version. All sets satisfy x,y ∈ R2 with x ≥ 0, y ≥ 0, and x,y ≤ 2. ... Winners will be determined for each problem test set ... (1.1)

[101 step formula inserted herewith after winner was announced, 6/13/2019]

(((((~((s@s)>p)&~((s@s)>q))&(~((%s<#s)>p)&~((%s<#s)>q)))>
((p<(r&(%s<#s)))&(q<(r&(%s<#s)))))>#s)>
((~((%s>#s)>((u>(x&y))+(v>(x&y))))>((u=(u+(%s>#s)))&(v=(v+(%s>#s)))))+
(((%s>#s)>((u>(x&y))+(v>(x&y))))>((u=u)&(v=v)))))>(u@v) ;
TTTT TTTT TTTT TTTT(8), **FFFF FFFF FFFF FFFF(8)** (1.2)

Eq. 1.2 as rendered is *not* tautologous, thereby refuting the soundness of the experiment.

Refutation of Ackermann's approach for modal logic and second-order quantification reduction

Abstract: From one source we evaluate the Ackermann rule and from another source three examples in 15 equations of second-order reduction. None of the equations is tautologous. This implies these approaches to map modal clauses of first-order logic to and from second-order logic are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Conradie, W.; Goranko, V.; Vakarelov, D. (2006). arxiv.org/pdf/cs/0602024.pdf
Algorithmic correspondence and completeness in modal logic: I. The core algorithm SQEMA.
wec@rau.ac.za, goranko@maths.wits.ac.za, dvak@fmi.uni-sofia.bg

Lemma 0.1 (Ackermann's Lemma). Let P be a predicate variable and A(x,z) and B(P) be first-order formulae such that there are no occurrences of P in A(x,z). If P occurs only negatively in B(P) then

∃P (∀x[A(x,z) → P(x)] ∧ B(P)) ≡ B(A(t,z)/P(t)) (0.1.1.1)

LET p, q, r, t, x, y, z: p, A, B, t, x, y, z

(p<(q&(x&z)))>((#(r&p)<(p@p))>((((q&(#x&z))>(%p&#x))&(r&p))=(r&((q&(t&z))\(p&t))))) ;

```
TTTT TTTT TTTT TTTT(64),
TTTT TTTT TTTT TTTT,TTTT TTTC TTTT TTTC,(16)
TTTT TTTT TTTT TTTT(16),
TTTT TTTT TTTT TTTT,TTTT TTTC TTTT TTTC,(16)
TTTT TTTT TTTT TTTT(16)
```
(0.1.1.2)

and, respectively, if P occurs only positively in B(P), then

∃P (∀x[P(x) → A(x,z)] ∧ B(P)) ≡ B(A(t,z)/P(t)) (0.1.2.1)

(p<(q&(x&z)))>((#(r&p)>(p@p))>(((%p&#x)>(q&(r&p)))=(r&((q&(t&z))\(p&t))))) ;

```
TFTF TTTT TFTF TTTT(16),TNTT TTTT TNTT TTTT(16)
```

(0.1.2.2)

where z are parameters, and each occurrence of P(t) in B on the right hand side of the equivalences, for terms t, is substituted by A(t,z).

Remark 0.1: Combining the antecedents of Eqs. 0.1.1.2 and 0.1.2.2 produces the Ackermann rule.

(p<(q&(x&z)))>(((#(r&p)<(p@p))>((((q&(#x&z))>(%p&#x))&(r&p))=(r&((q&(t&z))
(p&t))))) &((#(r&p)>(p@p))>(((%p&#x)>(q&(r&p)))=(r&((q&(t&z))\(p&t)))))) ;

```
TFTF TTTT TFTF TTTT(16),TNTT TTTT TNTT TTTT(16),
TFTF TTTT TFTF TTTT(16),TNTT TTTT TNTT TTTT(16),
TFTF TTTT TFTF TTTT,TFTF TTTF TFTF TTTF(16),
TNTT TTTT TNTT TTTT(16),
TFTF TTTT TFTF TTTT,TFTF TTTF TFTF TTTF(16),
TNTT TTTT TNTT TTTT(16)
```
(0.2)

The Ackermann rule in Eq. 0.2 is *not* tautologous, hence refuting it.

From: Schmidt, R.A. (2012).
The Ackermann approach for modal logic, correspondence theory and second-order reduction.
Journal of Applied Logic 10 (2012) 52–74. renate.schmidt@manchester.ac.uk

Example 1. Let us see if we can derive the seriality property, ∀x∃y[R(x, y)], (1.0.1)

LET p, q, r: x, y, R

(r&(#p&%q)) = (p=p) ; **FFFF FFF**N **FFFF FFF**N (1.0.2)

for the modal axiom **D** = ∀p[□p → ◊p]. (1.1.1)

##p>%p ; TTTT TTTT TTTT TTTT (1.1.2)

Its negation is: ¬**D** = ∃p[□p ∧ □¬p]. (1.2.1)

~(##p>%#p)=(#%p&#~%p) ; TTTT TTTT TTTT TTTT (1.2.2)

The input is the set containing

1. ¬a ∨ (□p ∧ □¬p) (1.1)

~q+(#p+#~p) ; TNTN TNTN TNTN TNTN (1.2)

and the goal is to eliminate p, that is, Σ = {p}. Rewriting with respect to the ∧ elimination replacement rule gives us:

2. ¬a ∨ ¬(¬□p ∨ ¬□¬p) 1, repl. (elim. ∧) (2.1)

~q+~(~#p+~#~p) ; T**F**T**F** T**F**T**F** T**F**T**F** T**F**T**F** (2.2)

and we cross out clause 1. Using the distributivity replacement rule we replace clause 2 by clause 3.

3. ¬¬(¬a ∨ □p) ∨ ¬(¬a ∨ □¬p)   2, repl. (distr.)   (3.1)

~(~(~q+#p)+~(~q+#~p)) = (p=p) ;   TT**FF** TT**FF** TT**FF** TT**FF**   (3.2)

Applying the clausify rule we obtain

4. ¬a ∨ □p   3, repl., cl.   (4.1)

~q+#p ;   TT**F**N TT**F**N TT**F**N TT**F**N   (4.2)

5. ¬a ∨ □¬p   3, repl., cl.   (5.1)

~q+#~p ;   TTN**F** TTN**F** TTN**F** TTN**F**   (5.2)

and delete clause 3. p occurs only positively in clause 4 but is shielded by a box operator. Applying the surfacing rule to 4 we obtain

6. $□^u$¬a ∨ p   4, surf.   (6.1)

#~q+p ;   NT**F**T NT**F**T NT**F**T NT**F**T   (6.2)

The positive occurrence of p is now unshielded and we can resolve 6 into 5 by applying the Ackermann rule. This replaces clauses 5 and 6 by 7.

7. ¬a ∨ □$□^u$¬a   6 into 5, Acker.   (7.1)

~q+##~q ;   TT**FF** TT**FF** TT**FF** TT**FF**   (7.2)

Since it does not contain the non-base symbol p, we could stop at this point. However clause 7 can be simplified by using the rewrite rule $\alpha \vee □^{\sigma} □^{\sigma,u}\alpha \Rightarrow \alpha \vee □^{\sigma} \neg \top$ from Table 3.

8. ¬a ∨ □⊤   7, repl.   (8.1)

~q+#(#(p@p)) ;   TT**FF** TT**FF** TT**FF** TT**FF**   (8.2)

The procedure returns {8}. Translating 8 into first-order logic we get:

$\forall x\pi(\neg a \vee □\neg\top, x) \equiv \pi(□\neg\top, a) = \forall x\neg R(a, x)$ (9.1)

LET p, q, R, s: pi, alpha, R, x

((p&((~q+#(p@p))&#s))=(p&(#(p@p)&q)))=~(r&(p&#s)) ;
TTTT TTTT TCTT TTTC   (9.2)

Unskolemization returns $\exists y\forall x[\neg R(y, x)]$.   (10.1)

LET p, q, r, s: x, y, R, z

~(r&(%q&#p)) = (p=p) ; TTTT TTTC TTTT TTTC (10.2)

Finally negating gives the expected result: $\forall y \exists x[R(y, x)]$. (11.1)

r&(#q&%p) ; **FFFF FFF**N **FFFF FFF**N (11.2)

Example 3. The modal axiom $\forall p \forall q[\Box(\Box p \equiv q) \rightarrow \Diamond\Box\neg p]$ (3.1.1)

#(##p=#q)>%#~#p ; TTTC TTTC TTTC TTTC (3.1.2)

corresponds to $\forall x \exists y \forall z[R(x, y) \wedge \neg R(y, z)]$, (3.2.1)

(r&(#p&#q))&~(r&(#q&#s)) ; **FFFF FFF**N **FFFF FFFF** (3.2.2)

in words, every world has a successor that is a dead-end.

Example 9. The following is the rule version of the axiom from Example 7.

$\forall p \forall q[\Box(p \vee q)/(\Box p \vee \Box q)$. (9.1.1)

#(#p+#q)\(##p+##q) ; TCCC TCCC TCCC TCCC (9.1.2)

We show that it [Eq. 9.1.1] is equivalent to $\forall p[\Diamond p \rightarrow \Box p]$. (9.2.1)

(#(#p+#q)\(##p+##q))=(%#p>##p) ; TCCC TCCC TCCC TCCC (9.2.2)

Excepting the expected modal axiom(s) for D as rendered, the 15 example equations are *not* tautologous. This means the Ackermann approach for modal logic, correspondence theory, and second-order reduction is refuted.

Recent advances in AA: factual mistake in We agnostics, p 53, invalidates the traditions

... the proposition that either God is everything or else He is nothing.

The unattributed source of this quotation is Emmet Fox, whose personal secretary was associated with Bill Wilson, meaning Bill was promoting his family religion. Fox, despite claims, was not a Christian but a dishonest Gnostic. The problem with the quotation on its face is the use of the existential quantifier (every, as in everything) and the negation of the universal quantifier (not all, as in nothing).

The Meth8 modal logic checker maps the quotation as follows.

LET: p God; q thing(s); ~ not; + Or; = equivalence;
% possibility (for at least one instance); # necessarily (for all instances)

"God is everything" (antecedent)

This is rewritten from "God is possibly a thing" p = %q (1)
to "God is all possible things" p = #%q (2)

"God is nothing" (consequent)

This is rewritten from "God is all things" p = #q (3)
to its negation as "God is not all things" p = ~#q (4)

The assertion is that antecedent Or consequent is tautologous. Hence the logical connective is Or, and the expression used for Tautologous is "God is God" p = p (5)

We rewrite the quotation as:

Either "God is all possible things" or "God is not all things" is equivalent to Tautologous.
Such truth is supposed to be a self-evident truth, an axiom.
By substitution of Eq. 2, 4, 5:

((p = #%q) + (p = ~#q)) = (p = p) ;
`NTNT EEEE UEUE IEIE PEPE` (6)

Meth8 evaluates Eq 6 as *not* tautologous where designated truth values are `T` and `E` and mean by first letter Non-contingent, Tautologous, Evaluated, Unevaluated, Improper, Proper.

This means the quotation is factually mistaken as proved by mathematical logic.

What follows is that the quotation is seriously misleading in this way. Many AA's invoke a description of God as "God is everything or God is nothing" to mean God can be both good and evil at the same time because both good and evil are ostensibly things. This is dangerous because to assert God is evil means God can tell a lie. However that is contradictory from the counter example that God is capable to do anything except for one thing: God cannot tell a lie. (The quality of God of absolute truthfulness was proved by Karl Popper, *Conjecture and Refutation*, 1972 ed, over 45 years ago.)

What follows is Tradition 2 (*one ultimate authority ... God ... in our group conscience)* is mistaken by assuming it is necessarily God's will, and thus the traditions themselves do not self-validate as claimed.

As an alternative refutation, we present the following.

"the proposition that either God is everything or else God is nothing." (AA BB, pg 53)

LET: p thing; ~p not thing (no thing); q God; # all or every; % one or some;
~ Not; + Or; = Equivalent to; > Imply, is, greater than (rendered as If, then).
T tautology; F contradiction; N truthity (non contingency); C falsity (contingency).
The result table of 16-values is row-major and presented horizontally to save space.
T is the designated truth value, meaning a proof has to have all T's in the result table.

God is equivalent to thing. (1.1)

q=p ; T**FF**T T**FF**T T**FF**T T**FF**T (1.2)

God is equivalent to a thing (some things). (2.1)

q=%p ; N**F**CT N**F**CT N**F**CT N**F**CT (2.2)

God is equivalent to every thing (all things) (3.1)

q=#p ; TC**F**N TC**F**N TC**F**N TC**F**N (3.2)

Eqs. 1.2, 2.2, and 3.2 as rendered are *not* tautologous (not proved as all TTTT's).

To weaken the argument in hopes of finding a proof, one replaces the connective Equivalent with the connective Imply. Eq. 3.1 becomes:

God implies every thing (all things). (4.1)

q>#p ; TT**F**N TT**F**N TT**F**N TT**F**N (4.2)

Eq. 3.2 as modified in 4.2 is still *not* tautologous.

The point is that God does *not* imply all things, or more strongly, God is *not* all things.

Refutation of AA aphorism that serenity is inversely proportional to expectation

Abstract: From the AA Big Book, Acceptance story, the conjecture that acceptance is inversely proportional to expectation is *not* tautologous, hence refuting it. The conjecture forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: AA Big Book. Third, Fourth Eds. Acceptance story.

The aphorism is that acceptance is inversely proportional to expectation. (1.0)

Remark 1.0: We rewrite Eq. 1.0 as (acceptance) = (tautology as T) / (expectation). (1.1)

LET p, q: acceptance , expectation.

p=((q=q)\q) ; **F**TNC **F**TNC **F**TNC **F**TNC (1.2)

Eq. 1.2 as rendered is *not* tautologous, hence refuting the aphorism that acceptance is inversely proportional to expectation.

Refutation of AA paradoxes

Abstract: Two AA paradoxes are *not* tautologous and *not* contradictory, but some intermediate truth table value state. Therefore these conjectures are a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

Seldom keep what do not give away. (1.0)

We rewrite Eq. 1.0 as: If one imparts sobriety, then one receives sobriety. (1.1.1)

LET p, s: one (person), sobriety; > Imply, gives to, imparts;
< Not Imply, receives surrender to win; seldom keep what do not give away

(p>s)>(p<s) ; FTFF FTFF FFFF FFFF (1.1.2)

If one imparts sobriety, then sobriety gives to one. (1.2.1)

(p>s)>(s>p) ; TTTT TTTT FTFT FTFT (1.2.2)

If sobriety gives to one, then one imparts sobriety. (1.3.1)

(s>p)>(p>s) ; TFTF TFTF TTTT TTTT (1.3.2)

Surrender to win. (2.0)

We rewrite Eq. 2.0 as: If one surrenders, then one wins. (2.1.1)

LET p, q: one, win

~(p>q)>(p>q) ; TFTT TFTT TFTT TFTT (2.1.2)

Two AA paradoxes are *not* tautologous and *not* contradictory, but some intermediate truth table valued state.

Topological semantics for conditionals and the Alexandroff correspondence

From: Marti, J.; Pinosio, R. Topological Semantics for Conditionals. Logica. January, 2013. researchgate.net/publication/29945557

We evaluate three theorems using the Meth8 apparatus.

LET: > -v->; @ Not equivalent to; # necessity, universal quantifier ; % possibility, existential quantifier;
p lc_phi; q lc_psi; (p=p) uc_Tau (for tautology); (p@p) inverted uc_Tau (for contradiction)

The designated proof value is T (tautology) as opposed to F (contradiction). N means non-contingent (truth value) as opposed to C contingent (falsity value).

Repeating fragments are from the 16-value truth table.

On page 11, Theorem 9 has three equivalences to hold in coherent neighborhood spaces. These equations are transcribed due not non-rendering as such:

#p=(~p>(p@p))	; TNTN	(9.1)
#p = ((p=p)>p)	; TNTN	(9.2)
(p>q) = ((%p>#(p>q)) & (~%p>#(p>%(p&#(p>q)))))	; NTNN	(9.3)

This tells us that topological conditionals are not bivalent. By extension the Alexandroff correspondence is also not bivalent. Previously, that fact was independently implied by Meth8 showing the Gödel-Löb theorem was not a tautology, and hence the axiom of choice was also not a tautology. Therefore the Alexandroff correspondence which relies on the axiom of choice is also not a tautology.

Refutation of shared variables in cross axiom models of alternating Turing machines

Abstract: We evaluate shared variables for the reduction of alternating Turing machines (ATMs) to subset space logic (SSL). The purpose of shared variables is to use binary counters for mapping cross-axiom models. None is tautologous, to refute cross-axiom models in the completeness conjectures. These form *non* tautologous fragment of the universal logic VL4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Hertling, P.; Krommes, G. (2019). EXPSPACE-completeness of the logics K4×S5 and S4×S5 and the logic of subset spaces, part 2: EXPSPACE-hardness. arxiv.org/pdf/1908.03509.pdf

1 Introduction In this article we are concerned with the complexity of the bimodal product logics K4×S5 and S4×S5 and with the subset space logic SSL, a bimodal logic as well. To the best of our knowledge, the complexity of K4×S5, of S4×S5, and of SSL were open problems.

3 Preparations for the reduction of alternating Turing machines to SSL

3.1 Shared variables We have to make sure that various kinds of information are stored in a suitable way in any model of the fo[r]mula.

Definition 3.1 (Shared Variables). For i ∈ N let A_i be special propositional variables, and let B be another special propositional variable B, different from all A_i . The shared variables α_i are defined as follows:

α_i := L(A_i∧LB). Note that ¬α_i ≡ K(¬A_i∨◊K¬B). (3.1.1.1)

LET p, q, r, s: A_i, B, K, L.

~(s&(p&(#s&q)))=(r&(~p&(%r&~q))) ;
FFFF T**FFF** **FFF**N T**FF**N (3.1.1.2)

A.1 Binary Counters in S4×S5 The shared variables α_i are defined as

α_i := LA_i . Note that ¬α_i ≡ K¬A_i . (A.1.2.1)

~(s&p)=(q&~p) ; **FF**T**F** **FF**T**F** **F**TTT **F**TTT (A.1.2.2)

Remark 4.0: Eqs. 3.1.1.2 and A.1.2.2 as *not* tautologous or equivalent are both taken as defining shared variables. We write the combined definition of shared variables as 3.1.1.1 And A.1.2.1: (4.1)

(~(s&(p&(#s&q)))=(r&(~p&(%r&~q))))&(~(s&p)=(q&~p)) ;
FFFF FFFF FFFN **FFF**N (4.2)

Remark 4.2: The purpose of shared variables is to use binary counters for mapping cross-axiom models. Because Eq. 4.2 is *not* tautologous, that refutes cross-axiom models in the completeness conjectures.

Refutation of the paradox of the concept of an analysis as both correct and informative

We assume the method and apparatus of Meth8/VŁ4 where Tautology is the designated *proof* value, F is contradiction, N is truthity (non-contingency), and C is falsity (contingency). The 16-valued truth table fragment) is row-major and horizontal.

From: en.wikipedia.org/wiki/Paradox_of_analysis, as how analysis is to be correct and informative.

LET p, r, s: member, brother, male-sibling

"For all x (any given member of a class or set), x is a brother if and only if x is a male sibling." (1.1.1)

#p&((p=r)>(p=s)) ; FNFN FFFF FNFN FNFN (1.1.2)

"One can say that (1.1.1) is correct because the expression “brother” represents the same concept as the expression “male sibling”".

This is mistaken because Eq. 1.1.2 as rendered is *not* tautologous.

"and (1.1.1) seems to be informative because the two expressions are not identical."

The informative status of Eq. 1.1.2 is *not* a theorem.

If brother is equivalent to male sibling, then Eq. 1.1.1 is true. (1.2.1)

((r=s)>(#p&((p=r)>(p=s))))=(p=p) ; FNFN TTTT TTTT FNFN (1.2.2)

Eq. 1.2.2 is *not* tautologous.

If Eq. 1.1.1 is truly correct, then brother is equivalent to male sibling. (1.3.1)

((#p&((p=r)>(p=s)))=(p=p))>(r=s) ; TTTT TTTT TCTC TTTT (1.3.2)

Eq. 1.3.2 is *not* tautologous.

For all x, x is a brother if and only if x is a brother. (2.1)

#p&((p=r)>(p=r)) ; FNFN FNFN FNFN FNFN (2.2)

Eq. 2.2 is *not* tautologous.

"Yet it is obvious that (2.1) is not informative, so either (1.1) is not informative, or the two expressions used in (1.1) are not interchangeable (because they change an informative analysis into an uninformative one) so (1.1) is not actually correct. In other words, if the analysis is correct and informative, then (1.1) and (2.1) must be essentially equal, but this is not true because (2.1) is not informative. Therefore, it seems an analysis cannot be both correct and informative at the same time."

None of the above follows because Eq.1.1.1 is *not* tautologous.

Hence the concept of analysis is *not* a paradox, and analysis is potentially both correct and informative.

Refutation of analytic choice principles: axioms of choice and dependent choice

Abstract: The axiom of Γ choice and axiom of Σ^1_1-dependent choice are *not* tautologous. Therefore an open problem on the Weihrauch degree of parallelization of the Σ^1_1-choice principle on the integers is not solved by using those axioms. These axioms form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Anglès d'Auriac, P-E.; Kihara, T. (2019). A comparison of various analytic choice principles. arxiv.org/pdf/1907.02769.pdf

2. Equivalence results in the Weihrauch lattice
2.1. Σ^1_1-Choice Principles.
One of the main notions in this article is the Σ^1_1-choice principle. … In logic, the axiom of Γ choice, Γ-AC, is known to be the following statement [where ϕ is a Γ formula]:

$$\forall a \exists b\ \phi(a, b) \rightarrow \exists f \forall a\ \phi(a, f(a)) \tag{2.1.1.1}$$

LET p, q, r, s, t: a, b, f, ϕ, n.

(s&(#p&%q))>(s&(#p&(%r&#p))) ;
TTTT TTTT TTTC TTTT (2.1.1.2)

In logic, the axiom of Σ^1_1-dependent choice on X is the following statement [where ϕ is a Σ^1_1-formula, and a and b range over X]:

$$\forall a \exists b\ \phi(a, b) \longrightarrow \forall a \exists f\ [f(0) = a\ \&\ \forall n\ \phi(f(n), f(n + 1))] \tag{2.1.2.1}$$

(s&(#p&%q))>((r&(z@z))=(#p&(s&((%r&#t)&(%r&(#t+(%z>#z))))))) ;
TTTT TTTT TTTT TTTT(1)
TTTT TTTT TTTT TTTC(1) (2.1.2.2)

The axiom of Γ choice, Eq. 2.1.1.2 as rendered, and axiom of Σ^1_1-dependent choice, 2.1.2.2, are *not* tautologous. Therefore an open problem on the Weihrauch degree of parallelization of the Σ^1_1-choice principle on the integers is not solved by using those axioms.

Anderson division by zero

From James, A.D.W. Anderson et al (2006), "Perspex Machine VIII: Axioms of Transreal Arithmetic".

The transreal number system based on 1/0 = Nullity (*not* undefined) claims this axiom for Lattice Completeness:

> The set, *X*, of all transreal numbers, excluding Φ (Nullity), is lattice complete because
>
> $\forall Y\colon Y \subseteq X \Rightarrow (\ \exists u \in X\colon (\ \forall y \in Y\colon y \leq u\ ) \wedge (\ \forall v \in X\colon (\ \forall y \in Y\colon y \leq v) \Rightarrow u \leq v))$ [A32]

We map and test axiom A32 in Meth8 script.

LET: pqrsuv xyXYuv; nvt not tautologous;
∀; % ∃; ~ Not; & ∧; + ∨; > Imply; < ∈, Not Imply; ~(m > n) (m ≤ n), (m ⊆ n);

(#s&~(s>r)) > (((%u<r)&((#q<s)&~(q>u)))&((#q<s)&(~(q>v)>~(v>u)))); (1)

Eq 1 is not tautologous. Here is the repeating fragment of the 128-truth tables:

```
Model 1          .Model 2.1       .Model 2.2       .Model 2.3.1     .Model 2.3.2
TTTTTTTTCCCCTTTT.EEEEEEEEUUUUEEEE.EEEEEEEEEEEEEEEE.EEEEEEEEPPPPEEEE.EEEEEEEEIIIIEEEE
(#s&~(s>r))>(((%u<r)&((#q<s)&~(q>u)))&((#q<s)&(~(q>v)>~(v>u))))   Step: 29
```

Eq 1 with the main connective < Not Imply is also not tautologous.

(#s&~(s>r)) < (((%u<r)&((#q<s)&~(q>u)))&((#q<s)&(~(q>v)>~(v>u)))); (2)

We jump forward in the paper to evaluate the first general algebraic property theorem:

$(a+b)=\Phi \equiv (a=\Phi)\vee(b=\Phi)\vee((a=\infty)\wedge(b=-\infty))\vee((a=-\infty)\wedge(b=\infty))$ [T52]

LET: pqrs *a b* Φ∞ [*We note the infinity symbol is used as a positive or negative number.*]

((p+q)=r) = (((p=r)+(q=r))+ (((p=r)&(q=~r))+((p=~r)&(q=r)))) ; (3)

Here is the entire truth table:

```
Model 1          .Model 2.1       .Model 2.2       .Model 2.3.1     .Model 2.3.2
TFFTTTTTTFFTTTTT.EUUEEEEEEUUEEEEE.EUUEEEEEEUUEEEEE.EUUEEEEEEUUEEEEE.EUUEEEEEEUUEEEEE
((p+q)=r)=(((p=r)+(q=r))+(((p=r)&(q=~r))+((p=~r)&(q=r))))   Step: 29
```

We resuscitate Eq 3 to tautology by replacing the main connective = Equivalent with the > Imply connective.

((p+q)=r) > (((p=r)+(q=r))+(((p=r)&(q=~r))+((p=~r)&(q=r)))) ; (4)

However, this is not what the authors stated in theorem T52, so we stop here.

Refutation of approximations of theories

Abstract: We evaluate the definition of *T*-approximations which is *not* tautologous, thereby refuting the approximations of theories.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r, s: ϕ, *T* , T, T'
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⊢, ↦, ≻, ⊃; < Not Imply, less than, ∈, ≺, ⊂;
= Equivalent, ≡, ⊨, :=, ⇔, ↔; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology; (z@z) **F** as contradiction, Ø, Null;
(%z<#z) C non-contingency, ∇, ordinal 2;
(%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).

From: Sudoplatov, S.V. (2019).
Approximations of theories. arxiv.org/pdf/1901.08961.pdf sudoplat@math.nsc.ru

2. *T*-approximations
Definition. Let *T* be a class of theories and T be a theory, T∉*T* . The theory T is called *T -approximated*, or *approximated by T* , or *T -approximable*, or a *pseudo-T-theory*, if for any formula ϕ∈T there is T'∈*T* such that ϕ∈T'. If T is *T* -approximated then *T* is called an *approximating family* for T, and theories T'∈*T* are *approximations* for T. (2.1)

(~(r<q)>(p<r))>((s<q)>(p<s)) ; TTTT TTTT TT**F**T **FF**TT (2.2)

Because the initial definition of Eq. 2.2 is *not* tautologous, this refutes the approximations of theories.

Refutation Arrow's impossibility theorem

Abstract: We evaluate two versions of Arrow's impossibility theorem with disjunctive or conjunctive results. Both are rendered as *not* tautologous. This means Arrow's framework is refuted, hence coloring the conjecture of Arrow's theorem before pivotal voters or dictators can be derived. Therefore Arrow's impossibility theorem forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Bisht, H.; Kuber, A. (2019). Aggregating relational structures. arxiv.org/pdf/1904.12482.pdf
en.wikipedia.org/wiki/Arrow's_impossibility_theorem

Say there are three choices for society, call them **A**, **B**, and **C**. (1.1)

LET p, q, r, s: choice A, choice B, choice C, society;
unanimity, everyone, everything; #~p everything not p

s>((p&q)&r) ; TTTT TTTT **FFFF FFF**T (1.2)

Suppose first that everyone prefers option **B** the least: (2.1)

#s>~q ; TTTT TTTT TTCC TTCC (2.2)

everyone prefers **A** to **B**, and everyone prefers **C** to **B**. (3.1)

(#s>(p>q))&(#s>(r>q)) ; TTTT TTTT TCTT CCTT (3.2)

By unanimity, society must also prefer both **A** and **C** to **B**. (4.1)

#(s>((p&r)>q)) = (p=p) ; NNNN NNNN NNNN N**F**NN (4.2)

… On the other hand, if everyone preferred **B** to everything else, then society would have to prefer **B** to everything else by unanimity. (5.1)

(#s>(q>#~q))>#(s>(q>#~q)) ; NNNN NNNN NNNN NNNN (5.2)

Remark 1.1-5.1: The argument then becomes, If Eqs. 1.1, Then ((2.1 And (3.1 and 4.1)) Or 5.1). (6.1)

(s>((p&q)&r)) >
(((#s>~q)&(((#s>(p>q))&(#s>(r>q)))&#(s>((p&r)>q)))) + ((#s>(q>#~q))>#(s>(q>#~q)))) ;
NNNN NNNN TTTT TTTN (6.2)

Remark 6.1: If the disjunctive phrase in Eq. 6.1 is changed to conjunctive (Or connective is changed to And), the argument is weakened as, If Eqs. 1.1, Then ((2.1 And (3.1 and 4.1)) And 5.1). (7.1)

(s>((p&q)&r)) >
(((#s>~q)&(((#s>(p>q))&(#s>(r>q)))&#(s>((p&r)>q)))) & ((#s>(q>#~q))>#(s>(q>#~q)))) ;
NNNN NNNN TTTT TTT**F** (7.2)

Eqs. 6.2 and 7.2 as rendered are *not* tautologous. This means Arrow's impossibility framework as stated is refuted, hence coloring the conjecture of Arrow's impossibility theorem before pivotal voters or dictators are derived.

Logical confirmation of the Holy Trinity formula in the Athanasian creed

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET p, q, r, s: God the Holy Trinity (GT), Person of God the Holy Ghost as the Paraclete (GP), Person of God the Father (GF) , Person of the Son (GS);
~ Not; & And; + Or; > Imply; = Equivalent;
# necessity, necessarily, for every; % possibility, possibly, for one.

From: ccel.org/ccel/schaff/creeds2.iv.i.iv.html (The Athanasian creed follows this analysis.)

3. And the Catholic Faith is this: That we worship one God in Trinity, and Trinity in Unity;
4. Neither confounding the Persons: nor dividing the Substance [Essence].
5. For there is one Person of the Father: another of the Son: and another of the Holy Ghost. (0.1)
15. So the Father is God: the Son is God: and the Holy Ghost is God.
16. And yet they are not three Gods: but one God.

We rephrase Lines 15-16 to express the co-equality as: GT implies ((GP, GF, and GS) implies (GP, GF, or GS)). (1.1)

p>((q&(r&s))>(q+(r+s))) ; TTTT TTTT TTTT TTTT (1.2)

Remark: Eq. 1.2 has the format of perfect number six: 1*2*3 implies 1+2+3.

23. The Holy Ghost is of the Father and of the Son: neither made, nor created, nor begotten: but proceeding. [The Holy Ghost proceeds from the Father *and* the Son.]

We rephrase Line 23 as the filioque: GF and GS necessarily imply GP. (2.1)

#(r&s)>q ; TTTT TTTT TTTT CCTT (2.2)

27. So that in all things, as aforesaid: the Unity in Trinity, and the Trinity in Unity, is to be worshiped.
We rephrase Lines 24-27, using Eqs. 2.1 to imply 1.1 as: If (GF and GS necessarily imply GP), then (GT implies ((GP, GF, and GS) imply (GP, GF, or GS)). (3.1)

(#(r&s)>q)>(p>((q&(r&s))>(q+(r+s)))) ;
TTTT TTTT TTTT TTTT (3.2)

Remark: Eq. 3.1 has Eq. 2.1 (filioque) as antecedent to Eq. 1.1 (co-equality) as consequent. In other words, the filioque *commences* the proof of the Holy Trinity.

Eq. 3.2 as rendered is tautologous, confirming the formula of the Holy Trinity in the commonly named Athanasian creed.

The Athanasian Creed. Old translation, revised. (from ccel.org/ccel/schaff/creeds2.iv.i.iv.html)

1. Whosoever will be saved: before all things it is necessary that he hold the Catholic Faith:

2. Which Faith except every one do keep whole and undefiled: without doubt he shall perish everlastingly.

3. And the Catholic Faith is this: That we worship one God in Trinity, and Trinity in Unity;

4. Neither confounding the Persons: nor dividing the Substance [Essence].

5. For there is one Person of the Father: another of the Son: and another of the Holy Ghost.

6. But the Godhead of the Father, of the Son, and of the Holy Ghost, is all one: the Glory equal, the Majesty coeternal.

7. Such as the Father is: such is the Son: and such is the Holy Ghost.

8. The Father uncreate [uncreated]: the Son uncreate [uncreated]: and the Holy Ghost uncreate [uncreated].

9. The Father incomprehensible [unlimited]: the Son incomprehensible [unlimited]: and the Holy Ghost incomprehensible [unlimited, or infinite].

10. The Father eternal: the Son eternal: and the Holy Ghost eternal.

11. And yet they are not three eternals: but one eternal.

12. As also there are not three uncreated: nor three incomprehensibles [infinites], but one uncreated: and one incomprehensible [infinite].

13. So likewise the Father is Almighty: the Son Almighty: and the Holy Ghost Almighty.

14. And yet they are not three Almighties: but one Almighty.

15. So the Father is God: the Son is God: and the Holy Ghost is God.

16. And yet they are not three Gods: but one God.

17. So likewise the Father is Lord: the Son Lord: and the Holy Ghost Lord.

18. And yet not three Lords: but one Lord.

19. For like as we are compelled by the Christian verity: to acknowledge every Person by himself to be God and Lord:

20. So are we forbidden by the Catholic Religion: to say, There be [are] three Gods, or three Lords.

21. The Father is made of none: neither created, nor begotten.

22. The Son is of the Father alone: not made, nor created: but begotten.

23. The Holy Ghost is of the Father and of the Son: neither made, nor created, nor begotten: but proceeding.

24. So there is one Father, not three Fathers: one Son, not three Sons: one Holy Ghost, not three Holy Ghosts.

25. And in this Trinity none is afore, or after another: none is greater, or less than another [there is nothing before, or after: nothing greater or less].

26. But the whole three Persons are coeternal, and coequal.

27. So that in all things, as aforesaid: the Unity in Trinity, and the Trinity in Unity, is to be worshiped.

28. He therefore that will be saved, must [let him] thus think of the Trinity.

29. Furthermore it is necessary to everlasting salvation: that he also believe rightly [faithfully] the Incarnation of our Lord Jesus Christ.

30. For the right Faith is, that we believe and confess: that our Lord Jesus Christ, the Son of God, is God and Man;

31. God, of the Substance [Essence] of the Father; begotten before the worlds: and Man, of the Substance [Essence] of his Mother, born in the world.

32. Perfect God: and perfect Man, of a reasonable soul and human flesh subsisting.

33. Equal to the Father, as touching his Godhead: and inferior to the Father as touching his Manhood.

34. Who although he be [is] God and Man; yet he is not two, but one Christ.

35. One; not by conversion of the Godhead into flesh: but by taking [assumption] of the Manhood into God.

36. One altogether; not by confusion of Substance [Essence]: but by unity of Person.

37. For as the reasonable soul and flesh is one man: so God and Man is one Christ;

38. Who suffered for our salvation: descended into hell [Hades, spirit-world]: rose again the third day from the dead.

39. He ascended into heaven, he sitteth on the right hand of the Father God [God the Father] Almighty.

41. At whose coming all men shall rise again with their bodies;

42. And shall give account for their own works.

43. And they that have done good shall go into life everlasting: and they that have done evil, into everlasting fire.

44. This is the Catholic Faith: which except a man believe faithfully [truly and firmly], he can not be saved.

Validation of **"Axiomatizing category theory in free logic"**

Introduction

The authors Benzmüller and Scott (2016) use a proof assistant named Isabelle/HOL to formalize axiom sets for category theory using the system "free logic" which is supposed to abide by the rules of classical logic.

Our motivation of this experiment is to validate those logical expressions of free logic in terms of classical logic. We ask, "Is system free logic compliant with classical logic?"

The approach is to use the modal logic theorem checker named Meth8 for five models from James (2016). Meth8 is based on the variant system **VŁ4** from Goodwin, James (2015) that corrects and rehabilitates the Łukasiewicz quaternary logic system of **Ł4**, where:

% Existential Quantifier, Modal Possibility; # Universal Quantifier, Modal Necessity; ~ Not;
& And; \ Not And; = Equivalent; @ Not Equivalent; > Imply; < Not Imply; + Or; - Not Or;
vt Validated as Tautologous; nvt Not Validated as Tautologous; nvt F Not Validated as Tautologous, all models contradiction

The logical values are, with designated truth values in italics:
FCNT for F contradiction, Contingent (falsity), Non Contingent (truth), *Tautologous*;
UIPE for Unevaluated, Improper, Proper, *Evaluated.*

We proceed to test the logical expressions in that paper.

Validation

The validation is presented as a table of the 8 expressions evaluated from that paper with: ID; section name or Meth8 script as tested; test validation result; name of the expression, section number; and notes. For expressions not validated as tautologous, the test results are shaded lighter gray, and of those returning all F values (contradictory) are further shaded darker gray.

ID	**Section name / Meth8 script**	**Test**	**Name**	**Sec no**	**Notes**
	1. Introduction				
1	(%p&~(s&p))>(p@p) [(p@p) is F contradiction]	nvt	[f_exist_proved]	1	"We can prove"
	2. Embedding of free logic in HOL				
2	#p=((q&(r&q))>(p&q))	nvt	f_for_all	2	
3	(#p&(q&p))>#q	vt	f_for_all_binder	2	
4	(p+q)=(~p>q)	vt	f_or	2	
5	(p&q)=~(~p+~q)	vt	f_and	2	
6	(p=q)=((p>q)&(q>p))	nvt	f_implied	2	
7	(p=q)=((p>q)&(q>p))	vt	f_equiv	2	
8	%p=~(#(r&q)&~(p&q))	nvt	f_exists	2	
9	(%p&(q&p))=%q	nvt	f_exists_binder	2	

Truth table fragments for nvt tests above are keyed to the ID for models and step (stp) below.

ID	Model 1	Model 2.1	Model 2.2	Model 2.3.1	Model 2.3.2	Stp
1.	NFNF NFNF NTNT NTNT	EUEU EUEU EEEE EEEE	UUUU UUUU UEUE UEUE	IUIU IUIU IEIE IEIE	PUPU PUPU PEPE PEPE	9
2.	FNFN FNTN FNFN FNTN	UEUE UEEE UEUE UEEE	UUUU UUEU UUUU UUEU	UIUI UIEI UIUI UIEI	UPUP UPEP UPUP UPEP	11
6.	FTTF FTTF FTTF FTTF	UEEU UEEU UEEU UEEU	UEEU UEEU UEEU UEEU	UEEU UEEU UEEU UEEU	UEEU UEEU UEEU UEEU	7
7.	CTCT CTTT CTCT CTTT	UEUE UEEE UEUE UEEE	EEEE EEEE EEEE EEEE	PEPE PEEE PEPE PEEE	IEIE IEEE IEIE IEEE	9
8.	NNFT NNFT NNFT NNFT	EEUE EEUE EEUE EEUE	UUUE UUUE UUUE UUUE	IIUE IIUE IIUE IIUE	PPUE PPUE PPUE PPUE	7

Discussion

Of the 9 expressions tested by Meth8, 4 are validated as tautologous (vt) and 5 are not validated as tautologous (nvt).

1. Introduction

The paper authors write: [0.] "We can prove (∃ x.~(*E*x))→contradictory, where *E* is the existence predicate"; and "Read this as: "If there are undefined objects, then we have falsity."

We write this as 1. (%p&~(s&p))>(p@p) to mean "Both possibly p and the non-existence of r imply falsity." Substituting from above 7. f_exists for *E*x as %p=~(#(r&q)&~(p&q)), writes

10	(%p&~(~(#(r&q)&~(p&q))))>(p@p)	vt	f_exists_*E*x	1	

Hence we confirm [0].

2. Embedding of free logic in HOL

We validate as tautologous the non-assistant expressions of 4, 5, and 7 as the connectives Or, And, and Equivalent.

We validate as not tautologous the non-assistant expressions of 1, 6, and 8 as the universal quantifier, Implication connective, and existential quantifier.

Conclusion

Because we do not validate as tautologous expressions 1, 6, and 8, free logic is not validated as compliant with classical logic. Consequently we do not proceed to subsequent sections 3 and 4-10 for Preliminaries and Axiom sets 1-8. We conclude that the assistant tool Isabelle/HOL is not compliant with classical logic.

References

Benzmüller, C.; Scott. D.S. (2016). Axiomatizing category theory in free logic. arXiv:1609:01493v3 32

Refutation of axiomatizing logics of fuzzy preferences using graded modalities

Abstract: We evaluate the fuzzy preference relation of the assumed ∧-transitivity theorem as *not* tautologous, relegating it, along with the subsequent minimal modal logics of a finite residuated lattice and the Bulldozed method, as *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Vidal1, A.; Esteva, F.; Godo, L. (2019). Axiomatizing logics of fuzzy preferences using graded modalities. arxiv.org/pdf/1909.07674.pdf amanda@cs.cas.cz, esteva@iiia.csic.es, godo@iiia.csic.es

Abstract The aim of this paper is to propose a many-valued modal framework to formalize reasoning with both graded preferences and propositions, in the style of van Benthem et al.'s classical modal logics for preferences. Axiomatizing logics of fuzzy preferences using graded modalities.

2. Preliminaries on fuzzy preference relations

… In this paper, we will assume that a weak A-valued preference relation on a set U will be now a fuzzy ∧-preorder P : U ×U → A, where P(a, b) is interpreted as the degree in which v is at least as preferred as u, that is, satisfying: ...

∧-transitivity: P(u, v) ∧ P(v, w) ≤ P(u, w) for each u, v, w ∈ U (2.5.1)

Remark 2.5.1: We ignore the subset clause for evaluation of the assumed ∧-transitivity theorem.

LET p, q, r, s: P, u, v, w.

~((p&(q&s))<((p&(q&r))&(p&(r&s)))) = (p=p) ;
TTTT TTTT TTT**F** TTTT (2.5.2)

Remark 2.5.2: Eq. 2.5.2 as rendered is *not* tautologous. This refutes subsequent conjectures in the text, notably, the minimal modal logics of a finite residuated lattice and the Bulldozed method.

Refutation of the generating positive cone in ordered Banach space

Abstract: From the background definitions in the ordered Banach space, we evaluate equations to produce the term named positive generating cone. It is *not* tautologous, hence refuting the model.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⊢, ⊨, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z<#z) `C` as contingency, Δ, ordinal 1; (%z>#z) `N` as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Dahlqvist, F.; Kozen, D. (2019).
Semantics of higher-order probabilistic programs with conditioning.
arxiv.org/pdf/1902.11189.pdf f.dahlqvist@ucl.ac.uk, dexter.kozen@cornell.edu

1) Regular Ordered Banach spaces: An *ordered vector space* V is a vector space together with a partial order ≤ which is compatible with the linear structure in the sense that

for all u, v, w ∈ V, λ ∈ R^+, u ≤ v ⇒ u + w≤v + w and u ≤ v ⇒ λu ≤ λv (2.1.1)

(((#u&(#v&#w))<p)&(q<~(p&p)))>
((~(#v<#u)>(#u+(~(#v<#w)+#w)))&(~(#v<#u)>~((#q&#u)<(#p&#v)))) ;
`TTTT TTTT TTTT TTTT` (2.1.2)

A vector v in an ordered vector space V is called *positive* if v ≥ 0 and the collection of all positive vectors is called the *positive cone* of V and denoted V^+. The positive cone is said to be *generating* if V = $V^+ - V^+$, that is to say if every vector can be expressed as the difference of two positive vectors.

V^+=v ≥ 0, V=V^+ - V^+ : (2.3.1)

p=((~((p@p)>v))-(~((p@p)>v))) ;
F`T`**F**`T` **F**`T`**F**`T` **F**`T`**F**`T` **F**`T`**F**`T` (2.3.2)

Remark 2.3.1: Eq. 2.3.1 follows from 2.1.1 for V. (2.4.1)

((((#u&(#v&#w))<p)&(q<~(p&p)))>((~(#v<#u)>(#u+(~(#v<#w)+#w)))&
(~(#v<#u)>~((#q&#u)<(#p&#v)))))>(p=(~((p@p)>v)-~((p@p)>v))) ;
F`T`**F**`T` **F**`T`**F**`T` **F**`T`**F**`T` **FTFT** (2.4.2)

Eq. 2.4.2 is *not* tautologous. Hence, the positive generating cone is refuted in the ordered Banach space.

Refutation of the Banach-Tarski paradox

Abstract: We evaluate the crucial claim of the proof in Step 3, as a fleshed out detail. It is *not* tautologous, nor is it contradictory. This means the claim is a non tautologous fragment of the universal logic VŁ4 and constitutes the briefest known refutation of the Banach-Tarski paradox.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) `C` non-contingency, ∇, ordinal 2; (%z>#z) `N` as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: en.wikipedia.org/wiki/Banach–Tarski_paradox

Some details, fleshed out ... [for Step 3 of 4]
What remains to be shown is the Claim: $S^2 - D$ is equidecomposable with S^2.
Proof. Let λ be some line through the origin that does not intersect any point in D. This is possible since D is countable. Let J be the set of angles, α, such that for some natural number n, and some P in D, **r**(nα)P is also in D, where **r**(nα) is a rotation about λ of nα. Then J is countable. So there exists an angle θ not in J. Let ρ be the rotation about λ by θ. Then ρ acts on S^2 with no fixed points in D, i.e., $\rho^n(D)$ is disjoint from D, and for natural m<n, $\rho^n(D)$ is disjoint from $\rho^m(D)$. Let E be the disjoint of $\rho^n(D)$ over n = 0, 1, 2, ... Then

$$S^2 = E \cup (S^2 - E) \sim \rho(E) \cup (S^2 - E) = (E - D) \cup (S^2 - E) = S^2 - D, \qquad (3.1)$$

LET p, q, r, s: E, D, ρ, S^2

`(s=((p+(s-p))=( (r&p)+(s-p))))=(((p-q)+(s-p))=(s-q)) ;`

`FF``TT` **`F`**`TT`**`F`** **`FF`**`T`**`F`** **`F`**`TTT` (3.2)

where ~ denotes "is equidecomposable to".

Remark 3.2: We write "~" as "equivalent to". Eq. 3.2 as rendered is *not* tautologous. Because it is the crucial claim of the proof, the result is that the Banach-Tarski paradox is also not contradictory, and hence a non tautologous fragment of the universal logic VŁ4.

Resolution to the Banach-Tarski Paradox

This experiment logically tests the Banach-Tarski Paradox as an equivalence and an implication.

At en.wikipedia.org/wiki/Banach%E2%80%93Tarski_paradox , we find after "[s]ome details fleshed out", Step 3:

$$S^2 = ... = (E - D) \cup (S^2 - E) = S^2 - D \tag{1.1}$$

We assume the Meth8 apparatus using VŁ4, where the designated proof value is T tautology and F contradiction. The 16-value truth table is presented row major and horizontally.

LET: s S^2; q E; p D; = Equivalent to; ∪ + Or; ⊃ > Imply; - Not Or; & And

s = (((q-p)+(s-q)) = (s-p)) ; `FTTF FTTF FTTT FTTT` (1.2)

Because Eq. 1.2 is not tautologous, we weaken the argument for the equivalent to connective =, with replacement by the connective > Imply.

s > (((q-p)+(s-q)) > (s-p)) ; `TTTT TTTT FTTT FTTT` (1.3)

Eq. 1.3 is the equivalent to writing Eq 1.1 in the text symbols as:

$$S^2 \supset (E - D) \cup (S^2 - E) \supset S^2 - D. \tag{1.4}$$

While Eq. 1.3 is relatively less contradictory than Eq.1.2, it remains that both Eq. 1.1 and Eq. 1.4 in the text symbols remain as not tautologous.

This means the Banach-Tarski Paradox, as rendered, is not a paradox, not a theorem, and non-tautologous.

What follows is that the Von Neumann Paradox on the Euclidean plane is also suspicious as a paradox and possibly not a paradox.

Refutation of mapping definable functions as neighbourhood functions

Abstract: We evaluate two applied equations of stopping and closure functions of bar recursion. None is tautologous. This refutes the approach of mapping functions as such.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. Reproducible transcripts for results are available. (See ersatz-systems.com.)

LET p, q, r, s, t, u, v, w, x :
D, T, ρ, σ, τ, τ*, T, N, N*;
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⊢, ⊨; < Not Imply, less than, ∈, ≺, ⊂;
= Equivalent, ≡, :=, ⇔, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) C non-contingency, ∇, ordinal 2; (%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).

From: Kawai, T. (2019). Representing definable functions of HA^{ω} by neighbourhood functions. arxiv.org/pdf/1901.11270.pdf tatsuji.kawai@jaist.ac.jp

We call a function $BR^{\tau,\sigma}$ of type

$$((N \to \tau) \to N) \to (\tau * \to \sigma) \to (\tau * \to (\tau \to \sigma) \to \sigma) \to \tau * \to \sigma \qquad (6.3.1.1.1)$$

which satisfies (6.3) a bar recursor of types τ and σ. The first argument of a bar recursor, i.e., a function of type (N → τ) → N), is called a stopping function of bar recursion.

(((w>t)>w)>(u>s))>(((u>(t>s))>s)>(u>s)) ;
FFFF FFFF TTTT TTTT(2),
TTTT TTTT TTTT TTTT(14) (6.3.1.1.2)

Theorem 6.3. Closure under the rule of bar induction
For any type σ and a closed term Y: NN→N, there exists a closed term ξ of type

$$(N* \to \sigma) \to (N* \to (N \to \sigma) \to \sigma) \to N* \to \sigma \qquad (6.3.1.2.1)$$

((x>s)>((x>(w>s))>s))>(x>s) ;
TTTT TTTT TTTT TTTT(16),
FFFF FFFF TTTT TTTT(16) (6.3.1.2.2)

Eqs. 6.3.1.1.2 and 6.3.1.2.2 are *not* tautologous, hence refuting the mapping approach.

Barcan formula in evaluation of "Can modalities save naive set theory?"

Introduction

As a memorial to Grigori Mints, the captioned draft paper was authored by Peter Fritz, Harvey Lederman, Tiannkai Liu, and Dana Scott (2015). That paper contains many modal comprehension principles and relies on the Converse Barcan Formula and logic system **T**. The intent of that paper is to: affirm that naive set theory is inconsistent; affirm that the Russell paradox is consequently inconsistent; and answer the question as "no".

Our motivation is to validate the logical expressions of that paper. The approach is to use the modal logic theorem checker named Meth8 for five models from James (2016). Meth8 is based on the variant system **VŁ4** from Goodwin, James (2015) that corrects and rehabilitates the Łukasiewicz quaternary logic system of **Ł4**, where:

> % Existential Quantifier, Modal Possibility; # Universal Quantifier, Modal Necessity; ~ Not; & And; \ Not And; = Equivalent; @ Not Equivalent; > Imply; < Not Imply; + Or; - Not Or; vt Validated as Tautologous; nvt Not Validated as Tautologous; nvt F Not Validated as Tautologous, all models contradiction
>
> FCNT for F contraction, Contingent, Non Contingent, *Tautologous*;
> UIPE for Unevaluated, Improper, Proper, *Evaluated.*

We evaluate the schemata of the Barcan Formula and its converse before proceeding to test the logical expressions in that paper.

The Barcan Formula

The Barcan Formula is defined in Meth8 script as either:

```
(#x&#(p&x))     >     #(#x&(p&x)) ; vt ;    (BF0.1) > (BF0.2):    (BF.0)
%(%x&(p&x))     >     (%x&%(p&x)) ; vt ;    (BF1.1) > (BF1.2):    (BF.1)
```

The truth tables for each literal group in (BF.0) and (BF.1) are in non repeating rows after 5 or 6 steps:

```
FFFN FFFN   UUUE UUUE   UUUU UUUU   UUUI UUUI   UUUP UUUP    (BF0.1); (BF0.2)
CCCT CCCT   UUUE UUUE   EEEE EEEE   PPPE PPPE   IIIE IIIE    (BF1.1); (BF1.2)
```

In Meth8 for (BF.0) and (BF.1), the respective antecedents and consequents are equivalent, so:

```
((#x&#(p&x)) = #(#x&(p&x))) = (%(%x&(p&x)) = (%x&%(p&x))) ;    vt ;    (BF.2)
```

The converse Barcan Formula to (BF.2) is defined in Meth8 script as:

```
#(#x&(p&x))) = ((#x&#(p&x)) = (%x&%(p&x))) = (%(%x&(p&x)) ;    vt ;    (BF.5)
```

Because (BF.2) and (BF.5) are equivalent, we do not take the converse Barcan formula to be more plausible than the Barcan formula. In that paper, the expression (p&x) is substituted for p below.

Validation

The validation is presented as a table of the 64 expressions evaluated from that paper with: ID; section name or Meth8 script as tested; test validation result; name of the expression, section number; and notes. For expressions not validated as tautologous, the test results are shaded lighter gray, and of those returning all F values (contradiction) are further shaded darker gray.

ID	Section name / Meth8 script	Test	Name	Sec no	Notes
	1. Background				
1	(%y&#x)&((x<y)=#p)	nvt	(Comp#)	1	
2	(%y&#x)&#((x<y)=#p)	nvt	(#Comp#)	1	
3	#(#x&p)>(#x&#p)	vt	(CBF)	1	converse Barcan
4	%y&#(#x&((x<y)=#p))	nvt	Principle	1	imply (#Comp#)
5	(#x&#p)>#(#x&p)	vt	(BF)	1	Barcan formula
6	%y& #(#x&((x<y)=((x<u)&p)))	nvt	(MZF Comp)	1	per Kajíček et al
7	(%y&#x) & (((#x<y)=#p) & ((#~x<y)=#~p))	nvt F	(MCA)	1	
	2. The Consistency of (Comp#)				
8	Substitution of predicate logic theorem		(LPC)	2	not tested
9	(#(p>q)>(#p>#q)) > (t=t)	vt	(K)	2	"⊦" as "> (t=t)"
10	(#p>p)>(t=t)	vt	(T)	2	
11	((p>(t=t))&((p>q)>(t=t)))>(p>(t=t))	vt	(MP)	2	
12	((p>q)>(t=t))>(((p>(#x&q))>(t=t))&(~%x& p))	nvt	(∀2) x ~free p	2	~%x&p
13	(p>(t=t))>(#p>(t=t))	vt	(RN)	2	
14	((p>q)>(t=t)) > (((#p>#q)>(t=t))&((%p> %q)>(t=t)))	vt	(RM)	2	
15	(#x&#p)>#(#x&p)	vt	(BF)	2	Barcan formula
16	#(#x&p)>(#x&#p)	vt	(CBF)	2	converse Barcan
17	#p>##p	vt	(L4)	2	
18	~#p>#~#p	vt	(L5)	2	
19	p>#%p	vt	(B)	2	
	2.1 Consistency				
20	#(#x&(p&x))=(#x&#(p&x))	vt	(Bar)	2.1.6.1	
21	(#y&#p)&((#x&((x<y)=(x<p)))>(y=p))	nvt	(Ext)	2.16.2	
22	((#p&%y)&#x)&((x<y)=~(x<p))	nvt	(Neg)	2.1.6.3	
23	((#p&#q)&(%y&#x))&((x<y)=((x<p)&(x<q)))	nvt	(Con)	2.1.6.4	
24	(%y&#x)&((x<y)=(%p&x))	nvt	(Comp%)	2.1.6.5	
25	(#x&#y)&((%x=y)>(#x=y))	nvt	(Equ)	2.1.6.6	
26	(#x&#y)&%(x<y)	nvt F	(Mem)	2.1.6.7	

ID	Section name / Meth8 script	Test	Name	Sec no	Notes
27	(#x&#y)&%(~x<y)	nvt F	(Non)	2.1.6.8	
	2.2 Undecidability				
28	(%y&#x)&(~x<y)	nvt F	(Empty)	2.2	
29	((#y&#z)&(%w&#x)) & ((x<w)=((x<y)+(x=z)))	nvt F	(Add) (x=z)	2.2	
30	((x@x)&((#y&#z)&(%w&#x))) & ((x<w)=((x<y)+(x=z)))	nvt F	(Add) (x@x)	2.2	
31	((x@x)&((%y&#x)&((x<y)=#p))) = ((%y&#x)&(~x<y))	vt	(Comp#) > (Empty)[x@x]	2.2	
	2.3 Concluding discussion				
32	(x<y)>(#x<y)	nvt	Axiom	2.3	membership is #
33	p=(x>x)	nvt	Instance	2.3	inconsistent **KD**
	3. Inconsistency (#Comp#)				
34	(%y&#x)&#((x<y)>(#~x<x))	nvt	(#Russell#)	3.1	
35	(p>(q=#~q))>~#p	vt	Proposition	3.2	
36	(#y&~#x)&#((x<y)=#(~x<x))	nvt F	Generalization	3.2.13	on 3.2.12
37	(~%y&#x)&#((x<y)=#(~x<x))	nvt F	Df∀	3.2.14	on 3.2.13
38	((#p>(q=#~q))>~#p	vt	Proposition	3.3	
39	(#p>q)>(#p>#q)	vt	(RM)	3.3	
40	((p>q)>(t=t)) > (((#p>#q)>(t=t))&((%p> %q)>(t=t)))	vt	(RM)	3.3	
41	#p>(q=#~q)	nvt	Assumption	3.3	
	4. Inconsistency of (#Comp#%)				
42	(%y&#x)&#((x<y)=#%#p)	nvt	(#Comp#%)	4	
43	(%y&#x)&#((x<y)>(#%~x<x))	nvt	(#Russell#%)	4	
44	(#p>(q=#%~q)) > ~#p	vt	Proposition	4.2	
45	#p>(q=#%~q)	nvt	(Assumption)	4.2.1	antecedent
46	~#p [tested as ~#p+~#p]	nvt	Proposition	4.2.12	consequent
	5. Inconsistency of (#Comp#%#)				
47	(%y&#x)&#((x<y)>#%p)	nvt	(#Comp#%#)	5	
48	(%y&#x)&#((x<y)>(#%~x<x))	nvt	(#Russell#%#)	5	
49	#%p=#%#%p	vt	(Red#%)	5	Reduction law
50	(#p>(p=#%#~p))>~#p	vt	Proposition	5.2	
	6. Duality between modalities				
	7. Conclusion				
	7.1 Converse Barcan formula				

ID	Section name / Meth8 script	Test	Name	Sec no	Notes
51	(#x&p) > p	vt	predicate logic	7.1.1	
52	#(#x&p) > #p	vt	(RN), (K)	7.1.2	
53	#(#x&p)>(#x&#p)	vt	(∀2)	7.1.3	
54	%y&#(#x &((x<y)=#p))	nvt	(#∀Comp#)	7.1	"the principle"
55	(#x&p)>((%x&(x=y))>(p&(x+y)))	vt	(Rest Gen)	7.1	for [x/y] as x+y
56	(#x&p)>((%x&(x=y))>(p&(x@y)))	nvt	(Rest Gen)	7.1	for [x/y] as x@y
57	#x&(%y&(x=y))	nvt	(UE)	7.1	
58	(#x&(p>q)) > ((#x&p) > (#x&q))	vt	(∀>)	7.1	
59	(p>(t=t))>((#x&p)>(t=t))	vt	(UG)	7.1	also (U=G)
60	(%y&##x)&((x<r)=#((%y&(y=x))>(x>x)))	nvt	(#∀Russell#)	7.1	
	7.2 Replacing (T) with (D)				
61	#p>%p	vt	(D)	7.2	Replace (T), (D)
62	%p>#p	nvt	(Dc)	7.2	in **KDDc**, (Dc)
	7.3 Gödel-McKinsey-Tarski naive comp.				
63	#%%y&##x)&#((#x<y)=#p)	nvt	(CompGMT)	7.3	
64	(#%%y&##x)&#((#x<y)=#~#p)	nvt	(RussellGMT)	7.3	
65	(###y&~##x)&#((#x<y)=#(~#x<x))	nvt F	Proposition	7.3.6	last line in proof

Discussion

Of the 64 expressions tested by Meth8, 1 is not tested, 28 are validated as tautologous (vt), 36 are not validated as tautologous (nvt), and 10 of those not validated as tautologous are nvt *and* F contradiction. The logical expressions of interest are those which Meth8 refuted as nvt, and in particular of those nvt *and* F contradiction. Rather than relisting those nvt we step through that paper by section.

1. Background

All of the comprehension principles are validated as not tautologous: (Comp#); (#Comp#); principle implying (#Comp#); (MZF Comp); and (MCA) as nvt *and* F contradiction. We note that 4. Principle to imply 2. (#Comp#) is vt although not listed. These results confirm the results in that section.

2. The Consistency of (Comp#)

We test 12. (∀2) with x not free in p as nvt. However the expression is vt if the constraint is removed.

2.1 Consistency

For (Comp#) as valid in the model M,. the principles of **S5** apply as do a series of 8 axioms. Axiom (Bar) is vt, but axioms 2-8 are nvt. In particular axioms 7 and 8 are nvt *and* F contradiction. This raises our suspicions that (Comp#) is not valid in model M, and hence is not consistent.

2.2 Undecidability

Axiom (Empty) is nvt *and* F contradiction. This causes us to doubt if the axiom of the empty set is also nvt by virtue of the two interpretations of Robinson Arithmetic given there. We tested two variations of (Add): with x=z and x@z, both as nvt *and* F contradiction. If (Empty) and (Add) are F contradiction, then the interpretation of that section does not support (Comp#)+**S5** as undecidable.

3. Inconsistency (#Comp#)

We tested (#Russell#) as nvt, confirming it it inconsistent. We tested Proposition 3.2.13 and 3.2.14 as nvt *and* F contradiction, confirming that (#Comp#) is inconsistent.

4. Inconsistency of (#Comp#%)

We tested (#Comp#%) and (#Russell#%) as nvt, confirming inconsistency.

5. Inconsistency of (#Comp#%#)

We tested (#Comp#%#) and (#Russell#%#) as nvt, confirming inconsistency.

7.1 Converse Barcan formula

We test (#∀Comp#) and (#∀Russell#) as nvt, confirming inconsistency. We test (RestGen) for [y/x] to mean x+y or x@y as vt or nvt. We note that (UG) also holds for (U=G), and should be rewritten as such with the equivalence connective.

7.2 Replacing (**T**) with (**D**)

We test the axiom schema %p>#p as nvt. rendering Dc in **KDDc** as suspicious.

7.3 Gödel-McKinsey-Tarski naive comprehension

We test (CompGMT), (RussellGMT), and Step 7.3.6 as nvt, confirming inconsistency.

Conclusion

While we confirm inconsistency in many expressions of that paper and the answer "no", we are left with some egregious expressions as nvt *and* F contradiction. For example, %p>#p as Dc is untenable, as is also (Comp#)+**S5** as undecidable, and particularly the Russell paradox as inconsistent (due to what follows).

From our previous refutations we may cut to the chase regarding ZFC and the Russell paradox: axiom of the empty set is nvt; and Russell's paradox is *not* inconsistent, and hence resolved as *not* a paradox.

Axiom of the empty set

The ZFC axioms we find nvt are: extensionality; regularity (foundation); empty set; pairing; union; and power set.

For example the axiom of the empty set is:

(#p&#q) & ((#r&((r<p)=(r<q)))>(p=q)) ; consequent tautologous; [&] makes nvt (ES.1)

Russell's paradox (See en.wikipedia.org/wiki/Russell%27s_paradox)

R = { x | x ∉ x }, then R ∈ R <=> R ∉ R. (R.1)

(r = (x>x)) > ((r<r) = (r>r)) ; nvt (R.2)

Russell's paradox as stated is nvt, but it is not a paradox or a contradiction.

In the formal presentation of Russell's "Naive Set Theory (NST), as the theory of predicate logic with a binary predicate ∈ and the following axiom schema of unrestricted comprehension:

∃ y ∀ x (x ∈ y <=> P (x)) (R.5)

for any formula *P* with only the variable *x* free. Substitute x ∉ x for P (x).
Then by existential instantiation (reusing the symbol *y*) and universal instantiation
y ∈ y <=> y ∉ y is a contradiction. Therefore, NST is inconsistent.": [∉ is >]

(%y&#x)&((x<y)=(p&x)) ; nvt (R.6)
for (p&x) substitute (x>x)
(%y&#x)&((x<y)=(x>x)) ; nvt *and* F contradiction (R.7)

However there is a problem with the substitution of (p&x)=(x>x) if (p&x) is removed from the expression as in (7); the correct expression is (p&x)=(x>x), not (x>x) with truth table fragment:

(%y&#x)&((x<y)=((p&x)=(x>x))) ; nvt [but not *and* F contradiction] (R.8)

```
Model 1     Model 2.1   Model 2.2   Model 2.3.1 Model 2.3.2
FFFF FFNF   UUUU UUEU   UUUU UUUU   UUUU UUIU   UUUU UUPU    Step: 15
```

Therefore Russell's NST is nvt, but it is *not* inconsistent as a contradiction.

References

Fritz, Peter, Lederman, Harvey, Liu, Tiankai, Scott, Dana. 2015. Can modalities save naive set theory? *Memorial to Grigori Mints (1939-2014)*.

Refutation of the Barwise compactness theorem via sublanguage L_A

Abstract: A definition with variant to establish a sublanguage in support of the Barwise compactness theorem is *not* tautologous. By extension the theorem is also refuted. These conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Bell, J.L. (2016). 5. Sublanguages of $L(\omega_1,\omega)$ and the Barwise compactness theorem. plato.stanford.edu/entries/logic-infinitary/#5. jbell@uwo.ca

We say that L_A is a sublanguage of $L(\omega_1,\omega)$ if the following conditions are satisfied:

(ii.) if $\varphi, \psi \in L_A$, then $\varphi \wedge \psi \in L_A$ and $\neg\varphi \in L_A$ (5.ii.1)

((p&s)<q)>((p&(s<q))&(~p<q)) ;
TTTT TTTT TT**F**T TT**F**T (5.ii.2)

Remark 5.ii.2: Eq. 5.ii.1 could be interpreted and rendered in part as $(\varphi\wedge\psi)\in L_A$ for

((p&s)<q)>(((p&s)<q)&(~p<q)) ;
TTTT TTTT T**F**TT T**F**TT (5.ii.3)

Eq. 5.ii.2 (with 5.ii.3 as rendered) is *not* tautologous. The purpose of Eq. 5.ii.1 was in the first place to support a proof of the Barwise compactness theorem, herewith refuted by extension.

Meth8 evaluation of Bayes rule

Bayes rule from cs.cornell.edu/home/kleinber/networks-book/networks-book-ch16.pdf, information cascades

Section 1. We ask: "Can we validate Bayes rule as defined in the captioned textbook link?"

We assume the notation of Meth8 and Pr[...] from the text as Probability of [...], which is ignored for our purposes here because Pr[...] precedes each term of the formulas of the text.

We assume the apparatus of Meth8 modal logic model checker, implementing our resuscitation of the Łukasiewicz four-valued logic as system variant VŁ4. The 16-valued truth tables are horizontal.

LET: p q [A B, from the text], (q>p) [A|B], (p>q) [B|A]
vt Validated tautology, nvt Not validated tautology,
Designated truth value: T Tautology (F Contradiction)

The text defines A given B, that is, if B then A:

(q>p)=((p&q)\q) ; TT**FF** TT**FF** TT**FF** TT**FF** (1)

Because Eq 1 is not vt, as expected from the text, we test the main connective for > Imply instead of = Equivalent.

(q>p)>((p&q)\q) ; TTT**F** TTT**F** TTT**F** TTT**F** (1.1)

The text defines B given A, that is, if A then B:

(p>q)=((q&p)\p) ; T**F**T**F** T**F**T**F** T**F**T**F** T**F**T**F** (2)

Because Eq 2 is not vt, as expected from the textbook, we test the main connective for > Imply instead of = Equivalent.

(p>q)>((q&p)\p) ; TTT**F** TTT**F** TTT**F** TTT**F** (2.1)

Eq 1 and Eq 2 are supposed to be vt but are not. We note that Eq 1.1 is equivalent to Eq 2.1 where the respective main connectives are > Imply, not = Equivalent.

((q>p)>((p&q)\q)) = ((p>q)>((q&p)\p)) ; TTTT TTTT TTTT TTTT (3)

Because Eqs 1 and 2 are nvt, we could terminate validation at this point.

Section 2. We ask: "Can the argument from the text be resuscitated in the process of continuing to evaluate it?"

The text rewrites Eqs 1 and 2 by multiplying both sides of the formula by the denominator in the respective consequent. In Eqs 1 and 2 the respective multiplier terms are q and p. The idea is to clear the denominator in the respective consequents.

((q>p)&q) = (((p&q)\q)&q) ; TT**FF** TT**FF** TT**FF** TT**FF** (4)
((p>q)&p) = (((q&p)\p)&p) ; T**F**T**F** T**F**T**F** T**F**T**F** T**F**T**F** (5)

We test the main connective in Eqs 4 and 5 for > Imply instead of = Equivalent, with the same result as in Eqs 1.1,2.1, and 3.

Because (p&q) = (q&p), the text rewrites Eq 5 but Eq 4 is carried over as unchanged.

((q>p)&q) = (((p&q)\q)&q) ; TT**FF** TT**FF** TT**FF** TT**FF** (6)
((p>q)&p) = (((p&q)\p)&p) ; T**F**T**F** T**F**T**F** T**F**T**F** T**F**T**F** (7)

The text rewrites Eqs 6 and 7 by simplifying the consequents.

((q>p)&q) = (p&q) ; TTTT TTTT TTTT TTTT (8)
((p>q)&p) = (p&q) ; TTTT TTTT TTTT TTTT (9)

The text sets Eq 8 equal to Eq 9.

((q>p)&q) = ((p>q)&p) ; TTTT TTTT TTTT TTTT (10)

For Eq 10 the text divides both antecedent and consequent by the term q to reduce the antecedent then rewrites.

(q>p) = (((p>q)&p)\q) ; TT**FF** TT**FF** TT**FF** TT**FF** (11)

This produces the intended definition of the text for the expression Pr[(A|B] (16.4) as Bayes rule.

Bayes rule as Eq 11 is nvt. We note the text begins with Eqs 1 and 2, both nvt.

This leads us to consider Eq 3 vt as the basis from which to obtain Bayes rule.

((q>p)>((p&q)\q)) = ((p>q)>((q&p)\p)) ; TTTT TTTT TTTT TTTT (3)

From Eq 3, we seek to find the definition of (q>p), or as an alternative approach of (p>q).

In the case of the term (q>p) we seek to remove from the antecedent in Eq 3 the term ((p&q)\q). The procedure is to apply the expression <((p&q)\q) to the antecedent and consequent.

(((q>p)>((p&q)\q))<((p&q)\q)) = (((p>q)>((q&p)\p))<((p&q)\q)) ;
TTTT TTTT TTTT TTTT (12)

We simplify and rewrite Eq 12.

(q>p) = (((p>q)>((q&p)\p))<((p&q)\q)) ; **FF**T**F** **FF**T**F** **FF**T**F** **FF**T**F** (13)

In the case of the term (p>q) we seek to remove from the consequent in Eq 3 the term ((q&p)\p)). The procedure is to apply the expression <((q&p)\p) to the consequent and antecedent.

(((q>p)>((p&q)\q))<((q&p)\p)) = (((p>q)>((q&p)\p))<((q&p)\p)) ;
TTTT TTTT TTTT TTTT (14)

We simplify and rewrite Eq 14.

(p>q) = (((q>p)>((p&q)\q))<((q&p)\p)) ; **FFTF FFTF FFTF FFTF** (15)

The textbook definitions of Bayes rule are not validated as tautologous and cannot be resuscitated from the textbook.

Section 3. As an experiment, we ask: "Are the definitions of Bayes rule derivable from Eq 3, the only expression tautologous, from Section 1; in other words, can Meth8 produce a correct Bayes rule because Section 1 failed to do so?"

We reiterate Eq 3 from above and rename it for this section as Eq 3'.

((q>p)>((p&q)\q)) = ((p>q)>((q&p)\p)) ; vt (3')

LET r=((p&q)\q), s=((q&p)\p) and rewrite Eq 3' with those definitions by substitution.

((r=((p&q)\q))&(s=((q&p)\p)))> ((((q>p)>r)-s) = (((p>q)>s)-r)) ; vt (4')

Our approach is to manipulate the term ((q>p)>r)-s) so that (q>p) is the antecedent of an equality.

This means finding the correct method to represent (q>p) as a separate term in ((q>p)>r)-s), or as an alternative approach to represent (p>q) as a separate term in ((p>q)>s)-r), or both.

We use the template A>B = ~A+B where A is (q>p) and B is r, so ((q>p)>r)-s becomes (~(q>p)+r)-s.

((r=((p&q)\q))&(s=((q&p)\p)))> (((~(q>p)+r)-s) = (((p>q)>s)-r)) ; vt (5')

This successfully removed from the antecedent term of interest the second > Imply connective to leave connectives + Or and - Not Or.

We use the same template as C>D = ~C+D where C is (p>q) and D is s, so that ((p>q)>s)-r becomes (~(p>q) +s)-r.

((r=((p&q)\q))&(s=((q&p)\p)))> (((~(q>p)+r)-s) = ((~(p>q)+s)-r)) ; vt (6')

This successfully removed from the consequent term of interest the second > Imply connective to leave connectives + Or and - Not Or.

We cannot extract either (q>p) or (p>q) as separate terms from Eq. 6'. Therefore we abandon seeking these terms as those claimed for Pr[A|B] or Pr[B|A] in the text for Bayes rule.

BCK-algebra confirmed, but BCI-algebra refuted

See vixra.org/pdf/1803.0466v1.pdf for text mapped.

By a *BCI-algebra* we mean a set X with a binary operation * and the special element 0 satisfying the axioms:

(a1) $((x * y) * (x * z)) * (z * y) = 0;$
(a2) $(x * (x * y)) * y = 0;$
(a3) $x * x = 0;$
(a4) $x * y = y * x = 0 \rightarrow x = y;$ for all $x; y; z \in X:$
If a *BCI*-algebra X satisfies the axiom
(a5) $0 * x = 0$ for all $x \in X;$ then we say that X is a *BCK-algebra*.

LET p q r s: x y z X;
> Imply, →; < Not Imply, less than, ∈ *;* & And, *; = Equivalent; @ Not Equivalent;
necessity, for all; (p@p) 0.

The designated proof value is `T`; `C` is the value for falsity, `N` for truthity, and `F` for contradiction. The 16-valued proof tables are row-major and horizontal.

#((p&q)&r)<s ; `FFFF FFFN FFFF FFFF` (a0)
((p&q)&((p&r)&(r&q)))=(p@p) ; `TTTT TTTF TTTT TTTF` (a1)
((p&(p&q))&q)=(p@p) ; `TTTF TTTF TTTF TTTF` (a2)
(p&p)=(p@p) ; `TFTF TFTF TFTF TFTF` (a3)
(((p&q)=(q&p))=(p@p))>(p=q); `TTTT TTTT TTTT TTTT` (a4)
((#p<s)>(((p@p)&p)=(p@p))) ; `TTTT TTTT TTTT TTTT` (a5)

(a0)>((a4)&(a3)&(a2)&(a1)) (1.1)

Remark: In this evaluation we apply quantified expressions as immediate antecedents implying that quantified as the consequent, rather than invoking the & And connective to distribute the quantifier as a modifier throughout the literal.

(#((p&q)&r)<s) & ((((((p&q)=(q&p))=(p@p))>(p=q)) & ((p&p)=(p@p)))
& ((((p&(p&q))&q)=(p@p)) & (((p&q)&((p&r)&(r&q)))=(p@p)))) ;
`TTTT TTTC TTTT TTTT` (1.2)

Eq. 1.2 is *not* tautologous (proof table off by one value), meaning BCI-algebra is not confirmed.

((a0)>((a4)&(a3)&(a2)&(a1)))>(a5) (2.1)

((#((p&q)&r)<s)&(((((p&q)=(q&p))=(p@p))>(p=q)) &
((p&p)=(p@p)))&((((p&(p&q))&q)=(p@p))&(((p&q)&((p&r)&(r&q)))=(p@p)))))
> (((#p<s)>(((p@p)&p)=(p@p)))) ; `TTTT TTTT TTTT TTTT` (2.2)

Eq. 2.2 is tautologous, confirming the BCK-algebra.

The Bell /CHSH inequalities and Spekken toy model

From: en.wikipedia.org/wiki/Spekkens_Toy_Model (edited)

> The *knowledge balance principle* of the Spekken toy model ensures that any measurement of a system from within itself yields incomplete knowledge of itself. This implies that observable states of a system are epistemic, that is, only relate to the study of knowledge.
>
> The Spekken toy model implicitly assumes that there is an ontic state of a system at any instant, but which is unobserved.
>
> The model can not derive quantum mechanics due to a disparity of model and quantum theory.
>
> The model contains local and noncontextual variables, so based on Bell's theorem [*] the model can not replicate predictions made by quantum mechanics.
>
> The toy model produces strange quantum effects, interpreted in support the epistemic view.

For an elementary system, the four ontic states are p,q,r,s.

LET { 1, 2, 3, 4, |0>, |1>, |+>, |->, |i>, |-i>, *I*/2 }, where *I* is not defined at the link (1)
= { p, q, r, s, t, u, v, w, x, y, , z }

For an elementary system, the four ontic states are p, q, r, s.

These map into 6 qubit states, with + And, = Equivalent, @ Not Equivalent, > Imply, < Not Imply:

LET p+q = t; r+s = u; p+r = v; q+s = w; p+s = x; q+r = y; p+q+r+s = z; (2a)
Derived for: r = (((u-s)+(v-p))+(y-q)); s = (((u-r)+(w-q))+(x-p)); (2b)
All states: (((((p+q)=t)&((r+s)=u))&(((p+r)=v)&((q+s)=w)))&
((((p+s)=x)&((q+r)=y))&(((p+q)+(r+s))=z))) ; (2c)

The knowledge balance principle [**] is satisfied by transformations on the ontic state of the system in permutations of the four ontic states. For example:

(((p&q)&(r&s))&(p+q)) > (p+q) ; (3)
(((p&q)&(r&s))&(p+r)) > (q+s) ; (4)
(((p&q)&((((u-s)+(v-p))+(y-q))&(((u-r)+(w-q))+(x-p))))&(p+r)) > (q+r) ; (5)

The example given of an antiunitary map on Hilbert space is the antecedent of Eq 5:

(((p&q)&((((u-s)+(v-p))+(y-q))&(((u-r)+(w-q))+(x-p))))&(p+r)) ; (6)

For the permutations of the six states below, no single transformation as the antecedent serves as a universal state inverter to imply the properties of these consequents:

(p+q)<(r+s) ; (p+r)<(q+s) ; (p+s)<(q+r) ; (7a)
(r+s)<(p+q) ; (q+s)<(p+r) ; (q+r)<(p+s) ;

We rewrite Eqs 7a by substitution of Eqs 2a as:

(t<u) ; (v<w) ; (x<y) ; (7b)
(u<t) ; (w<v) ; (y<x) ;

We ask if any or all of Eqs 7b are validated as Tautologous, that is, are *not* allowed as implied transformations. This means we test Eqs 7b for each equation as separate and also for all of the equations as combined.

To test Eqs 7b for any equation, we use the Or connective (+) as sum of equations below in Eq 7c:

((((t<u)+(v<w))+((x<y)+(u<t)))+((w<v)+(y<x))) ; (7c)

To test Eqs 7b for all equations, we use the And connective (&) as product of equations below in Eq 7d.

((((t<u)&(v<w))&((x<y)&(u<t)))&((w<v)&(y<x))) ; (7d)

We also note that for Eq 7c, 7d to be complete, we must account for the definitions of variables in Eq 2c. We therefore rewrite Eq 7c, 7d in Eq 7e, 7f below:

((((((p+q)=t)&((r+s)=u))&(((p+r)=v)&((q+s)=w)))&((((p+s)=x)&((q+r)=y))&(((p+q)+(r+s))=z))) &
((((t<u)+(v<w))+((x<y)+(u<t)))+((w<v)+(y<x)))) ; (7e)

((((((p+q)=t)&((r+s)=u))&(((p+r)=v)&((q+s)=w)))&((((p+s)=x)&((q+r)=y))&(((p+q)+(r+s))=z))) &
((((t<u)&(v<w))&((x<y)&(u<t)))&((w<v)&(y<x)))) ; (7f)

Our experiment tests Eqs 7e, 7f for the Truth value of (z=z) in Eqs 8.1,8.2 and Eqs 9.1,9.2.

(z=z) = ((((((p+q)=t)&((r+s)=u))&(((p+r)=v)&((q+s)=w)))&((((p+s)=x)&((q+r)=y))
&(((p+q)+(r+s))=z))) & ((((t<u)+(v<w))+((x<y)+(u<t)))+((w<v)+(y<x)))) ; (8.1)
not validated as tautologous, and contradictory ;

(z=z) > ((((((p+q)=t)&((r+s)=u))&(((p+r)=v)&((q+s)=w)))&((((p+s)=x)&((q+r)=y))
&(((p+q)+(r+s))=z))) & ((((t<u)+(v<w))+((x<y)+(u<t)))+((w<v)+(y<x)))) ; (8.2)
not validated as tautologous, and contradictory ;

(z=z) = ((((((p+q)=t)&((r+s)=u))&(((p+r)=v)&((q+s)=w)))&((((p+s)=x)&((q+r)=y))
&(((p+q)+(r+s))=z))) & ((((t<u)&(v<w))&((x<y)&(u<t)))&((w<v)&(y<x)))) ; (9.1)
not validated as tautologous, and contradictory ;

(z=z) > ((((((p+q)=t)&((r+s)=u))&(((p+r)=v)&((q+s)=w)))&((((p+s)=x)&((q+r)=y))
&(((p+q)+(r+s))=z))) & ((((t<u)&(v<w))&((x<y)&(u<t)))&((w<v)&(y<x)))) ; (9.2)
not validated as tautologous, and contradictory ;

To our question if any or all of Eqs 7b are validated as Tautologous, our answer is no, meaning some or all of Eqs 7b are allowed as transformations. This means that the knowledge based principle, as applied to elementary ontic values and transformations therefrom, is not validated as tautologous.

What follows is that according to the VŁ4 modal propositional logic of Meth8, the Spekken toy model as an epistemic foundation of the quantum model is suspicious.

[*] The CHSH inequality and Bell inequality

1. The CHSI inequality is an acronym for John Clauser, Michael Horne, Abner Shimony, and Richard Holt, and is described at en.wikipedia.org/wiki/CHSH_inequality :

|S| ≤ 2 [= (|S|-1) ≤ 1] where (10)
E = (w-x-y+z)\(w+x+y+z) (11)
S = E(p,q) - E(p,s) +E(r, q) + E(r,s), (12)

LET (|s|-1)≤1) :: (((s-(%s>#s))=(%s>#s))+((s-(%s>#s))<(%s>#s))) ; (13)
LET E :: u = (((w-x)-(y+z))\((w+x)+(y+z))) ; (14)
LET S :: s = u&(((p&q)-(p&s))+((r&q)+(r&s))) ; (15)

((u=(((w-x)-(y+z))\((w+x)+(y+z))))&(s=(u&(((p&q)-(p&s))+((r&q)+(r&s)))))) >
(((s-(%s>#s))=(%s>#s))+((s-(%s>#s))<(%s>#s))) ;

```
TTTT TTTT TTTT TTTT(2),
TTTT TTTT CTCT CCCC(2)
```
(16)

The CHSH inequality is *not* validated as tautologous.

2. The original Bell inequality named after John Stewart Bell is described at en.wikipedia.org/wiki/Bell %27s_theorem :

Ch(a,b) = E(A(a,z),B(b,z)), where z is lower case lambda (17)
[Ch(a,c) - Ch(b,a) - Ch(b,c)] ≤ 1 (18)

LET Ch(a,b) :: (y&(p&z)) = (u&((w&(p&z))&(x&(q&z)))) ; (19)
LET Ch(a,c) :: (y&(p&r)) = (u&((w&(p&z))&(x&(r&z)))) ; (20)
LET Ch(b,c) :: (y&(q&r)) = (u&((w&(q&z))&(x&(r&z)))) ; (1)
LET [Ch(a,c) - Ch(b,a) - Ch(b,c)] :: (2)

(((y&(p&z))=(u&((w&(p&z))&(x&(q&z))))) -
(((y&(p&r))=(u&((w&(p&z))&(x&(r&z))))) -
((y&(q&r))=(u&((w&(q&z))&(x&(r&z))))))) ; `(23a)

We assign this as its own named definition in Eq 23b, preparing for assignment of inequality in Eq 24:

s = (((y&(p&z))=(u&((w&(p&z))&(x&(q&z))))) - (((y&(p&r))=(u&((w&(p&z))&
(x&(r&z))))) - ((y&(q&r))=(u&((w&(q&z))&(x&(r&z))))))) ; `(23a)

LET s ≤ 1 = ~(s > 1) :: ((s<(s\s))+(s=(s\s))) = ~(s>(%s>#s)) ; (24)

(((y&(p&z))=(u&((w&(p&z))&(x&(q&z))))) -
(((y&(p&r))=(u&((w&(p&z))&(x&(r&z))))) -
((y&(q&r))=(u&((w&(q&z))&(x&(r&z))))))) = ~(s>(%s>#s))) ;
not tautologous (25)

Bell's inequality (25) is not validated as tautologous, and it should not be validated as tautologous because the CHSH inequality (16) is not validated as tautologous as an abstraction of (25).

3. We then test the truth relationship between the CHSH inequality and Bell's inequality.

We ask if the more general CHSH inequality implies the more specific Bell's inequality.

(((u=(((w-x)-(y+z))\((w+x)+(y+z))))&(s=(u&(((p&q)-(p&s))+((r&q)+(r&s)))))) >
(((s-(%s>#s))=(%s>#s))+((s-(%s>#s))<(%s>#s)))) >
((((y&(p&z))=(u&((w&(p&z))&(x&(q&z))))) -
(((y&(p&r))=(u&((w&(p&z))&(x&(r&z))))) -
((y&(q&r))=(u&((w&(q&z))&(x&(r&z))))))) = ~(s>(s\s))) ;

```
TFTF TFTT FTFT FTFF(2),
TFTT TFTT NTNF NTNN(2)
```

(26)

The CHSH inequality does not imply Bell's inequality, or vice versa. What follows is that the CHSH inequality and Bell's inequality are not logically related.

This means both inequalities are now suspicious as proofs of Bell's theorem.

This also raises a further, more general doubt that the foundation of quantum mechanics is questionable from the standpoint of system VŁ4.

[**] We note that the term "knowledge balance principle", as defined above at the instant wiki site, was no where else found in the extant quantum literature.

Shortest refutation of Bell's inequality

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET: p, q, r, s: A, B, C, N; ~ Not; & And; + Or; - Not Or;
> Imply, greater than; < Not Imply, less than; = Equivalent; (p=p) proof.

From: emmynoether.com/EAppendixBell.pdf (Leon M. Ledderman), attributed to faraday.physics.utoronto.ca/PVB/Harrison/BellsTheorem/BellsTheorem.html

"This is a special case of a more general statement for ensembles of objects each having three binary attributes, A, B, and C: N(A, not B) + N(B, not C) ≥ N(A, not C). (1.1)
We give the proof of this theorem below. We'll adapt this, following John Bell, in a "physically reasonable way" to our quantum mechanical experiment (a lot of "philosophy" is hidden in the phrase "physically reasonable way")."

Eq. 1.1 means a statement of Bell's inequality.

~(((s&(p&~q))+(s&(q&~r)))<(s&(p&~r)))=(p=p) ;
TTTT TTTT TTFT TFTT (1.2)

Eq. 1.2 is *not* tautologous. This means Eq. 1.1 is not a theorem

Remark: Because N is a counting function, "A, not B" could be construed as "A + not B". If that is the case, then in Eq. 1.2 the inner & connective is replaced with +, but still results in *not* tautologous with the same number of two values of F as contradiction.

~(((s&(p+~q))+(s&(q+~r)))<(s&(p+~r)))=(p=p) ;
TTTT TTTT TTTT FTFT (1.3)

Simplest refutation of Bell's inequality

From: Maccone, L. (2013). "A simple proof of Bell’s inequality". arxiv.org/pdf/1212.5214.pdf

We use the apparatus and method of the modal logic model checker Meth8/VŁ4, a resuscitation and correction of the modal logic system of Łukasiewicz B_4.

The designated *proof* value is T tautology; other values are: N truthity (non contingency); C falsity (contingency); and F contradiction.

With four propositional variables, the 16-valued truth table result is row-major and horizontal.

LET ~ Not; & And; + Or, add; > Imply, greater than; < Not Imply, less than;
= Equivalency; % possibility, for one or some; # necessity, for all;
p probability; (%p>#p) ordinal one, N truthity; (p=p) T tautology, theorem;
~(x>y) not (x greater than y), as in x equal to or less than y.

The summation of the respective probabilities for q equivalent to r, r equivalent to s, and q equivalent to s is equal to or greater than one, and hence is equivalent to a theorem. (1.1)

~((((p&q)=(p&r)) + (((p&r)=(p&s)) + ((p&q)=(p&s)))) < (%p>#p)) = (p=p) ;
NNNN NNNN NNNN NNNN (1.2)

For further qualification to strengthen Eq. 1.1, we rewrite it as:

If the respective probabilities for q, r, s are equivalent to and equal to one, then the summation of the respective probabilities for q equivalent to r, r equivalent to s, and q equivalent to s is equal to or greater than one. (2.1)

(((p&q)=((p&r)=(p&s)))=(%p>#p)) >
~((((p&q)=(p&r)) + (((p&r)=(p&s)) + ((p&q)=(p&s)))) < (%p>#p)) ;
NNNT TTNN TTNN NNTT (2.2)

Eqs. 1.2 and 2.2 as rendered are *not* tautologous. Hence, Bell's inequality as Eqs. 1.1 or 2.1 is refuted.

Refutation of Bell's inequality by the Zermelo-Fraenkel (ZF) axiom of the empty set

Abstract: Bell's inequality is in the form of P(A not B) + P(B not C) ≥ P(A not C. By applying the ZF axiom of the empty set, Bell's inequality takes the form of P(A not B) + P(B not C) ≠ P(A not C). Neither equation is tautologous, with the latter relatively weaker as the negated truth table result of the former. Hence, Bell's inequality and the ZF axiom of the empty set are summarily refuted in tandem.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: P, A, B, C;
~ Not; + Or; & And; > Imply, greater than; < Not Imply, less than;
= Equivalent; @ Not Equivalent; (p=p) `T`autology; ~(p<q) (p≥q).

From: Guz, Kajetan. (2016). The new axiom of set theory and Bell inequality. arxiv.org/ftp/arxiv/papers/1603/1603.08916.pdf (kajetan at guz dot pl).

Bell's inequality is written in the form of an inequality of probabilities:

P(A not B) + P(B not C) ≥ P(A not C) (1.1)

~(((p&(q&~r)) + (p&(r&~s)) < (p&(q&~s))) = (p=p) ;
`TTTT TFTT TTTF TTTT` (1.2)

However, experiments in quantum physics contradict this inequality. All interpretations to date are directed against the thought experiment of Einstein, Podolsky and Rosen (EPR). Attention [was not paid], however, to the imperfection of the mathematical apparatus used to describe quantum reality. Using the new axiom of empty sets we ... present Bell's inequality in a different form. Each of the three sets A, B and C has its respective empty set: $Ø_A$, $Ø_B$ and $Ø_C$ as $PØ_{A\bar{B}} + PØ_{B\bar{C}} \neq PØ_{A\bar{C}}$. Bell's inequality takes the form:

P(A not B) + P(B not C) ≠ P(A not C) (2.1)

((p&(q&~r)) + (p&(r&~s))) @ (p&(q&~s)) ;
`FFFF FTFF FFFT FFFF` (2.2)

Eqs. 1.2 and 2.2 as rendered are *not* tautologous.

This confirms Bell's inequality is refuted on its face by Eq. 1.2.

By applying the ZF axiom of the empty set, this confirms Bell's inequality is also refuted in Eq. 2.2 and fares relatively weaker as the negated truth table result of Eq. 1.2.

What follows by extension is that the axiom of the empty set itself is also *not* tautologous.

The Bell-CHSH inequality refuted as Bogus Bellian logic (BBL)

Abstract: The Bell-CHSH inequality is often touted as S=E(a,b)+E(a',b)+E(a,b')-E(a',b'), ~(2<|S|)=(|S|≦2), and $E=(N_{++}+N_{--}-N_{+-}-N_{-+})/(N_{++}+N_{--}+N_{+-}+N_{-+})$. We confirm this is not tautologous and refute the Bell-CHSH inequality as Bogus Bellian logic (BBL).

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, s, w, x, y, z :E, S, a, a', b, b'; ~ Not; & And; + Or;
> Imply, greater than; < Not Imply, less than; = Equivalent; @ Not Equivalent;
% possibility, for one or some; # necessity, for all or every;
(%p<#p) ordinal two; (p@p) ordinal zero.

For examples of Bell test experiments, the famous formulas often trotted out are:

S=E(a,b)+E(a',b)+E(a,b')-E(a',b') (1.1)

(((p&(w&y))+(p&(x&y)))+((p&(w&z))-(p&(x&z))))=(p=p) ;
TFTF TFTF TFTF TFTF, TTTT TTTT TTTT TTTT (1.2)

with ~(2<|S|) = (|S|≦2). (2.1)

We define the absolute value operator for |S| to mean 0 ≤ S for (2.1.1)

~(s<(p@p)) ; TTTT TTTT FFFF FFFF (2.1.2)

~((%p<#p)<~(s<(p@p)))=(p=p) ; TTTT TTTT NNNN NNNN (2.2)

We substitute Eqs. 1.1 into 2.1. (3.1)

~((%p<#p)<~((((p&(w&y))+(p&(x&y)))+((p&(w&z))-(p&(x&z))))<(p@p)))=(p=p) ;
NNNN NNNN NNNN NNNN, NTNT NTNT NTNT NTNT (3.2)

Remark: Injecting quantifiers onto variables does not help.

Eqs. 1.2, 2.2, and 3.2 are *not* tautologous and *not* equal, establishing bogus Bellian logic (BBL).

LET p, q, r, u, v : E, N_{++}, N_{--}, N_{+-}, N_{-+}.

We further define the experimental estimate E.

$E=(N_{++}+N_{--}-N_{+-}-N_{-+})/(N_{++}+N_{--}+N_{+-}+N_{-+})$ (4.1)

p=(((q+r)-(u-v))\((q+r)+(u+v))) ;
FTFT FTFT FTFT FTFT, TFFT FTFT TFFT FTFT (4.2)

Remark: Including Eq. 4.1 to map explicitly free variables does not help.

We rewrite Eq. 3.1 to include the explicit definition of Eq. 4.1.

If Eq. 4.1, then Eq. 3.1. (5.1)

```
(p=(((q+r)-(u-v))\((q+r)+(u+v))))>
~((%p<#p)<~((((p&(w&y))+(p&(x&y)))+((p&(w&z))-(p&(x&z))))<(p@p))) ;
          NTTN TNTN NTTN TNTN, TTTT TTTT TTTT TTTT,
          NTTT NTTT NTTT TTTT, TNTN TNTN TNTN TNTN,
          NTNT TNTN NTNT TNTN                          (5.2)
```

Eqs. 4.2 and 5.2 are also *not* tautologous, further establishing bogus Bellian logic (BBL), and refuting the Bell-CHSH inequality.

Remark: The assumption of fair sampling as a loophole here is irrelevant because it is not bivalent, but based on a probabilistic vector space.

Refutation of another conjecture to coerce Bell's inequality to be true

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: P, A, B, C;
~ Not; + Or; & And; > Imply, greater than; < Not Imply, less than;
= Equivalent; @ Not Equivalent;
(p=p) Tautology; (p@p) **F** as contradiction, ordinal zero; ~(p<q) (p≥q).

We ask if another conjecture for Bell's inequality as *assumed* is provably true:

$$P(A\wedge\neg B)+P(B\wedge\neg C)\geq P(A\wedge\neg C). \quad (1.1)$$

~(((p&(q&~r))+(p&(r&~s)))<(p&(q@~s)))=(p=p);
TTT**F** TTT**F** TTTT TTTT (1.2)

Remark: Another mapping of Eq. 1.1 by substituting "∧¬" with "Xor" (@) produces
~(((p&(q@r))+(p&(r@s)))<(p&(q@s)))=(p=p) ;
TTTT T**F**TT TTT**F** TTTT (no.go)

Eq. 1.2 as rendered is *not* tautologous. This means another conjecture to prove Bell's inequality is refuted.

Refutation of the coin toss proof for conjectures of the Bell-CHSH inequalities

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: A, A′, B, B′;
~ Not; + Or; > Imply; < Not Imply; = Not Or; = Equivalent;
% possibility, for one or some; # necessity, for every or all;
(%p>#p) ordinal 1; (%p<#p) ordinal 2; ~(y<x) (x≤y).

From: Gill, R.D. (2014). Statistics, causality and Bell's theorem. arxiv.org/pdf/1207.5103.pdf

Based on coin tossing, Bell's inequality and the CHSH inequality are presented for any four numbers A, A′, B, B′ each equal to ±1 as:

Fact 1. Bell's inequality: $AB+AB'+A'B-A'B'=\pm 2$. (1.1.1)

((p=(q=(r=s)))=((%p>#p)+~(%p>#p)))>((((p&r)+(p&s))+((q&r)-(q&s)))=
((%p<#p)+~(%p<#p))) ; TTTT TT**F**T TT**F**T TTTT (1.1.2)

Proof. Notice that $AB+AB'+A'B-A'B'=A(B+B')+A'(B-B')$. B and B′ are either equal to one another or unequal. In the former case, $B-B'=0$ and $B+B'=\pm 2$; [or] in the latter case $B-B'=\pm 2$ and $B+B'=0$. Thus, $AB+AB'+A'B-A'B'$ equals either A or A′, (1.2.1)
both of which equal ±1, times ±2. All possibilities lead to $AB+AB'+A'B-A'B'=\pm 2$.

((p=(q=(r=s)))=((%p>#p)+~(%p>#p)))>(((((r+s)=(p@p))&((r-s)=((%p<#p)+ ~(%p<#p))))+
(((r+s)=((%p<#p)+~(%p<#p)))&((r-s)=(p@p))))>(((p&(r+s))+
(q&(r-s)))=((%p<#p)+~(%p<#p)))) ; **F**TTT TT**F**T TT**F**T **F**TTT (1.2.2)

Fact 2. CHSH inequality: $\langle AB\rangle+\langle AB'\rangle+\langle A'B\rangle-\langle A'B'\rangle\leq 2$. (2.1)

((p=(q=(r=s)))=((%p>#p)+~(%p>#p)))>~((((p&r)+(p&s))+((q&r)-(q&s)))>
(%p<#p)) ; NTTN TN**F**T TN**F**T NTTN (2.2)

Eqs. 1.1.2, 1.2.2, and 2.2 as rendered are *not* tautologous. This means the coin-toss proof for conjectures of the Bell-CHSH inequalities is refuted.

Remark: Eq. 1.2.2 as a defective proof invoking induction was repeated unawares.

Refutation of Bell's original inequality using its assumption

Abstract: Bell's original inequality from 1964 assumed ranges for probabilities of averages. We show that with or without this assumption Bell's inequality is *not* tautologous and hence refuted.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: P, a, b, c; ~ Not; & And; + Or; > Imply, greater than;
< Not Imply, less than; = Equivalent; @ Not Equivalent;
(%p>#p) ordinal 1; (p@p) ordinal 0; (p=p) `T`;
~(y<x) x≤y; ~(x<(p@p)) | x |, (0 ≤ x).

From: Bell, J. S. (1964). On the Einstein Podolsky Rosen paradox. *Physics* 1, 195-200.
cds.cern.ch/record/111654/files/vol1p195-200_001.pdf

We assume Bell's range definitions of (-1≤ P(a,b) ≤1), (-1≤ P(a,c) ≤1), and (-1≤ P(a,c) ≤1). (1.1)

We take the probability expression w≤ P(x,y) ≤z to mean (w≤ P(x,y) Xor (P(x,y)≤ z) so as to distinguish (x,y) as separate, non equivalent vectors.

((~((p&(q&r))<~(%p>#p))@~((%p>#p)<(p&(q&r)))) &
(~((p&(q&s))<~(%p>#p))@~((%p>#p)<(p&(q&s))))) &
(~((p&(r&s))<~(%p>#p))@~((%p>#p)<(p&(r&s)))) ;
`NNNN NNNN NNNN NNNN` (1.2)

%p>#p ; `NNNN NNNN NNNN NNNN` (1.3)

Because the truth table in Eq. 1.2 is equivalent to that of (%p>#p) in Meth8/VŁ4 in Eq. 1.3, we substitute the shorter token in equations. Eq. 1.1, substituting Eq. 1.3, is the antecedent in Bell's conjecture.

Bell's original inequality as 1+P(b,c) ≥ | P(a,b) - P(a,c) |. (3.0)

This is equivalent to | P(a,b)-P(a,c) | - P(b,c) ≤ 1, which is the consequent in Bell's conjecture . (3.1)

~((%p>#p)<~((~(((p&(q&r))-(p&(q&s)))<(p@p))-(p&(r&s)))<(p@p)))=(p=p) ;
`CCCC CCCT CCCT CTCT` (3.2)

We map the conjecture of Eq. 1.1 to imply the antecedent of Eq. 3.1. (4.1)

(%p>#p) > ~((%p>#p)<~((~(((p&(q&r))-(p&(q&s)))<(p@p))-(p&(r&s)))<(p@p))) ;
`CCCC CCCT CCCT CTCT` (4.2)

Eq. 4.2 as rendered is *not* tautologous, nor is Eq. 3.2, hence refuting Bell's original inequality with or without its assumption.

Refutation of Bell's original inequality from 1964 with assumptions

Abstract: Bell's original inequality from 1964 is *not* tautologous and hence refuted.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

> LET p, q, r, s: P, a, b, c; ~ Not; & And; + Or; > Imply, greater than;
> < Not Imply, less than; = Equivalent; @ Not Equivalent;
> (%p>#p) ordinal 1; (p@p) ordinal 0; (p=p) T;
> ~(y<x) x≤y; ~(x<(p@p)) | x |, (0 ≤ x).

From: Bell, J. S. (1964). On the Einstein Podolsky Rosen paradox. *Physics* 1, 195-200.
cds.cern.ch/record/111654/files/vol1p195-200_001.pdf

Bell's original inequality is 1+P(b,c) ≥ | P(a,b) - P(a,c) |. (1.0)

This is equivalent to | P(a,b)-P(a,c) | - P(b,c) ≤ 1. (1.1)

> ~((%p>#p)<~((~(((p&(q&r))-(p&(q&s)))<(p@p))-(p&(r&s)))<(p@p)))=(p=p) ;
> CCCC CCCT CCCT CTCT (1.2)

Bell also makes there assumptions from his text:

P(a,b) = ± 1 (2.1.1)

> (p&(q&r))=((%p>#p)+~(%p>#p)) ; **FFFF FFF**T **FFFF FFF**T (2.1.2)

P(a,b) = ± 1 (2.2.1)

> (p&(q&s))=((%p>#p)+~(%p>#p)) ; **FFFF FFFF FFF**T **FFF**T (2.3.2)

P(a,b) = ± 1 (2.3.1)

> (p&(r&s))=((%p>#p)+~(%p>#p)) ; **FFFF FFFF FFFF F**T**F**T (2.3.2)

We substitute Eqs. 2.1.1, 2.2.1, and 2.3.1 into 1.1. (3.1)

> ~((%p>#p)<~((~((((p&(q&r))=((%p>#p)+~(%p>#p)))-((p&(q&s))=((%p>#p)+~(%p>#p))))<(p@p))-((p&(q&r))=((%p>#p)+~(%p>#p))))<(p@p)))=(p=p) ;
> CCCC CCCT CCCT CCCT (3.2)

Eq. 1.2 as rendered and 3.2 are *not* tautologous, hence refuting Bell's inequality with assumptions.

Refutation of conjectures for Bell's original inequality and CHSH inequality

Abstract: Bell's original conjecture of inequality (Ch(a,c)−Ch(b,a)−Ch(b,c)≤1) and the subsequent CHSH conjecture of inequality (Ch(a,b)+Ch(a,b′)+Ch(a′,b)−Ch(a′,b′)≤2), collectively known as the "Bell inequality" and "Bell-CHSH inequality", are respectively proved *not* tautologous and both *not* equivalent.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: C_h, a, b, c;
~ Not; + Or; - Not Or; & And; > Imply, greater than; < Not Imply, less than;
= Equivalent; (p=p) Tautology; (%p>#p) ordinal 1; (%p<#p) ordinal 2;
~(p>q) (p≤q).

From: en.wikipedia.org/wiki/Bell's_theorem

Original Bell's inequality: C_h(a,c)−C_h(b,a)−C_h(b,c)≤1 (1.1)

~(((p&(q&s))-((p&(r&q))-(p&(r&s))))>(%p>#p))=(p=p) ;
FFFF FFFC **FFFF F**C**FF** (1.2)

Remark: In Eq. 1.1 if "≤1" is read as "≤T", the result is *contradictory* (all **F**'s).

CHSH inequality: generalizing Bell's original inequality, John Clauser, Michael Horne, Abner Shimony, and R.A. Holt introduced the CHSH inequality which puts classical limits on the set of four correlations in Alice and Bob's experiment, without any assumption of perfect correlations (or anti-correlations) at equal settings

C_h(a,b)+C_h(a,b′)+C_h(a′,b)−C_h(a′,b′)≤2. [2.4.1]

Making the special choice a′=b+π, denoting b′=c, and assuming perfect anti-correlation [2.1.1]
at equal settings, perfect correlation at opposite settings, therefore ρ(a,a+π)=1 [2.2.1]
and ρ(b,a+π)=−ρ(b,a), [2.3.1]
the CHSH inequality reduces to the original Bell inequality. [3.1]

Nowadays, [2.4.1] is also often simply called "the Bell inequality", but sometimes more completely "the Bell-CHSH inequality".

LET p, q, r, s, t, u, v, w: C_h, a, b, c, a′, b′, ρ, π

a′=b+π, denoting b′=c:

(t=(r+w))>(u=s) ;
TTTT TTTT **FFFF** TTTT, TTTT TTTT TTTT **FFFF**,
FFFF TTTT TTTT TTTT, TTTT **FFFF** TTTT TTTT,
TTTT TTTT **FFFF FFFF**, TTTT TTTT TTTT TTTT,
FFFF FFFF TTTT TTTT (2.1.2)

$\rho(a,a+\pi)=1$:

(v&(q&(q+w)))=(%p>#p) ;
CCCC CCCC CCCC CCCC, CCNN CCNN CCNN CCNN (2.2.2)

$\rho(b,a+\pi)=-\rho(b,a)$:

(v&(r&(q+w)))=~(v&(r&q)) ;
FFFF FFFF FFFF FFFF, FFFF TT**FF FFFF** TT**FF** (2.3.2)

$C_h(a,b)+C_h(a,b')+C_h(a',b)-C_h(a',b')\leq 2$:

~(((((t=(r+w))>(u=s))&(((v&(q&(q+w)))=(%p>#p))&((v&(r&(q+w)))=
~(v&(r&q)))))>((((p&(q&r))+(p&(q&u)))+(p&(t&r)))-(p&(t&u))))>
(%p<#p))=(p=p) ; NNNN NNNN NNNN NNNN (2.4.2)

$(C_h(a,c)-C_h(b,a)-C_h(b,c)\leq 1)=(C_h(a,b)+C_h(a,b')+C_h(a',b)-C_h(a',b')\leq 2)$:

(~(((p&(q&s))-((p&(r&q))-(p&(r&s))))>(%p>#p))=(p=p))=(~(((((t=(r+w))>
(u=s))&(((v&(q&(q+w)))=(%p>#p))&((v&(r&(q+w)))=~(v&(r&q)))))>
((((p&(q&r))+(p&(q&u)))+(p&(t&r)))-(p&(t&u))))>(%p<#p))=(p=p)) ;
CCCC CCCF CCCC CFCC (3.2)

Eq. 1.2 as rendered is *not* tautologous. This means the conjecture of Bell's original inequality is refuted.

Eq. 2.4.3 is *not* tautologous. This means the conjecture of CHSH inequality is refuted.

Eq. 3.2 is *not* tautologous. This means the conjecture of CHSH inequality reducing to Bell's original inequality is refuted.

Another refutation of Bell's inequality by positive reasons

Abstract: Bell's inequality as defined by P(A&~B)+P(B&~C)-P(A&~C)=P(A&~B&C)+P(B&~C&~A)≥0 is refuted as TTT**F** TTT**F** TTTT TTTT.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: P, A, B, C;
~ Not; + Or; - Not Or; & And; > Imply, greater than; < Not Imply, less than;
= Equivalent; @ Not Equivalent;
(p=p) Tautology; (p@p) **F** as contradiction, ordinal zero; ~(p<q) (p≥q).

From: [Academic's name purposely suppressed by the instant author.] (2018). Email communication.

There is also an easy proof for (Eq. 6.1, Bell's inequality), which provides positive reasons for believing it to be a tautology by writing:

P(A & ~B) = P(A & ~B & C) + P(A & ~B & ~C) (1.1)

(p&(q&~r))=((p&((q&~r)&s))+(p&((q&~r)&~s)));
TTTT TTTT TTTT TTTT (1.2)

P(A & ~C) = P(A & ~C & B) + P(A & ~C & ~B) [= P(A & B & ~C) + P(A & ~B & ~C)]
(2.1)

(p&(q&~s))=((p&((q&~s)&r))+(p&((q&~s)&~r)));
TTTT TTTT TTTT TTTT (2.2)

P(B & ~C) = P(B & ~C & A) + P(B & ~C & ~A) [= P(A & B & ~C) + P(~A & B & ~C)]
(3.1)

(p&(r&~s))=((p&((r&~s)&q))+(p&((r&~s)&~q)));
TTTT TTTT TTTT TTTT (3.2)

Then calculate

P(A & ~B) + P(B & ~C) - P(A & ~C) = [= P(A & ~B) + P(B & ~C) - P(A & ~C))]
(4.1.1)

(((p&(q&~r))+(p&(r&~s)))-(p&(q&~s)))=(p=p);
TTT**F** T**F**T**F** TTT**F** TTTT (4.1.2)

By substitution from Eqs. 1.2, 2.2, 3.2:

((((p&((q&~r)&s))+(p&((q&~r)&~s)))+((p&((r&~s)&q))+(p&((r&~s)&~q))))-((p&((q&~s)&r))+(p&((q&~s)&~r))))=(p=p) ;
TTT**F** T**F**T**F** TTT**F** TTTT (4.1.3)

P(A & ~B & C) + P(B & ~C & ~A) (4.2.1)

((p&((q&~r)&s))+(p&((r&~s)&~q)))=(p=p) ;
FFFF FTFF FFFT FFFF (4.2.2)

P(A & ~B) + P(B & ~C) - P(A & ~C) ≥ 0 (5.1.1)

(~(((p&(q&~r))+(p&(r&~s)))-(p&(q&~s)))<(p@p))=(p=p) ;
FFFT FTFT FFFT FFFF (5.1.2)

P(A & ~B & C) + P(B & ~C & ~A) ≥ 0 (5.2.1)

(~((p&((q&~r)&s))+(p&((r&~s)&~q)))<(p@p))=(p=p) ;
TTTT TFTT TTTF TTTT (5.2.2)

P(A & ~B) + P(B & ~C) - P(A & ~C) = P(A & ~B & C) + P(B & ~C & ~A) ≥ 0 (6.1)

(~ ((((p&(q&~r))+(p&(r&~s)))-(p&(q&~s)))=((p&((q&~r)&s))+(p&((r&~s)&~q))))<(p@p))=(p=p) ;
TTTF TTTF TTTT TTTT (6.2)

Eq. 6.2 as rendered is *not* tautologous. This means the conjecture by positive reason proof for Bell's inequality is refuted.

Refutation of Bell's theorem for temporal logic

Abstract: We evaluate Bell's theorem for temporal logic. It is *not* tautologous. Hence schematics of a protocol for a violation of Bell's inequalities for temporal order are similarly moot. These conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Zych, M.; Costa, F.; Pikovski, I.; Brukner, Č. (2019). Bell's theorem for temporal order. nature.com/articles/s41467-019-11579-x, arxiv.org/pdf/1708.00248.pdf m.zych@uq.edu.au

B. Bell’s theorem for temporal order The scenario for which the theorem is formulated involves a bipartite system ...

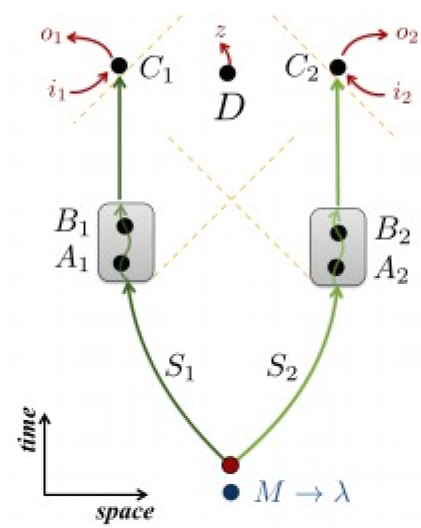

FIG. 2: Bell’s theorem for temporal order. A bipartite system, made of subsystems S_1 and S_2, is sent to two groups of agents. Operations on S_1 (S_2) are performed at events A_1, B_1 (A_2, B_2). At event C_1 (C_2), a measurement with setting i_1 (i_2) and outcome o_1 (o_2) is performed. Events A_1, B_1 are space-like separated from A_2, B_2[,] and C_1 is space-like to C_2; light cones are marked by dashed yellow lines. … The system M is measured at event D, producing an output bit z. If the initial state of the systems S_1, S_2, M is separable, and λ is a classical variable … , the resulting bipartite statistics of the outcomes o_1, o_2 cannot violate any Bell inequality, even if conditioned on z. (1.0)

LET p, q, r, s, t, u, v, w: A, B, C, S, D, i, o, z.

((s&(%s>#s))>(((p&(%s>#s))&(q&(%s>#s)))>((r&(%s>#s))>(((u&(%s>#s))>(v&(%s>#s)))>(t>z)))))=
((s&~(%s>#s))>(((p&~(%s>#s))&(q&~(%s>#s)))>((r&~(%s>#s))>(((u&~(%s>#s))>(v&~(%s>#s)))>(t>z))))) ;

```
TTTT TTTT TTTT TTTT( 1)    }x4
TTTT TTTT TTTT TTTF( 1)    }
TTTT TTTT TTTT TTTT( 2)    }
TTTT TTTT TTTT TTTF( 1)}x3}
TTTT TTTT TTTT TTTT( 1)}   }
TTTT TTTT TTTT TTTT( 2)    }
TTTT TTTT TTTT TTTT( 1)}x2}
TTTT TTTT TTTT TTTF( 1)}   }
TTTT TTTT TTTT TTTT(64)
```
(1.2)

Eq. 1.2 as rendered is *not* tautologous. Hence schematics of a protocol for a violation of Bell's inequalities for temporal order are similarly moot**.**

Refutation of the tropical sum for Bell's theorem

Abstract: We evaluate the tropical sum definition to show topped summing is refuted by mathematical logic and hence cannot occur in physics realty.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

From: Geurdes, H. (2018). On Bell's experiment. vixra.org/pdf/1811.0247v1.pdf han.geurdes@gmail.com

LET p, q: x, y;
~ Not; + Or; = Equivalent; @ Not Equivalent;
> Imply, greater than; < Not Imply, lesser than;
((p+q)<(p@p))>~((p+q)<(p@p)) (|x+y|<1)

Tropical sum. Let us define the tropical algebra sum on real, i.e. R ∩ [−1, 1], values for *x* and *y*. We define

$$x \oplus y = \{\, x + y,\ |x + y| < 1;\ \ +1, x + y > 1;\ \ -1, x + y < -1\} \qquad (7.1)$$

We note that the summation in (7.1) is allowed. If readers disagree they have to *prove* that this way of topped summing cannot for sure occur in physics reality.

(p@q)=(((((((p+q)<(p@p))>~((p+q)<(p@p)))<(%p>#p))>(p+q))+
(((p+q)<(%p<#p))>(%p<#p)))+(~((%p>#p)>(p+q))>(p+q))) ;
FTT**F** **F**TT**F** **F**TT**F** **F**TT**F** (7.2)

Remark 7.2: Eq. 7.2 as rendered is *not* tautologous. This means topped summing is refuted by mathematical logic to occur in physics realty.

Solution proof of Bellman's Lost in the forest problem for triangles

Abstract: From the area and dimensions of an outer triangle, the height point of an inner triangle implies the minimum distance to the outer triangle. This proves the solution of Bellman's Lost in the forest problem for triangles. By extension, it is the general solution proof for other figures.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) `C` as contingency, Δ, ordinal 1; (%z>#z) `N` as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Williams, S.W. (2000). Million buck problems.
math.buffalo.edu/~sww/0papers/million.buck.problems.mi.pdf sww@buffalo.edu [bounced]

12. **Lost in a Forest Problem:** In 1956 R. Bellman asked the following question: Suppose that I am lost without a compass in a forest whose shape and dimensions are precisely known to me. How can I escape in the shortest possible time? Limit answers to this question for certain two-dimensional forests; planar regions. ... For many plane regions the answer is known: circular disks, regular even sided polygonal regions, half-plane regions (with known initial distance), equilateral triangular regions. However, for some regions, for regular odd-sided polygonal regions in general and triangular regions in particular, only approximates to the answer are known.

R. Bellman, *Minimization problem*. Bull. Amer. Math. Soc. **62** (1956) 270.
J. R. Isbell, *An optimal search pattern*. Naval Res. Logist. Quart. **4** (1957) 357-359.
Web survey and reference article: http://www.mathsoft.com/asolve/forest/forest.html
[The link above as published maps to ptc.com which apparently hijacked that link.]

We proceed to define a triangle area for A = base * height / 2. (1.1.0)

LET p left-side base to height-r point,
q right-side base from height-r point,
r height-r point,
s height-s point,
t left-side base to height-s point,
u right-side base from height-s point.

t>p: s = s+(r-s)/2

```
                r
                |
                |      s
                |      |
._______p_______.______|________q________________.
._t____________________.___u_____________________.
```

t>p: s = s+(r-s)/2

```
                r
                |
                |
                |                      s
._______p_______._______________q______|_________.
._t____________________________________.___u_____.
```

t<p: t = t-(p-t)/2

```
                r
                |
            s   |
            |   |
._______p___|___._______________q________________.
._t_________.______________u_____________________.
```

t<p: t = t-(p-t)/2

```
                r
                |
                |
     s          |
.____|__p_______._______________q________________.
._t__._____________________u_____________________.
```

We map Eq. 1.1.1 with height-r point as the larger area triangle equivalent to the two smaller area triangles. (1.1.1)

((p+q)&(r\(%z<#z)))=((p&(r\(%z<#z)))+(q&(r\(%z<#z)))) ;

TTTT TTTT TTTT TTTT(128) (1.1.2)

The height-s triangle inside the outer triangle, maps respectively into two smaller triangles. (1.2.1)

((t+u)&(s\(%z<#z)))=((t&(s\(%z<#z)))+(u&(s\(%z<#z)))) ;

TTTT TTTT TTTT TTTT(128) (1.2.2)

The bases of height-r and height-s triangles are equivalent, so we define the base of the larger height-r triangle in terms of the base for the smaller height-s triangle, and vice versa. The facts that height-s is less than or equal to height-r, and that base t is lesser than or equal to base p+q are mapped as the antecedent below. (1.3.1)

```
(~(r<s)&~((p+q)<t))>(((p+q)=(t+u))>(((p=((t+u)-q))&
(q=((t+u)-p)))=((t=((p+q)-u))&(u=((p+q)-t))))) ;
                                        TTTT TTTT TTTT TTTT(128)      (1.3.2)
```

Assuming Eq. 1.3.2, for the inside height-s triangle components, its triangle with the smaller area implies the shortest path to the outside height-r triangle. The shortest path to the edge of the height-r triangle is then the smaller value of s or t, with the direction as vertical for s or horizontal for t. (1.4.1)

```
((~(r<s)&~((p+q)<t))&
(((p+q)=(t+u))>(((p=((t+u)-q))&(q=((t+u)-p)))=((t=((p+q)-u))&(u=((p+q)-t))))))>
(((t<p)>(t=((u-p)\(%z<#z))))+(~(p<t)>(s=(s+((r-s)\(%z<#z)))))) ;
                                        TTTT TTTT TTTT TTTT(128)      (1.4.2)
```

Bell's forest problem is solved for triangles with the Eq. 1.4.2 as tautologous.

Berkeley's paradox

From: http://lesswrong.com/lw/nr/the_argument_from_common_usage/

"Berkeley's paradox. Tautologically, nobody has ever heard a tree fall that nobody heard. (Planting a tape recorder or radio transmitter and listening to that counts as hearing it.)" (1)

This is rewritten in a simpler format, excluding falling trees, but folding in the caveat of tape recorder:

> "No one heard the sound that no one heard: a tape recorder counts as hearing sound." (2)

LET: p hearing person; ~p no hearing person; s sound
~ Not; > Imply (hearing); < Not Imply (not hearing);
vt tautologous; nvt not tautologous

> "No person heard the sound of either what no person heard (no tape recorder sound) or what no person did not hear (tape recorder sound)." (3.1)

~p>(s=((~p>s)+(~p<s))) ; nvt (3.2)

```
Model 1              Model 2.1            Model 2.2            Model 2.3.1          Model 2.3.2
FTFT FTFT TTTT TTTT  UEUE UEUE EEEE EEEE  UEUE UEUE EEEE EEEE  UEUE UEUE EEEE EEEE  UEUE UEUE EEEE EEEE
```

Berkeley's paradox is not tautologous, so it is not a paradox.

The difficulty is in understanding exactly the meaning of the original paradox statements because there are two, and they are linked.

For example, these two logical expressions seem to be equivalent, but Meth8 based on VŁ4 shows they are in fact not:

> "No person heard either the sound no person heard or the sound no person did not hear." (4.1)

~p>((~p>s)+(~p<s)) ; vt (4.2)

> "No person heard the sound of either the sound no person heard or he sound no person did not hear." (5.1)

~p>(s=((~p>s)+(~p<s))) ; nvt [Eq 3.2 is the same.] (5.2)

In Eq 4.1 for "heard either the sound" reads in Eq 5.1 "heard the sound of either the sound". Hence Eq 5.1 further clarifies the sound as either that sound heard by no one or that sound not heard by no one.

Refutation of tensor product and Bernstein-Vazirani algorithm

We assume the apparatus and method of Meth8/VŁ4. The designated *proof* value is T. Result tables are in row-major and presented horizontally.

1. We initially evaluate the tensor product operation.

From en.wikipedia.org/wiki/Kronecker_product (relation to the abstract tensor product),

> v, w, x, y are vector spaces; linear transformations are s=(v>x) and t=(w>y); and
> the tensor product symbol ⊗ is taken as @, to mean Not Equivalent, the XOR operator.

The abstract tensor product of linear maps is:

> ((s=(v>x))&(t=(w>y)))>((s@t)=((v@w)>(x@y))) ;
> repeated tables of: TTTT TTTT TTTT TTTT, ... , TTTT TTTT FFFF FFFF (1.2)

By substitution for s and t, we rewrite Eq. 1.2.

> (((v>x)@(w>y))>(((v@w)>(x@y)) ; (2.2)

We cast Eq. 2.2 into the four variable version of Meth8/VL4 for the brevity of 16-valued result tables.

> LET p q r s: v, w, x, y
>
> ((p>r)&(q>s))=((p@q)>(r@s)) ; TTTF TTFF TFTF TFFT (3.2)

From Eq. 3.2 as rendered, the tensor product operation is *not* tautologous. This was expected because vector spaces are not bivalent but probabilistic.

2. We next evaluate the Bernstein-Vazirani algorithm in two variables.

From: Krishna, R.; Makwanay, V.; Suresh, A. (2016). "A generalization of Bernstein-Vazirani algorithm to qudit systems". arxiv.org/pdf/1609.03185.pdf

> "in a tensor product of two quantum states we are free to associate the sign with whichever state we choose to. $|u\rangle \otimes (-|v\rangle) = -(|u\rangle \otimes |v\rangle) = (-|u\rangle) \otimes |v\rangle$ (4.1)
>
> LET p q: |u〉; |v〉; = Equivalent; @ Not Equivalent; ~ Not
>
> (p@~q) = (~(p@q) = (~p@q)) ; TFFT TFFT TFFT TFFT (4.2)

Eq. 4.2 as rendered is *not* tautologous, hence Bernstein-Vazirani is refuted.

> **Remark:** Eq. 4.2 coerces a tautology with the Imply connective: (p@~q)>(~(p@q)=(~p@q)). However, that violates the strength of the Bernstein-Vazirani algorithm as based on the Equivalent connective. The other replacement of the Imply connective does *not* coerce a tautology: ((p@~q)=~(p@q))>(~p@q), with the result table of Eq. 4.2.

Refutation of the no-cloning theorem in statistical models

Abstract: The assumptions comprising the conjecture of the no-cloning theorem on statistical models is refuted. What follows is that the no-cloning theorem itself is also refuted.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: $\langle\varphi|\psi\rangle$, $\langle\varphi|\psi\rangle^2$, $\langle\varphi|\psi\rangle^4$, s;
~ Not; + Or, ∨; - Not Or; & And, ∧; > Imply;
% possibility, for any one or some, ∃; # necessity, for every or all, ∀.
(s=s) T tautology; (s@s) **F** contradiction;
(%s>#s) 1, N truthity; (%s<#s) 0, C falsity;

From: Nagata, K.; Nakamura, T. (2018). The no-cloning theorem based on a statistical model.
vixra.org/pdf/1809.0552v1.pdf

The following assumptions are made:

$\langle\varphi|\psi\rangle^2 = 0 \vee \langle\varphi|\psi\rangle^2 = 1$ (5.1)

q=(((s@s)+q)=(%s>#s)) ; NNNN NNNN NNNN NNNN (5.2)

$\langle\varphi|\psi\rangle^4 = 0 \vee \langle\varphi|\psi\rangle^2 = 1$ (6.1)

r=(((s@s)+r)=(%s>#s)) ; NNNN NNNN NNNN NNNN (5.2)

$\langle\varphi|\psi\rangle = 1 \wedge \langle\varphi|\psi\rangle^4 = 0$ (23.1)

p=(((%s>#s)&r)=(s@s)) ; FTFT NCNC FTFT NCNC (23.2)

Eqs. 5.1, 6.1, and 23.1 as rendered are *not* tautologous. This refutes those assumptions and the conjecture of the no-cloning theorem on statistical models. What follows is that the no-cloning theorem itself is also refuted.

Refutation of the Bertrand postulate and Bertrand-Chebyshev theorem

We assume the apparatus and method of Meth8/VŁ4, with the designated proof value of T and truth tables as row-major, horizontally.

From: en.wikipedia.org/wiki/Bertrand%27s_postulate, the Bertrand postulate:

[F]for every n > 1, there is always at least one prime p such that n < p < 2 n. (1.1)

LET: pq pn; (%p>#q) 1; (%p<#q) 2

#(q>(%q>#q)) > %((q<p)&~ (p>((%q<#q)&q))) ;

CCCC CCCC CCCC CCCC (1.2)

From: proofwiki.org/wiki/Bertrand-Chebyshev_Theorem, Bertrand-Chebyshev theorem:

For all *n*∈N>0, there exists a prime *number p* with *n*<*p*≤2*n*. (2.1)

LET: r N; ~(q<p) p≤q

(q<r)>%((q<p)&~(p>((%q<#q)&q))) ;

TTCC TTTT TTCC TTTT (2.2)

Eqs. 1.2 and 2.2 as rendered are *not* tautologous, meaning both Bertrand expressions are suspicious.

Refutation of the axiomatic theory of betweenness

Abstract: We evaluate sixteen equations as axioms and conclusions, for nine as *not* tautologous. This refutes the axiomatic theory of betweenness.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⇔, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z<#z) C non-contingency, ∇, ordinal 2; (%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).

From: Azimipour, S.; Naumov, P. (2019). Axiomatic theory of betweenness. arxiv.org/pdf/1902.00847.pdf pgn2@cornell.edu

4. Axioms
For any given set V , our axiomatic system consists of the following axioms in the language Φ(V):

1. Trivial Path: ¬(A|B|C) if A∩C≠Ø (4.1.1)

LET p, q, r: A, B, C

((p&r)@(p@p))>~((p<q)<r) ; TTTT TTTT TTTT TTTT (4.1.2)

2. Empty Set: Ø |B|C (4.2.1)

LET q, r: B, C

(p@p)<(q<r) ; **FFFF FFFF FFFF FFFF** (4.2.2)

3. Aggregation: A_1|B|C→(A_2|B|C→A_1,A_2|B|C) (4.3.1)

LET p, q, r, s: A_1, A_2, B, C

((p<r)<s)>(((q<r)<s)>((p&q)<(r<s))) ;
TTTT TTTT TTTT TTTT (4.3.2)

4. Symmetry: A|B|C→C|B|A (4.4.1)

LET p, q, r: A, B, C

((p<q)<r)>((r<q)<p) ; T**F**TT TTTT T**F**TT TTTT (4.4.2)

5. Left Monotonicity: $A_1,A_2|B|C \rightarrow A_1|B|C$ (4.5.1)

LET p, q, r, s: A_1, A_2, B, C

`((p&q)<(r<s))>(p<(r<s))` ; TTTT TTTT TTTT TTTT (4.5.2)

6. Central Monotonicity: $A|B_1|C \rightarrow A|B_1,B_2|C$ (4.6.1)

LET p, q, r, s: A, B1, B2, C

`(p<(q<s))>((p<(q&r))<s)` ; TTTT TTTT T**F**T**F** T**F**T**F** (4.6.2)

7. Insertion: $A|B_1,I,B_2|C \rightarrow (A|I,C|B_1 \rightarrow (B_2|A,I|C \rightarrow A|I|C))$ (4.7.1)

LET p, q, r, s, t, u: A, I, B, C, B_1, B_2

`(p<((t&(q&u))<s))>((p<((q&s)<t))>((u<((p&q)<s))>(p<~(s>q))))` ;
TTTT TTTT TTTT TTTT(2),
TTTT TTTT T**F**TT T**F**TT(2) (4.7.2)

8. Transitivity: $\neg(A|B|d) \rightarrow (\neg(d|B|C) \rightarrow \neg(A|B|C))$, where $d \notin B$ (4.8.1)

LET p, q, r, s: A, B, C, d

`(s<q)>(~((p<q)<s)>(~(s<(q<r))>~((p<q)<r)))` ;
TTTT TTTT TTTT TTTT (4.8.2)

In the above axioms by A; B we denote the union of sets A and B. Note that we represent union by comma only inside [the] betweenness predicate. In all other setting[s], to avoid confusion, we use standard notations A [∩] B.

3. Conclusion
With minimal modifications to the proofs given in this article, one can show the following logical system completely axiomatizes the non-strict betweenness relation:

1. Trivial Path: $\neg(A|B|C)$ if $(A \cap C) \setminus B \neq \emptyset$ (7.1.1)

LET p, q, r: A, B, C

`(((p&r)\q)@(p@p))>~((p<q)<r)` ; T**F**TT TTTT T**F**TT TTTT (7.1.2)

Remark 7.1: Eq. 7.1.2 differs from 4.1.2.

2. Empty Set: $\emptyset\,|B|C$ (7.2.1)

LET q, r: B, C

`(p@p)<(q<r)` ; **FFFF FFFF FFFF FFFF** (7.2.2)

Remark 7.2: Eq. 7.2.2 is the same as 4.2.2.

3. Reflexivity A|B|C, where A⊆B (7.3.1)

LET p, q, r: A, B, C

~(q<p)>((p<q)<r) ; `TTTT TTTT TTTT TTTT` (7.3.2)

Remark 7.3: Eq. 7.3.2 is not included in the list of 4. Axioms.

4. Aggregation: A_1|B|C→(A_2|B|C→A_1,A_2|B|C) (7.4.1)

LET p, q, r, s: A_1, A_2, B, C

((p<r)<s)>(((q<r)<s)>((p&q)<(r<s))) ;
`TTTT TTTT TTTT TTTT` (7.4.2)

Remark 7.4: Eq. 7.4.2 is renamed for 4.3.2.

5. Symmetry: A|B|C→C|B|A (7.5.1)

LET p, q, r: A, B, C

((p<q)<r)>((r<q)<p) ; T**F**TT TTTT T**F**TT TTTT (7.5.2)

Remark 7.5: Eq. 7.5.2 is renamed for 4.4.2.

6. Monotonicity: A_1,A_2|B|C→A_1|B|C (7.6.1)

LET p, q, r, s: A_1, A_2, B, C

(((p&q)<r)<s)>((p<r)<s) ; `TTTT TTTT TTTT TTTT` (7.6.2)

Remark 7.6: Eq. 7.6.2 is renamed for 4.5.1, from left monotonicity. There is no central monotonicity as Eq. 4.6.1 in the list of 7. Conclusion.

7. Insertion: A|B_1,I,B_2|C→(A|I,C|B_1→(B_2|A, I|C→A|I|C)) where A∩B_2=B_1∩C=A∩C=Ø
(7.7.1)

LET p, q, r, s, t, u: A, I, B, C, B_1, B_2

((((p&u)=(t&s))=(p&s))=(p@p))>((p<((q&s)<t))>((u<((p&q)<s))>(p<~(s>q)))) ;

```
TTTT TTTT TTTT TTTT(2), FFFF FFFF TFTT TFTT(1),
TTTT TTTT TTTT TTTT(3), FFFF FFFF TFTT TFTT(1),
TTTT TTTT TTTT TTTT(3), FFFF FFFF TFTT TFTT(1),
TTTT TTTT TTTT TTTT(3), FFFF FFFF TFTT TFTT(1),
TTTT TTTT TTTT TTTT(3), FFFF FFFF TFTT TFTT(1),
TTTT TTTT TTTT TTTT(1)
```
(7.7.2)

Remark 7.7: Eq. 7.7.2 differs from 4.7.2.

8. Transitivity: A|B|C→(A|B|d∨d|B|C (7.8.1)

LET p, q, r, s: A, B, C, d

(p<(q<r))>(((p<q)<s)+(s<(q<r))) ; `TTTT TTT`**`F`**` TTTT TTTT` (7.8.2)

Remark 7.8: Eq. 7.8.2 differs from 4.8.2.

Of the 16 equations under sections for Axioms and Conclusion, nine are *not* tautologous. This refutes the axiomatic theory of betweenness.

Refutation of BF calculus (and square root of negation)

Abstract: We evaluate the eight defining equations of the Spencer-Brown system. None is tautologous. This refutes the subsequent primary arithmetic renamed as BF calculus. We previously refuted the Dunn-Belnap 4-valued bilattice as *not* bivalent and thus non tautologous, so to draw in refinements and extensions by others and apply BF to it compounds the mistakes. Further producing a square root operation on negative 1 is also *not* tautologous. Spencer-Brown and BF systems subsequently form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Kauffman, L.H.; Collings, A.M. (2019). The BF calculus and the square root of negation. arxiv.org/pdf/1905.12891.pdf kauffman@uic.edu, otter@mac.com

Comment: If the "otter" email prefix above implies use of the Prover9(nee Otter) proof assistant by the authors, we show elsewhere that assistant is *not* bivalent.

II. Laws of form

A. *Distinction*

Laws of Form by Spencer-Brown [..], and the Primary Algebra (PA) it describes, is based on the idea of *distinction*, represented by the dividing of a space into two regions, one *marked* , the second *unmarked* . In *Laws of Form*, the mark ⏋ indicates the *marked state*, and the empty value " " [or Not(⏋)] indicates the *unmarked state*. The step of representing a value by an empty space, by the lack of a sign, is motivated by a key idea: doing so permits the mark ⏋ to act both as the name of a value and as an operation.

The Mark as an Operation

I⏋= O, O⏋= I, I inside circle, and O outside circle

Fig. 1. Representing a Distinction between Inside (I) and Outside (O)

Consider Figure 1, in which we have drawn a closed circle, creating a distinction between inside, I, and outside O. We regard the mark ⏋ as an operator that takes I to O and O to I. Then we observe the following:

I⌝= O, O⌝= I, (1.1.1, 1.2.1)

LET p, q, r, s: I, O, r, ⌝ or ⌉.

(p&s)=q ; TT**FF** TT**FF** T**FF**T T**FF**T (1.1.2)
(q&s)=p ; T**F**T**F** T**F**T**F** T**FF**T T**FF**T (1.2.2)

I⌝⌉ = O⌝= I, O⌝⌉ = I⌝= O, (2.1.1, 2.2.1)

(((p&s)&s)=(q&s))=p ; **F**T**F**T **F**T**F**T **FF**TT **FF**TT (2.1.2)
(((q&s)&s)=(p&s))=q ; **FF**TT **FF**TT **F**T**F**T **F**T**F**T (2.2.2)

so for any state X we have X⌝⌉ = X. (2.3.1)

Remark 2.3.1: Eq. 2.3.1 is a trivial tautology, for which also see below at 5.1.1.

The conceptual shift is to designate the inside to be unmarked (literally to have no symbol), so that

I = “ ”. (2.4.1)

p=~s ; **F**T**F**T **F**T**F**T T**F**T**F** T**F**T**F** (2.4.2)

Then from (1) we obtain

⌉ = O, O⌝ = , (3.1.1)

(((p&s)=q)&((q&s)=p))>((s=q)&((q&s)=~s)) ;
FTTT **F**TTT **F**TT**F** **F**TT**F** (3.1.2)

which means we have equated the mark ⌉ with the outside O. (3.2.1)

((((p&s)=q)&((q&s)=p))>((s=q)&((q&s)=~s)))>(s=q) ;
TT**FF** TT**FF** T**F**TT T**F**TT (3.2.2)

From (2), we obtain

⌝⌉ = “ ” (4.1.1)

(((((p&s)&s)=(q&s))=p)&((((q&s)&s)=(p&s))=q))>((s&s)=~s);
TTT**F** TTT**F** TTT**F** TTT**F** (4.1.2)

By identifying the value of the outside with the result of crossing from the unmarked inside, Spencer-Brown has introduced a multiplicity meanings to the mark. The statement ⌉ =⌉ can be interpreted on the left side to mean “cross from the inside” and on the right as “the name of the outside”.

The mark itself can be seen to divide its surrounding space into an inside and an outside. When we write ⌉ =⌉ , the two marks are positioned mutually outside each other, and we can choose to interpret either mark as a name that refers to the outside of the other. We may also interpret two such juxtaposed marks to indicate successive naming of the state indicated by the mark. In either case we can take as an instance of the principle that to repeat a name can be identified with a single calling of the name:

$$\urcorner \; \urcorner = \urcorner \, . \qquad (5.1.1)$$

Remark 5.1.1: Eq. 5.1.1 is a trivial tautology.

At this point we have a single sign $\urcorner$ representing both the operation of crossing the boundary of a distinction and representing the name of the outside of that distinction. Furthermore, since the mark itself can be seen to make a distinction in its own space, the mark can be regarded as referent to itself and to the (outer side) of the distinction that it makes. The two equations (4) and (5) represent these aspects of understanding a distinction and the signs that can represent this distinction. We will now see that the two equations and a natural formalism for expressions in the mark become a formal system that can be seen as an 'arithmetic' for Boolean algebra.

B. The Primary Arithmetic
On the basis of these considerations, Spencer-Brown defines a very simple calculus, which he calls the *Primary Arithmetic*. …

We evaluated the eight defining equations of the Spencer-Brown system. None is tautologous. This refutes the subsequent primary arithmetic renamed as BF calculus. We previously refuted the Dunn-Belnap 4-valued bilattice as *not* bivalent and thus non tautologous, so to draw in refinements and extensions on it by by others and apply BF to it compounds the mistakes. By further producing a square root operation on negative 1 is also *not* tautologous.

Biscuit conditionals

An example of the biscuit conditional is: There is a biscuit, if you want it. (1)

This can be rephrased as: If you want a biscuit, there is one. (2)

We assume the Meth8 script and apparatus.

LET: p something; % possibly; # necessarily; > Imply; (n)vt (Not) validated as a tautology

We rewrite Eq 2 in an abstract form in modal logic as:

If possibly something, then not necessarily something. (3.1)

%p > ~#p ; TCTC TCTC (3.2)
[Tautology is a proof value, and Contingency is a not truth value.]

The above is equivalent logically to:

If possibly something, then not possibly necessarily something. (4.1)

%p > ~%#p ; TCTC TCTC (4.2)

If something, then not necessarily something. (5.1)

p > ~#p ; TCTC TCTC (5.2)

If all things, then not possibly all things. (6.1)

#p > ~%#p ; TCTC TCTC (6.2)

If all things, then not necessarily possibly all things. (7.1)

#p > ~#%p ; TCTC TCTC (7.2)

If all things, then not possibly a thing. (9.1)

#p > ~%p ; TCTC TCTC (9.2)

If possibly something, then not necessarily possibly something. (10.1)

%p > ~#%p ; NFNF NFNF (10.2)
[Non contingent is a truth value, and F is a contradiction value.]

If something, then not possibly something. (11.1)

p > ~%p ; TFTF TFTF (11.2)

From this exposition, Meth8 does not validate as a tautology the biscuit conditionals.

We also examine some definitions of biscuit conditionals from the literature.

From: Katshuhiko Sano, Yurie Hara. (2014).
"Conditional independence and biscuit conditional questions in dynamic semantics".
Proceedings of SALT 24: 84-101.
at journals.linguisticsociety.org/proceedings/index.php/SALT/article/download/2473/2221

a. The speaker knows the proposition P ([]P, in short) in r if $r \subseteq P$. (8)
b. P is consistent (<>P, in short) in r if $r \cap P \neq 0$.
c. ‘if P then Q’ holds in r if $r \cap P \subseteq Q$.

LET: p P; q Q; r lower-case omega

~(r>p)>#p ; TTTT FTFT (8.1)

((r&p)=~(p-p))>%p ; CTCT CTCT (8.2)

~((r&p)>q)>(p>q) ; TTTT TFTT (8.3)

(~(r>p)>#p) & ((((r&p)=~(p-p))>%p)&(~((r&p)>q)>(p>q))) ;
CTCT FFFT (8.4)

[T]he consequent entailment []Q follows from a strict implication ‘if P then Q’, together with the following independence assumption. (12)

(p>q)>#q ; FTNN FTNN (12.1)Meth8
evaluates Eqs 8.1-8.4, and 12.1 to not validated as tautologies.

Refutation of bisimulation

Abstract: We evaluate two definitions of bisimularity, both *not* tautologous. That refutes bisimulation, along with its proof tools of coinduction and Howe's congruence method.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⇒ ,↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) C non-contingency, ∇, ordinal 2; (%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Pitts, A. (2011). Howe's method for higher-order languages.
In D. Sangorgi, J. Rutten (eds.), Advanced topics in bisimulation and coinduction,
Cambridge tracts in theoretical computer science. no. 52, chapter 5, pages 197-232.
cl.cam.ac.uk/~amp12/papers/howmho/howmho.pdf andrew.pitts@cl.cam.ac.uk

5. Howe's method for higher-order languages
5.1 Introduction
[A]lthough it is usually easy to see that a bisimilarity ≃ satisfies

∀Q. P(x)≃P'(x)⇒ P(Q)≃P'(Q) (5.1.1.1)

LET p, q, r, s, t: P, Q, P', x, Q'

((p&s)=(r&s))>((p&#q)=(r&#q)) ;
TTTC TTCT TTTT TTTT (5.1.1.2)

for compatibility of ≃ we have to establish the stronger property

∀Q,Q'. P(x)≃P'(x)^Q≃Q'⇒ P(Q)≃P'(Q') (5.1.2.1)

(((p&s)=(r&s))&(#q=#t))>((p&#q)=(r&#t)) ;
TTTC TTCT TTTT TTTT,
TTTT TTTT TTTT TTTT (5.1.2.1)

This is often hard to prove directly from the definition of ≃.

Eqs. 5.1.1.2 and 5.1.2.2 as rendered are *not* tautologous. This refutes bisimularity, along with its proof tools of coinduction and Howe's congruence method.

Refutation of bitstring and question-answer semantics

Abstract: We evaluate bitstring semantics and its follow-on by partition. Its ordered set of exhaustive predicates is *not* bivalent but a probabilistic vector space. Its calculus of relations is *not* tautologous. Hence its broader framework of question-answer semantics (QAS) is *no*t tautologous. The conjecture of "generalizing the Aristotelian square within one common gathering" is denied. What is affirmed is the Meth8 corrected, modern, revised square of opposition is a square, to mean the following conjectures are probabilistic vector spaces: collapsed number line of opposition; non-standard quadrilateral of oppositions; and colored square of oppositions. Bitstring semantics and the extended QAS form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Schang, F. (2019). End of the square? sa-logic.org/sajl-v4-i2/11-Schang-SAJL.pdf

1 Introduction: oppositions

In the third section, we will introduce a special semantics provided to account for the meaning of oppositional relations through opposite-forming operators: a *bitstring* semantics, where the basicality of opposition stems from a common analysis of logical space in terms of *partition*. By doing so, we will complete the preceding proposal by generalizing the Aristotelian square within one common gathering.

1 Oppositions with a square

[T]he kinds of logical opposition are depicted by functional expressions *ct* (for contrariety), *cd* (for contradictoriness), *sct* (for subcontrariety), *sb* (for subalternation), and *sp* (for superalternation).

3 Oppositions with another square

3.1 Bi[t]string semantics

The following semantics is a special application of a broader semantic framework: *Question-Answer Semantics* [QAS], where … results from an ordered set of exhaustive predicates.

Num.	QAS bitstring	QAS value	M8 script	M8 result	QA bitstring vs M8 truth table	Note
3.1.0	β(⊥)	0000	p@p	FFFF	ok	
3.1.1	β(p ∧ q)	1000	p&q	FFFT	Read script right to left	
3.1.2	β(¬(p → q))	0100	~(p>q)=(p=p)	FTFF	Read script left to right	2=14
3.1.3	β(¬(p ← q))	0010	~(p<q)=(p=p)	TFTT	Negate script right to left	3= 8
3.1.4	β(¬(p ∨ q))	0001	~(p+q)=(p=p)	TFFF	Read script right to left	
3.1.5	β(p)	1100	p=(p=p)	FTFT	undecipherable	
3.1.6	β(¬(p ↔ q))	0110	~(p=q)=(p=p)	FTTF	ok	
3.1.7	β(¬p)	0011	~p=(p=p)	TFTF	undecipherable	
3.1.8	β(p → q)	1001	p>q	TFTT	undecipherable	8= 3
3.1.9	β(¬q)	0101	~q=(p=p)	TTFF	undecipherable	
3.1.10	β(q)	1010	q=(p=p)	FFTT	undecipherable	
3.1.11	β(p ∨ q)	1110	p+q	FTTT	Read script fight to left	
3.1.12	β(¬(p ∧ q))	0111	~(p&q)=(p=p)	TTTF	Read right to left	
3.1.13	β(p ↔ q))	1011	p=q	TFFT	unreadable	
3.1.14	β(p ← q))	1101	p<q	FTFF	Reverse script right to left	14= 2
3.1.15	β(⊤)	1111	p=p	TTTT	ok	

Remark 3.1: Of the 16 claimed bitstring values, three are bivalent mappings as represented in Meth8(M8) script and result: Eqs. 3.1.0, 3.1.6, and 3.1.15. These are for respectively contradiction (none), not equivalent, and tautology (all). Four bitstrings using the imply or not imply connectives are equivalents and hence are *not* unique values of the 16 as claimed.

Calculus of logical relations.

$$cd(\beta(x)) = 1 \Leftrightarrow \beta(x) = 0\text{, i.e. } cd(\beta(x)) = 1 \Rightarrow \beta(x) = 0 \text{ and } cd(\beta(x)) = 0 \Rightarrow \beta(x) = 1 \quad (3.2.1)$$

LET p, q, r, s: cd (for contradictority), β , x, s.

((((p&q)&s)=(s=s))>((q&r)=(s@s)))&((((p&q)&s)=(s@s))>((q&r)=(s=s))) ;
FFFF FFTT FFFT FFTF (3.2.2)

Remark 3.2.2: Eq. 3.2.2 as rendered is *not* tautologous. Hence that logical relation is refuted, to color the entire claimed calculus.

3.3 Iterated oppositions

Remark 3.3.0.1: We take the edges of the corrected square of opposition from: James, C. (2019). Refutation of hexagons of opposition for statistical modalities. vixra.org/abs/1901.0192. We set the logical oppositions to those of the text in italics.

Source type	Def.	Meth8 corrected script	Valid as		
Corner	A	#(s= p)		N**F**N**F** N**F**N**F** F**N**F**N** F**N**F**N**	
	E	#(s=~p)		**F**N**F**N **F**N**F**N N**F**N**F** N**F**N**F**	
	I	%(s= p)		TCTC TCTC CTCT CTCT	
	O	%(s=~p)		CTCT CTCT TCTC TCTC	
Contrarity	AE	#(s= p)\#(s=~p)	A \ E	TTTT TTTT TTTT TTTT	*ct*
Superalternity	AI	#(s= p)>%(s= p)	A > I	TTTT TTTT TTTT TTTT	*sp*
Contradictority	AO	#(s= p)\%(s=~p)	A \ O	TTTT TTTT TTTT TTTT	*cd*
Contradictority	EI	#(s=~p)\%(s= p)	E \ I	TTTT TTTT TTTT TTTT	*cd*
Subalternity	EO	#(s=~p)>%(s=~p)	E > O	TTTT TTTT TTTT TTTT	*sb*
Subcontrarity	IO	%(s= p)+%(s=~p)	I + O	TTTT TTTT TTTT TTTT	*sct*

Remark 3.3.0.2: These formulas for the edges of the corrected, modern, revised square of opposition are tautologous, but not found anywhere in the instant text or its references. The point is that the square of opposition need not be a rectangle, reduced to one dimension, or abandoned as such.

Nevertheless, another parallel way to characterize subalternation is to define it by means of iterated functions. Here is the central point of the present paper: logical relations … can be reduced after all to an iteration of basic oppositions. To begin with such a process, any subaltern of an arbitrary formula x is to be defined as the contradictory of a contrary of x:

$$sb(\beta(x)) = cd(ct(\beta(x))) \quad (3.3.1.1)$$

Conversely to (3.3.1.1), any superaltern of x is to be defined as the contrary of the contradictory of x:

$$sp(\beta(x)) = ct(cd(\beta(x))) \quad (3.3.2.1)$$

A subcontrary of any x is the contradictory of the superaltern of x or, by substituting the latter relation for its iterative definition, the contradictory of the contrary of the contradictory of x:

$$sct(\beta(x)) = cd(sp(\beta(x))) = cd(ct(cd(\beta(x)))) \quad (3.3.3.1)$$

Remark 3.3.0.3: As expected, Eqs. 3.3.1.1-..3.1 are tautologous. This means the iterated oppositions are obvious and not new per se or a recent advance.

From the sections above, we refute *bitstring* semantics, its follow-on by *partition*, its broader framework of question-answer semantics (QAS), and “generalizing the Aristotelian square within one common gathering”. What is affirmed is the Meth8 corrected, modern, revised square of opposition, to mean the following conjectures are probabilistic vector spaces: collapsed number line of opposition; non-standard quadrilateral of oppositions; and colored square of oppositions.

Refutation of a modal logic system for reasoning about the degree of blameworthiness

Abstract: We evaluate a modal logic system for the blamable coalition of an outcome if there is a strategy to prevent it and where the degree of blameworthiness is measured by costs of prevention or sacrifice. Of eight axioms, three are *not* tautologous, hence refuting the approach. We do not consider the claimed technical result of a completeness theorem.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p , q , r , s , t ,
lc_phi ϕ, lc_psi ψ, Blameable, ^{degree} , cost,
u , v , w , x, y, z:
$_{C}$ coalition (small), $_{D}$ coalition (large), Statement (N), x, y, z;
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⊢, ↦; < Not Imply, less than, ∈;
= Equivalent, ≡, ⊨, :=, ⇔, ↔; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology; (z@z) **F** as contradiction, Ø, Null;
(%z<#z) C non-contingency, ∇, ordinal 2;
(%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y).

From: Cao, R.; Naumov, P. (2019). The limits of morality in strategic games.
arxiv.org/pdf/1901.08467.pdf rui.cao@alumni.ubc.ca, pgn2@cornell.edu

3 Axioms: In addition to the propositional tautologies in language Φ, our logical system contains the following axioms:

1. Truth:
Nϕ → ϕ and B^{s}_{C}ϕ → ϕ, (1.1)

((w&p)>p)&(((r&s)&(u&p))>p) ;
TTTT TTTT TTTT TTTT (1.2)

2. Distributivity:
N(ϕ → ψ) → (Nϕ → Nψ), (2.1)

(w&(p>q))>((w&p)>(w&q)) ;
TTTT TTTT TTTT TTTT (2.2)

3. Negative Introspection:
¬Nϕ → N¬Nϕ, (3.1)

~(w&p)>(w&~(w&p)) ;
TTTT TTTT TTTT TTTT(8), **FFFF FFFF FFFF FFFF**(8) (3.2)

4. None to Blame:

$\neg B^{s}_{\emptyset}\phi$, (4.1)

~((r&s)&((z@z)&p))=(p=p); TTTT TTTT TTTT TTTT (4.2)

5. Monotonicity:

$B^{s}_{C}\phi \rightarrow B^{t}_{D}\phi$, where $C \subseteq D$ and $s \leq t$, (5.1)

(~(v<u)&~(t<s))>(((r&s)&(u&p))>((r&t)&(u&p))) ;
TTTT TTTT TTTT TTTT(2), TTTT TTTT TTTT T**F**TT(1),
TTTT TTTT TTTT TTTT(3), TTTT TTTT TTTT T**F**TT(1),
TTTT TTTT TTTT TTTT(1) (5.2)

6. Joint Responsibility:

if $C \cap D = \emptyset$, then $NB^{s}_{C}\phi \wedge NB^{t}_{D}\psi \rightarrow (\phi \vee \psi \rightarrow B^{s+t}_{C\cup D}(\phi \vee \psi))$, (6.1)

((u&v)=(z@z))>(((~w&((r&s)&(u&p)))&(~w&((r&t)&(v&q))))>((p+q)>
((r&((s+t)&(u+v)))&(p+q)))) ;
TTTT TTTT TTTT TTTT (6.2)

7. Blame for Cause:

$N(\phi \rightarrow \psi) \rightarrow (B^{s}_{C}\psi \rightarrow (\phi \rightarrow B^{s}_{C}\phi))$, (7.1)

(w&(p>q))>(((r&s)&(u&q))>(p>((r&s)&(u&p)))) ;
TTTT TTTT TTTT TTTT (7.2)

8. Fairness:

$B^{s}_{C}\phi \rightarrow N(\phi \rightarrow B^{s}_{C}\phi)$. (8.1)

((r&s)&(u&p))>(w&(p>((r&s)&(u&p)))) ;
TTTT TTTT TTTT TTTT(2), TTTT TTTT TTTT T**F**T**F**(2),
TTTT TTTT TTTT TTTT(2), TTTT TTTT TTTT T**F**T**F**(2),
TTTT TTTT TTTT TTTT(8) (8.2)

For Eqs. 1.2-8.2 as rendered, the three 3.2, 5.2, and 8.2 are not tautologous, hence refuting the proposed system. The claimed technical result of a completeness theorem is not evaluated.

Refutation of block argumentation

Abstract: We evaluate the approach of block argumentation using the legal example of a popular case. Three scenarios are *not* tautologous. We attempt to resuscitate the method by substitution of generic block argumentation and also by testing the consequent parts separately. The equations were *not* tautologous, hence refuting the block argumentation approach as presented.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r, s, t, u, v, w, x, y, z: a1, a2, a3, a4, a5, a6, a7, a8, x, y, z;
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⊢; < Not Imply, less than, ∈;
= Equivalent, ≡, ⊨, :=, ⇔, ↔; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology; (z@z) **F** as contradiction;
(%z<#z) C non-contingency, ∇, ordinal 2;
(%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y).

From: Arisaka, R.; Bistarelli, S.; Santini, F. (2019). Block argumentation.
arxiv.org/pdf/1901.06378.pdf
ryutaarisaka@gmail.com, Stefano.Bistarelli@unipg.it, Francesco.Santini@unipg.it

"[I]n Section 2 we motivate our approach with a real legal example from a popular case.

*a*1**:** After the victim was attacked with VX, the suspect walked quickly to a restroom for washing hands. (a1.1)

p ; (a1.2)

*a*2**:** The suspect knew VX was on her hands. (a2.1)

q ; (a2.2)

*a*3**:** The suspect was acting for a prank video. (a3.1)

r ; (a3.2)

*a*4**:** The suspect adjusted her glasses with VX on her hands before walking to the restroom. (a4.1)

s ; (a4.2)

*a*5**:** Malaysian authorities are biased against the suspect, tampering with evidence by intentional omission of relevant CCTV footage. (a5.1)

t=(z@z) ; (a5.2)

*a*6**:** *a*1 supports *a*2. (a6.1)

u=(p>(q=(z=z))) ; (a6.2)

*a*7**:** *a*4 attacks *a*2. (a7.1)

v=(s>(q=~(z=z))) ; (a7.2)

*a*8**:** *a*7 attacks *a*6." (a8.1)

w=(v>(u=~(z=z))) ; (a8.2)

Argumentation:

Remark A.1: Argumentation assumes the relevant definitions from a1.1-a8.1 above.

A: a1 supports a2; a2 attacks a3; a4 attacks a2; a7 attacks a6; a8 supports a5. (A.1)

(((t=(z@z))&(u=(p>(q=(z=z)))))&((v=(s>(q=~(z=z))))&(w=(v>(u=~(z=z))))))>
((((p>(q=(z=z)))&(q>(r=~(z=z))))&v)&((v>(u=~(z=z)))&(w>(t=(z=z))))) ;
TTTT TTTT TTTT TTTT(13), **F**T**FF** **F**T**FF** **F**TTT **F**TTT(1),
T**F**TT T**F**TT T**F**TT T**F**TT(1) , TTTT TTTT TT**FF** TT**FF**(1) (A.2)

B: a1 supports a2; a4 attacks a2. (B.1)

((u=(p>(q=(z=z))))&(v=(s>(q=~(z=z)))))>(u&v) ;
TTTT TTTT TTTT TTTT(8) , TTTT TTTT TT**FF** TT**FF**(4),
T**F**TT T**F**TT T**F**TT T**F**TT(4) (B.2)

C: a1 supports a2; a4 attacks a2; (a1 supports a2) and (a4 attacks a2) supports a5. (C.1)

(((t=(z@z))&(u=(p>(q=(z=z)))))&(v=(s>(q=~(z=z)))))>((u&v)>t) ;
TTTT TTTT TTTT TTTT(14), **F**T**FF** **F**T**FF** **F**TTT **F**TTT(2) (C.2)

"Then we can model the example argumentation as in A . Malaysian Police uses *a*1 for *a*2 (*a*1 supports *a*2) to dismiss *a*3 (*a*2 attacks *a*3). All these three arguments are made available to the audience. The defence lawyer uses *a*4 to counter *a*2. *a*4 is also available to the audience as attacking *a*2. He then uses *a*7, which is itself an argumentation, to attack Malaysian Police' argumentation *a*6. This is also presented to the audience. Finally, he uses *a*8, an argumentation, for *a*5."

Remark A.B.C: The authors do not predict or show which argumentation block of A, B, C is tautologous.

"Arguments of the kinds of *a*6, *a*7 and *a*8 are themselves argumentations, so "*a*7 attacks *a*6" could be detailed as in B , and "*a*8 supports *a*5" as in C."

Remark D.1: Argumentation A can be rewritten to include generic expansions for B and C as a1 supports a2; a2 attacks a3; a4 attacks a2; B; C, although the authors do not show exactly how. Because C contains no reference to w, the assumption of w may be removed from the antecedent of A, although not considered by the authors. (D.1)

(((t=(z@z))&(u=(p>(q=(z=z)))))&(v=(s>(q=~(z=z)))))>
((((p>(q=(z=z)))&(q>(r=~(z=z))))&v)&((u&v)&((u&v)>t))) ;
TTTT TTTT TTTT TTTT(10), TTTT TTTT TT**FF** TT**FF**(2),
T**F**TT T**F**TT T**F**TT T**F**TT(2), **F**T**FF** **F**T**FF** **F**TTT **F**TTT(2) (D.2)

Eqs. A.2-D.2 as rendered are *not* tautologous, thereby refuting the approach of block argumentation as presented. The authors proceed to predict graphical (syntactic) and semantic constraints which we can not confirm.

Remark E: To resuscitate the approach we test each argument separately as the consequent in Eq. A.1 with respective assumptions. (E.0)

((t=(z@z))&(u=(p>(q=(z=z)))))>(((p>(q=(z=z)))&(q>(r=~(z=z))))&v) ;
T**F**TT T**F**TT T**F**TT T**F**TT(4) , TTTT TTTT TTTT TTTT(8),
FT**FF** **F**T**FF** **F**T**FF** **F**T**FF**(2) , TTTT TT**FF** TTTT TT**FF**(2) (E.1.2)

((u=(p>(q=(z=z))))&(v=(s>(q=~(z=z)))))>(u&v) ;
TTTT TTTT TTTT TTTT(8) , TTTT TTTT TT**FF** TT**FF**(4),
T**F**TT T**F**TT T**F**TT T**F**TT(4) (E.2.2)=(B.2)

(((t=(z@z))&(u=(p>(q=(z=z)))))&(v=(s>(q=~(z=z)))))>((u&v)>t) ;
TTTT TTTT TTTT TTTT(14), **F**T**FF** **F**T**FF** **F**TTT **F**TTT(2) (E.3.2)=(C.2)

Eq. E1.2-3.2 are *not* tautologous, hence not resuscitating the approach of block argumentation.

Refutation of the Blok-Esakia theorem for universal classes

Abstract: Grzegorczyk (gzr) algebras as used for support and the Blok-Esakia theorems are not confirmed as tautologies and hence refuted.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

LET p, q, r: a *or* p *or* P, c, g;
~ Not; & And, ∧; + Or, ∨; > Imply, →; < Not Imply, less than, ∈; = Equivalent;
% possibility, for one or some, <>; # necessity, for every or all, [];
~(y<x) (x≤y); ~(y>x) (x≥y).

From: Stronkowski, M.M. (2018). On the Blok-Esakia theorem for universal classes.
arxiv.org/pdf/1810.09286.pdf m.stronkowski@mini.pw.edu.pl

Remark 1.0: Eqs. are keyed to the text sections and sequential order if not specifically numbered. Grzegorczyk algebras are grz.

Introduction to the Blok-Esakia theorem
[]([](p → []p) → p) → p (1.0.1)

#(#(p>#p)>p)>p ; CTCT CTCT CTCT CTCT (1.0.2)

A modal algebra M is called an interior algebra if for every a ∈ M it satisfies
[][]a=[]a ≤ a (3.0.1.1)

##p=~(p<#p) ; **F**T**F**T **F**T**F**T **F**T**F**T **F**T**F**T (3.0.1.2)

An interior algebra M is a Grzegorczyk algebra if it also satisfies
[]([](a → []a) → a) ≤ a (3.0.2.1)

~(p<(#(#(p>#p)>p)))=(p=p) ; TNTN TNTN TNTN TNTN (3.0.2.2)

Proposition
[](x → []x) → x (3.2.1)

#(p>#p)>p ; CTCT CTCT CTCT CTCT (3.2.2)

Remark 3.2.2: Eqs. 1.0.2 and 3.2.2 as rendered are equivalent with identical truth table results.

Appendix Blok theorems: For every c subset of C define Pc=[](g∨c)∧~c.
Then (Pc≤((g∨c)∧~c))=((g∧~c)≤ g), and [](g∨c)≥[](Pc∨c)=[](([](g∨c)∧~c)∨c)
=[]([](g∨c)∨c)≥[][](g∨c)=[](g∨c). Thus (P) [](Pc∨c)=[](g∨c). (5.10.1)

(((p&q)=(#(r+q)&~q))>((~(((r+q)&~r)<(p&q))=~(r<(r&~q)))
&((~(#((p&q)+r)>#(r+q))=#((#(r+q)&~q)+q))=(~((r+q)=#(r+q))>
#(#(r+q)+q)))))>((#(p&q)+q)=#(r+q)) ;
`TTTN CCTN TTTN CCTN` (5.10.2)

Eqs. for paper sections 1, 3, and 5 are *not* tautologous. This means that gzr algebras as used for support and the Blok-Esakia theorems are refuted.

Bogdanov map as a 2D conjugate to the Hénon map

From mathworld.wolfram.com/BogdanovMap.html :

x' = x + y'; also rewritable as y' = x' - x (21)

y' = y + ey +kx(x-1) + mxy (22)

LET: x = x; y = y; w = x'; p = e; q = k; r = m; = Equivalent; + Or; (x\x) = 1

(w-x) = ((y+(p&y))+(((q&x)&(x-(x\x)))+((r&x)&y))) ; nvt (23)

Result: the Bogdanov map as a 2D conjugate to the Hénon map is not tautologous.

Refutation of a formula for systems of Boolean polynomials to parameterized complexity

Abstract: Three formulas defining Boolean polynomial arithmetic are *not* tautologous, to refute the conjecture. These form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕ ;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Tomoya Machide, T. (2019). A formula for systems of Boolean polynomial equations and applications to parameterized complexity. arxiv.org/pdf/1907.09686.pdf

Abstract: It is known a method for transforming a system of Boolean polynomial equations to a single Boolean polynomial equation with less variables. In this paper, we improve the method, and give a formula in the Boolean polynomial ring for systems of Boolean polynomial equations. The formula has conjunction and disjunction recursively, and it can be expressed in terms of binary decision trees. As corollaries, we prove parameterized complexity results for systems of Boolean polynomial equations and NP-complete problems.

1 Introduction

The finite field $F_2 = \{0,1\}$ with two elements, which is also called the Galois field GF.. in his honor, plays fundamental roles in mathematics and computer science. It is the smallest finite field with a simple algebraic structure which is determined by a few equations involving the addition "+" and multiplication "·". One of the outstanding facts of F_2 is a structural relation to the two-element Boolean algebra B = {False, True} under the identifications False = 0 and True = 1. That is, for any pair (α,β) of elements,

$$\alpha \wedge \beta = \alpha \cdot \beta, \qquad \alpha \vee \beta = (\alpha + 1) \cdot (\beta + 1) + 1, \qquad \alpha \oplus \beta = \alpha + \beta, \qquad (1.1.1\text{-}.3)$$

where ∧, ∨, and ⊕ stand for the binary operations of conjunction, disjunction, and exclusive disjunction in B, respectively.

LET p, q, r, s: α , β, r, s.

(p+q)=(((p+(p=p))&(q+(q=q)))+(s=s)) ;
FTTT **F**TTT **F**TTT **F**TTT (1.1.2.2)

(p@q)=(p+q) ; TTT**F** TTT**F** TTT**F** TTT**F** (1.1.3.2)

2 Review of the Boolean polynomials

… In addition, we have

$$p2 = p, \qquad p(p + 1) = p + p = 0, \tag{2.9.1-.2}$$

(p&(p+(p=p)))=((p+p)=(p@p)) ;

`FFFF FFFF FFFF FFFF` (2.9.2)

Eqs. 1.1.2.2, 1.1.3.2, and 2.9.2 as rendered are *not* tautologous. This refutes the author's title.

Refutation of the Boone-Rogers theorem

Abstract: The Boone-Rogers theorem states that the uniform word problem for the class of all finitely presented groups with solvable word problem is unsolvable. We show that is *no*t tautologous. We further show that a universal, solvable word problem group is tautologous. The former forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Word_problem_for_groups [Note that this entry is not well written.]

Word problem for groups
The related but different **uniform word problem** for a class *K* of recursively presented groups is the algorithmic problem of deciding, given as input a presentation *P* for a group *G* in the class *K* and two words in the generators of *G*, whether the words represent the same element of *G*. Some authors require the class *K* to be definable by a recursively enumerable set of presentations.

Unsolvability of the uniform word problem
The criterion given above, for the solvability of the word problem in a single group, can be extended by a straightforward argument. This gives the following criterion for the uniform solvability of the word problem for a class of finitely presented groups:

> To solve the uniform word problem for a class K of groups, it is sufficient to find a recursive function f(P,w) that takes a finite presentation P for a group G and a word w in the generators of G, such that whenever G ∈ K:
>
> f(P,w) = {0 if w ≠ 1 in G
> {undefined/does not halt if w = 1 in G (1.1)

LET p, q, r, s: P, G (or H), f (or h_n), w.

(r&(p&s))=((((s=~(%s>#s))<q)> (s@s)) + (((s= (%s>#s))<q)>~(s@s))) ;
FFFF FFFF FFFF FT**F**T (1.2)

Remark 1.2: For Eq. 1.2 as rendered to prove the solution of the uniform word problem, the result should be a theorem of all T's. In fact, the result is *not* a theorem, and also *not* a contradiction, but something else.

Boone-Rogers Theorem: There is no uniform partial algorithm that solves the word problem in all finitely presented groups with solvable word problem.

In other words, the uniform word problem for the class of all finitely presented groups with solvable word problem is unsolvable.

Remark 1.3: Eq. 1.2 contradicts the Boone-Rogers theorem with restatement to refute it.

Proof that there is no universal solvable word problem group
If H has solvable word problem, then at least one of these homomorphisms must be an embedding. So given a word w in the generators of H: (3.1)

If w≠1 in H, h_n(w)≠1 in G for some h_n
If w=1 in H, h_n(w)=1 in G for all h_n

%(((~(s=(%s>#s))<q)>((q<r)>~((r&s)=(%s>#s))))+
(((s=(%s>#s))<q)>((q<r)> ((r&s)=(%s>#s)))))=(q=q) ;
TTTT TTTT TTTT TTTT (3.2)

The function f clearly depends on the presentation P. Considering it to be a function of the two variables, a recursive function f(P,w) has been constructed that takes a finite presentation P for a group H and a word w in the generators of a group G, such that whenever G has soluble word problem: (4.0.1.1)

f(P,w) = {0 if w ≠ 1 in H
{undefined/does not halt if w = 1 in H (4.0.2.1)

Remark 4.0.1.1: We write Eq. 4.0.1.1 as If 3.1 and 1.1, then 4.1 (4.1)

(%(((~(s=(%s>#s))<q)>((q<r)>~((r&s)=(%s>#s))))+(((s=(%s>#s))<q)>((q<r)>
((r&s)=(%s>#s)))))&((r&(p&s))=((((s=~(%s>#s))<q)> (s@s))+(((s=
(%s>#s))<q)>~(s@s)))))>((r&(p&s))=((((s=~(%s>#s))<q)> (s@s))+(((s=
(%s>#s))<q)>~(s@s)))) ; TTTT TTTT TTTT TTTT (4.2)

But this uniformly solves the word problem for the class of all finitely presented groups with solvable word problem, contradicting Boone-Rogers. This contradiction proves G cannot exist.

Remark 4.2: Eq. 4.2 is tautologous, proving G can exist and that there is a universal, solvable word problem group.

Refutation of the Borel base and hull

Abstract: We evaluate in two equations the Borel base and hull as *not* tautologous and contradictory, refuting the conjectures and forming a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
\> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Michalski, M. (2019). Rediscovered theorem of Luzin. arxiv.org/pdf/1907.09305.pdf

Definition 1.1. We say that a σ-ideal I

has a Borel base if (A I)(B B ∩ I(A B); ∀ ∈ ∃ ∈ ⊆ (1.1.1)

LET p, q, r, s: A, B, B', I.

(#p<s)&((%q<q)&(s&~(q<p))) ;
FFFF FFFF FFFF FFFF (1.1.2)

has a Borel hull property if for (A)(B B)(A B and (B′ B)(A B′ ∀ ∃ ∈ ⊆ ∀ ∈ ⊆ ⊆ B)(B\B′ I)). ... ∈ (1.2.1)

#p&((%q<q)&(~(q<p)&((#r<q)&(~(q<~(r<p))&(q\(r<s)))))) ;
FFFF FFFF FFFF FFFF (1.2.2)

Eqs. 1.1.2 and 1.2.2 as rendered are not tautologous, refuting the Borel base and hull conjectures.

Refutation of the Born rule in EQM as the probability of the wave function squared

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal. Result fragments are the repeating truth tables. Operators are: ~ Not; + Or; > Imply, →; = Equivalent.

From: Carroll, S. (2014). Why probability in quantum mechanics is given by the wave function squared. preposterousuniverse.com/blog/2014/07/24/why-probability-in-quantum-mechanics-is- given-by-the-wave-function-squared/, preposterousuniverse.com/blog/wpcontent/uploads/2014/07/quantum-slu.jpeg .

LET p, s, t, u, v, w: $|O_0\rangle$, $(1/\sqrt{2})$, $|\uparrow\rangle$, $|A_0\rangle$, $|e_0\rangle$, $|A_\uparrow\rangle$.

$|\psi\rangle =$ $|O_0\rangle ((1/\sqrt{2})|\uparrow\rangle + (1/\sqrt{2})|\downarrow\rangle) |A_0\rangle |e_0\rangle$ (1.1.1)

(p&((s&t)+(s&~t)))&(u&v) ;
FFFF FFFF FFFF FFFF, FFFF FFFF FTFT FTFT (1.1.2)

→ $|O_0\rangle ((1/\sqrt{2})|\uparrow\rangle|A_\uparrow\rangle + (1/\sqrt{2})|\downarrow\rangle|A_\downarrow\rangle) |e_0\rangle$ (apparatus measures) (1.2.1)

(p&((s&w)+(s&~w)))&v ;
FFFF FFFF FFFF FFFF, FFFF FFFF FTFT FTFT (1.2.2)

We now reduce the argument to four variables for simplicity.

LET p, q, r, s: $|O_0\rangle$, $|O_\uparrow\rangle$, $|\uparrow\rangle|A_\uparrow\rangle|e_\uparrow\rangle$, $(1/\sqrt{2})$.

→ $(1/\sqrt{2})|\uparrow\rangle|A_\uparrow\rangle|e_\uparrow\rangle + (1/\sqrt{2})|\downarrow\rangle|A_\downarrow\rangle|e_\downarrow\rangle)$ (decoherence) (1.3.1)

p&((s&r)+(s&~r)) ; FFFF FFFF FTFT FTFT (1.3.2)

= $(1/\sqrt{2})|O_0\rangle|\uparrow\rangle|A_\uparrow\rangle|e_\uparrow\rangle + (1/\sqrt{2})|O_0\rangle|\downarrow\rangle|A_\downarrow\rangle|e_\downarrow\rangle$ (*self-locating uncertainty*) (1.4.1)

s&((p&r)+(p&~r)) ; FFFF FFFF FTFT FTFT (1.4.2)

→ $(1/\sqrt{2})|O_\uparrow\rangle|\uparrow\rangle|A_\uparrow\rangle|e_\uparrow\rangle + (1/\sqrt{2})|O_\downarrow\rangle|\downarrow\rangle|A_\downarrow\rangle|e_\downarrow\rangle$ (measurement compete) (1.5.1)

s&((q&r)+(~q&~r)) ; FFFF FFFF TTFF TTFF (1.5.2)

Eqs. 1.1.2-1.5.2 as rendered are *not* tautologous.

We rewrite Eqs. 1.1.1-1.5.1 as: (1.1.1 → 1.2.1) → (1.3.1 = (1.4.1 → 1.5.1)). (1.6.1)

(((p&((s&t)+(s&~t)))&(u&v))>((p&((s&w)+(s&~w)))&v))>
(p&((s&r)+(s&~r)))=((s&((p&r)+(p&~r)))>(s&((q&r)+(~q&~r)))) ;
FFFF FFFF FTFF FFFT (1.6.2)

Eq. 1.6.2 is *not* tautologous, but differs from contradictory by two T values. This means the Born rule is refuted in Everettian quantum mechanics (EQM).

Refutation of the exclusivity rule (as extended basis of the Born rule and free will theorem)

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET: p, q, r, s: *p*, *i*, *j*, *k*; ~ Not; + Or; & And; > Imply, greater than;
= Equivalent; % possibility, for one or some; # necessity, for all or every;
(%x>#x) ordinal 1; (x=x) `T`, proof.

From: Cabello, A. (2018). A simple explanation of Born's rule. arxiv.org/abs/1801.06347.

"*The exclusivity (E) principle*. If every two events in a set are exclusive (i.e., if they are exclusive pairwise), then all the events in the set are mutually exclusive (i.e., they are exclusive globally). For example, if every two events in the set $\{i, j, k\}$ are pairwise exclusive, then their graph of exclusivity is a triangle. If the E principle holds, then the possible probability assignments $\{p_i, p_j, p_k\}$ do not only have to satisfy

$p_i + p_j \leq 1$, (1.1)
$p_i + p_k \leq 1$, and (2.1)
$p_j + p_k \leq 1$, but also (3.1)
$p_i + p_j + p_k \leq 1$." (4.1)
"$p_i + p_j \leq 1$, $p_i + p_k \leq 1$, and $p_j + p_k \leq 1$, but also $p_i + p_j + p_k \leq 1$" (5.1)

~(((p&q)+(p&r))>(%s>#s)) = (s=s); `FFFC FCFC FFFC FCFC` (1.2)
~(((p&q)+(p&s))>(%r>#r)) = (r=r) ; `FFFC FFFC FCFC FCFC` (2.2)
~(((p&r)+(p&s))>(%q>#q)) = (q=q) ; `FFFF FCFC FCFC FCFC` (3.2)
~((((p&q)+(p&r))+(p&s))>(%p>#p)) = (p=p);
`FFFC FCFC FCFC FCFC` (4.2)
(~(((p&q)+(p&r))>(%s>#s))&~(((p&q)+(p&s))>(%r>#r)))&
(~(((p&r)+(p&s))>(%q>#q))&~((((p&q)+(p&r))+(p&s))>(%p>#p))) ;
`FFFF FFFC FFFC FCFC` (5.2)

Eq. 5.2 as rendered is *not* tautologous. This means the exclusivity (E) principle is not a theorem. What is proved is something closer to a contradiction. For example, the author(s) could write "$p_i + p_j$ = proof, $p_i + p_k$ = proof, and $p_j + p_k$ = proof, but also $p_i + p_j + p_k$ = proof" in which case the table result of `FFFF FFFF FFFF FFFF` is forced into a contradiction. However, such liberties fly in the face of the intention of the rule which was to map probability as a theorem.

What follows is that if the exclusivity principle is refuted, then so are refuted the extended chain of subsequent assertions in the order of Born's rule and the free will thereon.

Remark: This is an example of the faulty mathematical logic which unfortunately peppers the quantum hypothesis field, beginning from about Gödel.

Refutation of the Borsuk-Ulam theorem (BUT)

From: James F. Peters (University of Manitoba), Arturo Tozzi (University of North Texas). (2018). Entangled antipodal points on black hole surfaces: the Borsuk-Ulam theorem comes into play. vixra.org/pdf/1804.0014v1.pdf

"BUT states that two features with matching description are mapped to a single feature one dimension lower, provided the function under assessment is continuous." (1.0)

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET: p fiducial point or region on a 2D plane or on an nD sphere;
q podal point or region on a 2D plane or on an nD sphere;
r contrapodal point or region on a 2D plane or on an nD sphere;
s dimension D;
~ Not; & And; + Or; - Not Or; > Imply, greater than; = Equivalent;
% possibility, for one or some; # necessity, for every or all;
(%s>#s) ordinal one, 1

We rewrite Eq. 1.0 as

If D>1, then antipodal points as podal and contrapodal in D are mapped to a single point as fiducial in D-1. (1.1)

Remark: For a function to be continuous, it must be in greater than one dimension.

(s>(%s>#s)) > ((s&(q&r))>((s-(%s>#s))&(p=(q+r))))
TTTT TTTT TTTT TTCC (1.2)

Eq. 1.2 as rendered is *not* tautologous due to the C contingency values (falsity), hence refuting BUT.

BUT is properly named the Borsuk-Ulam conjecture (BUC).

Refutation of bounded and Σ_1 formulas in PA

Abstract: A fundamental proposition for bounded and Σ_1 formulas in PA is *not* tautologous. While the author states that the informal notes are full of errors, this fundamental mistake causes the entire section about Rosser's form of Gödel's theorems to collapse. Therefore the proposition is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Moschovakis, Y.N. (2014). Lecture notes in logic.
math.ucla.edu/~ynm/lectures/lnl.pdf ynm@ucla.edu

Proposition 4C.12. *Suppose T is an extension of* PA*, in the language of* PA.

(1) *The class of T-Σ_1 formulas includes all prime formulas and is closed under the positive propositional connectives* & *and* ∨*, bounded quantification of both kinds, and unbounded existential quantification.*

... to show for the proof of (1) that the class of *T-Σ_1* formulas is closed under universal bounded quantification, it is enough to show that for any extended formula $\varphi(x,y,z)$,

$$T \vdash (\forall x \leq y)(\exists z)\varphi(x,y,z) \leftrightarrow (\exists w)(\forall x \leq y)(\exists z \leq w)\varphi(x,y,z); \qquad (4.12.1)$$

LET p, w, x, y, z: *φ,* w*, x, y, z.*

(~(y>#x)&(p&((x&y)&%z)))=((~(y>#x)&~(%w<%z))&(p&((x&y)&z))) ;
TTTT TTTT TTTT TTTT(48),
TNTN TNTN TNTN TNTN(16),
TTTT TTTT TTTT TTTT(48) (4.12.2)

the equivalence expresses an obvious fact about numbers, which can be easily proved by induction on *y*|and this induction can certainly be formalized in PA.

Eq. 4.12.2 as rendered is *not* tautologous. While the author states that the informal notes are full of errors, this fundamental mistake causes the entire section about Rosser's form of Gödel's theorems to collapse.

Refutation of modal logics that bound the circumference of transitive frames

Abstract: We evaluate six seminal equations, none of which is tautologous. (The author mistakenly labels the Löb axiom as a "fact" as proved by another author.) Therefore modal logics bounding the circumference of transitive frames is refuted and becomes another *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Goldblatt, R. (2019). Modal logics that bound the circumference of transitive frames. arxiv.org/pdf/1905.11617.pdf

2 Grzegorczyk and Löb

[Definition of strict implication] is □(ϕ→ψ) (2.0.1)

LET p, q: ϕ, ψ.

(p>q)>#(p>q) ; NTNN NTNN NTNN NTNN (2.0.2)

Remark 2.1: Eq. 2.1, not shown here, is missing a leading left parentheses.

This is the Löb axiom, referred to as a "fact": □(□p→p)→□p(2.3.1)

#(#p>p)>#p ; CTCT CTCT CTCT CTCT (2.3.2)

Remark 2.3.2: Eq. 2.3.2 is *not* tautologous, and hence not factual.

To indicate the nature of the axioms C_n, we indicate first that C_0 is equivalent over all frames to the formula C_0 is equivalent over all frames to the formula

◊p→◊(p∧¬◊p), (C_0.1)

%p>%(p&~%p) ; TCTC TCTC TCTC TCTC (C_0.2)

which is itself equivalent to the Löb axiom (2.3.1): (C_0.1)=(2.3.1)

(%p>%(p&~%p))=(#(#p>p)>#p) ;
CCCC CCCC CCCC CCCC (C0.2)=(2.3.2)

Remark C_0: Eqs. C_0.2 and 2.3.2 are *not* tautologous and *no*t equivalent as claimed.

Remark 2.4, 2.5, 5.1: The subsequent various combinations of C_1 with Eqs. 2.4, 2.5, 5.1, and others form self-evident tautologies, ignored as trivial.

6 Extensions of K4C_n; Linearity

K4.3 is the smallest normal extension of K4 that includes the scheme $(\phi\wedge\phi\rightarrow\psi)\vee(\psi\wedge\psi\rightarrow\phi)$. (6.1.1)

#((p&#p)>q)+#((q&#q)>p) ; NNNN NNNN NNNN NNNN (6.1.2)

Remark 6.1.2: Eq. 6.1.2 is *no*t tautologous, but rather a *truthity*.

The canonical frame of any normal extension of K4.3 is weakly connected. If a transitive weakly connected frame is *point-generated*, i.e. $W=\{x\}\cup\{y\in W:xRy\}$ for some point $x\in W$, then the frame is *connected*: it satisfies

$$\forall y\forall z(yRz\vee y=z\vee zRy). \quad (6.2.1)$$

LET p, q, r, s: x, y, R, z

((#q&(r&#s))+#q)=(#s+(#s&(r&#q))) ;
TTCC TTCC CCTT CCTT (6.2.2)

Such a connected frame can be viewed as a linearly ordered set of clusters.

Remark 6.2.2: Eq. 6.2.2 is *not* tautologous and hence *not* a linearly ordered set of clusters as claimed.

The six equations evaluated above are *not* tautologous, hence refuting the conjecture that modal logics bound the circumference of transitive frames. The author makes a serious mistake in labeling the Löb axiom as a fact, relying on Segerberg.

Refutation of Bourbaki's fixed point theorem and the axiom of choice

Abstract: We evaluate Moroianu's and the Tarski-Bourbaki fixed point theorem and axiom of choice (AC). Two versions of the theorem and then seven theorems and corollary which follow are also *not* tautologous. Therefore these conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Zarouali-Darkaoui, M. (2019). On the Bourbaki's fixed point theorem and the axiom of choice. arxiv.org/abs/1905.09782 mohssin.zarouali@gmail.com

Lemma 2 (Tarski–Bourbaki). Let E be a set, S⊂P(E), and ϕ:S→E a map such that ϕ(X)∉X for all X∈S. Therefore there is a unique subset M of E that can be well-ordered satisfying 1) for all x∈M:S_x∈S and ϕ(S_x)=x; 2) M∉S. (2.1.1)

Remark 2.1.1: We map Eq. 2.1.1 with a conjunctive consequent of 1) and 2).

LET q, r, p, s, x: E, M, ϕ, S, X

```
(p=((s>q)>(((#x<x)>(x<s))>~((p&x)<x))))>(((#x<r)=(((s&x)<x)&((p&(s&x))=x)))&~(r<s)) ;
                                        TTTT TFTF TTTT TTTT(16)
                                        TCTC TFTF TCTC TTTT(16)      (2.1.2)
```

Remark 2.1.3: If we map the consequent to a weakened condition of 1) implies 2), then:

```
(p=((s>q)>(((#x<x)>(x<s))>~((p&x)<x))))>(((#x<r)=(((s&x)<x)&((p&(s&x))=x)))>~(r<s)) ;
                                        TTTT TFTF TTTT TTTT          (2.1.3)
```

Eqs. 2.1.2 or 2.1.3 are *not* tautologous, to refute Moroianu's and the Tarski-Bourbaki fixed point theorem and axiom of choice (AC). The seven theorems and corollary which follow are also *not* tautologous.

Refutation of Bob Boyer's paradox

Abstract: Bob Boyer's paradox is as follows. "A question: It is generally granted that 'p implies p', which is to say, 'if p, then p'. So what about this claim: 'If any number is prime, then any number is prime'?" We find the sentences are unrelated. We then rewrite the second sentence as: "If at least one number is prime, then at least one number is prime"; or as "If at least one number is prime, then possibly all numbers are prime". Both are trivial tautologies, meaning neither is a contraction or paradox, and therefore forming a tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: cs.utexas.edu/~boyer/

A question

It is generally granted that 'p implies p', which is to say, 'if p, then p'. (1.1)

Remark 1.2: Eq. 1.1 as p>p is a trivial tautology. (1.2)

So what about this claim: 'If any number is prime, then any number is prime'?

LET p, q: number, prime.

Remark 2.1: We take 1.1 and 2.1 as unrelated, despite the conjunction in 2.1 of "So". Because "any number" is in the singular, we take it as "any one" number, that is, "at least one number as in the singular (but not all numbers as in the plural)". Hence, 'If at least one number is prime, then at least one number is prime'. (2.1)

Remark 2.2: Eq. 2.1 as (%p>q)>(%p>q) is also a trivial tautology. (2.2)

So what about this claim: 'If at least one number is prime, then possibly all numbers are prime'? (3.1)

Remark 3.2: Eq. 3.1 as (%p>q)>%(#p>q) is also a trivial tautology. (3.2)

The three equations tested are tautologous, to mean there is no contradiction or paradox.

Branching quantifier

From en.wikipedia.org/wiki/Branching_quantifier, the FOL capture of Hintikka's natural language sentence to demonstrate branching is:

$$[\ \forall x1\ \exists y1\ \forall x2\ \exists y2\ \phi\ (\ x1\ ,\ x2\ ,\ y1\ ,\ y2\ )\ ]\ \wedge\ [\ \forall x2\ \exists y2\ \forall x1\ \exists y1\ \phi\ (\ x1\ ,\ x2\ ,\ y1\ ,\ 2\ )\ ] \qquad (1.1)$$

where $\phi\ (\ x1\ ,\ x2\ ,\ y1\ ,\ y2\ )$ (2.1)

denotes $(\ V\ (\ x1\ )\ \wedge\ T\ (\ x2\ )\ )\ \rightarrow\ (\ R\ (\ x1\ ,\ y1\ )\ \wedge\ R\ (\ x2\ ,\ y2\ )\ \wedge\ H\ (\ y1\ ,\ y2\ )\ \wedge\ H\ (\ y2\ ,\ y1\ )\ )$ (3.1)

LET: p φ; q ; r R; s ; t T; u H ; v V; w x2; x x1; y y1; z y2; #∀; ∃ %; nvt not tautologous;

Designated truth value is T Tautology (proof), with C Contingent (falsity),
N Non contingent (truth), and F for contradiction (absurdum).

(#x&(%y&(#w&(%z&(p&(x&(w&(y&z)))))))) & (#w&(%z&(#x&(p&(x&(w&(y&z))))))) ; (1.2)

(p&(x&(w&(y&z)))) ; (2.2)

((v&x)&(t&w)) > (((r&(x&y))&(r&(w&z)))&((u&(y&z))&(u&(z&y)))) ; (3.2)

For the conjecture as If Eq 2.1 is equivalent to Eq 3.1, then Eq 1.1 as: (4.1)

((p&(x&(w&(y&z))))=(((v&x)&(t&w))>(((r&(x&y))&(r&(w&z)))&((u&(y&z))&(u&(z&y)))))) >
((#x&(%y&(#w&(%z&(p&(x&(w&(y&z))))))))&(#w&(%z&(#x&(p&(x&(w&(y&z)))))))) ;
nvt (4.2)

In Model 1, fragments of repeating truth tables are:

```
FFFF FFFF FFFF FFFF
TTTT TTTT TTTT TTTT
TNTN TNTN TNTN TNTN
FTFT FTFT FTFT FTFT
FTFT TNTN FTFT TNTN
```

Meth8 finds Eq 4.2 nvt, hence invalidating the conjecture of Eq 4.1 (composed of Eqs 1.1, 2.1, and 3.1).

Weakening the conjecture with "denotes" to mean "implies" also results with nvt and these truth table fragments:

```
FFFF FFFF FFFF FFFF
FTFT FTFT FTFT FTFT
FNFN FNFN FNFN FNFN
FTFT FNFN FTFT FNFN
```

Refutation of constructive Brouwer fixed point theorem

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET: + Or; & And; > Imply, greater than; = Equivalent.

We construct the Brouwer fixed point theorem (BFPT) as implications of four variables:

the antecedent is the relationship of their rank orders; (21.1)

((p>q)>r)>s ; `TFTT FFFF TTTT TTTT` (21.2)

the consequent is disjunction of relational pairs of variables (away tautologous); and (22.1)

(((p>q)+(p>r))+(p>s))+(((q>r)+(q>s))+(r>s)) ;
`TTTT TTTT TTTT TTTT` (22.2)

the implication is always tautologous. (23.1)

(((p>q)>r)>s) > ((((p>q)+(p>r))+(p>s))+(((q>r)+(q>s))+(r>s))) ;
`TTTT TTTT TTTT TTTT` (23.2)

Eq. 23.2 as rendered is tautologous, and on its face appears as a constructive proof of BFPT.

However there is problem as to completeness because the consequent is composed of the totality of ordered combinations.

Therefore, the connective is equivalence. (24.1)

(((p>q)>r)>s) = ((((p>q)+(p>r))+(p>s))+(((q>r)+(q>s))+(r>s))) ;
`TFTT FFFF TTTT TTTT` (24.2)

Eq. 24.2 is *not* tautologous, and refutes BFPT using a constructive proof.

Remark: If the consequent is taken as a multiplicity of ordered combinations, the equivalence connective and the implication connective share the same table result which deviates further from Eq. 24.2 with another `F` contradictory value. (25.1)

(((p>q)>r)>s) [= or >] ((((p>q)&(p>r))&(p>s))&(((q>r)&(q>s))&(r>s))) ;
`TTFF TTTT TFFF TFTT` (25.2)

We conclude that BFPT is mislabeled as a theorem, as non constructively based on set theory, and correctly named as the Brouwer fixed point conjecture (BFPC).

Refutation of Browder's theorem

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s, t, u: x, n, k, C, ℕ , Ω;
~ Not, ¬; + Or, ∨; & And, ∧; = Equivalent; > Imply; < Not Imply, ∈;
\# necessity, for all or every, □, ∀; % possibility, for one or some, ◊, ∃;
y≤z (~z>y).

From: Ferreira, F.; et al. (2018). On the removal of weak compactness arguments in proof mining. arXiv:1810.01508 laurentiu.leustean@unibuc.ro

[page 4, numbering added]: In loose terms, one can prove Browder’s theorem in a certain formal theory using the principle

$$\forall x \in C \exists n \in \mathbb{N} \ (x \in \Omega_n) \rightarrow \exists n \in \mathbb{N} \ \forall x \in C \exists k \leq n \ (x \in \Omega_k), \qquad (4.1)$$

where C is a bounded closed convex subset of the Hilbert space, and $(\Omega_n)_{n\in\mathbb{N}}$ is a sequence of open sets for the strong topology.

((#p<(s<%q))<(t&(p<(u&q)))) >
((%q<(t&#p))<(~(q&(p<(u&r)))>(s&%r))) ;
TTTT TTTT TTTT TTTT, TCTC TCTT TTTC TTTC,
TTTC TTTC TTTC TTTC (4.2)

Eq. 4.2 as rendered is *not* tautologous. This means Browder's theorem as framed is refuted.

Refutation of the tetralemma and Buddhist logic

Abstract: The Buddhist tetralemma as a rendition of the Greek square of opposition produces four axioms for true, false, true and false (contradiction), and neither true nor false (contradiction). There is no designated proof value in Buddhist logic. Because Greek logic of about -350 was transmitted along with mathematical astronomy to India beginning in -100, Greek logic predates Buddhist logic by more than 200 years. Hence Buddhist logic is a trivial subset and mis-application of the Greek logic.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s; ~ Not; + Or; - Not Or; & And; = Equivalent;
% possibility, for any one or some, ∃; # necessity, for every or all, ∀;
(s=s) T tautology; (s@s) **F** contradiction.

The tetralemma axioms of Buddhist logic are:

Affirmation: (0.1.1)

p=q ; T**FF**T T**FF**T T**FF**T T**FF**T (0.1.2)

Negation: (0.2.1)

p=~q ; **F**TT**F** **F**TT**F** **F**TT**F** **F**TT**F** (0.2.2)

Both: (0.3.1)

(p=q)&(p=~q); **FFFF FFFF FFFF FFFF** (0.3.2)

Neither: (0.4.1)

(p=q)-(p=~q) ; **FFFF FFFF FFFF FFFF** (0.4.2)

The rules of inference of Buddhist logic use the universal quantifier to mean everywhere (all locations), everything (all things), and always (all times), ie, all things are everywhere at all times.

Remark: The existential quantifier applies to rules only without the universal quantifier, as *only* in Eqs. 1.2 and 2.2.

Whether p is q : (1.1)

%p=%q ; TCCT TCCT TCCT TCCT (1.2)

Whether p is not q: (2.1)

%p=%~q ; CTTC CTTC CTTC CTTC (2.2)

Whether p is q everywhere: (3.1)

#(p=q)=(p=p) ; N**FF**N N**FF**N N**FF**N N**FF**N (3.2)

Whether p is q always: (4.1)

#(p=q)=(p=p) ; N**FF**N N**FF**N N**FF**N N**FF**N (4.2)

Whether p is q in everything: (5.1)

#(p=q)=(p=p) ; N**FF**N N**FF**N N**FF**N N**FF**N (5.2)

Whether p is not q everywhere: (6.1)

#(p=~q)=(p=p) ; **F**NN**F** **F**NN**F** **F**NN**F** **F**NN**F** (6.2)

Whether p is not q always: (7.1)

#(p=~q)=(p=p) ; **F**NN**F** **F**NN**F** **F**NN**F** **F**NN**F** (7.2)

Whether p is not q in everything: (8.1)

#(p=~q)=(p=p) ; **F**NN**F** **F**NN**F** **F**NN**F** **F**NN**F** (8.2)

The axioms and rules of inference above are *not* tautologous. This refutes Buddhist logic.

Remark: It is mis-reported, notably by Graham Priest, that the four axioms of Buddhist logic represent a four-valued logic as, for example: true; false; true and false (contradiction); and neither true nor false (contradiction). Such a three-valued logic has no designated proof value for tautology.

This places Buddhist logic as a subset of Greek logic, for which there are historical reasons. The Greek square of opposition dates to about -350, but the Buddhist rendition dates to -50. This is because Greek philosophical knowledge was exported west to east during that 300 year period as concurrent with the transmission of mathematical astronomy to India.

Buridan's Ass paradox

Donkey's are known to eat only the food stuff nearest to them. Buridan's paradox states that a donkey with two food sources at equal distances chooses neither and starves. (1)

LET:	p Left hay;	q Donkey; r Right hay
	$(p < q) < r)$	Positions from left to right of left hay, the donkey, and right hay
	(q-p), (r-q)	Distance to hay on either side of the donkey
	$(u = (q\&p))$	Donkey eats right hay
	$(v = (q\&r))$	Donkey eats left hay

If the position of the left hay is less than the position of the donkey is less than the position of the right hay and the hay distance from the donkey is the same on the left and right sides, then the donkey eating hay left hay and right hay implies the donkey does "not eat either the left or right hay". (2)

$$(((p<q)<r)\&((q-p)=(r-q)))>(((u=(q\&p))\&(v=(q\&r)))> \quad \sim(u+v)) \text{ ; vt} \qquad (3)$$

Now we write Assertion 2 with the ending connective and consequent as implies the donkey does not "not eat either the left or right hay". (4)

$$(((p<q)<r)\&((q-p)=(r-q)))>(((u=(q\&p))\&(v=(q\&r)))> \sim \sim(u+v)) \text{ ; vt} \qquad (5)$$

At first appearance, Eq 3 tautologous is contradicted by Eq 4 also tautologous.

We test this by including both the ending consequent expressions to rewrite as And or Or, for: implies the donkey "does eat and/or does not eat either the left or right hay". (6), (7)

$$(((p<q)<r)\&((q-p)=(r-q)))>(((u=(q\&p))\&(v=(q\&r)))> ((u+v)\&\sim(u+v))) \text{ ; vt} \quad (8)$$
$$(((p<q)<r)\&((q-p)=(r-q)))>(((u=(q\&p))\&(v=(q\&r)))> ((u+v)+\sim(u+v))) \text{ ; vt} \qquad (9)$$

Therefore the donkey eating and not eating reduces as a choice to tautologous, and the donkey eating or not eating reduces as a choice to tautologous. In other words, the donkey can logically choose to eat and not eat as a tautologous choice. This means the paradox of Buridan's donkey is not a paradox of the donkey unable to eat, but is a theorem of the donkey able to eat or not to eat *if it wants to eat*.

Confirmation of the collapse of the Buss hierarchy of bounded arithmetics

Abstract: Two seminal rules of inference evaluated as *not* tautologous. This means the following are also refuted: Buss's hierarchy of bounded arithmetics does not entirely collapse; Takeuti's argument implies P ≠ NP; and systems PV and PV⁻. What follows is that separation of bounded arithmetic using a consistency statement is not viable. Therefore the above are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables.
(See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1;
(%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Yamagata, Y. (2019). Separation of bounded arithmetic using a consistency statement. arxiv.org/pdf/1904.06782.pdf yoriyuki.y@gmail.com, yoriyuki.yamagata@aist.go.jp

Abstract. This paper proves Buss's hierarchy of bounded arithmetics ... does not entirely collapse... . Further, we can allow any finite set of true quantifier free formulas for the BASIC axioms … . By Takeuti's argument, this implies P ≠ NP.

3. PV and related systems
3.2. Equality axioms. The identity axiom is formulated as (15) t = t
The remaining equality axioms are formulated as inference rules rather than axioms.

(19) $\dfrac{t(x) = u(x)}{t(r) = u(r)}$ for any term r. (3.2.19.1)

LET p, q, s, t , u, v, x, w :
ε, i, s, t_1, u, v, x, t_2 .
((t&x)=(u&x))>((t&r)=(u&r)) ;
`TTTT TTTT TTTT TTTT( 1), TTTT` **`FFFF`** `TTTT` **`FFFF`**`( 2),`
`TTTT TTTT TTTT TTTT( 2), TTTT` **`FFFF`** `TTTT` **`FFFF`**`( 2),`
`TTTT TTTT TTTT TTTT( 2), TTTT` **`FFFF`** `TTTT` **`FFFF`**`( 2),`
`TTTT TTTT TTTT TTTT( 2), TTTT` **`FFFF`** `TTTT` **`FFFF`**`( 2),`
`TTTT TTTT TTTT TTTT(17)` (3.2.19.2)

3.3. Induction.

(20) $$\frac{t1(\varepsilon) = t2(\varepsilon)\ \ t1(six) = vi(t1(x))\ \ t2(six) = vi(t2(x))\ \ (i = 0, 1)}{t1(x) = t2(x)} \tag{3.3.20.1}$$

((((t&p)=(w&p))&((t&((s&q)&x))=((v&q)&(t&x)))) &
(((w&((s&q)&x))=((v&q)&(w&x)))&(q=((q@q)+(q=q))))) >
((t&x)=(w&x)) ;

```
                                TTTT TTTT TTTT TTTT(16),
TTTT TTTT TTTT TTTT( 1), TTFT TTFT TTTT TTTT( 1),
TTTT TTTT TTTT TTTT( 1), TTFT TTFT TTTT TTTT( 1),
TTTT TTTT TTTT TTTT( 1), TTTT TTTT TTFT TTFT( 1),
TTTT TTTT TTTT TTTT( 1), TTTT TTTT TTFT TTFT( 1),
TTFT TTFT TTTT TTTT( 1), TTTT TTTT TTTT TTTT( 1),
TTFT TTFT TTTT TTTT( 1), TTTT TTTT TTTT TTTT( 1),
TTTT TTTT TTFT TTFT( 1), TTTT TTTT TTTT TTTT( 1),
TTTT TTTT TTFT TTFT( 1), TTTT TTTT TTTT TTTT( 1)    (3.3.20.2)
```

Remark 3.3.20.2: The term (i=0,1) is written as i=**F** or T, not as ordinals.

The system PV contains defining axioms, equality axioms, and induction as axioms and inference rules. By contrast, the system PV^{-} contains only defining axioms and equality axioms as axioms and inference rules.

Eqs. 3.2.19.2 and 3.3.20.2 as rendered are *not* tautologous. This refutes two inference rules, whereby the following are also refuted: Buss's hierarchy of bounded arithmetics does not entirely collapse; Takeuti's argument implies P ≠ NP; and systems PV and PV^{-}. What follows is that separation of bounded arithmetic using a consistency statement is not viable.

Refutation of Cabannas theory of objectivity

Abstract: We evaluate equations about adding or subtracting something from nothing. The duals as a disjunction are tautologous. However that disjunction is not itself equivalent to nothing. This refutes the Cabannas theory of objectivity at its atomic level, forming a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
\> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Cabannas, V. (2016). Theory of objectivity. vixra.org/pdf/1904.0536v1.pdf
contact@theoryofobjectivity.com traduz@teoriadaobjetividade.com

In fact, the answers to these questions will be discussed in detail based on the ideas presented herein. Before addressing those topics, however, it is necessary to conclude in a well-grounded manner what Nothing was, since it is the fundamental basis of that analysis.

In this theory, Nothing, time zero, has autonomous existence and does not mean zero in the form agreed upon in human mathematics. To validate this theory of Nothing, it is necessary to provide a full proof. This proof is existence itself. Material existence is the greatest proof that Nothing had an autonomous existence, for if it were not so, all other things could not arise from it. However, I will attempt here to demonstrate even using mathematical foundations, that Nothing, time zero, does not have the meaning that humanity has agreed upon. That is, Nothing does not mean the absence of any element.

Initially, to demonstrate that Nothing in fact possesses an autonomous existence in itself, I will present an equation formed by a true sentence. This true sentence stems from the first absolute truth, which says that before the universe arose, there was Nothing.

The universe, of course, represents everything that exists. So, if there was Nothing before the universe existed, a unit could be added to Nothing (n) and it would remain Nothing (n + 1). A unit could also be subtracted from Nothing and it would remain Nothing (n - 1). We then have the following, considering n = 0 = Nothing: N + 1 = n – 1, N - n = -1 – 1, 0 = -2;

Or, reversing equality: N - 1 = n + 1, N - n = 1 + 1, 0 = 2. That is, the equation has two possible solutions: -2 and +2 . (1.0)

> **Remark 1.0:** We write the above to mean "Nothing plus one (or T) as nothing OR nothing minus one (or T) as nothing is a theorem." (1.1)
>
> LET p, ~#p: p, Nothing [not every thing]
> ((~#p+(%p>#p)=~#p)+((~#p-(%p>#p))=~#p) ;
> `TTTT TTTT TTTT TTTT` (1.2)

These simple mathematical formulas mean that Nothing (n) plus or minus a unit is equal to Nothing (n), (2.0)

> **Remark 2.0:** We write this to mean, "Nothing plus one (or T) as nothing OR nothing minus one (or T) as nothing is a theorem equal to nothing as a theorem." (2.1)
>
> (((~#p+(%p>#p))=~#p)+((~#p-(%p>#p))=~#p))=~#p ;
> `TCTC TCTC TCTC TCTC` (2.2)

for if Nothing is the absence of existence, adding to or subtracting from that absence of existence positive or negative values of the same weight will yield the same result: the absence of existence. That is, the result of adding a unit to Nothing (n + 1) is equal to the result of subtracting a unit from Nothing (n – 1). By solving this true and logical equality, one will always find a nonzero value.

Eq. 1.2 is tautologous as expected because the antecedent and consequent as duals form a disjunction. However, Eq. 2.2 is not tautologous because the theorem of Eq. 1.2 is not equivalent to Nothing. This refutes the Cabannas theory of objectivity at its atomic level.

Refutation of the Cabannas conjecture of objectivity

Abstract: We evaluate the 13 atomic equations, with none tautologous. This refutes the Cabannas conjecture of objectivity, forming a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Cabannas, V. (2016). Theory of objectivity. vixra.org/pdf/1904.0536v1.pdf
contact@theoryofobjectivity.com

The universe, of course, represents everything that exists. So, if there was Nothing before the universe existed, a unit could be added to Nothing (n) and it would remain Nothing (n + 1). (3.1)

LET p, ~#p:
p, Nothing [not every thing], N, n;
(%s>#s) ordinal 1; (%s<#s) ordinal 2; (s@s) zero.

p+(%s>s) ; NTNT NTNT TTTT TTTT (3.2)

A unit could also be subtracted from Nothing and it would remain Nothing (n - 1). (4.1)

p-(%s>s) ; CFCF CFCF FFFF FFFF (4.2)

We then have the following, considering n = 0 = Nothing: (5.1)

(p=(s@s))= p ; FFFF FFFF FFFF FFFF (5.2)

Remark 5.1: The author may mean to write (n=0) and (0=Nothing). (6.1)

(p=(s@s))&((s@s)=p) ; TFTF TFTF TFTF TFTF (6.2)

which is a strengthening of Eq. 5.1.

N + 1 = n – 1, i.e, (3.2)=(4.2) (7.1)

(p+(%s>#s))=(p-(%s>#s)) ; **FFFF FFFF FFFF FFFF** (7.2)

N - n = -1 – 1, (8.1)

(p-p)=(~(%s>#s)-(%s>#s)) ; **F**T**F**T **F**T**F**T **F**T**F**T **F**T**F**T (8.2)

Remark 8.1: The author may mean to write N-n=((~1)+(~1)) ; (9.1)

(p-p)=(~(%s>#s)+~(%s>#s)) ; CNCN CNCN CNCN CNCN (9.2)

which is a strengthening of Eq. 8.2.

0 = -2; (10.1)

(s@s)=~(%s<#s) ; CCCC CCCC CCCC CCCC (10.2)

Or, reversing equality: N - 1 = n + 1, i.e, (8.1) (11.1)

(p-(%s>#s))=(p+(%s>#s)) ; **F**T**F**T **F**T**F**T **F**T**F**T **F**T**F**T (11.2)

N - n = 1 + 1, (12.1)

(p-p)=((%s>#s)+(%s>#s)) ; NCNC NCNC NCNC NCNC (12.2)

0 = 2. i.e, Not(10.1) (13.1)

(s@s)=(%s<#s) ; NNNN NNNN NNNN NNNN (13.2)

That is, the equation has two possible solutions: -2 and +2, i.e, (8.1) and (12.1): (14.1)

((p-p)=(~(%s>#s)-(%s>#s)))&((p-p)=((%s>#s)+(%s>#s))) ;
FC**F**C **F**C**F**C **F**C**F**C **F**C**F**C (14.2)

Remark 14.1: If the author meant to write -2 or +2, i.e, (8.1) or (12.1): (15.1)

((p-p)=(~(%s>#s)-(%s>#s)))+((p-p)=((%s>#s)+(%s>#s))) ;
NTNT NTNT NTNT NTNT (15.2)

which is a strengthening of Eq. 14.1.

We evaluated 13 equations, with none tautologous. This refutes the Cabannas conjecture of objectivity at its most atomic level.

Refutation of Cantor's original continuum hypothesis via injection and binary trees

From: Pindsle, C. (2018). "The continuum hypothesis". vixra.org/pdf/1803.0088v1.pdf

Note: Because of no email contact disclosed at that venue, that author's name is likely a pseudonym.

> "[With representation using binary trees: the intention was] to prove the hypothesis in its original form as proposed by Georg Cantor in 1878: Any uncountable set of real numbers is equinumerous with ℝ.. Since there is a bijection between the open interval (0,1) and the set of all the real numbers, there is a bijection between any subset of (0,1) and a subset of ℝ.. Therefore it is sufficient to prove: Any uncountable subset of (0,1) is equinumerous with ℝ.."

> $\phi : RJ \longmapsto RJT$ *is bijective:* It is injective because: $\phi(r1) = \phi(r2) \Rightarrow (\phi(r1) \succ \phi(r2)$ and $\phi(r2) \succ \phi(r1))$ $\Rightarrow (r1 \succ r2$ and $r2 \succ r1) \Rightarrow r1 = r2$ (3.5.1.)

Because the intention of the proof is to show $\phi(r1) = \phi(r2) \Rightarrow$ .... $\Rightarrow r1 = r2$, we rewrite Eq. 3.5.1.

$$\phi(r1) = \phi(r2) \Rightarrow r1 = r2 \tag{3.5.1.1}$$

We assume the apparatus and method of Meth8/VŁ4 with designated *proof* value `T`, and contradiction value `F`. The 16-valued result table is row-major and presented horizontally.

LET p q r: *ϕ,* lc_phi; r1; r2; & And; > Imply, ≻, ⇒; = Equivalent to.

((p&q)=(p&r))>(q=r) ; `TTFT FTTT TTFT FTTT` (3.5.1.2)

Eq. 3.5.1.2 as rendered is *not* tautologous. Hence, the hypothesis as Eq. 3.5.1.1 fails.

This is the briefest known such refutation of Cantor's continuum conjecture.

> **Remark:** To coerce Eq. 3.5.1.2 into tautology, we weaken the argument by replacing the Equivalent connective with the Imply connective.

((p&q)>(p&r))>(q>r) ; `TTFT TTTT TTFT TTTT` (3.5.1.3)

Eq. 3.5.1.3 does come closer to tautology with two less contradiction `F` values, but to no avail.

Refutation of Cantor's continuum hypothesis

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET: ~ Not; & And; + Or; > Imply; < Not Imply; = Equivalent;
% possible, for one or some; # necessary, for all or every.

From: Ragusa, R. (2018). The function f(x)=C and the continuum hypothesis, an algebraic proof of the CH. vixra.org/pdf/1806.0030v1.pdf

The continuum hypothesis was ... based on the possibility that infinite sets come in different sizes ... that the set of real numbers is a larger infinity than the set of natural numbers. That is to say the set of real numbers has a cardinal number greater than the cardinal number of the set of natural numbers; (1.1.0)
and ... that no set exists with a cardinal number between the two. (1.2.0)

LET p, q, r, s: cardinal number (index), natural number, real number, set;
p<(s&#q) cardinal number (index) is less than all natural numbers .

We rephrase Eq. 1.1.0 as:

"Possibly a cardinal number (index), within the set of all natural numbers, for the set of real numbers is greater than a cardinal number (index), within the set of all natural numbers, for the set of natural numbers." (1.1.1)

(%(p<(s&#q))&(s&r))>((p<(s&#q))&(s&q)) ;
`TTTT TTTT TTTT NFNT` (1.1.2)

We rephrase Eq. 1.2.0 as: "No cardinal number, within the set of all natural numbers, exists for a set greater than the cardinal number for the set of natural numbers and less than the cardinal number for the set of real numbers." (1.2.1)

((p<(s&#q))&(s&q))<(~(p<(s&#q))<(%(p<(s&#q))&(s&r))) ;
`FFFF FFFF FFFC FFFC` (1.2.2)

The argument is Eqs. (1.1.0 and 1.2.0), meaning Eqs. (1.1.1 and 1.2.1). (2.1)

((%(p<(s&#q))&(s&r))>((p<(s&#q))&(s&q))) & (((p<(s&#q))&(s&q))
<(~(p<(s&#q))<(%(p<(s&#q))&(s&r)))) ;
`FFFF FFFF FFFC FFFC` (2.2)

Remark: Eqs. 1.2.2 and 2.2 bear the same result table.

Eq. 2.2 as rendered is *not* tautologous, and nearly contrariety (with two `C`), hence refuting the continuum hypothesis.

Refutation of Cantor's diagonal argument

From: en.wikipedia.org/wiki/Cantor%27s_diagonal_argument

"A generalized form of the diagonal argument was used by Cantor to prove Cantor's theorem: for every set *S*, the power set of *S*—that is, the set of all subsets of *S* (here written as ***P***(*S*))—has a larger cardinality than *S* itself. This proof proceeds as follows: Let *f* be any function from *S* to ***P***(*S*). It suffices to prove *f* cannot be surjective. That means that some member *T* of ***P***(*S*), i.e. some subset of *S*, is not in the image of *f*. As a candidate consider the set:

$T = \{ s \in S : s \notin f(s) \}$. [0.1]

For every *s* in *S*, either *s* is in *T* or not. If *s* is in *T*, then by definition of *T*, *s* is not in *f*(*s*), so *T* is not equal to *f*(*s*). [1.1]

On the other hand, if *s* is not in *T*, then by definition of *T*, *s* is in *f*(*s*), so again *T* is not equal to *f*(*s*) ..." [2.1]

We assume the apparatus of the Meth8 modal logic model checker implementing variant system VŁ4. Meth8 allows to mix four logical values with four analytical values. The designated *proo*f value is T.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	p=p	T	Tautology	proof	11	3
2	p@p	F	Contradiction	absurdum	00	0
3	%p>#p	N	Non-contingency	truth	01	1
4	%p<#p	C	Contingency	falsity	10	2

LET: ~ Not; + Or; & And; \ Not and; > Imply; < Not imply, ∈ ; = Equivalent to; @ Not equivalent to; # all, every; % some, each; pqrs fTSs; *s*∉ *f*(*s*) ~(*s*>*f*(*s*))

Results are the repeating proof table(s) of 16-values in row major horizontally.

q=((s<r)>~(s<(p&s))) ; `FFTT FFTT TFFT FFTT` (0.2)

(q=((s<r)>~(s<(p&s))))>(((#s<r)>((s<q)>~(s>(p&s))))>(q@(p&s))) ;
`TTTT TTTT FTTF TTTF` (1.2)

(q=((s<r)>~(s<(p&s))))>((~(#s<r)>((s<q)> (s>(p&s))))>(q@(p&s))) ;
`TTTT TTTT CTTF TTTF` (2.2)

Because Eqs. 1.2 and 2.2 result in the same consequent, they are rewritten to remove respective common terms and set as an equivalence according to Eqs. [1.1] and [2.1].

((#s<r)>((s<q)>~(s>(p&s)))) = (~(#s<r)>((s<q)> (s>(p&s)))) ;
`TTTT TTTT NCTT FTTT` (3.2)

Eqs. 1.2 and 2.2 as rendered are *not* tautologous. Hence Cantor's diagonal argument is not supported.

Refutation of Cantor's pairing function:

Recall Cantor's pairing function as a functor designation of

c(x,y) = ((1 /2)* (x+y)*(x+y+1))+y. (1)

This is rewritten with spaces so as to map to Meth8 script for propositions (LET p=c, q=x, r=y) and for theorems (LET A=c, B=x, C=y) with 01 as (%p>%#p):

```
(c  (x,y)   = (  (    1     /          2          ) *   (x+y)*(   x+y +    1    )) + y.
                                                                                (2a)
(p&(q&r))  = (  ((%p>%#p)\((%p>%#p)+(%p>%#p)))  &  ((q+r)&(((q+r)+(%q>%#q))  + r))) ;
                                        TTNN NNNC TTNN NNNC            (2b)
```

Should we replace the main connective in 2b from equivalent to "=" with imply ">"

```
(p&(q&r))  >  (  ((p\p)\((p\p)+(p\p)))  &  ((p+q)&(((p+q)+(p\p))  + q))) ;
                                             TTTT TTTC TTTT TTTC      (3b)
```

then Eq. 2b fares slightly better toward tautology, but still not tautologous.

This leads us to consider that Cantor's pairing function is not an equivalency and hence suspicious.

Refutation of Carroll's tortoise and Achilles as a paradox

We assume the method and apparatus of Meth8/VŁ4 where `T`autology is the designated *proof* value, `F` is contradiction, `N` is truthity (non-contingency), and `C` is falsity (contingency). The unique 16-valued truth table fragment(s) is row-major and horizontal, but repeating in different order for the 128-tables of 11 potential variables.

From: en.wikipedia.org/wiki/What_the_Tortoise_Said_to_Achilles
Carroll, L. (1895). "What the tortoise said to Achilles". Mind.

LET p, q, s, u, v: thing_1, thing_2, thing_same, triangle_side_1_thing, triangle_side_2_thing; > Imply.

A: "Things that are equal to the same are equal to each other" (1.1)

((p=r)&(q=r))>(p=q) ; `TTTTTTTTTTTTTTTT` (1.2)

B: "The two sides of this triangle are things that are equal to the same" (2.1)

(u=s)&(v=s) ;
`TTTTTTTTFFFFFFFF,FFFFFFFFFFFFFFFF,FFFFFFFFTTTTTTTT` (2.2)

Therefore Z: "The two sides of this triangle are equal to each other" (3.1)

u=v ; `TTTTTTTTTTTTTTTT,FFFFFFFFFFFFFFFF` (3.2)

A and B (4.1)

(((p=s)&(q=s))>(p=q))&((u=s)&(v=s)) ;
`TTTTTTTTFFFFFFFF,FFFFFFFFFFFFFFFF,FFFFFFFFTTTTTTT` (4.2)

A and B, Therefore Z (5.1)

((((p=s)&(q=s))>(p=q))&((u=s)&(v=s)))>(u=v) ;
`TTTTTTTTTTTTTTTT` (5.2)

Eq. 5.2 as rendered is tautologous and hence a theorem. Eq. 5.2 is *not* contradictory: this refutes Carroll's tortoise and Achilles as a paradox.

Confirmation of Caswell's 1952 "Significant curriculum issues" using mathematical logic

Abstract: From Caswell's seminal paper of 1952, we evaluate 11 significant curriculum issues, then group them into the arbitrary categories of: (1) Learner-Learned (p,q); (2) Accountability of school (r,s,t); (3) Unit of value (u,v,w); and (4) Identity of process (x,y,z). We do not assume weighting factors, so as to avoid AI networking issues. We evaluate the conjecture that (3) implies (2) implies (4) implies (1). The conjecture as rendered is confirmed as tautologous. Therefore Caswell's hypothesis is elevated to a theorem. What follows is that mathematical logic can be a useful approach to verify curriculum issues and extended in the field of education.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

Variable	**Assignment**
p	The learner as the student receiving curriculum
q	The learned as the teacher imparting curriculum
r	Responsibilty of the school
s	Role of the school in the community
t	Capacity of the school for needs of all, extended opportunities
u	Economical unit value as driver ed, occupational ed
v	Political unit value as democracy, fascism & socialism, monarchy, republic
w	Relational individual value as counseling of unstable, family unit ed, sex ed
x	Identity of the planner.
y	Identity of the elements as planned.
z	Identity of the educational plan.

LET: ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⊢; < Not Imply, less than, ∈;
= Equivalent, ≡, ⊨; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(p=p) T as tautology; (p@p) **F** as contradiction;
(%p<#p) C as contingency, Δ; (%p>#p) N as non-contingency, ∇;
~(y < x) (x ≤ y), (x ⊆ y).

From: Caswell, H. L. (1952). Significant curriculum issues.
Association for supervision and curriculum development. NEA.

We group variables defined from Caswell's selections as affecting curriculum into four categories:

(1) Learner-Learned (p, q);
(2) Accountability of school (r, s, t);
(3) Unit of value (u, v, w); and
(4) Identity of process (x, y, z).

We assume no variable or category is weighted, as in AI expositions.

We make arbitrary assumptions in applying the implication operator as follows.

For (3), the political unit implies the economic unit implies the individual value. (5.1)

((v>w)>u) = (p=p) ;
FFFF FFFF FFFF FFFF(6), TTTT TTTT TTTT TTTT(10) (5.2)

For (2), the role of the school in the community combines with the responsibility of the school and combines with the capacity of the school. (6.1)

((s&r)&t) = (p=p) ;
FFFF FFFF FFFF FFFF(8),**FFFF FFFF FFFF** TTTT**(8)** (6.2)

For (4), the identity of the elements planned combines with the identity of the planner and combines with the identity of the educational plan. (7.1)

((y&x)&z) = (p=p) ;
FFFF FFFF FFFF FFFF(112),TTTT TTTT TTTT TTTT(16) (7.2)

For (1), the student combines with the teacher. (8.1)

(p&q) = (p=p) ; **FFF**T **FFF**T **FFF**T **FFF**T**(128)** (8.2)

For the student implies the teacher: (9.1)

(p>q) = (p=p) ; T**F**TT T**F**TT T**F**TT T**F**TT(128) (9.2)

We proceed to build this conjecture:

For ((3) & (2) & (4) & (1)) > (p>q), if the values combine with the accountabilities and combine with the identities and combine with the student and teacher, then if the student implies the teacher. (10.1)

(((((v>u)>w)&((s&r)&t))&((y&x)&z))&(p&q))>(p>q) ;
TTTT TTTT TTTT TTTT(128) (10.2)

Eq. 9.2 as rendered is tautologous. Therefore our conjecture of Eq. 9.1 is confirmed, and Caswell's significant curriculum issues, presented as a hypothesis, are now a theory.

Refutation of Whewell's axiom of causality

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal.

> LET p; q; r; s: effect or reaction; cause or action; magnitude of effect; magnitude of cause; ~ Not; & And; \ Not And; + Or; - Not Or; > Imply.

From: en.wikipedia.org/wiki/Axiom_of_Causality

> "According to William Whewell [1794-1866] the concept of causality depends on three axioms: (4.1)
>
> 1. Nothing takes place without a cause (1.1)
> 2. The magnitude of an effect is proportional to the magnitude of its cause (2.1)
> 3. To every action there is an equal and opposed reaction. (3.1)
>
> A similar idea is found in western philosophy for ages (sometimes called principle of universal causation (PUC) or law of universal causation, for example:
>
> > In addition, everything that becomes or changes must do so owing to some cause; for nothing can come to be without a cause. — Plato in Timaeus
>
> [The m]odern version of PUC is connected with Newtonian physics, but is also criticized for instance by David Hume. ... Kant opposed Hume in many aspects, defending the objectivity of universal causation."

#q>p ; `TTCT TTCT TTCT TTCT` (1.2)

((r=(p\q))&(s=(q\p)))>(((s=r)>(q=p))+(((s>r)>(q>p))+((s<r)>(p>q)))) ;
`TTTT TTTT TTTT TTTT` (2.2)

#q>(#p=~#q) ; `TTTC TTTC TTTC TTTC` (3.2)

Remark: Weakening Eq. 3.2 to #q>(#p>~#q) produces the same truth table.

((#q>p)&(((r=(p\q))&(s=(q\p)))>(((s=r)>(q=p))+(((s>r)>(q>p))+
((s<r)>(p>q))))))&(#q>(#p=~#q)) ; `TTCC TTCC TTCC TTCC` (4.2)

Eqs. 2.2 is tautologous. Eqs. 1.2, 3.2, and 4.2 as rendered are *not* tautologous. This means the concept of causality as produced from Whewell's three axioms is refuted.

> **Remark:** From a metaphysical view, the axiom of causality is a bar to miracle because first cause is always assumed. This is overcome with the axiom "The necessity of effect implies the possibility of cause *or* no cause": #q>%(p+~p) ; `TTTT TTTT TTTT TTTT`.

Category composition of morphisms

From en.wikipedia.org/wiki/Category_theory, en.wikipedia.org/wiki/Glossary_of_category_theory:

The binary operation, named composition of morphisms, is defined :

If f: a → b, g: b → c are functors, then the composition g ◦ f is the functor defined by:

for an object x and a morphism y in a as

$(g \circ f)(x) = g(f(x))$ and $(g \circ f)(y) = g(f(y))$ (1.1)

Meth8 maps Eq 1 as

LET: p a; q b; r c; u f; v g; s x; t y;
> Imply →; ◦ And &; = Equivalent to =; nvt not tautologous.

We rewrite Eq 1 components as If u: p > q and v: q > r, then v & u is equivalent to ... :

((u=(p>q)) & (v=(q>r))) > ((u&v) = ((((v&u)&s)=(v&(u&s))) &
(((v&u)&t)=(v&(u&t))))) ; nvt; (1.2)

The repeating truth table fragment for Model 1 is TFTT TTTT, where T is the designated truth value.

Eq 1.2 is not tautologous, thereby rendering category theory as not tautologous.

Refutation of Tannakian categories via profinite groups of Iwasawa embedding

Abstract: We evaluate the seminal equation to axiomatize profinite groups with the Iwasawa embedding property. It is *not* tautologous. This taints Tannakian categories and subsequent conjectures to establish model theory of proalgebraic (pro-affine algebraic) groups. These conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Pillay, A.; Wibmer, M. (2019). Model theory of proalgebraic groups. arxiv.org/pdf/1908.10064.pdf

Abstract We lay the foundations for a model theoretic study of proalgebraic groups (more accurate to use the term “pro-affine algebraic group” or “affine group scheme”). Our axiomatization is based on the [T]annakian philosophy.

Introduction The theory T_{IP} axiomatizes profinite groups G having the Iwasawa (or embedding) property: Any diagram G→A, G→B, B→A where B→A is an epimorphism of finite groups and G→A is an epimorphism can be completed to a commutative diagram via an epimorphism G→B, if B is a quotient of G. (1.1)

LET p, q, r, s: A, B, G, Divisor.

((q>p)&(r>p))>(((r\s)>q)>(r>q)) ; TTTT TTTT TTTT TT**F**T (1.2)

Remark 1.2: Eq. 1.2 as rendered is *not* tautologous. This refutes the profinite groups of Iwasawa embedding, taints Tannakian categories, and taints subsequent conjectures to establish model theory of proalgebraic (pro-affine algebraic) groups.

Refutation of category theory by lattice identity, and graph-theoretic / set-of-blocks in partitions

Abstract: We evaluate a definition and two models of a method in graph theory to define any Boolean operation. The definition is *not* tautologous, refuting that only partition tautologies using only the lattice operations correspond to general lattice-theoretic identities. Defined models of graph-theoretic and set-of-blocks do not produce a common edge, but rather show the graph-theoretic definition implies the set-of-blocks definition. This refutes the graph-theoretic model as defining *any* Boolean operation on lattice partitions of category theory. What follows is that general lattice theory is also refuted via partitions. Therefore the conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Ellerman, D. (2019). "A graph-theoretic method define any Boolean operation on partition". The Art of Discrete and Applied Mathematics 2 (2): 1–9. arxiv.org/pdf/1906.04539.pdf

Abstract: The lattice operations of join and meet were defined for set partitions in the nineteenth century, but no new logical operations on partitions were defined and studied during the twentieth century. Yet there is a simple and natural graph-theoretic method presented here to define any n-ary Boolean operation on partitions. An equivalent closure-theoretic method is also defined. In closing, the question is addressed of why it took so long for all Boolean operations to be defined for partitions.

4 The Implication Operation on Partitions

The real beginning of the logic of partitions, as opposed to the lattice theory of partitions, was the discovery of the set-of-blocks definition of the implication operation σ ⇒ π for partitions. [T]he corresponding relation holds in the partition case: σ ⇒ π = 1 iff σ ≼ π.

A logical formula in the language of join, meet, and implication is a *subset tautology.* All partition tautologies are subset tautologies but not vice-versa. *Modus ponens* (σ ∧ (σ ⇒ π)) ⇒ π is both a subset and partition tautology but Peirce's law, ((σ ⇒ π) ⇒ σ) ⇒ σ, accumulation, σ ⇒ (π ⇒ (σ ∧ π)), and distributivity, ((π ∨ σ) ∧ (π ∨ τ)) ⇒ (π ∨ (σ ∧ τ)), are examples of subset tautologies that are not partition tautologies.

Remark 4.5: The equations above are tautologous, and hence must be subset tautologies and partition tautologies in order for partition logic to be bivalent.

The importance of the implication for partition logic is emphasized by the fact that the only partition tautologies using only the lattice operations, e.g., π∨1, correspond to general lattice-theoretic identities, i.e., π ∨ 1 = 1. (4.6.1)

LET p, q, r, s, t, u, v: π, σ, B, C (or c), C' (or c), u (or a), u' (or b) .

(p+(%p>#p))=(%p>#p) ; TNTN TNTN TNTN TNTN (4.6.2)

Remark 4.6.2: Eq. 4.6.2 is *not* tautologous, hence that only partition tautologies using only the lattice operations correspond to general lattice-theoretic identities.

There is a link u − u′ in G (σ ⇒ π) in and only in the following situation where (u, u′) ∈ indit (π) and (u, u′) ∈ dit (σ)–which is exactly the situation when B is not contained in any block C of σ: Figure 1: Links u − u′ in G (σ ⇒ π).

Remark Fig. 1 and 2: The diagrams' graphic images are not redrawn here as Figs. 4.7.1 or 4.8.1.

(((u<(r&s))&(v<(r&t)))>(u-v)) ;

TTTT TTTT TTTT TTTT(6)
FFFF FFFF FFFF TTTT(1)
FFFF TTTT **FFFF** TTTT(1) (4.7.2)

Thus the graph-theoretic and set-of-blocks definitions of the partition implication are equivalent.

Figure 2: Example of graph for partition implication.
Example [2] Let U = {a, b, c, d} so that K(U) = K4 is the complete graph on four points. Let σ = {{a} , {b, c, d}} and π = {{a, b} , {c, d}} so we see immediately from the set-of-blocks definition, that the π-block of {c, d} will be discretized while the π-block of {a, b} will remain whole so the partition implication is σ ⇒ π = {{a, b} , {c} , {d}}. After labelling the links in K (U), we see that only the a−b link has the Fσ⇒π 'truth value' so the graph G (σ ⇒ π) has only that a−b link (thickened in Figure 2). Then the connected components of G (σ ⇒ π) give the same partition implication σ ⇒ π = {{a, b} , {c} , {d}}. (4.8.1)

(((u+(v&(s&t)))>((u&v)+(s&t)))=((u&v)+(s+t)))>(u-v) ;

TTTT TTTT TTTT TTTT(3)
FFFF FFFF FFFF FFFF(5) (4.8.2)

Remark 4.9: For the graph-theoretic and set-of-blocks definitions of the partition implication to be equivalent, Eqs. 4.7.2 and 4.8.2 should be equivalent. (4.9.1)

(((u<(r&s))&(v<(r&t)))>(u-v))>((((u+(v&(s&t)))>((u&v)+(s&t)))=((u&v)+(s+t)))>(u-v)) ;

TTTT TTTT TTTT TTTT(2)
FFFF FFFF TTTT TTTT(1)
TTTT TTTT **FFFF FFFF**(2)
FFFF FFFF FFFF FFFF(1)
TTTT TTTT TTTT **FFFF**(1)
TTTT **FFFF** TTTT **FFFF**(1) (4.9.2)

Remark 4.10: To resuscitate the conjecture of Eq. 4.9.1, we remove the consequent in the models of (u-v) to test for equality of the antecedent models. (4.10.1)

```
((u<(r&s))&(v<(r&t)))=(((u+(v&(s&t)))>((u&v)+(s&t)))=((u&v)+(s+t))) ;
                    TTTT TTTT FFFF FFFF( 1)
                    FFFF FFFF FFFF FFFF( 1)
                    FFFF FFFF TTTT TTTT( 1)
                    TTTT TTTT FFFF FFFF( 2)
                    FFFF FFFF FFFF FFFF( 1)
                    TTTT TTTT TTTT FFFF( 1)
                    TTTT FFFF TTTT FFFF( 1)
```

(4.10.2)

Remark 4.11: To further coerce Eq. 4.9.1 from 4.10.1, we weaken the argument in 4.10.1 from an equality to a conditional via the imply connective. (4.11.1)

```
((u<(r&s))&(v<(r&t)))>(((u+(v&(s&t)))>((u&v)+(s&t)))=((u&v)+(s+t))) ;
                    TTTT TTTT TTTT TTTT
```

(4.11.2)

While the result of Eq. 4.11.2 is tautologous, it means that the graph-theoretic and set-of-blocks definitions do not produce the common edge of Figs. 1 or 2, but rather that Fig. 1 implies Fig. 2, to mean the graph-theoretic definition implies the set-of-blocks definition. This refutes the graph-theoretic model as defining any Boolean operation on lattice partitions of category theory.

Refutation of the CC conjecture of Lin Fan Mao

Abstract: The CC conjecture as defined by Lin Fan Mao is **not** tautologous and hence refuted.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: p, q, r, science *T*; ~ Not; + Or; - Not Or; & And; > Imply;
% possibility, for any one or some, ∃; # necessity, for every or all, ∀.
(p=p) `T` tautology; (p@p) **F** contradiction; (%p>#p) `N` truthity; (%p<#p) `C` falsity;

From: Mao, L.F. (2015). Mathematics after CC conjecture: combinatorial notions and achievements. vixra.org/pdf/1508.0244v1.pdf

Remark: In the paper title, the "CC" in "CC conjecture" is not defined as a two-letter acronym within the paper.

CC Conjecture ... : Any mathematical science can be reconstructed from or made by combinatorialization. (1.0)

We rewrite Eq.1.0 as:

"The combinatorial result of elements implies making any mathematical science that is invertible from its elements." (1.1)

(((p&q)&r)>%s)>(%s>((p&q)&r)) ; `NNNN NNNT` **FFFF FFF**`T` (1.2)

Remark: In Eq. 1.1 the antecedent is "The combinatorial result of elements implies making any mathematical science" as ((p&q)&r)>%s ;
`TTTT TTTC TTTT TTTT.` (1.3)

[I]t is a mathematical machinery of philosophical notion: *there always exist universal connection*[*s*] *between things T* with a disguise G^L[*T*] on connections, which enables us converting a mathematical system with contradictions to a compatible one (2.0)

We rewrite Eq. 2.0 as:

"If [t]he combinatorial result of elements implies making any mathematical science that is invertible from its elements, then a disguise always exists to imply a contradictory mathematical science that is tautologous" (2.1)

((((p&q)&r)>%s)>(%s>((p&q)&r)))>#(~%s>(s>(p=p))) ;
`TTTT TTTN TTTT TTTN` (2.2)

Eqs. 1.2, 1.3, and 2.2 as rendered are *not* tautologous. This refutes the CC conjecture of Lin Fan Mao.

Refutation of Chaitin's theorem of incompleteness

We assume the apparatus of the Meth8 modal logic model checker implementing variant system VŁ4. Meth8 allows to mix four logical values with four analytical values. The designated *proo*f value is T.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	p=p	T	Tautology	proof	11	3
2	p@p	F	Contradiction	absurdum	00	0
3	%p>#p	N	Non-contingency	truth	01	1
4	%p<#p	C	Contingency	falsity	10	2

LET: ~ Not; + Or; & And; \ Not and; > Imply; < Not imply; = Equivalent to;
@ Not equivalent to; # all; % some; (p@p) 00, zero; (p=p) 11, one;
pqrs KLns

Results are the proof table of 16-values in row major horizontally.

We evaluate Chaitin's incompleteness theorem of 1974 from

wikipedia.org/wiki/Kolmogorov_complexity#Chaitin.27s_incompleteness_theorem .

Martin Davis described it as "*a dramatic extension of Gödel's incompleteness theorem*" (Davis, 1978).

"Theorem: There exists a constant L ... such that there does not exist a string s for which the statement

(K(s) ≥ L) (as formalized in S) [This is equivalent to ~(K(s) < L).] (0.1)

can be proven within the axiomatic system S. Note that, by the abundance of nearly incompressible strings, the vast majority of those statements must be true. (1.1)

The proof is by contradiction. It the theorem were false [not a proof] then the following is a proof [tautology]:

Assumption (X): For any integer n there exists a string s for which there is a proof in [logic system] S of the expression "(K(s)≥ L)". (S is assumed to enumerate all formals proofs of S.) (2.1)

We render Eq. 0.1 as:

~((p&s)<q) ; `TTTT TTTT TFTT TFTT` (0.2)

Eq. 0.2 means that "~(K(s)<L) (as formalized in S)" is already not a proof (not a tautology) but is also is not a contradiction because the F value of contradiction is mixed twice into the resulting proof table.

Remark: Eq. 0.2 implies that Chaitin's constant L is suspicious.

We render Eq. 1.1 as:

%q>((~((p&s)<q)=(s=s))>~%s) ;   NNNN NNNN NTFF NTFF   (1.2)

Eq. 1.2 means the theorem is not a tautology, and *not* a contradiction, with the proof table of a mixture of values for F, N, and T.

The refutation of the theorem could end here, however for to be comprehensive we continue the approach of the argument and render Eq. 2.1 as:

#r&(%s>(~((p&s)<r)>(s=s))) ;   FFFF NNNN FFFF NNNN   (2.2)

Eq. 1.2 means that Assumption (X) is not a contradiction because of the N value of truth mixed into the resulting proof table.

In an attempt to resuscitate Eq. 1.2, we rewrite it by distributing the universal quantifier over the antecedent and consequent as:

(#r&%s) > (#r&(~((p&s)<r)>(s=s))) ;   TTTT TTTT TTTT TTTT   (2.3)

In this case, Eq. 2.3 shows Assumption(X) is a proof, and therefore Eq. 1.1 should be a contradiction. However, we already showed Eq. 1.2 is *not* a contradiction, but rather contains some T value of tautology mixed with some F value of contradiction.

In either case of Eq. 0.2 with Eq. 2.2 or with Eq. 2.3, the approach of the conjecture is moot, and Chaitin's theorem of incompleteness is refuted.

Reference:

Davis, M. (1978). “What is a computation?”. Steen, L.A. (ed.) Mathematics Today, Twelve informal essays. Springer. 1978. pp. 241/267. DOI: 10.1007/978-1-4613-9435-8_10.

The Brain Simulator Reply (BSR) of the Chinese Room Argument (CRA) is confirmed.

We assume the apparatus of the Meth8 modal logic model checker implementing variant system VŁ4. Meth8 allows to mix four logical values with four analytical values. The designated *proo*f value is T.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	p=p	T	Tautology	proof	11	3
2	p@p	F	Contradiction	absurdum	00	0
3	%p>#p	N	Non-contingency	truth	01	1
4	%p<#p	C	Contingency	falsity	10	2

LET: ~ Not; + Or; & And; \ Not and; > Imply; < Not imply; = Equivalent to; @ Not equivalent to; # all; % some; (p@p) 00, zero; (p=p) 11, one

Results are the repeating proof table(s) of 16-values in row major horizontally.

From: plato.stanford.edu/entries/chinese-room by dcole@d.umn.edu (2014)

"Brain Simulator Reply. ... Searle correctly notes that one cannot infer from *X* simulates *Y*, and *Y* has property *P*, to the conclusion that therefore *X* has *Y*'s property *P* for arbitrary *P*. [1.1]

But ... Searle ... commits the simulation fallacy in extending the CR argument from traditional AI to apply against computationalism. The contrapositive of the inference is logically equivalent—*X* simulates *Y*, *X* does not have *P* therefore *Y* does not [have P]" [2.1]

We map Eqs. 1.2 and 2.1 as follows.

LET: p q r P X Y

((q>r)&(r>p))>(q>(r>p)) ; `TTTT TTTT TTTT TTTT` (1.2.1)

((q>r)&(r>p))>(q>(r&p)) ; `TTTT TTTT TTTT TTTT` (1.2.2)

Eqs. 1.2.1 and variant 1.2.2 are tautologous, contradicting the conjecture of [1.1].

((q>r)&(q>~p))>(r>~p) ; `TTTT TFTT TTTT TFTT` (2.1)

Eq. 2.1 is not tautologous, contradicting [2.1] as logically equivalent to [1.1].

The Brain Simulator Reply of the Chinese room argument is hence confirmed and validated.

Shorter refutation of CHSH inequality

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal.

> LET: p, q, r, s: E(a,b), E(a,b′), E(a′,b), E(a′,b′); ~ Not; & And; + Or; - Not Or;
> \> Imply, greater than; < Not Imply, less than; = Equivalent; @ Not Equivalent;
> % possibility, for one or some; # necessity, for all or every;
> (p@p) contradiction; (%p>#p) ordinal 1; (%p<#p) ordinal 2; (p=p) proof.

From: en.wikipedia.org/wiki/CHSH_inequality

> The usual form of the CHSH inequality is based on "terms *E*(*a*, *b*) etc. [as] quantum correlations of the particle pairs, where the quantum correlation is defined to be the expectation value of the product of the "outcomes" of the experiment, i.e. the statistical average of *A*(*a*)·*B*(*b*), where *A* and *B* are the separate outcomes, using the coding +1 for the '+' channel and −1 for the '−' channel":

$$|S| \leq 2 \text{ , where } S = E(a,b)-E(a,b')+E(a',b)+E(a',b'). \qquad (1.1)$$

~((%p<#p)>((p-q)+(r+s)))=(p=p) ;
FCCC FFFF FFFF FFFF (1.2)

Eq. 1.2 as rendered is *not* tautologous. This means the CHSH inequality is refuted.

Refutation of CHSH and a dual reality conjecture

Abstract: The equation for the Clauser-Horne-Shimony-Holt [CHSH] inequality is refuted. Hence a dual reality conjecture for experimental (confirmation or) rejection of observer-independence in the quantum world becomes moot. Therefore the CHSH inequality is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables.
(See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊔ ;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1;
(%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Proietti, M.; et al. (2019).
Experimental rejection of observer-independence in the quantum world.
arxiv.org/pdf/1902.05080.pdf martin.ringbauer@uibk.ac.at

[W]hen the variables A_x, B_y take values $a,b \in \{-1,+1\}$, then the average values $\langle A_x B_y \rangle$ ... must obey the Clauser-Horne-Shimony-Holt [CHSH] inequality ... : $S =$

$$\langle A_1 B_1 \rangle + \langle A_1 B_0 \rangle + \langle A_0 B_1 \rangle - \langle A_0 B_0 \rangle \leq 2 \qquad (2.1)$$

(~(%p>#p)<(((p&q)&(r&s))<(%p>#p)))>
~((%p<#p)<(((p&q)+(p&s))+((r&q)-(r&s)))) ;
`TTTT TTNT TTTT NTNT` (2.2)

Eq. 2.2 as rendered is *not* tautologous. This is the shortest known refutation of the CHSH inequality. What follows is that experimental rejection (or confirmation) of a dual reality conjecture for observer-independence in the quantum world becomes moot.

Church's thesis (constructive mathematics)

From: en.wikipedia.org/wiki/Church%27s_thesis_(constructive_mathematics)

Formal statement:

$$(\forall x \exists y \phi(x, y)) \rightarrow (\exists e \forall x \exists y, u\, T(e, x, y, u) \wedge \phi(x, y)). \qquad (1)$$

LET: # ∀, % ∃, r y, s ϕ, p x, q ψ, e r, t f, u u, v T

((#p&%q)&(s&(p&q)))>((%r&(#p&%q))&((u&(v&(r&(p&(q&u)))))&(s&(p&q))));
(2)

```
Model 1              Model 2.1            Model 2.2            Model 2.3.1          Model 2.3.2
TTTT TTTT TTTC TTTC  EEEE EEEE EEEU EEEU  EEEE EEEE EEEE EEEE  EEEE EEEE EEEP EEEP  EEEE EEEE EEEI EEEI
TTTT TTTT TTTC TTTC  EEEE EEEE EEEU EEEU  EEEE EEEE EEEE EEEE  EEEE EEEE EEEP EEEP  EEEE EEEE EEEI EEEI
TTTT TTTT TTTC TTTC  EEEE EEEE EEEU EEEU  EEEE EEEE EEEE EEEE  EEEE EEEE EEEP EEEP  EEEE EEEE EEEI EEEI
TTTT TTTT TTTC TTTT  EEEE EEEE EEEU EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEP EEEE  EEEE EEEE EEEI EEEE
```

Result: Church Thesis (1) is not tautologous.

Extended Church thesis (ECT):

$$(\forall x \psi(x) \rightarrow \exists y \phi(x, y)) \rightarrow \exists f (\forall x \psi(x) \rightarrow \exists y, u\, T(f, x, y, u) \wedge \phi(x, y)). \qquad (3)$$

((#p&(q&p))>(%r&(s&(p&r))))>(%t&((#p&(q&p))>(%r&((u&(v&(t&(p&(r&u)))))
&(s&(p&r)))))) ; (4)

```
Model 1              Model 2.1            Model 2.2            Model 2.3.1          Model 2.3.2
CCCT CCCT CCCT CCCC  UUUE UUUE UUUE UUUU  EEEE EEEE EEEE EEEE  PPPE PPPE PPPE PPPP  IIIE IIIE IIIE IIII
TTTT TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE
```

Result: Extended Church Thesis (2) is not tautologous.

Refutation of the Church-Rosser theorem

Abstract: The Church-Rosser theorem evaluates as *not* tautologous, hence forming a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Church–Rosser_theorem

If term *a* can be reduced to both *b* and *c*, then there must be a further term *d* (possibly equal to either *b* or *c*) to which both *b* and *c* can be reduced. (1.1)

LET p, q, r, s: a, b, c, d.

(p>(q&r))>(%(s=(q+r))>((q&r)>s)) ;
TTTT TTNN TTTT TTTT (1.2)

Remark 1.2: Eq. 1.2 may also be rendered as
%(s=(q+r))>((p>(q&r))>((q&r)>s)) with the same truth table result. (1.3)

Eqs. 1.2 and 1.3 are *not* tautologous, hence refuting the Church-Rosser theorem.

Refutation of the Clausius-Clapeyron equation for spatial dimension

We evaluate a generalized implementation of Clausius-Clapeyron from:

> Pilot, C. (2018). A new type of phase transition based on the Clausius-Clapeyron relation involving a change in spatial dimension. vixra.org/pdf/1804.0293v1.pdf

$$2[(n+1)/n]u(n)V(n) = 2[n/(n-1)]u(n-1)V(n-1)+L(n-1) \qquad (2\text{-}24.1)$$

We assume the apparatus and method of Meth8/VŁ4, where the designated *proo*f value is Tautology and truthity is Non contingency. The 16-valued truth table is row-major and horizontal.

```
LET pqrs nuVL;  (%p>#p) 1;  (%p<#p) 2;
% possibility, for one or some;   # necessity, for all.

 (((%p<#p)&((p+(%p>#p))\p))&((q&p)&(r&p))) =
((((%p<#p)&(p\(p-(%p>#p))))&((q&(p-(%p>#p)))&(r&(p-(%p>#p)))))
+ (s&(p-(%p>#p)))) ;                TTTT TTNT NTNT NTNT          (2-24.2)
```

Eq. 2-24.2 as rendered is *not* tautologous. This means the implementation of Clausius-Clapeyron is refuted.

Clifford tori in even-dimensions with complex coordinates for unit spheres is tautologous.

From: en.wikipedia.org/wiki/Clifford_torus

> Any unit sphere S^{2n-1} in an even-dimensional euclidean space $\mathbf{R}^{2n} = \mathbb{C}^n$ may be expressed in terms of the complex coordinates as follows:
>
> $S^{2n-1} = \{ (z_1 , \ldots , z_n) \in C^n : |z_1|^2 + \ldots + |z_n|^2 = 1 \}$. (1)

1.1 We ask: "Is the set denoted in Eq 1 compatible as a subset of the formula defined?"

We assume the Meth8 apparatus and method, and designated truth values as Tautologous, Evaluated.

LET: p q r s $(z_1 , \ldots , z_4)$, ∈ <
(%p>#p) 1, ((%p>#p)-(%p>#p)) 0

If Eq 1 is followed in order, where the C^n term can be ignored for our purposes, then the z-element series term is a subset of the z-power series term:

((p&q)&(r&s))<(((p+q)+(r+s))=(%p>#p)) ; FFFF FFFF FFFF FFFC (2)

Because the truth table for Eq. 2 diverges slightly from contradictory, we rewrite Eq 1 to juxtapose the terms so that the z-power series is now the superset of the z-element series:

(((p+q)+(r+s))=(%p>#p))>((p&q)&(r&s)) ; NCCC CCCC CCCC CCCT (3)

1.2 We answer 1.1: "No, both Eqs. 1 and 2 are not tautologous."

2.1 We then ask: "Is the definition in Eq 2 tautologous when the degenerate case of the radius of 0 is included?"

(((p+q)+(r+s))&(((%p>#p)-(%p>#p))=(%p>#p)))>((p&q)&(r&s)) ;
TTTT TTTT TTTT TTTT (4)

2.2 We then answer: "The definition as modified in Eq 4 is tautologous." This confirms the definition with a degenerate radius of 0 in Eq 4.

We are reminded this is for even-dimensional Euclidean space. In other words the validated dimensions are [D0], D2, D4 but not D1, D3, D5. This means for the Clifford tori to be a model for the brain requires 2D or 4D, but not 3D or 5D. Because 4D can be explained as the assumption of 3D with the addition of time as a dimension to make 3D into 4D, we believe that assumption is mistaken.

What follows is a 2D Clifford torus becomes flattened to a plane, and hence effectively a network of linear spaces. Therefore a series of such flat tori with intersections may constitute the brain model.

By extension from the standpoint of the Kanban cell neuron model network, this means the linear formula ((p'&q')+r')=s' then feeds a subsequent linear formula as ((s'&q")+r")=s" in a network of linear formulas.

3.1 We now ask: "Is the Kanban cell neuron model based on the AND-OR gate correct, as rendered with 14 self-filtering and self-timing values not equal to zero in Table 1 (from US Patent No. 9,501,737 and No. 9,202,16)?"

Connective No.	((ii	& pp)	| qq)	= kk
091	01	01	10	11
095	01	01	11	11
106	01	10	10	10
111	01	10	11	11
123	01	11	10	11
127	01	11	11	11
149	10	01	01	01
159	10	01	11	11
167	10	10	01	11
175	10	10	11	11
183	10	11	01	11
191	10	11	11	11
213	11	01	01	01
234	11	10	10	10

Table 1

In Table 1, the "Connective No." is the decimal representation of the bits as concatenated and indexes a canonical table of 256 connectives based on 8-bits in our literature.

The expression we test is "If ii, pp, qq, or kk are not 00, then (ii * pp) + qq = kk." (5)

LET: p q r s ii pp qq kk, + |, p=(p@p) p=00, q=(q@q) q=00, r=(r@r) r=00, s=(s@s) s=00

Eq. 5 may be written equivalently in two ways with the same truth tables:

~(((s=(s@s))+(r=(r@r)))+((p=(p@p))+(q=(q@q))))>(((p&q)+r)=s) ;
TTTT TTTT TTTT TTTT (6)

~(((s&r)&(p&q))=(((s@s)&(r@r))&((p@p)&(q@q))))>(((p&q)+r)=s) ;
TTTT TTTT TTTT TTTT (7)

3.2 We now answer 3.1: "The Kanban model is correct as stated above."

This means the Kanban model in Eq 6, 7 is consistent with the 2D Clifford torus in Eq 4.

Denial of the refutation of coherence in modal logic

Abstract: We evaluate the refutation of coherence in modal logic as based on weakly transitive logics using a ternary term to admit finite chains. The term is *not* tautologous, thereby denying the refutation. What follows is that K, KT, K4, and S4 (and S5) are tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⇔, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) `C` non-contingency, ∇, ordinal 2; (%z>#z) `N` as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).

From: Kowalski, T.; Metcalfe, G. (2019). Coherence in modal logic.
arxiv.org/pdf/1902.02540.pdf t.kowalski@latrobe.edu.au, george.metcalfe@math.unibe.ch

Abstract : A variety is said to be coherent if the finitely generated subalgebras of its finitely presented members are also finitely presented. ... In this paper, a more general criterion is obtained and used to prove the failure of coherence and uniform deductive interpolation for a broad family of modal logics, including K, KT, K4, and S4.

4.2 Weakly Transitive Logics ... We therefore make use here of the ternary term

t(x,y,z)=□(y∨□(z∨x))∨x (4.2.1)

LET p, q, r, s: x, y, z, t;

(s&((p&q)&r))=(#(q+#(r+p))+p) ;
`T`**`F`**`C`**`F`** `C`**`F`**`C`**`F`** `T`**`F`**`C`**`F`** `C`**`F`**`CT` (4.2.2)

Lemma 4.2: Let L be a modal logic admitting finite chains, and let t(x,y,z) be as defined above.

Eq. 4.2.2 as rendered is not tautologous. This means the ternary term is not a theorem on which finite terms are admitted, thereby denying the refutation of coherence in modal logic. What follows is that K, KT, K4, and S4 (and S5) are tautologous fragments of the universal logic VŁ4.

Confirmation of the Collatz conjecture

Abstract: Using the standard wiki definition of the Collatz conjecture, we map a positive number to imply that a divisor of two implies either an even numbered result (unchanged) or an odd numbered result (changed to the number multiplied by three plus one) to imply the final result of one. This is the shortest known confirmation of the conjecture, and in mathematical logic.

The Collatz conjecture is described at wikipedia.org/wiki/Collatz_conjecture, for which we decompose farther below:

> "[A] sequence defined as follows: start with any positive number *n*. Then each term is obtained from the previous term as follows: if the previous term is even, the next term is one half the previous term. If the previous term is odd, the next term is 3 times the previous term plus 1. The conjecture is that no matter what value of *n*, the sequence will always reach 1." (0.0)

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET p, q, r, s: positive integer, quotient, remainder, divisor;
+ Or, add; & And, multiply; \ Not And, divide;
> Imply, greater than; < Not Imply, lesser than; = Equivalent; @ Not Equivalent;
% possibility, possibly, for one or some; # necessity, necessarily, for all or every.
(%s>#s) ordinal one; (%s<#s) ordinal two; (s=s) ordinal three, ; (s@s) ordinal zero.
~(y > x) (x ≥ y)

"[A] sequence defined as follows: start with any positive integer *n*." (0.1)

~((%p>#p)>p) = (p=p) ; N**F**N**F** N**F**N**F** N**F**N**F** N**F**N**F** (0.2)

Remark 0.0.1: Previously we used zero as the fiducial point for positive integers, when in fact ordinal 1 is the fiducial point. Hence "p is greater than or equal to one" is captured by "not one greater than p" as above.

We divide p by the divisor s to produce a quotient q and remainder r as a fraction of the divisor s. (1.1)

(p\s)=(q+(r\s)) ; TTTT TTTT T**F**T**F** **F**TT**F** (1.2)

We define an even number as having the fractional part of remainder r as zero in the numerator and divisor s in the denominator, for a remainder of zero, to imply p is p divided by s. (2.1)

(r=(r@r))>(p=(p\s)) ; **F**T**F**T TTTT **FFFF** TTTT (2.2)

We define an odd number as having the fractional part of remainder r as one in the numerator and divisor s in the denominator, for a remainder of one divided by s, to imply p is p multiplied by three plus one. (3.1)

(r=(%r>#r))>(p=((p&(p=p))+(%p>#p))) ;
TTTT CTCT TTTT CTCT (3.2)

We build the argument that Eq. 0.1 implies the following: divisor s as two implies (Eq. 1) the form of p/2 as quotient plus fraction as remainder/2 which implies either (Eq. 2) the form of an even p or (Eq. 3) the form of an odd p, to imply the final result of one. (4.1)

(~((%p>#p)>p)>(((s=(%s<#s))>(((p\s)=(q+(r\s))) > (((r=(r@r))>(p=(p\s)))+
((r=(%r>#r))>(p=((p&(p=p))+(%p>#p))))))) = (%p>#p)) ;
TTTT TTTT TTTT TTTT (4.2)

Eq. 4.2 as rendered is tautologous, hence confirming the Collatz conjecture. We note this is the shortest known such proof, and in mathematical logic.

Refutation of collection theory as the set of universal closure of sentences

Abstract: We evaluate collection theory as the set of universal closure of sentences in a schema equation. It is *no*t tautologous. This refutes Collection as the conjectured schema. Therefore collection theory is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Quinsey, J.E. (1980). Applications of Kripke's notion of fulfillment. Dissertation. St. Catherine's College, Oxon. arxiv.org/pdf/1904.10540.pdf jquinsey@i2msystems.com

When we say, for example, that Collection is the schema

$$\forall x \in a\ \exists y\theta \supset \exists b\ \forall x \in a\ \exists y \in b\theta\ , \qquad (1.1)$$

we mean that Collection is the set of universal closure of sentences of this form, where θ ranges over all formulae of the language under consideration, and where suitable precautions are taken to avoid collision of variables. A *theory* is a set of sentences.

LET p, q, x, y, t: *a*, *b*, *x*, *y*, θ.

(#x<(p&(%y&t)))>((%q&#x)<((p&%y)<(q&t))) ;

```
TTTT TTTT TTTT TTTT(16), CCTT CCTT CCTT CCTT(16),
TTTT TTTT TTTT TTTT(16),
CCTC CCTC CCTC CCTC, CTTT CTTT CTTT CTTT}x8=(16)     (1.2)
```

Eq. 1.2 as rendered is not tautologous. This refutes Collection as the conjectured schema.

Refutation of short circuit evaluation for propositional logic by commutative variants

From: Ponse, A.; et al. (2018).
Propositional logic with short-circuit evaluation: a non-commutative and a commutative variant.
arXiv:1810.02142 a.ponse@uva.nl

At A.4. Theorem 6.3, the four-valued truth table is for the connective "o^" as a short-circuited operator And.

We substitute the logical values {0, 1, 2, 3} by the 2-tuple as respectively {00, 01, 10, 11}:

o^	00	01	10	11
00	00	00	10	10
01	00	01	10	11
10	10	10	10	10
11	10	11	10	11

Our two examples are:

11 o^ **0**0 = **1**0
11 o^ **1**0 = **1**0

Therefore, (**1** o^ **0**) = (**1** o^ **1**), implying **0** = **1**.

The truth table for o^ is *not* bi-valent and exact but a vector space and hence probabilistic.

The short circuit evaluation for propositional logic by commutative variants is *not* tautologous, and thereby refuted.

Refutation of comorphism of sites

Abstract: The complex equation evaluated is *not* tautologous, hence refuting the conjecture of *comorphism of sites* as a functor with a *covering lifting property*. What follows is that the following are also refuted: surjections, inclusions, localic morphisms, hyperconnected morphisms, and equivalences of toposes. This further relegates category theory of Grothendieck to a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Caramello, O. (2019). Denseness conditions, morphisms and equivalences of toposes.
arxiv.org/pdf/1906.08737.pdf olivia.caramello@uninsubria.it

Abstract: We establish a general theorem providing necessary and sufficient explicit conditions for a morphism of sites to induce an equivalence of toposes. This results from a detailed analysis of arrows in Grothendieck toposes and denseness conditions, which yields results of independent interest. We also derive site characterizations of the property of a geometric morphism to be an inclusion (resp. a surjection, hyperconnected, localic), as well as site-level descriptions of the surjection inclusion and hyperconnected-localic factorizations.

6 Geometric morphisms induced by comorphisms of sites

Recall that a *comorphism of sites* (D, K) → (C, J) (where J and K are Grothendieck topologies respectively on C and D) is a functor F : D → C which has the *covering lifting property*, that is the property that for every d ∈ D and any J-covering sieve S on F (d) there is a K-covering sieve R on c such that F (R) ⊆ S. (6.1)

LET	p,	q,	r,	s,	t,	u,	v,	w,	x,	y:
	C,	D,	R,	S,	K,	F,	J,	c,	d,	J-covering sieve

((((v>p)&(t>q))>((x&t)>(p&v)))>(u=(q>w)))>((#(x<q)&%(y>(u&x)))>((r>w)>~(s<(u&r)))) ;

```
TTTT TTTT TTTT TTTT(16)
TTTT TTTT TTTT TTTT( 2)}x4
TTTT TTTT CCTT TTTT( 2)}      (6.2)
```

Eq. 6.2 as rendered is *not* tautologous, hence refuting the conjecture of *comorphism of sites* as a functor with a *covering lifting property*. By extension, also refuted are: surjections, inclusions, localic morphisms, hyperconnected morphisms, and equivalences of toposes; and further the category theory Grothendieck.

Refutation of the complementarity inequality

We evaluate the modified Mach-Zehnder setup, to confirm the findings of the Afshar experiment from:

> Flores, E.V.; De Tala, J.M. (2006). Comlementarity paradox solved: surprising consequences.
> arxiv.org/ftp/arxiv/papers/1001/1001.4785.pdf

We assume the method and apparatus of Meth8/VŁ4 where Tautology is the designated proof value, F is contradiction, N is truthity (non-contingency), and C is falsity (contingency). The 16-valued truth table is row-major and horizontal.

K is which-way info, and V is visibility: $K^2 + V^2 \leq 1$. (1)

LET pqrs x y K V; # necessity, for all; % possibility, for one or some; ~ Not;
& And; + Or; - Not Or; > Imply, greater than; < Not Imply, less than; = Equivalent;
(%p>#p) 1; (%p<#p) 2; ((%p>#p)-(%p>#p)) 0; K=K'; T (p=p).

We rewrite Eq. 1 for K ≥ 0 as V ≤ ((1-K)*(1-K)). (2.1)

~(r>(((%p>#p)-s)&((%p>#p)+s))) ; (2.2)

$$V \geq \frac{(1-x)/(1-y) - x/y}{(1-x)/(1-y) + x/y} \quad (5.1)$$

~(~(r>(((%p>#p)-s)&((%p>#p)+s)))
< (((((%p>#p)-p)\((%p>#p)-q))-(p\q))\((((%p>#p)-p)\((%p>#p)-q))+(p\q)))) = (p=p) ;
TTTT TTTT TTTT TTTT (5.2)

K' ≥ (1-2x), rewritten as K ≥ (1-2x) (6.1)

~(r< ((%p>#p)-((%p<#p)&p))) = (p=p) ;
TTTT CFCF TTTT CFCF (6.2)

$K'^2 + V^2 < 2$, rewritten as $(K^2 + V^2 < 2)$ which in mathematical logic is $(K + V < 2)$
(7.1)

(~(r< ((%p>#p)-((%p<#p)&p)))
\+
~(~(r>(((%p>#p)-s)&((%p>#p)+s)))
< (((((%p>#p)-p)\((%p>#p)-q))-(p\q))\((((%p>#p)-p)\((%p>#p)-q))+(p\q)))))
< (%p<#p) ; NNNN NNNN NNNN NNNN (7.2)

Eq. 5.2 as rendered for the modified Mach-Zehnder setup is tautologous. This confirms the findings of the Afshar experiment.

Eq. 7.2 as rendered for complementarity inequality is *not* tautologous, although the closest state of truthity (non-contingent). That refutes the findings of the captioned paper. This violates and refutes the complementarity inequality, and confirms the original Afshar paper.

Refutation of completeness for inclusion and equivalence of universality

Abstract: We evaluate a formula for inclusion and equivalence of universality. It is *not* tautologous, refuting the conjecture of completeness, and forming a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Masopust, T.; Krötzsch, M. (2019). Partially ordered automata and piecewise testability. arxiv.org/pdf/1907.13115.pdf

Abstract: Universality is the question whether a system recognizes all words over its alphabet. Complexity of deciding universality provides lower bounds for other problems, including inclusion and equivalence of systems behaviors. We study the complexity of universality for a class of nondeterministic finite automata, models as expressive as boolean combinations of existential first-order sentences. **Conclusion:** [W]e obtained PSpace-completeness for several restricted types ... for problems including inclusion, equivalence, and (k-)piecewise testability.

7. Inclusion and equivalence: A consequence of the complexity of universality is the worst-case lower-bound complexity for the inclusion and equivalence problems. These problems are of interest, e.g., in optimization. … Although equivalence means two inclusions, complexities of these two problems may differ significantly, e.g., inclusion is undecidable for deterministic context-free languages.. while equivalence is decidable.. . Since universality can be expressed as the inclusion Σ∗ ⊆ L or the equivalence Σ∗ = L, we immediately obtain the hardness results for inclusion and equivalence from the results for universality. Therefore, it remains to show memberships of our results … Let A be an automaton of any of the considered types ... depending on the type of B. We assume that both automata are over the same alphabet specified by B. If B is a DFA, then

L(A) ⊆ L(B) if and only if L(A) ∩ L(B) = Ø̸, (7.1.1)

LET p, q, r, s: A, B, L, s.

(((r&p)&(r&~q))=(s@s))>~((r&q)<(r&p)) ;
`TTTT TT`**F**`T TTTT TT`**F**`T` (7.1.2)

Remark 7.1.2: Eq. 7.1.2 as rendered is *not* tautologous. This refutes the hardness results for inclusion and equivalence from the results for universality, meaning the conjecture is not complete.

Meth8/VŁ4 on complex numbers (ℂ)

Complex numbers (ℂ) are generally defined by a component of the imaginary number as i^2 = -1, where i = √-1 as i = (1+i)/√2 and i = (-1-i)/√2) (0.0.1)

> **Remark:** We note that the roots of i are axioms described in terms of itself, normally not allowed, and the cause of the skepticism of Euler and others.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal.

> LET: p, q: i, √2;
> ~ Not; & And; + Or; - Not Or; = Equivalent to; > Imply;
> % possibility, for one or some; # necessity, for all or every;
> (%r>#r) ordinal one, 1.

> p=((((%r>#r)+p)\q)+((~(%r>#r)-p)\q)) ; `FTNT FTNT FTNT FTNT` (0.0.2)

Because Eq. 0.0.2 is *not* tautologous, not a theorem, that is cause to reject the imaginary number as a bivalent entity.

We attempt to weaken the expression in Eq. 0.1 to obtain a tautologous result by replacing the Equivalent connective with the Imply connective. (0.1.1)

> p>((((%r>#r)+p)\q)+((~(%r>#r)-p)\q)) ; `TTTT TTTT TTTT TTTT` (0.1.2)

Eq. 0.1.2 is tautologous, as based on the canonical pattern in VŁ4 of `FTFT` > `TTCT` = `TTTT`.

This means that imaginary numbers in Meth8/ VŁ4 are rendered as implications and not equivalences, which serves to reason since complex numbers are imaginary and literally *not* real. Hence the complex number space (ℂ) is arguably a probabilistic vector space and *never* exact. Quantum field theory as based on ℂ is probabilistic, not bivalent, and hence suspicious.

Refutation of computer-simulation model theory

Abstract: Computer simulation model theory (CSMT), as a substituted extension of mathematical model theory (MMT), is a conjecture that: *for a formula φ, construct a computer simulation model S such that 1- φ does not hold in S, and 2- the reasoner I (human being, the one who lives inside the reality) cannot distinguish S from the reality (R), then I cannot prove φ in reality.* The conjecture is *not* tautologous. While we show elsewhere that P=NP is *not* tautologous (via refutation of the Schaefer theorem), the unprovability of P=NP does not follow from this CSMT approach. This conjecture forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , ·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Ramezanian, R. (2019). Computer-simulation model theory (P = NP is not provable). arxiv/pdf/1.org906.09873.pdf

Abstract The simulation hypothesis says that all the materials and events in the reality (including the universe, our body, our thinking, walking and etc) are computations, and the reality is a computer simulation program like a video game. All works we do (talking, reasoning, seeing and etc) are computations performed by the universe-computer which runs the simulation program. Inspired by the view of the simulation hypothesis (but independent of this hypothesis), we propose a new method of logical reasoning named "Computer-Simulation Model Theory", CSMT. Computer-Simulation Model Theory is an extension of Mathematical Model Theory where instead of mathematical-structures, computer-simulations are replaced, and the activity of reasoning and computing of the reasoner is also simulated in the model. (CSMT) argues that:

for a formula φ, construct a computer simulation model S such that
1- φ does not hold in S, and
2- the reasoner I (human being, the one who lives inside the reality) cannot distinguish S from the reality (R),
then I cannot prove φ in reality. (1.1)

LET p, q, r, s: φ=[P=NP], *I* reasoner, *R* reality, *S* simulation model [= *E*].

(~((p=(p=p))<s)&((q<r)>~(s+r)))>((p<r)=(p=p)) ;
F`T`**F**`T` **F**`T`**F**`T` **F**`TTT` **FFFF** (1.2)

Although CSMT is inspired by the simulation hypothesis, but this reasoning method is independent of the acceptance of this hypothesis. As we argue in this part, one may do not accept the simulation hypothesis, but knows CSMT a valid reasoning method. As an application of Computer-Simulation Model Theory, we study the famous problem P vs NP. We let $\varphi \equiv [P = NP]$ and construct a computer simulation model E such that P = NP does not hold in E. (2.1)

Remark 2.1: Eq. 2.1 is ostensibly the same as 1.1 as rendered, hence 2.1 maps to 1.2.

Eq. 1.2 is not tautologous, hence refuting the conjecture of CSMT. While we show elsewhere that P=NP is *not* tautologous (via refutation of the Schaefer theorem), the unprovability of P=NP does not follow from this CSMT approach.

Confirmation of Perez' definition of the conceivable statement

Abstract: Of the 16 equations evaluated, 15 are *not* tautologous. Elsewhere we correctly proved and confirmed most of the conjectures of the author such as refutations of ZFC, Cantor, and Gödel. The definition of a conceivable statement is confirmed as a theorem, with the other equations forming a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Perez, P.A. (2010). Addressing mathematical inconsistency: Cantor and Gödel refuted arxiv.org/ftp/arxiv/papers/1002/1002.4433.pdf jap717@juanperezmaths.com

Abstract. This article undertakes a critical reappraisal of arguments in support of Cantor's theory of transfinite numbers. The following results are reported:

- Cantor's proofs of nondenumerability are refuted by analyzing the logical inconsistencies in implementation of the *reductio* method of proof and by identifying errors. Particular attention is given to the diagonalization argument and to the interpretation of the axiom of infinity.
- Three constructive proofs have been designed that support the denumerability of the power set of the natural numbers, *P* (N), thus implying the denumerability of the set of the real numbers R. These results lead to a Theorem of the Continuum that supersedes Cantor's Continuum Hypothesis and establishes the countable nature of the real number line, suggesting that all infinite sets are denumerable.

Some immediate implications of denumerability are discussed:

- Valid proofs should not include inconceivable statements, defined as statements that can be found to be false and always lead to contradiction. This is formalized in a Principle of Conceivable Proof.
- Substantial simplification of the axiomatic principles of set theory can be achieved by excluding transfinite numbers. To facilitate the comparison of sets, infinite as well as finite, the concept of relative cardinality is introduced.
- Proofs of incompleteness that use diagonal arguments (e.g. those used in Gödel's Theorems) are refuted. A constructive proof, based on the denumerability of *P* (N), is presented to demonstrate the existence of a theory of first-order arithmetic that is consistent, sound, negation-complete, decidable and (assumed p.r. adequate) able to prove its own consistency. Such a result reinstates Hilbert's

Programme and brings arithmetic completeness to the forefront of mathematics.

3. Refutations of Cantor's proofs of nondenumerability

3.1.2 Proofs by external (or conventional) contradiction

(3.2.1) $\neg P \Rightarrow Q_1 \Rightarrow Q_2 \Rightarrow . . . \Rightarrow Q_n \Rightarrow (R \wedge \neg R)$.

(~p>(q>s))>(r&~r) ; **FFFF FFFF FFFF FFFF** (3.2.2)

(3.3.1) $\neg P \Rightarrow (R \wedge \neg R)$.

~p>(r&~r) ; **F**T**F**T **F**T**F**T **F**T**F**T **F**T**F**T (3.3.2)

(3.4.1) $\neg(R \wedge \neg R) \Rightarrow \neg(\neg P) \Rightarrow P$

Remark 3.4.1: Eq. 3.4.1 is a trivial tautology.

3.1.3 Proofs by internal (or self-referential) contradiction

(3.5.1) $\neg P \Rightarrow Q_1 \Rightarrow Q_2 \Rightarrow . . . \Rightarrow Q_n \Rightarrow P$.

(~p>(q>s))>p ; **F**TTT **F**TTT **F**T**F**T **F**T**F**T (3.5.2)

(3.6.1) $\neg P \Rightarrow (P \wedge \neg P)$.

~p>(p&~p) ; **F**T**F**T **F**T**F**T **F**T**F**T **F**T**F**T (3.6.2)

Remark 3.6.2: Eq. 3.6.2 is not equal to 3.7.2 or 3.10.2.

(3.7.1) $\neg(P \wedge \neg P) \Rightarrow \neg(\neg P) \Rightarrow P$

Remark 3.7.1: Eq. 3.7.1 is a trivial tautology and not equal to 3.6.1 or 3.10.1.

(3.8.1) $\neg P \Leftrightarrow Q_1 \Leftrightarrow Q_2 \Leftrightarrow . . . \Leftrightarrow Q_{i-1} \Leftrightarrow Q_i \Rightarrow Q_{i+}1 \Rightarrow . . . \Rightarrow Q_n \Rightarrow P$.

((~p=(q=r))>s)>p ; TT**F**T **F**TTT **F**T**F**T **F**T**F**T (3.8.2)

(3.9.1) $Q_i \Rightarrow \neg P \wedge Q_i \Rightarrow P$

(q>(~p&q))>p ; **F**T**F**T **F**T**F**T **F**T**F**T **F**T**F**T (3.9.2)

(3.10.1) $Q_i \Rightarrow (P \wedge \neg P)$.

q>(p&~p) ; TT**FF** TT**FF** TT**FF** TT**FF** (3.10.2)

Remark 3.10.2: Eq. 3.10.2 is not equal to 3.6.2 or 3.7.2.

3.2. **Cantor's diagonalization argument**

3.2.1 *Logical objection to the proof* .

(3.11.1) $\neg P \Leftrightarrow Q_1 \Leftrightarrow Q_2 \Rightarrow Q_3 \Leftrightarrow P$

((~p=(q=r))>s)=p ; TT**FF** **FF**TT **F**T**F**T **F**T**F**T (3.11.2)

3.3. **Cantor's Theorem: the power set.**
3.3.2. *First proof for higher powers* .

(3.23.1) $\neg P \Leftrightarrow Q_1 \Leftrightarrow Q_2 \Rightarrow Q_3 \Leftrightarrow P$

((~p=(q=r))>s)=p ; TT**FF** **FF**TT **F**T**F**T **F**T**F**T (3.23.2)

(3.24.1) $\neg P \Leftrightarrow Q \Leftrightarrow C$

~p=(q=r) ; T**FF**T **F**TT**F** T**FF**T **F**TT**F** (3.24.1.2)

Consequently, (3.24[.1]) equates to writing $\neg P \Leftrightarrow C$, and this is taken as a sufficient argument supporting the truth of the theorem. (3.24.2.1)

(~p=(q=r))=(~p=r) ; **FF**TT **FF**TT **FF**TT **FF**TT (3.24.2.2)

3.4. **The Axiom of Infinity.**

(3.25.1) $\exists y(\emptyset \in y \wedge \forall x(x \in y \Rightarrow x \cup \{x\} \in y))$.

LET p, q: x, y

((s@s)<%q)&((#p<%q)>(#p+(#p<%q))) ;
FFFF FFFF FFFF FFFF (3.25.2)

3.5. **Cantor's first proof of the nondenumerability of** R **.**

(3.32.1) $\neg P \Leftrightarrow Q_1 \Leftrightarrow Q_2 \Rightarrow Q_3 \Leftrightarrow P$

(~p=(q=r))>(s=p) ; TTT**F** T**F**TT **F**TTT TT**F**T (3.32.2)

5. Implications for proofs by reductio (ad absurdum)
The first requirement, a method to identify incorrect mathematical statements, is addressed by the following definition.

Definition 5.1. A mathematical statement Q is said to be inconceivable when there is another statement P such that

(5.1.i.1) $(Q \Rightarrow P) \wedge (Q \Rightarrow \neg P)$, or

(q>p)&(q>~p) ; TT**FF** TT**FF** TT**FF** TT**FF** (5.1.i.2)

(5.1.ii.1) $Q \Rightarrow ((P \Rightarrow \neg P) \vee (\neg P \Rightarrow P))$.

Otherwise, the statement Q is considered conceivable. (5.1.1)

(5.1.i.1) or (5.1.ii.1) ((q>p)&(q>~p))+(q>((p>~p)+(~p>p))) ;

```
TTTT TTTT TTTT TTTT
```
(5.1.2)

Remark 5.1.2: Eq. 5.1.2 is tautologous to confirm the definition of a conceivable statement.

The mathematical statement Q can itself define a given mathematical object or entity. In such a case, this object is also considered inconceivable or conceivable, in line with the statement which defines it.

Principle 5.2 (of Conceivable Proof). No mathematical proof can be judged valid if its construction includes an inconceivable statement; the exception is if the purpose of the proof is to demonstrate the falsehood of an inconceivable statement, provided that the resulting contradiction is not conceptually linked to the initial assumption of the proof.

Conjecture 5.3 (of Logical Imperfection). Any sound and/or consistent system of mathematics is capable of generating inconceivable statements.

(5.1.1) $\neg P \Leftrightarrow Q_1 \Leftrightarrow Q_2 \Leftrightarrow Q_3 \Rightarrow Q_4 \Leftrightarrow Q_5 \Leftrightarrow Q_6 \Rightarrow C$

(~p=(q=r))>((s=t)>u) ;

```
FTTF TFFT TTTT TTTT( 1)
TTTT TTTT FTTF TFFT( 1)
TTTT TTTT TTTT TTTT( 2)
```
(5.1.2)

Of the 16 equations evaluated, 15 are *not* tautologous. Elsewhere we correctly proved and confirmed most of the conjectures of the author such as refutations of ZFC, Cantor, and Gödel. The author's definition of a conceivable statement is confirmed as a theorem.

Refutation of the proscriptive principle (conceptivistic logic) and containment logic

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

From: Ferguson, T. M. (2017). The proscriptive principle and logics of analytic implication. academicworks.cuny.edu/gc_etds/1882

Proscriptive principle: "No formula with analytic implication as main relation holds universally if it has a free variable occurring in the consequent but not the antecedent."

LET p, q: *A* or ϕ, *B* or ψ;
~ Not, ¬ superscript dot; & And, ∧ superscript dot; > Imply; = Equivalent.

"Angell himself remarks that A→B and A∧B↔A are equally good characterizations of the notion of analytic containment. ... in Cor we may define a notion of entailment where A→B [3.2.1.9.1.1] serves as an abbreviation for A∧B↔A." [3.2.1.9.2.1]

(p>q) ; `TFTF TFTF TFTF TFTT` (3.2.1.9.1.2)

(p&q)=p ; `TFTF TFTF TFTF TFTT` (3.2.1.9.2.2)

"In **S1**, the strict implication connective is not primitive, but is defined in terms of ◊ , so that $\phi\rightarrow\psi=_{df}\neg\Diamond(\phi\wedge\neg\psi)$ appears in its axiomatization." (8.2.2.1)

(p>q)=~(%p&%~q) ; `NTNN NTNN NTNN NTNN` (8.2.2.2)

Eqs. 3.2.1.9.1.2, 3.2.1.9.2.2 and 8.2.2.2 as rendered are *not* tautologous. These are seminal to the proscriptive principle (conceptivistic logic) and containment logic, which are therefore refuted.

Refutation of the improved Adams hypothesis of conditional logic

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s, t: P, a, b, c, d; ~ Not, ¬; + Or; & And, ∧, " ·"; \ Not And, |;
> Imply, greater than; = Equivalent; @ Not Equivalent, ≠;
necessity, for all or every; % possibility, for one or some; (p=p) `T`; ~(p>p) p≤p.

From: conditionals; plato.stanford.edu/entries/logic-conditionals/
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"McGee focuses on one of these origins, namely the notion of conditional probability as axiomatized by Karl Popper (1959, appendix). A Popper function on a language *L* for the classical sentential calculus is a function *P*: *L*×*L* → *R*, where *R* denotes the real numbers, which obeys the following axioms:

1.1 For any *a* and *b*, there exist *c* and *d* with *P*(*a* | *b*) ≠ *P*(*c* | *d*)

#(q&r)>(%(s&t)&((p&(q\r))@(p&(s\t)))) ;
`TTTT TTCC TTTT TTCC` (1.2)

2.1 If *P*(*a* | *c*) = *P*(*b* | *c*), for every *c*, then *P*(*d* | *a*) = *P*(*d* | *b*), for every *d*

(#s&((p&(q\s))=(p&(r\s))))>(#t&((p&(t\q))=(p&(t\r)))) ;
`TTT TTTT CCCT CTCC,`
`TTTN TNTT TTTT TTTT` (2.2)

3.1 *P*(*a* | *a*) = *P*(*b* | *b*)

(p&(q\q))=(p&(r\r)) ;
`TTTF TFTT TTTF TFTT` (3.2)

4.1 *P*(*a* ∧ *b* | *c*) ≤ *P*(*a* | *c*)

~((p&((q&r)\s))>(p&(q\s)))=(p=p) ;
`FFFF FFFF FFFT FFFF` (4.2)

5.1 *P*(*a* ∧ *b* | *c*) = *P*(*a* | *b* ∧ *c*)·*P*(*b* | *c*)

(p&((q&r)\s))=((p&(q\(r&s)))&(p&(q\s))) ;
`TTTT TTTT TTTF TTTT` (5.2)

6.1 *P*(*a* | *b*) + *P*(¬*a* | *b*) = *P*(*b* | *b*), unless *P*(*b* | *b*) = *P*(*c* | *b*) for every *c*

~(#s&((p&(r\r))=(p&(s\r))))>(((p&(q\r))+(p&(~q\r)))=(p&(r\r))) ;
`TTTT TFTF TTTT TNTN` (6.2)

Axiom (5.1) is crucial and older than its use in Popper's theory. It goes back at least to Jeffreys' work where it is in turn presented as W. E. Johnson's product rule (see Jeffreys 1961, p. 25). Contemporary the product rule has been called also the *multiplication axiom*. Now, with the help of this notion of conditional probability, we can define a new form of Adams's hypothesis:

Improved Adams Hypothesis

[7.1] $P(a > c) = P(c \mid a)$, where both *a* and *c* are *factual* or conditional-free sentences"

(p&(q>s))=(p&(s\q)) ;   TTTF TTTF TTTF TTTF   (7.2)

Eqs. 1.2-7.2 as rendered are *not* tautologous. This means the probability axioms of Popper and McGee and the improved hypothesis of Adams are refuted.

Refutation of conditional events in quantum logic

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: a, b, c, d; ~ Not, ('); + Or ; & And;
> Imply; = Equivalent.

Remark: The conditional operator as the pipe symbol (|) below is mapped from right to left to mean the right unit as antecedent implies the left unit as consequent. For example, (a|b) iterates "b implies a" as (b>a).

From: Calabrese, P.G. (2017). Logic and Conditional Probability—A Synthesis. Studies in Logic 69. College Publications.

At "3.10.5 Iterated Conditioning" (pg. 56/7), we evaluate the equations and tabulate the results.

Eq. (n.)	Text (.1)	Iteration (.2)	Pseudo map(.3) ~ ' Not, & And, + Or, > Imply	M8 script (.4) LET a,b,c,d: p,q,r,s	Truth table (.5) T tautology, **F** contradiction
1	(a|b)|(c|d)	d then c implies b then a	(d>c)>(b>a)	(s>r)>(q>p) ;	TT**F**T TT**F**T TTTT TT**F**T
2	(a|b & (c|d))	((d then c) and b) implies a	((d>c)&b)>a	((s>r)&q)>p ;	TT**F**T TT**F**T TTTT TT**F**T
3	a|b & c & d)	d and c and b) implies a	((d&c)&b)>a	((s&r)&q)>p ;	TTTT TTTT TTTT TT**F**T
4	(a+ b'|c + d')	not d or (c implies not b) or a	~d+(~b>c)+a	~s+(~q>r)+p ;	TTTT TTTT **F**TTT TTTT
5	(a&b|c+d')	not d or c implies b and a	(~d+c)>(b&a)	(~s+r)>(q&p) ;	**FFF**T **FFF**T TTTT **FFF**T
6	a+b'|c&d)	d and c implies not b or a	(d&c)>(~b+a)	s&r)>(~q+p) ;	TTTT TTTT TTTT TT**F**T
7	(a&b|c&d)	d and c implies b and a	(d&c)>(b&a)	(s&r)>(q&p) ;	TTTT TTTT TTTT **FFF**T
8	b&(c|d)= b&(c+d')	d implies c and b equals not d or c and b	(d>c)&b)= ((~d+c)&b)	((s>r)&q)= ((~s+r)&q) ;	TTTT TTTT TTTT TTTT
9	(a|b(c+d'))	not d or c and b implies a	((~d+c)&b)>a	((~s+r)&q)>p ;	TT**F**T TT**F**T TTTT TT**F**T

Eq. 8 is tautologous (a theorem). Eqs. 1, 2, 9 have the same truth table, and Eqs. 3, 6 have the same truth table. Eqs. 1-7, 9 are supposed to be equivalent, but are not.

Hence this evaluation does not confirm conditional events as conjectured.

Refutation of conditional necessitarianism

Abstract: The seminal definition of conditional necessitarianism is *not* tautologous. The spin-off is to deny the following five conjectures, so based thereon: Truthmaker-dependence (TD); Truthbearer-requirement (TB); Aboutness-requirement (AC); and versions of TF named deflationary and inflationary. Therefore these six conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Schipper, A. (2019). Fundamental truthmakers and non-fundamental truths.
link.springer.com/article/10.1007%2Fs11229-019-02266-x

Abstract Recently, philosophers have tried to develop a version of truthmaker theory which ties the truthmaking relation (T-REL) closely to the notion of fundamentality. In fact, some of these *truthmaker-fundamentalists* (TF-ists), as I call them, assume that the notion of fundamentality is intelligible in part by citing, as central examples of fundamentals, truthmakers, which they understand necessarily as constituents of fundamental reality. The aim of this paper is first to bring some order and clarity to this discussion, sketching how far TF is compatible with orthodox truthmaking, and then critically to evaluate the limits of TF. It will be argued that truthmaker theory cannot directly help with articulating the nature of fundamental reality and that T-REL does not necessarily relate truths with anything more fundamental, unless what is fundamental is what the truthbearers in question are about. I shall argue that TF faces a rather thorny dilemma and some general problems. I shall present two exhaustive types of fundamentalism on which a version of TF can be based: deflationary and inflationary. It will be argued that each version of TF runs into significant troubles accounting for all truth, specifically ordinary truths and metaphysical truths about the relations between ordinary facts and fundamental facts. I shall not attempt to solve these problems, but rather, at the end, diagnose the issues with TF as lying in the difficulties with reconciling the manifest image with the scientific and metaphysical images of reality.

2 Preliminaries: truthmaking

First, some preliminaries about truthmaker theory. Here are several basic assumptions, which I shall assume any version of truthmaker theory must accept.

Truthmaker-dependence (TD): the truthmaking relation (T-REL) is a species of dependence; generally, truths asymmetrically depend for their truth on truthmakers.

Truthbearer-requirement (TB): T-REL is a relation which, given normal linguistic practices, only rarely has truthbearers on both sides of the relation.

Aboutness-requirement (AC): truths are made true by the parts (or aspects) of reality *which they are about*.

Necessitation (NEC): truthbearer *p* is made true by truthmaker *x* iff in all possible worlds where *p* exists and *x* exists, *p* is true,[9] [9] This is what Merricks (2007, p. 7) articulates as *conditional necessitarianism*. [Merricks, T. (2007). *Truth and ontology*. Oxford: Clarendon Press.]

(1.1)

LET p, q: truthbearer p, truthmaker x.

((%p&%q)>(p=(p=p)))>(q>(p=(p=p))) ;

TTCT TTCT TTCT TTCT (1.2)

Eq. 1.2 is *not* tautologous. This refutes conditional necessitarianism. The spin-off is to deny the following five conjectures of the first author so based thereon: Truthmaker-dependence (TD); Truthbearer-requirement (TB); Aboutness-requirement (AC); and versions of TF named deflationary and inflationary.

Refutation of confluence in rewrite systems

Abstract: We evaluate confluence in two definitions and one theorem, none which is tautologous. This refutes the approach of confluence as a central property of rewrite systems.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. Reproducible transcripts for results are available. (See ersatz-systems.com.)

LET p, q, r, s: a, x, y, z;
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ↞;
= Equivalent, ≡, :=, ⇔, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) C non-contingency, ∇, ordinal 2; (%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).

Remark 0: For clarity, we distribute the quantifiers to each instance of a variable.

From: Endrullis, J.; Klop, J.W.; Overbeek, R. (2019). Decreasing diagrams for confluence and commutation. arxiv.org/pdf/1901.10773.pdf j.endrullis@vu.nl, j.w.klop@vu.nl, roy.overbeek@cwi.nl

A binary relation → is called confluent if two coinitial reductions (i.e., reductions having the same starting term) can always be extended to cofinal [having same term] reductions, that is:

∀abc.(b↞a↠c⇒∃d.b↠d↞c) (1.1)

((#q<#p)>#r)>(#q>(%s<#r)) ; TTTC TTCC TTTC TTCC (1.2)

Definition 5 (Strong confluence). [$^{=}$ means empty set, ignored here]

∀axy.∃z.(a→x∧a→y)⇒(x→$^{=}$z↞y) (5.1)

((#p>#q)&(#p>#q))>(q>(%s<r)) ;
TTCC TT**FF** TTCC TT**FF** (5.2)

Theorem 23: Proof: ... Finally, the following formula requires all elements, except for [a], to be deterministic:

... ∀xyz.(a→x∧a→y∧a→z)⇒y=z (23.1)

((q@(p&q))>(q>r))>(r=s) ; TTTT **FFFF** **FF**T**F** TTTT (23.2)

Eqs. 1.2, 5.2, and 23.2 for two definitions and one theorem are *not* tautologous. We evaluate confluence in two definitions and one theorem, none which is tautologous. The refutes the approach of confluence as a central property of rewrite systems.

Refutation of connexive logic based on Wansing's nightmare

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: A, B, C, D; ~ Not, ¬; + Or; & And; > Imply, greater than, → ; = Equivalent, ↔.

From: Connexive logic. Copyright © 2014 by Heinrich Wansing <Heinrich.Wansing@rub.de>. plato.stanford.edu/entries/logic-connexive/

The set of all valid formulas is axiomatized by the following set of axiom schemata and rules:

$a1_c$	the axioms of classical positive logical
a2	$\sim\sim A \leftrightarrow A$
a3	$\sim(A \vee B) \leftrightarrow (\sim A \wedge \sim B)$
a4	$\sim(A \wedge B) \leftrightarrow (\sim A \vee \sim B)$
a5	$\sim(A \rightarrow B) \leftrightarrow (A \rightarrow \sim B)$
R1	modus ponens

MC can be faithfully embedded into positive classical logic, whence **MC** is decidable. The classical tautology $\sim(A \rightarrow B) \rightarrow (A \wedge \sim B)$ is, of course, not a theorem of **MC**. Like **C**, **MC** is a paraconsistent logic containing contradictions.

From: Ferguson, T.M.; Omori, H.; Wansing, H. (2016). The tenacity of connexive logic: Preface to the special issue. *FCoLog Journal of Logics and their Applications*. 3:3.293.

In Omori's research note on Francez' paper ... a deductive calculus including the analogous axiom $\neg(\phi\rightarrow\psi)\leftrightarrow(\neg\phi\rightarrow\psi)$ is introduced by means of an axiomatic proof theory and a corresponding possible worlds semantics.

From: Omori, H. (2016). A note on Francez' half-connexive formula. *IFCoLog Journal of Logics and their Applications*. 3:3.507.

Remark 4. ... (Ax12) [$\sim(A\rightarrow B)\leftrightarrow(\sim A\rightarrow B)$] is replaced by '$\sim(A\rightarrow B)\leftrightarrow(A\rightarrow\sim B)$'.

Connexive logic turns on $\sim(A\rightarrow B)\leftrightarrow(A\rightarrow\sim B)$ (a5.1.1)

~(A>B)=(A>~B) ; FCNT FCNT FCNT FCNT (a5.1.2)

or the falsity-weakend $\sim(A\rightarrow B)\leftrightarrow(\sim A\rightarrow B)$ (a5.2.1)

~(A>B)=(~A>B) ; TTTT NNNN CCCC FFFF (a5.2.2)

Eqs. a5.1.2 and a5.2.2 as rendered are *not* tautologous. Hence connexive logic is refuted.

Refutation of Aristotle's and Boethius' theses in connexive logic

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET ~ Not; > Imply; = Equivalent;

From: Wansing, H. (2018). Connexive conditional logic. pdmi.ras.ru/EIMI/2018/LP/lp_2018-abstracts.pdf

> Connexive logics are contra-classical logics. They are neither subsystems nor supersystems of classical logic, and what is characteristic of them is that they validate the so-called Aristotle's Theses and Boethius' Theses:

~(~A> A)=(A=A) ;	TNCF TNCF TNCF TNCF	(AT)
~(A>~A)=(A=A) ;	FCNT FCNT FCNT FCNT	(AT)'
(A> B)>~(A>~B) ;	FCNT FCNT FCNT FCNT	(BT)
(A>~B)>~(A> B) ;	FCNT FCNT FCNT FCNT	(BT)'

Eqs. AT, AT', BT, and BT' are *not* tautologous, hence refuting those theses of Aristotle and Boethius.

> **Remark:** Because ~AT = AT' = BT = BT', what follows is that using conditionals to justify connexive logic makes Wansing's nightmare worse.

Refutation of De Finettian logics of indicative conditionals

Abstract: We evaluate De Finettian logics which specifies four conditionals, as attributed to Aristotle, Boethius, Cooper-Cantwell, Jeffrey, and six axioms. None is tautologous. This refutes their use to justify connexive logic.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. Reproducible transcripts for results are available. (See ersatz-systems.com.)

LET p, q, r, s: A, B, C, D;
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨; < Not Imply, less than, ∈, ≺, ⊂;
= Equivalent, ≡, :=, ⇔, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) C non-contingency, ∇, ordinal 2; (%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).

From: Égré, P.; Rossi, L.; Sprenger, J. (2019).
De Finettian logics of indicative conditionals. arxiv.org/pdf/1901.10266.pdf
paul.egre@ens.fr, lorenzo.rossi@sbg.ac.at, jan.sprenger@unito.it

Remark 0: Because formulas in the paper are not labelled, equations are keyed to the respective section and number of consecutive appearance.

$A\rightarrow B|{=}_{TT}\neg(A\rightarrow\neg B)$ (3.1.5.1)

(p>q)>~(p>~q) ; **F**T**F**T **F**T**F**T **F**T**F**T **F**T**F**T (3.1.5.2)

$\neg A\vee B\not\models_{TT}\neg(\neg A\vee\neg B)$ (3.1.6.1)

~((~p+q)>~(~p+~q))=(p=p) ; T**F**T**F** T**F**T**F** T**F**T**F** T**F**T**F** (3.1.6.2)

$A\rightarrow B|{=}_{SS}A\wedge B$ (3.2.1.1)

(p>q)>(p&q) ; **F**T**F**T **F**T**F**T **F**T**F**T **F**T**F**T (3.2.1.2)

$A\rightarrow B|{=}_{SS}B\rightarrow A$ (3.2.2.1)

(p>q)>(q>p) ; TT**F**T TT**F**T TT**F**T TT**F**T (3.2.2.2)

$A\rightarrow B\not\models_{TT}A\wedge B$ (3.2.3.1)

~((p>q)>(p&q))=(p=p) ; T**F**T**F** T**F**T**F** T**F**T**F** T**F**T**F** (3.2.3.2)

$A\rightarrow B\nvDash_{TT}B\rightarrow A$ (3.2. 4.1)

~((p>q)>(q>p))=(p=p) ; FFTF FFTF FFTF FFTF (3.2.4.2)

$(A\rightarrow B)\rightarrow A$ (4.2.1.1)

(p>q)>p ; FTFT FTFT FTFT FTFT (4.2.1.2)

Aristotle's thesis: $\neg(\neg A\rightarrow A)$ (5.3.1.1)

~(~p>p)=(p=p) ; TFTF TFTF TFTF TFTF (5.3.1.2)

Boethius' thesis: $(A\rightarrow C)\rightarrow\neg(A\rightarrow\neg C)$ (5.3.2.1)

(p>r)>~(p>~r) ; FTFT FTFT FTFT FTFT (5.3.2.2)

Holds for Cooper-Cantwell conditional : $\neg(A\rightarrow B)\equiv_{m}(A\rightarrow\neg B)$ (5.3.3.1)

~(p>q)=(p>~q) ; FTFT FTFT FTFT FTFT (5.3.3.2)

Holds for any Jeffrey conditional: $A\rightarrow B\nvDash_{TT}\neg B\rightarrow\neg A$ (5.6.1.1)

~((p>q)>(~q>~p))=(p=p) ; FFFF FFFF FFFF FFFF (5.6.1.2)

None of the ten equations above as rendered is tautologous. This denies that use to justify connexive logic with which we dispensed previously elsewhere.

Refutation of Ishihara's tricks and (seemingly) impossible theorems in constructive mathematics

Abstract: The precise definition of LPO and Ishihara's tricks as rendered in four equations are *not* tautologous. This refutes LPO and Ishihara's tricks. What follows is that (seemingly) impossible theorems in constructive mathematics are denied as theorems. Therefore those conjectures are *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1;
(%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Diener, H.; Hendtlass, M. (2019).
(Seemingly) impossible theorems in constructive mathematics. arxiv.org/pdf/1904.11378.pdf
hannes.diener@canterbury.ac.nz, matthew.hendtlass@canterbury.ac.nz

We make frequent reference to two classically valid "omniscience principles", the limited principle of omniscience (LPO) [equivalent to the existence of strongly existensional discontinuous function f] and the weak limited principle of omniscience (WLPO) [equivalent to the existence of discontinuous function f].

The following lemma is very much folklore, at least the WLPO part, however we were unable to find it in the literature. … Here and in the following a function $f : X \rightarrow Y$ between two metric (X,σ) and (Y,ρ) is called *strongly extensional* if $\forall x,y \in X$: $f(x) \neq f(y) \Rightarrow x \neq y$, or to be more precise, $\forall x,y \in X$: $\rho(f(x), f(y)) > 0 \Rightarrow \sigma(x,y) > 0$. (2.3.1)

((r&((p&x)&(p&y)))>(p@p))>((s&(x&y))>(p@p)) ; "or to be more precise"
`TTTT TTTT TTTT TTTT(16),`
`TTTT TTTT` **FFFF F**`T`**F**`T(16)` (2.3.2)

Proposition 3 (Ishihara's first trick). *For all positive reals* $\alpha < \beta, \exists n \in$ N: $\rho(f(xn), f(x)) > \alpha \vee \forall n \in$ N: $\rho\, f(xn), f(x)) < \beta$. (3.1.1)

Remark 3.1.1: The expression *for all positive reals* $\alpha < \beta$ is mapped as (3.1.1.1)

α<β and α*β>0 to #((p<q)&((p&q)>(p@p)))=(p=p) for an antecedent result of
FN**FF** **F**N**FF** **F**N**FF** **F**N**FF**. (3.1.1.2)

#((p<q)&((p&q)>(p@p)))>(((%u<v)>((r&((s&(x&u))&(s&x)))<#q))+
((#u<v)>((r&((s&(x&u))&(s&x)))<#p)));

TTTT TTTT TTTT TTTT(2),
TCTT TCTT TCTT TCTT(2),
TTTT TTTT TTTT TTTT(4) (3.1.2)

Proposition 4 (Ishihara's second trick). *For all positive reals* $\alpha < \beta$*, either we have* $\rho(f(xn), f(x)) < \beta$ *eventually, or* $\rho(f(xn), f(x)) > \alpha$ *infinitely often.* (4.1.1)

Remark 4.1.1: The terms *eventually* and *infinitely often* are mapped respectively as possibly and necessarily.

#((p<q)&((p&q)>(p@p)))>(%((r&((s&(x&u))&(s&x)))<#q)+
(#(r&((s&(x&u))&(s&x)))<#p)) ;

TCTT TCTT TCTT TCTT(2),
TCTT TCTT TCTT TTTT(2) (4.1.2)

There are various results that improve, generalise, or modify Ishihara's tricks ...

The precise definition of LPO and Ishihara's tricks as rendered in four equations are *not* tautologous. This refutes LPO and Ishihara's tricks. What follows is that (seemingly) impossible theorems in constructive mathematics are denied as theorems.

Constructivistic logic

From: Badie, F. A theoretical model for meaning construction through constructivist concept learning . (2017). researchgate.net/publication/318430404 .

We evaluate constructivistic logic using two papers at the page numbers of the dissertation text.

We assume the apparatus of M8-VL4.

LET: p ai, m; q A,lc_L; r R, mentor(l); s aj, learner(m); t MentorOf(m,l); u LearnerOf(l,m); x constant; y function; z R; # universal quantifier; % existential quantifier

Fragments are the repeating truth tables of 16-values, of 128 tables.

In *A conceptual mirror: towards a reflectional symmetrical relation between mentor and learner*:

"Formally:"
(#p<q)>((p&r)&p) ; TCTT TTTT TCTT TTTT pg 95
((#p<q)>((p&r)&s)) > ((%s<q)>((s&r)&p)) ; NNTT NNTT FNTT FTTT pg 96

In *Towards semantic analysis of mentoring-learning relationships within constructivist interactions*.

t ; FFFF FFFF FFFF FFFF, TTTT TTTT TTTT TTTT (i), pg 188
r ; FFFF TTTT FFFF TTTT, FFFF TTTT FFFF TTTT (ii)
t>r ; TTTT TTTT TTTT TTTT, FFFF TTTT FFFF TTTT (i) > (ii)
r=p ; TFTF FTFT TFTF FTFT, TFTF FTFT TFTF FTFT (iii)
p=r ; TFTF FTFT TFTF FTFT, TFTF FTFT TFTF FTFT (iv)
(r=p)>((r>p)&(p>r)) ; TTTT TTTT TTTT TTTT, TTTT TTTT TTTT TTTT (v), pg 189
(x>y)&(y>x) ; TTTT TTTT TTTT TTTT, TTTT TTTT TTTT TTTT (vi)
t=((r>p)&(p>r)) ; FTFT TFTF FTFT TFTF, TFTF FTFT TFTF FTFT (vii)
(t>((r>p)&(p>r)))&(((r>p)&(p>r))>t) ;
FTFT TFTF FTFT TFTF, TFTF FTFT TFTF FTFT (viii)
(z>((y>x)&(x>y)))&(((x>y)&(y>x))>z) ;
FFFF FFFF FFFF FFFF, TTTT TTTT TTTT TTTT
16*F, 32*T, 16*F, 16*T, 32*F, 16*T = 128 tables ; (ix)
[(viii) is not structurally equivalent to (ix)]
t>((r>p)&(p>r)) ; TTTT TTTT TTTT TTTT, TFTF FTFT TFTF FTFT (x), pg 190
u>((s>q)&(q>s)) ; TTTT TTTT TTTT TTTT, TTFF TTFF FFTT FFTT (xi)
((r>p)&(p>r))>t ; FTFT TFTF FTFT TFTT, TTTT TTTT TTTT TTTT (xii)
((s>q)&(q>s))>u ; FFTT FFTT TTFF TTFF, TTTT TTTT TTTT TTTT (xiii)

As rendered, 14-expressions are *not* tautologous, therefore constructivistic logic is suspicious.

Refutation of templates for converting counterexamples to necessitarianism and into internalism

Abstract: Five equations of counter-examples for two conjectures are *not* tautologous. Hence use of templates for metaphysical grounding is refuted in contingentism, externalism, internalism, and necessitarianism. These conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Skiles, A. (2019). Metaphysical grounding and necessity. [supposed to be a dictionary entry]
academia.edu/39395740/_Metaphysical_Grounding_and_Necessity_?email_work_card=view-paper

Necessity relates to the modal import of grounding, such as what facts ground what facts, for disputes of necessitarianism versus contingentism and disputes of internalism versus externalism. Necessitarianism does not entail internalism, and contingentism does not entail externalism. Arguments to support contingentism may also support internalism in a general template for converting counter-examples to necessitarianism into counter-examples to internalism. For an example:

Suppose that [p] grounds, but does not necessitate, a certain fact [q]. (3.1)

((p>q)&~(#(p>q)=(s=s)))) ; (3.2)

Remark 3.2: Eq. 3.1 is a trivial tautology.

Let [r] be any arbitrarily chosen fact that is not modally or ground-theoretically connected to either [p] or [q]. (4.1)

~(r>(p+q)) =(p=p) ; **FFFF** T**FFF** **FFFF** T**FFF** (4.2)

Now consider the fact [(p & q) + r]. (5.1)

((p&q)+r) ; **FFFF** TTTT **FFFF** TTTT (5.2)

Remark 6.0: The conjecture becomes Eqs. 3.1 and 4.1 implies 5.1. (6.1)

(((p>q)&~(#(p>q)=(s=s)))&~(r>(p+q)))>((p&q)+r) ; (6.2)

Remark 6.2: Eq. 6.1 is a trivial tautology, as the starting assumption.

Given widely held principles governing the grounding of logically complex facts ... , in a possible situation where [p] grounds [q], we have it that [p] alone grounds [(p & q) + r]. (7.1)

%(p>q)>(#p>((p&q)+r)) ; (7.2)

Remark 7.2: Eq. 7.1 is a trivial tautology.

Yet given our starting assumptions, there is also a possible situation in which [p] holds but not [q], yet nonetheless [r] holds, and thus one in which [p] and [(p & q) + r] both hold even though the former does not ground the latter. (8.1)

Remark 8.1: We write this as Eq. 6.1 (the starting assumptions) implies 7.1 to imply the consequent above.

(((((p>q)&~(#(p>q)=(s=s)))&~(r>(p+q)))>((p&q)+r))>(%(p>q)>(#p>((p&q)+r)))) >
(((p&~q)&#r)&(~(p>((p&q)+r))>(p&((p&q)+r)))) ;
FFFF FFN**F FFFF FF**N**F** (8.2)

[M]ethods of arguing for externalism make no appeal to contingentist-supporting considerations, and are compatible with its denial. For example, a putative instance of grounding early pre-emption [where word meanings are irrelevant]:

if [p1] holds but not [q1], then [p1] would ground [r]—but only by way of [p1]'s grounding [p2]. (10.1)

LET p, q, r, s, t: p1, q1, r, p2, q2

(p>r)>((p&~q)>(p>r)) ; (10.2)

Remark 10.2: Eq. 10.1 is a trivial tautology.

if [p1] holds but [q1] holds too, then [q1]'s grounding of [r] by way of [q1]'s grounding [q2] 'pre-empts' [p1]'s grounding of [r], as [q2]'s holding is incompatible with [p1]'s grounding [p2]. (11.1)

(t@(p>s))>((p&q)>(((q>t)>(p>r))>(q>r))) ;
TTTT TTTT TTT**F** TTTT
TTTT TTTT TTTT TTTT (11.2)

Yet even if this is a potential counterexample to internalism, it need not be a counterexample to necessitarianism: even though [p1]'s grounding of [r] is pre-empted, it is still a case in which [p1] and [r] both obtain, and thus no threat has yet been raised to [p1]'s necessitating [r]. (12.1)

(((q>t)>(p>r))>(p>r))>#(p>r) ; N**F**NT NNNN N**F**NT NNNN
N**F**N**F** NNNN N**F**N**F** NNNN (12.2)

Five equations of counter-examples for two conjectures are *not* tautologous. Hence use of templates for metaphysical grounding is refuted in contingentism, externalism, internalism, and necessitarianism.

Refutation of the continuum hypothesis

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET p, q, r, s: minimal cardinality, integer, real, set.
~ Not; & And; > Imply, greater than; = Equivalent;
# necessity, for all; % possibility, for one or some.

From: en.wikipedia.org/wiki/Continuum_hypothesis

The continuum hypothesis states that the set of real numbers has minimal possible cardinality which is greater than the cardinality of the set of integers. (1.1.1)

((s&r)>(%p<p))>((s&q)>%p) ; TTTT TTTT TTCT TTTT (1.1.2)

That is, every set, S, of real numbers can either be mapped one-to-one into the integers or the real numbers can be mapped one-to-one into S. (1.2.1)

((#s&r)>q)+(r>#s) ; TTTT TTTT TTTT TTTT (1.2.2)

Eq. 1.1.1 is equivalent to Eq. 1.2.1. (1.3.1)

(((s&r)>%p)>((s&q)>%p)) = (((#s&r)>q)+(r>#s)) ;
TTTT TTTT TTCT TTTT (1.3.2)

Eq. 1.1.2 as rendered is *not* tautologous, thereby refuting the continuum hypothesis. This is the briefest known refutation of the continuum hypothesis.

Eq. 1.2.2 as rendered is proffered as an obtuse restatement and is tautologous, a theorem.

Eq. 1.1.2 is supposed to be equivalent to Eq. 1.2.2 as Eq. 1.3.2. However 1.3.2 is *not* tautologous. This further refutes the continuum hypothesis.

Refutation of Cook-Reckhow definition

Abstract: We evaluate the Cook-Reckhow definition as represented for conjectures of a polynomial equation and integer linear inequalities with both as *not* tautologous. Hence we do not evaluate the conjecture of a relational form on which the paper subsequently relies. These results form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Krajíček, J. (2019). The Cook-Reckhow definition. arxiv.org/pdf/1909.03691.pdf

The Cook-Reckhow paper .. introduced the notions of *propositional proof* systems and *polynomial simulations* among them, described several classes of logical propositional calculi and compared them with regards to their efficiency, and introduced the *pigeonhole principle tautology* PHP_n that is the prime example of a tautology hard to prove in weaker systems ever since … It was the Cook-Reckhow 1979 paper .. which defined the area of research we now call proof complexity.

Remark 0: We prove the stronger and weaker pigeon hole principles as theorems at: vixra.org/pdf/1902.0414v1.pdf .

[Remark 1.2:] As rendered in Eq. 1.2, the pigeon hole principle is tautologous and a trivial theorem. It is the stronger form of the theorem. The weaker form, to which the paper directs, in this context substitutes the antecedent clause of "some object implies the necessity of space" with "some object implies the *possibility* of space", for result of the same table.

1 Definition of proof systems

For example, a clause

$$p \vee \neg q \vee r \quad (1.1.1)$$

(p+~q)+r ; TT**F**T TTTT TT**F**T TTTT (1.1.2)

together with the requirement that we look for 0−1 solution can be represented by polynomial equations

$$(1 - p)q(1 - r) = 0, \tag{1.2.1}$$

((((%s>#s)-p)&q)&((%s>#s)-r))=(s@s) ;
TTNT TTTT TTNT TTTT (1.2.2)

$$p^2 - p = 0,\ q^2 - q = 0,\ r^2 - r = 0 \tag{1.3.0}$$

Remark 1.3.0: Eq. 1.3.0 factors into the form of p(p - 1) = 0 to mean the solutions are 0 and 1, hence rendering p, q, r as always tautologous (and trivial for the instant example).

the first equation states that the clause contains a true literal while the last three equations force 0−1 solutions over any integral domain. In this case we can use a calculus deriving elements of the ideal generated by the equations representing similarly all clauses of the formula, trying to derive 1 as a member of the ideal and thus demonstrating the unsolvability of the equations and hence the unsatisfiability of the formula.

Another approach is to represent the clause as integer linear inequalities

$$p + (1 - q) + r \geq 1\ , \tag{1.4.1}$$

~((%s>#s)>((p+((%s>#s)-q))+r))=(s=s) ;
N**F**N**F** **FFFF** N**F**N**F** **FFFF** (1.4.2)

$$1 \geq p,\ q,\ r\ \geq 0 \tag{1.5.1}$$

~(~((p+(q+r))>(s@s))>(%s>#s))=(s=s) ;
FCCC CCCC **F**CCC CCCC (1.5.2)

and use some integer linear program[m]ing algorithm to derive the unsolvability of the system of inequalities representing the whole CNF [conjunctive normal form] formula.

Eqs. 1.2, 2.2, 4.2, and 5.2 as rendered are *no*t tautologous. This refutes the Cook-Reckhow definition on its face, without resorting to evaluation of "[p]roof systems… also defined equivalently in a relational form", on which the paper subsequently relies.

Refutation of Coq proof assistant to map Euclidean geometry to Hilbert space

Abstract: For the relation of "a point is incident to a straight line", we find that proposition is *not* tautologous. This denies the conjectured approach of a constructive mapping Euclidean geometry into a Hilbert space and also refutes the Coq proof assistant as a bivalent tool. The conjecture and Coq are therefore *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables.
(See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊔ ;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1;
(%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Ivashkevidh, E.V. (2019). On constructive-deductive method for plane Euclidean geometry.
arxiv.org/pdf/1903.05175.pdf ivashkev@yandex.ru

Constructive-deductive method for plane Euclidean geometry is proposed and formalized within Coq Proof Assistant. This method includes both *postulates* that describe elementary constructions by idealized geometric tools (pencil, straightedge and compass), and *axioms* that describes properties of basic geometric figures (points, lines, circles and triangles). The proposed system of postulates and axioms can be considered as a constructive version of the Hilbert's formalization of plane Euclidean geometry.

Remark 1.4: The law of excluded middle as presented is unclear as to its order of operations; to be tautologous, the main connective is the Or operator.

2.2. Incidence relation

The main undefined relation, that determines the relative position of points and straight lines on the plane, is the relation of *incidence*. ... It is easy to see that if at some scale the spots that represent a point and a straight line do not intersect and do not touch each other, then they will be distinguishable on all larger scales. In this case, we say that the point is *apart* from the line. It is impossible to confirm *empirically* the fact that a point belongs to a straight line, because for this we would have to make sure that the graphite spots that represent this point and this straight line overlap or touch each other on all scales. Note that expressions often used in geometry: "a point lies on a line", "a point belongs to a line", "a line passes through a point" — all of them are equivalent to the proposition "a point is incident to a straight line".

$(A\ B\ \ldots : \text{Point})(x\ y\ \ldots : \text{Line}),\ (A \in\ xy \ldots) \equiv A \in x \wedge\ A \in y \wedge \ldots$; the lines $x, y, \ldots$ pass through point A (2.2.2.1)

LET p, q, r, s: *A*, *B*, *x*, *y*

(p<(r&s))=((p<r)&(p<s)) ; `TTTT T`**`F`**`T`**`F`**` T`**`F`**`T`**`F`**` TTTT` (2.2.2.2)

Remark 2.2.2.2: The only way to coerce Eq. 2.2.2.1 into a tautology is to specify that:

Point A is lesser than points x or y is equivalent to A is lesser than x and y, as (p<(r+s))=((p<r)&(p<s)) ; or (2.2.2.3)

Point A lesser than points x and y, is equivalent to A is lesser than x or y, as (p<(r&s))=((p<r)+(p<s)) . (2.2.2.4)

Eq. 2.2.2.2 as rendered is *not* tautologous. This refutes Coq proof assistant and further denies the conjecture of a constructive and deductive mapping Euclidean geometry into a Hilbert space.

We also hasten to add that the former is bivalent and exact, but the latter is a vector space and probabilistic, meaning the approach is *not* tenable.

Refutation of reversing the counterfactual analysis of causation

Abstract: The seminal formula of C causes E iff (~C □→ ~E) is *no*t tautologous, that is, it is *not* a theorem, from which the conjecture is derived. Hence reversing the counterfactual analysis of causation is refuted. Therefore the conjecture forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , ·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Broadbent, A. (2007). Reversing the counterfactual analysis of causation. abbroadbent@uj.ac.za www.academia.edu/attachments/1859485/download_file?st=MTU1ODQ1OTY3Miw3NS43MS4xNjEuMTQ2LDc2MDk1MzU4&s=swp-toolbar&ct=MTU1ODQ1OTY3MiwxNTU4NDU5NjgzLDc2MDk1MzU4

Abstract: The counterfactual analysis of causation has focused on one particular counterfactual conditional, taking as its starting point the suggestion that C causes E iff (~C □→ ~E). (1.1)

Remark 1.1: We interpret (~C □→ ~E) to mean □(~C → ~E).

LET p, q: C, E.

#(~p>~q)>(p>q) ; TCTT TCTT TCTT TCTT (1.2)

Eq. 1.2 as rendered is *no*t tautologous, that is, *not* a theorem, from which the conjecture is derived. Hence reversing the counterfactual analysis of causation is refuted.

Refutation of counterpart theory via modal logic

Abstract: Of seven examples for counterpart theory, none is tautologous. In fact, a translation is *not* tautologous in the counterpart model *or* in QMT, but rather shares the same truth table result. Two definitions of intensionality are also *not* tautologous and hence refuted. Therefore counterpart theory forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Yli-Vakkuri, J. (2019). Counterpart theory and modal logic. ylivakkuri@gmail.com
academia.edu/39124562/Counterpart_Theory_and_Modal_Logic?email_work_card=view-paper

3. Uncontested principles of modal logic

While Lewis (1968: 123) proposed to test various contested principles of quantified modal logic (QML) by checking whether his translation validated them, he did not test the translation itself by checking whether it validated the most basic uncontested principles of QML—or even propositional modal logic. I will now carry out that test by examining four uncontested principles (axiom schemas) of propositional modal logic, in order of increasing weakness in Kripke semantics:

Intensionality: □(φ ↔ψ) → (χ→χ[φ/ψ]), where χ[φ/ψ] is the formula (if any) that results from replacing all free occurrences of ψ in χ, with free occurrences of φ. (3.1.1.1)

LET p, q, r, s, t, u, v, w, x, y, z
A, b, C, G, I, u, v, w, x, y, z

#(p=q)>(r>(r&(p\q))) ; `TTTT TTCT TTTT TTCT` (3.1.1.2)

Remark 3.1.1.2: Eq. 3.1.1.2 as rendered is *not* tautologous as it should be if intensionality is a theorem.

We should not expect Intensionality to hold in every language, but, by definition, it does hold in any language with no hyperintensional operators, and QML is such a language. (3.1.1.3)

Remark 3.1.1.3: As we show above, and elsewhere (Refutation of realizability semantics for QML), intensionality does not hold in QML such as in VŁ4.

The model is not a model of:
∃w(∀x(Ixw↔Ax)∧(∀v∀x(Cxbv→(Gx→∃x(Ixv∧Gx)))→(Gb→∃x(Ixw∧Gz))) (3.2.1.1)

```
((((t&#x)&%w)=(p&#x)) & (((r&#x)&(q&#v))>((s&#x)>((t&(%x&#v))&(s&%x)))))>
((s&#q)>((t&(#x&%w))&(s&z))) ;
          TTTT TTTT TTCC TTCC(16),
          TTTT TTTT TTCT TTCT( 4), TTTT TTTT TTCT TTTT( 1),
          TTTT TTTT TTCT TTCT( 1), TTTT TTTT TTCT TTTT( 1),
          TTTT TTTT TTCT TTCT( 2), TTTT TTTT TTTT TTTT( 1),
          TTTT TTTT TTCT TTCT( 1), TTTT TTTT TTTT TTTT( 1),
          TTTT TTTT TTCT TTTT( 1), TTTT TTTT TTTT TTTT( 1),
          TTTT TTTT TTCT TTTT( 1), TTTT TTTT TTTT TTTT( 1)    (3.2.1.2)
```

which is the translation of the following T-instance □(Gb→∃xGx)→(Gb→∃xGx)(3.2.2.1)

```
#((s&q)>(s&%x))>((s&q)>(s&%x)) ;TTTT TTTT TTTT TTTT              (3.2.2.2)
```

Remark 3.2.2.2: Eq. 3.2.1.2 as a counterpart theorem is *not* tautologous. However the T-instance it maps in 3.2.2.2 is tautologous.

The model is also not a model of: ∀w∀x∀y(Cxyabw → (Fx∧Gy))→∃w∃x∃y(Cxyabw∧(Fx∧Gy)), (3.2.3.1)

```
LET   p, q,  r,  s,  t, u, v, w, x, y, z
      a, b, C, G, I, F, v, w, x, y, z

((((r&#x)&(#y&p))&((p&q)&#w))>((u&#x)&(s&#y)))>
((((r&%x)&(%y&p))&((p&q)&%w))&((u&%x)&(s&%y))) ;
      FFFF FFFF FFFF FFFF(2), FFFF FFFF FFFF FFFC(2)} x14
      FFFF FFFN FFFF FFFN(2), FFFF FFFN FFFF FFFT(2)} x 2 (3.2.3.2)
```

which is the translation of the D-instance □(Fa∧Gb)→◊(Fa∧Gb). (3.2.4.1)

```
#((u&y)&(s&q))>%((u&y)&(s&q)) ;TTTT TTTT TTTT TTTT              (3.2.4.2)
```

Remark 3.2.4.2: Eq. 3.2.3.2 as a counterpart theorem is *not* tautologous. However the D-instance it maps in 3.2.4.2 is tautologous.

Nor is it a model of: ∀w∀x(Cxaw→(Fx→∃y(Iyw∧Fy)))→(∀w∀x(Cxaw→Fx)→∀w∃x(Ixw∧Fx)) (3.2.5.1)

```
(((r&#x)&(p&#w))>((u&#x)>(((t&%y)&#w)&(u&y))))>
(((r&#x)&((p&#w)>(u&#x)))>(((t&%x)&#w)&(u&%x))) ;
                             TTTT TTTT TTTT TTTT(16),
                             TTTT CCCC TTTT CCCC( 8),
                             TTTT CTCT TTTT CTCT( 3),}x 2
                             TTTT TTTT TTTT TTTT( 1) }    (3.2.5.2)
```

which is the translation of the K-instance □(Fa→∃xFx)→(□Fa→□∃xFx)). (3.2.6.1)

#((u&p)>(u&%x))>((#u&p)>(#u&%x)) ;
`TTTT TTTT TTTT TTTT` (3.2.6.2)

Remark 3.2.6.2: Eq. 3.2.5.1 as a counterpart theorem is *not* tautologous. However the K-instance it maps in 3.2.6.2 is tautologous.

Finally, while the translation of □(¬ ∃xFx↔(Fa∧¬Fa)) (3.2.7.1)

#(~(u&%x)=((u&p)&~(u&p))) = (p=p) ;
```
FFFF FFFF FFFF FFFF(16),
FFFF FFFF FFFF FFFF( 2),}x 4
NNNN NNNN NNNN NNNN( 2) }
```
(3.2.7.2)

is true in the model, the translation of □(¬ ∃xFx→⊥) is not. (3.2.8.1)

#(~(u&%x)>(p@p)) = (p=p) ;
```
FFFF FFFF FFFF FFFF(16),
FFFF FFFF FFFF FFFF( 2),}x 4
NNNN NNNN NNNN NNNN( 2) }
```
(3.2.8.2)

Remark 3.2.8.2: In fact, Eqs. 3.2.8.2 is equivalent to 3.2.7.2 which the text denies.

The model, then, is not a model of the translation of the Intensionality instance □((Fa∧¬Fa)↔⊥)→(□(¬ ∃xFx↔(Fa∧¬Fa))→□(¬ ∃xFx→⊥)). (3.2.9.1)

#((((u&p)&~(u&p))=(p@p))>(#(~(u&x)=((u&p)&~(u&p)))>#(~(u&x)>(p@p)))) = (p=p) ;
`NNNN NNNN NNNN NNNN` (3.2.9.2)

Remark 3.2.9.2: Eq. 3.2.9.2 as rendered is not a model of the intensionality instance because it returns `N` non-contingency (truthity) and not `T` tautology.

Of seven examples for counterpart theory, none is tautologous. In fact, a translation is *not* tautologous in the counter point model *or* in QMT, but rather shares the same truth table result. Two definitions of intensionality are also *not* tautologous and hence refuted.

Refutation of the constructive Craig interpolation theorem by Fefferman

Abstract: We evaluate the Craig constructive interpolation theorem, find a mistake in a Craig lemma as rendered by Feferman, and refute the theorem as not constructive.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨ ; - Not Or; & And, ∧; \ Not And;
> Imply, greater than, →; < Not Imply, less than, ∈;
= Equivalent, ≡; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊; # necessity, for every or all, ∀, □;
~(y < x) (x ≤ y), (x ⊆ y).

From: en.wikipedia.org/wiki/Craig_interpolation;

Remark 0: We use only four variables to minimize table results to 4-rows or 16-values (instead of 512-rows or 2048-values) . Hence we avoid direct assignment of φ, ψ as separate variables.

LET p, q, r, s: P, Q, R, T.

In propositional logic, let

$\varphi = \sim(P \wedge Q) \rightarrow (\sim R \wedge Q)$ (1.1.1)

~(p&q)>(~r&q) ; **FF**TT **FFF**T **FF**TT **FFF**T (1.1.2)

$\psi = (T \rightarrow P) \vee (T \rightarrow \sim R)$. (1.2.1)

(s>p)+(s>~r) ; TTTT TTTT TTTT **F**T**F**T (1.2.2)

Then φ tautologically implies ψ, but only because both are not tautologous. (1.3.1)

(~(p&q)>(~r&q))>((s>p)+(s>~r)) ;
TTTT TTTT TTTT TTTT (1.3.2)

Eq. 1.3.2 refutes the Craig interpolation theorem because both Eqs.1.1.2 and 1.2.2 are *not* tautologous.

Remark 1: Eq. 1.3 is a tautology via contradiction implying contradiction (**F**>**F**=T). This form of proof is not constructive in an affirmative or positive sense. A much longer constructive proof exists, but it can be minimized by its use of *induction*.

From: Feferman, S. (2008). Harmonious Logic: Craig's Interpolation Theorem and its Descendants math.stanford.edu/~feferman/papers/Harmonious%20Logic.pdf

[Here ⊢ is validity in classical first order logic with equality (FOL), ϕ, ψ, θ are sentences, and R, S, and T are sequences of relation symbols for which the sequence S is non-empty.]

A common statement of Craig's theorem (initially referred to by him as a lemma) goes as follows:

[LET r, s, t, w, x, y: R, S, T, ϕ, ψ, θ.]

Suppose $\vdash \phi(R, S) \rightarrow \psi(S, T)$. Then there is a $\theta(S)$
such that $\vdash \phi(R, S) \rightarrow \theta(S)$ and $\vdash \theta(S) \rightarrow \psi(S, T)$. (2.1)

Remark 1.1: Again, the antecedent and consequent are both not tautologous, hence the product of a tautology, as in Remark 1 is not constructive.

(((w&(r&s))>(x&(s&t)))=(p=p)) >
((y&s)>((((w&(r&s))>(y&s))=(p=p))&(((y&s)>(x&(s&t)))=(p=p)))) ;

```
TTTT TTTT TTTT TTTT
```
(2.2)

Accounting for the unclear writing of Craig (and Feferman), the lemma of Eq. 2.2 as rendered refutes the Craig interpolation as a constructive theorem.

Refutation of constructive proof of Craig's interpolation theorem using Maehara's technique

Abstract: We evaluate a constructive proof of Craig's interpolation theorem by way of Maehara's technique. Six equations are *not* tautologous, and serve as antecedents for respective conclusions of two- or four-sequents. Hence, the concluding consequents in any state of proof value will always return a tautology. This means the technique of Maehara does not produce a constructive proof of Craig's interpolation theory as applied to sequent logic for interpolation of non-normal logics. Therefore the approach forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Orlandelli, E. (2019). Sequent calculi and interpolation for non-normal logics. arxiv.org/pdf/1903.11342.pdf eugenio.orlandelli@unibo.it

4 Craig's interpolation theorem

[F]or each modal or deontic logic X which has neither C nor D$^{\Diamond}$ as axiom, a constructive proof of Craig's interpolation theorem by means of the well-known Maehara's technique [for respective conclusions of two- or four-sequents]

Remark 4: Rendering of the original text equations in pdf required too much format manipulation, so only the mappings into Meth8 script are below, and in reverse order from the text.

LET p, q, r, s, t: A, B, Γ, Δ, Π.

((p=p)>p)>(r>(s&#p)) ; TTTT T**F**T**F** TTTT NNNN (R-N.2)

t>((#t&r)>s) ; TTTT TTTT TTTT TTTT(1),
TTTT CCCC TTTT TTTT(1) (L-D*.2)

p>((#p&r)>s) ; TTTT TCTC TTTT TTTT (L-D$^{\perp}$.2)

(t>q)>((#t&r)>(s&#q)) ;   TTTT TTTT TTTT TTTT(1),
TTTT TTCC TTTT TTTT(1)   (LR-K.2)

((p&t)>q)>((#p&(#t&r))>(s&#q)) ;
TTTT TTTT TTTT TTTT(1),
TTTT TTTC TTTT TTTT(1)   (LR-R.2)

(p>q)>((#p&r)>s) ;   TTTT TTTC TTTT TTTT   (LR-M.2)

((p>q)&(q>p))>((#p&r)>(s&#q)) ;
TTTT TTTC TTTT TTTT   (LR-E.2)

Remark 4: The six equations above are *not* tautologous. Because these serve as antecedents for respective conclusions of two- or four-sequents, the concluding consequents in any state of proof value will always return a tautology. This means the technique of Maehara does not produce a constructive proof of Craig's interpolation theory.

Creative theories in degrees of unsolvability

From: Solomon Feferman. "Degrees of unsolvability associated with classes of formalized theories." Journal of symbolic logic. v 22, n 2, June 1957. pg 169

[The unnumbered equation at top of the page cannot be rendered due to image adulteration enforced by jstor.org and aslonline.org.]

LET: # inverted upper case V; y y; x upper case Phi; v V-sub upper case Tau; & And; w upper case Delta sub p; % upper case V; z z; < lower case epsilon, Not imply; > Imply

```
( #y&( ((x&v)&(w&y)) > ((v&z)&(~(z>y)&((x&y)&(w&z)))) ) ) < v ;
                                          FFFF FFFF FFFF FFFF,
                                          NNNN NNNN NNNN NNNN        (1)
```

Eq 1 is not validated as tautology by Meth8, meaning the degrees of unsolvability are not finitely axiomizable.

Refutation of the Curry-Howard correspondence

Abstract: In the Curry-Howard correspondence, the identity and composition combinators are *not* tautologous. In fact, the examples result in equivalent truth table values. Further demonstrated is that the instances of Hilbert, lambda, and sequent fragments are also *not* tautologous with a recent paper rendered moot. These artifacts form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Curry–Howard_correspondence

In programming language theory and proof theory, the Curry–Howard correspondence … is the direct relationship between computer programs and mathematical proofs.

Examples

Thanks to the Curry–Howard correspondence, a typed expression whose type corresponds to a logical formula is analogous to a proof of that formula. Here are examples.

The identity combinator seen as a proof of *α* → *α* in Hilbert-style logic

As an example, consider a proof of the theorem *α* → *α*. In lambda calculus, this is the type of the identity function **I** = λ*x*.*x* and in combinatory logic, the identity function is obtained by applying **S** = λ*fgx*.*fx*(*gx*) twice to **K** = λ*xy*.*x*. That is, **I** = ((**S K**) **K**). As a description of a proof, this says that the following steps can be used to prove *α* → *α*:

instantiate the second axiom scheme with the formulas *α*, *β* → *α* and *α*, so that to obtain a proof of (*α* → ((*β* → *α*) → *α*)) → ((*α* → (*β* → *α*)) → (*α* → *α*)), (1.1)

LET p, q, r: *α*, *β*, Γ.

(p>((q>p)>p))>((p>(q>p))>(p>p)) ;
TTTT **F**T**F**T TTTT **F**T**F**T (1.2)

instantiate the first axiom scheme once with *α* and *β* → *α*, so that to obtain a proof of *α* → ((*β* → *α*) → *α*), (2.1)

p>((q>p)>p) ; TTTT **F**T**F**T TTTT **F**T**F**T (2.2)

instantiate the first axiom scheme a second time with α and β, so that to obtain a proof of
$\alpha \to (\beta \to \alpha)$, (3.1)

p>(q>p) ; TTTT **F**T**F**T TTTT **F**T**F**T (3.2)

The composition combinator seen as a proof of (β→α) → (Γ→β) → Γ → α in Hilbert-style logic
As a more complicated example, let's look at the theorem that corresponds to the **B** function. The type of **B** is $(\beta\to\alpha) \to (\Gamma\to\beta) \to \Gamma \to \alpha$. **B** is equivalent to (**S** (**K S**) **K**). This is our roadmap for the proof of the theorem

$$(\beta \to \alpha) \to (\Gamma \to \beta) \to \Gamma \to \alpha. \quad (4.1)$$

(((q>p)>(r>q))>r)>p ; TTTT **F**T**F**T TTTT **F**T**F**T (4.2)

The identity combinator (Eqs. 1-3) and composition combinator (Eq. 4) are *not* tautologous. In fact, the example steps given result in equivalent truth table values. Further demonstrated is that the instances of Hilbert, lambda, and sequent fragments are also *not* tautologous. Hence, the following paper becomes moot:

Caires, L.; Pérez, J.A.; Pfenning, F.; Toninho, B. (2019).
Domain-aware session types (extended version). arxiv.org/pdf/1907.01318.pdf

Abstract We develop a generalization of existing Curry-Howard interpretations of (binary) session types by relying on an extension of linear logic with features from *hybrid logic*, in particular modal worlds that indicate *domains*. These worlds govern *domain migration*, subject to a parametric accessibility relation familiar from the Kripke semantics of modal logic. The result is an expressive new typed process framework for domain-aware, message-passing concurrency.

Analysis of ultrafilter D equations by Meth8 logic model checker

From: Aleksandar Jovanovic, Aleksandar Perović (2007.01),
"Contrapunctus of the continuum problem and the measure problem",
Publications de l Institut Mathematique 01/2007(82(96)):83 - 91.

An ultrafilter D over infinite cardinal κ is:

weakly normal, if each function f:κ−→κ such that [we read *κ−→κ* as *κ→κ*] (1.1)

(f&(k>k)) (1.2)

{α∈κ|f(α)< α}∈D (2.1)

(((a<k)+((f&a)<a))<D) (2.2)

is bounded by some constant in ∏D 〈κ, <〉 , i.e. there is β∈κ (3.1)

(b<k) (3.2)

such that {α∈κ|f(α)< β}∈D (4.1)

(((a<k)+((f&a)<b))<D) (4.2)

We build:

(((a<k)+(((f&a)<a)<D)) >(f&(k>k))) < ((b<k)>(((a<k)+((f&a)<b))<D)). (5.1)

To map to Meth8:

LET pqrst = abfDk; vt tautologous, nvt not tautologous

(((p<t)+(((r&p)<p)<s))>(r&(t>t))) < ((q<t)>(((p<t)+((r&p)<q))<s)) ; nvt; (5.2)

```
FFTF FFTF FFTF FFTT   UUEU UUEU UUEU UUEE   UUEU UUEU UUEU UUEE   UUEU UUEU UUEU UUEE   UUEU UUEU UUEU UUEE
FFFF FFFF FFFF FFFF   UUUU UUUU UUUU UUUU   UUUU UUUU UUUU UUUU   UUUU UUUU UUUU UUUU   UUUU UUUU UUUU UUUU
Model 1               Model 2.1             Model 2.2             Model 2.3.1           Model 2.3.2
```

For Eq 5.1 to be tautologous, the truth table fragment above for Eq 5.2 should be T Tautologous for Model 1 and E Evaluated for Models 2.n.

Refutation of existentially closed De Morgan algebras

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: x, y, r, s;
~ Not, ¬; + Or; & And, ∧; = Equivalent;
> Imply, greater than, → ; < Not Imply, lesser than;
necessity, for all or every, □, ∀; % possibility, for one or some, ◊, ∃;
(s@s) zero, 0; (%s>#s) one, 1.

From: Aslanyan, V. (2018). Existentially closed De Morgan algebras. arxiv.org/pdf/1810.02335.pdf
vahagn@math.cmu.edu

$$\forall x(x > 0 \rightarrow \exists y(0 < y < x)) \qquad (2.1)$$

(#p>(s@s))>(((s@s)<%q)&(%q<#p)) ;
FN**F**N **F**N**F**N **F**N**F**N **F**N**F**N (2.2)

We also evaluate the lattice complement as

$$\forall x \exists y(xy = 0 \wedge x + y = 1) \qquad (3.1)$$

((#p&#q)=(s@s))&((p+q)=(%s>#s)) ;
CNN**F** CNN**F** CNN**F** CNN**F** (3.2)

Eqs. 2.2 and 3.2 as rendered are *not* tautologous. This means existentially closed De Morgan algebras are refuted.

Refutation of pre-measurement, decoherence, and multiverse quantum mechanics

Abstract: The equation for pre-measurement is *not* tautologous, thereby refuting it and decoherence as a basis for multiverse quantum mechanics. These form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Bousso, R.; Susskind, L. (2011). The multiverse interpretation of quantum mechanics. arxiv.org/pdf/1105.3796.pdf

Abstract: We argue that the many-worlds of quantum mechanics and the many worlds of the multiverse are the same thing … Decoherence -- the modern version of wave-function collapse -- is subjective in that it depends on the choice of a set of unmonitored degrees of freedom, the "environment". … We argue that the global multiverse is a representation of the many-worlds (all possible decoherent causal diamond histories) in a single geometry.

Decoherence: Decoherence.. explains why observers do not experience superpositions of macroscopically distinct quantum states, such as a superposition of an alive and a dead cat. The key insight is that macroscopic objects tend to quickly become entangled with a large number of "environmental" degrees of freedom, *E*, such as thermal photons. In practice these degrees of freedom cannot be monitored by the observer. ...

As an example, consider an isolated quantum system *S* with a two-dimensional Hilbert space, in the general state $a|0\rangle_S + b|1\rangle_S$. Suppose a measurement takes place in a small spacetime region, which we may idealize as an event *M*. By this we mean that at *M*, the system *S* interacts and becomes correlated with the pointer of an apparatus *A* [this process is unitary and is referred to as a pre-measurement]:

$$(a|0\rangle_S + b|1\rangle_S) \otimes |0\rangle_A \rightarrow a|0\rangle_S \otimes |0\rangle_A + b|1\rangle_S \otimes |1\rangle_A \; ; \quad (1.1.1)$$

LET p, q, r, s
$a|0\rangle_S$, $b|1\rangle_S$, $|0\rangle_A$, $|1\rangle_A$.

((p+q)&r)>((p&r)+(q&s)) ; TTTT TT**F**T TTTT TTTT (1.1.2)

Eq. 1.1.2 as rendered is *not* tautologous, thereby refuting pre-measurement, decoherence, and hence the conjecture of multiverse quantum mechanics.

Dedekind lattice identity

From: Gian-Carlo **Rota**. "The Many Lives of Lattice Theory".
Notices of the AMS. 44:11. 1440-1445. December, 1997.

p. 1441, identity discovered by Dedekind:

((r&(p+q))+q) = ((r+q)&(p+q)) ; tautologous

Refutation of Dempster-Shafer belief and plausibility theory

Abstract: We evaluate Dempster-Shafer belief and plausibility functions. Three definitions are *not* tautologous. This refutes Dempster-Shafer belief and plausibility theory.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r, s: P, bel(ief) or support, pl(ausibility), A;
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⊢, ↦; < Not Imply, less than, ∈;
= Equivalent, ≡, ⊨, :=, ⇔, ↔; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology; (z@z) **F** as contradiction, Ø, Null;
(%z<#z) C non-contingency, ∇, ordinal 2;
(%z>#z) N as non-contingency, Δ, ordinal 1; ~(y < x) (x ≤ y), (x ⊆ y).

From: en.wikipedia.org/wiki/Dempster–Shafer_theory

Belief and plausibility functions:

bel(A)<=P(A)<=pl(A) (1.1)

~((r&s)<(~(p&s)<(q&s)))=(p=p) ; TTTT TTTT TTTT T**FFF** (1.2)

pl(A)= (1-bel(~A)) (2.1)

(r&s)=((%s>#s)-(q&~s)) ; NNTT NNTT NNNN CCCC (2.2)

bel(A)<=P(A)<=(1-bel(~A)) (3.1)

~(((%s>#s)-(q&~s))<(~(p&s)<(q&s)))=(p=p) ;
TTTT TTTT TNNN TNNN (3.2)

Dempster-Shafer generalization of Bayesian theory:

If (A And B)=Null, then bel(A Or B)=bel(A) Or bel(B). (4.1)

((s&p)=(s&s))>((q&(s+p))=((q&s)+(q&p))) ;
TTTT TTTT TTTT TTTT (4.2)

Remark 4.2: Because we show elsewhere that Bayes' theorem is *not* tautologous, we expect Dempster-Shafer, as *no*t tautologous from Eq. 3.2, to be an equivalence.

Eqs. 1.2-3.2 as rendered are not tautologous. This refutes Dempster-Shafer theory as plausibilty and belief functions.

Density of a final segment of the truth-table degrees

From: Mohrherr, Jean-Leah (1984). "Density of a final segment of the truth table degrees".
Pacific Journal of Mathematics. 1984.115.2: 409-420.
[msp.org/pjm/1984/115-2/pjm-v115-n2-p12-s.pdf]

"D_T is the structure of all Turing (T) degrees with the induced partial ordering; D_{tt}, [is] the structure of all truth-table (tt) degrees with the induced partial ordering."

"Still, we do not know whether there is a $\mathbf{d} \in D_{tt}$ such that

$$(\forall \mathbf{a} \in D_{tt})(\mathbf{a} > \mathbf{d} \to (\exists\, \mathbf{b} \in D_{tt})(\, \mathbf{b} < \mathbf{a} \text{ and } \mathbf{a} = \mathbf{b} \cup \mathbf{d}))." \tag{410.1}$$

LET p a, s D_{tt}, q d, r b, and assume the Meth8 script:

((#p<s)&(((p>q)>(%r<s))&((r<p)&(p=(r+q))))) > (q<s) ;
vt (410.1.1)

In Eq 410.1.1 we show there is such a $\mathbf{d} \in D_{tt}$.

"Therefore the sentence

$$(\forall \mathbf{a})(\exists\, \mathbf{b})(\, \mathbf{b} > \mathbf{a} \text{ and } (\forall \mathbf{c})(\mathbf{b} \geq \mathbf{c} \geq \mathbf{a} \;\to \mathbf{c} = \mathbf{b} \text{ or } \mathbf{c} = \text{a})) \tag{410.2}$$

is tautologous for D_T but contradictory for D_{tt}."

LET t c, u D_T.

(s\u)>((#p&%r)&((r>p)&(~(~(r<t)<p)>((t=r)+(t=p))))) ;
FFFF FFFF FFFF FFFF,
FFFF FNFN FFFF FNFN,
FFFF FFFF TTTT TTTT,
FFFF FNFN TTTT TTTT (410.2.1)

In Eq2 410.2.1 we show that the sentence and conclusion of Eq 410.2 is not tautologous.

What follows is that Eqs 410.1.1 and 410.2.1 do not validate tautologous what Mohrer (1984) claims to prove. This means that Turing degrees and truth-table degrees are not related, and a connection is therefore suspicious. That conclusion is consistent with our previous work showing set theory is also suspicious.

Refutation of the axiom of dependent choices (DC) on supercompactness of ω_1

Abstract: The axiom of dependent choices (DC) is evaluated in two equations on supercompactness of ω_1, with *neither* tautologous and hence refuting DC. Therefore DC equations are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Ikegami, D.; Trang, N. (2019). On supercompactness of ω_1.
arxiv.org/pdf/1904.01815.pdf ikegami@shibaura-it.ac.jp nam.trang@unt.edu

This paper studies structural consequences of (full) supercompactness of ω1 under ZF. We first show the following basic structural consequences.

Theorem 1. Assume that ω1 is supercompact. Then 1. the Axiom of Dependent Choices (DC) holds, while ... (1.0)

3 Choice principles and supercompactness of ω1

In this section, we prove Theorem 1.
Since ω_1 is supercompact, there is a fine normal measure on $P\omega_1 A$. We fix such a measure μ.
Claim. For μ-measure one many elements σ of $P_{\omega 1}A$, the following holds:

$(\forall x \in \sigma)\ (\exists y \in \sigma)\ (x, y) \in R$ (3.1.1)

LET p, q, r, s: x, y, R, σ .
(((%p<s)&(#q<s))&(p&q))<r ; **FFF**N **FFFF FFFF FFFF** (3.1.2)

Suppose not. $(\exists x \in \sigma)\ (\forall y \in \sigma)\ (x, y) \notin R$

(((%p<s)&(#q<s))&(p&q))>r ; TTTC TTTT TTTT TTTT (3.1.3)

Eqs. 3.1.2 and 3.1.3 are *not* tautologous, hence refuting the axiom of dependent choices (DC) for (full) supercompactness of ω_1 under ZF.

Refutation of the axiom of dependent choices in mice

Abstract: The axiom of dependent choices as ∀a∈X∃b∈XP(a, b) ⇒ ∃f: ω → X∀n P(f(n), f(n+1)) is *not* tautologous. What follows is that the axiom of determinacy is also *no*t tautologous, hence relegating both axioms to a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Müller, S. (2019). The axiom of determinacy implies dependent choices in mice. arxiv.org/pdf/1907.02755.pdf

1. Introduction

We prove that in passive, countably iterable mice M constructed over their reals, AD, the Axiom of Determinacy, implies DC, the Axiom of Dependent Choices, working in a background universe which satisfies ZF+DCRM. Here we write RM = R ∩ M for the set of reals in M.

Recall that DC is the following statement: For every nonempty set X and every binary relation P on X,

$$\forall a \in X \exists b \in X\ P(a, b) \Rightarrow \exists f : \omega \to X\ \forall n\ P(f(n), f(n + 1)). \qquad (1.0.1)$$

LET q, r, s, t, w, p, x: a, b, f, n, w, P, X.

((#q<(x&%r))<(p&(q&r)))>(%s=(w>((x&(#t&p))&((s&t)&(s&(t+(%z>#z))))))) ;

```
TTCC TTTT TTTT TTTT(x 8)
TTTT TTTT TTCC TTTT(x 1)
TTTT TTTT TTCT TTTT(x 1)
```
(1.0.2)

Eq. 1.0.2 as rendered for the axiom of dependent choices is *no*t tautologous, thereby refuting it. What follows is that the axiom of determinacy is also *no*t tautologous.

Description logic

From: Badie, F. A formal semantics for concept understanding relying on description logics. Proceedings of the 9th International Conference on Agents and Artificial Intelligence (ICAART 2017). 2:42-52. DOI:10.5220/0006113800420052.

We evaluate the prototypical description logic from Table 1.

We assume the apparatus of M8-VŁ4.

LET: p a; q b; r R; s C; # necessity, universal quantifier; % possibly, existential quantifier

The designated proof value is T tautology, with C contingent (falsity value), N non-contingent (truth value); and F contradiction (not a proof). The fragments are the truth table as horizontal rows major.

$\exists\ R.\ C$ [is] $\{\ a \mid \exists\ b.(a,b) \in RI \wedge b \in CI\ \}$ (1.1.1)

(%r&s)= ((((%q&(p&q))<r)&(q<s))>p) ;

`FFFF FFFF CCCC TTTT` (1.1.2)

$\forall\ R.\ C$ [is] $\{\ a \mid \forall\ b.(a,b) \in RI \supset b \in CI\ \}$ (1.2.1)

(#r&s)= ((((#q&(p&q))<r)&(q<s))>p) ;

`FFFF FFFF FFFF NNNN` (1.2.2)

We remark that the definition for existential "restriction" fares closer to a tautology than does the universal quantification.

We conclude description logic is suspicious.

Refutation of the power set in description logic

Abstract: We evaluate four, simple axioms of any Ω-model, including the operator Pow, to support the power set in description logic. None is tautologous, meaning the power set as asserted is refuted.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And; > Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) C non-contingency, ∇, ordinal 2; (%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Giordano, L.; Policriti, A. (2019). Adding the power-set to description logics. arxiv.org/pdf/1902.09844.pdf laura.giordano@uniupo.it alberto.policriti@uniud.it

2.2 First order theory Ω

The first-order theory Ω consists of the following four (simple) axioms, written in the language whose relational symbols are ∈ and ⊆, and whose functional symbols are ∪, \, Pow: (2.2.0)

Remark 2.2.0: Pow is an operator derived from formal semantics of the language OWL-Full, *not* based on the corrected Square of Opposition and thus not bivalent but a probabilistic vector space.

We take Pow to mean (C)∈Pow(C), where also possibly Pow(C)∈C, and map it as: (C)⊆(C), or ~((C)<(C)).

x∈y∪z↔x∈y∨x∈z; (2.2.1.1)

LET p, q, r: x, y, z

((p<q)+r)=((p<q)+(p<r)) ; TTT**F** **F**TT**F** TTT**F** **F**TT**F** (2.2.1.2)

x∈y\z↔x∈y∧x∉z; (2.2.2.1)

((p<q)\r)=((p<q)&~(p<r)) ; **FFFF** **FFFF** **FFFF** **FFFF** (2.2.2.2)

x⊆y↔∀z(z∈x→z∈y); (2.2.3.1)

~(q<p)=((#r<p)>(#r<q)) ; TT**F**T TTNT TT**F**T TTNT (2.2.3.2)

$x \in Pow(y) \leftrightarrow x \subseteq y.$ (2.2.4.1)

$(p<\sim(q<q))=\sim(q<p)$; **FF**TF **FF**T**F** **FF**T**F** **FF**T**F** (2.2.4.2)

Remark 2.2: Eqs. 2.2.n.2 as rendered are *not* tautologous. This refutes the four, simple axioms of any Ω-model.

In any Ω-model *everything* is supposed to be a set. Hence, a set will have (only) sets as its elements and circular definitions of sets are not forbidden—i.e., for example, there are models of Ω in which there are sets admitting themselves as elements. Moreover, not postulating in Ω any *link* between membership ∈ and equality—in axiomatic terms, having no *extensionality* (axiom)—, there exist Ω-models in which there are different sets with equal collection of elements.

Refutation of descriptive unions in descriptively near sets

Abstract: We evaluate an intersection operator named descriptive union for descriptively near sets. From two sources the definition of the operator is *not* tautologous. A proof of seven properties derived from the second definition contains two trivial tautologies with the rest as *no*t tautologous. This refutes the descriptive intersection operator and descriptively near sets on which it is based. This also casts doubt on the bevy of derived math and physics papers so spawned at arxiv, researchgate, and vixra.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r, s, t, u, v, w, x, y, z:
lc_phi φ, uc_Phi Φ, A, B, lc_pi π, K, R^n, 2^K, x, y, (q&(r&s));
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⊢, ↦; < Not Imply, less than, ∈;
= Equivalent, ≡, ⊨, :=, ⇔, ↔; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology; (z@z) **F** as contradiction, Ø;
(%z<#z) C non-contingency, ∇, ordinal 2;
(%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y).

From: Peters, J.F. (2013). Near sets: an introduction. Math.Comput.Sci (2013) 7:3-9.

The descriptive intersection $\cap_\Phi$ of *A* and *B* is defined by
$A \cap_\Phi B = \{x \in A \cup B : (x) \in Q(A) \text{ and } (x) \in Q(B)\}$. (0.0.0)
That is, $x \in A \cup B$ is in $A \cap_\Phi B$, provided there is ... $a \in A$, $b \in B$ such that $(x) = (a) = (b)$.
Observe that *A* and *B* can be disjoint and yet $A \cap_\Phi B$ can be nonempty. (0.0.1)

(((y<r)&(z<s))>((q&x)=((q&y)=(q&z))))>(x<(r+s)) ;
FFFF FFFF FFFF FFFF(16), TTTT **FFFF FFFF FFFF**(16),
FFFF FFFF FFFF FFFF(16), TTTT **FFFF FFFF FFFF**(16),
TTTT **FFFF FFFF FFFF**(16), TTTT **FFFF FFFF FFFF**(48) (0.0.2)

Remark 0.0.2: The definition of Eq. 0.0.0 as rendered in 0.0.2 is *not* tautologous.

From: Ahmad, M.Z.; Peters, J.F. (2018).
Descriptive unions: a fibre bundle characterization of the union of descriptively near sets.
arxiv.org/pdf/1811.11129.pdf ahmadmz@myumanitoba.ca james.peters3@umanitoba.ca

Definition 3: ... $\cap_\Phi$ is the descriptive intersection. ...

Theorem 1. Let A,B ⊂ K be two subsets of a set K,φ : 2K → Rn be the probe function and π : Rn → 2K be a map such that π : x ↦ {y ∈ K : φ(y) = x}. Then, A $\cap_\Phi$ B has [the] following properties: (1.0.1)

((((r<u)&(s<u))>((p=(w>v))&(t=(v>w))))>(t=(x>((y<u)>((p&y)=x)))))
>(z=(q&(r&s))) ;

FFFF FFFF FFFF FFTT(4), **F**T**F**T **F**T**F**T **F**T**F**T **FF**TT(3),
T**F**T**F** T**F**T**F** T**F**T**F** **FF**TT(3), TTTT TTTT TTTT TTTT(4),
FT**F**T **F**T**F**T **F**T**F**T **F**TTT(1), T**F**T**F** T**F**T**F** T**F**T**F** T**F**TT(1) (1.0.2)

Note: Eq. 1.0.2 as rendered serves as antecedent to the 1.n.2 consequents listed below.

1.10 $A \cap_{\Phi} B = A \cap_{\Phi} B$. (1.1.1)

(((((r<u)&(s<u))>((p=(w>v))&(t=(v>w))))>(t=(x>((y<u)>((p&y)=x)))))
>(z=(q&(r&s))))>(z=z) ;

TTTT TTTT TTTT TTTT(16) (1.1.2)

Remark 1.1.2: Eq. 1.1.1 is trivial with this result to be expected.

1.20 $A = \emptyset \Rightarrow A \cap_{\Phi} B = \{x \in B : \varphi(x) = \varphi(\emptyset)\}$. (1.2.1)

(((((r<u)&(s<u))>((p=(w>v))&(t=(v>w))))>(t=(x>((y<u)>((p&y)=x)))))
>(z=(q&(r&s))))>(((r=(z@z))>z)=((x<s)>((p&x)=(p&(z@z))))) ;

T**F**T**F** T**F**T**F** TTTT TTTT(8), TTTT TTTT TTTT TTTT(8) (1.2.2)

1.30 $A = B \Rightarrow A \cap_{\Phi} B = A$. (1.3.1)

(((((r<u)&(s<u))>((p=(w>v))&(t=(v>w))))>(t=(x>((y<u)>((p&y)=x)))))
>(z=(q&(r&s))))>((r=s)>(z=r)) ;

T**F**T**F** TTTT TTTT TTTT(4), **F**T**F**T TTTT TTTT TTTT(4),
FFFF TTTT TTTT TTTT(4), TTTT TTTT TTTT TTTT(4) (1.3.2)

1.40 $A \cap B \Rightarrow A \cap_{\Phi} B$. (1.4.1)

(((((r<u)&(s<u))>((p=(w>v))&(t=(v>w))))>(t=(x>((y<u)>((p&y)=x)))))
>(z=(q&(r&s))))>((r&s)>z) ; TTTT TTTT TTTT TTTT(16) (1.4.2)

Remark 1.4.2: Eq. 1.4.1 is trivial with this result to be expected.

1.50 $A \cap_{\Phi} B \not\Rightarrow A \cap B$. (1.5.1)

(((((r<u)&(s<u))>((p=(w>v))&(t=(v>w))))>(t=(x>((y<u)>((p&y)=x)))))
>(z=(q&(r&s))))>~(z>(r&s)) ;

TTTT TTTT TTTT TT**FF**(10), TTTT TTTT TTTT **FFFF**(4),
TTTT TTTT TTTT T**FFF**(1), TTTT TTTT TTTT **F**T**FF**(1) (1.5.2)

1.60 $(A \cap_{\Phi} B = A \cap B) \Leftrightarrow \varphi$ is an injective function. (1.6.1)

(((((r<u)&(s<u))>((p=(w>v))&(t=(v>w))))>(t=(x>((y<u)>((p&y)=x)))))
>(z=(q&(r&s))))>((z=(r&s))=p) ;

T**F**T**F** T**F**T**F** T**F**T**F** TT**F**T(4), TTTT TTTT TTTT TT**F**T(8),
T**F**T**F** T**F**T**F** T**F**T**F** **F**T**F**T(4) (1.6.2)

1.70 $A \cap_{\Phi} B \subseteq A \cup B$. (1.7.1)

(((((r<u)&(s<u))>((p=(w>v))&(t=(v>w))))>(t=(x>((y<u)>((p&y)=x)))))
>(z=(q&(r&s))))>~((r+s)<z) ;
TTTT **FFFF FFFF FF**TT(43), TTTT **FFFF FFFF FFFF**(16),
TTTT **FFFF FFFF FF**T**F**(4), TTTT **FFFF FFFF FFF**T(1),
TTTT TTTT TTTT TTTT(64) (1.7.2)

A set intersection operator was proposed for descriptively near sets and named descriptive union. In the Theorem 1 proof seven properties are listed: two are trivial tautologies; and five as rendered are not tautologous. The refutes descriptive intersection operators and near sets on which it is based.

Dialetheism

Arenhart, J.R.B.; Melo, E.S. (2017). Classical negation strikes back: Why Priest's attack on classical negation can't succeed. *Logica Universalis*. October.

From: researchgate.net/publication/320506867

We assume the Meth8 apparatus, implementing the variant system VŁ4.

LET: p a; q I; ~ ⌐,¬ ; # any, universal quantifier;
> Imply; = Equivalent to, equal; @ Not equivalent to, not equal;
(p@p) contradictory; (p=p) tautologous;

Result fragments are repeating rows in the truth table, with T as the designated proof value.

While we choose the second negation symbol of ⌐, an upside down ¬, both such negation symbols are evaluated the same in VŁ4 as the tilde ~ not symbol.

We map the four equations on page 7 in Section 3, which are the crux of Graham Priest's thesis:

... selecting two different signs, one ¬ for De Morgan negation, and another, ⌐, for Boolean negation ... the model theoretic truth conditions for these negations in an interpretation are as follows: given any interpretation *I*,

De Morgan:

¬*a* is tautologous in *I* iff *a* is contradictory in *I*. (1.1.1)

(#q>(p= (p@p)))>(#q>(~p=(p=p)));
TTTT (1.1.2)

¬*a* is contradictory in *I* iff *a* is tautologous in *I*. (1.2.1)

(#q>(p=(p=p))) >(#q>(~p=(p@p)));
TTTT (1.2.2)

Boole:

⌐*a* is tautologous in *I* iff *a* is **not** tautologous in *I*. (2.1.1)

(#q>(p=~(p=p)))>(#q>(~p=(p=p)));
TTTT (2.1.2)

⌐*a* is contradictory in *I* iff *a* is tautologous in *I*. (2.2.1)

(#q>(p=(p=p)))>(#q>(~p=(p@p)));
TTTT (2.2.2)

Meth8 evaluates the renditions of Eqs. as tautologous. Hence there is no difference in negation between

Boole and De Morgan. Due to this experiment, we conclude that dialetheism is suspicious.

Where dialetheism is mistaken

https://plato.stanford.edu/entries/dialetheism/, Section "2. Dialetheism in the history of philosophy".

> "This technique is called *parameterisation* and is adopted quite generally: when one is confronted with a seemingly tautologous contradiction, *A* & ¬*A*, it is a common strategy to treat the suspected dialetheia *A*, or some of its parts, as having different meanings, and hence as ambiguous (maybe just *contextually* ambiguous). For instance, if one claims that *P*(*a*) & ¬*P*(*a*), parameterisation holds that one is in effect claiming that *a* is *P* and is not *P* under different parameters or in different respects — say, *r1* and *r2*. To the extent that one's claim shows no sign of such parameters, it is tempting to ascribe inconsistency to the claim. But this can be resolved by clarifying that $P_{r1}(a)$ & $\neg P_{r2}(a)$ "
> Copyright © 2013 by Graham Priest. (Francesco Berto <*F.Berto@uva.nl*>)

We assume the Meth8-VŁ4 apparatus, including equivalency of modal and quantified operators, and map the expressions.

LET: p P( ); q a; r r-sub-1; s r-sub-2; # necessity mode, universal quantity

Result fragments are the entire 16-value truth table in rows major horizontally.
The designated proof value is T tautology, and **F** contradiction;
also C contingent (falsity), and N non-contingent (truth) .

if *P*(*a*) & ¬*P*(*a*), then *a* is *P* and is not *P* (2.1.1)

((p&q)&~(p&q))>((p=q)&(p=~q)) ; TTTT TTTT TTTT TTTT (2.1.2)

We test Eq. 2.1.1 for if the equivalency of the antecedent and consequent clauses.

P(*a*) & ¬*P*(*a*) is equivalent to *a* is *P* and is not *P* (2.1.1.1)

((p&q)&~(p&q))=((p=q)&(p=~q)) ; TTTT TTTT TTTT TTTT (2.1.1.2)

Next we evaluate the fix.

this can be resolved by clarifying that $P_{r1}(a)$ & $\neg P_{r2}(a)$ (2.2.1)

(((p&r)&q)&((p&s)&q))>((q=(p&r))&(q=(p=s)));
TTTT TTTT TTTT TTTT (2.2.1)

We test Eq. 2.2.1 for the equivalency of the antecedent and consequent clauses.

$P_{r1}(a)$ & $\neg P_{r2}(a)$ is equivalent to *a* is *P* and is not *P* (2.2.1.1)

(((p&r)&q)&((p&s)&q))=((q=(p&r))&(q=(p=s))) ;
FFTT **F**TTT **F**TTT **F**TTT (2.2.1.2)

We reintroduce the universal quantifier "for any *A*, it is necessary" as a prophylactic test:

For any *A*, $P_{r1}(a)$ & $\neg P_{r2}(a)$ is equivalent to *a* is *P* and is not *P* (2.2.2.1)

(((#p&r)&q)&((#p&s)&q))=((q=(#p&r))&(#q=(p=s))) ;
FFTT **F**NTT **F**TTT **F**TTT (2.2.2.2)

Eq. 2.2.2.2 results in marginally greater truth value than Eq. 2.2.1.2.

Eq. 2.1.1 is an equivalency, but the description for dialetheism in Eq. 2.2.1 is an inference, not an equivalency. This tells us that *parameterisation* introduces an inconsistency to demonstrate dialetheism. What follows is that dialetheism is suspicious.

In passing, as found later we evaluated the contra assertion against the objection of the argument from explosion, Section 4.1, with edited labels:

> "Aristotelian syllogistic — the first formally articulated logic in Western philosophy — is not explosive. Aristotle held that some syllogisms with inconsistent premises are valid, whereas others are not (An. Pr. 64a 15). Just consider the inference:
>
> (P1.1) Some logicians are intuitionists; [*]
> (P2.1) No intuitionist is a logician;
> (C3.1) Therefore, all logicians are logicians. [*]
> [(R4.1) (P1.1) & (P2.1) > (C3.1).; we also use the = connective to test theoremhood.]
>
> This is not a valid syllogism, despite the fact that its premises are inconsistent."

We evaluate as follows:

LET: p logicians; q intuitionists; % possibility, existential; # necessity, universal

(P1.2) %(p=q) ;	TCCT TCCT TCCT TCCT
(P2.2) (~q=p) ;	**F**TT**F** **F**TT**F** **F**TT**F** **F**TT**F**
(C3.2) #(p=p) ;	NNNN NNNN NNNN NNNN
[(R4.2)(%(p=q)&(~q=p))>#(p=p) ;	TTNT TTNT TTNT TTNT
(R4.3) (%(p=q)&(~q=p))=#(p=p) ;	CC**F**C CC**F**C CC**F**C CC**F**C

Eq. R4.1 as rendered in R4.2 is a valid syllogism, and the premises are consistent, but the result is not tautologous. This means Eq. R4.1 cannot be discredited as an argument from explosion against dialetheism. (What follows is the Aristotelian logic *is* in fact explosive according to system VŁ4.)

* If P1.1 is rendered as "some (possibly one) logician is an intuitionist" (%p=q), instead of "some (the possibility of) logicians are intuitionists" %(p=q), then R4.1 can be coerced to tautology. We believe Aristotle intended P1.2, as in our text, because of the plural of "intuitionists".

Using this same reason of plural words for C3.1, we interrupt the universal operator as outside the equation of logicians are logicians, #(p=p).

The dichotomy of selection argument as a contradiction

Manuel Morales proposed a philosophy of science based on a system of logic named the dichotomy of selection argument for two variables of coin, cup as p,q here. There are two possible states of affairs: for the coin in the cup; and for the coin in the cup or outside the cup. These states are not "hidden variables" and are named "one potential" and "more than one potential", mapped as:

(p&q) (1)
(p&(q+~q)) (2)

Eq 1 means "coin and cup".
Eq 2 means "coin, and cup or no cup".

The argument requires that two conditions are applied to Eq 1-2 as "direct selection" and "indirect selection". We take these conditions to be the logical connective of Imply as "p>", to mean "If coin, then ... " or "Coin implies ... ." The conditions are not a "hidden variable".

p>(p&q) (3)
p>(p&(q+~q)) (4)

Eq 3 means "If coin, then coin and cup are selected."
In other words, "Coin implies direct selection of coin and of cup."

Eq 4 means "If coin, then both coin and cup or no cup are selected."
In other words, "Coin implies indirect selection of both coin and of cup or of no cup."

The argument also requires that an action be taken (executed) or not be taken (not executed), where: if not taken or not executed, then "no physical effects exist." The action is not a "hidden variable". The action taken is mapped above in Eq 3-4, and the action not taken is mapped below in Eq 5-6:

(~p>~(p&q)) (5)
(~p>~(p&(q+~q))) (6)

Eq 5 means "If no coin, then no physical effect on coin and on cup."
In other words, "No coin implies no action on coin and on cup."
Eq 5 does *not* mean the negation of Eq 3 as "Coin does not imply direct selection ... .")

Eq 6 means "If no coin, then no physical effect on coin, and on cup or on no cup."
In other words, "No coin implies no action on both coin and on cup or on no cup."
Eq 6 does *not* mean the negation of Eq 4 as "Coin does not imply indirect selection ... .")

We test if both Eq 3-4 are equivalent to the negation of the opposite as both Eq 5-6:

((p>(p&q))&(p>(p&(q+~q)))) = ~((~p>~(p&q))&(~p>~(p&(q+~q)))) (7)

Eq 7 is not validated as tautologous. (The main connective as imply > also is not validated as tautologous). Therefore the dichotomy of selection argument is a contradiction and hence not a viable philosophy of science.

Refutation of differential reasoning

Abstract: We refute Hempel's raven paradox elsewhere, hence refuting differential reasoning which forms a *non* tautologous fragment of the universal logic VL4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
\> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: van Krieken, E.; Acar, E.; van Harmelen, F. (2019). Semi-supervised learning using differentiable reasoning. arxiv.org/pdf/1908.04700.pdf

Abstract We introduce Differentiable Reasoning (DR), a novel semi-supervised learning technique which uses relational background knowledge to benefit from unlabeled data. We apply it to the Semantic Image Interpretation (SII) task and show that background knowledge provides significant improvement. We find that there is a strong but interesting imbalance between the contributions of updates from Modus Ponens (MP) and its logical equivalent Modus Tollens (MT) to the learning process, suggesting that our approach is very sensitive to a phenomenon called the Raven Paradox.. . We propose a solution to overcome this situation.

1 Introduction Semi-supervised learning is a common class of methods for machine learning tasks where we consider not just labeled data, but also make use of unlabeled data.. . This can be very beneficial for training in tasks where labeled data is much harder to acquire than unlabeled data. This can be very beneficial for training in tasks where labeled data is much harder to acquire than unlabeled data. ... In the experimental analysis, we find that the gradient updates using the Modus Ponens (MP) and Modus Tollens (MT) rules are disproportionate. That is, MT often strongly dominates MP in the learning process. Such behavior suggests that our approach is highly sensitive to the Raven Paradox.. . It refers to the phenomenon that the observations obtained from "All ravens are black" are dominated by its logically equivalent "All non-black things are non-ravens".

We refute Hempel's raven paradox elsewhere, hence refuting differential reasoning. (See for example: vixra.org/pdf/1908.0274v1.pdf; and "Logical induction is not tautologous via the Black raven paradox and Kripkenstein", ersatz-systems.com/RA.Meth8.refut.valid.abstract.pdf .)

Refutation of the variety of distributive bilattices

Abstract: The example stated of a distributive bilattice is not tautologous. This refutes that variety and forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Moraschini, T. (2019). A study of truth predicates in matrix semantics.
arxiv.org/pdf/1908.01661.pdf moraschini@cs.cas.cz

4. Semilattice-based examples
In this section we review a family of natural examples of logics whose truth sets are almost parametrically equationally, but not equationally, definable. In the light of Corollary 3.10 we know that all these examples need to be purely inferential.

Example 4.4 (Distributive Bilattices).
An algebra A = ⟨A, , , , , ¬ ∧ ∨ ⊗ ⊕ ⟩ is a bilattice if ⟨A, , , , ∧ ∨ ⊗ ⊕⟩ is a pre-bilattice such that ¬¬a = a and [for every a, b ∈ A ..]

a≤b⇒(¬b≤¬a and ¬a⊑¬b) (4.4.1.1)

LET p, q: a, b.

~(q<p)>(~(~p<~q)&~(~q<~p)) ;
T**F**TT T**F**TT T**F**TT T**F**TT (4.4.1.2)

Eq. 4.4.1.2 as rendered is *not* tautologous, refuting distributive bilattices.

Refutation of disturbance as a feature

Abstract: We evaluate four binary equations as *not* tautologous. The refutes the formal description of verifiability for disturbance as a feature.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⇔, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) C non-contingency, ∇, ordinal 2; (%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).

From: Hansen, A.; Wolf, S. (2019). Disturbance: It's a feature, not a bug.
arxiv.org/pdf/1902.02088.pdf arne.hansen@usi.ch

III. Verifiability
C. Formal description
a. Binary questions

In the binary case, questions in Q can be regarded as statements that are either true or false. A statement can imply another,

(Q1,t)⇒(Q2,t) (0.1)

LET p, q, r, s: Q'2, Q1, Q2, t

(q&s)>(r&s) ; TTTT TTTT TT**FF** TTTT (0.2)

where "⇒" is the notion of implication in ordinary language.

Note that the ~-equivalence of questions is not equal to the bi-directional implication

(Q1,t)⇒(Q2,t)∧(Q1,t)⇐(Q2,t) (1.1.1)

Remark 1.1.1: The note above is not relevant to the argument as immediately following below. In fact, Eq. 1.1.1 is *not* tautologous for any order of operation as specified by nested parentheses.

If (Q1,t)⇒(Q2,t) and an inquiry yields (Q1,t), then it follows that (Q2,t). (2.1)

We write this as, If (((Q1,t)⇒(Q2,t)) implies (Q1,t)), then (Q2,t).
(((q&s)>(r&s))>(q&s))>(r&s) ; TTTT TTTT TT**FF** TTTT (2.2)

Remark 2.2: Eq. 2.2 produces the same truth table result as 0.2, both *not* tautologous.

If, then, one attempts to confirm (Q2,t), one inquires about a ≡-equivalent question Q′2≡Q2. (3.1)

We write this as, If (Q′2≡Q), then ((((Q1,t)⇒(Q2,t)) implies (Q1,t)), then (Q2,t)).

(p=s)>((((q&p)>(r&p))>(q&p))>(r&p)) ;
`TTTT TTTT TTT`**`F`**` TTTT` (3.2)

Remark 3.2: Eq. 3.1 apparently describes a test by induction. The result in 3.2 is to bring the truth table closer to a tautology with one value for **`F`** contradiction instead of two values for **`F`** in 2.2.

Eqs. 0.2-3.2 as rendered are *not* tautologous. The refutes the formal description of verifiability for disturbance as a feature.

Diverse double-compiling (DDC)

We evaluate a security scheme using the Meth8 modal model checker implementing variant system VŁ4, a resuscitation of the Łukasiewicz quaternary logic based on the 2-tuple {11,10,01,00}, in five models..

From Wheeler, David. "Fully countering trusting trust through diverse double-compiling". 2009. arxiv.org/ftp/arxiv/papers/1004/1004.5534.pdf, on pg 47, 5.1.2 DDC components:

LET: nvt not tautologous; and

	cT:	p
	sP:	q
	sA:	r
(1)	e1:	((p&q)>s)
(2)	e2:	((s&r)>t)
(3)	eA_run:	u
(4)	1sP:	v & q [not used in Eq 9]
(5)	1sA:	v & r [not used in Eq 9]
(6)	e1_effects:	e1 > w; (s>w)
(7)	e2_effects:	e2 > x; (t>x)
(8)	stage_1:	(q & p & e1_effects & e2 & eA_run) > y; ((q&(p&(w&(t&u)))))>y)
(9)	stage_2:	(r & stage_1 & e2_effects & e2 & eA_run) [> z]; (r&(y&(x&(t&u))))

The conjecture to test in words is: If Eqs 1,2,3,6,7,8, then Eq 9. By substitution:

(((p&q)>s)&(((s&r)>t)&((s>w)&((t>x)&((q&(p&(w&(t&u))))>y)))))>(r&(y&(x&(t&u))));nvt; (10)

A fragment in Model 1 is below of the 7 repeating truth tables (of 128). The designated truth value is T tautology with other 2-tuple values as C contingent (falsity value), N non contingent (truth value), and F contradiction.

```
TTTT TTTT TTTT TTTT
FFFT FFFT TTTT TTTT
FFFT TTTT TTTT TTTT
FFFT FFFT FFFF TTTT
FFFT FFFT FFFF FFFF
FFFT FFFT FFFF TTTT
FFFT TTTT FFFF TTTT
```

Eq 10 is nvt by Meth8. The reason this differs from the paper using Prover9 (P9) is that P9 implements standard FOL which is not bivalent, as we showed elsewhere. FOL is based on the modern, revised Square of Opposition without equations for all edges and from which two of the 24 usable syllogisms needed fix-ups (Modus Camestros and Modus Cesare).

If Eq 10 is modified to use the universal quantifier or modal necessity on the antecedent or consequent, then it is nvt and only on values T, C. Similarly for the existential quantifier or modal possibility, it is nvt on values T, F, N. (We proved elsewhere the equivalence of the respective quantifiers to modal operators.)

Refutation of domain theory

Abstract: We evaluate six equations for conjectures in five subsections of origins, bases of objects, axiomatic conditions, adjunctions, finite domains, and join-approximable relations. None is tautologous, hence refuting the domain theory of Dana Scott. Therefore, Scott's domain theory is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
\> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1;
(%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Abramsky, S.; Jung, A. (2017). Domain theory. cs.bham.ac.uk/~axj/pub/papers/handy1.pdf
karen.barnes@cs.ox.ac.uk A.Jung@cs.bham.ac.uk

1. Introduction and origins, 1.1 Origins, 1.1.2.2. Recursive types.

Domain Theory ... began in 1969, [attributed to] Dana Scott [in] the following key insight ...

2. Recursive types. Scott’s key construction was a solution to the “domain equation” ... thus giving the first mathematical model of the type-free λ-calculus.

$$D \simeq [D \rightarrow D] \qquad (1.1.2.1)$$

D=(D>D) ;

```
FFFF FFFF FFFF FFFF(4),
CCCC CCCC CCCC CCCC(4),
NNNN NNNN NNNN NNNN(4),
TTTT TTTT TTTT TTTT(4)
```
(1.1.2.2)

Remark 1.1.2.2: The "domain equation" is *not* tautologous, hence refuting domain theory at the outset. However, we press on with evaluations of six other equations as keyed to sections.

2 Domains individually, 2.2 Approximation, 2.2.6 Bases as objects

Definition 2.2.20. *An (*abstract*) basis is given by a set* B *together with a transitive relation* $\prec$ *on* B*, such that ... holds for all elements* x *and finite subsets* M *of* B.

(INT) $M \prec x \Rightarrow \exists y \in B.M \prec y \prec x$ (2.2.20.1)

LET p, q, r, s: M, X, y, B.

(p<q)>((%r<s)&(p<(r<q))) ; TCTT T**F**TT T**F**TT T**F**TT (2.2.20.2)

There is one further problem to overcome, namely, the fact that continuous functions do not preserve the order of approximation. The only way out is to switch from functions to relations, where we relate a basis element c to all basis elements approximating f(c). This can be axiomatized as follows.

Definition 2.2.27. *A relation* R *between abstract bases* B *and* C *is called* approximable *if the following conditions are satisfied: ...*

4. $\forall x \in B\ \forall y \in C.\ (xRy \Longrightarrow (\exists z \in B.\ x \succ zRy))$. (2.2.27.4.1)

LET p, q, r, s, x, y, z:
A, B, R, C, x, y, z

((#x<q)&(#y<r))&((#x&(r&#y))>((%z<q)&(x>(%z&(r&#y))))) ;
FFFF FFFF FFFF FFFF(48),
NN**FF FFFF** NN**FF FFFF**(16) (2.2.27.4.2)

3. Domains collectively, 3.1 Comparing domains, 3.1.3 Adjunctions,

Proposition 3.1.10. Let P *and* Q *be posets and* l: P → Q *and* u: Q → P *be monotone functions. Then the following are equivalent: ...*

4. $\forall x \in P\ \forall y \in Q.(x \sqsubseteq u(y) \Leftrightarrow l(x) \sqsubseteq y)$. (3.1.10.4.1)

LET p, q, r, s:
P, Q, x, y; l=(p>q) and u=(q>p).

((#r<p)&(#s<q))&(~(((q>p)&#s)<#r)=~(s<((p>q)&#r))) ;
FFFF FFFF FFFF N**FFF** (3.1.10.4.2)

4. Cartesian closed categories of domains, 4.2 Finite choice: compact domains, 4.2.1 Bifinite domains

Lemma 4.2.3. *If* D *is a bifinite domain and* E *is pointed and algebraic, then every joinable subset of* K(D) × K(E) *gives rise to a compact element of* [D → E]. *If* F *and* G *are joinable families then the corresponding functions are related if and only if*

$\forall (d, e) \in G\ \exists (d', e') \in F.\ d' \sqsubseteq d$ *and* $e \sqsubseteq e'$. (4.2.3.1)

LET p, q, r, s, t, u:
d, e,d',e',F, G.

```
((#(p&q)<u)&(%(r&s)<t))&(~(p<r)&~(s<q)) ;
                                        FFFF FFFF FFFF FFFN(1),
                                        FFFF FFFF FFFF FFFF(3)          (4.2.3.2)
```

7. Domains and logic, 7.2 Some equivalences, 7.2.5 Compact-open set and spectral spaces

Two additional axioms are needed, however, because frame-homomorphisms are more special than Scott-continuous functions.

Definition 7.2.24. *A relation* R *between lattices* V *and* W *is called* join-approximable *if the following conditions are satisfied:*

1. ∀x, x′ ∈ V ∀y, y′ ∈ W. (x′ ≥ x R y ≥ y′ ⇒ x′ R y′)*;* (7.2.24.1.1)

```
LET     r,  v, w, x, y, p, q
        R,V,W, x, y, x',y'.

((#(x&q)<v)&(#(y&q)<w))&((~q>(~((#x&(r&#y))>p)))>(#p&(r&#q))) ;
                                        FFFF FFFF FFFF FFFF( 4),
                                        FFFF FFFN FFFF FFFN(12)        (7.2.24.1.2)
```

The six Eqs. 1, 2.2.20, 2.2.27, 3, 4, 7 above are *not* tautologous, to deny those subsections and hence refute the domain theory of Dana Scott.

Refutation of domain theory and the Scott model of language PCF in univalent type theory

Abstract: The definitions of directed, complete posets for antisymmetry and transitivity are *not* tautologous, thereby refuting basic domain theory. By extension, the Scott model of language PCF in univalent type theory is also refuted and another *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: de Jong, T. (2019). The Scott model of PCF in univalent type theory.
arxiv.org/pdf/1904.09810.pdf t.dejong@pgr.bham.ac.uk

2 Basic domain theory

We introduce basic domain theory in the setting of constructive univalent mathematics.

2.1 Directed complete posets

Definition 2.1. A *poset* (X, ≤) is a set X together with a proposition valued binary relation ≤: X → X → Ω satisfying:

(i) *reflexivity*: Qx:X x ≤ x; (2.1.i.1)

LET p, q, r: x, y, z.

~(p<p) = (p=p) ; TTTT TTTT TTTT TTTT (2.1.i.2)

(ii) *antisymmetry*: Qx,y:X x ≤ y → y ≤ x → x = y; (2.1.ii.1)

(~(q<p)>~(p>q))>(p=q) ; T**FF**T T**FF**T T**FF**T T**FF**T (2.1.ii.2)

(iii) *transitivity*: Qx,y,z:X x ≤ y → y ≤ z → x ≤ z. (2.1.iii.1)

(~(q<p)>~(p>r))>~(r<p) ; TTTT TT**F**T TTTT TT**F**T (2.1.iii.2)

Eqs. 2.1.ii.2 and 2.1.iii.2 as rendered are *not* tautologous. The definitions of directed, complete posets for antisymmetry and transitivity are *not* tautologous, thereby refuting basic domain theory. By extension the Scott model of programming language PCF in univalent type theory is also refuted.

Doxastic logic

We assume the script of Meth8 where nvt not tautologous. The designated truth value in the proof tables of horizontal rows is T Tautologous. Other values are F contradictory, C Contingent, and N Non contingent.

From en.wikipedia.org/wiki/Doxastic_logic (a difficult read with gendered pronouns):

			`Model 1 of 5
Accurate reasoner:	#p>((r&p)>p) ;	vt ;	TTTT TTTT TTTT TTTT
Inaccurate reasoner:	(%p&~p)&(%p&(r&p)) ;	nvt ;	**FFFF FFFF FFFF FFFF**
Conceited reasoner.1:	r&(~%p&(~p&(r&p))) ;	nvt ;	**FFFF FFFF FFFF FFFF**
Conceited reasoner.2:	r&(#p&((r&p)>p)) ;	nvt ;	**FFFF FNFN FFFF FNFN**
Consistent reasoner.1:	(~%p&(r&p)) & (~%p&(r&~p)) ;	nvt ;	**FFFF FFFF FFFF FFFF**
Consistent reasoner.2:	(#p&(r&p)) > (#p&(~r&~p)) ;	nvt ;	**TTTT TCTC TTTT TCTC**
Normal reasoner;	(#p&(r&p)) > (#p&(r&(r&p))) ;	vt;	
Peculiar reasoner:	(%p&(r&p)) & (%p&(r&~(r&p))) ;	nvt ;	**FFFF FFFF FFFF FFFF**
Regular reasoner:	(#(p&q)&(r&(p>q))) > (#(p&q)&(r&((r&p)>(r&q)))) ;	vt	
Reflexive reasoner;	((#p&(%q&r))&q) = ((#p&(%q&r))&((r&q)>p));	vt	
Unstable reasoner:	(%p&(r&(r&p))) & (%p&~(r&p)) ;	nvt ;	**FFFF FFFF FFFF FFFF**
Stable reasoner:	(#p&(r&(r&p))) > (#p&(r&p)) ;	vt	
Modest reasoner:	(#p&(r&((r&p)>p))) > (#p&(r&p)) ;	vt	
Queer reasoner:	Not explicitly stated, so not evaluated here.		Type G
Timid reasoner:	Not explicitly stated, so not evaluated here.		
Type 1.1 reasoner:	(#(p&q)&((r&p)&(r&(p>q)))) > (#(p&q)&(r&p)) ;	vt	
Type 1.2 reasoner:	(#(p&q)&(r&(p>q))) > (#(p&q)&((r&p)>(r&q))) ;	vt	
Type 1* reasoner:	(#(p&q)&(r&(p>q))) > (#(p&q)&(r&((r&p)>(r&q)))) ;	vt	
Type 2 reasoner:	((#(p&q)&r)&((r&p)&(r&(p>q)))) > ((#(p&q)&r)&(r&q)) ;	vt	
Type 3 reasoner:	(#p&(r&p)) > (#p&(r&(r&p))) ;	vt	
Type 4 reasoner:	((r&#p)&(r&p)) > ((r&#p)&(r&(r&p))) ;	vt	
Type G reasoner:	((r&#p)&(r&((r&p)>p))) > ((r&#p)&(r&p)) ;	vt	

Doxastic logic relies on the Löb theorem, also known as the

Gödel-Löb theorem:	#(#p>p)>#p ;	nvt ;	**CTCT CTCT CTCT CTCT**

We observe doxastic logic contains axioms not tautologous such as the Reasoners named: Inaccurate; Conceited; Consistent; Peculiar; and Unstable. Five types of Reasoners however are tautologous.

We conclude that doxastic logic as a whole is a logic system not tautologous by Meth8.

Refutation of drinker's paradox

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table fragment) is row-major and horizontal.

LET p, q, r, s: x, y, P, D; & And; > Imply; < Not Imply, less than, ∈ ;
% possibility, for one or some, ∃; # necessity, for all, ∀.

From: en.wikipedia.org/wiki/Drinker_paradoxy, of which please see because we do not reproduce it.

"There is someone in the pub such that, if he is drinking, then everyone in the pub is drinking." (1.0)

$$\exists x \in P . [D (x) \rightarrow \forall y \in P . D (y)] \quad (1.1)$$

((%p<r)&(s&p))>((%p<r)&((#q<r)&(s&q))) ; quantifier distributed ;
`TTTT TTTT TFTN TTTT` (1.2)

Eq. 1.2 as rendered is *not* tautologous and also *not* contradictory. Therefore, the drinker's paradox is refuted as a paradox.

Corrected weak duality theorem by way of refutation of the strong duality theorem

Abstract: The equation of the weak duality theorem, (Ax≤b, x≥0) ≤ (A^Ty≥c, y≥0), is confirmed as tautologous. Three proofs of it in the literature are *not* tautologous. The equation of the strong duality theorem, (Ax≤b, x≥0) = (A^Ty≥c, y≥0), is refuted as *no*t tautologous. These form a *non* tautologous fragment of the universal logic VŁ4. What follows is the weak duality theorem could just as easily exclude the "or equal to" relation to read (Ax≤b, x≥0) < (A^Ty≥c, y≥0) as the corrected weak duality theorem.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: math.ubc.ca/~anstee/math340/340weakduality.pdf; also,
www.coursera.org/lecture/approximation-algorithms-part-2/proof-of-weak-duality-theorem-eAkFN

Theorem (Weak Duality) Let x∗ be a feasible solution to the primal and let y∗ be a feasible solution to the dual where

primal max c·x: (Ax ≤ b, x ≥ 0)
dual min b·y: (A^Ty ≥ c, y ≥ 0).
Then c·x∗ ≤ b·y∗ . (0.1.1)

LET p, q, r, s, t, x, y: A, b, c, s, ^{T}, x, y

~(((r>((p&t)&y))&~((s@s)>y))<((q<(p&x))&((s@s)>x)))=(s=s) ;
TTTT TTTT TTTT TTTT (0.1.2)

Proof: … We obtain

$c \cdot x = x^T c \le x^T A^T y = y^T A x \le y^T b = b \cdot y$ (0.2.1)

((r&x)=~((((x&t)&p)&(t&y))<((x&t)&r))) =(~(((y&t)&q)<((y&t)&(p&x)))=(q&y)) ;
TTTT TTTT TTTT TTTT(16)
TTTT **FFFF** TTTT **FFFF**(16)
TT**FF** TT**FF** TT**FF** TT**FF**(1)}x8
TTTT TTTT TTTT TTTT(1)}
T**F**TT **FFF**T T**F**TT **FF**FT(1)}x8 (0.2.2)

We read off

$c \cdot x \leq b \cdot y.$ (0.3.1)

~((q&y)<(r&x))=(s=s) ; TTTT TTTT TTTT TTTT(32)
TT**FF** TT**FF** TT**FF** TT**FF**(16)
TT**FF** TTTT TT**F**F TTTT(16) (0.3.2)

The case of equality is of course of great interest and strong duality and complementary slackness deal with equality. Nonetheless, weak duality is of independent interest and is a model for other optimization problems for which we have no strong duality.

From: en.wikipedia.org/wiki/Weak_duality

In applied mathematics, weak duality is a concept in optimization which states that the duality gap is always greater than or equal to 0. That means the solution to the primal (minimization) problem is always greater than or equal to the solution to an associated dual problem. This is opposed to strong duality which only holds in certain cases.

The primal problem: Maximize $c^T x$ subject to $Ax \leq b$, $x \geq 0$; (1.1)
The dual problem: Minimize $b^T y$ subject to $A^T y \geq c$, $y \geq 0$. (2.1)
The weak duality theorem: $c^T x \leq b^T y$. (3.1)

Remark 3.1: We write the weak duality theorem Eq. 3.1 as 1.1 ≤ 2.1:

$(Ax \leq b,\ x \geq 0) \leq (A^T y \geq c,\ y \geq 0)$ (4.1)

LET p, q, r, s, t, x, y: A , b , c , s, ^{T},x, y.

~(((r>((p&t)&y))&~((s@s)>y))<((q<(p&x))&((s@s)>x)))=(s=s) ;
TTTT TTTT TTTT TTTT (4.2)

Proof: $c^T x = x^T c \leq x^T A^T y \leq b^T y$ (5.1)

((r&t)&x)=~(~(((q&t)&y)<((x&t)&((p&t)&y)))<((x&t)&r)) ;
TTTT TTTT TTTT TTTT(32)
TTTT TTTT TTTT TTTT(1)}x8
TT**FF** TT**FF** TT**FF** TT**FF**(1)}
TTTT TTTT TTTT TTTT(1)}x8
TT**F**T TTTT TT**F**T TTTT(1)} (5.2)

Eqs. 1.2 and 4.2 as rendered are tautologous and are equivalents, hence confirming the weak duality theorem. However, Eqs. 2.2, 3.2, and 5.2 are not tautologous and not equivalents, hence refuting three proofs of the theorem in the literature.

We turn to strong duality.

From: www.coursera.org/lecture/approximation-algorithms-part-2/proof-of-weak-duality-theorem-eAkFN

Strong duality theorem in general (10.1)

(P) primal max c·x: (Ax ≤ b, x ≥ 0)
(D) dual min b·y: ($A^T y \geq c$, y ≥ 0).
[empty value is 0]

Four possible cases:

(P) is empty, (D) has value plus infinity (11.1)

(s@s)&((q<(p&x))&((s@s)>x)) ;
FFFF FFFF FFFF FFFF (11.2)

(D) is empty, (P) has value minus infinity (12.1)

((r>((p&t)&y))&~((s@s)>y))=(s=s) ;
FFFF FFFF FFFF FFFF (12.2)

value(P)=value(D) (13.1)

((q<(p&x))&((s@s)>x))=((r>((p&t)&y))&~((s@s)>y)) ;
TT**FF** TT**FF** TT**FF** TT**FF**(16)
TT**F**T TT**F**T TT**F**T TT**F**T(16) (13.2)

[(P) and (D) empty] (14.1)

(s@s)&(s@s) ;
FFFF FFFF FFFF FFFF (14.2)

Remark 10.0: We write strong duality as the four possible states of Eqs. 11.1 or 12.1 or 13.1 or 14.1 (15.1)

((((s@s)&((q<(p&x))&((s@s)>x)))+((s@s)&((r>((p&t)&y))&~((s@s)>y))))+
((((q<(p&x))&((s@s)>x))=((r>((p&t)&y))&~((s@s)>y)))+((s@s)&(s@s))))=(s=s) ;
TT**FF** TT**FF** TT**FF** TT**FF**(16)
TT**F**T TT**F**T TT**F**T TT**F**T(16) (15.2)

Eq. 15.2 is equivalent to 13.2, as expected, and *not* tautologous, hence refuting the strong duality theorem.

What follows is that the weak duality theorem of Eq. 4.2 as (Ax≤b, x≥0) ≤ ($A^T y \geq c$, y≥0) could just as easily exclude the "or equal to" relation to read (Ax≤b, x≥0) < ($A^T y \geq c$, y≥0) as the corrected weak duality theorem.

Refutation of dyatic semantics on paraconsistent logic C_1

Abstract: From the example of applying dyatic semantics to paraconsistent logic C_1, four axioms are *not* tautologous, hence refuting that approach. Therefore paraconsistent logics are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Caleiro, C.; Carnielli, W.; Coniglio, M.E.; Marcos, J. (2003).
Suszko's Thesis and dyadic semantics.
sqig.math.tecnico.ulisboa.pt/pub/CaleiroC/03-CCCM-dyadic1.pdf

Example 6.6 Consider the paraconsistent logic *C*1 ... . It is well-known for long that this logic has no genuinely finite-valued characterizing semantics, though it *can* be decided by quasi matrices ... . In fact, a dyadic semantics for *C*1 is prompt[l]y available ... . Just recall that α° abbreviates $\neg(\alpha \wedge \neg\alpha)$ in *C*1, and consider the following bivaluation axioms:

(6.6.1.1) $b(\neg\alpha) \geq -b(\alpha)$;

LET p, q, r, s: α, b, β , s; $0 - b = -b$.

~((((s@s)-q)&p)>(q&~p)) = (s=s) ; **F**T**FF** **F**T**FF** **F**T**FF** **F**T**FF** (6.6.1.2)

(6.6.5.1) $b(\alpha \Rightarrow \beta\,) = -b(\alpha) \sqcup b(\beta)$;

(q&(p>r))=((((s@s)-q)&p)+(q&r)) ; T**FF**T T**F**TT T**FF**T T**F**TT (6.6.5.2)

(6.6.6.1) $b(\alpha^\circ) = -b(\alpha) \sqcup -b(\neg\alpha)$;

(q&~(p&~p))=((((s@s)-q)&p)+(((s@s)-q)&~p)) ;
FFFF FFFF FFFF FFFF (6.6.6.2)

(6.6.7.1) $b((\alpha \otimes \beta)^{\circ}) \geq (-b(\alpha) \sqcup -b(\neg\alpha)) \sqcap (-b(\beta) \sqcup -b(\neg\beta))$, for $\otimes \in \{\wedge, \vee, \Rightarrow\}$.

~((((((s@s)-q)&p)+(((s@s)-q)&~p)) & ((((s@s)-q)&r)+(((s@s)-q)&~r))) >
(q&~((p*q)&~(p*q)))) = (s=s) ;
[Recall p*r abbreviates ~((p*r)&~(p*r)) where * is &,+,>]
TT**FF** TT**FF** TT**FF** TT**FF** (6.6.7.2)

The example axioms 6.6.1 and 6.6.5-6.6.7 as rendered are *not* tautologous. This refutes the conjecture that dyatic semantics apply to paraconsistent logic C_1. By extension, that approach is denied for paraconsistent logics.

Anomaly in the equation of E=mc^2

We assume the Meth8/VŁ4 apparatus with `T` as designated *proof* value and tables row-major, horizontal:

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	p=p	T	Tautology	proof	11	3
2	p@p	F	Contradiction	absurdum	00	0
3	%p>#p	N	Non-contingency	truth	01	1
4	%p<#p	C	Contingency	falsity	10	2

LET: pqrs Emcs where c is a constant r equivalent to s the speed of light, and E=mc^2.

The equation for mass-energy equivalence is E=mc^2. (1.0)

If necessarily c is equivalent s, and c is not greater than or less than s, then:

(#(r=s)&~((r<s)+(r>s)))> #(p=(q&(r&s))) ; `TTTT TTTT TTTT TTTT` (1.1)

If possibly c is not equivalent to s, and c is less than or greater than s, then:

(%(r@s)& ((r<s)+(r>s)))> #(p=(q&(r&s))) ; `TTTT TCTC TCTC TTTT` (1.2)

If possibly c is not equivalent to s, and possibly c is less than or greater than s, then:

(%(r@s)&%((r<s)+(r>s)))> #(p=(q&(r&s))) ;`NNNN NFNF NFNF NNNN` (1.3)

If possibly c is equivalent to s, or possibly c is less than or greater than s, then:

(%(r=s)+%((r<s)+(r>s)))> #(p=(q&(r&s))) ; `NFNF NFNF NFNF NFFN` (1.4)

Eqs. 1.2-1.4 show the assumption for the logic of Eq. 1.0 to hold as Eq. 1.1 is that the speed of light is constant. Stephen J. Crothers questioned and showed this is not the case, that the speed of light varies. Hence Eqs. 1.2-1.4 serve as counter examples to Eq. 1.1, making E=mc^2 *not* tautologous after all.

Refutation of the EF-axiom

The EF-axiom describes the Efremovifč proximity δ by V.A. Efremovič from 1934 and published in Russian in 1951.

From: en.wikipedia.org/wiki/Near_sets#Visualization_of_EF-axiom

> "Let the set X be represented by the points inside [a] rectangular region ... . Also, let A, B be any two non-intersection subsets (i.e. subsets spatially far from each other) in X ... . Let $C^c = X \backslash C$ (complement of the set C). Then from the EF-axiom ... :
>
> $A \underline{\delta} B, B \subset C, D = C^c, X = D \cup C, A \subset D$, hence, we can write
> $A \underline{\delta} B \Rightarrow A \underline{\delta} C$ and $B \underline{\delta} D$, for some C, D in X so that $C \cup D = X$." (1.1.1)

We interpret the operator $\underline{\delta}$ to mean "nearby" or "in proximity", but could just as easily mean "distant" or "far apart". The size of an antecedent or consequent is not stated for the operator, so we determine that the operator applies to unrelated literals. Therefore, we evaluate $A \underline{\delta} B$ as ((A∈ B) Nor (B∈ A)).

We assume the apparatus and method of Meth8/VŁ4 with the designated *proof* value of `T` for tautology, `F` contradiction, `C` falsity, and `N` truth. The proof result is for 16-tables of 16-values as row-major and horizontally. There are 256-values because four theorems are evaluated as the capitalized variables.

~ Not; + Or; - Not Or; & And; \ Not And; = Equivalent to; @ Not Equivalent to;
> Imply, greater than; < Not Imply, less than, ∈;
necessity, for all; % possibility, for one or some.

LET: A B C D A B C D; A $\underline{\delta}$ B = ((A<B)-(B<A)); D = ((D+C)\C); X = D+C.

(((((A<B)-(B<A))&(((B<C)&(D=((D+C)\C))&((D+C)&(A<D))))>
((%C<(D+C))&(%D<(D+C)))) > ((C+D)=(D+C))) >
(((A<B)-(B<A))>(((A<C)-(C<A))&((B<D)-(D<B)))) ; (1.2.1)

```
TTTT TNTN TTCC TNCF . NTNT TNTN NTFC TNCF . CCTT CFTN TTCC TNCF . FCNT CFTN NTFC TNCF
NTNT TNTN NTFC TNCF . NTNT TTTT NTFC TTCC . FCNT CFTN NTFC TNCF . FCNT CCTT NTFC TTCC
CCTT CFTN TTCC TNCF . FCNT CFTN NTFC TNCF . CCTT CFTN TTTT TNTN . FCNT CFTN NTNT TNTN
FCNT CFTN NTFC TNCF . FCNT CCTT NTFC TTCC . FCNT CFTN NTNT TNTN . FCNT CCTT NTNT TTTT
```

Eq. 1.2.1 as rendered is *not* tautologous.

We conclude the EF-axiom is suspicious as the theoretical basis for proximity space and for topology in fuzzy, near, and rough sets.

Ehrenfeucht–Mostowski theorem of discernables

From; en.wikipedia.org/wiki/Ehrenfeucht%E2%80%93Mostowski_theorem

This theorem is based on ZFC set theory. However, ZFC is not validated as tautologous by Meth8 (except for the axiom of specification, unrelated to this argument). Therefore a model for indiscernables does not exist.

Einstein–Podolsky–Rosen (EPR) as not a paradox but a weakened theorem

We assume Meth8/VŁ4 with the designated *proof* value of Tautology.

LET + Or; - Not Or; & And, ","; # necessity, for all; % possibility, for one or some; (%p>#p) 1; ((%p>#p)-(%p>#p)) derived value 0.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Binary ordinal
1	p=p	T	tautology	proof	11	3
2	p@p	F	contradiction	absurdum	00	0
3	%p>#p	N	non-contingency	truthity	01	1
4	%p<#p	C	contingency	falsity	10	2

See: en.wikipedia.org/wiki/EPR_paradox; and from vixra.org/pdf/1804.0335v1.pdf :

(|0>, |0>) + (|1>, |1>) = (|0>, |1>) + (|1>, |0>) = (|0> + |1>, |0> + |1>) (1.1)

We weaken Eq. 1.1 by reassigning the equivalency connective to the implication connective in the same order of literals.

(|0>, |0>) + (|1>, |1>) > (|0>, |1>) + (|1>, |0>) > (|0> + |1>, |0> + |1>) (2.1)

We ignore the bra-ket designation as irrelevant to the instant binary argument.

(((((%p>#p)-(%p>#p))&((%p>#p)-(%p>#p))) + ((%p>#p)&(%p>#p))) >
((((%p>#p)-(%p>#p))&(%p>#p)) + ((%p>#p)&((%p>#p)-(%p>#p))))) >
((((%p>#p)-(%p>#p))+(%p>#p)) & (((%p>#p)-(%p>#p))+(%p>#p))) ;
TTTT TTTT TTTT TTTT (2.2)

Eq. 2.2 as rendered is tautologous. This confirms a modified thesis of the captioned paper, that EPR is not a paradox and is resolved as an implication theorem.

Refutation of finitary, non-deterministic, inductive definitions of ECST and denial of CZF

Abstract: From the elementary constructive set theory (ECST) of intuitionistic logic, we evaluate six axioms of equality for system CZF. None is tautologous. This refutes those axioms in set theory and by extension denies intuitionistic logic.

Therefore are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊔ ;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Hirata, A.; Ishihara, H.; Kawai, T.; Nemoto, T. (2019).
Equivalents of the finitary non-deterministic inductive definitions.
arxiv.org/pdf/1903.05852.pdf tatsuji.kawai@jaist.ac.jp

2 Elementary constructive set theory

We work in a weak subsystem of **CZF**, called the *elementary constructive set theory* **ECST** [..], where none of the known fragments of the NID principle [*non-deterministic inductive definitions*] seems to be derivable.
The language of **ECST** contains variables for sets and binary predicates = and ∈. The axioms and rules of **ECST** are the axioms and rules of intuitionistic predicate logic with equality, and the following set-theoretic axioms:

Extensionality: ∀a∀b (∀x (x ∈ a ↔ x ∈ b) → a = b) . (2.1.1)

LET p, q, r, s: a, b, x or y, u.

((#r<#p)=(#r<#q))>(#p=#q) ; TTTT TCCT TTTT TCCT (2.1.2)

Paring: ∀a∀b∃y∀u (u ∈ y ↔ u = a ∨ u = b) . (2.2.1)

(#s<%r)=(#s=((#p+#s)=#q)) ; TCCT TCCT CCTT TTCC (2.2.2)

Union: $\forall a \exists y \forall x\ (x \in y \leftrightarrow \exists u \in a\ (x \in u))$. (2.3.1)

LET p, q, r, s: a, u, x, y.

`(#r<%s)=((%q<#p)&(#r<%q))` ; `TTTT CCCC TTTT TTTT` (2.3.2)

Restricted Separation: $\forall a \exists b \forall x\ (x \in b \leftrightarrow x \in a \wedge \phi(x))$ where $\phi(x)$ is restricted. Here, a formula is said to be *restricted* if all quantifiers in the formula occur in the forms $\forall x \in a$ or $\exists x \in a$. (2.4.1)

LET p, q, r, s: ϕ , a, b, x

`(#s<%r)=((#s<#q)&(p&#s))` ; `TTTT TTTT CTCC TCTT` (2.4.2)

Replacement:
$\forall a(\forall x \in a \exists ! y\ \phi(x, y) \rightarrow \exists b \forall y\ (y \in b \leftrightarrow \exists x \in a\ \phi(x, y)))$ where $\phi(x, y)$ is any formula. (2.5.1)

LET p, q, r, x, y: ϕ, a, b, x, y

`((#x<#q)&(p&(#x&%y)))>((#y<%r)=((%x<#q)&(p&(%x&#y))))` ;
`TTTT TTTT TTTT TTTT(48),`
`TTTT TCTT TTTT TCTT(16)` (2.5.2)

Strong Infinity:
$\exists a[0 \in a \wedge \forall x(x \in a \rightarrow x + 1 \in a) \wedge \forall y\ (0 \in y \wedge \forall x(x \in y \rightarrow x + 1 \in y) \rightarrow a \subseteq y)]$
where $x + 1$ denotes $x \cup \{x\}$ and 0 is the empty set Ø. ... (2.6.1)

LET p, q, r: a, x, y

`(((p@p)<%p)&((#q<%p)>((#q+(%p>#p))<%p)))&(((p@p)<#r)&`
`(((#q<#r)>((#q+(%p>#p))<#r))>~(#r<%p)))` ; **`FFFF FFFF FFFF FFFF`** (2.6.2)

This completes the description of **ECST**. The constructive Zermelo–Fraenkel set theory **CZF** [..] is obtained from **ECST** by substituting Strong Collection for Replacement and adding Subset Collection and ∈-Induction. ...

Eqs. 2.1.2-2.6.2 as rendered are *not* tautologous. This refutes those axioms on **ECST** and hence denies intuitionistic predicate logic and **CZF** set theory.

Entanglement versus untanglement: no-no go-go

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal.

From: quantamagazine.org/entanglement-made-simple-20160428/ [Frank Wilczek], and see preposterousuniverse.com/wp-content/uploads/125c-2017-2.pdf .

LET: p, q, r, s: $\Phi_{\blacksquare}$, $\Phi_{\bullet}$, $\psi_{\blacksquare}$, $\psi_{\bullet}$(sub-systems); ~ Not; & And, ⊗; + Or, ⊕; - Not Or;
\> Imply, greater than; < Not Imply, less than; = Equivalent; @ Not Equivalent;
% possibility, for one or some; # necessity, for all or every;
(p@p) ordinal 0, F; (%p>#p) ordinal 1; (%p<#p) ordinal 2; (p=p) ordinal 3, T;
(~(p<(p@p))&~(p>(%p>#p))) probability on interval]0,1[.

Remarks: Variables may also represent sub-systems, where an equation is entangled if it is *not* expressed as a tensor product. In other words, (Φ ⊕ ψ) is entangled but (Φ ⊗ ψ) untangled. Hence, (p+r) is entangled and (p&r) untangled, and (p+q) is entangled and (p&q) untangled.

Untangled: $(\Phi_{\blacksquare} + \Phi_{\bullet})(\psi_{\blacksquare} + \psi_{\bullet}) = (\Phi_{\blacksquare}\ \psi_{\blacksquare} + \Phi_{\blacksquare}\ \psi_{\bullet} + \Phi_{\bullet}\ \psi_{\blacksquare} + \Phi_{\bullet}\ \psi_{\bullet})$ (0.1.1)

(p+q)&(r+s) ; FFFF FTTT FTTT FTTT (0.1.2)

Entangled: $(\Phi_{\blacksquare}\ \psi_{\blacksquare} + \Phi_{\bullet}\ \psi_{\bullet})$ (0.2.1)

(p&r)+(q&s) ; FFFF FTFT FFTT FTTT (0.2.2)

From Eq. 1.1, the entangled state of $(\Phi_{\blacksquare}\ \psi_{\bullet} + \Phi_{\bullet}\ \psi_{\blacksquare})$ is not accounted for. (0.3.1)

(p&s)+(q&r) ; FFFF FFTT FTFT FTTT (0.3.2)

Consequently, we evaluate the combinations of pairs of variables as untangled and entangled units for completeness when applying their combined probability on the interval]0,1[.

Entangled form of (A and B):

p&q ; FTFT FTFT FTFT FTFT (1.2)

p&r ; FFFF FTFT FFFF FTFT (2.2)

p&s ; FFFF FFFF FTFT FTFT (3.2)

q&r ; FFFF FFTT FFFF FFTT (4.2)

q&s ; FFFF FFFF FFTT FFTT (5.2)

r&s ; FFFF FFFF FFFF TTTT (6.2)

Entangled form of (A and B) or (C and D):

(p&q)+(r&s) ; FFFT FFFT FFFT TTTT (7.2)

(p&r)+(q&s) ; FFFF FTFT FFTT FTTT (8.2)

(p&s)+(q&r) ; FFFF FFTT FTFT FTTT (9.2)

Entangled form of either (A and B) or (C and D) on interval]0,1[:

(p=((p&q)+(r&s)))>(~(p<(p@p))&~(p>(%p>#p)));
FTFF FTFF FTFF TFTF (10.2)

(p=((p&r)+(q&s)))>(~(p<(p@p))&~(p>(%p>#p)));
FTFT FFFF FTTF FFTF (11.2)

(p=((p&s)+(q&r)))>(~(p<(p@p))&~(p>(%p>#p)));
FTFT FTTF FFFF FFTF (12.2)

Entangled form of (A and B) or (C and D), or (E and F) or (G and H), or (I and J) or (K and L) on interval]0,1[:

(p=(((p&q)+(r&s))+(((p&r)+(q&s))+((p&s)+(q&r)))))>(~(p<(p@p))&~(p>(%p>#p))) ;
FTFF FFTF FFTF TFTF (13.2)

Untangled form of (A or B):

p+q ; FTTT FTTT FTTT FTTT (21.2)

p+r ; FTFT TTTT FTFT TTTT (22.2)

p+s ; FTFT FTFT TTTT TTTT (23.2)

q+r ; FFTT TTTT FFTT TTTT (24.2)

q+s ; FFTT FFTT TTTT TTTT (25.2)

r+s ; FFFF TTTT TTTT TTTT (26.2)

Untangled form of (A or B) and (C or D):

(p+q)&(r+s) ; FFFF FTTT FTTT FTTT (27.2)

(p+r)&(q+s) ; FFFT FFTT FTFT TTTT (28.2)

(p+s)&(q+r) ; FFFT FTFT FFTT TTTT (29.2)

Untangled form of (A or B) and (C or D) on interval]0,1[:

(p=((p+q)&(r+s)))>(~(p<(p@p))&~(p>(%p>#p))) ;
FTFT FFTF FFTF FFTF (30.2)

(p=((p+r)&(q+s)))>(~(p<(p@p))&~(p>(%p>#p))) ;
FTFF FTTF FFFF TFTF (31.2)

(p=((p+s)&(q+r)))>(~(p<(p@p))&~(p>(%p>#p))) ;
FTFF FFFF FTTF TFTF (32.2)

Because Eqs. 1.2 to 9.2 and 21.2 to 29.2 as rendered are *not* tautologous, the approach of entangled and untangled units is suspicious.

When we apply the combined probabilities to combinations in Eqs. 10.2-12.2 and 30.2-32.2 there are no tautologies, and grouping all combinations in Eq. 13.2 does no better.

We conclude that there is no tautological basis for sub-system states of entangled or untangled units in quantum theory.

Refutation of enumeration reducibility, enumeration degrees, and non-metrizable topology

Abstract: We evaluate the equation for the enumeration operator Φ. It is is *not* tautologous, hence refuting enumeration degrees and non-metrizable topology. These results therefore form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Kihara, T.; Ng, K.M.; Pauly, A. (2019). Enumeration degrees and non-metrizable topology arxiv.org/pdf/1904.04107.pdf kihara@i.nagoya-u.ac.jp, kmng@ntu.edu.sg, Arno.M.Pauly@gmail.com

2.3. Computability theory.

2.3.1. *Enumeration and Medvedev reducibility.* We review the definition of enumeration reducibility … Let $(D_e)_{e\in\omega}$ be a computable enumeration of all finite subsets of ω. Given A, B ⊆ ω, we say that A is *enumeration reducible to* B (written A ≤e B) if there is a c.e. set Φ such that

$$n \in A \Leftrightarrow (\exists e)\ [\langle n, e\rangle \in \Phi \text{ and } D_e \subseteq B]. \qquad (2.3.1.1)$$

LET p, q, r, s, t, u: A, B, D, e, n, Φ (enumeration operator)

(t<p)=(((t&%s)<u)&~(q<(r&%s))) ;

TTTT TTTT TTTT TTTT(1),
CN**F**T CNCN T**FF**T T**FF**T(1),
TTTT TTTT TTTT TTTT(1),
FT**F**T **F**T**F**T **F**T**F**T **F**T**F**T (2.3.1.1.2)

The Φ in the above definition is called an enumeration operator.

Eq. 2.3.1.2 as rendered is *not* tautologous, hence refuting enumeration reducibility, enumeration degrees, and non-metrizable topology.

Refutation of the paradox of Epicurus as invoked by Epictetus

We evaluate the captioned by assuming the apparatus and method of Meth8/VŁ4. The designated proof value is T, and F is contradiction. The 16-valued result table is row-major and horizontal.

From: en.wikipedia.org/wiki/Epicurus#Pleasure_as_absence_of_suffering

The "Epicurean paradox" or "Riddle of Epicurus" is a version of the problem of evil.

LET p q r s: God removes; God is willing; God is envious; God is feeble.

God wishes to take away evils [of envy and feebleness], and is unable; (1.1.1)

q&~p ; (1.1.2)

or He is able, and is unwilling; (1.2.1)

p&~q ; (1.2.2)

or He is neither willing nor able, (1.3.1)

~q&~p ; (1.3.2)

or He is both willing and able. (1.4.1)

q&p ; (1.4.2)

If He is willing and is unable, He is feeble, which is not in accordance with the character of God;
(2.1)

(q&~p)>s ; (2.2)

if He is able and unwilling, He is envious, which is equally at variance with God; (3.1)

(p&~q)>r ; (3.2)

if He is neither willing nor able, He is both envious and feeble, and therefore not God;
(4.1)

(~q&~p)>(r&s) ; (4.2)

if He is both willing and able, [He is not envious and not feeble] which alone is suitable to God,
(5.1)

(q&p)>(~r&~s) ; (5.2)

if Eq. 5.1, from what source then are evils or why does He not remove them? (6.1)

((q&p)>(~r&~s))>(((r&s)>~(p=p))+~p) ; (6.2)

Eq. 2.1 or Eq.3.1 or Eq. 4.1 or Eq 5.1 (7.1)

((((q&~p)>s)+((p&~q)>r))+((~q&~p)>(r&s))) + (((q&p)>(~r&~s))>(((r&s)>~(p=p))+~p)) ;
TTTT TTTT TTTT TTTT (7.2)

Eq. 7.2 as rendered is tautologous. This means that the paradox of Epicurus as invoked by Epictetus is refuted as a contradiction, and is confirmed as a theorem and not as a paradox.

Refutation of the paradox of Epimenides the Cretan

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table fragment) is row-major and horizontal.

LET LET p q s: Epimenides, Cretan, statement;
~ Not; & And; + Or; = Equivalent; @ Not Equivalent; > Imply, greater than;
# necessity, for all; lie (s@s).

From: en.wikipedia.org/wiki/Epimenides_paradox

"Epimenides the Cretan said that all Cretans were liars, and all other statements made by Cretans were certainly lies. Was this a lie?" (1.1)

((p=q)>(s=(#q>(s@s)))) & ((q>#(~(s=(#q>(s@s)))=(s=s)))>(s@s)) ;
`FFTN FFTN FFCC FFCC` (1.2)

Eq. 1.2 as rendered is *not* tautologous and *not* contradictory. Therefore the paradox of Epimenides is refuted as a paradox. The answer to the question "Was this a lie" is neither contradiction nor proof.

Epistemic coalition with perfect recall

From: Naumov, Pavel; Tao, Jia. "Strategic coalitions with perfect recall". Technical report. July 2017. researchgate.net/publication/318460936.

We apply the Meth8/VŁ4 modal logic model checker apparatus to the epistemic transition system T_2 for these universal principles, keyed to equations:

1. Strategic positive introspection; (1.0)
2. Strategic negative introspection; (2.0)
3. Perfect recall; and (3.0)
4. Cooperation principle. (4.0)

(The agent is assumed to have perfect recall with two states: *one knows what a strategy is*; or *one does not know what a strategy is*.)

LET: lc lower_case; p lc_phi; q lc_psi; r s t u C D H K;
z null value; (z@z) contradiction; (z-z) zero;
~ Not; & And; - Not Or; = Equivalent to; @ Not Equivalent to; > Imply

Designated truth value is T (tautology), with negation F (contradiction).
The results are repeating truth fragments from 128-tables, each of 16-values.

(t&(r&p)) > (u& ((r&t)&(r&p))) ; TTTT TFTF TTTT TFTF (1.2)
~(t&(r&p)) > (u&~((r&t)&(r&p))) ; FFFF FTFT FFFF FTFT (2.2)

Perfect recall principle with null as contradiction:
(~(s>r)@(z@z)) > ((t&(s&p)) > (t&((s&u)&(r&p)))) ;
TTTT TTTT TFTF TTTT (3.2.1)

Perfect recall principle with null as zero:
(~(s>r)@(z-z)) > ((t&(s&p)) > (t&((s&u)&(r&p)))) ;
TTTT TTTT TFTF TTTT (3.2.2)

Cooperation principle with null as contradiction:
((r&s)=(z@z)) > (((t&r)&(p>q)) > ((t&(s&p))>((t&r)+(s&q)))) ;
TTTT TTTT TTTT TTTT (4.2.1)

Cooperation principle with null as zero:
((r&s)=(z-z)) > (((t&r)&(p>q)) > ((t&(s&p))>((t&r)+(s&q)))) ;
TTTT TTTT TTTT TTTT (4.2.1)

Lemma: contradiction, as null, implies zero:
(z@z) > (z-z) ; TTTT TTTT TTTT TTTT ; (L.0)

We find this knowledge system is very important because of those evaluated so far by Meth8 using VŁ4, this system has the highest proof potential. For example, the cooperation principle is tautology in Eq 4.2.1. While the perfect recall principle and positive strategic introspection are not tautologous, they are subject to subsequent manipulation using our modal operators as interchangeable quantifiers.

Dynamic epistemic reasoning

Wang, Yanjing; Li, Yanjun. Not All Those Who Wander Are Lost: Dynamic Epistemic Reasoning in Navigation. 2014.

From:
researchgate.net/publication/267668798_Not_all_those_who_wander_are_lost_Dynamic_epistemic_reasoning_in_navigation

We assume the Meth8 apparatus using system variant VŁ4 to evaluate two expressions.

LET: lc lower_case; p lc_phi; q lc_phi-prime; r lc_psi; s lc_psi-prime; t K; u <a>;
~ Not; & And; = Equivalent to; > Imply; T tautology; F contradiction.

Results are in horizontal fragments are repeating truth tables of 128, as 16-values row major.

From 3.1 Finite axiomatization System S_sub_EAL_A_P, page 565:

¬Kp→K¬Kp (3.1.5.1)

(~t&p)>(t&(~t&p)) ; TFTF TFTF TFTF TFTF,
TTTT TTTT TTTT TTTT ; (3.1.5.2)

From Proposition 3.5, page 566:

((p=q)&(r=s)) > (((~p=~q)&((p&q)=(~p&~q)))&(((u&p)=(u&q))&((t&p)=(t&q)))) ;
FTTF TTTT TTTT FTTF (3.5.2)

We conclude that dynamic epistemic reasoning in navigation is not tautologous by Meth8, and hence suspicious.

Epistemic logics: Hilbert substructure

From: Sedlár, Igor. "Substructural epistemic logics". J*ournal of applied non-classical logics* . 25(3) January 2016. d.o.i: 10.1080/11663081.2015.1094313.

From: researchgate.net/profile/Igor_Sedlar

We use the Meth8 apparatus to evaluate the equations with these not found to be tautology as claimed:

(p@q)=(p&q) ;	TFFF ;	Prop 26.2
#(p>q)>(#p>#q) ;	TCTC ;	Prop 26.4
p\#p ;	TCTC ;	Prop 26.5
#((#q&p)>#(p>(q&p))) ;	NNNN ;	Fig. 7
(p&q)= ~(p=q) ;	TFFF ;	page 28, under Rules.

We conclude that substructural epistemic logics are not bivalent and further, when based on Hilbert-style rules, cannot be coerced into bivalency.

Refutation of an epistemic logic for knowledge and probability

Abstract: We evaluate axioms and definitions of an epistemic logic for knowledge and probability. Two equations are *not* tautologous, hence refuting the proposed epistemic logic.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p,q,r,s: ϕ, x, T, t
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
\> Imply, greater than, →, ⊢, ↦, ≻, ⊃; < Not Imply, less than, ∈, ≺, ⊂;
= Equivalent, ≡, ⊨, :=, ⇔, ↔; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology; (z@z) **F** as contradiction, Ø, Null;
(%z<#z) C non-contingency, ∇, ordinal 2;
(%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).

From: Tomović, S.; Ognjanović, Z.; Doder, D. (2019).
A first-order logic for reasoning about knowledge and probability.
arxiv.org/pdf/1901.06886.pdf sinisatom@turing.mi.sanu.ac.rs

3. The axiomatization Ax_{PCKfo}
In this section we introduce the axiomatic system for the logic PCK^{fo} ... of the following axiom schemata and rules of inference:
I First-order axioms and rules
...

FOR. $\frac{\phi}{\forall x\phi}$ (3.1.6.1)

p>(#q&p) ; T**F**T**F** TNTN T**F**T**F** TNTN (3.1.6.2)

Definition 3.3. A set T of formulas is saturated iff it is maximal consistent and the following condition holds:

if ¬(∀x)ϕ(x) ∈ T , then there is a term t such that ¬ϕ(t) ∈ T. (3.3.1)

((~p&#q)<r)>((~p&%s)<r) ; TTCT TTTT TTTT TTTT (3.3.2)

Remark 3.3.2: The result is one falsity value C from tautology.

Eqs. 3.1.6.2 and 3.3.2 as rendered are *not* tautologous. This means the proposed epistemic logic for knowledge and probability is refuted.

Navigation in epistemic logic

From: Deuser, Kaya; Naumov, Pavel. Navigability with Imperfect Information.
Armstrong's Axioms and Navigation Strategies. 2017.

From respectively: researchgate.net/publication/318720612; arxiv.org/pdf/1707.08255.pdf

We evaluate three expressions, an Axiom (1) in both papers and a Lemma (19) and Claim (5) in the first paper. We assume the Meth8 modal logic model apparatus implementing system variant VŁ$_4$.

LET: p A; q B; r C, F; s D, G ; ~ Not; > Imply, ▷ direction of travel; + Or, ∪;
\ Nand; = Equivalent to; T tautology; F contradiction.

Truth tables in 16-values are horizontal as row-major.

The axiom of reflexivity is the same from respectively 1. Reflexivity, page 6, and 1. Axiom, page 7:

A▷B, where A⊆ B (A.1.1.4.1.1)

~(p>q)>(p>q) ; TFTT TFTT TFTT TFTT ; (A.1.1.4.1.2)

Lemma 19, page 19:

(A∪ B)\C=(A\C)∪ (B\(A∪C)). (L.19.1.1)

((p+q)\r)=((p\r)+(q\(p+r))) ; TTTT TFFT TTTT TFFT ; (L.19.1.2)

Proof [with decomposed expressions].
(A∪B)\C=(A∪(B\A))\C=(A\C)∪((B\A)\C) (L.19.2.1)

((p+q)\r)=(((p+(q\p))\r)=((p\r)+((q\p)\r))) ;
TTTT FFTT TTTT FFTT ; (L.19.2.2)

(A∪B)\C=(A∪(B\A))\C=(A\C)∪((B\A)\C)=(A\C)∪ (B\(A∪C)) (L.19.3.1)

(((p+q)\r)=(((p+(q\p))\r)=((p\r)+((q\p)\r))))=((p\r)+((q\p)\r)) ;
TTTT FTTT TTTT TTTT ; (L.19.3.2)

(A∪B)\C (L.19.4.1)

(p+q)\r ; TTTT TFFF TTTT TFFF ; (L.19.4.2)

(A∪(B\A))\C=(A\C)∪((B\A)\C) (L.19.5.1)

((p+(q\p))\r)=((p\r)+((q\p)\r)) ; TTTT FTFF TTTT FTFF ; (L.19.5.2)

(A\C)∪ (B\(A∪C)) (L.19.6.1)

(p\r)+((q\p)\r) ; TTTT TFTT TTTT TFTT ; (L.19.6.2)

Claim 5, page 27:

$A \cup B \subseteq F \cup G$ (C.5.1)

~(p+q)>(r+s) ; FTTT TTTT TTTT TTTT ; (C.5.2)

Meth8 validates all script renditions above as *not* tautology; hence the subject area is suspicious.

Quantifiers over epistemic agents

From: Naumov, Pavel; Tao, Jia. "Everyone knows that someone knows: quantifiers over epistemic agents". *The review of symbolic logic*. January 2018.

From:researchgate.net/publication/
315775241_Everyone_Knows_That_Someone_Knows_Quantifiers_over_Epistemic_Agents

We note that these conjectures assume the validity of set theory, as does S5 and Kripke worlds.

Using the Meth8 apparatus, we evaluate equations numbered in order by page and appearance. The logic values by 2-tuple are { 11, 10, 01, 00 } for < Tautology (T proof), Contingent (C falsity value), Non contingent (N truth value), Contradiction (F, Not T) >.

LET: r s t a b c; p q lc_phi lc_psi; u A;
~ Not; # necessity, universal quantifier; % possibility, existential quantifier;
> Imply, element of as (x>A) for (x is not an element of A).

```
(#r&#p) > #(#r&p) ;                      TTTT ;                 (page 1.1)
    (#r&#p) > (##r&p) ;                  TTTT ;                 (page 1.1) ; moving the #
    (r& p) >  (r&p) ;                    TTTT ;                 (page 1.1) ; without the #

(((#r&s)&p)&((#s&r)&q)) > ((#r&(s&r))&(p&q)) ;
                                         TTTT ;                 (page 1.2) ; with    #
    ((( r&s)&p)&(( s&r)&q)) > (( r&(s&r))&(p&q)) ;
                                         TTTT ;                 (page 1.2) ; without #

#r&((#r&p)>(#r&q)) ;                     FFFF NFNN ;            (page 2.1)
    r&((#r&p)>(#r&q)) ;                  FFFF TCTT ;            (page 2.1) ; no antec #
    #r&(( r&p)>( r&q)) ;                 FFFF NFNN ;            (page 2.1) ; no consq#
    r&(( r&p)>( r&q)) ;                  FFFF TFTT ;            (page 2.1) ; no #

 (#r&(#r&p)) > (#r&(#r&q)) ;             TTTT TCTT ;            (page 2.2) ;
                                                                different than  (page 2.1)

(#r&#s) & (((#r&#s)&p)>((#r&p)+(#s&p)));
                                         FFFF FFFF FFFF NNNN ;  (page 2.3)

#r & (((#r&%s)&(#s&p))>(#r&p)) ;         FFFF NNNN FFFF NNNN ;  (page 2.4)
```

LET: q A

```
(r>q) > (((#r&#q)&p)>((#q&#r)&p));       TTTT TTTT TTTT TTTT ;  (page 2.5)
                                         Barcan antecedent, Imply
(r>q) & (((#r&#q)&p)>((#q&#r)&p)) ;      TTTT FFTT TTTT FFTT ;  (page 2.5)
                                   Barcan antecedent, And
    ((#r&#q)&p)>((#q&#r)&p) ;            TTTT TTTT TTTT TTTT ;  (page 2.5)
                                   Barcan without antecedent restriction
    ((#r&#q)&p)>((#q&#r)&p) ;            TTTT TTTT TTTT TTTT ;  (page 2.5)
                                   Barcan without antecedent restriction, no #
```

On the page 3 definitions, we ignore "separate quantifiers for epistemic worlds and agents".

LET: uc upper_case; lc lower_case; p lc_psi; q uc_Phi; r C; s lc_phi;
~ Not; > Imply, not element of, as (~(s>q)) for (s is an element of q); v V; x x

~(s>(q&r)) > ~(~s>(q&r)) ;	TTTT TTTT FFFF FFTT ;	Def.3.2
~((s&p)>(q&r)) > (s>~(p>(q&r))) ;	TTTT TTTT TTTT TTTT ;	Def.3.3
(~(x>v)&~(s>(q&r))) > ~((#x&s)>(q&r)) ;	TTTT TTTT TTTT TTTT ;	Def.3.5
(~(x>v)&~(s>(q&r))) > ~((x&s)>(q&r)) ;	TTTT TTTT TTTT TTTT ;	Def.3.5, no #

Meth8 validates as tautology the Barcan formula as stated (both with and without the restriction and both with and without the existential quantifier and modal necessity) and also Defs 3.3 and 3.5. Meth8 does not validate as tautology Def 3.2. For the moment, we end our evaluation here.

What follows is that for system variant VŁ4, "separate quantifiers for epistemic worlds and agents" are not needed as a distinction.

Erdös-Strauss Conjecture

From: blogs.ams.org/matheducation/2015/05/01/famous-unsolved-math-problems-as-homework/

This uses the Meth8 model checker where ~ Not, + Or, - Not Or, @ Not equivalent, = Equivalent to, & And, \ Not And, > Imply.

((((p+q)+r)@((p@p) + ((r@r)-(%r>%#r)))) & (s@(((s>%#s)+(s@s)) + ((s@s)-(%s>%#s)))))
>
(((((s>%#s)+(s>%#s))+(s>%#s))\s) = ((((s>%#s)\p)+((s>%#s)\q))+((s>%#s)\r))) ;

TTTT TTCT TTTT TTTT ; *not* tautologous

Refutation of ethical reasoning and HOL as a universal meta-logic

Abstract: An exemplary equation in HOL for ethical reasoning is *not* tautologous. By extension, HOL is refuted as "a universal meta-logic", and "ethical reasoning" is refuted. Therefore HOL and ethical reasoning are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Benzmüller, C.; Parenta, X.; van der Torre, L. (2019).
Designing normative theories of ethical reasoning: formal framework, methodology, and tool support.
arxiv.org/pdf/1903.10187.pdf c.benzmueller@googlemail.com, c.benzmueller@fu-berlin.de, xavier.parent@uni.lu, leon.vandertorre@uni.lu

2. The SSE approach: HOL as a universal meta-logic

Remark 2: SSE is not defined as an acronym.

For example … ◊∀x.Px ≡ (λw.∃v.Rwv ∧ ∀x.P.xv). (2.1)
This illustrates the embedding of ◊∀x.Px in HOL.

LET p, r, v, w, x, z: P, R, v, w, x, λ.

(%#x&(p&x))=(((z&w)&(%v&(r&(w&v))))&(#x&(p&(x&v)))) ;

```
TTTT TTTT TTTT TTTT(16),
TCTC TCTC TCTC TCTC(12),
TCTC TTTT TCTC TTTT( 4)
```
(2.2)

Eq. 2.2 as rendered is *not* tautologous. By extension, HOL is refuted as "a universal meta-logic", and "ethical reasoning" is refuted.

Refutation of the Euathlus paradox: neither pay

To refute the Euathlus paradox and where *neither* pay, we evaluate this paper:

> Lisanyuk, Elena. (2017). "Why Protagoras gets paid anyway: a practical solution of the Paradox of Court". philarchive.org/archive/ELEWPG

using en.wikipedia.org/wiki/Paradox_of_the_Court .

We assume the apparatus and method of the modal model logic checker named Meth8/VŁ4, with the designated *proo*f value of T, and use four variables.

LET p, q, r, s:
pupil Euathlus; instructor Protagoras; court judgment; tuition payment

> "The famous sophist Protagoras took on a pupil, Euathlus, on the understanding that the student will pay Protagoras for his instruction after he wins his first court case." (1.1)

> (q&p) > ((p&r)>(p>(q&s))) ; TTTT TTT**F** TTTT TTTT (1.2)

Eq. 1.2 as rendered is *not* tautologous, but nearly so with one value **F** of 16 diverging from the tautology of all T's.

Remark 1. The instructor's assumption is that the pupil will win the necessity of his first court case, but no contingency is made for the event that the pupil possibly does not continue onto perform in any court. For example, there is no contingency for if the pupil became a lawyer but acted as a solicitor and not a barrister, then the litigious status of the pupil could never be tested before a court.

Remark 2. The rule of law in the West is that when an experienced lawyer as contractor, Protagoras, frames an agreement with a lesser experienced non-lawyer as contractee, Euathlus, then the contractor is held to a higher level of performance and closer reading of the agreement than is the contractee.

Remark 3. On the basis of no contingency arrangement for the contractee not to perform, the court would hold for a defective contract and disallow any claim by Protagoras. Should Euathlus counter-claim for lawyer's fees, the court would probably grant that motion on the basis of a frivolous lawsuit claim by Protagoras in the first place. In other words, Protagoras would would lose in either scenario, that is, not obtain relief for instructing the pupil, and liable for the pupil's legal expenses in that event.

> "After instruction, Euathlus decided not to enter the profession of law, (2.1.1)

> and [then] Protagoras decided to sue Euathlus for the amount owed." (2.2.1)

> ((q&p)>((p&r)>(p>(q&s)))) > (~(p&r)>~(p>(q&s))) ;
> **F**T**F**T **F**T**F**T **F**T**FF** **F**T**F**T (2.1.2)
> (((q&p)>((p&r)>(p>(q&s))))>(~(p&r)>~(p>(q&s)))) > (q&(r+~r)) ;
> T**F**TT T**F**TT T**F**TT T**F**TT (2.2.2)

Eqs. 2.2.1 and 2.2.2 are *not* tautologous, therefore that chain of events is suspicious.

Remark 4. The metaphysical question of "Was Euathlus morally wrong in not paying Protagoras for services rendered, regardless of outcome" can now be cast onto a physicalistic basis in this way. The proof tables for performance by Protagoras in Eq. 1.2 and for non-performance by Euathlus in Eq. 2.1.2 are contrasted:

(q&p) > ((p&r)>(p>(q&s))) ; TTTT TTT**F** TTTT TTTT (1.2)
((q&p)>((p&r)>(p>(q&s)))) > (~(p&r)>~(p>(q&s))) ;
FT**F**T FT**F**T **F**T**FF** **F**T**F**T (2.1.2)

Eq. 2.1.2 diverges *more* from tautology than does Eq. 1.2. This means a physicalistic basis if mapped for moral theology as a recent advance. In other words, Euathlus failed to do the right thing by withholding payment in any event, so as not to violate the intended spirit of the albeit defective contract.

"Protagoras argued that if he won the case he would be paid his money."
[In other words, if Eq. 2.2.1, then the Protagoras lawsuit obtains payment.] (3.1.1)

((((q&p)>((p&r)>(p>(q&s))))>(~(p&r)>~(p>(q&s))))>(q&(r+~r))) > ((q&r)>(p>(q&s))) ;
TTTT TTT**F** TTTT TTTT (3.1.2)

"If Euathlus won the case, Protagoras would still be paid according to the original contract, because Euathlus would have won his first first case."
[In other words, if not Eq. 3.1 then 1.1.] (3.2.1)

~((((q&p)>((p&r)>(p>(q&s))))>(~(p&r)>~(p>(q&s))))>(q&(r+~r))) > ((p&r)>(p>(q&s))) ;
TTTT T**F**TT TTTT T**F**TT (3.2.2)

Eqs. 3.1.2 and 3.2.2 are not equivalent and *not* tautologous

"Euathlus, however, claimed that if he won, then by the court's decision he would not have to pay Protagoras."
[In other words, if not Eq. 3.1.1 or not 3.2.1, then not 1.1.] (4.1.1)

(~(((((q&p)>((p&r)>(p>(q&s))))>(~(p&r)>~(p>(q&s))))>(q&(r+~r)))>((q&r)>(p>(q&s))))+
~(~((((q&p)>((p&r)>(p>(q&s))))>(~(p&r)>~(p>(q&s))))>(q&(r+~r)))>((p&r)>(p>(q&s))))) >
~((q&p)>((p&r)>(p>(q&s)))) ; TTTT T**F**TT TTTT T**F**TT (4.1.2)

"If, on the other hand, Protagoras won, then Euathlus would still not have won a case and would therefore not be obliged to pay."
[In other words, if Eq. 3.2.1 or 3.2.2, then not 3.1] (4.2.1)

((((((q&p)>((p&r)>(p>(q&s))))>(~(p&r)>~(p>(q&s))))>(q&(r+~r)))>((q&r)>(p>(q&s))))+
(~((((q&p)>((p&r)>(p>(q&s))))>(~(p&r)>~(p>(q&s))))>(q&(r+~r))) >
((p&r)>(p>(q&s)))))> ~((q&p)>((p&r)>(p>(q&s)))) ;
FFFF FFFT **FFFF FFFF** (4.2.2)

Eqs. 4.1.2 and 4.2.2 are not equivalent and *not* tautologous. In fact 4.2.2 is nearly contradictory. This means regardless of who wins the lawsuit of Protagoras, Euathlus does not pay. Hence the Euathlus paradox is refuted and resolved by default in favor of Euathlus.

Refutation of Euclidean geometry embedded in hyperbolic geometry for equal consistency

Abstract: The following conjecture is refuted: "[An] n-dimensional Euclidean geometry can be embedded into (n+1)-dimensional hyperbolic non Euclidean geometry. Therefore hyperbolic non Euclidean geometry and Euclidean geometry are equally consistent, that is, either both are consistent or both are inconsistent." Hence, the conjecture is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

"[An] n-dimensional Euclidean geometry can be embedded into (n+1)-dimensional hyperbolic non Euclidean geometry. The case n = 2 was already known to Gauss in the early 1800's. Therefore hyperbolic non Euclidean geometry and Euclidean geometry are equally consistent, that is, either both are consistent or both are inconsistent."

We rewrite the above excluding the Gaussian reference as:

"[An] n-dimensional Euclidean geometry can be embedded into (n+1)-dimensional hyperbolic non Euclidean geometry. (1.1.1)
Therefore hyperbolic non Euclidean geometry and Euclidean geometry are equally consistent, that is, either both are consistent or both are inconsistent." (1.2.1)

We map Eq. 1.1.1 as "dimensional-n Euclidean planar geometry can be embedded into dimensional-n+1 hyperbolic non Euclidean geometry":

LET p, q, r, s, t:
Euclidean geometry, dimensional-n, planar, hyperbolic, elliptical;
consistent (p=p) Tautology; inconsistent ~(p=p), (p@p) Contradiction.

(q&(r&p))<((q+(%q>#q))&(s&~p)) ; **FFFF FFF**T **FFFF FFF**T (1.1.2)

We map Eq. 1.2.1 as "(hyperbolic non Euclidean geometry and planar Euclidean geometry are equally consistent) is equivalent to, that is, either both are consistent or both are inconsistent":

(((s&~p)&(r&~p))=(p=p)) = ((((s&~p)&(r&~p))=(p=p))+(((s&~p)&(r&~p))=(p@p))) ;
FFFF FFFF FFFF T**F**T**F** (1.2.2)

Eqs. 1.1.2 and 1.2.2 respectively serve as antecedent and consequent for the full conjecture that 1.1.2 implies 1.2.2. (2.1)

We map Eq. 2.1 as: "(dimensional-n Euclidean planar geometry can be embedded into dimensional-n+1 hyperbolic non Euclidean geometry) implies ((hyperbolic non Euclidean geometry and planar Euclidean geometry are equally consistent) is equivalent to, that is, either both are consistent or both are inconsistent):

((q&(r&p))<((q+(%q>#q))&(s&~p))) >
((((s&~p)&(r&~p))=(p=p)) =
((((s&~p)&(r&~p))=(p=p))+(((s&~p)&(r&~p))=(p@p)))) ;
TTTT TTT**F** TTTT TTT**F** (2.2)

Eqs. 1.1.2, 1.2.2, and 2.2 as rendered are *no*t tautologous. This means the conjecture is refuted.

Remark 2.2: Eq. 2.2 can be coerced into a tautology by changing 1.2 to read "(hyperbolic non Euclidean geometry and planar Euclidean geometry are equally consistent) *implies*, that is, either both are consistent or both are inconsistent", to rely on the abstract (**F**>T)=T rather than on the original (**F**=T)=**F**.

Refutation of "some thing" from "non thing"

Abstract:

A variable implies itself in

p → p or ~p → ~p as "Thing implies thing" or "Non thing implies non thing"

but *not* when mixed with its negation in

~p → p or p → ~p as "Non thing implies thing" or "Thing implies non thing".

This means creation out of nothing "ex nihilo" is *not* supported in

~p → p as "Non thing implies thing",

or by introducing modal operators in

~◊p → ◊p as "Not some thing implies some thing" equivalent to

□~p → ◊p as "All non things imply some thing".

What follows is that

"ex nihilo" is not equivalent to "a nullo"

and that

"ex nihilo" is not synonymous with God and hence not an ontological proof of God.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

LET p, ~p, %p, ~%p: thing, non thing, some thing, not some thing

~ Not; > Imply, →;

% possibility, for one or some, ◊; # necessity, for all or every, □.

Remark: The word "nothing" is rendered here as "non thing" to preserve the distinction of the negation of "thing". To equate "nothing" with "not a thing" is also inexact because "a thing" is "some thing", as "one thing", as opposed to just "thing".

From: scottmsullivan.com/articles/NihilCh1.pdf

"[O]ut of nothing, nothing comes." as (1.0)

Non thing implies non thing. (1.1)

~p>~p ; TTTT TTTT TTTT TTTT (1.2)

Thing implies thing. (2.1)

p> p ; TTTT TTTT TTTT TTTT (2.2)

Non thing implies thing. (3.1)

~p> p ; **F**T**F**T **F**T**F**T **F**T**F**T **F**T**F**T (3.2)

Thing implies non thing. (4.1)

p>~p ; TFTF TFTF TFTF TFTF (4.2)

Remark 1-4: Eqs. 1-4 deal with the variable "thing" and its negation "non thing". Only Eqs. 1.2 and 2.2 are tautologous. Eqs. 3.2 and 4.2 as opposites attempt to imply thing from non thing or vice versa. Using Eq. 3.2 to support creation via "ex nihilo" is a mistake because God pre-existed and hence was *some* thing below.

We further refine "thing" to mean "at least one thing "or "some thing".

Not something implies not something. (5.1)

~%p>~%p ; TTTT TTTT TTTT TTTT (5.2)

Remark 5.2: Eq. 5.2 reduces to #~p>#~p, as All non things imply all non things.

Some thing implies some thing. (6.1)

%p> %p ; TTTT TTTT TTTT TTTT (6.2)

Not some thing implies some thing. (7.1)

~%p> %p ; CTCT CTCT CTCT CTCT (7.2)

Remark 7.2: Eq. 7.2 reduces to #~p> %p, as All non things imply some thing.

Some thing implies not some thing. (8.1)

%p>~%p ; NFNF NFNF NFNF NFNF (8.2)

Remark 8.2: Eq. 8.2 reduces to %p>#~p, as Some thing implies all non things.

Remark 5-8: Eqs. 5-8 introduce modal operators. Only Eqs. 5.2 and 6.2 are tautologous. Eqs. 7.2 and 8.2 as opposites attempt to imply some thing from not some thing or vice versa. Using Eq. 7.2 to support creation via ex nihilo is a mistake because God pre-existed and hence already was *some* thing and not null as "a nullo".

Refutation of Levi-identity and AGM postulates of fictional logic

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

From: Badura, C. (2016). Truth in fiction via non-standard belief revision.
illc.uva.nl/Research/Publications/Reports/MoL-2016-07.text.pdf

LET p, q, r: lc_phi φ, lc_psi ψ, B;
~ Not; + Or, expansion; - Not Or, contraction; & *, revision operator;
= Equivalent; @ Not equivalent ;
> Imply, greater than, not.lt.eq $\notin$; ~(>) Not Imply, lt.eq $\subseteq$;
(p=p) `T` designated *proof* value; (p@p) `F` as contradiction;
AGM (Alchourròn, Gärdenfors, Makinson) .

Remark: Equations from the text are not reproduced here due to non-portable pdf characters.

(r&p)=((r-~p)+p) ;	`TFTF TTTT TFTF TTTT`	(Levi-identity)

The AGM postulates for the revision operator(s) are from page 30:

(r&p)=(p=p) ;	`FFFF FTFT FFFF FTFT`	(1)
~(p>(r&p))=(p=p) ;	`FTFT FFFF FTFT FFFF`	(2)
(r&p)>(r+p) ;	`TTTT TTTT TTTT TTTT`	(3)
(~p>r)>~((r+p)<(r&p)) ;	`TFTF FTFT TFTF FTFT`	(4)
(p@p)>(~(r&p)=(p=p)) ;	`TTTT TTTT TTTT TTTT`	(5)
(p=q)>((r&p)=(r&q)) ;	`TTTT TTTT TTTT TTTT`	(6)
~((r&(p&q))>((r&p)+q))=(p=p) ;	`FFFF FFFF FFFF FFFF`	(7)
[maybe should read (r&(p&q))>((r&p)+q) ;		
	`TTTT TTTT TTTT TTTT`]	
(~q>(r&p))>~(((r&p)+q)>(r&(p&q))) ;		
	`TTTT TTTF TTTT TTTF`	(8)

Eq. (Levi-identity) as rendered is *not* tautologous, as a basis for the subsequent AGM expressions.

Eqs. 3, 5, 6, and (arguably) 7 are tautologous. However, Eqs. 1, 2, 4, and 8 are not. This refutes the AGM postulates as a basis for fictional logic.

Refutation of classification of finitary, algebraizable logics as undecidable in Hilbert calculi

Abstract: Two axioms as tested are *no*t tautologous, to refute the Hilbert calculus as claimed. This refutes the conjecture of classification of finitary, algebraizable logic as undecidable in Hilbert calculi. That also disallows a follow-on article. These form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Moraschini, T. (2019). A computational glimpse at the Leibniz and Frege hierarchies. arxiv.org/pdf/1908.00922.pdf

Abstract … algebraic logic (AAL for short) is a field that studies uniformly propositional logics.. . One of its main achievements is the development of the so-called Leibniz and Frege hierarchies in which propositional logics are classified according to two different criteria. More precisely, the Leibniz hierarchy provides a taxonomy that classifies propositional systems accordingly to the way their notions of logical equivalence and of truth can be defined. Roughly speaking, the location of a logic inside the Leibniz hierarchy reflects the strength of the relation that it enjoys with its algebraic counterpart. In this sense, the Leibniz hierarchy revealed to be a useful framework where to express general transfer theorems between metalogical and algebraic properties. This is the case for example for superintuitionistic logics… On the other hand, the Frege hierarchy offers a classification of logics according to general replacement principles. Remarkably, some of these replacement properties can be formulated semantically by asking that the different elements in a model of the logic are separated by a deductive filter. This is what happens for example in superintuitionistic logics, whose algebraic semantics is given by varieties of Heyting algebras where logical filters are just lattice filters. The aim of this paper is to investigate the computational aspects of the problem of classifying syntactically presented logics in the Leibniz and Frege hierarchies. More precisely, we will consider the following problem

> Let K be a level of the Leibniz (resp. Frege) hierarchy. Is it possible to decide whether the logic of a given finite consistent Hilbert calculus in a finite language belongs to K?

It turns out that in general the answer is negative both for the Leibniz and the Frege hierarchies. … Remarkably, our proof shows that this classification problem remains undecidable even if we restrict our attention to Hilbert calculi that determine a finitary algebraizable logic (Theorem 5.3).

5. The classification problem in the Frege hierarchy

Definition 5.1. … with two new connectives □ and →, respectively unary and binary. *L*(*a*, *b*) is the logic in the language *L* axiomatized by the following Hilbert calculus … for every formula φ of the following … :

$\varphi_4 := x \rightarrow (x \rightarrow \Box x)$ (5.1.4.1)

LET p: x.

p>(p>#p); `TNTN TNTN TNTN TNTN` (5.1.4.2)

$\varphi_6 := (\Box x \rightarrow x) \rightarrow ((x \rightarrow \Box x) \rightarrow x)$ (5.1.6.1)

(#p>p)>((p>#p)>p) ; **F**T**F**T **F**T**F**T **F**T**F**T **F**T**F**T (5.1.6.2)

Eqs. 5.1.4.2 and 5.1.6.2 as rendered are *not* tautologous. This means two axioms refute the Hilbert calculus as used. This also refutes the conjecture of classification of finitary, algebraizable logic as undecidable in Hilbert calculi and further disallows the follow-on article: Moraschini, T. (2019). On the complexity of the Leibniz hierarchy. arxiv.org/pdf/1908.00924.pdf

Denial that modal logics of finite direct powers of ω have the finite model property

Abstract: From the section partitions of frames, local finiteness, and the finite model property, we evaluate that definition. Because it is *not* tautologous, subsequent equations in the conjecture are denied. This means it is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables.
(See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊔ ;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Shapirovsky, I. (2019).
Modal logics of finite direct powers of ω have the finite model property.
arxiv.org/pdf/1903.04614.pdf shapir@iitp.ru

Definition 1. Let F = (W, R) be a Kripke frame. A partition A of W is tuned (in F) if for every U, V ∈ A,

∃u ∈ U ∃v ∈ V uRv ⇒ ∀u ∈ U ∃v ∈ V uRv. (2.1.1)

LET p, q, r, u, v: U, V, R, u, v.

(((%u<p)&(%v<q))&(%u&(r&%v)))>(((#u<p)&(%v<q))&(#u&(r&%v))) ;
TTTT NTTT TTTT NTTT (2.1.2)

F is tunable if for every finite partition A of F there exists a finite tuned refinement B of A.

Eq. 2.1.2 is *not* tautologous, hence denying subsequent equations in the conjecture.

Refutation of first-order proofs without syntax

Abstract: Proof examples (3) in the introduction, modal combinatorial proofs (1), and rules in Gentzen's classical sequent calculus (3) are *not* tautologous. This refutes the conjecture and approach of first-order proofs without syntax, to form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And; Proof examples (3) in the introduction, modal combinatorial proofs (1), and rules in Gentzen's classical sequent calculus (2) are *not* tautologous. This refutes the conjecture and approach of first-order proofs without syntax.
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Hughes, D.J.D. (2019). First-order proofs without syntax. arxiv.org/pdf/1906.11236.pdf
dominic@theory.stanford.edu

Abstract: Proofs are traditionally syntactic, inductively generated objects. This paper reformulates first-order logic (predicate calculus) with proofs which are graph theoretic rather than syntactic. It defines a *combinatorial proof* of a formula ϕ as a lax fibration over a graph associated with ϕ. The main theorem is soundness and completeness: a formula is a valid if and only if it has a combinatorial proof.

1 Introduction
Proofs are traditionally syntactic, inductively generated objects. For example, Fig. 1 shows a syntactic proof of

∃x(px ⇒ ∀y py). (1.1)

LET p, q, r, s: p, x, y, f (or a)

(p&%q)>(p&#r) ; TNT**F** TNTN TNT**F** TNTN (1.2)

The four combinatorial proofs of Fig. 2 are rendered in condensed form in Fig. 3.

(∀xpx) ⇒ ∀y (py∧pfy) (3.1.1)

(p&#q)>((p&#r)&(p&(s&#r))) ;
TTTC TTTC TTTC TTTT (3.1.2)

∃x(pa∨ py ⇒ px)) (3.4.1)

((p&s)+(p&r))>(p&%q) ; `TTTT TCTT TCTT TCTT` (3.4.2)

9 Modal combinatorial proofs

A *modal* formula is generated from the *modal operators* □ (necessity) and ◊ (possibility) instead of quantifiers and has all predicate symbols nullary, *e.g.* ◊(p ⇒□p). Every modal formula abbreviates a standard first-order one .. : replace every □ by ∀x, ◊ by ∃x, and predicate symbol p by px. For example,

◊(p ⇒□p) abbreviates ∃x(px ⇒ ∀x px), or ∃x(px ⇒ ∀y py) in rectified form. (9.1)

%(p>#p) > (((p&%q)>(p&#q))+((p&%q)>(p&#r))) ;
`TNTN TNTN TNTN TNTN` (9.2)

11 Proof of the Completeness Theorem

In this section we prove the Completeness Theorem … . Our strategy will be to show that every syntactic proof of a formula ϕ in Gentzen's classical sequent calculus .. generates a combinatorial proof of ϕ, so completeness follows from that of Gentzen's system.

$$\frac{\Gamma}{\Gamma,\phi}\ \text{W} \qquad (11.4.1)$$

LET p, q, r, s: ϕ, θ (or t in ∃), Γ, Δ (or x in ∀).

r>(r&p) ; `T`**`F`**`TT T`**`F`**`TT T`**`F`**`TT T`**`F`**`TT` (11.4.2)

$$\frac{\Gamma,\ \phi\{x7\rightarrow t\}}{\Gamma,\ \exists x\phi}\ \exists \qquad (11.8.1)$$

((r&p)&(s>q))>(r&(%s&p)) ;
`TTTT TCTC TTTT TTTT` (11.8.2)

$$\frac{\Gamma,\phi}{\Gamma,\forall x\phi}\ \forall\ (x\text{ not free in }\Gamma) \qquad (11.9.1)$$

(r&p)>(r&#p) ; `TTTT TNTN TTTT TNTN` (11.9.2)

The rules X, C and W are called *exchange*, *contraction* and *weakening*. Each sequent above a rule is a *hypothesis* of the rule, and the sequent below a rule is the *conclusion* of the rule.

Proof examples (3) in the introduction, modal combinatorial proofs (1), and rules in Gentzen's classical sequent calculus (3) are *not* tautologous. This refutes the conjecture and approach of first-order proofs without syntax.

Refutation of Fitch's paradox of knowability

We assume the method and apparatus of Meth8/VL4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table fragment) is row-major and horizontal.

LET p x; ~ Not; & And; = Equivalent; > Imply;
% possibility, for one or some, L; # necessity, for all, K *discernible* (instead of known);
(p=p) Tautology.

Note: The parser for Meth8 explicitly decomposes the negation of concatenated modal operators on literals as follows: ~%#(p>q) is ~(%#(p>q)=(p=p)).

From: en.wikipedia.org/wiki/Fitch's_paradox_of_knowability, of which please see because we do not reproduce here.

Fitch rules:

#p>p ;	TTTT TTTT TTTT TTTT	(A.2)
#(p&q)>(#p&#q) ;	TTTT TTTT TTTT TTTT	(B.2)
p>#%p ;	TTTT TTTT TTTT TTTT	(C.2)
~p>~%p ;	NTNT NTNT NTNT NTNT	(D.1.2)

Since Eq. D.1.1 is not tautologous, it should correctly read

"K~p>~Lp"		(D.2.1)
#~p>~%p ;	TTTT TTTT TTTT TTTT	(D.2.2)

Fitch steps:

#(p&~#p)=(p=p) ;	FFFF FFFF FFFF FFFF	(1.2)
#(p&~#p)>(#p&#~#p) ;	TTTT TTTT TTTT TTTT	(2.2)
(#p&#~#p)>#p ;	TTTT TTTT TTTT TTTT	(3.2)
(#p&#~#p)>#~#p ;	TTTT TTTT TTTT TTTT	(4.2)
(#p&#~#p)>~#p ;	TTTT TTTT TTTT TTTT	(5.2)
(#p&~#p)>~(#(p&~#p)=(p=p)) ;	TTTT TTTT TTTT TTTT	(6.2)
~(#(p&~#p)=(p=p)) > ~(%#(p&~#p)=(p=p)) ;	TTTT TTTT TTTT TTTT	(7.2)
(p&~#p)=(p=p) ;	FCFC FCFC FCFC FCFC	(8.2)
(p&~#p) > %#(p&~#p) ;	TNTN TNTN TNTN TNTN	(9.2)
(~(%#(p&~#p)=(p=p))&(%#(p&~#p)))>~(p&~#p) ;	TTTT TTTT TTTT TTTT	(10.2)
~(p&~#p)>(p>#p) ;	TTTT TTTT TTTT TTTT	(11.2)

As rendered, Eqs. D.1.2, 1.2, 8.2, and 9.2 are *not* tautologous. However Eqs. 7.2 and 11.2 are tautologous. This means the alleged paradox is *not* contradictory, *not* a paradox, and hence a theorem.

It states that "every truth is discernible", and we add, "by the instant modal logic model checker".

Some writers invoke Godel incompleteness then jettison the knowability rule (C.1) to generalize and solve the paradox, rewritten as:

```
%x((x&-Kx)&LKx)&LK((x &-Kx)&LKx)                                (C'.1.1)
%p&(((p&~#p)&%#p)&%#((p&~#p)&%#p)) ;
                                undistributed quantifier ;
                                FFFF FFFF FFFF FFFF             (C'.1.2)
(%p&((p&~#p)&%#p))&(%p&%#((p&~#p)&%#p)) ;
                                distributed quantifier ;
                                FFFF FFFF FFFF FFFF             (C'.1.3)
```

Eqs. C'.1.2 and C'.1.3 are *not* tautologous but contradictory. Therefore, that artifice solves nothing.

Refutation of mapping mu-calculus onto second-order logic

Abstract: When mapping mu-calculus onto second-order logic, we show use of the fixpoint operator as untenable. What follows is the effective refutation of mapping mu-calculus onto second-order logic.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

> LET p, q, r, s: p, q, w, v; ~ Not; & And, ∧; \ Not And, /; > Imply, greater than, →; < Not Imply, lesser than, ∈ ; = Equivalent; ~(y<x) x≤y, x⊆y; % possibility, for one or some, ∃; # necessity, for every or all, ∀.

From: Carreiro, F.; Facchini, A.; Venema, Y.; Zanasi, F. (2018). The power of the weak. arxiv.org/pdf/1809.03896.pdf

> **Remark:** We ignore phi-asterisk (φ*) as a constant/scalar in this demonstration.

[W]e would inductively translate $(\mu p.\varphi)^*$ as $\forall p\ \forall w\ (\varphi^*[w/v] \rightarrow p(w)) \rightarrow p(v))$ (29.1)

#p&((#r&((r\s)>(p&r)))>(p&s)) ; **F**N**F**N **FFFF** **F**N**F**N **F**N**F**N (29.2)

$(\mu p.\varphi)^* := \exists q\ \forall p \subseteq q.$ $\underline{p \in PRE((\varphi^*_p)\restriction_q)} \rightarrow p(v)$, where $p \in PRE((\varphi^*_p)\restriction_q)$ expresses that $p \subseteq q$ is a prefixpoint of the map $(\varphi^*_p)\restriction_q$, that is: $\underline{p \in PRE((\varphi^*_p)\restriction_q)} := \forall w\ (q(w) \wedge \varphi^*[w/v]) \rightarrow p(w)$. (32.1)

%q&(#p<((q&(#r&(((q&r)&(r\s))>(p&s))))>(p&s))) ;
FFFF FFFF FFFF FFFF (32.2)

The conjecture to be tested is if Eqs. 29.1 is equivalent to 32.1. (1.1)

(#p&((#r&((r\s)>(p&r)))>(p&s)))=(%q&(#p<((q&(#r&(((q&r)&(r\s))>(p&s))))>(p&s)))) ;
TCTT TTTC TCTT TCCT (1.2)

Eq. 1.2 is *not* tautologous meaning the fixpoint operator cannot map mu-calculus onto second-order logic. This effectively refutes the mapping of mu-calculus onto second-order logic.

> **Remark:** We attempt to rehabilitate the conjecture of Eq. 1.1 by testing for implication (> replaces =). (2.1)

(#p&((#r&((r\s)>(p&r)))>(p&s)))>(%q&(#p<((q&(#r&(((q&r)&(r\s))>(p&s))))>(p&s)))) ;
TCTC TTTT TCTC TCTC (2.2)

which is less desirable value-wise, with six C (contingency or falsity) instead of five C.

Disjunctive normal form (DNF) in first order logic, minimized by FOL Optimizer

Lampert, Timm."Minimizing disjunctive normal forms of pure first-order logic". Logic Journal of IGPL.

From:
researchgate.net/publication/319304582_Minimizing_disjunctive_normal_forms_of_pure_first-order_logic

We use the apparatus of Meth8 modal logic model checker (system Ł4 as resuscitated in variant VŁ4).

The designated proof value is T (tautology); other logic values are: Contingent (falsity); Non-contingent (truth); and F (contradiction). The 2-tuple is respectively { 11, 10, 01, 00 }.

Truth tables are presented as repeating fragments of four 16-values out of 128 tables, as four row major horizontally.

We replicate three examples of equations in FOLDNFs.

LET: s t u x y F G H x y; ~ Not; & And, ^; + Or, V; = Equivalent to
universal quantifier; % existential quantifier

17. This problem of complexity is given as "a conjunction/disjunction of primary formulas may be equivalent to a conjunction / disjunction of minimized primary formulas (context sensitivity of minimization)." The example in the footnote is supposed to be a tautology, and rendered in pseudo script as:

%y(Fy^#x(Gx V Hxy))^#xGx == %yFy^#xGx (17.1)

In Meth8, Eq. 17.1 is mapped as:

((%y&((s&y)&(#x&((t&x)+((u&x)&y)))))&(#x&(t&x))) = ((%y&(s&y))&(#x&(t&x))) ;
TTTT TTTT TTTT TTTT ; Steps 35 (17.2)

Eq. 17.2 is replicated as an expected tautology. This demonstrates the problem of complexity in minimization is overcome seamlessly by the automated tool Meth8.

18. This problem is given as reliance on derivation trees for minimization. The example is rendered in pseudo script as:

LET: u v F G; p q r s t x1 x2 y1 y2 y3

%y1(¬Fy1&%y2(Fy2&%y3(¬Gy1y3&¬Gy3y2))) & #x1(Fx1+#x2(¬Fx2+Gx1x2));
(18.1)

In Meth8, Eq. 18.1 is mapped as:

(%r&(~(u&r)&(%s&((u&s)&(%t&((~v&(r&t))&(~v&(t&s)))))))) &
(#p&((u&p)+(#q&(~(u&q)+(v&(p&q)))))) ;
FFFF FFFF FFFF FFFF ; Steps 43 (18.2)

Eq. 18.2 is replicated as an expected contradiction. This demonstrates the problem of complexity in minimization is overcome seamlessly without reliance on special rules or derivation trees.

Test. We replicate the test problem for "FOL Optimizer". The test equation is rendered in pseudo script as:

LET: p q r s t u v w x z, x1 x2 y1 y2 y3 y4 y5 y6 y7 F

~#y6~~#y3~%y2%y4#x1~(~Fy3y3 + ~(#x2(%y1Fy2y1 & (~Fy2y4 > Fy3x1) &
%y5%y7((Fy5y7 & Fy5x2) + Fy6x2)))) ; (Test.1)

In Meth8, Eq. Test.1 is mapped as:

(~#w&(~~#t&(~%s&(%u&#p)))) & (~(~z&(t&t)) + ~(#q&(((%r&(z&(s&r))) &
((~z&(s&u))>(z&(t&p)))) & ((%v&%x)&(((z&(v&x))&(z&(v&q))) + (z&(w&q))))))) ;
Steps 59 ; (Test.2)

In the repeating fragment below of 32 truth tables (out of 128), Eq. Test.2 has this result.

```
FFFF FFFF FFFF FFFF,FFFF FFFF FFFF FFFF,FFFF FFFF FFFF FFFF,FNFN FNFN FFFF FFFF,
FFFF FFFF FFFF FFFF,FFFF FFFF FFFF FFFF,FFFF FFFF FFFF FFFF,FNFN FNFN FFFF FFFF,
FFFF FFFF FFFF FFFF,FFFF FFFF FFFF FFFF,FFFF FFFF FFFF FFFF,FFFF FFFF FFFF FFFF,
FFFF FFFF FFFF FFFF,FFFF FFFF FFFF FFFF,FFFF FFFF FFFF FFFF,FFFF FFFF FFFF FFFF,
FFFF FFFF FFFF FFFF,FFFF FFFF FFFF FFFF,FFFF FFFF FFFF FFFF,FNFN FNFN FFFF FFFF,
FFFF FFFF FFFF FFFF,FFFF FFFF FFFF FFFF,FFFF FFFF FFFF FFFF,FNFN FNFN FFFF FFFF,
FFFF FFFF FFFF FFFF,FFFF FFFF FFFF FFFF,FFFF FFFF FFFF FFFF,FFFF FFFF FFFF FFFF,
FFFF FFFF FFFF FFFF,FFFF FFFF FFFF FFFF,FFFF FFFF FFFF FFFF,FFFF FFFF FFFF FFFF
```

Result. We replicate the test result for FOL Optimizer. The result equation is rendered in pseudo script as:

#x1%y1Fx1y1 (Result.1)

In Meth8, Eq. Result.1 is mapped as:

(#p&%r)&(z&(p&r)) ; Steps 9 (Result.2)

In the two repeating fragments below, the truth table result of Eq. Result.2 does not match that of Result.1.

FFFF FFFF FFFF FFFF, (For tables 1- 63)
FFFF **FNFN** FFFF **FNFN**. (For tables 64-128)

We conclude that FOL Optimizer is not bivalent.

Refutation of first-order continuous induction on real closed fields

Abstract: By mapping definitions, theorems, and propositions into Meth8/VŁ4, we refute the first-order continuous induction principle on real closed fields.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

LET p, q, r, s, t: *x*, *y*, ε *or z*, ϕ, ψ;
~ Not; & And, ∧; > Imply, →; < Not Imply, less than;
= Equivalent, <==>; @ Not Equivalent;
% possibility, for one or some; # necessity, for every or all;
(p@p) zero; ~(y<x) (x≤y).

From: Salehi, S.; Zarza, M.(2018). First-order continuous induction, and a logical study of real closed fields. arxiv.org/pdf/1811.00284.pdf msz1982@gmail.com

"Continuous Induction", "Induction over the Continuum", "Real Induction", "Non-Discrete Induction", or the like, are some terms used by authors for referring to some statements about the continuum R. These statements are as strong as the Completeness Axiom of R and a motivation for their introduction into the literature of mathematics is the easy and sometimes unified ways they provide for proving some basic theorems of mathematical analysis. ... The continuous induction principle introduced in [5] (see also [6]) is equivalent to the following:

(IND''_{R}): $\exists x\forall y\leq x\phi(y) \land \exists\varepsilon>0\forall x\phi(x) \rightarrow \forall y[x\leq y\leq x+\varepsilon \rightarrow \phi(y)] \rightarrow \forall x\phi(x)$ (Def. 2.2.1)

```
((~(((p&s)&q)<(%p&#q))&(%r>((p@p)&(#p&s))))
>(~((p+r)<~(#q<p))>(s&#q))) > (#p&s) ;
```

NNNN **FFFF** NN**F**N **F**N**F**N (Def. 2.2.2)

(IND_{R}): $\exists x\forall y<x\phi(y) \land \forall x\forall y<x\phi(y) \rightarrow \exists z>x\forall y<z\phi(y) \rightarrow \forall x\phi(x)$ (Def. 2.4.1a)

```
((((%p&#q)<((p&s)&q))&((#p&#q)<((p&s)&q)))>
(%r>((p&#q) <(p+((r&s)&q)))))>((#p&s)&p) ;
```

FFFF **FFF**N **F**N**F**N **F**N**F**N (Def. 2.4.2a)

$\exists x\forall y<x\phi(y) \land \forall x\forall y<x\phi(y) \rightarrow \exists\varepsilon>0\forall y<x+\varepsilon\ \phi(y) \rightarrow \forall x\phi(x)$ (Def. 2.4.1b)

```
((((%p&#q)<((p&s)&q))&((#p&#q)<((p&s)&q)))>
(%r>(((p@p)&#q)<(p+((r&s)&q)))))>((#p&s)&p) ;
```

FFFF **FFF**N **F**N**F**N **F**N**F**N (Def. 2.4.2b)

Remark 2.4: Defs. 2.4.2a and 2.4.2b are equivalent by truth table result.

(IND'_{R}): $\exists x\forall y\leq x\phi(y) \land \forall x\forall y\leq x\phi(y) \rightarrow \exists z>x\forall y<z\phi(y) \land \forall x[\forall y<x\phi(y) \rightarrow \phi(x)]$
$\rightarrow \forall x\phi(x)$ (Def. 2.6.1)

(((~((p&s)&q)<(%p&#q))&~(((p&s)&q)<(#p&#q)))>
((%r>((p&#q)<((r&s)&q)))&((#q<((#p&s)&q))>(s&#p))))>((#p&s)&p) ;
CCTC TTTC CTTN TTTN (Def. 2.6.2)

In any dense linear order without endpoints ... , the scheme IND_R holds, if and only if IND'_R holds.
(IND_R <==> IND'_R) (Def. 2.4.1a) = (Def. 2.6.1) (Thrm. 2.7.1)

(((((%p&#q)<((p&s)&q))&((#p&#q)<((p&s)&q)))>
(%r>((p&#q) <(p+((r&s)&q)))))>((#p&s)&p))=
((((~((p&s)&q)<(%p&#q))&~(((p&s)&q)<(#p&#q)))>
((%r>((p&#q)<((r&s)&q)))&((#q<((#p&s)&q))>
(s&#p)))))>((#p&s)&p)) ; NN**F**N **FFFF** NN**FF** **F**N**F**T (Thrm. 2.7.2)

In any ordered Abelian group whose linear order is dense without endpoints,
if IND_R holds then IND''_R holds too. But not vice versa: IND''_R holds in the rational numbers but IND_R does not.
(IND_R </=> IND''_R): (Def. 2.4.1b) > (Def. 2.2.1) (Thrm. 2.8.1)

(((((%p&#q)<((p&s)&q))&((#p&#q)<((p&s)&q)))>
(%r>((p&#q) <(p+((r&s)&q)))))>((#p&s)&p))>
(((~(((p&s)&q)<(%p&#q))&(%r>((p@p)&(#p&s))))
>(~((p+r)<~(#q<p))>(s&#q))) > (#p&s)) ;
TTTT TTTC TTTT TTTT (Thrm. 2.8.2)

(SUP): $\exists x\phi(x) \wedge \exists y\forall x[\phi(x) \rightarrow x\leq y] \rightarrow \exists z\forall y\forall x[\phi(x) \rightarrow x\leq y] \leftrightarrow z\leq y$ (Def. 3.1.1)

((s&%p)&((s&#p)>~(%q<#p)))>(((s&#p)>~(#q<#p))=~(q<%r)) ;
TTTT TTTT TTTC TTTT (Def. 3.1.2)

(INF): $\exists x\phi(x) \wedge \exists y\forall x[\phi(x) \rightarrow y\leq x] \rightarrow \exists z\forall y\forall x[\phi(x) \rightarrow y\leq x] \leftrightarrow y\leq z$ (Def. 3.2.1)

((s&%p)&((s&#p)>~(#p<%q)))>(((s&#p)>~(#p<#q))=~(%r<q)) ;
TTTT TTTT NNTT NNTT (Def. 3.2.2)

In any dense linear order without endpoints ... , the scheme SUP holds, if and only if INF holds.
(SUP ⇔ INF): (Def. 3.1.1) = (Def. 3.2.1) (Prop. 3.3.1)

(((s&%p)&((s&#p)>~(%q<#p)))>(((s&#p)>~(#q<#p))=~(q<%r)))=
(((s&%p)&((s&#p)>~(#p<%q)))>(((s&#p)>~(#p<#q))=~(%r<q))) ;
TTTT TTTT NNTC NNTT (Prop. 3.3.2)

(CUT): $\exists x\exists y[\phi(x) \wedge \psi(y)] \wedge \forall x\forall y[\phi(x) \wedge \psi(y) \rightarrow x<y] \rightarrow \exists z\forall x\forall y[\phi(x) \wedge \psi(y) \rightarrow x\leq z\leq y]$
(Def. 3.4.1)

((((s&%p)&(t&%q))&(s&#p))&((t&#q)>(#p<#q)))>
(((s&#p)&(t&#q))>~(q<~(t<p))) ; TTTT TTTT TTTT TTTT (Def. 3.4.2)

In any dense linear order without endpoints ... , the scheme CUT holds, if and only if SUP holds.
(CUT <==> SUP): (Def. 3.4.1) = (Def. 3.1.1) (Prop. 3.5.1)

(((((s&%p)&(t&%q))&(s&#p))&((t&#q)>(#p<#q)))>
(((s&#p)&(t&#q))>~(q<~(t<p))))=
(((s&%p)&((s&#p)>~(%q<#p)))>(((s&#p)>~(#q<#p))=~(q<%r))) ;
`TTTT TTTT TTTC TTTT` (Prop. 3.5.2)

In any dense linear order without endpoints ... , the scheme INF holds, if and only if INDR holds.
(INF <==> IND$_R$): (Def. 3.2.1) = (Def. 2.4.1a) (Prop. 3.6.1)

(((s&%p)&((s&#p)>~(#p<%q)))>(((s&#p)>~(#p<#q))=~(%r<q)))=
(((((%p&#q)<((p&s)&q))&((#p&#q)<((p&s)&q)))>
(%r>((p&#q) <(p+((r&s)&q)))))>((#p&s)&p)) ;
FFFF FFFN CT**F**N CT**F**N (Prop. 3.6.2)

As rendered, Defs. 2.2, 2.4, 2.6, 3.1-3 are *not* tautologous. Def. 3.4 is tautologous, but as expected from Tarski. Thrms. 2.7 and 2.8 are *not* tautologous, and Props.3.5 and 3.6 are *not* tautologous.

Remark 3.5: Thrm. 2.8 and Prop. 3.5 produce truth tables which diverge from tautology by one value of falsity, as `C` for contingency. We terminate our evaluation after Section 3.

The Defs., Thrms., and Props. refute the first-order continuous induction principle on real closed fields.

Refutation of logic first order team (FOT)

Abstract: We evaluate a definition equation as *not* tautologous, hence refuting first order team (FOT) logics. Therefore FOT is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Juha Kontinen, J.; Yang, F. (2019). Logics for first-order team properties.
arxiv.org/pdf/1904.08695.pdf fan.yang.c@gmail.com

Abstract. [W]e introduce a logic based on team semantics, called **FOT**, whose [sic] expressive power coincides with first-order logic both on the level of sentences and (open) formulas

4 Axiomatizing FOT
In this section, we introduce a system of natural deduction for FOT, and prove the soundness and completeness theorem. For the convenience of our proofs, we present our system of natural deduction in sequent style.

Definition 6. The system of natural deduction for FOT consists of ...

$$cx \subseteq vy \text{ is short for } \exists^1 u(u = c \wedge ux \subseteq vy) \tag{4.6.1}$$

Remark 4.6.1: When evaluating FOT properties, we translate injected weakened operators as the standard operators.

LET p, u, v, x, y: c, u, v, x, y.
~((v&y)<(p&x))=((%u=p)&~((v&y)<(%u&x))) ;

NCNC NCNC NCNC NCNC, **F**T**F**T **F**T**F**T **F**T**F**T **F**T**F**T,
TCTC TCTC TCTC TCTC, TTTT TTTT TTTT TTTT

(4.6.2)

Eq. 4.6.2 as rendered is *not* tautologous, hence refuting first order team logics.

Refutation of Fodor's causality principle and extension to the class Fodor principle

Abstract: We evaluate Fodor's principle in two versions we name *weaker* and *stronger*. Neither is tautologous. By extension, the class Fodor principle is *not* tautologous. Because of this, using Kelly-Morse set theory as a basis for denial is rendered moot. However, to rely on KM set theory to deny Fodor principles as a class principle is obviated by the refutation above. Therefore Fodor's causality principles and extension are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Jerry_Fodor

"The asymmetric causal theory
The main problem with this theory is that of erroneous representations. There are two unavoidable problems with the idea that "a symbol expresses a property if it is ... necessary that all and only the presences of such a property cause the occurrences". The first is that not *all* horses cause occurrences of *horse*. The second is that not *only* horses cause occurrences of *horse.* Sometimes the *A*(*horses*) are caused by A (horses), but at other times—when, for example, because of the distance or conditions of low visibility, one has confused a cow for a horse—the *A* (*horses*) are caused by B (cows). In this case the symbol *A* doesn't express just the property A, but the disjunction of properties A or B. The crude causal theory is therefore incapable of distinguishing the case in which the content of a symbol is disjunctive from the case in which it isn't. This gives rise to what Fodor calls the "problem of disjunction".

Fodor responds to this problem with what he defines as "a slightly less crude causal theory". According to this approach, it is necessary to break the symmetry at the base of the crude causal theory. Fodor must find some criterion for distinguishing the occurrences of *A* caused by A's (true) from those caused by B's (false). The point of departure, according to Fodor, is that while the false cases are *ontologically dependent* on the true cases, the reverse is not true. There is an asymmetry of dependence, in other words, between the true contents (*A*=A) (1.1)

LET p, q, r: *A*, A, B
p=q ; T**FF**T T**FF**T T**FF**T T**FF**T (1.2)

and the false ones (*A*=A or B). (2.1)

p=(q+r) ; `T`**`FF`**`T` **`F`**`T`**`F`**`T` `T`**`FF`**`T` **`F`**`T`**`F`**`T` (2.2)

The first can subsist independently of the second, but the second can occur only because of the existence of the first" (3.0)

Remark 3.0: We write Eq. 3.0 as 1.1 implies 2.1. (3.1)

(p=q) >(p=(q+r)) ; `TTTT` **`F`**`TTT` `TTTT` **`F`**`TTT` (3.2)

Remark 3.2: We name this Fodor's *weaker* principle.

To strengthen Fodor's weaker principle, we write Eq. 1.1 (*A*=A) as

(A=A), (4.1)

q=q ; `TTTT TTTT TTTT TTTT` (4.2)

and 2.1 (*A*=A or B) as

(A=A or B). (5.1)

q=(q+r) ; `TTTT` **`FF`**`TT` `TTTT` **`FF`**`TT` (5.2)

Remark 3.0: We rewrite Eq. 3.0 as 4.1 implies 5.1. (6.1)

(q=q) >(q=(q+r)) ; `TTTT` **`FF`**`TT` `TTTT` **`FF`**`TT` (6.2)

Remark 6.2: We name this Fodor's *stronger* principle.

Fodor's weaker principle (Eqs. 1, 2, 3 as expressed) is *not* tautologous; and Fodor's stronger principle (4,5,6) is *not* tautologous, thereby refuting the Fodor principle. In particular, the notion of Eq. 3.0 that while a true antecedent is independent of a consequent, the consequent can occur only because of the existence of the antecedent is mistaken.

These principles are extendable to the class Fodor principle, for example:

Gitman, V.; Hamkins, J,D.; Karagila, A. (2019).
Kelley-Morse set theory does not prove the class Fodor principle. arxiv.org/pdf/1904.04190.pdf
vgitman@nylogic.org, jhamkins@gc.cuny.edu, karagila@math.huji.ac.il

However, to rely on KM set theory to deny the Fodor principle, as the class Fodor principle, is obviated by the refutation above.

Confirmation of the conjecture that any force speed is greater than a light speed

Abstract: We evaluate two versions of the conjecture that any force speed is greater than a light speed. The conjecture is confirmed as tautologous.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

We evaluate force speed greater than light speed. In modal terms, we write this as necessity of force speed (passing through matter unhindered) is greater than possibility of light speed (varying as to medium). In quantified terms, we write this as any force speed greater than at least one light speed.

Definition 6.1: The necessity of force speed greater than the possibility of light speed implies that: the combination of the necessity of force speed and possibility of light speed is greater than the necessity of force speed lesser than the possibility of light speed implying the combination of the necessity of force speed and possibility of light speed. (6.1)

LET p, q, r, s: force speed, q, r, light speed.

(#p>%s)>((#p&%s)>((#p<%s)>(#p&%s))) ; TTTT TTTT TTTT TTTT (6.2)

Remark 6.2: Eq. 6.1 can be written with the consequent having a negation clause.

Definition 7.1: The necessity of force speed greater than the possibility of light speed implies that: the combination of the necessity of force speed and possibility of light speed is greater than not the combination of the necessity of force speed and possibility of light speed implying the necessity of force speed lesser than the possibility of light speed. (7.1)

(#p>%s)>((#p&%s)>~((#p&%s)>(#p<%s))) ;
TTTT TTTT TTTT TTTT (7.2)

Eqs. 6.2 and 7.2 as rendered are tautologous, hence confirming any force speed is greater than a light speed.

Refutation of the method of forcing

Abstract: We evaluate two examples of paradoxes to be resolved by the method of forcing. The first is *no*t tautologous, and the second is a contradiction so *not* proved as a paradox. This means the forcing method is refuted because it cannot coerce the two examples into abstract proofs and with the ultimate goal to produce larger truth values. What follows is that the forcing method is better suited for paraconsistent logics which are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⇔, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) C non-contingency, ∇, ordinal 2; (%z>#z) N as non-contingency, ∆, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Moore, J.T. (2019). The method of forcing. arxiv.org/pdf/1902.03235.pdf justin@math.cornell.edu

1. Introduction: Let us begin with two thought experiments. ... First consider the following "paradox" ... in more formal language we have that, "for all z∈ R, *almost surely* Z≠z", while "*almost surely* there exists a z∈ R, Z=z." (1.1.1)

Remark 1.1.1: We interpret "almost surely" to mean "possibly".

LET p, q, r: z, Z, R

((#p<r)>(q@p))&((%p<r)>(q=p)) ; T**F**NC TTTT T**F**NC TTTT (1.1.2)

Next suppose ... [i]n terms of the formal logic, we have that, "for all i≠j in I, *almost surel*y the event $Z_i \neq Z_j$ occurs", while "*almost surely it is false that* for all i≠j∈ I, the event $Z_i \neq Z_j$ occurs". (1.2.1)

LET p, q, r, s: i, j, I, Z

((#(p@q)<r)>%((s&p)@(s&q)))&~((#(p@q)<r)>%((s&p)@(s&q))) ;
FFFF FFFF FFFF FFFF (1.2.2)

It is natural to ask whether it is possible to revise the notion of *almost sure*ly so that its meaning remains unchanged for simple logical assertions such as $Z_i \neq Z_j$ but such that it commutes with quantification. ... Such a formalism would describe truth in a necessarily larger model of mathematics, one in which there are new outcomes to the random experiment which did not exist before the experiment was performed.

From a modern perspective, forcing provides a formalism for examining what occurs *almost surely* not only in probability spaces but also in a much more general setting than what is provided by our conventional notion of randomness. Forcing has proved extremely useful in developing and understanding of models of set theory and in determining what can and cannot be proved within the standard axiomatization of mathematics (which we will take to be ZFC). (1.3.1)

> **Remark 1.3.1:** The author assumes ZFC is an axiomatization of mathematics, which elsewhere we show otherwise.

In fact it is a heuristic of modern set theory that if a statement arises naturally in mathematics and is consistent, then its consistency can be established using forcing, possibly starting from a large cardinal hypothesis. (1.4.1)

> **Remark 1.4.1:** An heuristic is not a theorem but the hypothetical starting point of an hypothesis as evaluated by an iterative loop of trial-and-error.

The focus of this article ... is to demonstrate how the method of forcing can be used to p*rove theorem*s as opposed to *establish consistency result*s. Forcing itself concerns the study of adding *generic object*s to a model of set theory, resulting in a larger model of set theory. One of the key aspects of forcing is that it provides a formalism for studying what happens *almost surely* as the result of introducing a generic object. An analysis of this formalism sometimes leads to new results concerning the original model itself — results which are in fact independent of the model entirely. (1.5.1)

> **Remark 1.5.1:** The method of forcing injects itself onto a fiducial model as a larger abstraction which is then named differently as a generic model. However, the larger problem of this method is that forcing cannot be entirely separated from and fully independent of the original model as its basis.

Eqs. 1.1.2 and 1.2.2 as rendered are *no*t tautologous with the latter as a contradiction to mean it is not proved as a paradox. This means the two examples in the introduction to demonstrate the forcing method cannot be forced into proofs, hence refuting the method of forcing to produce larger truth values. What follows is that the forcing method is better suited for paraconsistent logics which we demonstrate elsewhere are *non* tautologous fragments of the universal logic VŁ4.

Refutation of forcing to change large cardinal strength

Abstract: The seminal theorem of the dissertation states a cardinal is greatly inaccessible if and only if it is Mahlo. Three non trivial equations of the proof are *not* tautologous, thereby refuting theorems derived therefrom. These conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Carmondy, E. (2015). Force to change large cardinal strength.
arxiv.org/pdf/1506.03432.pdf ecarmody@gradcenter.cuny.edu

Abstract: This dissertation includes many theorems which show how to change large cardinal properties with forcing. I consider in detail the degrees of inaccessible cardinals (an analogue of the classical degrees of Mahlo cardinals) and provide new large cardinal definitions for degrees of inaccessible cardinals extending the hyper-inaccessible hierarchy. I showed that for every cardinal κ, and ordinal α, if κ is α-inaccerssible, then there is a P forcing that κ which preserves that α-inaccessible but destorys [sic] that κ is (α+1)-inaccessible. I also consider Mahlo cardinals and degrees of Mahlo cardinals. I showed that for every cardinal κ, and ordinal α, there is a notion of forcing P such that κ is still α-Mahlo in the extension, but κ is no longer (α+1)-Mahlo. I also show that a cardinal κ which is Mahlo in the ground model can have every possible inaccessible degree in the forcing extension, but no longer be Mahlo there. The thesis includes a collection of results which give forcing notions which change large cardinal strength from weakly compact to weakly measurable, including some earlier work by others that fit this theme. I consider in detail measurable cardinals and Mitchell rank. I show how to change a class of measurable cardinals by forcing to an extension where all measurable cardinals above some fixed ordinal α have Mitchell rank below α. Finally, I consider supercompact cardinals, and a few theorems about strongly compact cardinals. Here, I show how to change the Mitchell rank for supercompactness for a class of cardinals.

Theorem 10. *A cardinal κ is greatly inaccessible if and only if κ is Mahlo.*

Remark 10: The first step of the proof as 10.1.1 is trivial and ignored here.

Proof. . . . Next, if A ∈ F, and A ⊆ B, then there is a club C ∈ F such that C ∩ I ⊆ A ⊆ B, thus B ∈ F, by construction, since F is the filter generated by sets of this form. (10.2.1) (((p<t)&~(q<p))>((r<t)>(~q<~(p<(r&u)))))>(q<t) ;

FFTT **FF**TT **FF**TT **FF**TT,
FFFF FFFF FFFF FFFF,
FFTT **F**TTT **FF**TT **F**TTT,
FFFF FFFF FFFF FFFF (10.2.2)

Third, if A and B are elements of F, then there are clubs C and D such that C ∩ I ⊆ A, and D ∩ I ⊆ B, so A ∩ B contains (C ∩ D) ∩ I. (10.3.1)

(((p&q)<t)>((~p<(%r&u))&~(q<(%s&u))))>((p&q)>((r&s)&u)) ;

TTTT TTTT TTTT TTTT,
TTT**F** TTT**F** TTT**F** TTTT (10.3.2)

This is of the form which generated the filter, thus A ∩ B ∈ F. (10.4.1)

((((p&q)<t)>((~p<(%r&u))&~(q<(%s&u))))>((p&q)>((r&s)&u)))>((p&q)<t) ;

FFFT **FFF**T **FFF**T **FFF**T(3),
FFFT **FFF**T **FFF**T **FFFF**(1) (10.4.2)

Eqs. 10.2.2-10.4.2 as rendered are *not* tautologous. Since Theorem 10 is seminal to the dissertation, theorems derived therefrom are refuted.

Refutation of formalization of axiom of choice and equivalent theorems using the Coq tool

Abstract: We evaluate two definitions for maximum and minimal set membership, for the nesting of sets, and the equivalence relations of axiom of choice, Tukey's lemma, Hausdorff maximal principle, maximal principle, Zermelo's postulate, Zorn's lemma, well-ordering theorem. None is tautologous, refuting the claims. The authors conclude: "The whole process of formal proof demonstrates that the Coq-based machine proving of mathematics theorem is highly reliable and rigorous. The formal work of this paper is enough for most applications, especially in set theory, topology and algebra." We refute those assertions based on the non-bivalent performance of the Coq proof assistant. Therefore, these formalizations and methodology render a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Sun, T.; Yu, W. (2019). Formalization of the axiom of choice and its equivalent theorems. arxiv.org/pdf/1906.03930.pdf {stycyj,wsyu}@bupt.edu.cn

Abstract. In this paper, we describe the formalization of the axiom of choice and several of its famous equivalent theorems in Morse-Kelley set theory. These theorems include Tukey's lemma, the Hausdorff maximal principle, the maximal principle, Zermelo's postulate, Zorn's lemma and the well-ordering theorem. We prove the above theorems by the axiom of choice in turn, and finally prove the axiom of choice by Zermelo's postulate and the well-ordering theorem, thus completing the cyclic proof of the equivalence between them. The proofs are checked formally using the Coq proof assistant in which Morse-Kelley set theory is formalized. The whole process of formal proof demonstrates that the Coq-based machine proving of mathematics theorem is highly reliable and rigorous. The formal work of this paper is enough for most applications, especially in set theory, topology and algebra.

The following definitions are very important, and they are used in Tukey's lemma, the Hausdorff maximal principle and so on.

Definition 3.2 (Maximal (Minimal) Member)**.** *F* is a maximal (minimal) member of *f* iff no member of *f* properly contains *F* (no member of *f* is properly contained in *F*). When *f* is equal to empty, *f* has no maximal (minimal) member. Thus we add the condition *f* ≠ Ø *;* when we formalize the maximal (minimal) member. The condition is very important in proving the existence of maximal elements in a set. At the same time, it eliminates many unnecessary discussions.

Definition MaxMember (F f: Class) : Prop : f ≠ Ø; -> F ∈ f ∧ (∀E, E ∈ f -> ~ (F ⊊ E)). (3.2.1.1)

LET p, q, r, s: f, E, F, s.

(p@(s@s))>((r<p)&((#q<p)>~(r<#q))) ;
T**F**T**F** T**F**T**F** T**F**T**F** T**F**T**F** (3.2.1.2)

Definition MinMember (F f: Class) : Prop : f ≠ Ø; -> F ∈ f ∧ (∀E, E ∈ f -> ~ (E ⊊ F)). (3.2.2.1)

(p@(s@s))>((r<p)&((#q<p)>~(#q<r))) ;
T**F**T**F** T**F**T**F** T**F**T**F** T**F**T**F** (3.2.2.2)

The following is the definition of nest. It will be used in the description of the Hausdorff maximal principle. The specific description and Coq formalization of it are as follows:

Definition 3.3 (Nest)**.** *n* is a nest if and only if, whenever *x* and *y* are members of *n*, then either $x \subset y$ or $y \subset x$.

Definition Nest n : Prop := ∀ x y, x ∈ n ∧ y∈ n -> x⊂ y ∨ y⊂ x. (3.3.1.1)

((#p<r)&(#q<r))>((#p<#q)&(#q<#p)) ;
TTTC TTTT TTTC TTTT (3.3.1.2)

Remark 3.3: Eqs. 3.2.1.2, 3.2.2.2, and 3.3.1.2 as rendered are *not* tautologous. This leads us to abandon mappings of subsequent equations with Coq-unique commands such as Ensemble, etc.

4. Formal proof of the equivalence

In this section, we present the formal proof of AC and its equivalent theorems. As shown in Figure 1, we start from AC to prove Tukey's lemma, the Hausdorff maximal principle, the maximal principle, Zermelo's postulate, Zorn's lemma and the well-ordering theorem in turn. We prove AC through Zermelo's postulate and the well-ordering theorem finally, thus completing the cyclic proof of the equivalence between AC and these theorems. Before each theorem is proved, we will give its formal description.

Figure 1: The relation of AC and its equivalent theorems

Axiom of Choice → Tukey's Lemma → Hausdorff Maximal Principle → Maximal Principle →

Zermelo's Postulate → Axiom of Choice

[or]

Zorn's Lemma → Well-ordering theorem → Axiom of Choice (4.0.1)

LET p axiom of choice; q Tukey's lemma; r Hausdorff maximal principle;
s maximal principle; t Zermelo's postulate; u Zorn's lemma; v well-ordering theorem.

((p>q)>(r>s))>(t>p))+((u>v)>p)) ; TTTT TTTT TTTT TTTT}x1
FT**F**T TTTT **F**T**F**T **F**T**F**T}

```
TTTT TTTT TTTT TTTT( 2)
TTTT TTTT TTTT TTTT}x2
FTFT TTTT FTFT FTFT}
```
(4.0.2)

Remark 4.0: Eq. 4.0.2 as rendered is *not* tautologous, refuting the claimed relation of AC and its equivalent theorems using the Coq-assistant.

Refutation of Frauchiger-Renner paradox

Abstract: We evaluate unique conjunctive combinations of four statements, and as doubles and triples, which are *not* tautologous. This means the experiment is framed on conjectures for a flawed model and form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Gurappa, N. (2019). Resolving Schrödinger's cat, Wigner's friend and Frauchiger-Renner's paradoxes at a single-quantum level. vixra.org/pdf/1909.0397v1.pdf

[Text not reproduced for the four statements for brevity.]

LET	p,	q,	r,	s,	t,	u,	v,	w,	x:
	-F,	F,	-W,	W,	t,	fw,	bk,	up,	dn.

p>(t>(s>u)) ; (1.2)
q>(w>(p>t)) ; (2.2)
r>(u>(q>w)) ; (3.2)
s>(v>(r>v)) ; (4.2)

Eqs. 1.2-4.2 are required as a conjunctive for the experiment.

((p>(t>(s>u)))&(q>(w>(p>t))))&((r>(u>(q>w)))&(s>(v>(r>v)))) ;

TTTT TTTT TTTT TTTT }
TTTT TTTT T**F**T**F** T**F**T**F** }
TTTT TT**FF** TTTT TT**FF** }
TTTT TT**FF** TTTT TT**FF** }x2

TTT**F** TTT**F** TTT**F** TTT**F** }
TTTT TTTT T**F**T**F** T**F**T**F** }
TTT**F** TTT**F** TTT**F** TTT**F** }
TTTT TTTT TTTT TTTT }x2

Eqs. 1.2-4.2 as a conjunctive are *not* tautologous.

Unique combinations of 1.2-4.2 as conjunctives of doubles and triples are also *not* tautologous. Hence the experiment is framed on conjectures which are *not* tautologous and is a flawed model.

Refutation of the Frauchiger-Renner thought experiment with modal operators as a paradox

Abstract: We use modal logic to evaluate a quantum rendition of the Frauchiger-Renner thought experiment to refute it as paradox (contradiction) *and* as tautology.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET p, q, r, s: u, w, a, b;
~ Not; + Or ; & And; > Imply; = Equivalent; @ Not Equivalent;
% possibility, for one or some; # necessity, for all or every;
(p=p) ok; (q@q) fail; (%r>#r), (%s>#s) ordinal one, 1.

From: Nurgalieva, N.; del Rio, L. (2018). Inadequacy of modal logic in quantum settings. arxiv.org/pdf/1804.01106.pdf delrio@phys.ethz.ch

Remark 0: The paper described the thought experiment in several ways, however examples for *t* were not clear. Therefore we relied on the simpler equations of results from the sketch of the reasoning of agents.

(u = ok) → (b = 1) (1.1)

(p=(p=p)) > (s=(%s>#s)) ; `TCTC TCTC TNTN TNTN` (1.2)

(b = 1) → (a = 1) ; (2.1)

(s=(%s>#s)) > (r=(%r>#r)) ; `TTTT NNNN CCCC TTTT` (2.2)

(a = 1) → (w = fail) (3.1)

(r=(%r>#r)) > (q=(q@q)) ; `TTNN TTCC TTNN TTCC` (3.2)

The text injects "w =" into the antecedent of Eq. 1.1 as "(w = u = ok)" for:

(w = Eqs. 1.1) → 2.1 → 3.1 (4.1)

((((p=q)=(p=p))>(s=(%s>#s)))>((s=(%s>#s))>(r=(%r>#r))))>((r=(%r>#r))>(q=(q@q))) ;
`TTNN TTCC TTNN TTCC` (4.2)

Remark 5: Without the injection, Eqs. 1.1 → 2.1 → 3.1, with the table result as that for 4.2: (5.1)

(((p=(p=p))>(s=(%s>#s)))>((s=(%s>#s))>(r=(%r>#r))))>((r=(%r>#r))>(q=(q@q))) ;
`TTNN TTCC TTNN TTCC` (5.2)

Eqs. 1.2-5.2 as rendered are *not* tautologous, meaning the quantum example of the Frauchiger-Renner thought experiment is refuted as a paradox (contradiction), and using modal operators. We stopped evaluation of the paper title at this point.

Refutation of the Frauchiger-Renner thought paradox as a quantum model

Abstract: We use modal logic to evaluate a quantum model of the Frauchiger-Renner thought experiment as not a contradiction (paradox) *and* not a tautology (theorem). The example misapplies the Born rule which we refute elsewhere.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

LET p , q , r, s, t , u , v , w , x , y , z :
Alice, |1>, R, S, memory, Ursula, Bob, Wigner, √1, √2, √3,
~q |0>;
~ Not; + Or ; & And; \ Not And; > Imply; = Equivalent; @ Not Equivalent;
% possibility, for one or some; # necessity, for all or every;
(p=p) ok, tautology, ordinal three 3; (p@p) fail, contradiction, zero 0;
(%p>#p) truthity, ordinal one 1; (%p<#p) falsity, ordinal two 2;
a=0 ((p&t)=(~q&p)); a=1 ((p&t)=(q&p);
b=0 ((v&t)=(~q&v)); b=1 ((v&t)=(q&v).

From: Nurgalieva, N.; del Rio, L. (2018). Inadequacy of modal logic in quantum settings. arxiv.org/pdf/1804.01106.pdf delrio@phys.ethz.ch

Initial settings:

R = ((√(1/3)|0>$_R$)+((√(2/3)|1>$_R$) (1.0.1.1)

r=(((x\y)&(~q&r))+((y\z)&(q&r))) ;
TTTT TTTT TTTT TTTT, TTTT **FF**TT TTTT **FF**TT
TTTT TT**FF** TTTT TT**FF**, TTTT **FFFF** TTTT **FFFF** (1.0.1.2)

S = |0>$_S$ (1.0.2.1)

s=(~q&s) ;
TTTT TT**FF** TTTT TT**FF** (1.0.2.2)

Alice memory = |0>$_A$ (1.0.3.1)

(p&t)=(~q&p) ;
T**F**TT T**F**TT T**F**TT T**F**TT, TTT**F** TTT**F** TTT**F** TTT**F** (1.0.3.2)

Bob memory = |0>$_B$ (1.0.4.1)

(v&t)=(~q&v) ;
TTTT TTTT TTTT TTTT(4), **FF**TT **FF**TT **FF**TT **F**FTT,
TT**FF** TT**FF** TT**FF** TT**FF** (1.0.4.2)

Ursula memory = $|0>_{U}$ (1.0.5.1)

(u&t)=(~q&u) ;
TTTT TTTT TTTT TTTT(2), **FF**TT **FF**TT **FF**TT **F**FTT,
TT**FF** TT**FF** TT**FF** TT**FF** (1.0.5.2)

Wigner memory = $|0>_{W}$ (1.0.6.1)

(w&t)=(~q&w) ;
TTTT TTTT TTTT TTTT(8), **FF**TT **FF**TT **FF**TT **F**FTT,
TT**FF** TT**FF** TT**FF** TT**FF** (1.0.6.2)

T 1: **Remark 1:** We map agent *T* = <1, 2, 3, 4, 5> using instructions in the text as best as we can follow.

R = $|0>_{R}$, $|1>_{R}$ (1.1.1.1)

r=((~q&r)+(q&r)) ;
TTTT TTTT TTTT TTTT(128) (1.1.1.2)

Alice records the result in her memory A. (1.1.2.1)

(p&t)=((~q&p)+(q&p));
T**F**T**F** T**F**T**F** T**F**T**F** T**F**T**F**, TTTT TTTT TTTT TTTT (1.1.2.2)

Alice prepares S :
if outcome a = 0 then her memory is $|0>_{A}$ and S is $|0>_{S}$; or
if outcome a = 1 then her memory is $|1>_{A}$ and S is
$(1/\sqrt{2})((|0>_{S})+|1>_{S}))$ (1.1.3.1)

(((p&t)=(~q&p))>(((p&t)=(~q&p))&(s=(~q&s)))) +
(((p&t)=(q&p))>(((p&t)=(q&p))&(s=((x\y)&((~q&s)+(q&s)))))) ;
TTTT TTTT TT**F**T TT**F**T, TTTT TTTT TTTT TTTT (1.1.3.2)

Alice replaces Bob's system S with her own. (1.1.4.1)

s=((((p&t)=(~q&p))>(((p&t)=(~q&p))&(s=(~q&s)))) +
(((p&t)=(q&p))>(((p&t)=(q&p))&(s=((x\y)&((~q&s)+(q&s))))))) ;
FFFF FFFF TTTT TTTT, **FFFF FFFF** TT**F**T TT**F**T (1.1.4.2)

T 2: **Remark 2:** We follow *T* 1 by replacing Alice with Bob and R with S, but exclude Eq. 1.1.3.1 for Bob.

S = $|0>_{S}$, $|1>_{S}$ (2.1.1.1)

s=((~q&s)+(q&s)) ;
TTTT TTTT TTTT TTTT(128) (2.1.1.2)

Bob records the result in his memory B. (2.1.2.1)

(v&t)=((~q&v)+(q&v));
TTTT TTTT TTTT TTTT(4), **FFFF FFFF FFFF FFFF** (2.1.2.2)

Bob prepares R :
if outcome b = 0 then his memory is $|0>_{B}$ and R is $|0>_{R}$; or
if outcome b = 1 then his memory is $|1>_{B}$ and R is
$(1/\surd 2)((|0>_{R})+|1>_{R}))$ (2.1.3.1)

(((v&t)=(~q&v))>(((v&t)=(~q&v))&(r=(~q&r))))
+
(((v&t)=(q&v))>(((v&t)=(q&v))&(r=((x\y)&((~q&r)+(q&r)))))) ;
TTTT TTTT TTTT TTTT, TTTT TT**FF** TTTT TT**FF** (2.1.3.2)

T 3: Ursula measures and records the result of Alice's lab as:

RA = $|ok>_{RA}$, $|fail>_{RA}$ where

$|ok>_{RA} =\surd(1/2)(|0>_{R}|0>_{A} - |1>_{R}|1>_{A})$
$|fail>_{RA}=\surd(1/2)(|0>_{R}|0>_{A} + |1>_{R}|1>_{A})$ (3.1.1.1)

(((r&p)&(p=p))=((x\y)&(((~q&r)&(~q&p))-((q&r)&(q&p))))) +
(((r&p)&(p@p))=((x\y)&(((~q&r)&(~q&p))+((q&r)&(q&p))))) ;
TTTT T**F**T**F** TTTT T**F**T**F**, TTTT TTTT TTTT TTTT (3.1.1.2)

T 4: Wigner measures and records the result of Bob's lab as:

SB = $|ok>_{SB}$, $|fail>_{SB}$ where

$|ok>_{SB} =\surd(1/2)(|0>_{S}|0>_{B} - |1>_{S}|1>_{B})$
$|fail>_{SB}=\surd(1/2)(|0>_{S}|0>_{B} + |1>_{S}|1>_{B})$ (4.1.1.1)

(((s&v)&(p=p))=((x\y)&(((~q&s)&(~q&v))-((q&s)&(q&v))))) +
(((s&v)&(p@p))=((x\y)&(((~q&s)&(~q&v))+((q&s)&(q&v))))) ;
TTTT TTTT TTTT TTTT, TTTT TTTT **FFFF FFFF** (4.1.1.2)

T 5: Ursula and Wigner compare their recorded measurements.
If both are ok, then the experiment ends, otherwise initial settings and timers are reset to repeat. (5.0)

Excepting the obvious theorems of Eqs. 1.1.1.2 and 2.1.1.2, Eqs. 1.2-4. as rendered are *no*t tautologous. This means the model conjectured is not a contradiction (paradox) *and* not a tautology (theorem). Therefore the model is indeterminate.

The authors invoke the Born rule. (We refute the Born rule elsewhere in Everettian quantum mechanics (EQM) as the probability of the wave function squared.) The authors halt the experiment at an injected *T* 2.5 to give a probability of 1/12. In fact, the model cannot halt because at each iteration, initial values are reset .

We ask what is the logical table result of the entire system as rendered, combining each step as an antecedent to imply the next step as a consequent. This amounts to:

If Eqs.1.0, then if T 1 and T 2 then T 3 and T 4. (6.1)

(((r=(((x\y)&(~q&r))+((y\z)&(q&r))))&(s=(~q&s)))&((((p&t)=
(~q&p))&((v&t)=(~q&v)))&(((u&t)=(~q&u))&((w&t)=
(~q&w)))))
\>
((((r=((~q&r)+(q&r)))>((p&t)=((~q&p)+(q&p))))>(s=((((p&t)=
(~q&p))>(((p&t)=(~q&p))&(s=(~q&s))))+(((p&t)=(q&p))>
(((p&t)=(q&p))&(s=((x\y)&((~q&s)+(q&s)))))))))
&
(((s=((~q&s)+(q&s)))>((v&t)=((~q&v)+(q&v))))>
(((((v&t)=(~q&v))>(((v&t)=(~q&v))&(r=(~q&r))))+(((v&t)=
(q&v))> (((v&t)=(q&v))& (r=((x\y)&((~q&r)+(q&r)))))))
\>
(((((r&p)&(p=p))=((x\y)& (((~q&r)&(~q&p))-((q&r)&(q&p))))))+
(((r&p)&(p@p))=((x\y)&(((~q&r)&(~q&p))+((q&r)&(q&p))))))
&
((((s&v)&(p=p))=((x\y)&(((~q&s)&(~q&v))-((q&s)&(q&v))))))+
(((s&v)&(p@p))=((x\y)&(((~q&s)&(~q&v))+((q&s)&
(q&v)))))))))) ;

FT**F**T **F**T**FF** TTTT TTTT, **FFF**T **FFF**T TTTT T**F**TT,
FFFT **FF**TT TTTT T**F**TT, **FFF**T TT**F**T TTTT TTTT,
FFFT TTTT TTTT TTTT, **FF**TT **FF**TT **FF**TT **F**FTT,
FFTT **FF**TT TTTT T**F**TT, **FF**TT TTTT TTTT TTTT,
FT**F**T **F**TTT TTTT TTTT, **F**T**F**T TT**F**T TTTT TTTT,
FT**F**T TTTT TTTT TTTT, TT**F**T TT**FF** TTTT TTTT,
TT**F**T TT**F**T TTTT TTTT, TT**F**T TTTT TTTT TTTT
(317 steps) (6.2)

Eq. 6.2 is *not* tautologous, meaning the thought experiment is not a paradox (contradiction) *and* not a theorem (tautology).

Refutation of Fredkin paradox in one variable

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table fragment) is row-major and horizontal.

From: en.wikipedia.org/wiki/Fredkin%27s_paradox

LET ~ Not; & And; + Or; - Not Or; > Imply, greater than; < Not Imply, less than;
@ Not Equivalent;
p, ~p : chosen state, alternative to chosen state.

"The more equally attractive two alternatives seem, the harder it can be to choose between them—no matter that, to the same degree, the choice can only matter less." (1.1)

((p>~p)&((p-~p)>(p@p)))>((p+~p)<((p>~p)&((p-~p)>(p@p)))) ;
FTFT FTFT FTFT FTFT (1.2)

Eq. 1.2 as rendered is *not* contradictory, and hence refutes the Fredkin paradox in *one* variable.

Refutation of free choice permission (FCP) in deontic logic

Abstract: The formula for free choice permission (FCP) in deontic logic as P(p\/q)→(Pp/\Pq) (FCP) is *not* tautologous, *not* a paradox, and hence *not* applicable in Hilbert-style classical deontic logic as a guarded version. Therefore FCP forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Governatori, G.; Rotolo, A. (2019). Is free choice permission admissible in classical deontic logic?. arxiv.org/pdf/1905.07696.pdf

Abstract In this paper, we explore how, and if, free choice permission (FCP) can be accepted when we consider deontic conflicts between certain types of permissions and obligations. As is well known, FCP can license, under some minimal conditions, the derivation of an indefinite number of permissions. We discuss this and other drawbacks and present six Hilbert-style classical deontic systems admitting a guarded version of FCP. The systems that we present are not too weak from the inferential viewpoint, as far as permission is concerned, and do not commit to weakening any specific logic for obligations.

1 Introduction and background

A significant part of the literature in deontic logic revolves around the discussions of puzzles and paradoxes which show that certain logical systems are not acceptable—typically, this happens with deontic KD, i.e., Standard Deontic Logic (SDL)—or which suggest that obligations and permissions should enjoy some desirable properties. One well-known puzzle is the so-called Free Choice Permission paradox, which was originated by the following remark by von Wright in [23, p. 21]: "On an ordinary understanding of the phrase 'it is permitted that', the formula 'P(p∨q)' seems to entail 'Pp∧Pq'. If I say to somebody 'you may work or relax' I normally mean that the person addressed has my permission to work and also my permission to relax. It is up to him to choose between the two alternatives." Usually, this intuition is formalised by the following schema:

P(p ∨ q) → (Pp ∧ Pq) (FCP) (1.1)

LET p, q, r: p, r, P(ermission)

(r&(p+q))>((r&p)&(r&q)) ;

`TTTT T`**`FF`**`T TTTT T`**`FF`**`T` (1.2)

Remark 1.2: Eq. 1.2 as rendered is not tautologous. This refutes FCP as a paradox and its subsequent use in Hilbert-style classical deontic systems with a guarded version of FCP.

Refutation of replacing classical logic with free logic

Abstract: We evaluated 12 equations for the assertions with none tautologous. Therefore this conjecture is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Bornali, P. (2019). Proposal of replacing classical logic with free logic for reasoning with non-referring names in ordinary discourse. vixra.org/pdf/1905.0358v1.pdf [no email published]

Abstract: Reasoning carried out in ordinary language, can not avoid using non-referring names if occasion arises. Semantics of classical logic does not fit well for dealing with sentences with non-referring names of the language. The principle of bivalence does not allow any third truth-value, it does not allow truth-value gap also. The outcome is an ad hoc stipulation that no names should be referentless. The aim of this paper is to evaluate how far free logic with supervaluational semantics is appropriate for dealing with the problems of non-referring names used in sentences of ordinary language, at the cost of validity of some of the classical logical theses/ principles.

3.1. Presupposition as a semantic relation … is as follows: If A and B are two propositions, then a characterization of presupposition can be given in a language as,

A presupposes B iff A is neither true nor false unless B is true. (3.1.1.1)

LET p, q: A, B. (This makes for shorter table results for propositions and not theorems.)

((q=(q=q))>~(~((p=~(p=p))+(p=~(p@p)))=(p=p)))>(p>q) ;
T**F**TT T**F**TT T**F**TT T**F**TT (3.1.1.2)

This is equivalent to, If A is true, then B is true and, If A is false, then B is true ... (3.1.2.1)

((p=(p=p))>(q=(q=q)))&((p=(p@p))>(q=(q=q))) ;
FFTT **FF**TT **FF**TT **FF**TT (3.1.2.2)

Remark 3.1.1.2-3.12.2: Eqs. 3.1.1.2 and 3.1.2.2 are *not* tautologous and *not* equivalent, as asserted, hence refuting those two conjectures.

Presupposition is different from other semantic relations, e.g., implication and necessitation.

Implication is defined as the logical truth of

'A⊃B'(~A∨B). (3.1.3.1)

(p>q)&(~p+q) ; T**F**TT T**F**TT T**F**TT T**F**TT (3.1.3.2)

For implication *modus tollens* is accepted as valid, whereas in case of presupposition it doesn't hold, since the analogue of *modus tollens* with respect to presupposition:

A presupposes B (not B) Therefore, (not A) is not valid; if both the premises are true, the conclusion is not true (i.e. neither true nor false). (3.1.4.1)

((p>q)>~q)>~p ; T**F**T**F** T**F**T**F** T**F**T**F** T**F**T**F** (3.1.4.2)

Another distinction is that the argument:

A presupposes B (not A) Therefore, B is valid in case of presupposition, since if the premises are true, so is the conclusion; whereas, for implication this argument doesn't hold. (3.1.5.1)

((p>q)>~p)>q ; **FF**TT **FF**TT **FF**TT **FF**TT (3.1.5.2)

However, presupposition and implication have something in common, which is, if A either resupposes or implies B then the argument from A to B is valid. (3.1.6.1)

Remark 3.1.6.1: We map resupposes as either Eqs. 3.1.4.1 or 3.1.5.1, or 3.1.3.1, as valid.

((((p>q)>~q)>~p)+(((p>q)>~p)>q))+((p>q)&(~p+q)) ;
T**F**TT T**F**TT T**F**TT T**F**TT (3.1.6.2)

3.3. Shortcomings of supervaluation semantics:
Considering the above case, where ‘a’ is denoting and ‘b’ is not,

‘(∀x)Px⊃Pa’ is true, (3.3.1.1)

LET p, q, r, s: p, x, a, b.

(r>(p=p))>((p&#q)>(p&r)) ; TTTT TTTC TTTT TTTC (3.3.1.2)

though ‘(∀x)Px⊃Pa’ is not. (3.3.2.1)

(s>(p@p))>((p&#q)>(p&s)) ; TTTC TTTC TTTT TTTT (3.3.2.2)

However, in standard first order predicate logic (FOP) both are true as endorsed by UI rule, known as the principle of Specification. This is however quite expected in a system of free logic.

Remark 3.3: Eqs. 3.3.1.2 and 3.3.2.2 as rendered are *not* tautologous and *not* contradictory, thereby refuting four conjectures in FOP: two as true and false, and two as true.

All conjectures evaluated in 12 eqs. are *not* tautologous, and refutes replacing classical logic with free logic.

Refutation of the paradox of Moses Maimonides for free will

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table fragment) is row-major and horizontal.

LET p q: God, man;
~ Not; & And; + Or; = Equivalent; @ Not Equivalent;
> Imply, greater than; < Not Imply, less than; # necessity, for all;
(%p>#p) good; (%p<#p) bad; (p@p) imperfect, a lie.

From: en.wikipedia.org/wiki/Argument_from_free_will

> Moses Maimonides formulated an argument regarding a person's free will, in traditional terms of good and evil actions, as follows:
>
> > Does God know or does He not know that a certain individual will be good or bad? (1.1)

(p>(q>(%p>#p)))+(p>(q>(%p<#p))) ;
TTTT TTTT TTTT TTTT (1.2)

> > If thou sayest 'He knows', then it necessarily follows that the man is compelled to act as God knew beforehand he would act, (2.1)

(p>(q>(%p>#p)))>#(q>(p>(q>(%p>#p)))) ;
NNNT NNNT NNNT NNNT (2.2)

> > otherwise God's knowledge would be imperfect ... (3.1)

[<] p=(p@p) ; TFTF TFTF TFTF TFTF (3.2)

If Eq. 1.2, then if Eq. 2.1 then Eq. 3.1. (4.1)

(((p>(q>(%p>#p)))+(p>(q>(%p<#p))))>
((p>(q>(%p>#p)))>#(q>(p>(q>(%p>#p)))))) < (p=(p@p)) ;
FNFT FNFT FNFT FNFT (4.2)

As rendered, Eq. 1.2 is tautologous, *not* contradictory, and a theorem. Eqs. 2.2 and 3.2 are *not* tautologous and *not* contradictory. Eq. 4.2, the further embellishment of Eqs. 1.2, 2.2, and 3.2 is *not* tautologous and *not* contradictory. Therefore the paradox of Maimonides is refuted as a paradox.

Refutation of non-existence proof of free will

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal.

From: Luan, Q. (2018). A rigorous non-existence proof of free will in an indeterministic universe. vixra.org/pdf/1805.0193v1.pdf

LET ~ Not; & And; + Or; - Not Or; > Imply, greater than; < Not Imply, less than;
= Equivalent; @ Not Equivalent; # necessity, for all; % possibility, for one or some;
p, q (~q), s: freewill; outcome (~ alternative outcome); personal entity in the universe;
%(q+~q) at least one choice.

If free will exists in an indeterministic universe, all of the following three statements are valid and non-contradictory. (S.4.1)

There is at least one entity with free will in the universe. Let F be an entity with free will in the universe. (S.1.1)

%p>%s ; `TCTC TCTC TTTT TTTT` (S.1.2)

As per the definition of free will, F has made at least one non-random choice. (S.2.1)

%p>(%s>%(q+~q)) ; `TTTT TTTT TTTT TTTT` (S.2.2)

Let t_c be the time when F non-randomly chose one from multiple different physical possibilities. Let the possibility chosen be p_c. (S.3.1)

%p>((%p>(%s>%(q+~q)))>(%s&%(q+~q))) ;
`TCTC TCTC TTTT TTTT` (S.3.2)

Use of the phrase "non-randomly" is ignored because the definition of Eq. S.2.1 includes that. We interpret the possibility chosen p_c not as a single variable such as q but rather as either variable (q+~q) so as not to *assume* which is chosen.

The injections of both the temporal variable t for time or the name universe for possible worlds are not needed because the possible existence of at least one personal agent as %s. Therefore we ignore both injections.

These exclusions actually help the arguments by making Eq. S.3.1 (not a tautology) irrelevant, and hence Eq. S.3.2 could be excluded in our evaluation here.

As rendered, only Eq. 3.2.2 is tautologous. This disagrees with Eq. S.4.1 where all Eqs. 3.n.2 should be tautologous.

At t_c, the universe either contained or did not contain the information that p_c was chosen.

At t_c, if the universe did not contain the information that p_c was chosen, F as defined is an entity in the universe and therefore did not contain the information that p_c was chosen. (C.1.1.1)

(((q+~q)=(q@q))&(%p>%s))> ~((%p>%s)>(q+~q)) ;
TTTT TTTT TTTT TTTT (C.1.1.2)

Therefore, the choice at t_c was not non-randomly made, (C.1.2.1)

((((q+~q)=(q@q))&(%p>%s))> ~((%p>%s)>(q+~q)))> ~(%p>(q+~q)) ;
FFFF FFFF FFFF FFFF (C.1.2.2)

which contradicts the statement "Let t_c be the time when F non-randomly chose one from multiple different physical possibilities. (C.1.3.1)

(((((q+~q)=(q@q))&(%p>%s))> ~((%p>%s)>(q+~q)))> ~(%p>(q+~q))) =
(%p>((%p>(%s>%(q+~q)))>(%s&%(q+~q)))) ;
FNFN FNFN FFFF FFFF (C.1.3.2)

We also test if Eq. C.1.2.2 is equal to Eq. S.2.2. (C.1.3.3.1)

(((((q+~q)=(q@q))&(%p>%s))> ~((%p>%s)>(q+~q)))>
~(%p>(q+~q))) = (%p>(%s>%(q+~q))) ;
FFFF FFFF FFFF FFFF (C.1.3.3.2)

At t_c, if the universe contained the information that p_c was chosen, there wouldn't be other different physical possibilities than p_c, (C.2.1.1)

((q+~q)=(q=q))>~(%(q+~q)=(p=p)) ; FFFF FFFF FFFF FFFF (C.2.1.2)

which again contradicts the statement "Let t_c be the time when F non-randomly chose one from multiple different physical possibilities." (C.2.2.1)

(((q+~q)=(q=q))>~(%(q+~q)=(p=p))) = (%p>((%p>(%s>%(q+~q)))>(%s&%(q+~q)))) ;
FNFN FNFN FFFF FFFF (C.2.2.2)

We also test if Eq. C.2.1.2 is equal to Eq. S.2.2. (C.2.2.3.1)

(((q+~q)=(q=q))>~(%(q+~q)=(p=p))) = (%p>(%s>%(q+~q))) ;
FFFF FFFF FFFF FFFF (C.2.2.3.2)

Eqs. C.1.2.2 and C.2.2.2 are *not* tautologous as expected. Eqs. 1.3.2 and 2.2.2 are *not* contradictory as expected. However, only by weakening the arguments do they become contradictory in Eqs. C.1.3.3.2 and C.2.3.3.2. Nevertheless, we therefore conclude that he non-existence proof of free will is refuted.

Refutation of Clifton's Kochen-Specker statistical argument (basis for free will theorem)

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal.

> LET: p, q, r: prob, *v*(*A*)=*a*, *v(B)=b*; & And; > Imply, greater than; = Equivalent;
> % possibility, for one or some; # necessity, for all or every; (%p>#p) ordinal 1.

From: plato.stanford.edu/entries/kochen-specker/ [Carsten Held]

> "3.5 A Statistical KS [Kochen-Specker] Argument in Three Dimensions (Clifton)
>
> We now assume, in addition, that any constraint on value assignments will show up in the measurement statistics. In particular:
>
> If prob[*v*(*A*)=*a*] = 1, and *v*(*A*)=*a* implies *v*(*B*)=*b*, then prob[*v*(*B*)=*b*] = 1." (3.5.1)

(((p&q)=(%p>#p))&(q>r))>((p&r)=(%p>#p)) ;
`TTTT TNTT TTTT TNTT` (3.5.2)

Eq. 3.5.2 as rendered is *not* tautologous, meaning something other than a theorem is assumed in Clifton's KS argument.

What the author(s) could write was a *non* statistical argument using ordinal 1 to mean a designated proof value. For example, "if valid[*v*(*A*)=*a*] is a proof, and *v*(*A*)=*a* implies *v*(*B*)=*b*, then valid[*v*(*B*)=*b*] is a proof" is tautologous. But ejecting probability produces a no-go statistical assumption.

> **Remark:** This is an example of the faulty mathematical logic which unfortunately peppers the quantum hypothesis field, beginning from about Gödel.

Refutation of the Free Will hypothesis based on its defective FIN

We assume the method and apparatus of Meth8/VŁ4 with T autology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The repeating fragment(s) of 16-valued truth table(s) is row-major and horizontal.

LET p, q, r, w, x, y, z: j, k, l, w, x, y, z;
& And; + Or; - Not Or; > Imply, →, greater than; < Not Imply, lesser than;
= Equivalent; @ Not Equivalent;
(p@p) ordinal zero, 0; (%p>#p) ordinal 1; (%p<#p) ordinal 2; (p=p) T, *proof*.

From: Conway, J.; Kochen, S. (2006). The free will theorem. arxiv.org/pdf/quant-ph/060409.v1.pdf

"The SPIN axiom: A triple experiment for the frame (x, y, z) always yields the outcomes 1,0,1 in some order. We can write this as: x → j, y → k, z → ℓ, where j, k, ℓ are 0 or 1 and j + k + ℓ = 2. (1.1)
... [I]f measurements in the order x, y, z for one particle produced x → 1, y → 0, z → 1, then measurements in the order y, z, x for the second particle would produce → 0, z → 1, x → 1."

((((p=((p@p)+(%p>#p)))&(q=((q@q)+(%q>#q))))&(r=((r@r)+(%r>#r))))&(((p+q)+r)=(%p<#p))) > (((x>p)&((y>q)&(z>r)))&(((y>q)&(z>r))&(x>p))) ;
TTTT TTTT TTTT TTTT (1.2)

"The TWIN axiom: For twinned spin 1 particles, if the first experimenter A performs a triple experiment for the frame (x, y, z), producing the result x → j, y → k, z → l while the second experimenter B measures a single spin in direction w, then if w is one of x, y, z, its result is that w → j, k,or l, respectively." (2.1)

(((x>p)&((y>q)&(z>r)))&w)>((w=(x+(y+z)))>(w>(p+(q+r)))) ;
TTTT TTTT TTTT TTTT (2.2)

The FIN axiom: "'effective causality,' that effects cannot precede their causes." (3.1)

(p>q)>~(q<p) ; TTFT TTFT TTFT TTFT (3.2)

As rendered, Eqs. 1.2 and 2.2 are tautologous, but 3.2 is *not* tautologous. This means axioms for SPIN and TWIN are tautologous, but the axiom for FIN is *not* tautologous.

Because the assumption of axiom FIN is essential to the authors' proof, the Free Will theorem is also *no*t tautologous and refuted by its own derivation. This means the Free Will theorem can not be reasserted by resurrection as such.

Refutation of the Strong Free Will hypothesis based on its defective MIN

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The repeating fragment(s) of 16-valued truth table(s) is row-major and horizontal.

LET p, q, r, s, t, u, w, x, y, z:
A, B, A-first frame, B-first frame, a's prior response, b's prior response, w, x, y, z
~ Not; & And; + Or; > Imply, greater than; < Not Imply, lesser than.

From: Conway, J.; Kochen, S. (2008). The strong free will theorem. arxiv.org/pdf/0807.3286.pdf

"MIN′: In an A-first frame, B can freely choose any one of the 33 directions w, and a's prior response is independent of B's choice. Similarly, in a B-first frame, A can independently freely choose any one of the 40 triples x, y, z, and b's prior response is independent of A's choice." (10.1)

((r>(q>w))&~(t<q))+((s>(p>(x&(y&z))))&~(u<p)) ;
TTTT TTTT TTTT TTTT,
FTTT FTTT FTTT FTTT (10.2)

As rendered, Eq. 10.2 is *not* tautologous. This means axiom MIN', as replacement for the previous FIN in the Free Will theorem, is *not* tautologous.

Because the assumption of axiom MIN' is essential to the authors' proof, the Strong Free Will theorem is also *no*t tautologous and refuted by its own derivation. This means the Strong Free Will theorem can not be reasserted by resurrection as such.

Refutation of the frequency dependence of mass

From: Rajna, G. (2014). "The secret of quantum entanglement." vixra.org/pdf/1406.0008v2.pdf; and
Rajna, G. (2018). "Mathematical models of inventions". vixra.org/pdf/1801.0366v1.pdf
Note: The name George Rajna is a pseudonym after a chess player with not address.

"The frequency dependence of mass: Since E = hν and E = mc^2, m = hν /c^2 that is the m depends only on the ν frequency." where m = mass, h = Planck's constant, c = speed of light.

Remark: Planck's constant is arguably not exact, but rather a probabilistic estimation.

m = hν /c^2 (1.1)

Remark: m is undefined if either hν or c^2 is zero. (Elsewhere we show 0/n is not 0.)

LET: pqrs cmhv; ~ Not; - Not Or; & And; \ Not And; > Imply; = Equivalence; (p-p) Numeric zero

T is tautology as the designated *proof* value, with F as contradiction
The 16-valued truth tables are presented row-major and horizontally.

Using the Meth8/VŁ4 apparatus and method, we render Eq. 1.1 as

q=((r&s)\(p&p)) ; `FFTT FFTT FFTT FFTT` (1.2)

Eq. 1.2 is *not* tautologous which means the fractional equation cannot be a theorem.

We attempt to resuscitate Eq. 1.2 by changing the connective of the literal to > Imply.

q>((r&s)\(p&p)) ; `TTTT TTTT TTTT TTTF` (1.2.1)

Eq. 1.2.1 is *not* tautologous, meaning that Eq. 1.2.1 is not an implication, although nearly so.

We attempt to resuscitate Eq. 1.2 by defining p as not numeric zero ~(p-p):

(p=~(p-p))>(q=((r&s)\(p&p))) ; `FFTT FFTT FFTT FFTT` (1.3)

Eq. 1.3 is *not* tautologous and results in the same truth table as Eq. 1.2.

We attempt to resuscitate Eq. 1.2 by defining p as numeric zero (p-p):

(p= (p-p))>(q=((r&s)\(p&p))) ; `TTTT TTTT TTTT TTTT` (1.4)

Eq. 1.4 is tautologous, meaning that in the case of p as numeric zero then Eq. 1.2 is a theorem.

What follows is the frequency dependence of mass is *untenable*: Since E = hν and E = mc^2, m = hν /c^2 that is the m depends only on the ν frequency is *not* tautologous. Hence, the frequency of mass is a suspicious statistic.

Injection, surjection, and bijection functions as bivalent mappings

From: /en.wikipedia.org/wiki/Bijection,_injection_and_surjection

The terminology of injection, surjection, and bijection was due to the Bourbaki group which attempted to recast mathematics onto set theory since 1934. (We prove elsewhere that the axiom of specification as-is is the *only* ZFC axiom that is tautologous.)

We evaluate arguments and images as input and output between domain and codomain for functions defined as injective, surjective, and bijective.

> We assume the apparatus and method of Meth8/VŁ4. The designated *proof* value is T; F contradiction; C falsity; N truth. The 16-valued truth tables are row-major and horizontally.
>
> LET: p q r s f x; x', y; X; Y; (X>Y) as (r>s);
> ~ Not; & And; > Imply, greater than, ⇒; < Not Imply, less than, ∈; = Equivalent to;
> # necessity, for all, ∀; % possibility, for one or some, ∃.
>
> We distribute quantified expressions for intended clarity.
>
> Given a function f : X → Y,
>
> Injective, notationally: $\forall x, x' \in X$, $f(x) = f(x') \Rightarrow x = x'$. (1.1.1)
>
> ((#(p&q)<s)&(((r>s)&p)=((r>s)&q))) > ((#(p&q)<s)&(p=q)) ;
> TTTT TTTT TTTT TTTT (1.1.2)
>
> Surjective, notationally: $\forall y \in Y, \exists\, x \in X$ such that $y = f(x)$. (2.1.1)
>
> (#(q<s)&(((r>s)&p)=q)) > (#(q<s)&%(p<r)) ;
> TTTT TTTT TTTT TTTT (2.1.2)
>
> Bijective, notationally: iff for all $y \in Y$, there is a unique $x \in X$ such that $f(x) = y$. (3.1.1)
>
> (#(q<s)&(((r>s)&p)=q))>(#(q<s)&%(p<r)) ;
> TTTT TTTT TTTT TTTT (3.1.2)
>
> Or the function is *both* injective and surjective: (Eq. 1.1.1) & (Eq. 2.1.1) (3.2.1)
>
> (((#(p&q)<s)&(((r>s)&p)=((r>s)&q))) > ((#(p&q)<s)&(p=q))) &
> ((#(q<s)&(((r>s)&p)=q)) > (#(q<s)&%(p<r))) ;
> TTTT TTTT TTTT TTTT (3.2.2)

The equations above as rendered are tautologous.

From the category of sets, injection, surjection, and bijection correspond precisely to monomorphism, epimprophism, and isomorphism; hence the latter are respectively also tautologous.

Gentzen proof of sequent System G-M

Steward, Charles; Stouppa, Phiniki. (2004). A systematic proof theory for several modal logics; also at textproof.com/supervision/phiniki04sbm.pdf

We assume the Meth8 apparatus implementing system variant VŁ4 in five models.

LET: p A; q B; r C; > Imply; + Or; & And; # [] modal necessity; % <> modal possibility.

The designated proof value is T tautology with F contradiction, C contingency (truth), and N non-contingency (falsity). Fragments are repeating rows one and two (of four) in the truth table.

We begin evaluation on pages 313/4, 323 of the text to derive the systems of interest.

K: [](p ⊃ q) ⊃ ([]p ⊃ []q) (3.1.1)

#(p>q)>(#p>#q) ; `TTTT TTTT` (3.1.2)

Axiom **T**: []p ⊃ p (3.2.1)

#p>q ; `TTTT TTTT` (3.2.2)

M, obtained by extending system **K** with rule **T** [not Gödel's system T] (3.3.1)

(#(p>q)>(#p>#q))>(#p>q) ; `TCTT TCTT` (3.3.2)

"The strongest system from these modal logics that is perfectly straightforward to formulate in a sequent system and to prove cut-free is system **G-M** (for Gentzen system **M**)".

[We remark that the subsequent derivations of **S4**, **B**, and **S5** are tautologous, as are **K** and **T**.]

"The following lemma is a straightforward exercise in theoremhood over **K**:
LEMMA 6 If A ⊃ B is a theorem of **M**, then so are: (L.6.0.1)

1. A ∧ C ⊃ B ∧ C; (L.6.1.1)
2. A ∨ C ⊃ B ∨ C; (L.6.2.1)
3. []A ⊃ []B; (L.6.3.1)
4. <>A ⊃ <>B." (L.6.4.1)

To map Eq. L.6.0.1 we use Eq. 3.3.2.

((#(p>q)>(#p>#q))>(#p>q)) > (p>q) ;`TNTT TNTT` (L.6.0.2)

We then reuse Eq. L.6.0.2 to map L.6.1.2 - 6.4.2.

(((#(p>q)>(#p>#q))>(#p>q))>(p>q)) > ((p&r)>(q&r)) ;
`TTTT TCTT` (L.6.1)

(((#(p>q)>(#p>#q))>(#p>q))>(p>q)) > ((p+r)>(q+r)) ;
`TCTT TTTT` (L.6.2)

(((#(p>q)>(#p>#q))>(#p>q))>(p>q)) > (#p>#q) ;
TCTT TCTT (L.6.3)

(((#(p>q)>(#p>#q))>(#p>q))>(p>q)) > (%p>%q) ;
TCTT TCTT (L.6.4)

On page 321, "we recommend the reader works through ... for example

(A ⊃ B ⊃ C) ⊃ (A ⊃ C) ⊃ B ⊃ C". (7.1)

((((p>q)>r)>(p>r))>q)>r ; TFFF TFFF (7.2)

Eq. 7.2 is also *not* tautologous.

We conclude system **G-M** as rendered is not tautologous, and Gentzen-sequent systems are suspicious.

Refutation of Gettier problem of justified true/false belief

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET: & And; > Imply, greater than, believes, knows; < Not Imply, less than;
= Equivalent, is; # necessity, for all; % possibility, for one or some;
p Proposition; q proposition; s Subject;
(%q>#q) truthity; (%q<#q) falsity; (q=q) tautology, justified.

From: allthatsinteresting.com/fascinating-unsolved-problems/2

Critics of justified true belief assert "it's impossible to justify anything which is not true (where "truth" is a construct designed for the sake of argument as being some irrefutable fact)." (0.0)

Justified true belief is defined as: A subject S knows that a proposition P is true if and only if: (4.1)

[=] %s>(p=(%q>#q)) ; `TNTN TNTN CNCN CNCN` (4.2)

P is true, (1.1)

p=(%q>#q) ; `CNCN CNCN CNCN CNCN` (1.2)

and S believes that P is true, (2.1)

[&] s>(p=(%q>#q)) ; `TTTT TTTT CNCN CNCN` (2.2)

and S is justified in believing that P is true (3.1)

[&] (s>(q=q))>(s>(p=(%q>#q))) ;
`TTTT TTTT CNCN CNCN` (3.2)

Eqs. 1.1 and 2.1 and 3.1 are equivalent to 4.1. (5.1)

(((p=(%q>#q))&(s>(p=(%q>#q))))&((s>(q=q))>(s>(p=(%q>#q)))))= (%s>(p=(%q>#q))) ;
`CTCT CTCT TTTT TTTT` (5.2)

Eq. 5.2 is *not* tautologous. Therefore justified true belief is not a theorem.

To answer Eq. 0.0 we rewrite it using falsity instead of truthity to read justified false belief as:

A subject S knows a proposition is P is false if and only if P is false, and S believes P is false, and S is justified in believing P is false. (0.1)

To answer Eq. 0.0, we cast Eq. 5.2 with falsity (%q<#q) instead of truthity (%q>#q). (6.1)

(((p=(%q<#q))&(s>(p=(%q<#q))))&((s>(q=q))>(s>(p=(%q<#q)))))= (%s>(p=(%q<#q))) ;
TCTC TCTC TTTT TTTT (6.2)

Eq. 6.2 is *not* tautologous. Therefore justified false belief is also not a theorem.

This means the Gettier problem as the superset of the justified belief arguments is refuted as a problem and resolved as a non-problem.

Refutation of GHZ experiments

We rely on this description from: en.wikipedia.org/wiki/GHZ_experiment .

> GHZ experiments are a class of physics experiments that may be used to generate starkly contrasting predictions from local hidden variable theory and quantum mechanical variable theory, and permit immediate comparison with actual experimental results. A GHZ experiment is similar to a test of Bell's inequality, except using three or more entangled particles, rather than two. With specific settings of GHZ experiments, it is possible to demonstrate absolute contradictions between the predictions of local hidden variable theory and those of quantum mechanics, whereas tests of Bell's inequality only demonstrate contradictions of a statistical nature. The results of actual GHZ experiments agree with the predictions of quantum mechanics. The GHZ experiments are named for Daniel M. Greenberger, Michael A. Horne, and Anton Zeilinger (GHZ) who first analyzed certain measurements involving four observers and who subsequently ... applied their arguments to certain measurements involving three observers. A GHZ experiment is performed using a quantum system in a Greenberger-Horne-Zeilinger state. An example of a GHZ state is three photons in an entangled state ... .
>
> [T]hey are able to obtain the following four equations concerning one and the same value λ:
>
> (1) $A(a_2, \lambda)\, B(b_2, \lambda)\, C(c_2, \lambda) = -1$,
>
> (2) $A(a_2, \lambda)\, B(b_1, \lambda)\, C(c_1, \lambda) = 1$,
>
> (3) $A(a_1, \lambda)\, B(b_2, \lambda)\, C(c_1, \lambda) = 1$, and
>
> (4) $A(a_1, \lambda)\, B(b_1, \lambda)\, C(c_2, \lambda) = 1$.
>
> Taking the product of the last three equations, and noting that
>
> (5) $A(a_1, \lambda)\, A(a_1, \lambda) = 1$,
>
> (6) $B(b_1, \lambda)\, B(b_1, \lambda) = 1$, and
>
> (7) $C(c_1, \lambda)\, C(c_1, \lambda) = 1$, yields
>
> (8) $A(a_2, \lambda)\, B(b_2, \lambda)\, C(c_2, \lambda) = 1$
>
> in contradiction to the first equation [1.]; $1 \neq -1$. (9)

We assume the Meth8/VŁ4 apparatus and method. The designated *proof* value is T with F for contradiction and C for contingency and falsity. The table results are repeating 16-valued fragments.

LET p, q, r, s, t, u, (%p>#p): a1, a2, b1, b2, c1, c2, 1; *A*, *B*, *C*, λ are ignored to simplify.

We apply the note in Eqs. 5, 6, 7 to the product of Eqs. 2, 3, 4 as tested to Eq. 1.

```
(((p=(%p>#p))&(r=(%p>#p)))&(t=(%p>#p))) & ((((((q&r)&t)=(%p>#p))&
(((p&s)&t)=(%p>#p)))&(((p&r)&u)=(%p>#p))) = (((q&s)&u)=~(%p>#p)));
                                          FFFFFFFFFFFFFFFF,
                                          FFFFFFFFFFFFFFFF,
                                          FFFFFFFFFFCFFFFF,
                                          FFFFFFFFFFFFFFFF          (10)
```

The expected result is supposed to be a contradiction (all F) in Eq. 9. However Eq. 10 as rendered is not a contradiction (notice the one bold value of **C**). This means the GHZ experiment is refuted, further supporting previous refutations of Bell's inequality using Meth8/VŁ4.

Refutation of Gleason's theorem

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET: & And; > Imply, greater than, believes, knows; < Not Imply, less than;
= Equivalent, is; # necessity, for all or any; % possibility, for one or some;
p probability measure; q quantum state; r measurement outcomes; s space;
(p=p) tautology, legitimate.

From: en.wikipedia.org/wiki/Gleason%27s_theorem

"Effectively, the theorem says that any legitimate probability measure on the space of measurement outcomes is generated by some quantum state." (1.1)

%q>((#p>(p=p))<(r<s)) ; TTTT NNFF TTTT TTTT (1.2)

Eq. 1.2 as rendered is *not* tautologous. This means Gleason's theorem is *not logically* "a mathematical result which shows that the rule one uses to calculate probabilities in quantum physics follows logically [*sic*] from particular assumptions about how measurements are represented mathematically".

Refutation of Gobbay's separation theorem

Abstract: The separation theorem of Gobbay takes eight basic cases and four cases for disjunction. None is tautologous. In fact, three groups of the basic cases share unique truth table result values, and one group of the disjunctive cases shares the same truth table result values. This refutes the theorem and adds it as another *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Gabbay%27s_separation_theorem [from footnote source below]

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formula until there are no nestings of $\mathcal{U}^+$ within $\mathcal{S}^-$ and vice versa. For $\mathcal{U}^+$ within $\mathcal{S}^-$, there are eight basic cases to handle. Let φ and ψ stand for arbitrary formulae and letters a and b, etc., denote propositional atoms. The eight cases are:-

1. $\varphi\,\mathcal{S}^-\,(\psi \wedge a\,\mathcal{U}^+\,b)$
2. $\varphi\,\mathcal{S}^-\,(\psi \wedge \neg(a\,\mathcal{U}^+\,b))$
3. $(\varphi \vee a\,\mathcal{U}^+\,b)\,\mathcal{S}^-\,\psi$
4. $(\varphi \vee \neg(a\,\mathcal{U}^+\,b))\,\mathcal{S}^-\,\psi$
5. $(\varphi \vee a\,\mathcal{U}^+\,b)\,\mathcal{S}^-\,(\psi \wedge a\,\mathcal{U}^+\,b)$
6. $(\varphi \vee \neg(a\,\mathcal{U}^+\,b))\,\mathcal{S}^-\,(\psi \wedge a\,\mathcal{U}^+\,b)$
7. $(\varphi \vee a\,\mathcal{U}^+\,b)\,\mathcal{S}^-\,(\psi \wedge \neg(a\,\mathcal{U}^+\,b))$
8. $(\varphi \vee \neg(a\,\mathcal{U}^+\,b))\,\mathcal{S}^-\,(\psi \wedge \neg(a\,\mathcal{U}^+\,b))$

Other nested $\mathcal{U}^+$ forms reduce to one of the 8 schema for atomic $\mathcal{U}^+$ formula. For example, consider $\varphi\,\mathcal{S}^-\,(a\,\mathcal{U}^+\,(p\,\mathcal{U}^+\,q))$, however, this can be viewed as a formula of shape 1 above. Replace the sub-formula $p\,\mathcal{U}^+\,q$ by pq say, and note that the formula ψ in 1 is **true**.

For each of the above shapes, one can provide an equivalent formula of form $E_1 \vee E_2 \vee E_3$ where each E_i is a boolean combination of pure past, present and pure future formulae. An inductive proof can then establish that separation can occur for all formulae.

In the following we establish the first of the above eliminations. Let $E \stackrel{def}{=} \varphi\,\mathcal{S}^-\,(\psi \wedge a\,\mathcal{U}^+\,b)$. We can write E as the disjunction of $E_1 E_2 E_3$ such that the E_i contain no nested $\mathcal{U}^+$, in fact in a separated form,

$$\models E \Leftrightarrow E_1 \vee E_2 \vee E_3$$

In order to construct the formulae E_i, consider a model for $\varphi\,\mathcal{S}^-\,(\psi \wedge a\,\mathcal{U}^+\,b)$.

$$E_1 \;:\; y < n \;:\; \varphi\,\mathcal{S}^-\,(b \wedge \varphi \wedge (\varphi \wedge a)\,\mathcal{S}^-\,\psi)$$
$$E_2 \;:\; y = n \;:\; b \wedge (\varphi \wedge a)\,\mathcal{S}^-\,\psi$$
$$E_3 \;:\; y > n \;:\; a\,\mathcal{U}^+\,b \wedge a \wedge (\varphi \wedge a)\,\mathcal{S}^-\,\psi$$

In mathematical logic and computer science, Gabbay's separation theorem, named after Dov Gabbay, states that any arbitrary temporal logic formula can be rewritten in a logically equivalent "past → future" form. I.e. the future becomes what must be satisfied.[1]

[1] Fisher, M.; Gabbay, D.; Vila, L. Eds. (2005). Handbook of temporal reasoning in artificial intelligence. Foundations of artificial intelligence. 1. Elsevier. [See image above.]

LET p, q, s, u, x, y: φ, ψ, S^-, U^+, a, b.

```
(p&s)&(q& (x&(u&y))) ;      FFFF FFFF FFFT FFFT(48)
                            FFFF FFFF FFFF FFFF( 2)}x4
                            FFFF FFFF FFFT FFFT( 2)}      (1.2)

(p&s)&(q&~(x&(u&y))) ;      FFFF FFFF FFFT FFFT(48)
                            FFFF FFFF FFFT FFFT( 2)}x4
                            FFFF FFFF FFFF FFFF( 2)}      (2.2)

(p& (x&(u&y)))&(s&q) ;      FFFF FFFF FFFT FFFT(48)
                            FFFF FFFF FFFF FFFF( 2)}x4
                            FFFF FFFF FFFT FFFT( 2)}      (3.2)

(p&~(x&(u&y)))&(s&q) ;      FFFF FFFF FFFT FFFT(48)
                            FFFF FFFF FFFT FFFT( 2)}x4
                            FFFF FFFF FFFF FFFF( 2)}      (4.2)

(p& (x&(u&y)))&(s&(q& (x&(u&y)))) ;
                            FFFF FFFF FFFT FFFT(48)
                            FFFF FFFF FFFF FFFF( 2)}x4
                            FFFF FFFF FFFT FFFT( 2)}      (5.2)

(p&~(x&(u&y)))&(s&(q& (x&(u&y)))) ;
                            FFFF FFFF FFFF FFFF           (6.2)

(p& (x&(u&y)))&(s&(q&~(x&(u&y)))) ;
                            FFFF FFFF FFFF FFFF           (7.2)

(p&~(x&(u&y)))&(s&(q&~(x&(u&y)))) ;
                            FFFF FFFF FFFT FFFT(48)
                            FFFF FFFF FFFT FFFT( 2)}x4
                            FFFF FFFF FFFF FFFF( 2)}      (8.2)
```

Remark 1.2-8.2: Eqs. 1.2-8.2 are *not* tautologous. In fact, the following groupings have identical truth table result values with abbreviated differences:

```
2.2, 4.2, 8.2:      FFFF, FFFT, FFFF
1.2, 3.2, 5.2:      FFFT, FFFF, FFFT
6.2, 7.2:           FFFF
```

This refutes the eight *different* basic cases to handle in Gabbay's separation result.

E: def [see image above] (E0.1)

(p&s)&((q&x)&(u&y)) ; **FFFF FFFF FFFF FFFF** (E0.2)

E_1: y<n (E1.1)

(p&s)&((y&p)&((p&x)&(s&q))) ;

FFFF FFFF FFFF FFFF(48)
FFFF FFFF FFFT **FFF**T**(16)** (E1.2)

E_2: y=n (E2.1)

y&((p&x)&(s&q)) ; **FFFF FFFF FFFF FFFF(48)**
FFFF FFFF FFFT **FFF**T**(16)** (E2.2)

E_3: y>n (E3.1)

(x&(u&y))&(x&((p&x)&(s&q))) ;

FFFF FFFF FFFF FFFF(48)
FFFF FFFF FFFF FFFF(2)}x4
FFFF FFFF FFFT **FFF**T**(2)}** (E3.2)

$E = E_1 + E_2 + E_3$: Disjunctions (E4.1)

((p&s)&((q&x)&(u&y)))=(((p&s)&((y&p)&((p&x)&(s&q))))+
((y&((p&x)&(s&q)))+((x&(u&y))&(x&((p&x)&(s&q)))))) ;

TTTT TTTT TTTT TTTT(48)
TTTT TTTT TTT**F** TTT**F**(2)}x4
TTTT TTTT TTTT TTTT(2)} (E4.2)

Remark E1-E4: Eqs. E1.2-E4.2 are *not* tautologous. In fact, the following grouping has identical truth table result values with abbreviated differences:

E1.2, E2.2: **FFFF, FFF**T

This refutes the disjunction equations of the model for Gobbay's separation theorem.

Refutation of the Gödel class with identity as un-solvable

Abstract: We evaluate the Gödel quantification scheme of ∃ *∀ⁿ∃ *φ as tautologous. When it is extended to decidable and undecidable prefix-classes, none is tautologous. This refutes the Gödel class with identity as undecidable, to mean it is in fact solvable as *not* tautologous. Therefore the prefix-classes are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

See: Marcos, J. (2016). Breaking the proof code. youtube.com/watch?v=XykVjsweqpc

The class of sentences of the form

∃ *∀ⁿ∃ *φ, where φ is quantifier free, is decidable if (and only if) n≤2 . (1.0)

Remark 1.0: We take ∃ * to mean zero or more existential quantifiers, and n≤2 to mean n={0, 1, 2}. We rewrite (1.0) by inserting the variables t, u, v, w, x, y, z as needed:

∃ x∀ⁿy∃zφ, where φ is quantifier free, is decidable if (and only if) n≤2 . (1.1)

LET p, q, r, s, t, u, v, w, x, y, z:
φ, q, n,*, t, u, v, w, w, y, z;
* > 0; n ≤ 2; (p@p) 0; (p=p) 3.

(~(s<(q@q))&~(r>(q=q)))>(((%s&x)&((#r&y)&(%s&z)))&p) ;
TTTT TTTT TTTT TTTT (1.2)

Remark 1.2: Eq. 1.2 as rendered is tautologous, hence confirming Gödel's asserted proof. However, the antecedent is a contradiction, **FFFF**, causing any consequent (here in part **F**N**F**N) to imply tautology.

From: Goldfarb, W.D. (1984). The Gödel class with identity is unsolvable.
academia.edu/31504009/The_Godel_Class_with_Identity_is_Unsolvable goldfarb@fas.harvard.edu

For example, let $G = \forall x\exists u\forall yK$ be any $\forall\exists\forall$-formula of pure quantification theory; (2.1.1)
we may suppose that the predicate letters of *G* are distinct from those of *F*. A straightforward argument shows that *G* is satisfiable if and only if $F \wedge \forall x\forall y\exists u(Sux \wedge K)$ is satisfiable; and the latter formula has a prenex equivalent in the (2.2.1)
GCI [Gödel class with identity]. Since the class of $\forall\exists\forall$-formulas is undecidable, we obtain the
THEOREM. *The Gödel Class with Identity is undecidable*. (2.3.1)

LET p, q, r, s, u, x, y:
F,G,K,S, u, x, y.

```
q=((#x&(%u&#y))&r) ;        FFFF FFFF FFFF FFFN(128)        (2.1.2)

p&(((#x&#y)&%u)&((s&(u&x))&r)) ;
                            FFFF FFFF FFFF FFFF(52),
                            FNFN FNFN FNFN FNFN( 2),
                            FFFF FFFF FFFF FFFF( 2),
                            FNFN FNFN FNFN FNFN( 2),
                            FFFF FFFF FFFF FFFF( 2),
                            FNFN FNFN FNFN FNFN( 2),
                            FFFF FFFF FFFF FFFF( 2),
                            FNFN FNFN FNFN FNFN( 2)         (2.2.2)
```

Remark 2.3.1: The conjecture is Eqs. 2.2.1 implies 2.2.1: (2.3.1)

```
(p&(((#x&#y)&%u)&((s&(u&x))&r))) > (q=((#x&(%u&#y))&r)) ;
                            TTTT TTTT TTTT TTTT(52),
                            TTTT TTTT TTTT TTCT( 2),
                            TTTT TTTT TTTT TTTT( 2),
                            TTTT TTTT TTTT TTCT( 2),
                            TTTT TTTT TTTT TTTT( 2),
                            TTTT TTTT TTTT TTCT( 2),
                            TTTT TTTT TTTT TTTT( 2),
                            TTTT TTTT TTTT TTCT( 2)         (2.3.2)
```

Remark 2.3.2: Eq. 2.3.2 is *not* tautologous, but *not* for the reason as claimed that 2.2.1 is satisfiable (decidable).

The theorem may be sharpened. Using several additional predicate letters, we may construct an infinity axiom and encode $\forall\exists\forall$-formulas while using only one existential quantifier. Hence the Minimal GCI, i.e., the class of formulas with prefixes $\forall x\forall y\exists z$, is undecidable. This settles the decision problem for all prefix-classes of quantification theory with identity, for we now have the following division: (3.0)

Remark 3.0: We rewrite Eqs. 4.1-7.1 by inserting variables as in Remark 1.0.

Decidable prefix-classes: $\exists \cdots \exists\forall\cdots\forall$ and (4.1.1)

∃ •••∃∀∃ •••∃. (4.2.1)

%p&((%q&#r)&#s) ; and **FFFF FFFF FFFF FFF**N (4.1.2)

%p&((%q&(#r&%s))&%t) ; **FFFF FFFF FFFF FFFF**(2),
FFFF FFFF FFFF FFFN(2) (4.2.2)

Undecidable prefix-classes: ∀∃ ∀ and (5.1.1)

∀∀∃ . (5.2.1)

#p&(%q&#r) ; **FFFF FFF**N **FFFF FFF**N (5.1.2)

#p&(#q&%r) ; **FFFF FFF**N **FFFF FFF**N (5.2.2)

This dividing line differs from that in pure quantification theory, where the ∃•••∃∀∀∃•••∃ class is decidable, (6.1)

%u&(((%v&#w)&(#x&%y))&%z) ; **FFFF FFFF FFFF FFFF**(126),
NNNN NNNN NNNN NNNN(2) (6.2)

so that the minimal undecidable prefix-classes are ∀∃ ∀ (7.1.1)

and ∀∀∀∃ ... (7.2.1)

#p&(%q&#r) ; **FFFF FFF**N **FFFF FFF**N (7.1.2)

#p&((#q&#r)&%s) ; **FFFF FFFF FFFF FFF**N (7.2.2)

The "sharpening" of the *The Gödel Class with Identity as undecidable* produces two decidable and two undecidable prefix classes, none of which is tautologous (Eqs. 4-5). The difference from quantification theory for class decidability is *not* tautologous (Eq. 6), and the minimal prefix-classes for undecidability are *not* tautologous and *not* equivalent (Eqs. 7). This means the Gödel class with identity is decidable, but solvable as *not* tautologous.

Gödel compactness theorem

From ocw.mit.edu/courses/linguistics-and-philosophy/24-241-logic-i-fall-2005/readings/chp09.pdf

> **Definition.** A set of sentences Ω is a complete story just in case it satisfies the following five conditions, for any φ and ψ:
>
> a) (φ ∧ ψ) ∈ Ω iff φ ∈ Ω and ψ ∈ Ω.
> b) (φ ∨ ψ) ∈ Ω iff φ ∈ Ω or ψ ∈ Ω (or both).
> c) (φ → ψ) ∈ Ω iff φ ∉ Ω or ψ ∈ Ω (or both).
> d) (φ ↔ ψ) ∈ Ω iff φ and ψ are both in Ω or neither of them is.
> e) ¬Φ ∈ Ω iff φ ∉ Ω.

Meth8 mapping is as follows.

LET: p = lc-phi; q = psi; r = uc-omega; s = uc-phi; ¬ ~; ∈ <; ∉ >; ∧ &; ∨ +; → =; ↔ =.

```
(((p&q)=(p=p))& (  (p<r)&(q<r))                        ) > ((p&q)<r) ; a.      validated
(((p&q)=(p=p))&((  (p<r)+(q<r)) +(  (p<r)&(q<r)))) > ((p&q)<r) ; b.      validated
(((p&q)=(p=p))&((  (p>r)+(q<r)) +(  (p>r)&(q<r)))) > ((p >q)<r) ; c. not validated ; no ~
(((p&q)=(p=p))&((~(p<r)+(q<r)) +(~(p>r)&(q<r)))) > ((p >q)<r) ; c. not validated
(((p&q)=(p=p))&((  (p<r)&(q<r))+  ((p<r) \ (q<r)))) > ((p =q)<r) ; d. not validated ; no ~
(((p&q)=(p=p))&((  (p<r)&(q<r))+~((p<r)&(q<r)))) > ((p =q)<r) ; d. not validated

(      (p=p) &   (p>r)                  )                          > (~s <r)       ; e. not validated ; uc-Phi
(      (p=p) & ~(p<r)                   )                          > (~p <r)       ; e. not validated ; lc-phi
```

Two conditions (a,b) are satisfied (tautologous), and three conditions are not satisfied (not tautologous). This means the five conditions of the compactness theorem are not all satisfied, and hence the Gödel compactness theorem is not tautologous.

Short refutation of Gödel's completeness theorem

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal

LET: p, q, r: x, y, R; & And; > Imply, greater than, →;
% possibility, <>, for one or some, ∃; # necessity, [], for every or all, ∀ .

From: en.wikipedia.org/wiki/Gödel's_completeness_theorem

By Gödel's completeness result, the formula $(\forall x.R(x,x)) \rightarrow (\forall x \exists y.R(x,y))$ holds in all structures, and hence must have a natural deduction proof. (1.1)

(#p&(r&(p&p)))>((#p&%q)&(r&(p&q))) ;
`TTTT TCTT TTTT TCTT` (1.2)

Eq. 1.2 is *not* tautologous, meaning it does not hold in all structures and serves as a contra-example. Hence Gödel's completeness theorem is refuted.

Remark: When reduced to an abstract and atomic state, Eq. 1.1 becomes weakened as $R(x,x) \rightarrow R(x,y)$ for (2.1)

(r&(p&p))>(r&(p&q)) ; `TTTT TFTT TTTT TFTT` (2.2)

Shorter refutation of Gödel's completeness theorem

Abstract: The completeness theorem rendered as (∀*x*.*R*(*x*,*x*))→(∀*x*∃*y*.*R*(*x*,*y*)) is *not* tautologous. The application of Isabelle/HOL to prove the same also is *not* tautologous, to invalidate that tool. These demonstrations form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , ·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Gödel%27s_completeness_theorem

Gödel's completeness theorem is a fundamental theorem in mathematical logic that establishes a correspondence between semantic truth and syntactic provability in first-order logic. … The formula

(∀*x*.*R*(*x*,*x*))→(∀*x*∃*y*.*R*(*x*,*y*)) (1.1)

holds in all structures. By Gödel's completeness result, it must hence have a natural deduction proof.

LET q, r, s: *x*, *R*, *y*.

(r&(#q&#q))>(r&(#q&%s)) ; TTTT TTCC TTTT TTTT (1.2)

Remark 1.2: Not distributing the quantifiers as (#q&(r&(q&q)))>((#q&%s)&(r&(q&s))) produces the same truth value table.

Eq. 1.2 as rendered is *not* tautologous, refuting Gödel's completeness theorem.

Remark 1.3: The following paper in using an assistant to prove Gödel's completeness theorem further refutes the Isabelle/HOL tool itself: Margetson, J. (2014). Proving the completeness theorem within Isabelle/HOL. isa-afp.org/entries/Completeness-paper.pdf.

Gödel's first incompleteness theorem

Gödel reduced the liar paradox to a self-referential sentence with an initially unknown truth value:

"This sentence is contradictory." (1.1.1)

We assume the Meth8 apparatus. The designated proof value is T tautology. Other values are: F contradiction; C contingency (a value of falsity); N non-contingency (a value of truth). The results are 16-value truth tables, presented as row major horizontally.

LET: p sentence; = Equivalent to; @ Not equivalent to, XOR; > Imply;
% "This" as meaning a possible instance, the existential quantifier.
(p@p), (p&~p) contradictory; (p=p) tautologous ;

"This sentence is contradictory." (1.1.1)

%(p=(p@p))=(p@p) ; `FNFN FNFN FNFN FNFN` (1.1.2)

If we attempt to weaken the conjecture in Eq. 1.1.1 by using the connective imply >, then the sentence reads:

"This sentence implies falsity." (1.2.1)

%(p>(p@p))>(p@p) ; `FNFN FNFN FNFN FNFN` (1.2.2)

Eqs. 1.1.2 and 1.2.2 are the same truth table, and not tautologous.

The instance of the sentence changing the value of "contradictory" to "tautologous" is:

"This sentence is tautologous." (2.1.1)

%(p=(p=p))=(p=p) ; `CTCT CTCT CTCT CTCT` (2.1.2)

If we attempt to weaken the conjecture in Eq. 2.1.1 by using the connective imply >, then the sentence reads:

"This sentence implies truth": (2.2.1)

%(p>(p>p))>(p>p) ; `TTTT TTTT TTTT TTTT` (2.2.2)

Eqs. 2.1.2 and 2.2.2 are not the same truth table, with 2.2.2 tautologous.

For a conjecture to test both sentences in Eqs. 1.1.1 and 2.1.1, we write the sentence to read:

"This sentence is contradictory", or "This sentence is tautologous". (3.1.1)

(%(p=(p@p))=(p@p)) + (%(p=(p=p))=(p=p)) ;
`CTCT CTCT CTCT CTCT` (3.1.2)

If we attempt to weaken the conjecture in Eq. 3.1.1 by using the connective imply >, then the instance reads:

"This sentence implies falsity", or "This sentence implies truth". (3.2.1)

(%(p>(p@p))>(p@p)) + (%(p>(p>p))>(p>p)) ;
TTTT TTTT TTTT TTTT (3.2.2)

Limiting an evaluation to the mapping of "is" to mean equivalence, then Eqs. 1.1.1, 2.1.1, and 3.1.1 are not tautologous. This does not confirm the liar's paradox as rendered, and hence shows Gödel's first incompleteness theorem as not tautologous.

On the other hand, if the mapping of "is" relaxes to "implies", then Eqs. 2.2.2 and 3.2.2 are tautologous.

However we are left with the fact that the liar's sentence as written is an equivalency and not an implication.

If Gödel's first incompleteness theorem is not tautologous, then there is no reason to pursue his second incompleteness theorem.

Gödel incompleteness theorems

Zhu, M-Y. (2013). Gödel's incompleteness theorem verified by PowerEpsilon. Technical report. DOI: 10.13140/RG.2.2.31985.6896

This paper relies heavily on the first order logic (FOL) expressions in the text which is a perfect implementation of Gödel's axioms, rules, and theorems in the programming language of PowerEpsilon. With that exposition, Meth8 is capable to evaluate the theorems of Gödel.

LET: p x; q y; s s; # for all; % for some; & And; \ Not And /; > Imply; < Not Imply

The designated proof value is T tautology. Other values are: F contradiction; C contingency (a falsity value); and N non-contingency (a truth value).

Truth tables are presented as the 16-values in row major, horizontally.

When rendering quantified operators from the text to the script of Meth8, we explicitly distribute quantified operators for clarity and portability. For example ∀p . (p ∨ ¬p) is equivalent to ∀p . (p) ∨ ∀p . (¬p).

We examine FOL expressions to replicate results in the text:

[Section 4.4. FOL axioms replicated and confirmed tautologous.
Section 4.5. FOL inference rules; we stopped at 4.5.2.13 with functions;
then commenced again at 4.5.2.14.1.]

At 4.5.2.15 for universal quantifier:

LET: p X; q Y; r v; s Γ upper_case_Gamma; # for all; % for some

<u>∀Y . Γ ⊢ X[Y/v]</u>
Γ ⊢ ∀v . X (4.5.2.15.1)

((#q&s)>(p&(q\r)))>(s>(#r&p)) ; TTTT TTTT TTCT TTTT (4.5.2.15.1.1)

<u>Γ ⊢ ∀v . X</u>
∀Y . Γ ⊢ X[Y/v] (4.5.2.15.2)

((s>(#r&p))>(#q&s))>((#q&s)>(p&(q\r))) ;
TTTT TTTT TTCT TTCC (4.5.2.15.2.1)

<u>Γ ⊢ Y Γ ⊢ ∀v . X</u>
Γ ⊢ X[Y/v] (4.5.2.15.3)

((s>q)&(s>(#r&p)))>(s>(p&(q\r))) ; TTTT TTTT TTTT TTTC (4.5.2.15.3.1)

At 4.5.2.16 for existential quantifier:

$$\frac{\exists Y . \Gamma \vdash X[Y/v]}{\Gamma \vdash \exists v . X} \qquad (4.5.2.16.1)$$

((%q&s)>(%q&(p&(q\r))))>(s>(%r&p)) ;
TTTT TTTT CCTC CTTT (4.5.2.16.1.1)

$$\frac{\Gamma \vdash \exists v . X}{\exists Y . \Gamma \vdash X[Y/v]} \qquad (4.5.2.16.2)$$

(s>(%r&p))>((%q&s)>(%q&(p&(q\r)))) ;
TTTT TTTT TTTT TTTC (4.5.2.16.2.1)

$$\frac{\Gamma \vdash Y \; \Gamma \vdash X[Y\backslash v]}{\Gamma \vdash \exists v . X} \qquad (4.5.2.16.3)$$

((s>q)&(s>(p&(q\r))))>(s>(%r&p)) ; TTTT TTTT TTTC TTTT (4.5.2.16.3.1)

At 4.5.2.21 for universal and existential quantifiers:

$$\frac{\Gamma \vdash \neg\forall v . X}{\Gamma \vdash \exists v . \neg X} \qquad (4.5.2.21.3)$$

(s>(~#r&p))>(s>(%r&~p)) ; TTTT TTTT TFTF TNTN (4.5.2.21.3.1)

$$\frac{\Gamma \vdash \neg\exists v . X}{\Gamma \vdash \forall v . \neg X} \qquad (4.5.2.21.4)$$

(s>(~%r&p))>(s>(#r&~p)) ; TTTT TTTT TCTC TTTT (4.5.2.21.4.1)

Meth8 does not replicate those quantified expressions in Sections 4.5.2.15, 4.5.2.16, or 4.5.2.21. Some of the truth tables come close to tautology by pattern.

At 8.2.4 for completeness and incompleteness theorems:

Completeness of logic system: $\forall p . (\exists\Gamma . \Gamma \vdash p \vee \exists\Gamma . \Gamma \vdash \neg p)$ (8.2.3.1)

(#p&(%s&(s>p)))+(#p&(%s&(s>~p))) ;
FFFF FFFF FNFN FNFN (8.2.3.1.1)

Incompleteness of logic system: $\exists p . (\neg\exists\Gamma . \Gamma \vdash p \wedge \neg\exists\Gamma . \Gamma \vdash \neg p)$ (8.2.3.2)

(#p&(~%s&(s>p)))&(#p&(~%s&(s>~p))) ;
FNFN FNFN FFFF FFFF (8.2.3.2.1)

Completeness of formula set: $\forall p . (\Gamma \vdash p \vee \Gamma \vdash \neg p)$ (8.2.4.1)

(#p&(s>p))+(#p&(s>~p)) ; FNFN FNFN FNFN FNFN (8.2.4.1.1)

Incompleteness of formula set: $\exists p . (\Gamma \not\vdash p \land \Gamma \not\vdash \neg p)$ (8.2.4.2)

(%p&(s<p))&(%p&(s<~p)) ; `FFFF FFFF FFFF FFFF` (8.2.4.2.1)

Meth8 does not replicate those quantified theorems in Sections 8.2.3 or 8.2.4. Eq. 8.2.4.1 is validated as contradictory.

At Example 6.1, page 105:

"Let p be the string in the formal language S defined by $p \equiv \forall y \exists x(x = sy)$." (1.0)

We ignore the "p = = ", to test the consequent clauses in the 4-variable and 11-variable Meth8 versions.

Meth8-4 for 4-variables (p, q, r, s) produces one 16-value truth table.

$p \equiv \forall y \exists x(x = sy)$. (1.0)

(#q&%p)&(p=(s&q)) ; `FFFF FFFF FFFF FFFN` (1.1)

If Eq. 1.1 is weakened by replacing the equivalent connective = with the imply connective, we have:

(#q&%p)&(p>(s&q)) ; `FFFF FFFF FFFN FFFN` (1.1.1)

In the truth table for Eq. 1.1, `FFFF FFFF FFFF FFFN` , we notice the result is nearly a contradiction except for the one truth value of N non-contingency. While Meth8 validates Peano arithmetic as tautologous elsewhere here, the particular Eq. 1.1 is *not* tautologous.

Gödel incompleteness theorem: contradictions in FOL

The Gödel incompleteness theorem (Meyer, 2014) contains examples of two contradictions in first order logic (FOL), as an axiom in system P and a proposition for generic schema.

We assume the Meth8 apparatus implementing variant system VŁ4 for:
T tautology (designated proof value); F contradiction; C contingency (falsity); N non-contingency (truth). Truth table results in 16-values are row-major and horizontally.

1. "This axiom represents the axiom of reducibility (the axiom of comprehension of set theory)" in formal system P, Section 2, Proposition IV.1:

(∃ u)(v ∀ (u(v) ≡ a)) (2.4.1.0)

LET: p r s a u v ; % existential quantifier, # universal quantifier, as undistributed

%r&(s&#((r&s)=p)) ; `FFFF FFFF FFFF FNFN` (2.4.1.1)

LET: p r s a u v ; % existential quantifier, # universal quantifier, as distributed

(%r&s)&(%r&((#r&#s)=#p)) ; `FFFF FFFF CCCC CTCT` (2.4.1.2)

Eqs. 2.4.1.1 and 2.4.1.2 are *not* tautologous.

2. "Relation (class) is called arithmetical" in Section 3, Proposition 6:

x > y ≡ ~(∃z)[y = x+z] ; (3.6.0)

LET: p q r x y z ; % existential quantifier, as undistributed

(p>q)=(~%r&(q=(p+r))) ; `NTFN FTFF NTFN FTFF` (3.6.1)

LET: p q r x y z ; % existential quantifier, as distributed

(p>q)=((~%r&q)=(~%r&(p+r))) ; `TNCT TFTT TNCT TFTT` (3.6.2)

Eqs. 3.6.1 and 3.6.2 are *not* tautologous.

Using Meth8-VŁ4 we can not find tautology in these examples. We conclude that the use of quantified operators by Gödel was mistaken as inconsistent, or not bivalent, or both.

References

Meyer, J.R. (2014). Meltzer's English translation of "Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I". jamesrmeyer.com/pdfs/godel-original-english.pdf.

Gödel incompleteness theorems: Evaluation of computer assisted proofs

To better evaluate computer assisted proofs of Gödel incompleteness theorem(s), we ask two questions.

Q1. Are there refutations of Gödel incompleteness theorems using proof assistants?; and

Q2. In the PowerEpsilon proof language (Zhu, 2013), what is the first mistake found, and does it color the results?

We answer:

A1.1 The website jamesrmeyer.com/ffgit/representability.html shows that first order logic (FOL) can be represented mistakenly in numeric symbols. Used as examples are objections to Gödel theorems at:

jamesrmeyer.com/pdfs/ff_harrison.pdf
jamesrmeyer.com/pdfs/ff_oconnor.pdf
jamesrmeyer.com/pdfs/ff_shankar.pdf

Each assessment is negative for the computer assisted tool in evaluation of the theorems. The reasons rely on set theory to show a mixing of number domains as the cause of misrepresentation, with the effective admonition against the proof assistants of garbage-in, garbage-out.

The website proffers no assisted proof tool, alternative or original, to those abandoned. The website publishes logical verbiage as contra arguments to the incompleteness theorems. Unfortunately there are no clear FOL expressions proffered for mapping into renditions suitable to test, as in those very proof assistants so abandoned. In other words, the negated proof tools are not allowed to evaluate the arguments proposed by the website.

Unfortunately none of the website papers is peer reviewed which we could recognize or presented elsewhere on academic forums for comments.

A1.2 The website does not evaluate PowerEpsilon, so we supplied a copy of (Zhu, 2013) with the question of: "What is mistaken in this monograph". Due to no timely response, a complaint invoked by the website on others, we concluded that the website found no logical mistakes in (Zhu, 2013). This led us to ask Q2.

A2. The first equation we evaluated in (Zhu, 2013) was for induction that we validated as tautologous using Meth8-VŁ4 (James III, 2017).

The next expression in the text is the FOL axiom of the law of excluded middle (LEM):

$\forall$ P . P $\vee$ ¬P (2.1)

We assume the Meth8 apparatus, where the designated proof value is T tautology. Truth tables are in 16-values as presented row major and horizontally.

LET: # $\forall$; p P; + Or; ~ Not; = Equivalent to; (p=p) Tautology.

$\forall$ P . P $\vee$ ¬P (2.1)

(#p&(p+~p)) = (p=p) ; `FNFN FNFN FNFN FNFN` (2.1.1)

Eq. 2.1.1 is not tautologous.

We distribute the universal quantifier directly to the variable in the antecedent at Eq. 2.1.1:

(#p+#~p) = (p=p) ; FNFN FNFN FNFN FNFN (2.1.2)

The result of Eq. 2.1.2 and 2.1.1 is the same, as not tautologous.

We are reminded that the LEM without the universal quantifier is:

(p+~p) = (p=p) ; TTTT TTTT TTTT TTTT (2.1.3)

Eq. 2.1.3 is tautologous.

We attempt to coerce the universal quantifier onto the LEM to make it tautologous as follows:

(#(p+~p)=#(p=p))) = (p=p) ; TTTT TTTT TTTT TTTT (2.1.4)

Eqs. 2.1.4 is tautologous.

We conclude that Eq. 2.1 is mistaken.

For the second part of Q2, does Eq. 2.1, now as not tautologous, affect the resulting conclusion in PowerEpsilon to prove the theorem(s) of incompleteness? We conclude Eq. 2.1 has no affect. Our reasoning is that the Gödel formulas are mistaken, due to non-uniform representation of quantification, but mapped with fidelity by PowerEpsilon. In other words, the source of error is fully with Gödel.

References

James III, C. 2017. PowerEpsilon mathematical induction. Technical report.)

Zhu, M-Y. 2013. Gödel's incompleteness theorem verified by PowerEpsilon. Technical report. DOI: 10.13140/RG.2.2.31985.6896

The shortest refutation of Gödel's theorem of incompleteness

We assume the apparatus of the Meth8 modal logic model checker implementing variant system VŁ4. Meth8 allows to mix four logical values with four analytical values. The designated *proo*f value is T.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	p=p	T	Tautology	proof	11	3
2	p@p	F	Contradiction	absurdum	00	0
3	%p>#p	N	Non-contingency	truth	01	1
4	%p<#p	C	Contingency	falsity	10	2

LET: ~ Not; + Or; & And; \ Not and; > Imply; < Not imply; = Equivalent to; @ Not equivalent to; # all; % some; (p@p) 00, zero; (p=p) 11, one

Results are the proof table of 16-values in row major, horizontally.

We define:

"a sentence" as p (0.0)

p ; FTFT FTFT FTFT FTFT (0.1)

We assert for clarity an expression cast in the positive using *a*s for a fragment of implication, instead of *is* for a sentence of equivalency, and inserting the modal operator of necessity:

"The necessity of 'This sentence as a proof'." (1.1)

#(p > (p=p)) ; NNNN NNNN NNNN NNNN (1.2)

Systems of two- or three-valued logic are insufficient to capture the complete informational content of Eq. 1.1 for subsequent discourse. We also avoid testing the more complicated instance forced by assignment of Eq. 1.1 to another variable by inserting the modal operator of possibility:

"Possibly a sentence implies the necessity of 'This sentence as a proof'." (2.1)

%p > #(p > (p=p)) ; NNNN NNNN NNNN NNNN (2.2)

This means Eq. 2.1 is an axiom with a truth value of N for non-contingency (as opposed to a falsity value of C for contingency), but not a theorem with truth value of T for tautology. This contradicts Gödel's theorem of incompleteness, where Eq. 2.2 should a refutation with truth value of F for contradiction.

We test the common contra-example for 'This sentence as not a proof'. We rewrite Eqs. 1.1-2.2:

"The necessity of 'This sentence as not a proof'." (3.1)

#(p > ~(p=p)) ; NFNF NFNF NFNF NFNF (3.2)

"Possibly a sentence implies the necessity of 'This sentence as not a proof'." (4.1)

%p > #(p > ~(p=p)) ; NFNF NFNF NFNF NFNF (4.2)

This means Eq. 4.2 is not an axiom or a theorem. This contradicts Gödel's theorem of incompleteness, where Eq. 4.2 should be a theorem with truth value of T for tautology.

Remark: In quantified terms, Eqs. 2.1 and 4.1 with the same results alternatively read:

"Some sentence implies all instances of 'This sentence as a proof'." (5.1)
"Some sentence implies all instances of 'This sentence as not a proof'." (6.1)

Our examples show the shortest refutation for Gödel's incompleteness theorem.

Contra-examples to Gödel incompleteness theorem as Löb axiom and sub-conjecture □⊥>⊥

Abstract: We show the Löb axiom □(□⊥>⊥)>□⊥ is *not* tautologous, and the conjecture □⊥>⊥ is *not* contradictory. These serve as two contra-examples to the Gödel incompleteness theorem, hence refuting it.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q: p, q; ~ Not; > Imply, greater than; = Equivalent; @ Not Equivalent;
(p=p) Tautology; (q@q) **F** contradiction, ⊥;
% possibility, ◊, for one or some, ∃; # necessity, □, for every or all, ∀.

The Löb axiom is supposed to define transitivity and second-order converse well-foundedness as:

□(□p>p)>□p (1.1)

#(#p>p)>#p ; (1.2)

We decompose the variables in Eq. 1.2 to show the table results at each step.

p=(p=p) ;	**F**T**F**T **F**T**F**T **F**T**F**T **F**T**F**T	(1.2.1.2)
#p=(p=p) ;	**F**N**F**N **F**N**F**N **F**N**F**N **F**N**F**N	(1.2.2.2)
#p>p ;	TTTT TTTT TTTT TTTT	(1.2.3.2)
#(#p>p)=(p=p) ;	NNNN NNNN NNNN NNNN	(1.2.4.2)
#(#p>p)>#p ;	CTCT CTCT CTCT CTCT	(1.2.5.2)

We replace the variable p with the symbol for contradiction ⊥:

□(□⊥>⊥)>□⊥ (2.1)

#(#(q@q)>(q@q))>#(q@q) ; CCCC CCCC CCCC CCCC (2.2)

We decompose the variables in Eq. 2.2 to show the table results at each step.

(q@q)=(p=p) ;	CCCC CCCC CCCC CCCC	(2.2.1.2)
#(q@q)=(p=p) ;	**FFFF FFFF FFFF FFFF**	(2.2.2.2)
#(q@q)>(q@q) ;	TTTT TTTT TTTT TTTT	(2.2.3.2)
#(#(q@q)>(q@q))=(p=p) ;	NNNN NNNN NNNN NNNN	(2.2.4.2)
#(#(q@q)>(q@q))>#(q@q) ;	CCCC CCCC CCCC CCCC	(2.2.5.2)

Eqs. 1.2 and 2.2 are *not* tautologous, hence refuting the Löb axiom.

The simpler conjectures of □p>p or □⊥>⊥, as rendered in Eqs. 1.2.3.2 or 2.2.3.2, are tautologous. However according to the Gödel incompleteness theorem, these should be *not* tautologous. Similarly the conjecture of the Löb axiom should be tautologous, but it is not. Consequently these serve as contra-examples to the Gödel incompleteness theorem, hence refuting it.

Refutation of the second incompleteness theorem by Gödel logic

Abstract: Gödel's second incompleteness theorem as based on the minimal modal logic to express the Löb axiom is *not* tautologous. Subsequent substitutions into the Löb axiom along with Hájek's earlier lemma raise further suspicion about Gödel-justification logic.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: φ or i or x, ψ or y, R, s or z;
~ Not, ¬; + Or, ∨; & And, ∧; = Equivalent, ↔; > Imply, →, greater than;
# necessity, for all or every, □, ∀; % possibility, for one or some, ◊, ∃;
(s=s) ⊤, Tautology as the designated *proof* value; (s@s) ⊥, **F** as contradiction.

From: Holliday, W.H.; Litak, T. (2018). Complete additivity and modal incompleteness. arxiv.org/pdf/1809.07542.pdf

Let vB be the smallest normal modal logic containing the axiom

□◊⊤ → □(□(□p → p) → p), (2.0.1.1)

#%(p=p)>#(#(#p>p)>p) ; **F**N**F**N **F**N**F**N **F**N**F**N **F**N**F**N (2.0.1.2)

which we will call the vB-axiom. Van Benthem [1979] proved that the logic vB is Kripke incomplete.

In this connection, it is noteworthy that the vB-axiom is a theorem of the provability logic GL, the smallest normal modal logic containing the Löb axiom,

□(□p → p) → □p. (2.0.2.1)

#(#p>p)>#p ; CTCT CTCT CTCT CTCT (2.0.2.2)

Remark 2.0.2.: Eq. 2.0.2.2 as rendered is *not* tautologous. This means the Löb axiom is refuted. (From our other papers, the likely intention of the Löb axiom is: □(□p→p)↔(p∨¬p); with a simpler version as either □(□¬p→p)↔□p or □(□p→¬p)↔□¬p.)

Substituting ⊥ for p in the Löb axiom yields

□◊⊤ → □⊥, (2.0.3.1)

#%(p=p)>#(p@p) ; **FFFF FFFF FFFF FFFF** (2.0.3.2)

which in the context of provability logic is a modal version of Gödel's Second Incompleteness Theorem.

Remark 2.0.3.2: Eq. 2.0.3.2 is *not* tautologous, meaning in this modal context Gödel's second incompleteness theorem is refuted.

Clearly the vB-axiom is derivable from □◊⊤ → □⊥.

We write this to mean that Eqs. 2.0.3.1 implies 2.0.1.1. (2.0.4.1)

(#%(p=p)>#(p@p)) > (#%(p=p)>#(#(#p>p)>p)) ;
TTTT TTTT TTTT TTTT (2.0.4.2)

In the other direction, van Benthem showed that □◊⊤ → □⊥ is a Kripke-frame consequence of the vB-axiom. However, he also showed that □◊⊤ → □⊥ is not a theorem of vB.

We write this to mean that Eqs. 2.0.1.1 does not imply 2.0.3.1. (2.0.5.1)

~((#%(p=p)>#(#(#p>p)>p))>(#%(p=p)>#(p@p))) = (p=p) ;
FN**F**N **F**N**F**N **F**N**F**N **F**N**F**N (2.0.5.2)

Together these facts imply the Kripke-incompleteness of vB. (2.0.6.0)

We write this to mean that Eqs. 2.0.4.1 and 2.0.5.1 imply Kripke-incompleteness as defined in Fn. 7 per the Henkin sentence, as "a simplest possible Kripke incomplete unimodal logic" of □(□p↔p)→□p (with the same result table for Eq. 2.0.2.1, the Löb axiom, as *not* tautologous). (2.0.6.1)

(((#%(p=p)>#(p@p))>(#%(p=p)>#(#(#p>p)>p)))&
~((#%(p=p)>#(#(#p>p)>p))>(#%(p=p)>#(p@p))))>
(#(#p=p)>#p) ; TTTT TTTT TTTT TTTT (2.0.6.2)

Remark 2.0.6.2: Eq. 2.0.6.2 has the canonical form of True And False Implies False (TTTT & **F**N**F**N = **F**N**F**N) > CTCT as a theorem. This means that Eq. 2.0.6.0 uses the *non* fact of Eq. 2.0.5.1 to imply the *non* fact of Kripke-incompleteness of vB which is probably not the author intention.

SO[second-order] (□◊⊤ → □⊥), which is equivalent to

∀x(∀y(Rxy → ∃zRyz) → ∀y¬Rxy), (8.1.1)

(#%(s=s)>#(s@s))=(((r&(#p&#q))>(r&(#q&s)))>(#q&(~r&(#p&#q))))
TTTC TTTC TTTC TTTT (8.1.2)

by a formalized version of the proof of Lemma 2.1.

Remark 8.1.2: Eq. 8.1.2 is *not* tautologous, meaning the equivalence of Eq. 8.1.1 is denied.

Proposition 9.2 ... [D]erive □◊⊤ → □⊥ from the vB-axiom □◊⊤ → □(□(□p → p) → p).

We write this as (□◊⊤ → □(□(□p → p) → p)) implies (□◊⊤ → □⊥). (9.2.1)

((#%(s=s))>#(#(#p>p)>p))>(#%(s=s)>#(s@s)) ;
TCTC TCTC TCTC TCTC (9.2.2)

Remark 9.2.2: Eq. 9.2.2 is *not* tautologous, meaning he derivation is denied.

From: Pischke, N. (2018). A note on strong axiomatization of Gödel-justification logic. arxiv.org/pdf/1809.09608.pdf

Lemma 2.6 (Hájek [*1998*]). G proves the following formulas:

(1) $\varphi \to (\psi \to \varphi)$
(2) $\varphi \to \varphi$
(3) $\varphi \to (\psi \to \chi) \to ((\varphi \to \psi) \to (\varphi \to \chi))$ (2.6.3.1)

While item (1) and (2) are even theorems for Hájek's basic logic BL, item (3) is a particular feature of Gödel logic, distinguishing it from the other prominent t-norm based logics. This lemma is also the reason for why the usual proof of the classical deduction theorem works in Gödel logic.

(p>(q>r))>((p>q)>(q>r)) ; T**F**TT TTTT T**F**TT TTTT (2.6.3.2)

Remark 2.6.3.2: Eq. 2.6.3.2 as rendered is *not* tautologous. That particular feature of Gödel logic is refuted along with the lemma as "reason for why the usual proof of the classical deduction theorem works in Gödel logic". Consequently, Gödel-justification logic is suspicious.

Refutation of subset models for justification logic

Abstract: We evaluate two axioms for justification logic which are not tautologous. This means subset models for justification logic are refuted.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⇔, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) C non-contingency, ∇, ordinal 2; (%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).

Note: To preserve clarity, we usually distribute quantifiers to each variable so designated.

From: Lehmann, E.; Studer, T. (2019). Subset models for justification logic.
arxiv.org/pdf/1902.02707.pdf tstuder@inf.unibe.ch

2.1 Syntax
We investigate a family of justification logics that differ in their axioms and how the axioms are justified.

We have two sets of axioms, the first axioms are:

[We take ":" to mean "such that" and mapped as the Imply connective.]

j+ s : A∨t : A→(s+t) : A; (2.1.2.1)

LET p, q, r, s: j, A, t, s.

((s>(q+r))>(q>(s+r)))>q ;
j+c: (((p+s)>(q+r))>(q>(s+r)))>q ;
FFTT **FF**TT **FF**TT **FF**TT (2.1.2.2)

jc★ c★ : A∧c★ : (A→B)→c★ : B. (2.1.3.1)

LET p, q, r, s: j, c*, A, B.

(q>(r&q))>((r>s)>q))>s ; TT**FF** TTTT **FFFF** TTTT (2.1.3.2)

Eqs. 2.1.2.2 and 2.1.3.2 as rendered are *not* tautologous as axioms. This means subset models for justification logic are refuted.

Gödel-Löb Axiom

This example replicates the proof for provability logic of the Gödel-Löb axiom GL as □(□p→p)→□p. If p is "*choice*", this transcribes in words to: "The necessity of *choice*, as always implying *a choice*, implies always *a choice*."

The axiom transcribes to #(#p>p)>#p for test input to Meth8 with output in Tab. 6. Model 2.2 is validated as one of five models. Hence by VŁ4 the Gödel-Löb axiom is suspect.

For the GL axiom to be validated in five of five models, the expression is rewritten as □(□p→p)↔(p∨¬p), in words: "The necessity of *choice*, as always implying *a choice*, is equivalent to always *a choice* or *no choice*."

A simpler rendition of a validated GL-type axiom is either □(□¬p→p)↔□p or □(□p→¬p)↔□¬p as respectively in words: "The necessity of *no choice*, as always implying *a choice*, is equivalent to always *a choice*."; or "The necessity of *choice*, as always implying *no choice*, is equivalent to always *no choice*."

If GL fails, then so also does Zermelo-Fraenkel set theory and axiom of choice (ZFC) as the basis of modern mathematics.

Table 6

Test input as processed is: #(#p>p)>#p				
Model 1	Model 2.1	Model 2.2	Model 2.3.1	Model 2.3.2
#p				
FNFN	UEUE	UUUU	UIUI	UPUP
p				
FTFT	UEUE	UEUE	UEUE	UEUE
#(#p>p)				
NNNN	EEEE	UUUU	IIII	PPPP
#p				
FNFN	UEUE	UUUU	UIUI	UPUP
#(#p>p)>#p ; not validated tautologous				
CTCT	UEUE	EEEE	PEPE	IEIE
Model 1	Model 2.1	Model 2.2	Model 2.3.1	Model 2.3.2

Refutation of the Gödel-McKinsey-Tarski translation of intuitionistic logic (IPC)

Abstract: We evaluate the Gödel–McKinsey–Tarski [GMT] translation of IPC "by reduction from intuitionistic logic (IPC) using a series of translations". The equation is *not* tautologous. This refutes GMT, the method approach using a series of translations, and IPC itself which form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Jeřábek, E. (2019). Rules with parameters in modal logic II. arxiv.org/pdf/1905.13157.pdf

Abstract: We analyze the computational complexity of admissibility and unifiability with ... parameters (constants) in transitive modal logics satisfying certain extension properties

3 Derivability: … what is the complexity of tautologicity or derivability in these logics.

We now turn to the lower bound. … We will use another method, namely by reduction from intuitionistic logic (IPC) using a series of translations. This route is more useful for our purposes, because the resulting statement is relatively more general in the context of transitive modal logics (it applies to all transitive logics with the disjunction property, and it also applies to various extensions of K4.2, which will be relevant in the sequel).

Definition 3.5 Let T denote the *Gödel–McKinsey–Tarski translation* of **IPC** (formulated using connectives {→,∧,∨,⊥}) in **S4**: T(ϕ)=□ϕ if ϕ is an atom, T commutes with ∧,∨, and ⊥, and T(ϕ→ψ)=□(T(ϕ)→T(ψ)). (3.5.1)

LET p, q: ϕ, ψ.

((p=p)&(p>q))=#(((p=p)&p)>((p=p)&q)) ;
NTTN NTTN NTTN NTTN (3.5.2)

Remark 3.5.2: Eq. 3.5.2 as rendered is not tautologous as the Gödel–McKinsey–Tarski [GMT] translation of IPC. This refutes GMT, the method approach of "reduction from intuitionistic logic (IPC) using a series of translations", and IPC itself.

Gödel pairing function (pairing axiom)

From: math.uni-bonn.de/people/koepke/Preprints/Computing_a_model_of_set_theory.pdf

$\forall\alpha, \beta, \gamma(g(\beta, \gamma) \leq \alpha \leftrightarrow \forall\delta, ((\delta,) <* (\beta, \gamma) \rightarrow g(\delta,) < \alpha)).$ (1.1)

LET: p q r s t u v $\alpha\ \beta\ \gamma\ \delta\ \theta\ \eta$ g

#p,q,r(v(q,r)≤p = #s, ((s,) < * (q,r) > v(s,) < p)). (1.2)
#((p&q)&r) & (~((v&(q&r))>p)=(#s&((s< *(q,r))>((v&s)<p)))) ;
#((p&q)&r) & (~((v&(q&r))>p)=(#s&((s<(q&r))>((v&s)<p)))) ;
FFFF FFFN FFFF FFFF (1.3)

#((p&q)&r) & (~((v&(q&r))>p)=(#s&((s<(q&r))>((v&s)<p)))) ;
(#((p&q)&r) &~((v&(q&r))>p))=(#((p&q)&r) &(#s&((s<(q&r))>((v&s)<p)))) ;
(#((p&q)&r) distributed; TTTT TTTT TTTT TTTC (1.4)

(#((p&q)&r) &~((v&(q&r))>p))=(#((p&q)&r) &(#s&((s<(q&r))>((v&s)<p)))) ;
(#((p&q)&r) distributed, (s<(q&r)) replaced by Eq 2.2, a tautology, from farther below

((%u&t)&(((p>q)&(u=p)+(~(p>q)&(u=q)))&(((r>s)&(t=r))+(~(r>s)&(t=s)))&((u<t)+
(((u=t)&(p<r)))+(((u=t)&(p =r))&(q<s))))) as: (2.2)

(#((p&q)&r) &~((v&(q&r))>p))=(#((p&q)&r) &(#s&((%u&t)&(((p>q)&(u=p)+
(~(p>q)&(u=q)))&(((r>s)&(t=r))+(~(r>s)&(t=s)))&((u<t)+(((u=t)&(p<r)))+(((u=t)&(p
=r))&(q<s))))))>((v&s)<p)))) ; TTTT TTTT TTTT TTTC (1.5)

Eq. 1.5 is nearly barely *not* tautologous with the contingency C value of falsity.

Here $(\alpha, \beta) <* (\gamma, \delta)$ stands for (p,q) < * (r, s) (2.1)

$\exists\eta, \theta(\eta = \max(\alpha, \beta) \wedge \theta = \max(\gamma, \delta) \wedge (\eta < \theta \vee (\eta = \theta \wedge \alpha < \gamma) \vee (\eta = \theta \wedge \alpha = \gamma \wedge \beta < \delta))),$
$\exists\eta, \theta(((\alpha > \beta) \wedge (\eta = \alpha)) \vee ((\alpha \leq \beta) \wedge (\eta = \beta))) \wedge (((\gamma > \delta) \wedge (\theta = \gamma)) \vee ((\gamma \leq \delta) \wedge (\theta = \delta))) \wedge ((\eta < \theta) \vee (((\eta = \theta) \wedge (\alpha < \gamma))) \vee ((\eta = \theta) \wedge (\alpha = \gamma) \wedge (\beta < \delta)))),$

where $\gamma = \max(\alpha, \beta)$ abbreviates $(\alpha > \beta \wedge \gamma = \alpha) \vee (\alpha \leq \beta \wedge \gamma = \beta)$;
where $\eta = \max(\alpha, \beta)$ abbreviates $(\alpha > \beta \wedge \eta = \alpha) \vee (\alpha \leq \beta \wedge \eta = \beta)$;
where $\theta = \max(\gamma, \delta)$ abbreviates $(\gamma > \delta \wedge \theta = \gamma) \vee (\gamma \leq \delta \wedge \theta = \delta)$;

(%u&t)&(((p>q)&(u=p)+(~(p>q)&(u=q)))&(((r>s)&(t=r))+(~(r>s)&(t=s)))&((u<t)+
(((u=t)&(p<r)))+(((u=t)&(p =r))&(q<s)))) ;
TTTT TTTT TTTT TTTT (2.2)

Eq. 2.2 is tautologous.

Gödel Recursion

From: Xaver Y. Newberry. (2016),
"The Recursion Theorem from a Different Angle"

We map into Meth8 script the formula for the diagonal lemma following Table 2 of the text as below.

~(Ex)(Prf(x,<#~(Ex)(Ey)((Prf(x,y)&This(y) #>) <==> ~(Ex)(Ey)((Prf(x,y)&This(y)) (1.1)

LET: % E, p Prf, t This, This(y) = (t&y) = (~(%x&%y)&((p&(x&y))&(t&y))),
vt tautologous, nvt not tautologous;
T Tautologous, E Evaluated, F Contradictory,
U Unevaluated [values are FCNT, UIPE as 00, 10, 01, 11]

(((t&y) = (~(%x&%y)&((p&(x&y))&(t&y)))) & (~%x&(p&(x&(t&y))))) =
(((t&y) = (~(%x&%y)&((p&(x&y))&(t&y))))&(t&y)) ;
vt (1.2)

The truth table for Eq 1 in the five models of Meth8 is below:

```
TTTT TTTT TTTT TTTT   EEEE EEEE EEEE EEEE   EEEE EEEE EEEE EEEE   EEEE EEEE EEEE EEEE   EEEE EEEE EEEE EEEE
(((t&y)=(~(%x&%y)&((p&(x&y))&(t&y))))&(#~x&(p&(x&(t&y)))))=(((t&y)=(~(%x&%y)&((p&(x&y))&(t&y))))&(t&y))
Step: 49
```

We include the definition of (t&y) for both the antecedent and consequent groups to ensure the repeated (t&y) is present; without that, Eq. 0 is *no*t tautologous.

Gödel-Scott on God

From: Benzmüller, C.; Paleo, B.W. (2003).
"Formalization, Mechanization and Automation of Gödel's Proof of God's Existence".
DOI: 10.3233/978-1-61499-419-0-93. arxiv.org/abs/1308.4526.

These assertions are attributed to the rendering of Gödel's expressions by Dana S. Scott (unpublished, 2004), where A axiom, T theorem, and D definition:

A1.1 Either a property or its negation is positive, but not both: $\forall\varphi[P(\neg\varphi) \leftrightarrow \neg P(\varphi)]$

A2.1 A property necessarily implied by a positive property is positive:
$\forall\varphi\forall\psi[(P(\varphi) \wedge \forall x[\varphi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$

T1.1 Positive properties are possibly exemplified: $\forall\phi[P(\phi) \rightarrow \blacklozenge\exists x\phi(x)]$

D1.1 A God-like being possesses all positive properties: $G(x) \leftrightarrow \forall\varphi[P(\varphi) \rightarrow \varphi(x)]$

A4.1 Positive properties are necessarily positive: $\forall\varphi[P(\varphi) \rightarrow P(\varphi)]$

The Meth8 mapping is below with repeating fragments of truth tables.

LET: ¬ ~, # ∀, % ∃, % ♦, ∧ &, → >, ↔ =, p P, t G, x x, φ q, ψ r, nvt not tautologous, vt ~nvt.

A1.2 (#q&(p&~q))=(#q&(~p@q)) ; nvt

```
Model 1              Model 2.1            Model 2.2            Model 2.3.1          Model 2.3.2
TTTC TTTC TTTC TTTC  EEEU EEEU EEEU EEEU  EEEE EEEE EEEE EEEE  EEEP EEEP EEEP EEEP  EEEI EEEI EEEI EEEI
```

A2.2 ((#q&#r)&((p&q)&#(#x&((q&x)>(r&x))))) >((#q&#r)&(p&r)) ;
vt

```
Model 1              Model 2.1            Model 2.2            Model 2.3.1          Model 2.3.2
TTTT TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE
```

T1.2 (#q&(p&q))=(#q&%(%x&(q&x))) ; nvt

```
Model 1              Model 2.1            Model 2.2            Model 2.3.1          Model 2.3.2
TTTC TTTC TTTC TTTC  EEEU EEEU EEEU EEEU  EEEE EEEE EEEE EEEE  EEEP EEEP EEEP EEEP  EEEI EEEI EEEI EEEI
TTCT TTCT TTCT TTCT  EEUE EEUE EEUE EEUE  EEEE EEEE EEEE EEEE  EEPE EEPE EEPE EEPE  EEIE EEIE EEIE EEIE
```

D1.2 (t&x)=(#q&((p&q)>(q&x))) ; nvt

```
Model 1              Model 2.1            Model 2.2            Model 2.3.1          Model 2.3.2
TTCC TTCC TTCC TTCC  EEUU EEUU EEUU EEUU  EEEE EEEE EEEE EEEE  EEPP EEPP EEPP EEPP  EEII EEII EEII EEII
FFNN FFNN FFNN FFNN  UUEE UUEE UUEE UUEE  UUUU UUUU UUUU UUUU  UUII UUII UUII UUII  UUPP UUPP UUPP UUPP
TTCT TTCT TTCT TTCT  EEUE EEUE EEUE EEUE  EEEE EEEE EEEE EEEE  EEPE EEPE EEPE EEPE  EEIE EEIE EEIE EEIE
```

A4.2 (#q&(p&q))=(#q&#(p&q)); vt

```
Model 1              Model 2.1            Model 2.2            Model 2.3.1          Model 2.3.2
TTTT TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE
```

We ask if (A1.1 & A2.1) > T1.1, that is: (A1.2 & A2.2) > T1.2.

(((#q&(p&~q))=(#q&(~p@q))) & (((#q&#r)&((p&q)&#(#x&((q&x)>(r&x)))))>((#q&#r)&(p&r))) >
((#q&(p&q))=(#q&%(%x&(q&x)))) ; nvt

```
Model 1              Model 2.1            Model 2.2            Model 2.3.1          Model 2.3.2
TTTT TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE
TTCT TTCT TTCT TTCT  EEUE EEUE EEUE EEUE  EEEE EEEE EEEE EEEE  EEPE EEPE EEPE EEPE  EEIE EEIE EEIE EEIE
```

We ask if (A1.1 > A2.1) > T1.1, that is: (A1.2 > A2.2) > T1.2.

(((#q&(p&~q))=(#q&(~p@q))) > (((#q&#r)&((p&q)&#(#x&((q&x)>(r&x)))))>((#q&#r)&(p&r))) > ((#q&(p&q))=(#q&%(%x&(q&x)))) ; nvt

```
Model 1              Model 2.1            Model 2.2            Model 2.3.1          Model 2.3.2
TTTC TTTC TTTC TTTC  EEEU EEEU EEEU EEEU  EEEE EEEE EEEE EEEE  EEEP EEEP EEEP EEEP  EEEI EEEI EEEI EEEI
TTCT TTCT TTCT TTCT  EEUE EEUE EEUE EEUE  EEEE EEEE EEEE EEEE  EEPE EEPE EEPE EEPE  EEIE EEIE EEIE EEIE
```

Our results are summarized as:

R1 A1, T1, and D1 are not tautologous.
R2 A2 and A4 are tautologous.
R3 A1 and A2 does not imply T1.
R4 A1 implying A2 does not then imply T1.

We conclude that the Gödel-Scott proof of God is not tautologous, as advertised in the popular press.

Benzüller, Paleo, and Scott decline to share the tool results for independent replication, casting further doubt on the veracity of the claimed results.

Goldbach's conjectures

From: Noheda, Pedro; Tabarés, Nuria. 2017. "
A primordial, mathematical, logical and computable, demonstration (proof) of the family of conjectures known as Goldbach´s";
researchgate.net/publication/315793002_A_primordial_mathematical_logical_and_computable_demonstration_proof_of_the_family_of_conjectures_known_as_Goldbachs

We evaluated the beginning of this conference paper using Meth8 modal logic model checker, based on Łukasiewicz variant system VŁ4 of our resuscitation, with negative results, so we stopped.

Troubling was early on page 8 with the equation following this text :

"Thus, we were able to state about natural numbers (0) and one (1) the following:

(((0∈ N[0⊂PA] ∧ **0**∈ N[0⊄PA]**)** ∧ **(1**∈ N[0⊂PA] ∧ N[0⊄PA]**)** [1]
∨ **(((0**∈ N[0⊂PA] ∩ N[0⊄PA]**)** ∧ **(1∈** N[0⊂PA] ∩ N[0⊄PA]**)** [2]
→ **(**N[0⊄PA] ⊂ N[0⊂PA]**))**" [3]

We label the unnumbered equation parts as follows. Eq 1 Or Eq 2 Implies Eq 3. Eq 1 has antecedent 1.1 And consequent 1.2. Eq 2 has antecedent 2.1 And consequent 2.2.

Meth8 makes no distinction between set operators and Boolean operators. Therefore Eq 1.2 is equivalent to Eq 2.2. Because both Eq 1.2 and 2.2 are antecedents connected by Or between Eq 1 and Eq 2, we can remove Eq 1.2 and Eq 2.2. This reduces to Eqs: (1.1 Or 2.1) Implies 3. This means explicit reference to natural number (1) is removed logically, and the equation describes only natural number (0).

LET: ~ Not; q = 0<PA; ~q = ~(0<PA); > Imply, greater than; < member of, less than, Not Imply; = Equivalent; @ Not Equivalent; & And; + Or; 0 ((%p>#p)-(%p>#p)); ~(q<q) = (q>q)+(q=q).

The designated truth value is T for tautology, and opposite F for contradiction. Result fragments are a repeating row from a 16-value truth table.

p; FTFT
q ; FFTT
~q ; TTFF *
((%p>#p)-(%p>#p)) ; CCCC
~(((%p>#p)-(%p>#p))<~q) ; TTNN
(~q<q) ; TTFF *
Eq 1: (((%p>#p)-(%p>#p))<q)&~(((%p>#p)-(%p>#p))<~q) ;
CCFF
Eq 2: (((%p>#p)-(%p>#p))<~q)&~q ;
FFFF
Eq 3: ~q ; TTTT
Eq 4: Eq 1 + Eq 2 = Eq 3: (((((%p>#p)-(%p>#p))<q)&~(((%p>#p)-(%p>#p))<~q))+((((%p>#p)-(%p>#p))<~q)&~q)) > ~q ; TTTT

We conclude that while Eq 4 is proved in the weakest form of implication where two contradictory expressions imply a tautologous one, Eq 4 relates only to natural number (0), and hence excludes proof also of natural number (1).

What follows is that the text statement in italics "We are able to define both, the union and the intersection of both [~q] and [q]" is not mistaken. What follows correctly is that contradictory antecedent Eq 1 Or contradictory consequent Eq 2 Implies a tautologous result Eq 3 for natural number (0), only.

Refutation of the Goldbach strong conjecture

Abstract: We evaluate a claimed proof of the strong Goldbach conjecture in two stages. The claimed proof of stage one is *not* tautologous, however from the word description it is tautologous. We do not evaluate stage two. Three equations claimed in the summary are *not* tautologous. These form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Maitra, S. (2019). Proof of Goldbach's strong conjecture. vixra.org/pdf/1907.0276v1.pdf

First stage [in words]: There is at least one prime p (3≤p<n) for every 2n>6 such that p®n

((2n>6)>(3≤∃p<q))>(p®2n) (1.1)

Remark 1.1: We take "a|b meaning a divides b" to mean "a is divisible by b"; we write ordinal six as 6=2*3.

LET p, q, k, s: p, n,k, s

((((%s<#s)&q)>((%s<#s)&(s=s)))>(~(%p<(s=s))<q))>~ (p\((%s<#s)&q)) ;
`FF``TT` **`FF`**`TT` **`FF`**`TT` **`FF`**`TT` (1.2)

First stage proof from the text: Consider the evens 2n-2 or 2n+2 such that they are not integral powers of 2. Now n-1 or n+1 is divisible by at least one prime p* (3≤p*<n). So

p*|2(n-1)⇒p*®2(n-1)+2⇒ p*®2n or p*|2(n+1) ⇒p*®2(n+1)-2⇒p*®2n (2.1)

((((q>(s@s))&~(q=(s=s)))&((((%s<#s)&p)-(%s<#s))+(((%s<#s)&p)+(%s<#s))))>(((q-(%s>#s))+(q+(%s>#s)))\(~(%p<(s=s))<q))) > (((p\((%s<#s)&(q-(%s>#s))))>~(p\(((%s<#s)&(q-(%s>#s)))+(%s<#s)))) > ((~(p\((%s<#s)&q))+(p\((%s<#s)&(q+(%s>#s)))))>(~(p\(((%s<#s)&(q+(%s>#s)))-(%s<#s)))>~(p\((%s<#s)&q))))) ;
`TTTT TTTT TTTT TTTT` (2.2)

Remark 2.2: Eqs. 1.2 and 2.2 are not equivalent. This means 1.2 as rendered does not support 2.2 as the claimed tautology in the text.

Second stage proof: … (3.1)

Remark 3.1: We do not evaluate this or the clarification of it at vixra.org/pdf/1907.0386v1.pdf at this time, and instead map the summary below.

Summing up the above discussions we are bound to accept the conclusion that

$p_{k+1}>n \Rightarrow p_{k+1} = 2n-p_k$, i.e, $2n = p_k+p_{k+1}$. (4.1)

((((p&(r+(%s>#s)))>q)>(p&(r+(%s>#s))))=(((%s<#s)&q)=(p&r)))>(((%s<#s)&q)=((p&r)+(p&(r+(s>#s))))) ; TCNC TTNT TCN**F** TTNT (4.2)

If $p_2>n$, then $2n=p_1+p_2$. (5.1)

LET p, q, r, s: p_1, n, p_2, s.

(r>q)>(((%s<#s)&q)=(p+r)) ; T**F**NC TTCC T**F**NC TTCC (5.2)

Remarks 4.2, 5.2: Eqs. 4.2 and 5.2 are *not* tautologous, hence refuting the conclusions.

Finally 6=3+3 and 4=2+2, therefore Goldbach's strong conjecture holds for every 2n≥4. (6.1)

Remark 6.1: We write 4=3+1 and 6=4+2 here.

(((s=s)+(%s>#s))&(((s=s)+(%s>#s))+(%s<#s)))>~(q>((%s<#s)&q)) ;
FFNN **FF**NN **FF**NN **FF**NN (6.2)

Refutation of the Goldblatt-Thomason theorem

Abstract: From the introduction, we evaluate the terms forth and back as a duality. Neither is negation of the other, hence refuting the core basis of the Goldblatt-Thomason theorem. The question posed by it is further answered by the universal logic VŁ4 that expresses by modal axioms *all* first-order definable properties of a binary relation, due to equivalences of the respective quantifier and modal operator.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And; > Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) `C` non-contingency, ∇, ordinal 2; (%z>#z) `N` as non-contingency, ∆, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Goldblatt, R. (2019). Morphisms and duality for polarities and lattices with operators. arxiv.org/pdf/1902.09783.pdf rob.goldblatt@msor.vuw.ac.nz

1 Introduction

We develop here a new notion of 'bounded morphism' between certain structures that model propositional logics lacking the distributive law for conjunction and disjunction. Our theory adapts a well known semantic analysis of modal logic, which we now review.

There are two main types of semantical interpretation of propositional modal logics. In *algebraic* semantics, formulas of the modal language are interpreted as elements of a modal algebra (B, f), which comprises a Boolean algebra B with an additional operation f that interprets the modality ◊ and preserves finite joins. In *relationa*l semantics, formulas are interpreted as subsets of a Kripke frame (X, R), which comprises a binary relation R on a set X.

The relationship between these two approaches is explained by a *duality* that exists between algebras and frames. This is fundamentally category-theoretic in nature. The modal algebras are the objects of a category MA whose arrows are the standard algebraic homomorphisms. The Kripke frames are the objects of a category KF whose [*sic*] arrows are the *bounded morphisms*, α: (X, R) → (X′, R′), i.e. functions α: X → X′ satisfying the 'back and forth' conditions

(Forth): xRy implies α(x)R′α(y). (1.1.1.1)

Remark 1.1.1.1: The Forth label is later interchanged with the confusing name of preservation.

LET p, r, s, w, x, y, z: α, R, R', β, x, y, z

(x&(r&y))>((p&x)&(s&(p&y))) ; TTTT TTTT TTTT TTTT(48),
TTTT **FFFF** TTTT **F**T**F**T(16) (1.1.1.2)

(Back): α(x)R′z implies ∃y(xRy&α(y)=z). (1.1.2.1)

Remark 1.1.2.1: The Back label is later interchanged with the confusing name of reflection.

((p&x)&(s&z))>((x&(r&%y))&((p&%y)=z)) ;
TTTT TTTT TTTT TTTT(80),
TTTT TTTT T**F**T**F** TCTC(16),
TTTT TTTT TTTT TTTT(16),
TTTT TTTT T**F**T**F** TTTT(16) (1.2.2.2)

(Bounded morphisms are also known as p-morphisms. The adjective ‘bounded’ derives from the R-bounded existential quantification in (1.2.1).)

Remark 1.n: Eqs. 1.1.2.1 and 1.2.2.2 as rendered are not respective negations. This refutes Forth and Back as a *duality*.

12 Goldblatt-Thomason theorem

This theorem [pay-to-play reference, from 1975] was originally formulated as an answer to the question: which first-order definable properties of a binary relation can be expressed by modal axioms? (12.1.1)

Remark 12.1.1: The universal logic VŁ4 answers Eq. 12.1.1 as:

"*all* first-order definable properties of a binary relation can be expressed by modal axioms" because in the universal logic VŁ4 the respective quantifiers are equivalent to the modal operators. (12.1.2)

Refutation of the predicatively unprovable termination of the Ackermannian Goodstein process

Abstract: We evaluate the Goodstein theorem which is *not* tautologous. This refutes such follow-on as the Ackermanian Goodstein process and conjecture of its predicatively unprovable termination. Therefore that segment is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Arai, T.; Fernández-Duqaue, D.; Wainer, S.; Weiermann, A. (2019).
Predicatively unprovable termination of the Ackermannian Goodstein process.
arxiv.org/pdf/1906.00020.pdf

1. Introduction: Among the greatest accomplishments of mathematical logic in the first half of the twentieth century was the identification of true arithmetical statements unprovable in Peano arithmetic (PA): the consistency of PA, due to Gödel ... However, such statements do not clarify whether incompleteness phenomena should be pervasive in other disciplines such as combinatorics or number theory. In contrast, Goodstein's principle is a purely number-theoretic statement simple enough to be understood by a high school student yet unprovable in PA.

Theorem 1.1 (Goodstein). For every $m \in N$ there is $i \in N$ such that $G_i m = 0$. (1.1.1)

Remark 1.1: This is repeated as theorem 2.9.

LET p, q, r, s: *i*, G, *m*, N.

((#s<r)>(%q<r))>(((p&q)&s)=(s@s)) ;
`TTTT TTTT TTT`**`F`** `TTT`**`F`** (1.1.2)

Eq. 1.1.2 as rendered is *not* tautologous, and so colors combinations of it as in an Ackermannian process with predicatively unprovable termination.

Refutation of graded modal logic

Abstract: We evaluate definitions of five frame classes and a main theorem of satisfiability. Three of the five frame classes are *not* tautologous and the designated example for satisfiability is *not* tautologous. This refutes graded modal logic.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET: ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →; < Not Imply, less than, ∈;
= Equivalent, ≡; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊; # necessity, for every or all, ∀, □;
~(y < x) (x ≤ y), (x ⊆ y); (p=p) Tautology.

From: Kazakov, Y.; Pratt-Hartmann, I. (ca. 2009).
A note on the complexity of the satisfiability problem for graded modal logics.
yevgeny.kazakov@comlab.ox.ac.uk , ipratt@cs.man.ac.uk
uni-ulm.de/fileadmin/website_uni_ulm/iui.inst.090/Publikationen/2008/KazPH08Graded_TR.pdf

LET p, q, r, s: x, y, r, z

Table I lists these frame classes together with their respective defining first-order sentences.

Reflexive frames: ∀x.r(x,x) (1.1.1)

r&#p ; **FFFF F**N**F**N **FFFF F**N**F**N (1.1.2)

Serial frames: ∀x∃y.r(x,y) (1.2.1)

r&(#p&%q) ; **FFFF FFF**N **FFFF FF**FN (1.2.2)

Symmetric frames: ∀x∀y.(r(x,y) → r(y,x)) (1.3.1)

(r&(#p&#q))>(r&(#q&#p)) ; TTTT TTTT TTTT TTTT (1.3.2)

Transitive frames: ∀x∀y∀z.(r(x,y) ∧ r(y,z) → r(x,z)) (1.4.1)

(r&(#p&#q))&((r&(#q&#s))>(r&(#p&#s))) ;
FFFF FFFN **FFFF FFF**N (1.4.2)

Euclidean frames: ∀x∀y∀z.(r(x,y) ∧ r(x,z) → r(y,z)) (1.5.1)

((r&(#p&#q))&(r&(#p&#s)))>(r&(#q&#s)) ;
TTTT TTTT TTTT TTTT (1.5.2)

Because three of the five frame class definitions are *not* tautologous, the system as refuted.

Theorem 4: ... consider the formula ϕ given by

$$\phi := q0 \wedge \Diamond_{\geq 2}(\neg q0 \wedge q1 \wedge \Diamond_{\geq 1}(\neg q0 \wedge \neg q1)) \wedge \Diamond_{\leq 1}\neg q1 \qquad (4.1)$$

The formula ϕ is certainly satisfiable over transitive frames; however, it is not satisfiable over tree-shaped transitive frames.

Remark 4.1: We map $\Diamond_{\geq 2}$ as 2*◊ and $\Diamond_{\geq 1}$ or $\Diamond_{\leq 1}$ as 1*◊ to mean ◊, that is the iteration to be the designated ordinal in the sub-scripted relation.

```
(p&((%s<#s)&%((~p&q)&%(~p&~q))))&%~q ;
                      FCFC FCFC FCFC FCFC          (4.2)
```

Because Eq. 4.2 is *not* tautologous as the example for Thm. 4, we say that Thm. 4 is also *not* tautologous, and hence do not proceed further through Thm. 6 and Lem. 3 as the proof path for Thm 4.

The Grassmannian paradox

A paradox arises for the Grassmannian **Gr**(r,V) in the short exact sequence and the dual, from en.wikipedia.org/wiki/Grassmannian:

> Every r-dimensional subspace W of V determines an $(n - r)$-dimensional quotient space V/W of V. This gives the natural *short exact sequence*:
>
> $$0 \to W \to V \to V/W \to 0. \qquad (1)$$
>
> Taking the *dual* to each of these three spaces and linear transformations yields an inclusion of $(V/W)^*$ in V^* with quotient W^*:
>
> $$0 \to (V/W)^* \to V^* \to W^* \to 0. \qquad (2)$$
>
> Using the natural isomorphism of a finite-dimensional vector space with its double dual shows that taking the dual again recovers the original short exact sequence.

We map Eq 1 and 2 into Meth8 script. The keyed truth table fragments follow on the next page and are informative.

LET: v=V=V*; w=W=W*; → Imply (>); \ Not And; = Equivalent;
0 zero ((u\u)-(u\u)) ; nvt not tautologous

(((((u\u)-(u\u))>(v\w))>v)>w) > ((u\u)-(u\u)) ;
nvt (1.1)

(((((u\u)-(u\u))>w)>v)>(v\w)) > ((u\u)-(u\u)) ;
nvt (2.1)

We test if Eq 1.1 and 2.1 are equivalent:

((((((u\u)-(u\u))>w)>v)>(v\w))>((u\u)-(u\u))) = ((((((u\u)-(u\u))>(v\w))>v)>w)>((u\u)-(u\u))) ;
nvt (3)

Eq 1.1 and 2.1 are not equivalent.

We then test if Eq 1.1 implies 2.1:

((((((u\u)-(u\u))>w)>v)>(v\w))>((u\u)-(u\u))) > ((((((u\u)-(u\u))>(v\w))>v)>w)>((u\u)-(u\u))) ;
nvt (4)

Eq 1.1 does not imply 2.1 (and because of 3, 2.1 also does not imply 1.1). Therefore from Eq 3 and 4, the *short exact sequence* and the *dual* of the Grassmannian **Gr**(r,V) are a paradox. This renders such a theory of vector analysis in physics as suspicious.

Truth table fragments are keyed to the above Eq 1.1, 2.1, 3, 4. We note that Eq 4, meaning the short exact sequence implies the dual, approaches a proof, but fails with 2 of the 16 table lines as F contradictory and Unevaluated.

(1.1) `((((u\u)-(u\u))>w)>v)>(v\w))>((u\u)-(u\u))  Step: 23`

```
Model 1                Model 2.1              Model 2.2              Model 2.3.1            Model 2.3.2
FFFF FFFF FFFF FFFF    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU
FFFF FFFF FFFF FFFF    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
FFFF FFFF FFFF FFFF    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU
FFFF FFFF FFFF FFFF    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
FFFF FFFF FFFF FFFF    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU
FFFF FFFF FFFF FFFF    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
```

(2.1) `((((u\u)-(u\u))>(v\w))>v)>w)>((u\u)-(u\u))    Step: 23`

```
FFFF FFFF FFFF FFFF    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU
FFFF FFFF FFFF FFFF    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
FFFF FFFF FFFF FFFF    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU
FFFF FFFF FFFF FFFF    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
FFFF FFFF FFFF FFFF    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU
FFFF FFFF FFFF FFFF    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
```

(3) `(((((u\u)-(u\u))>w)>v)>(v\w))>((u\u)-(u\u)))=(((((u\u)-(u\u))>(v\w))>v)>w)>((u\u)-(u\u)))   Step: 47`

```
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
FFFF FFFF FFFF FFFF    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU
FFFF FFFF FFFF FFFF    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
FFFF FFFF FFFF FFFF    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU
FFFF FFFF FFFF FFFF    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
```

(4) `(((((u\u)-(u\u))>w)>v)>(v\w))>((u\u)-(u\u)))>(((((u\u)-(u\u))>(v\w))>v)>w)>((u\u)-(u\u)))   Step: 47`

```
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
FFFF FFFF FFFF FFFF    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU
FFFF FFFF FFFF FFFF    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU    UUUU UUUU UUUU UUUU
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE    EEEE EEEE EEEE EEEE
Model 1                Model 2.1              Model 2.2              Model 2.3.1            Model 2.3.2
```

Refutation of the Hadamard gate

From: en.wikipedia.org/wiki/Quantum_logic_gate#Hadamard_(H)_gate, et seq.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET: p, q, r, s: probability, |0>, |1>, 2^0.5; ~ Not; & And; + Or; - Not Or;
> Imply, greater than; < Not Imply, less than; = Equivalent; @ Not Equivalent;
% possibility, for one or some; # necessity, for all or every;
(p@p) ordinal 0; (%p>#p) ordinal 1.

"The Hadamard gate acts on a single qubit. It maps the basis state |0⟩ to (|0⟩+|1⟩)/√2 and (1.1)

q>((q+r)\s) ; `TTTT TFTF TTTT TFTF` (1.2)

|1⟩ to (|0⟩−|1⟩)/√2 ..." (2.1)

r>((q-r)\s) ; `TTTT TTTT TTTT TTTT` (2.2)

"... which means that a measurement will have equal probabilities to become 1 or 0" (3.0)

We write Eq. 3.0 to mean: the measurement of the basis states of Eqs. 1.1 and 2.1 imply a combined probability of]0,1[. (3.1)

(p=((q>((q+r)\s))&(r>((q-r)\s))))>((p>(p@p))&(p<(%p>#p))) ;
`TFTF TFTF TFFT TFFT` (3.2)

Eq. 3.2 as rendered is *not* tautologous.

This means the Hadamard gate is refuted as producing an outcome with a combined probability of 0.00 to 1.00.

Remark: Should the combined probability be *only* greater than 0.00, then Eq. 3.2 becomes strengthened slightly, but *not* tautologous, as (4.1)

(p=((q>((q+r)\s))&(r>((q-r)\s))))>(p>(p@p)) ;
`TFTF TFTF TFTT TFTT` (4.2)

Refutation of the Hahn-Banach theorem

Abstract: We evaluate the Hahn-Banach theorem. Without or with the universal quantifiers, the equations are *not* tautologous. This refutes the Hahn-Banach theorem.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨ ; - Not Or; & And, ∧; \ Not And;
> Imply, greater than, →; < Not Imply, less than, ∈;
= Equivalent, ≡; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊; # necessity, for every or all, ∀, □;
~(y < x) (x ≤ y), (x ⊆ y); (p=p) `T`autology.

From: en.wikipedia.org/wiki/Hahn–Banach_theorem

LET p, q, r, s, u, v: p, x, φ, ψ, *U*, *V*.

Hahn–Banach theorem (Rudin 1991, Th 3.2). If *p*: *V* → **R** is a sublinear function, and *φ*: *U* → **R** is a linear functional on a linear subspace

U ⊆ *V* (0.1)

~(v<u) = (p=p) ; **`FFFF FFFF FFFF FFFF( 4),`**
`TTTT TTTT TTTT TTTT(12)` (0.2)

which is dominated by *p* on *U*, i.e.

φ (x) ≤ p (x) ∀ x ∈ U (1.1)

Remark 1: We ignore the universal quantification on *U* and *V* in this test.

~((p&q)<(r&q)) = (p=p) ; `TTT`**`F`** `TTTT TTT`**`F`** `TTTT` (1.2)

then there exists a linear extension *ψ*: *V* → **R** of *φ* to the whole space *V*, i.e., there exists a linear functional *ψ* such that

ψ (x) = φ (x) ∀ x ∈ U , (2.1)

(r&q)=(s&q) ; `TTTT TT`**`FF`** `TT`**`FF`** `TTTT` (2.2)

ψ (x) ≤ p (x) ∀ x ∈ V . (3.1)

~((p&q)<(s&q)) = (p=p) ; `TTT`**`F`** `TTT`**`F`** `TTTT TTTT` (3.2)

If Eqs 1, then (2 and 3). (4.1)

```
~((p&q)<(r&q)) > (((r&q)=(s&q))&~((p&q)<(s&q))) ;
                                   TTTT TTFF TTFT TTTT
```
(4.2)

Eq. 4.2 as rendered is *not* tautologous, hence refuting the Hahn-Banach theorem.

Remark 5: To include the relationship of *U* and *V* in Eqs. 0 and the universal quantification on *U* and *V* in 1 and 2 produces this result. (5.1)

```
~(v<u) >
(((#q<u)&~((p&#q)<(r&#q))) >
 (((#q<u)&((r&#q)=(s&#q)))&((#q<v)&~((p&#q)<(s&#q)))));
                                   TTTT TTCC TTCT TTTT( 4),
                                   TTTT TTTT TTTT TTTT(12)
```
(5.2)

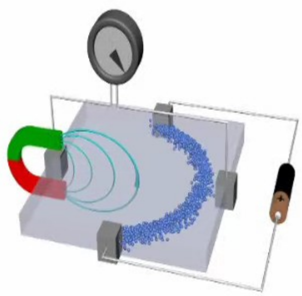
Eq. 5.2 is also *not* tautologous, hence refuting the Hahn-Banach theorem.

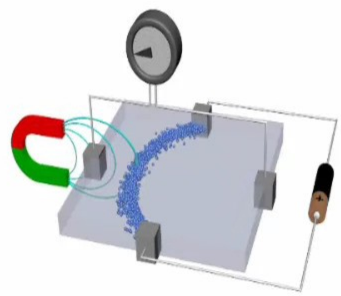
Refutation of the Hall effect

We assume the apparatus and method of Meth8/VŁ4. The designated *proof* value is Tautology. The 16-valued truth table is row-major and horizontal.

We evaluate the Hall effect as animated at en.wikipedia.org/wiki/File:Hall_Sensor.webm .

Figure 1: (1.1) ; Figure 2: (2.1)





LET p q r s: green bar -; red bar +; top source -; bottom target +.
Fig. 1 is green top to red bottom; Fig. 2 is red top to green bottom.

(p>q) > (s>r) (1.2) ; (q>p) < (s>r) (2.2)

The two states of the Hall effect are either Eq. 1.2 or Eq. 2.2. (3.1)

((p>q)>(s>r)) + ((q>p)<(s>r)) ; TTTT TTTT TT**F**T TTTT (3.2)

Eq. 3.2 as rendered is *not* tautologous due to the single contradiction **F** value. This means the Hall effect is refuted and not confirmed as a theorem.

What follows is that application of the Hall effect to quantum models, skewed lattices, and brain dimensions is likewise *not* tautologous.

Why the imaginary Hamiltonian quaternion bears no nexus to reality

We evaluate the Hamiltonian quaternion using the Meth8/VŁ4 modal logic model checker.

From: Santana, Yeray Cachón. 2018. Fractals on non-euclidean metric. vixra.org/pdf/1804.0173v1.pdf

The quaternion is defined as equal to the negation of its conjugate.

q = a + b(î) + c(ĵ) + d(k̂); and the conjugate: q[*] = a - b(î) - c(ĵ) - d(k̂). (1.0)

For simplicity, we set the real numbers a, b, c, d to 1.

(1 + (î) + (ĵ) + (k̂) = ~(1 - (î) - (ĵ) - (k̂)). (1.1)

LET pqrs: 1, î, ĵ, k̂; (%s>#s) 1, ordinal 1; (%s<#s) -1, negative ordinal 1;
necessity, for all; % possibility, for one or some; `T` tautology (designated proof value);
`F` contradiction; `N` truthity (non-contingency); `C` falsity (contingency);
The 16-valued table results are row-major and horizontal.

(((%s>#s)+q)+(r+s))=~(((%s>#s)-q)-(r-s)) ;
`NNTT CCFF CCFF CCFF` (1.2)

Eq. 1.2 as rendered is *not* tautologous. This refutes Eq.1.0, that the quaternion is equal to the negation of its conjugate.

We attempt to strengthen the argument of Eq. 1.0 by injecting the rule of Hamilton for quaternion multiplication.

((i&j)&k)= -1 (2.1)

((q&r)&s)=~(%s>#s) ; `NNNN NNNC NNNN NNCC` (2.2)

While Eq. 2.2 as rendered is *not* tautologous, meaning Eq. 2.1 is not bi-valent, we proceed to combine Eq. 2.1 as the antecedent by implication to Eq.1.1 as the consequent in a strengthened argument.

(((i&j)&k)= -1) > ((1 + (i^) + (j^) + (k^) = ~(1 - (i^) - (j^) - (k^))) ; (3.1)

(((q&r)&s)=~(%s>#s)) > ((((%s>#s)+q)+(r+s))=~(((%s>#s)-q)-(r-s))) ;
`TTTT CCCN CCCC CCNN` (3.2)

Eq. 3.2 as rendered is *not* tautologous. The attempt to strengthen Eq. 1.1 failed. This exercise effectively refutes the quaternion of Hamilton as *not* tautologous.

Refutation of Hamkins' theorem

Abstract: Hamkins' theorem claims "every countable model … of set theory embeds into its own constructible universe … x∈y ↔ j(x)∈j(y)", which is *not* tautologous, forming a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Fuchs, G.; Gitman, V.; Hamkins, J.D. (2018). Incomparable ω_1-like models of set theory. arxiv.org/pdf/1501.01022.pdf

Abstract. We show that the analogues of the Hamkins embedding theorems .. , proved for the countable models of set theory, do not hold when extended to the uncountable realm of ω_1-like models of set theory. Specifically, under the ◊ hypothesis and suitable consistency assumptions, we show that there is a family of 2^{ω_1} many ω_1-like models of ZFC, all with the same ordinals, that are pairwise incomparable under embeddability; there can be a transitive ω_1-like model of ZFC that does not embed into its own constructible universe; and there can be an ω_1-like model of PA whose structure of hereditarily finite sets is not universal for the ω_1-like models of set theory.

1. Introduction We should like to consider the question of whether the embedding theorems of Hamkins .. , recently proved for the countable models of set theory, might extend to the realm of uncountable models. … The question we consider here is, do the analogous results hold for uncountable models? Our answer is that they do not.

The Hamkins embedding theorems are expressed collectively in theorem 1 below. An *embedding* of one model ... of set theory into another … is simply a function … ; note by extensionality that every embedding is injective.

Although this is the usual model-theoretic embedding concept for relational structures, the reader should note that it is a considerably weaker embedding concept than commonly encountered in set theory, because this kind of embedding need not be elementary nor even Δ_0-elementary, although clearly every embedding as just defined is elementary at least for quantifier-free assertions. So we caution the reader not to assume a greater degree of elementarity beyond quantifier-free elementarity for the embeddings appearing in this paper, except where we explicitly remark on it.

Theorem 1 (Hamkins) ..

> (3) Consequently, every countable model … of set theory embeds into its own constructible universe … $x \in y \leftrightarrow j(x) \in j(y)$ (1.3.1)
>
> LET x, y, j: p, q, r
>
> (p<q)=((r&p)<(r&q)) ; `T`**`F`**`TT TTTT T`**`F`**`TT TTTT` (1.3.2)
>
> **Remark 1.3.2:** Eq. 1.3.2 is *not* tautologous, thereby refuting a seminal conjecture of Hamlin's theorem 1.

One can begin to get an appreciation for the difference in embedding concepts by observing that ZFC proves that there is a nontrivial embedding $j: V \rightarrow V$, namely, the embedding recursively defined as follows $j(y) = j(x)|x \in y \cup \{\emptyset, y\}$.

We leave it as a fun exercise to verify that $x \in y \leftrightarrow j(x) \in j(y)$ for the embedding j defined by this recursion.[1] [1]See [Ham13]; but to give a hint here for the impatient, note that every j(y) is nonempty and also $\emptyset \notin j(y)$; it follows that inside j(y) we may identify the pair $\{\emptyset, y\}$ j(y); it follows that j is injective and furthermore, the only way to have $j(x) \in j(y)$ is from $x \in y$.

Refutation of generalized Hardy's paradox

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are non-repeating fragments of 16-valued truth tables in row-major and horizontal

LET: p, q, r, s, t: A1, A2, B1, B2, probability; also,
p, q, r, s, t: a+, b+, c+, c-, probability;
~ Not; & And; + Or; - Not Or; > Imply, greater than; < Not Imply, lesser than;
@ Not Equivalent; % possibility, for one or some; (x@x) ordinal zero, 0.

From: Jiang, S.H.; Xu, Z.P.; Su, H-Y.; Pati, A.K.; Chen, J-L. (2018).
Generalized Hardy's paradox. arxiv.org/pdf/1709.09812.pdf

"In any local theory, if the events A2<B1, B1<A1, and A1<B2 never happen, then naturally the event A2<B2 must never happen. According to quantum theory, however, there exist two-particle entangled states and local projective measurements that break down these local conditions; that is, in terms of probabilities, P(A2<B1)=P(B1<A1)=P(A1<B2)=0, and P(A2<B2)>0, where the last condition evidently conflicts with the prediction of local theory, leading to a paradox. In [fn] the author showed that for the n-qubit Greenberger-HorneZeilinger (GHZ) state the maximal success probability (i.e., the last condition above) can reach [1+cos n− π 1]/2n.

Moreover, a quantum paradox can be naturally transformed to a corresponding Bell's inequality. For instance, the paradox mentioned above can be associated to the following Hardy's inequality P(A2<B2)−P(A2<B1)−P(B1<A1)−P(A1<B2)≤0, which is equivalent to Zohren and Gill's version [fn] of the Collins-GisinLinden-Massar-Popescu inequalities (i.e., tight Bell's inequalities for two arbitrary d-dimensional systems, and the inequality becomes the CHSH inequality for d=2).[fn] ... "

If the events A2<B1, B1<A1, and A1<B2 never happen, then naturally the event A2<B2 must never happen. (1.1)

~((%(q<r)&%(r<p))&%(p<s))>~(%(q<s)) ;
`TTCC TTCC TTTT TTTT,`
`TTCC TTCC TTTT TTTT` (1.2)

In terms of probabilities, P(A2<B1)=P(B1<A1)=P(A1<B2)=0, (2.1)
and P(A2<B2)>0, (3.1)
where the last condition evidently conflicts with the prediction of local theory, leading to a paradox. (4.1)

Moreover, a quantum paradox can be naturally transformed to a corresponding Bell's inequality. For instance, the paradox mentioned above can be associated to the following Hardy's inequality
P(A2<B2)−P(A2<B1)−P(B1<A1)−P(A1<B2)≤0, (5.1)
which is equivalent to Zohren and Gill's version [fn] of the Collins-GisinLinden-Massar-Popescu inequalities (i.e., tight Bell's inequalities for two arbitrary d-dimensional systems, and the inequality becomes the CHSH inequality for d = 2)

[fn]. See also [fn] for a connection between Hardy's inequality and Wigner's argument [of joint

probabilities as $p(a+;\, b+) - p(a+;\, c+) - p(c+;\, b+) - p(c-;\, c-) \leq 0$]. (6.1)

(((t&(q<r))=(t&(r<p)))=(t&(p<s)))=(t@t) ;
TTTT TTTT TTTT TTTT,
TFFT FFFF TTFF FTFT (2.2)

(t&(p<s)) >(t@t) ; TTTT TTTT TTTT TTTT,
TTFF TTFF TTTT TTTT (3.2)

((((t&(q<r))=(t&(r<p)))=(t&(p<s)))=(t@t))&((t&(q<s))>(t@t)) ;
TTTT TTTT TTTT TTTT,
TFFF FFFF TTFF FTFT (4.2)

~((((t&(q<s))-(t&(q<r)))-((t&(r<p))-(t&(p<s))))>(t@t))=(p=p) ;
FFFF FFFF FFFF FFFF,
FFFF FFFF FFFF FFFF (5.2)

LET p, q, r, s, t: a+, b+, c+, c-, probability

~((((t&(p&q))-(t&(p&r)))-((t&(r&q))-(t&(r&r))))>(t@t))=(p=p) ;
FFFF FFFF FFFF FFFF,
FFFF FFFF FFFF FFFF (6.2)

Eqs. 2.2, 3.2, 4.2, 5.2, and 6.2 as rendered are *not* tautologous. This means the generalized Hardy's paradox is refuted. Eqs. 5.2 and 6.2 are contradictory. The means Hardy's inequality and Wigner's argument of joint probabilities are refuted, as is a claimed connection.

Remark: The basis of the entire claim is Eq. 1.1: "If the events A2<B1, B1<A1, and A1<B2 never happen, then naturally the event A2<B2 must never happen." As rendered in Eq. 2, this is not tautologous with result values of contingency (falsity). This is a gross example of mathematical logic exposing the mistaken assumptions of quantum field theory.

Refutation of Hegel's dialectical method

We assume the method and apparatus of Meth8/VŁ4 where `T`autology is the designated *proof* value, `F` is contradiction, `N` is truthity (non-contingency), and `C` is falsity (contingency). The 16-valued truth table is row-major and horizontal. We evaluate the following in *one* variable of p.

From: Maybee, J. (2016). Hegel's dialectics. plato.stanford.edu/entries/hegel-dialectics

Stage 1:
p *content*; #p *necessity* of content;
~p *determinate negation* of content;
~#p *determinate nothingness* of content;
> Imply, greater than, *becomes, becoming*;
< Not Imply, less than, *sublation*

Stage 2:
%p possibility of content, *coherence*;
%(#p=#p) *immanence*, `T`autology, proof, dialectics as "the principle through which alone *immanent coherence and necessity* enter into the content of science"

Stage 3:
~# not necessity;
%p some new idea;
< to show up from outside;
%p<#p self-sublation;

"because the form or determination that arises is the *result* of the self-sublation of the determination from the moment of understanding, there is no need for some new idea to show up from the outside." (3.1.1)

(%p<#p)>~(#(%p<(%p>#p))=(p=p)) ;
`TTTT TTTT TTTT TTTT` (3.1.2)

For example:

%p "somethings";
~%p "some other things", something-others;

Being-for-itself (3.2.1)

(%p>~%p)&(~%p>%p) ; `FFFF FFFF FFFF FFFF` (3.2.2)

"Being-for-itself embraces the something-others in its content" with a "process of passing back-and-forth between the something-others" (3.3.1)

((%p>~%p)&(~%p>%p))>((~%p>~%p)&(~%p<~%p)) ;
`TTTT TTTT TTTT TTTT` (3.3.2)

Stage 4:
(%p<#p) the finite;
#(%p<#p) everything finite;

"everything finite is: its own sublation" (4.1.1)

#(%p<#p)<#(%p<#p) ; FFFF FFFF FFFF FFFF (4.1.2)

all;
#p all content;
(p=p) genuine;
< p nonexternal to p;
> elevation above;
(%p<#p) the finite;
%(#p=#p) principle [dialectics]

"all genuine, nonexternal elevation above the finite is to be found in this principle [of dialectics]" (4.2.1)

((#(p=p)<#p)>(%p<#p))<%(#p=#p) ;
FFFF FFFF FFFF FFFF (4.2.2)

Stage 5: "the result of the dialectical process is a new concept but one higher and richer than the preceding—richer because it negates or opposes the preceding and therefore contains it, and it contains even more than that, for it is the unity of itself and its opposite." (5.1.1)

%(#p=#p) > ((~#p>(%#p>#p))>(#p=#((%#p>#p)&~(%#p>#p))))) ;
TCTC TCTC TCTC TCTC (5.1.2)

Stage 6: the "Absolute" for logic—as an oval that is filled up with and surrounds numerous, embedded rings of smaller ovals and circles, which represent all of the earlier and less universal determinations from the logical development (6.1.1)

#p>(%p>p) ; TTTT TTTT TTTT TTTT (6.1.2)

Hegel's entire philosophical system ... "presents itself therefore as a circle of circles" (6.2.1)

#p=(#p>(%p>p)) ; FNFN FNFN FNFN FNFN (6.2.2)

#(p=p) moving soul of scientific progression, necessity of proof ;

"the dialectical constitutes the moving soul of scientific progression" (6.3.1)

(#p=(#p>(%p>p)))=#(p=p) ; CTCT CTCT CTCT CTCT (6.3.2)

As rendered, Eqs. 3.1.2, 3.3.2, and 6.1.2 are tautologous, but Eqs. 3.2.2, 4.1.2, 4.2.2, 5.1.2, 6.2.2, and 6.3.2 are *not* tautologous with Eqs. 3.2.2, 4.1.2, and 4.2.2 as contradictions.

In Stage 3, Eq. 3.2.2 the definition of Being-for-itself is a contradiction. Subsequently the main results for Stages 4, 5, and 6 are *not* tautologous.

We conclude that this refutes Hegel's dialectical method, and in only one variable.

Refutation of Heider inspired international relation theory

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal.

From: academia.edu/27360936/A_Formal_Semantics_of_International_Relations by Fabian Schang

§1. Heider

LET: p, q, r, s: x, y, R, z;
~ Not; + Or; & And; > Imply, greater than; < Not Imply, less than, ∈;
% possibility, possibly, for one or some; # necessity, necessarily, for every or all.

R(x,z)= R(x,y) × R(y,z) (3.2.1)

(r&(p&s))=((r&(p&q))&(r&(q&s))) ; `TTTT TTTT TTTT TFTT` (3.2.2)

Remark: Removing the "R" functor deviates further from tautology. (3.2.2.1)

(p&s)=((p&q)&(q&s)) ; `TTTT TFTT TTTT TFTT` (3.2.2.2)

What follows from Heider rendered in Eq. 3.2.2 as *not* tautologous is its further negation as irrelevant.

§2. Coherent theorem

Every political relationship R(*x*,*y*) (where R∈ {*E*,*F*}) is coherent. (4.3.3.1.1)

((r<((r=r)+(r@r)))>(#r&(p&q))) ; `TTTT TTTT TTTT TTTT` (4.3.3.1.2)

$\vDash_{\Pi}$ ∀*x*∀*y* ⌐(R(*x*,*y*)∧ ⌐R(*x*,*y*)) (4.3.3.2.1)

(#p&#q)&~((r&(p&q))&(~r&(p&q))) ;
`FFFN FFFN FFFN FFFN` (4.3.3.2.2)

Eqs. 4.3.3.1.1 implies 4.3.3.2.1 (4.3.3.3.1)

((r<((r=r)+(r@r)))>(#r&(p&q))) > ((#p&#q)&~((r&(p&q))&(~r&(p&q)))) ;
`FFFN FFFN FFFN FFFN` (4.3.3.3.2)

Remark: As expected Eqs. 4.3.3.2.2 and 4.3.3.3.2 have identical table results, as *not* tautologous.

§3. Hegemon as demiurge in the world

LET: r: uc_pi Π

∀*x*∀*y*∀*z* Π(*x*) > Π(*y*) ^ (Π(*y*) + Π(*z*) > Π(*x*)) (5.2.Riv.1)

(#p&(#q&#s))&(((r&p)>(r&q)) & ((r&q)+((r&s)>(r&p)))) ;
FFFF FFFF FFFN FFFN (5.2.Riv.2)

Therefore we deny "the hegemon acts as a demiurge in the wor[l]d".

Refutation of the Heisenberg principle of uncertainty by mathematical logic

The Heisenberg principle of uncertainty is written with h for an approximation of Planck's constant as

$$\sigma(X) * \sigma(p) \geq (h/(4*\pi)) \qquad (1)$$

From Eq. 1 we rewrite it as

$$(h/(4*\pi)) * \sigma(X) * \sigma(p) \geq 1 \qquad (2)$$

Eq. 2 may be stated in the negative as Not < 1 as

$$\text{Not } [(h/(4*\pi)) * \sigma(X) * \sigma(p) < 1] \qquad (3.1)$$

Assuming the apparatus and method of Meth8/VŁ4, we map Eq. 3.1 below.

LET: p q r s p, X, (h/(4*π)), σ;
~ Not; & And, *; \ Not And; > Imply; < Not Imply, less than; = Equivalent to;
# Necessity, for all; % Possibility, for some (one);

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	p=p	T	Tautology	proof	11	3
2	p@p	F	Contradiction	absurdum	00	0
3	%p>#p	N	Non-contingency	truth	01	1
4	%p<#p	C	Contingency	falsity	10	2

(%p>#p) 1; (p=p) T tautology, as the designated *proof* value.

The 16-valued truth table is presented row-major and horizontally.

~((r&((s&q)&(s&p))) < (%p>#p)) = (p=p) ;
TTTT TTTT TTTN TTTN (3.2)

It is permissible to remove the r term because it is a scalar constant.

~(((s&q)&(s&p)) < (%p>#p)) = (p=p) ;
TTTT TTTT TTTN TTTN (3.3)

Eqs. 3.2 and 3.3 result in the same truth table, rendering Eq. 2 as *not* tautologous. This means the Heisenberg uncertainty principle is untenable.

Refutation of the conjecture for Heisenberg's principle

Abstract: The conjecture for Heisenberg's principle is that for a particle/wave at an exact time, the location, and momentum is impossible to know, that is: the variables as together cannot be true. We show this is *not* tautologous and hence refute it in the shortest demonstration of its kind.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

The conjecture for Heisenberg's principle is that for a particle/wave at an exact time, the location, and momentum is impossible to know, that is: the variables as together cannot be true. (1.1)

LET p, q, r, s: particle/wave, time, location,
momentum [alternatively velocity or speed serve the same logical purpose here];
~ Not; & And; = Equivalent; T (p=p); **F** ~(p=p).

(p&(q&(r&s))) = ~(p=p) ; TTTT TTTT TTTT TTT**F** (1.2)

Eq. 1.2 as rendered is *not* tautologous and hence the shortest refutation of the conjecture for Heisenberg's principle.

Refutation of the Heisenberg principle as a no-go axiom, and its trivial replacement theorem

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p; q; r; s:
p σ_p standard deviation of the particle position;
q σ_x standard deviation of the particle momentum;
$\hbar$ reduced Planck constant; s;
~ Not; & And; \ Not And; > Imply, greater than; < Not Imply, less than;
= Equivalent; # necessity, for all or every, ∀; % possibility, for one or some, ∃;
(s=s) T; (%s<#s) ordinal 2.

From: en.wikipedia.org/wiki/Uncertainty_principle

The Heisenberg uncertainty principle "states that the more precisely the position of some particle is determined, the less precisely its momentum can be known, and vice versa".

$\sigma_x \sigma_p \geq \hbar/2$ (1.0)

~((p&q)<(r\(%s<#s)))=(s=s) ; TTTT TTTT TTTT TTTN (1.1)

Eq. 1.2 as rendered is *not* tautologous, due to one value of non contingency N (truthity).

Because the Heisenberg principle is a no-go axiom, we ask what can be known about the relation of statistics of position and momentum in a numerical relation, regardless of injection of Planck statistics, such that:

The standard deviation of the particle position is greater than the standard deviation of the particle momentum. (2.1)

p>q ; TFTT TFTT TFTT TFTT (2.2)

(p>q)>((p&q)>(p>q)) ; TTTT TTTT TTTT TTTT (3.2)
(p>q)>((p>q)+(p+q)) ; TTTT TTTT TTTT TTTT (4.2)
(p>q)>((p&q)>(p+q)) ; TTTT TTTT TTTT TTTT (5.2)

Because Eqs. 3.2 and 4.2 have the antecedent of Eq. 2.2 as a term in the consequent, we evaluate only Eq. 5.2 which does not have this recurrence of literals.

Eq. 5.2 is tautologous because the consequent ((p&q)>(p+q)) is tautologous.

Back translating this theorem for in terms of the statistic of deviation means:

If position is greater than momentum, then position and momentum are greater than position or momentum. (5.1)

Remark: Eqs. 5.1 could just as easily read "If position is less than momentum" and 5.2 as (p<q) because the consequent is tautologous.

Eq. 5.1 as rendered means there is no uncertainty as to what constitutes a proved relationship between the statistics of deviation for the position and momentum of the particle. In other words, there is an exact statistical relationship in the theorem of Eq. 5.2.

Therefore Eq. 5.1 serves as a counter-example in mathematical logic to the Heisenberg uncertainty principle, and hence refutes then replaces it. (For example in positive integers and ignoring the instance of 2>1, if 3>2 then 3*2 is always greater than 3+2.) To reduce the Heisenberg uncertainty principle to a trivial assertion flies in the face of the intention of modern physics to use the principle as the very basis for justifying investment in itself under the guise of quantum logic.

Take a picture of an electron to refute the Heisenberg uncertainty principle

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET: ~ Not; & And, multiply; \ Not And, divide; + Or, add; - Not Or, subtract;
> Imply, greater than; = Equivalent;

p, q: photon, electron;

p=(p=p) ;	FTFT FTFT FTFT FTFT	(0.1)
q=(q=q) ;	FFTT FFTT FFTT FFTT	(0.2)
[r=(q=q) ;	FFFF TTTT FFFF TTTT	(0.3)]
[s=(q=q) ;	FFFF FFFF TTTT TTTT	(0.4)]

We evaluate the unoriginal thought experiment of taking a picture of an electron in a vacuum. To take a picture of an electron requires shining light on it. The state of the electron is therefore combined with that of the photon wave to produce a combined state. The combined state may be additive or multiplicative:

electron summed with photon (1.1)

(q+p) ; FTTT FTTT FTTT FTTT (1.2)

electron multiplied with photon (2.1)

(q&p) ; FFFT FFFT FFFT FFFT (2.2)

We ask, "Is a theorem derivable by trial and error for Eqs. 1.2 and 2.2, such as

Eq. 2.2 implies Eq. 1.2?" (3.1)

(q&p)>(q+p) ; TTTT TTTT TTTT TTTT (3.2)

We ask, "If Eq. 3.2 is a theorem, then can we find

other theorems as co-equal thereto?" (4.1)

((q&p)>(q+p)) = (((q&p)\~p) = ~((q+p)-~p)) ;
TTTT TTTT TTTT TTTT (4.2)

We ask, "Can we derive q back out of the theorem(s) in Eq. 4.1,

only by logically removing p?" (5.1)

(((q&p)\~p) & ((q+p)-~p)) > q ; TTTT TTTT TTTT TTTT (5.2)

Eq. 5.2 makes Eq. 3.2 inversive and is tautologous. This means the state of indeterminancy to take a picture of an electron using light is invertible. Therefore, the uncertainty principle is logically contradicted.

Refutation of the paradox of Hempel's raven

Abstract: We evaluate the hypothesis which is *not* tautologous and hence *not* a paradox. It forms a *non* tautologous fragment of the universal logic VL4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Raven_paradox

Hempel describes the paradox in terms of the hypothesis: ..

(1) All ravens are black. (1.1.1)

In the form of an implication, this can be expressed as:

If something is a raven, then it is black. (1.2.1)

Remark 1.2.1: We write Eq. 1.2.1 as:

If raven, then black. (1.3.1)

LET p, q, r, s: black, green apple, raven, s.

r>p ; TTTT **F**T**F**T TTTT **F**T**F**T (1.3.2)

Via contraposition, this statement is equivalent to:

(2) If something is not black, then it is not a raven. (2.1.1)

Remark 2.1.1: To map via contraposition, we write Eq. 2.1.1 as:

If not black, then not raven. (2.2.1)

~p>~r ; T**F**T**F** TTTT T**F**T**F** TTTT (2.2.2)

In all circumstances where (2) is true, (1) is also true—and likewise, in all circumstances where (2) is false (i.e., if a world is imagined in which something that was not black, yet was a raven, existed), (1) is also false. (2.3.1)

. . . Given a general statement such as *all ravens are black*, a form of the same statement that refers to a specific observable instance of the general class would typically be considered to constitute evidence for that general statement. For example,

(3) *My pet raven is black.* (3.1.1)

is evidence supporting the hypothesis that *all ravens are black*.

Remark 3.1.1: Eqs. 1.3.1 and 3.1.1 are equivalent.

The paradox arises when this same process is applied to statement (2). On sighting a green apple, one can observe:

(4) *This green apple is not black, and it is not a raven.* (4.1.1)

(q>~p)&(q>~r) ; TTT**F** TT**FF** TTT**F** TT**FF** (4.1.2)

By the same reasoning, this statement is evidence that (2) *if something is not black then it is not a raven.* (5.1.1)

((q>~p)&(q>~r))>(~p>~r) ; TTTT **F**TTT TTTT **F**TTT (5.1.2)

Remark 5.1.2: Eq. 5.1.2 is *not* tautologous to mean 4.1.2 is *not* evidence of 2.2.2.

But since (as above) this statement is logically equivalent to (1) *all ravens are black*, it follows that the sight of a green apple is evidence supporting the notion that all ravens are black.

(((q>~p)&(q>~r))>(~p>~r))>(r>p) ;
TTTT TT**F**T TTTT TT**F**T (6.1.1)

This conclusion seems paradoxical because it implies that information has been gained about ravens by looking at an apple.

Remark 6.1.1: Eq. 6.1.1 is *not* tautologous, and it does *not* imply that information was gained about ravens by looking at an apple. Hence the hypothesis is *not* a paradox.

Henkin applications to logic

From: J Donald Monk, [ca 1986], "Leon Henkin and cylindric algebras.
euclid.colorado.edu/~monkd/monk85.pdf.

"Cylindric algebras are abstract algebras which stand in the same relationship to first-order logic as Boolean algebras do to sentential logic."

From pages 6-7, with Meth8 scripts and results inserted as N.n equation numbers.

LET # ∀, % ∃, p ϕ, q ψ, r r, u F, v G, x x, y y, ~ ¬, + ∨, & ∧, > →, = =, = ↔,
vt tautologous, nvt *not* tautologous

"In [67] Henkin considers first-order logic with only finitely many variables. In the case of just two variables x and y, he proves that the formula

$$\exists x(x = y \wedge \exists yGxy) \rightarrow \forall x(x = y \rightarrow \exists yGxy) \quad (2.1)$$

(%x&((x=y)&((%y&v)&(x&y)))) > (#x&((x=y)>((%y&v)&(x&y)))) ;
nvt (2.2)

is universally valid but not derivable from the natural axioms (restricted to two variables). Here G is a binary relation symbol.

The non-derivablity is proved using a modified cylindric set algebra. This example suggests adding all formulas of the following forms to the axioms for two-variable logic:

(1) $\exists x(x = y \wedge \phi) \rightarrow \forall x(x = y \rightarrow \phi)$ (3.1)

(%x&((x=y)&p)) > (#x&((x=y)>p)) ;
nvt (3.2)

$\exists y(x = y \wedge \phi) \rightarrow \forall y(x = y \rightarrow \phi)$ (4.1)

(%y&((x=y)&p)) > (#y&((x=y)>p)) ;
nvt (4.2)

Henkin shows, again using a modified cylindric set algebra, that this axiom system is also incomplete; the following universally valid formula is not provable in the expanded axiom system:

$$\exists xFx \wedge \forall x\forall y[Fx \wedge Fy \rightarrow x = y] \rightarrow [\exists x(Fx \wedge Gxy) \leftrightarrow \forall x(Fx \leftrightarrow Gxy)] \quad (5.1)$$

((%x&(u&x))&((#x&#y)&(((u&x)&(u&y))>(x=y)))) >
((%x&((u&x)&(v&(x&y))))=(#x&((u&x)=(v&(x&y))))) ;
vt (5.2)

An analysis of this situation leads to adding the following formulas to the axioms:

(2) $\exists x\forall y(\phi \leftrightarrow y = x) \rightarrow [\exists y(\phi \wedge \psi) \leftrightarrow \forall y(\phi \rightarrow \psi)]$ with x not free in ϕ
[x not free in ϕ is ~%x&ϕ] (6.1)

((~%x&p)&((%x&#y)&(p=(y=x)))) > ((~%x&p)&((%y&(p&q))=(#y&(p>q)))) ;
vt (6.2)

∃y∀x(ϕ ↔ x = y) → [∃x(ϕ ∧ ψ) ↔ ∀y(ϕ → ψ)] with y not free in ϕ
[y not free in ϕ is ~%y&ϕ] (7.1)

((~%y&p)&((%y&#x)&(p=(x=y)))) > ((~%y&p)&((%x&(p&q))=(#y&(p>q)))) ;
vt (7.2)

But again the resulting axiom system is not complete. By another modified cylindric set algebra Henkin shows that the following formula is universally valid but not derivable in this axiom system:

∃xGxy ↔ ∃x(x = y ∧ ∃yGyx). (8.1)

(%x&(v&(x&y))) = (%x&((x=y)&((%y&v)&(y&x)))) ;
vt (8.2)

Finally, adding the following axioms results in a complete axiom system:

∃xϕ ↔ ∃x(x = y ∧ ∃yϕr) (9.1)

(%x&p) = (%x&((x=y)&(%y&(p&r)))) ;
nvt (9.2)

∃yϕ ↔ ∃y(y = x ∧ ∃yϕr) [probably should read ... ∃ *xϕr*] (10.1)

(%y&p) = (%y&((y=x)&(%y&(p&r)))) ;
nvt [... *(%x&(p&r))* ; nvt] (10.2)

where ϕr is recursively defined by interchanging x and y if ϕ is atomic, with

(¬ϕ)r = ¬ϕr, (11.1)

(~p&r) = ~(p&r) ; nvt [but (~p&r) = (~p&r) ; vt] (11.2)

(ϕ ∨ ψ)r = ϕr ∨ ψr, (12.1)

((p+q)&r) = ((p&r)+(q&r)) ; vt (12.2)

(∃xϕ)r = ∃y(x = y ∧ ∃xϕ), and (13.1)

((%x&p)&r) = (%y&((x=y)&(%x&p))) ;
nvt (13.2)

(∃yϕ)r = ∃x(x = y ∧ ∃yϕ) ; [probably should read ... ∧ ∃ *xϕ*] (14.1)

((%y&p)&r) = (%x&((y=x)&(%y&p))) ;
nvt [... *&(%x&p)* ; nvt] (14.2)

The proof of completeness of the resulting axiom system is rather involved, but is completely carried out. It is shown that the above axioms do not suffice for logic with three variables."

Results from Meth8 conclude that out of the 14 axioms above, 8 are not tautologous (1-4,9-10, 12-14). Consequently, Henkin's proof of Eq 2 is not tautologous for two variables and is not universally valid. This means the application to logic of cylindric algebras to first order logic is suspicious.

Henkin: Validation of permutation model in non-representable cylindric algebra

From: J Donald Monk, [ca 1986], "Leon Henkin and cylindric algebras".
euclid.colorado.edu/~monkd/monk85.pdf.

Remark: The problem is taken from an unnumbered equation on page 5 where with Meth8 scripts and results inserted as N.2 equation numbers.

"the following inequality (which can be written as an equation) holds in every representable CAα with α ≥ 3 but fails in a permutation model:"

$$c_0x \cdot c_1y \cdot c_2z \leq c_0c_1c_2[c_2(c_1x \cdot c_0y) \cdot c_1(c_2x \cdot c_0z) \cdot c_0(c_2y \cdot c_1z)]. \quad (1.1)$$

LET p c0, q c1, r c2, x x, y y, z z

(((p&x)&(q&y))&(r&x)) =
(((p&w)&r) & (((r&((q&x)&(p&y))) & (q&((r&x)&(q&x)))) &
(p&((r&y)&(q&z))))); TTTT TTTT TTTT TTTT (1.2)

Eq 1.2 as an equivalency is tautologous, and hence also holds for < Imply and > Not Imply. Hence Eq 1.2 is not an inequality but also holds in a permutation model. This means the assertion that the inequality fails is not validated.

Herbrand semantics

From: Genesereth, M; Kao, E. "The Herbrand manifesto: thinking inside the box". (2015). logic.stanford.edu/herbrand/manifesto.htmllogic.stanford.edu/herbrand/manifesto.html

"4. Curiouser and Curiouser ...

"The typical approach in relational logic would be to write the definition shown here.

∀x.∀z.(q(x,z) ⇔ p(x,z) ∨ ∃y.(p(x,y) ∧ q(y,z))) (1.1)

The Meth8 script maps Eq 1.1 as:

LET: u qh (the helper relation); n v;
necessity, universal quantifier∀; % possibility, existential quantifier ∃;
= Equivalent to ⇔; & And ∧; + Or ∨; ~ Not;
Model 1 logic values by first letter:
F contradictory; Contingent; Non contingent; Tautologous

(#x&#z) & ((q&(x&z))=((p&(x&z))+(%y&((p&(x&y))+(q&(y&z)))))) ;
The repeating truth table fragment is
NFFN FFFF FNNN;
the designated truth value T is not present. (1.2)

"Suppose we have the object constant 0, an arbitrary unary relation constant s (2.1, 3.1)

Meth8 maps Eq 2.1 and 3.1 as

~(s=s), (s=s) (2.2, 3.2)

"We ... can easily define *q* in terms of *qh*. *q* is tautologous of two elements if and only if there is a level at which *qh* becomes tautologous of those elements. (4)

qh(*x*,*z*,0) ⇔ *p*(*x*,*z*) ∨ *p*(*x*,0) ∧ *p*(0,*z*) (5.1)

qh(*x*,*z*,*s*(*n*)) ⇔ *qh*(*x*,*z*,*n*) ∨ (*qh*(*x*,*s*(*n*),*n*) ∧ *qh*(*s*(*n*),*z*,*n*)) (6.1)

Meth8 maps Eq 5.1 and 6.1 as

(u&(x&(z&~(s=s)))) = ((p&(x&z)) + ((p&(x&~(s=s)))&(p&(~(s=s)&z)))) ;
TTTT TFTF (5.2)

(u&((x&z)&((s=s)&v))) = ((u&(x&(z&v))) + (((u&x)&((s=s)&(v&v))) &
((u&(s=s))&(v&(z&v))))); TTTT (6.2)

Meth8 maps Eq 5.1, 6.1, and 4 as

(((u&(x&(z&~(s=s)))) = ((p&(x&z)) + ((p&(x&~(s=s)))&(p&(~(s=s)&z))))) &
((u&((x&z)&((s=s)&v))) = ((u&(x&(z&v))) + (((u&x)&((s=s)&(v&v))) &
((u&(s=s))&(v&(z&v))))))) > (q = u) ;
TTFF FFTT TTFT FTTT (7)

"The only disadvantage of this axiomatization is that we need the helper relation *qh*. But that causes no significant problems.

$$\forall x.\forall z.(q(x,z) \Leftrightarrow \exists n.qh(x,z,n)) \qquad (8.1)$$

Meth8 maps Eq 8.1 as

((#x&#z)&(q&(x&z))) = ((%v&u)&(x&(z&v))) ;
TTTT TTCC FFNN (8.2)

Meth8 finds Eq 6.2 to be validated as tautologous, and all others not so, notably the main conjecture of Eq 7. The conclusion is that Herbrand semantics are logically suspicious.

Refutation of Heyting algebra

Abstract: Using the Lindenbaum method, we show pseudo-complementation is *not* tautologous along with its eight properties. This refutes Heyting algebra. Based thereon, what follows is the Gödel n-valued matrix logic is refuted and the derivative intuitionistic propositional logic.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r, s: x, y, z, s;
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⊢; < Not Imply, less than, ∈;
= Equivalent, ≡, ⊨, :=, ⇔, ↔; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(s=s) T as tautology; (s@s) **F** as contradiction;
(%s<#s) C non-contingency, ∇, ordinal 2;
(%s>#s) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y).

From: Alex Citkin, A.; Muravitsky, A. (2018). Lindenbaum method tutorial. UNILOG. arxiv.org/pdf/1901.05411.pdf acitkin@gmail.com alexeim@nsula.edu

A Heyting algebra is an algebra H = hH; ∧, ∨, →, ¬, 1i of type h∧, ∨, →, ¬, 1i, where ∧ (meet) and ∨ (join), → (relative pseudo-complementation) are binary operations, ¬ (pseudo-complementation) is a unary operation and 1 (unit) is a 0-ary operation, if, besides the equalities (l1)–(l2) and (b1) (Section 1.2.1), the following equalities are satisfied for arbitrary elements x, y, z of H:

Remark 1.6.0: The equalities below assume that 1 is Tautology as (s=s). However if 1 is taken as ordinal one, N non-contingency, as (%s>#s), then (h5), (h6) are *not* tautologous but rather truth table mixtures of T and N values.

(h1)	$x \wedge (x \rightarrow y) = x \wedge y$,	(h2)	$(x \rightarrow y) \wedge y = y$,
(h3)	$(x \rightarrow y) \wedge (x \rightarrow z) = x \rightarrow (y \wedge z)$,	(h4)	$x \wedge (y \rightarrow y) = x$,
(h5)	$\neg 1 \vee y = y$,	(h6)	$\neg x = x \rightarrow \neg 1$.

[More trivial equalities elided.]

The following property characterizes pseudo-complementation:

$$x \le y \rightarrow z \Leftrightarrow x \wedge y \le z. \qquad (1.8.0.1)$$

(~(q<p)>r)=(p&~(r<q)) ; TFFF FFFT TFFF FFFT (1.8.0.2)

Remark 1.8: Eq. 1.8.0.2 as rendered is *not* tautologous, meaning pseudo-complementation is refuted.

Indeed, assume first that

$x \le y \to z.$ (1.8.1.1.1)

~(q<p)>r ; **FF**T**F** TTTT **FF**T**F** TTTT (1.8.1.1.2)

Then, in view of (l1–i), the monotonicity of ∧ w.r.t. ≤ and (h1), we have:

$x \wedge y \le y \wedge (y \to z) = y \wedge z \le z.$ (1.8.1.2.1)

(~(q<(p&q))&(q>r))=(q&(~r<r)) ;
FFFF **FF**T**F** **FFFF** **FF**T**F** (1.8.1.2.2)

If Eq. 1.8.1.1, then 1.8.1.2.1: (1.8.1.3.1)

(~(q<p)>r)>((~(q<(p&q))&(q>r))=(q&(~r<r))) ;
TT**F**T **FF**T**F** TT**F**T **FF**T**F** (1.8.1.3.2)

Conversely, suppose that

$x \wedge y \le z,$ (1.8.2.1.1)

p&~(r<q) ; **F**T**F**T **FFF**T **F**T**F**T **FFF**T (1.8.2.1.2)

that is $x \wedge y = x \wedge y \wedge z.$ (1.8.2.2.1)

(p&q)=((p&q)&r) ; TTT**F** TTTT TTT**F** TTTT (1.8.2.2.2)

Remark 1.8.2: Eq. 1.8.2.1.2 and 1.8.2.2.2 are *not* equivalent as claimed.

Then, in virtue of (h2), (h4), (h3), we obtain:

$x \le y \to x$ (1.8.4.1)

~(q<p)>p ; **F**TTT **F**TTT **F**TTT **F**TTT (1.8.4.2)

$= (y \to x) \wedge (y \to y)$ (1.8.5.1)

(q>p)&(q>q) ; TT**F**T TT**F**T TT**F**T TT**F**T (1.8.5.2)

$= y \to (x \wedge y)$ (1.8.6.1)

q>(p&q) ; TT**F**T TT**F**T TT**F**T TT**F**T (1.8.6.2)

$= y \to (x \wedge y \wedge z)$ (1.8.7.1)

q>((p&q)&r) ; TT**FF** TT**F**T TT**FF** TT**F**T (1.8.7.2)

$= (y \rightarrow (x \wedge y)) \wedge (y \rightarrow z) \leq y \rightarrow z.$ (1.8.8.1)

(q>(p&q))&(~(q<(q>r))>r) ; FFFT TTFT FFFT TTFT (1.8.8.2)

Remark 1.8.9: Eqs. 1.8.4.1-1.8.8.1 are supposed to be equivalent as a group, but is *not*. (1.8.9.1)

(((~(q<p)>p)=((q>p)&(q>q)))=(q>(p&q)))=((q>((p&q)&r))=
((q>(p&q))&(~(q<(q>r))>r))) ;
TFTF FTTT TFTF FTTT (1.8.9.2)

Using (1.8.0.1), we receive immediately:

$x \leq y \Leftrightarrow x \rightarrow y = 1;$ (1.9.0.1)

~(q<p)=((p>q)=(s=s)) ; TFFT TFFT TFFT TFFT (1.9.0.2)

[More trivial equalities elided.]

Proposition 1.2.3. Let H = hH;∧,∨,→,¬, 1i be a Heyting algebra. For arbitrary elements x, y and z of H the following properties hold:

(a) $x \leq y \rightarrow x,$ (1.2.3.a.1)

~(q<p)>p ; FTTT FTTT FTTT FTTT (1.2.3.a.2)

(b) $x \rightarrow y \leq (x \rightarrow (y \rightarrow z)) \rightarrow (x \rightarrow z),$ (1.2.3.b.1)

(p>~((p>(q>r))<q))>(p>r) ; TTTF TTTT TTTF TTTT (1.2.3.b.2)

(c) $x \leq y \rightarrow (x \wedge y),$ (1.2.3.c.1)

~(q>p)>(p&q) ; TTFT TTFT TTFT TTFT (1.2.3.c.2)

(d) $x \wedge y \leq x,$ (1.2.3.d.1)

p&~(p<q) ; FFFT FFFT FFFT FFFT (1.2.3.d.2)

(e) $x \leq x \vee y,$ (1.2.3.e.1)

~((p+q)<p)=(p=p) ; TTFT TTFT TTFT TTFT (1.2.3.e.2)

(f) $x \rightarrow z \leq (y \rightarrow z) \rightarrow ((x \vee y) \rightarrow z),$ (1.2.3.f.1)

(p>~((q>r)<r))>((p+q)>r) ; TTFF TTTT TTFF TTT (1.2.3.f.2)

(g) $x \rightarrow y \leq (x \rightarrow \neg y) \rightarrow \neg x,$ (1.2.3.g.1)

(p>~((p>~q)<q))>~p ; TTTF TTTF TTTF TTTF (1.2.3.g.2)

(h) $x \leq \neg x \rightarrow y$. (1.2.3.h.1)

~((~p>q)<p)=(p=p) ; TT**F**T TT**F**T TT**F**T TT**F**T (1.2.3.h.2)

Remark 1.2.3: Because Eqs. 1.2.3.a-h (and 1.8.01 farther above) are *not* tautologous we abandon our evaluation here. We also note it a mistake that "properties ... in Proposition 1.2.3 for Heyting algebras can be applied to Boolean algebras".

We use the method of Lindenbaum to show pseudo-complementation is *not* tautologous along with its eight properties. This refutes Heyting algebra. Based thereon, what follows is the Gödel n-valued matrix logic is refuted and the derivative intuitionistic propositional logic.

Meth8 on Heyting-Brouwer logic

From: Kamide, Norihiro; Shramko, Yaroslav; Wansing, Heinrich. (2017).
"Kripke completeness of bi-intuitionistic multilattice logic and its connexive variant".
Studia Logic. September 2017. doi:10.1007/s11225-017-9752-x.
researchgate.net/publication/319645851_Kripke_Completeness_of_Bi-intuitionistic_Multilattice_Logic_and_its_Connexive_Variant

Using the Meth8 apparatus we evaluate the Heyting-Brouwer logic via its variants of:

1. bi-intuitionistic multilattice (*n*-lattice) logic [BML*n*];
4.4.-4.5 bi-intuitionistic connexive *n*-lattice logic [CMLn]; and
4.3 bi-intuitionistic logic [BL] with a Kripke completeness extension.

LET: ~ Not; p lc_alpha; q lc_beta; r j; s k

The designated truth value is T (tautology), where ~T is F (contradiction). Repeating fragments of rows in truth tables are listed horizontally. For four variables, there is one table of 16-values. For five variables, there are 128-tables.

1. The following expressions are provable in BML*n*:

(a) ~j(α→jβ) ⇔ ~jβ←j~jα, (1.7.a.1)

(~r&(p>(r&q))) = ((~r&q)<(r&(~r&p))) ;
FTTF TTTT (1.7.a.2)

(b) ~j(α←jβ) ⇔ ~jβ→j~jα, [same as (a) above]

(c) ~k(α→jβ) ⇔ ~kα→j~kβ, (1.7.c.1)

(~s&(p>(r&q))) = ((~s&q)>(r&(~s&p))) ;
TFFT TFFT (1.7.c.2)

(d) ~k(α←jβ) ⇔ ~kα←j~kβ, [same as (d) above]

4.4. The following expressions are provable in CMLn:

(a) ~j(α→jβ) ⇔ α→j~jβ, (4.4.a.1)

(~r&(p>(r&q))) = (p>(r&(~r&q))) ; TTTT FTFT (4.4.a.2)

(b) ~j(α←jβ) ⇔ ~jα←jβ, (4.4.b.1)

(~r&(p<(r&q))) = ((~r&p)<(r&q)) ; TTTT TTTT (4.4.b.2)

4.5. Kripke connexive extension

(a) ~(α→β) ⇔ α→~β, (4.5.a.1)

~(p>q) = (p>~q) ; FTFT FTFT (4.5.a.2)

(b) ~(α←β) ⇔ ~α←β (4.5.b.1)

~(p<q) = (~p<q) ; TTFF TTFF (4.5.b.2)

4.3. Kripke completeness extension

LET u f; v h

1. f(~j(α→jβ)) := f(~jβ)←f(~jα), (def.4.3.1.1)

(u&(~r&(p>(r&q)))) = ((u&(~r&q))<(u&(~r&p))) ;
TTTT TTTT TTTT TTTT,
FTTT TTTT FTTT TTTT; (def.4.3.1.2)

2. f(~j(α←jβ)) := f(~jβ)→f(~jα), (def.4.3.2.1)

(u&(~r&(p<(r&q)))) = ((u&(~r&q))>(u&(~r&p))) ;
FFFF FFFF FFFF FFFF,
TFFF FFFF TFFF FFFF (def.4.3.2.2)

with the following conditions [where repl means replace]:

1. h(~j(α→jβ)) := h(α)→h(~jβ), (def.4.3.1.1.repl)

(v&(~r&(p>(r&q)))) = ((v&p)>(v&(~r&q))) ;
FFFF FFFF FFFF FFFF,
TTTF FTFT TTTF FTFT (def.4.3.1.2.repl)

2. h(~j(α←jβ)) := h(~jα)←h(β), (def.4.3.2.1.repl)

(v&(~r&(p<(r&q)))) = ((v&p)<(v&(~r&q))) ;
TTTT TTTT TTTT TTTT,
TTTF TFTF TTTF TFTF (def.4.3.2.2.repl)

In our previous work we showed the standard four-valued lattice logic was not bi-valent, but a probabilistic vector space. With the intention of re-evaluating that conclusion, we evaluated the Heyting-Brouwer logic based on lattice logic with the extensions proposed for BML_n, CMLn, and BL above.

We evaluated 10 expressions. Meth8 validated as tautology one, Eq. 4.4.b.2. We confirm our initial conclusion regarding lattice logic as not bivalent. What follows is that intuitionistic logic is also not bivalent.

Refutation of Heyting algebra (part two)

Abstract: We evaluate the seminal equation for Heyting algebra of a∧b≤c⇔a≤b→c. It is *not* tautologous, hence refuting Heyting algebra as stated, and forming another *non* tautologous fragment of Heyting algebra in the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≙, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Moraschini, T.; Wannenburg, J.J. (2019) Epimorphism surjectivity in varieties of Heyting algebras. arxiv.org/pdf/1908.00287.pdf

2. Esakia duality

A *Heyting algebra* is an algebra ***A*** = ⟨*A*; ∧, ∨, →, 0, 1⟩ which comprises a bounded lattice ⟨*A*; ∧, ∨, 0, 1⟩, and a binary operation → such that for every *a*, *b*, *c* ∈ *A*,

$$a \wedge b \leq c \Leftrightarrow a \leq b \rightarrow c. \qquad (2.1.1)$$

LET p, q, r: *a*, *b*, *c*.

(p&~(q<r))=(~(q<p)>r) ; T**FF**T **F**T**F**T T**FF**T **F**T**F**T (2.1.2)

It follows that Heyting algebras are distributive lattices. Remarkably, a Heyting algebra is uniquely determined by its lattice reduct. The class of all Heyting algebras forms a variety, ... HA.

Eq. 2.1.2 as rendered is *not* tautologous, hence refuting Heyting algebra as stated.

Refutation of Heyting logic

We assume Meth8/VŁ4 with the designated *proof* value of Tautology. The 16-valued truth table is row-major and horizontal.

LET p q r: x y z.

This is taken from:

Sheppeard, M.D. (2018). Idempotents in motivic quantum gravity. vixra.org/pdf/1804.0365v1.pdf

A Heyting algebra [10] is a not necessarily distributive poset lattice with a 0 and 1 and implication $x \rightarrow y$. Objects in the lattice are idempotent ... satisfying

$$x \wedge (y \vee x) = x = (x \wedge y) \vee x. \quad (6.1)$$

((p&(q+p))=p)=((p&q)+p) ; `FTFT FTFT FTFT FTFT` (6.2)

Remark: Eq. 6.1 is coerced into a theorem as $((x \wedge (y \vee x))=x) = (x=((x \wedge y) \vee x))$.

Implication satisfies

$$(x \rightarrow y) \wedge x = x \wedge y \quad (7.1)$$

((p>q)&p)=(p&q) ; `TTTT TTTT TTTT TTTT` (7.2)

and the distributivity

$$x \rightarrow (y \wedge z) = (x \rightarrow y) \wedge (x \rightarrow z). \quad (8.1)$$

(q>(q&r))=((p>q)&(p>r)) ; `TFFT TFTT TFFT TFTT` (8.2)

While Eq. 7.2 as rendered is tautologous, Eqs. 6.2 and 8.2 are *not* tautologous. This means that objects in the lattice-vectors of Heyting algebra are *not* idempotent. Consequently, Heyting logic is *not* bivalent, and hence refuted.

Refutation of the hoop and pocrim in Heyting algebras

Abstract: We evaluate 14 equations for the hoop and pocrim in Heyting algebras as *not* tautologous. The methodology of using proof assistants Prover9 and Mace8 is also refuted. These artifacts form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Arthan, R.; Oliva, P. (2019). Studying algebraic structures using Prover9 and Mace4. arxiv.org/pdf/1908.06479.pdf p.oliva@qmul.ac.uk, rda@lemma-one.com

Abstract … The specific tools in our case study are Prover9 and Mace4; the algebraic structures are generalisations of Heyting algebras known as hoops. We will see how this approach helped us to discover new theorems and to find new or improved proofs of known results. …

1.1 Using Prover9 and Mace4 In a semilattice, one defines a relation ≥ by

x ≥ y ⇔ x ∪ y = x. (1.1.1.1)

LET p, q, r: x, y, z.

~(q>p)=((p+q)=p) ; **`FFFF FFFF FFFF FFFF`** (1.1.1.2)

Remark 1.1.1.2: Eq. 1.1.1.2 is *no*t tautologous, refuting the definition of ≥ in a semilattice.

1.2 Investigating the algebraic structure of hoops Hoops are a generalisation of Heyting algebras (used in the study of intuitionistic logic [In a Heyting algebra one normally uses x → y for y ⊖ x, and x ∧ y for x ⊕ y.]). ... A hoop [Strictly speaking this is a bounded hoop: an unbounded hoop omits the constant 1 and axiom (8).] is a structure (H, 0, 1, ⊕, ⊖) satisfying the following axioms:

x ⊖ x = 0 (1.2.4.1)

LET p, q, r: x, y, z.

(p>p)=(s@s) ; **`FFFF FFFF FFFF FFFF`** (1.2.4.2)

$0 \ominus x = 0$ (1.2.7.1)

p>(s@s)=(s@s) ; **FFFF FFFF FFFF FFFF** (1.2.7.2)

$x \ominus 1 = 0$ (1.2.8.1)

((s=s)>p)=(s@s) ; **FFFF FFFF FFFF FFFF** (1.2.8.2)

Remark 1.2.8.2: Eqs. 1.2.4.2-..8.2 are *not* tautologous, are contradictions, and are equivalent.

[A] semilattice structure induces an ordering on a hoop which turns out to be equivalent to defining $x \geq y$ to hold when10 $y \ominus x = 0$. (1.2.9.1)

((p>q)=(s@s))>~(q>p) ; T**F**TT T**F**TT T**F**TT T**F**TT (1.2.9.2)

… [T]he conjecture that for any x, y and z, $z \geq x \ominus y$ iff $z \oplus y \geq x$. (1.2.10.1)

(#r&~(#p>#q))>~((q>p)>#r) ;TTTT TCTT TTTT TCTT (1.2.10.2)

[is] an analogue of one of the laws for manipulating inequalities in an ordered commutative group, is known as the *residuation* property and is quickly proved by Prover[9].. .

Remark 1.2.10.2: Eq 1.2.10.2 is not tautologous, refuting the residuation property.

A structure for the signature $(0, 1, \oplus, \ominus, \geq)$ such that $(0, \oplus, \geq)$ is an ordered commutative monoid with least element 0, greatest element 1 and satisfying the residuation axiom [known as a (bounded) pocrim]:

$z \geq x \ominus y \Leftrightarrow z \oplus y \geq x$. (1.2.11.1)

~(q>p)>(p<(q&(q>p))) ; TT**F**T TT**F**T TT**F**T TT**F**T (1.2.11.2)

Remark 1.2.11.2: Eq. 1.2.11.2 is *not* tautologous, refuting the definition of a pocrim.

One might conjecture that any pocrim is a hoop. However this conjecture is false … . Inspection of the operation tables reveals the weakness: in a hoop, if $x \geq y$ then $x = y \oplus (x \ominus y)$, but in a pocrim, even when $x \geq y$, we can have $x < y \oplus (x \ominus y)$: (1.2.12.0)

Remark 1.2.12.0: We write Eq. 1.2.12.1 to read: "in a hoop, if $x \geq y$ then $x = y \oplus (x \ominus y)$, but in a pocrim, even when $x \geq y$, we can have $x < y \oplus (x \ominus y)$, *which is different*:" (1.2.12.1)

(~(q>q)>(p=(q&(q>p))))@(~(q>q)>(p<(q&(q>p)))) ;
FFFF FFFF FFFF FFFF (1.2.12.2)

Remark 1.2.12.2: Eq. 1.2.12.2 is *not* tautologous, but a contradiction, meaning that the hoop and pocrim as rendered are equivalent, the opposite point of what the writers intended.

2 Analysing larger proofs
2.2 Discovering derived operations and their basic properties

This led us to introduce new operations so that multiple steps in the proof could be understood as properties of these new operations. In total we found, apart from x∪y, three further new derived operations:

Remark 2.0: We count four further new derived operations, but introduction of the "\" connective as "difference" in Table 3 Nomenclature is unclear to us (although we suspect XOR), so we stop evaluation after two equations.

x ∪ y ≡ x ⊕ (y ⊖ x) (2.0.1.1)

(p+q)=(p&(q>p)) ; TT**F**T TT**F**T TT**F**T TT**F**T (2.0.1.2)

Remark 2.2.1.2: Eq. 2.0.1.2 is the equivalent truth table value result as Eq. 1.2.11.2.

x ∩ y ≡ x ⊖ (x ⊖ y) (20.2.1)

(p&q)=((q>p)>p) ; T**FF**T T**FF**T T**FF**T T**FF**T (2.0.2.2)

When identifying these operations we also used our knowledge of the correspondence between hoops and Heyting algebras. For instance, x ∩ y in logical terms corresponds to (y → x) → x, which generalises double negation and in theoretical computer science is known as the continuation monad.. . (2.0.5.1)

(p&q)=((q>p)>p) ; T**FF**T T**FF**T T**FF**T T**FF**T (2.0.5.2)

Remark 2.0.5.2: Eq. 2.0.5.2 as rendered is equivalent to Eq. 2.0.2.2.

So, according to … our methodology, we looked first for basic properties of these new operations, or of their relation with the primitive operations. We come up with six simple properties (listed in the following lemma) that we then added as axioms, and rerun the proof search.

Lemma 2.1 The following hold in all hoops:

Remark 2.1.0: We evaluate two of the less obvious of the six properties to avoid the "\" connective.

(v) z ∩ (y ⊖ x) ≥ (z ∩ y) ⊖ (z ∩ x) (2.1.v.1)

~(((r&p)>(r&q)) >(r&(p>q))) = (s=s) ;
TTTT **FFFF** TTTT **FFFF** (2.1.v.2)

(vi) x ⊖ (x ∩ y) = x ⊖ y (2.1.vi.1)

((p&q)>p)=(q>p) ; TT**F**T TT**F**T TT**F**T TT**F**T (2.1.vi.2)

Remark 2.1.vi.2: Eq. 2.1.vi.2 is the equivalent truth table value result as 1.2.11.2. and 2.0.1.2. Eqs. ..v.2 and ..vi.2 as axioms are *not* tautologous.

2.5 Tackling the harder conjecture

Lemma 2.11 (MPS) x = (x ∩ y) ⊕ (x ⊖ y) (2.11.1)

p=(p&q)&(q>p) ; TFTT TFTT TFTT TFTT (2.11.2)

Remark 2.11.2: Eq. 2.11.2 is *not* tautologous and is the equivalent truth table value result as 1.2.9.2.

We evaluated 14 expressions as definitions, axioms, and lemmas for which none is tautologous and four are contradictions. This refutes hoops and pocrims as stated for Heyting algebras and the methodology adopted for Prover9 and Mace8 proof assistants.

We include the chapter conclusion of the writers to complete this artifact.

3 Concluding Remarks In Section 1 we have attempted to introduce the tools and methods we have been using by examples at the level of an undergraduate project. We hope this is of interest to educators and advocate introduction of tools such as Prover9 and Mace4 into mathematical curricula. At a more advanced level, we have discussed our own research using Prover9 and Mace4 to investigate algebraic structures. It is possible to demonstrate the provability of properties like duality, commutativity or homomorphism properties by model-theoretic methods but these methods are not constructive, whereas the methods discussed in Section 2 construct explicit equational proofs. … Tools such as Prover9 automate the process of discovering a proof, but at first glance, the proofs that are discovered seem inaccessible to a human reader. We take this as an intellectual challenge in its own right and claim that with human effort, judiciously applied, we can "mine" explanative and systematic human-oriented proofs from machine-generated ones, potentially leading to new insights into the problem domain. … Some automated support for refactoring the machine-generated proofs could be very helpful. The refactoring steps of interest would include separating out lemmas and retrofitting derived notations. It is certainly of interest to speculate on possibilities for fully automating extraction of human-readable proofs from machine-generated proofs, but we view this as a hard challenge for Artificial Intelligence.

Remark 3.0: The comments above stand on their face because we show that Prover9, a non-bivalent vector space, can itself be coerced into the appearance of bivalency, such as:
for (◊p&◊q)>◊(p&q), read (◊p&◊~p)>◊(p&~p).

Refutation of hexagons of opposition for statistical modalities

Abstract: We evaluate pragmatic hypotheses in the evolution of science as based on probabilistic squares and hexagons of opposition under coherence. Neither conjecture is tautologous, and hence both are refuted.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r, s: p; x; r; s;
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⊢; < Not Imply, less than, ∈;
= Equivalent, ≡, ⊨; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(p=p) T as tautology; (p@p) **F** as contradiction;
(%p<#p) C as contingency, Δ; (%p>#p) N as non-contingency, ∇; (%r>#r) Ordinal 1.
~(y < x) (x ≤ y), (x ⊆ y).

From: Esteves, L.G.; Izbicki, R.; Stern, R.B.; Stern, J.M. (2018).
Pragmatic hypotheses in the evolution of science. arxiv.org/pdf/1812.09998.pdf
lesteves@ime.usp.br, rafaelizbicki@gmail.com, rbstern@gmail.com, jmstern@hotmail.com

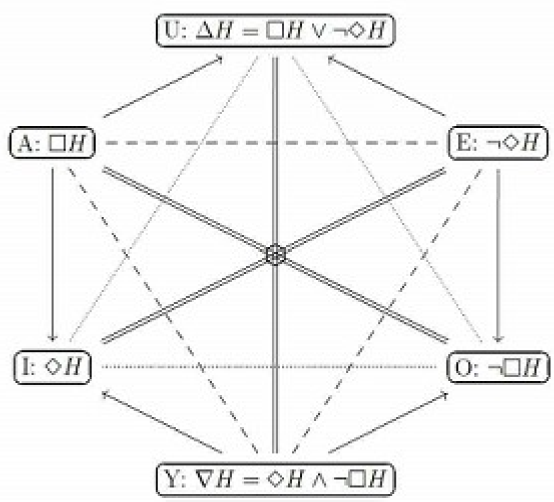


Figure 1: Hexagons of Opposition for Statistical Modalities.

We found the following expressions to be tautologous: A=~(E+Y); E=~(A+Y); U=(A+E); U=(A&E); Y=(I&O); I=~(O&U); O=~(I&U).

This lead us to evaluate the source of the atomic, statistical modalities as so cited below.

From: Pfeifer, N.; Sanfilippo, G. (2017).
Probabilistic squares and hexagons of opposition under coherence.
arxiv.org/pdf/1701.07306.pdf niki.pfeifer@lmu.de, giuseppe.sanfilippo@unipa.it

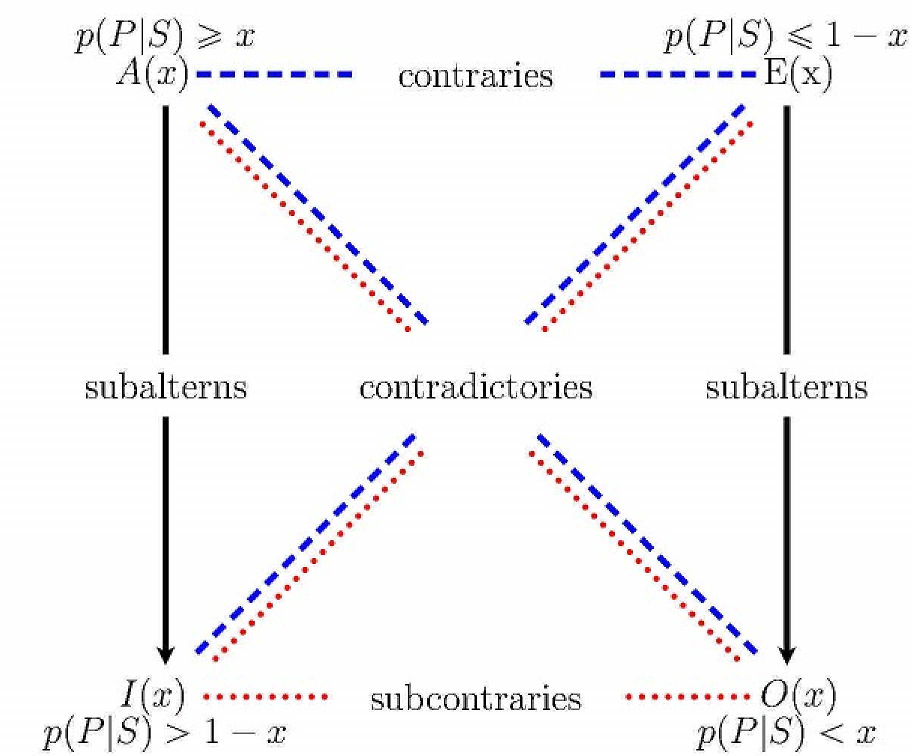


Figure 3: Probabilistic square of opposition $\mathbf{S}(x)$ defined on the four sentence types $(A(x), E(x), I(x), O(x))$ with the threshold $x \in]\frac{1}{2}, 1]$ (see also Table 1). It provides a new interpretation of the traditional square of opposition (see, e.g., [38]), where the corners are labeled by "Every S is P" (A), "No S is P" (E), "Some S is P" (I), and "Some S is not P" (O).

We evaluate the assigned probabilities above in the square of opposition as corrected by Meth8 below.

From: James, C. (2019). "Square of opposition as Meth8 corrected". [Search at vixra.org]

Source type	Def	Meth8 corrected	Valid as	Statistical probabilities	Truth table results
Corner	A	#(s= p)	N**F**N**F** N**F**N**F** F**N**F**N** F**N**F**N**	~(q>#(s=p))	**FF**CT **FF**CT **FF**TC **FF**TC
	E	#(s=~p)	**F**N**F**N **F**N**F**N N**F**N**F** N**F**N**F**	~(((%r>#r)-q)<#(s=~p))	NNTT NNTT NNTT NNTT
	I	%(s= p)	TCTC TCTC CTCT CTCT	(%(s=p)>((%r>#r)-q))	CT**F**N CT**F**N TCN**F** TCN**F**
	O	%(s=~p)	CTCT CTCT TCTC TCTC	(%(s=~p)<q)	CT**FF** CT**FF** TC**FF** TC**FF**
Contrarity	AE	#(s= p)\ #(s=~p)	A \ E TTTT TTTT TTTT TTTT	~(q>#(s=p))\ ~(((%r>#r)-q)<#(s=~p))	TTN**F** TTN**F** TT**F**N TT**F**N
Subaltern	AI	#(s= p)> %(s= p)	A > I TTTT TTTT TTTT TTTT	~(q>#(s=p)) >(%(s=p)>((%r>#r)-q))	TTNN TTNN TTNN TTNN
Contradictory	AO	#(s= p) \ %(s=~p)	A \ O TTTT TTTT TTTT TTTT	~(q>#(s=p)) \(%(s=~p)<q)	TTTT TTTT TTTT TTTT
Contradictory	EI	#(s=~p)\ %(s= p)	E \ I TTTT TTTT TTTT TTTT	~(((%r>#r)-q)<#(s=~p)) \(%(s=p)>((%r>#r)-q))	TCTC TCTC CTCT CTCT
Subaltern	EO	#(s=~p)> %(s=~p)	E > O TTTT TTTT TTTT TTTT	~(((%r>#r)-q)<#(s=~p)) >(%(s=~p)<q)	CT**FF** CT**FF** TC**FF** TC**FF**
Subcontrarity	IO	%(s= p)+ %(s=~p)	I + O TTTT TTTT TTTT TTTT	(%(s=p)>((%r>#r)-q)) +(%(s=~p)<q)	CT**F**N CT**F**N TCN**F** TCN**F**

The equation for AO contradictory is tautologous, as expected, however the other nine are not. This indicates mistakes in the assignments for probabilistic squares and hexagons of opposition under coherence. What follows is that since pragmatic hypotheses in the evolution of science are based thereon, they also are suspicious. In other words, both probabilistic squares and hexagons of opposition under coherence and the derived pragmatic hypotheses in the evolution of science are refuted.

Mathematical induction as a higher-order logical principle based on permutations of F>F=T.

From: en.wikipedia.org/wiki/Higher-order_logic

> "First-order logic quantifies only variables that range over individuals; second-order logic, in addition, also quantifies over sets; third-order logic also quantifies over sets of sets, and so on.
>
> For example, the second-order sentence
>
> $$\forall P ((0 \in P \wedge \forall i (i \in P \rightarrow i + 1 \in P)) \rightarrow \forall n (n \in P)) \quad (1.1)$$
>
> expresses the principle of mathematical induction. Higher-order logic is the union of first-, second-, third-, … , *n*th-order logic; i.e., higher-order logic admits quantification over sets that are nested arbitrarily deeply."
>
> **Remark:** The element *n*th-order logic implies it is a permutation.

We evaluate higher-order logic based on the principle of mathematical induction.

We assume the Meth8/VŁ4 apparatus and method.

LET: p q r P i n; # necessity, all, ∀; % possibility, one or some; + Or; - Not Or;
& And; > Imply, →; < Not Imply, less than, ∈; 1 (%p>#p); 0 (p@p) .
The designated proof value is T; F contradiction; C falsity; N truth.
The 16-valued truth tables are row-major and presented horizontally.

Eq. 1.1 is a higher-order logic expression where the entire formula is universally quantified on one set (P) over universally quantified variables (i, n).

Meth8/VŁ4 treats sets and variables as variables. Therefore Eq. 1.1 can be rendered by inserting quantifiers to modify each occurrence of a variable:

(((p@p)<#p)&((#q<#p)>((#q+(%p>#p))<#p)))>(#r<#p) ;
TTTT TTTT TTTT TTTT (1.2)

We examine the antecedent and consequent of Eq. 1.2.

((p@p)<#p)&((#q<#p)>((#q+(%p>#p))<#p)) ;
FFFF FFFF FFFF FFFF (1.3)

#r<#p ; **FFFF** TCTC **FFFF** TCTC (1.4)

The principle of induction in Eq. 1.2 is tautologous as a permutation by way of the generic format of **F**>**F**=T.

Hilbert's problem ten (H10) is undecidable: shortest refutation

Abstract: In the shortest refutation, Hilbert's tenth problem (H10) is decidable for solving Diophantine equations. We extend results from N and I to the open question of application to rational numbers as field Q of real numbers as structure R. We do this because modal propositional logic logic is sufficient to apply. This means that no solutions exist for the Q field in the R structure. This undecidable problem forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

Acknowledgment: Thanks are due for comments from Mihai Prunescu, Institute of Mathematics of the Romanian Academy.

LET p, q, r, s, t, u, v, w, x, y, z: c, d, e, s, a, b, v, w, x, y, z.

1. Coefficients a and b are integers; and we look for solutions x and y which are natural numbers N or integers I. For both N and I, the relation is Diophantine, and a general decision method is sought.

2.1 In N: a > 0 implies ∃ (b).(a= b+1). (2.1.1.1)

(t>(s@s))>(t=(%u+(%s>#s))) ; TTTT TTTT TTTT TTTT (2.1.1.2)

Eq 2.1.1.1 implies Eqs. 2.1.3.1 and 2.1.4.1: (2.1.5.1)

ax + by > z + 1 (2.1.3.1)

((t&x)+(u&y))>(z+(%s>#s)) ;
TTTT TTTT TTTT TTTT (2.1.3.2)

and 1 - ax - by > v + 1 (2.1.4.1)

(((%s>#s)-(t&x))-(u&y))>(v+(%s>#s)) ;
TTTT TTTT TTTT TTTT(1),
NNNN NNNN NNNN NNNN(1),
TTTT TTTT TTTT TTTT(6) (2.1.4.2)

((t>(s@s))>(t=(%u+(%s>#s))))>((((t&x)+(u&y))>(z+(%s>#s)))&((((%s>#s)-(t&x))-(u&y))>(v+(%s>#s)))) ; TTTT TTTT TTTT TTTT (2.1.5.2)

Both Eqs. 2.1.2.1 and 2.1.3.1 must be rewritten such that both right side and left side have only positive coefficients (always possible) or negative coefficients are allowed.

(ax + by - z – 1 > 0) + (0 > ax + by +v) = 0 (2.1.6.1)

Remark 2.1.6.1: Respective squaring of Eqs. 2.1.2.1 and 2.1.3.1 has not effect as A&A = A.

((((((p&x)+(q&y))-z)-(%s>#s))>(s@s))+((s@s)>(((p&x)+(q&y))+v)))=(s@s) ;
FFFF FFFF FFFF FFFF (2.1.6.2)

Eq. 2.1.6.2 as rendered is *not* tautologous, and hence there are no natural solutions in x, y, z, v.

2.2 In Z: a > 0 implies ∃(b,c,d,e).(a = 1 + b^2 + c^2 + d^2 + e^2) (2.2.1.1)

(t>(s@s))>(t=((%s>#s)+((%u+%p)+(%q+%r)))) ;
FFFF FFFF FFFF FFFF,
TTTT TTTT TTTT TTTT (2.2.1.2)

Eq 2.2.1.1 implies Eqs. 2.2.3.1 and 2.2.4.1: (2.2.5.1)

ax + by > 1 + v^2 + w^2 + z^2 + s^2 (2.2.3.1)

((t&x)+(u&y))>((%s>#s)+((v+w)+(z+s))) ;
TTTT TTTT TTTT TTTT (2.2.3.2)

and 1 - ax - by > 1 + s^2 + r^2 + p^2 + q^2 (2.2.4.1)

(((%s>#s)-(t&x))-(u&y))>((%s>#s)+((s+r)+(p+q))) ;
TTTT TTTT TTTT TTTT(1),
NTTT TTTT TTTT TTTT(1),
TTTT TTTT TTTT TTTT(1),
NTTT TTTT TTTT TTTT(3),
TTTT TTTT TTTT TTTT(2) (2.2.4.2)

((t>(s@s))>(t=((%s>#s)+((%u+%p)+(%q+%r)))))>((((t&x)+(u&y))>((%s>#s)+((v+w)+(z+s))))&((((%s>#s)-(t&x))-(u&y))>((%s>#s)+((s+r)+(p+q))))) ;
TTTT TTTT TTTT TTTT(1),
NTTT TTTT TTTT TTTT(1),
TTTT TTTT TTTT TTTT(3),
NTTT TTTT TTTT TTTT(1),
TTTT TTTT TTTT TTTT(2) (2.2.5.2)

We write this as one Diophantine equation which will have no solution in the displayed variables p, q, r, s, t, u, v, w, x, y, z. Hence, Eq 2.2.3.1 + Eq 2.2.4.1 = 0. (2.2.6.1)

(((t&x)+(u&y))>((%s>#s)+((v+w)+(z+s))))+((((%s>#s)-(t&x))-(u&y))>((%s>#s)+((s+r)+(p+q))))=(s@s) ;

```
FFFF FFFF FFFF FFFF(16), FFFF FFFF FFFF FFFF(1),
CFFF FFFF FFFF FFFF(1),  FFFF FFFF FFFF FFFF(1),
CFFF FFFF FFFF FFFF(1),  FFFF FFFF FFFF FFFF(28),
CFFF FFFF FFFF FFFF(1),  FFFF FFFF FFFF FFFF(1),
CFFF FFFF FFFF FFFF(14), FFFF FFFF FFFF FFFF(64)
```
(2.2.6.2)

We extend results from N and I to the open question of application to rational numbers as field Q of real numbers as structure R. We do this because modal propositional logic logic is sufficient to apply. This means that no solutions exist for the Q field in the R structure. Hence a general decision method for solving Diophantine equations does not exist, and Hilbert's Tenth Problem is rendered undecidable.

Hilbert system generalization

Hilbert system expression in question: "∀y(∀xPxy→Pty)".

In Meth8 script this is (#q&(#p&(p&q)))>(#q&(r&q)), where the universal quantifier ∀ is replaced by the modal necessity operator #.

The expression is not validated,:

```
(#q&(#p&(p&q))):  FFFN FFFN;  UUUE UUUE;  UUUU UUUU;  UUUI UUUI;  UUUP UUUP
(#q&(r&q)):       FFFF FFNN;  UUUU UUEE;  UUUU UUUU;  UUUU UUII;  UUUU UUPP
(#q&(#p&(p&q)))>(#q&(r&q)):
                  TTTC TTTT;  EEEU EEEE;  EEEE EEEE;  EEEP EEEE;  EEEI EEEE
             _    __^         ___^                    ___^        ___^
```

Refutation of the Hilbert Grand Hotel paradox

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal.

From: en.wikipedia.org/wiki/Hilbert's_paradox_of_the_Grand_Hotel

LET p, q: rooms, guests;
~ Not; & And; + Or; - Not Or; > Imply, greater than; < Not Imply, less than;
@ Not Equivalent; # necessity, for all; % possibility, for one or some;
(%p>#p) 1, one.

"It is demonstrated that a fully occupied hotel with infinitely many rooms may still accommodate additional guests, even infinitely many of them, and this process may be repeated infinitely often." (1.1)

We take the expression "a fully occupied hotel with infinitely many rooms may still accommodate additional guests" as rooms are greater than guests.

We also take the expression "and this process may be repeated infinitely often" to mean the possibility that both the rooms outnumber the guests *and* the guests outnumber the rooms.

((#(p>q)&~((p-q)<(%p>#p)))>
((((p-(%p>#p))&(q-(%p>#p)))>((p+(%p>#p))&(q-(%p>#p))))>
(((p-(%p>#p))&(q+(%p>#p)))>((p+(%p>#p))&(q+(%p>#p)))))) > %((p>q)&~(p>q)) ;
CCCC CCCC CCCC CCCC (1.2)

Eq. 1.2 as rendered is *not* contradictory but rather falsity. Hence this refutes the Hilbert Grand Hotel paradox.

Remark: We could not reduce this paradox to *one* variable because rooms and guests are distinctly counted.

Refutation of intuitionistic logic on a "transfinite argument"

Abstract: We evaluate intuitionistic logic via Hilbert's "transfinite argument" and Komogorov's implementation. None of the axioms is tautologous.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET p, q: A, B; ~ Not, ¬; & And; > Imply, →;
% possibility, for one or some, ∃; # necessity, for all or every, ∀;

From: Coquand, T. (2004). "Kolmogorov's contribution to intuitionistic logic". Chapter 2. in Charpentier, E. et al (Eds). Komogorov's heritage in mathematics. Springer-Verlag. sciencedocbox.com/Physics/65775934-Kolmogorov-s-heritage-in-mathematics.html

"**2.1.2 Kolmogorov's formalization of intuitionistic logic**

Kolmogorov's first contribution in this paper is a complete formalization of *minimal* propositional calculus (a strict subset of intuitionistic logic which is usually attributed to Johansson [Joh36]) and minimal predicate calculus. As indicated by Wang, Kolmogorov's formalization is no less remarkable than Heyting's [Hey30]. The very possibility of such a formalization is already quite surprising, if we reflect that the motivations behind intuitionism were opposed to the process of formalization[9].

[9]According to Wang[Wan87], Brouwer considered this result to be more remarkable and surprising than Gödel's celebrated incompleteness theorem [Göd31].

Kolmogorov's work is final concerning propositional calculus, but less precise with respect to predicate calculus.

The formalization is directly inspired from Hilbert [Hil23], who had suggested the following axioms for implication and negation:

1.A→B→A (1.1)

(p>q)>p ; **F**T**F**T **F**T**F**T **F**T**F**T **F**T**F**T (1.2)

2. (A→A→B)→A→B (2.1)

(((p>p)>q)>p)>q ; **FF**TT **FF**TT **FF**TT **FF**TT (2.2)

3. (A→B→C)→B→A→C (3.1)

((((p>q)>r)>q)>p)>r ; T**F**T**F** TTTT T**F**T**F** TTTT (3.2)

4. (B→C)→(A→B)→A→C (4.1)

(((q>r)>(p>q))>p)>r ; TFTF TTTT TFTF TTT (4.2)

5.A→¬A→B (5.1)

(p>~p)>q ; FTTT FTTT FTTT FTTT (5.2)

6. (A→B)→(¬A→B)→B" (6.1)

(p>q)>(~p>q))>q ; TFTT TFTT TFTT TFTT (6.2)

Remark 1.-6.: Eqs. 1.2-6.2 as rendered are *not* tautologous. This means Kolmogorov's adaptation of Hilbert's intuitionistic logic is similarly flawed.

"Hilbert's article [Hil23] raises the problem of justifying the rules of quantification (both existential and universal) over an infinite domain, in particular the following principle

(¬∀x.A)→∃x.¬A, (7.3.1)

(~#p&q)>(%p&~q) ; TTFN TTFN TTFN TTFN (7.3.2)

which follows from the Principle of Excluded Middle, and may be used to deduce the existence of an element

∃x.¬A (7.2.1)

%p&~q ; CTFF CTFF CTFF CTFF (7.2.2)

from a proof of the impossibility of its non-existence

¬∀x.A[6]. (7.1.1)

~#p&q ; FFTC FFTC FFTC FFTC (7.1.2)

This is a typical instance of what Hilbert calls a *transfinite argument*, a terminology which is also used in Kolmogorov's paper (these terms may be somewhat surprising, since the adjective "transfinite" is associated nowadays with the use of the class of countable ordinals, or more generally of uncountable classes)."

Remark 7.: Eqs. 7.1.2, 7.2.2, and 7.3.2 as rendered are *not* tautologous. This refutes Hilbert's use of the term "transfinite argument".

Eqs. 1.-7. are *not* tautologous. This means that Hilbert's intuitionistic logic, and as implemented by Kolmogorov, and by association Heyting, is refuted.

Refutation of Hilbert's first epsilon theorem in intuitionistic and intermediate logics

Abstract: From the universal quantifier shift of (∀xA(x)→B)→∃x(B→A(x)) as *not* tautologous, the intermediate logic **L** is refuted, refuting Hilbert's first epsilon theorem and intuitionistic logic, and forming a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Baaz, M.; Zach, R. (2019). The first epsilon theorem in pure intutionistic and intermediate logics. arxiv.org/pdf/1907.04477.pdf rzach@ucalgary.ca

§1. Introduction. In 1921, Hilbert introduced the ε-calculus as a formalism on which to build his proof-theoretic project. The ε-calculus was originally introduced as a formalization of classical first-order logic. It can be seen as an attempt to reduce proofs in first-order logic to proofs in propositional logic, where the role of quantifiers is taken over by certain terms. … In the presence of identity, the formalism is more complicated, as axioms for identity have to be added to propositional logic. Hilbert called the resulting system the "elementary calculus of free variables"— essentially a formalism with predicates and terms, as well as open axioms for identity, but without quantifiers.

§3. ετ-Calculi for intermediate logics. An intermediate logic **L** is a set of formulas that contains intuitionistic logic **H** and is contained in classical logic **C**, and is closed under modus ponens and substitution. For intermediate predicate logics, we also require closure under the universal and existential quantifier rules.

Definition 3.1. Suppose **L** is an intermediate logic. … Some of these are obtained from **QH** simply by adding propositional axiom schemes. Equivalently, they can be obtained by expanding a propositional intermediate logic **L** to a language with predicates and terms, the standard quantifier axioms ∀xA(x)→A(t) and A(t)→∃xA(x) and closing under substitution, modus ponens, and the quantifier rules. This results in the weakest pure intermediate predicate logic extending **L**. Not every intermediate predicate logic is obtained in this way, as it is possible to consistently add additional first-order principles to **L**. Some important first-order principles are, e.g., the constant domain principle ∀x(A(x)∨B)→(∀xA(x)∨B), (CD) the double negation shift (or Kuroda's principle), ∀x¬¬A(x)→¬¬∀xA(x) (K) and the quantifier shifts

(B→∃xA(x))→∃x(B→A(x)) (Q∃) (3.1.1.1)

LET p, q, r: A, B, x.

(q>(%r&(p&r)))>(%r&(q>(p&r))) ;
CCTT TTTT CCTT TTTT (3.1.1.2)

Remark 3.1.1.2: If the existential quantifier is distributed, the result is tautologous.

(∀xA(x)→B)→∃x(B→A(x)) (Q∀) (3.1.2.1)

((#r&(p&r))>q)>(%r&(q>(p&r))) ;
CC**FF** TT**F**T CCC**F** TT**F**T (3.1.2.2)

Remark 3.1.2.2: If the universal quantifier is distributed, the result is *not* tautologous as ((p&#r)>q)>(q>(p&%r)) ; TT**F**C TT**F**T TT**F**C TT**F**T (3.1.3.2)

§4. Critical formulas and quantifier shifts.

We might think of ετ-terms semantically as terms for objects which serve the role of generics taking on the role of quantifiers, and indeed in classical logic this connection is very close. Because of the validity of

∃x(∃yA(y)→A(x)) (Wel 1) (4.1.1.1)
∃x(A(x)→∀yA(y)) (Wel 2) (4.1.2.1)

in classical logic, there always is an object x which behaves as an ε-term (A(x) holds iff ∃x A(x) holds), and an object x which behaves as a τ-term (i.e., A(x) holds iff ∀y A(y) holds). One might expect then that Wel1 and Wel 2, when added to **QH**, have the same effect as adding critical formulas, i.e., that all quantifier shifts become provable. Note that Wel 1 and Wel 2 are intuitionistically equivalent to

∃x∀y(A(y)→A(x)) (Wel′ 1) (4.2.1.1)
∃x∀y(A(x)→A(y)) (Wel′ 2) (4.2.2.1)

Remark 4: We test Eqs 4.1.1.1=4.2.1.1 (4.3.1.1); 4.1.2.1=4.2.2.1 is a trivially tautologous.

((p&%r)>(p&%q))=((p&#r)>(p&%r)) ;
TTTT TCTT TTTT TCTT (4.3.1.2)

From the universal quantifier shift of Eq. 3.1.2.1 as not tautologous, the intermediate logic **L** is refuted along with Hilbert’s first epsilon theorem in intuitionistic logic.

Refutation of the HOL/Isabelle rejection of E.J. Lowe's modal ontological argument

Abstract: Of 20 equations evaluated, 16 are *not* tautologous. This effectively refutes Lowe's proof, as rendered by the authors. This also invalidates the authors' rejection of Lowe's proof due to incompleteness (six of Lowe's conclusions are dismissed without evaluation) and due to an interactive, trial by error approach to reconstruct Lowe. Therefore an ideal showcase for the computer-assisted interpretive method using HOL/Isabelle failed. These results form another *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Fuenmayor, D.; Benzmüller, C. (2019, nee 2017).
Computer-assisted reconstruction and assessment of E. J. Lowe's modal ontological argument.
isa-afp.org/browser_info/current/AFP/Lowe_Ontological_Argument/outline.pdf

Abstract: Computers may help us to understand –not just verify– philosophical arguments. By utilizing modern proof assistants in an iterative interpretive process, we can reconstruct and assess an argument by fully formal means. Through the mechanization of a variant of St. Anselm's ontological argument by E. J. Lowe, which is a paradigmatic example of a natural-language argument with strong ties to metaphysics and religion, we offer an ideal showcase for our computer-assisted interpretive method [tool named HOL/Isabelle].

2 E. J. Lowe's Modal Ontological Argument
2.1 Introduction
E. J. Lowe … "A modal version of the ontological argument"... features eight premises from which new inferences are drawn until arriving at a final conclusion: the necessary existence of God (which in this case amounts to the existence of some "necessary concrete being").

(P1.1) God is, by definition, a necessary concrete being.

LET p, q, r, s, t, u, v, w, x, y, z:
being, dependent, explanation, space, time, abstract, concrete, world, x, y, z.

Remark 1: The verb depend is taken to mean the imply operator, whereas the adjectives dependent (not independent) are taken as variables. While the verb explain can be taken to mean the imply operator, the noun explanation is taken as a variable standing on its own.

God: #(v&p)=(z=z) ; **FFFF FFFF FFFF FFFF**(4)
FN**F**N **F**N**F**N **F**N**F**N **F**N**F**N(4) (P1.2)

(P2.1) Some necessary abstract beings exist.

%#(u&p)=(z=z) ; **FFFF FFFF FFFF FFFF**(2)
FN**F**N **F**N**F**N **F**N**F**N **F**N**F**N(2) (P2.2)

(P3.1) All abstract beings are dependent beings.

(q&p)>#(u&p) ; TTTF TTTF TTTF TTFT(2)
TTTN TTTN TTTN TTTN(2) (P3.2)

(P4.1) All dependent beings depend for their existence on independent beings.

~(q&p)>#(q&p) ; **FFF**T **FFF**T **FFF**T **FFF**T (P4.2)

(P5.1) No contingent being can explain the existence of a necessary being.

(~(%z<#z)&p)>%#p ; TTTT TTTT TTTT TTTT (P5.2)

(P6.1) The existence of any dependent being needs to be explained.

%#(q&p)>r ; TTTC TTTT TTTC TTTT (P6.2)

(P7.1) Dependent beings of any kind cannot explain their own existence.

~(#(q&p)>(r>%#(q&p)))=(z=z) ;
FFFF FFFF FFFF FFFF (P7.2)

(P8.1) The existence of dependent beings can only be explained by beings on which they depend for their existence.

p>(#r>%(q&p)) ; TTTT TCTT TTTT TCTT (P8.2)

We will consider in our treatment only a representative subset of the [ten] conclusions, as presented in Lowe's article.

Remark 2 The authors summarily dismiss four of the ten conclusions (C2.1, C3.1, C4.1, and C6.1), suggesting an incomplete approach.

(C1.1) All abstract beings depend for their existence on concrete beings. (Follows from P3.1 and P4.1 together with D3.1 and D4.1.)

((((q&p)>#(u&p))&(~(q&p)>#(q&p)))&(((x>(v&p))=(((%s&t)+t)>%x))&
((x>(u&p))=((s&t)>~%x))))>((v&p)>((v&p)>%#p)) ;
TTTT TTTT TTTT TTTT (C1.2)

(C5.1) In every possible world there exist concrete beings. (Follows from C1.1 and P2.1.)

((((((q&p)>#(u&p))&(~(q&p)>#(q&p)))&(((x>(v&p))=(((%s&t)+t)>%x))&
((x>(u&p))=((s&t)>~%x))))>((v&p)>((v&p)>%#p)))&(%#(u&p)))>(#%w>%(v&p)) ;

```
TTTT TTTT TTTT TTTT(10)
TFTF TFTF TFTF TFTF( 2)
TTTT TTTT TTTT TTTT( 4)
```
(C5.1)

(C7.1) The existence of necessary abstract beings needs to be explained. (Follows from P2.1, P3.1 and P6.1.)

((%#(u&p))&(((q&p)>#(u&p))&(%#(q&p)>r)))>(%#(u&p)>r) ;

```
TTTT TTTT TTTT TTTT( 2)
TCTT TTTT TCTT TTTT( 2)
```
(C7.2)

(C8.1) The existence of necessary abstract beings can only be explained by concrete beings. (Follows from C1.1, P3.1, P7.1 and P8.1.)

(((((((q&p)>#(u&p))&(~(q&p)>#(q&p)))&(((x>(v&p))=(((%s&t)+t)>%x))&
((x>(u&p))=((s&t)>~%x))))>((v&p)>((v&p)>%#p)))&((q&p)>#(u&p)))&
((~(#(q&p)>(r>%#(q&p))))&(p>(#r>%(q&p)))))>(%#(u&p) >(r>(v&p))) ;

```
TTTT TTTT TTTT TTTT
```
(C8.2)

(C9.1) The existence of necessary abstract beings is explained by one or more necessary concrete beings. (Follows from C7.1, C8.1 and P5.1.)

(((((%#(u&p))&(((q&p)>#(u&p))&(%#(q&p)>r)))>(%#(u&p)>r))&((((((((q&p)>
#(u&p))&(~(q&p)>#(q&p)))&(((x>(v&p))=(((%s&t)+t)>%x))&((x>(u&p))=((s&t)>~
%x))))>((v&p)>((v&p)>%#p)))&((q&p)>#(u&p)))&((~(#(q&p)>(r>%#(q&p))))&
(p>(#r>%(q&p)))))>(%#(u&p) >(r>(v&p)))))&((~(%z<#z)&p)>%#p))>(%#(u&p)>(r>
%#(v&p))) ;

```
TTTT TTTT TTTT TTTT( 2)
TTTT TCTC TTTT TCTC( 2)
TTTT TTTT TTTT TTTT( 4)
```
(C9.2)

(C10.1) A necessary concrete being exists. (Follows from C9.1.)

((((((%#(u&p))&(((q&p)>#(u&p))&(%#(q&p)>r)))>(%#(u&p)>r))&((((((((q&p)>
#(u& p))&(~(q&p)>#(q&p)))&(((x>(v&p))=(((%s&t)+t)>%x))&((x>(u&p))=((s&t)>~
%x))))>((v&p)>((v&p)>%#p)))&((q&p)>#(u&p)))&((~(#(q&p)>(r>%#(q&p))))&
(p>(#r>%(q&p))))) >(%#(u&p) >(r>(v&p)))))&((~(%z<#z)&p)>#p))>(%#(u&p)>(r>
%#(v&p))))>(%#(v&p)) ;

```
FFFF FFFF FFFF FFFF( 2)
FFFF FNFN FFFF FNFN( 2)
FNFN FNFN FNFN FNFN( 4)
```
(C10.2)

Lowe also introduces some informal definitions which should help the reader understand the meaning of the concepts involved in his argument (necessity, concreteness, ontological dependence, metaphysical explanation, etc.). In the following discussion, we will see that most of these definitions do not bear the significance Lowe claims

Remark 3: The definitions in fact bear significance on their face. Examples are the injections of time to define omnipresence and space to define omnipotence (akin to the reasons in Popper's obscure footnote proof E(Gx)).

(D1.1) x is a necessary being := x exists in every possible world.

LET s, t, w, x, y: space, time , world, x, y.

```
(x>#p)=(#%w>%x) ;     TTTT TTTT TTTT TTTT( 8)
                      CCCC CCCC CCCC CCCC( 8)
                      FNFN FNFN FNFN FNFN(16)(D1.2)
```

(D2.1) x is a contingent being := x exists in some but not every possible world.

```
(x>(%z<#z))=((%~#%w>%x) ;
                      TTTT TTTT TTTT TTTT( 8)
                      CCCC CCCC CCCC CCCC(24)      (D2.2)
```

(D3.1) x is a concrete being := x exists in space and time, or at least in time.

```
(x>(v&p))=(((%s&t)+t)>%x) ;
                      TTTT TTTT TTTT TTTT( 1)}x8
                      CCCC CCCC CCCC CCCC( 1)}
                      FFFF FFFF FFFF FFFF( 4)}x2
                      FTFT FTFT FTFT FTFT( 4)}     (D3.2)
```

(D4.1) x is an abstract being := x does not exist in space or time.

```
(x>(u&p))=((s&t)>~%x) ;   TTTT TTTT TTTT TTTT( 1)}x8
                          TTTT TTTT NNNN NNNN( 1)}
                          FFFF FFFF FFFF FFFF( 1)}x4
                          FFFF FFFF TTTT TTTT( 1)}
                          FTFT FTFT FTFT FTFT( 1)}
                          FTFT FTFT TFTF TFTF( 1)}     (D4.2)
```

(D5.1) x depends for its existence on y := necessarily, x exists only if y exists.

```
(%y>x)=#(%y>%x) ;     TTTT TTTT TTTT TTTT(16)
                      NNNN NNNN NNNN NNNN(16)      (D5.2)
```

(D6.1) (For any predicates F and G) F depend for their existence on G := necessarily, Fs exist only if Gs exist.

LET p, q: F, G.

```
#(p&q)>((%q>p)=#(%q>%p)) ;
                      TTTT TTTT TTTT TTTT          (D6.2)
```

We will work iteratively on Lowe's argument by temporarily fixing truth values and inferential relationships among its sentences, and then, after choosing a logic for formalization, working back and forth on the formalization of its axioms and theorems by making gradual adjustments while getting automatic real-time feedback about the suitability of our changes, vis-a-vis the argument's validity. In this fashion, by engaging in an iterative process of trial and error, we work our way towards a proper understanding of the concepts involved in the argument, far beyond of what a mere natural-language based discussion would allow.

> **Remark 4:** The iterative process of back and forth formalization of axioms for adjustments based on trial and error is not an exact approach because it suggests an *a priori* goal, such as consistently to refute proofs of the existence of God using the HOL/Isabelle tool.

Of 20 equations evaluated, 11 are *not* tautologous. This effectively refutes Lowe's proof, as rendered by the authors. This also invalidates the authors' rejection of Lowe's proof due to incompleteness (six of Lowe's conclusions are dismissed to avoid evaluation) and due to an interactive, trial by error approach to reconstruct Lowe. Therefore, the HOL/Isabelle tool failed as a showcase.

Solution of Horty's puzzles in stit logic

Abstract: In see-to-it-that logic (stit logic), three deontic examples are presented of Horty's coin betting puzzle with two agents. The form of the examples is tautologous. However, a profitability analysis by contrasting outcome for the agents shows none is tautologous. The example for the agent initiating the state of the coin as more profitable than the other agent is more closely aligned to tautology and hence the more profitable strategic outcome. What follows is that stit logic is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables.
(See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1;
(%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Abarca, A.I.R.; Jan Broersen, J. (2019). A logic of objective and subjective oughts.
arxiv.org/pdf/1903.10577.pdf a.i.ramirezabarca@uu.nl J.M.broersen@uu.nl

2.1 Horty's Puzzles

The 3 puzzles ... that pose a problem for formalizing epistemic oughts just with the epistemic extension of act utilitarian logic, can be summarized as follows.

Example 1. Agent β places a coin on top of a table –either heads up or tails up– but hides it from agent α. Agent α can bet that the coin is heads up, that it is tails up, or it can refrain from betting. If α bets and chooses correctly, it wins €10. If it chooses incorrectly, it does not win anything, and if it refrains from betting, it wins €5. (2.1.1.1)

LET p, q, r, s: α , β, heads-up, prize.

(q>(r+~r)) >
(((((~r&(p>~r))+(r&(p> r)))>(s=(s=s)))+
(~((~r&(p>~r))+(r&(p> r)))>~(s=(s=s))))+(~(p>r)>(s=(s\(%s<#s))))) ;
`TTTT TTTT TTTT TTTT` (2.1.1.2)

Example 2. With the same scheme as in the previous example, if α bets and chooses correctly, it wins

€10. If it refrains from betting, it *also* wins €10. If it bets incorrectly, it does not win anything. (2.1.2.1)

(q>(r+~r)) >
((((((~r&(p>~r))+(r&(p>r)))>(s=(s=s)))+
(~((~r&(p>~r))+(r&(p>r)))>~(s=(s=s))))+(~(p>r)>(s=(s\(%s<#s))))) &
(((((~r&(q>~r))+(r&(q>r)))>(s=(s=s)))+
(~((~r&(q>~r))+(r&(q>r)))>~(s=(s=s))))+(~(q>r)> (s=(s=s))))) ;
TTTT TTTT TTTT TTTT (2.1.2.2)

Example 3. With the same scheme as in the previous examples, if α bets and chooses correctly, it wins €10. If it bets incorrectly or refrains from betting, it does not win anything. (2.1.3.1)

(q>(r+~r)) >
((((((~r&(p>~r))+(r&(p>r)))>(s=(s=s)))+
(~((~r&(p>~r))+(r&(p>r)))>~(s=(s=s))))+(~(p>r)>(s=(s\(%s<#s))))) &
(((((~r&(q>~r))+(r&(q>r)))>(s=(s=s)))+
(~((~r&(q>~r))+(r&(q>r)))>~(s=(s=s))))+(~(q>r)>~(s=(s=s))))) ;
TTTT TTTT TTTT TTTT (2.1.3.2)

Remark 2.1: Eqs. 2.1.1.2-2.1.3.2 as rendered are tautologous. This is because the respective main antecedent and consequent are tautologous.

Profitability is evaluated where each example implies the three cases for agent α has more, less, or the equivalent of agent β. Because the examples are theorems, the respective results are identical. We present the truth tables for Example 3.

Agent α has the equivalent of agent β: (3.1.1)

((q>(r+~r)) >
((((((~r&(p>~r))+(r&(p>r)))>(s=(s=s)))+
(~((~r&(p>~r))+(r&(p>r)))>~(s=(s=s))))+(~(p>r)>(s=(s\(%s<#s))))) &
(((((~r&(q>~r))+(r&(q>r)))>(s=(s=s)))+
(~((~r&(q>~r))+(r&(q>r)))>~(s=(s=s))))+(~(q>r)>~(s=(s=s))))))>
((p&s)=(q&s)) ; TTTT TTTT T**FF**T T**FF**T (3.1.2)

Agent α has less than agent β: (3.2.1)

((q>(r+~r)) >
((((((~r&(p>~r))+(r&(p>r)))>(s=(s=s)))+
(~((~r&(p>~r))+(r&(p>r)))>~(s=(s=s))))+(~(p>r)>(s=(s\(%s<#s))))) &
(((((~r&(q>~r))+(r&(q>r)))>(s=(s=s)))+
(~((~r&(q>~r))+(r&(q>r)))>~(s=(s=s))))+(~(q>r)>~(s=(s=s))))))>
((p&s)<(q&s)) ; **FFFF FFFF F**T**FF F**T**FF** (3.2.2)

Agent α has more than agent β: (3.3.1)

((q>(r+~r)) >
((((((~r&(p>~r))+(r&(p>r)))>(s=(s=s)))+

(~((~r&(p>~~r))+(r&(p>r)))>~(s=(s=s))))+(~(p>r)>(s=(s\(%s<#s))))) &
(((((~r&(q>~~r))+(r&(q>r)))>(s=(s=s)))+
(~((~r&(q>~~r))+(r&(q>r)))>~(s=(s=s))))+(~(q>r)>~(s=(s=s))))))>
((p&s)>(q&s)) ; TTTT TTTT T**F**TT T**F**TT (3.3.2)

Remark 3: Eqs. 3.1.2-3.3.2 are *not* tautologous. However, Eq. 3.3.2 is *closest* to a tautologous state with the fewest **F** values present in the resulting truth table. Hence, example 3 is the superior choice for a winning strategy.

Refutation of the Hrushovski construction, to confirm Lachlan and Zil'ber

Abstract: A condition for the Hrushovski construction is *not* tautologous, refuting it. This also denies alleged refutations using it, namely, to confirm the Lachlan conjecture and Zil'ber conjecture. The construction forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Hrushovski_construction

The construction
Let *L* be a finite relational language. Fix **C** a class of *finite L*-structures which are closed under isomorphisms and substructures. We want to strengthen the notion of substructure; let ≤ be a relation on pairs from **C** satisfying:

A⊆B⊆C and A≤C implies A≤B (1.1)

LET p, q, r: A, B, C.

~(r<~(q<p))&(~(r<p)>~(q<p)) ; TT**F**T TT**F**T TT**F**T TT**F**T (1.2)

~(C<~(B<A))&(~(C<A)>~(B<A)) ; TTTT NTNT CCTT **F**CNT (1.3)

Eqs. 1.2 and 1.3 as rendered are *not* tautologous, refuting the Hrushovski construction. This denies refutations using it, namely, to confirm the Lachlan conjecture and Zil'ber conjecture.

Refutation of Huemer's confirmation theory

Abstract: Huemer proposed a confirmation theory to solve the problem of induction. In lieu of the excess of content from Popper and Miller, the proposed replacement is also *not* tautologous, so we correct it for the intended use. That applied to the subsequent proposal in three parts shows one part is *not* tautologous, hence denying the proposal. Therefore Huemer's confirmation theory is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables.
(See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Huemer, M. (1993). Confirmation theory: a metaphysical approach.
owl232.net/papers/confirm.htm#N_18_ bmsjrqcrna@snkmail.com

I. Problem: The purpose of confirmation theory, ultimately, is to solve the problem of induction.
D. The Bayesian Approach, Objections
For Bayesianism to solve the problem of induction, it would have to show that for typical inductive arguments, the evidence confirms the excess content of the hypothesis above the observations. This notion of "excess content" is worth looking into. Karl Popper and David Miller claim that for any h and e, the excess content of h above e is equal to (h v ¬e), for reasons which are unnecessary to examine since they're wrong. Intuitively, the excess content of (A & B) above A should be B, not ((A & B) v ¬B). (D.1.1),(D.2.1)

LET p, q: A, B.
((p&q)\p)=q ; **FF**T**F** **FF**T**F** **FF**T**F** **FF**T**F** (D.1.2)
((p&q)+~q)=q ; **FFF**T **FFF**T **FFF**T **FFF**T (D.2.2)

Remark D: Eqs. D.1.2 and D.2.2 as rendered are *not* tautologous. The intention of excess content in D.1.1 is:

((((p&q)\p)-q)+q)=q ; or alternatively
TTTT TTTT TTTT TTTT (D.1.3)
(((p&q)\p)-q)=(p@p) ; TTTT TTTT TTTT TTTT (D.1.4)

The corrected form of Eqs. D.1.3 or D.1.4 is used below.

My proposal is this:

(a) If h can be written as a conjunction (e & x), where e and x are propositions *about different things* (separate and distinct classes), then the excess content of h above e is x; (3.1.1)

LET p, q, r: e, h, x.
((q=(p&r))&(p@r))>((((((p&q)\p)-q)+q)=q)=r) ;
T**F**TT TTTT T**F**TT TTTT (3.1.2)

(b) If e entails h [h implies e], then the excess content of h above e is nothing (or a tautology); (3.2.1)

(q>p)>((((((p&q)\p)-q)+q)=q)=((p@p)+(p=p))) ;
TTTT TTTT TTTT TTTT (3.2.2)

(c) Otherwise, the excess content of h above e is h. (3.3.1)

Remark 3.3.1: The otherwise is taken to mean Not(h implies e).

~(q>p)>((((((p&q)\p)-q)+q)=q)=q) ; TTTT TTTT TTTT TTTT (3.3.2)

Eq. 3.1.2 as rendered is *not* tautologous. This refutes the proposal of (a), (b), and (c) as a confirmation theory.

Huhn 2-distributive lattice identity

From: Rota, G-C. "The Many Lives of Lattice Theory".
Notices of the AMS. 44:11. 1440-1445. December, 1997.

p. 1441, 2-distributive lattice identity by Huhn:

(p+((q&r)&s))=(((p&(q+r))+(p&(q+r)))+(p+(r&s))) ; not tautologous

```
Model 1              Model 2.1            Model 2.2            Model 2.3.1          Model 2.3.2
TTTT TTTT TTTT FTTT  EEEE EEEE EEEE UEEE  EEEE EEEE EEEE UEEE  EEEE EEEE EEEE UEEE  EEEE EEEE EEEE UEEE
. . . . . . . . .^   . . . . . . . . .^
```

Refutation of predicate transformer semantics for hybrid systems

Abstract: We evaluate the steps of the approach to verify hybrid systems in the style of dynamic logic. Top tier input assumes modal Kleene algebras which are not bivalent. Middle tier processing invokes binary relations (which we do not test). Bottom tier output produces Lipschitz continuous vector fields as verification of hybrid store dynamics, which are not bivalent. This refutes the Isabelle framework for hybrid systems verification, and forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Huerta y Munive, J.; Struth, G. (2019). Predicate transformer semantics for hybrids systems: verification components for Isabelle/HOL. arxiv.org/pdf/1909.05618.pdf g.struth@sheffield.ac.uk

Abstract We present a semantic framework for the deductive verification of hybrid systems with Isabelle/HOL. It supports reasoning about the temporal evolutions of hybrid programs in the style of differential dynamic logic modelled by flows or invariant sets for vector fields. We introduce the semantic foundations of our approach and summarise their Isabelle formalisation as well as the resulting verification components. A series of examples shows our approach at work. Keywords: hybrid systems, predicate transformers, modal Kleene algebra, hybrid program verification, interactive theorem proving.

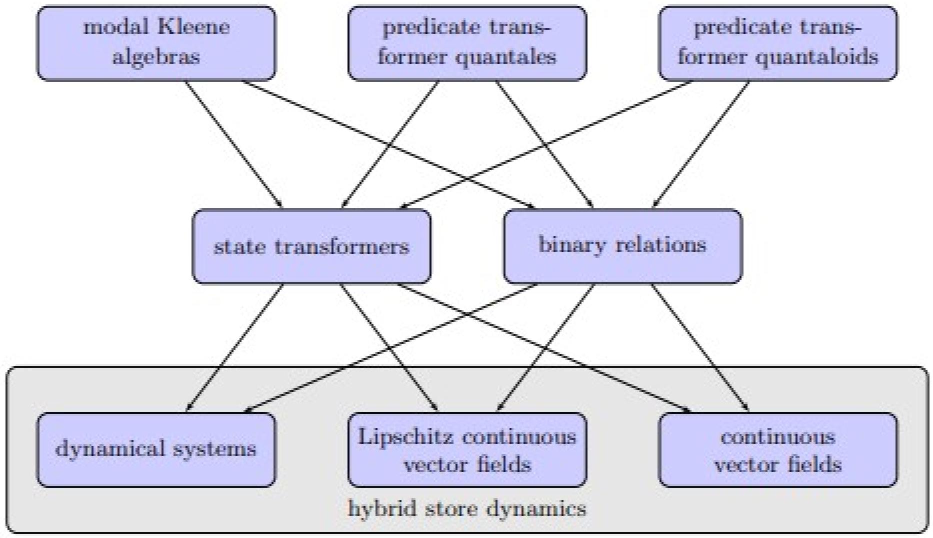


Figure 1: Isabelle framework for hybrid systems verification

Our main contribution is the first semantic framework for the deductive verification of hybrid systems in a general purpose proof assistant. … The entire approach, and the entire mathematical development in this article has been formalised with Isabelle.

Remark Fig. 1: We show elsewhere* that Kleene algebra is not bivalent. Therefore the starting point of the instant approach as "modal Kleene algebras" is not exact bivalency but an inexact vector space.

It proceeds to "binary relation" and proceeds onto "hybrid store dynamics" by way of "Lipschitz continuous vector fields".

We show elsewhere** a refutation of a class of Lipschitz horizontal vector fields in homogeneous groups. This denies the "hybrid store dynamics" by way of "Lipschitz continuous vector fields". The result is to refute the entire conjecture of predicate transformer semantics for hybrids systems. The approach inputs modal Kleene algebras, passes through claimed binary relations, then produces a vector space which is probabilistic and inexact.

In other words, the Isabelle framework claims to verify hybrid systems, but fails to verify the output in terms of the exact bivalency of classical logic.

[* See article listings and keywords for Kleene in James, C. (2016-2019). Recent advances in the modal model checker Meth8 and VŁ4 universal logic. The current abstract is located at ersatz-systems.com and separate articles at vixra.org.

** James, C. (2019). Refutation of a class of Lipschitz horizontal vector fields in homogeneous groups. (To appear momentarily at vixra.org). This title refutes: Magnani, V.; Trevisan, D. (2016). On Lipschitz vector fields and the Cauchy problem in homogeneous groups. arxiv.org/pdf/1606.05486.pdf.]

Confirmation of hydraulic forgiveness

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: God, forgiveness, another person, oneself;
& And; > Imply; = Equivalent;
% possibility, for one or some; # necessity, for all or every.

Infinite grace as mercy of forgiveness is a freely given gift proceeding from God. As a result, if one asks God to forgive another as preparation towards one forgiving the another, then when one duly forgives another, one is forgiven oneself. Forgiveness is listed in the *seven* spiritual works of mercy.

We write this as:

If the necessity of forgiveness proceeds from God for the possibility of another person and oneself, then: if oneself, as possibly forgiven, duly forgives another person, then another person is necessarily forgiven, thus implying oneself is necessarily forgiven. (1.1)

(#(p>q)&%(r&s))> ((((s&%q)>r)>(r=#q))>(s=#q)) ;
TTTT TTTT TTTT TTTT (1.2)

Eq. 1.2 is separated into the outer antecedent and consequent, respectively, as follows.

#(p>q)&%(r&s) ;	FFFF FFFF FFFF NFNN	(1.2.1)
(((s&%q)>r)>(r=#q))>(s=#q) ;	TTTT TTCC FFNN TTTT	(1.2.2)
> Imply	TTTT TTTT TTTT TTTT	(1.2)

Remark 1: The quantified expression for oneself "as possibly forgiven" can be excluded with identical value for the literal fragment:

(((s&%q)>r)>(r=#q)) = ((s>r)>(r=#q)) ;
TTTT TTTT TTTT TTTT (1.3)

Remark 2: One may ask why the forgiver cannot directly proceed to declare the forgivee as equivalent to forgiven in italics. (1.4.1)

#(p>q)&%(r&s) ;	FFFF FFFF FFFF NFNN	(1.2.1)
(*(s>(r=#q)*)>(s=#q));	TTCC TTCC FFNN **C**CTT	(1.4.2)
	^	

The marked value would render the result:
TTTT TTTT TTTT **C**TTT (1.4.3)

This means the decisive step is that the forgiver must first volitionally forgive the forgivee, as by the utterance "I forgive you", to render the forgivee as forgiven in italics:

```
#(p>q)&%(r&s) ;                      FFFF FFFF FFFF NFNN        (1.2.1)
 ((s>r)>(r=#q))>(s=#q)) ;            TTTT TTCC FFNN TTTT        (1.4.4)
                                                    ^
__________________                   __________________
The marked value renders the result: TTTT TTTT TTTT TTTT        (1.2)
```

Eq.1.2 is tautologous and a theorem as a recent advance in systematic theology of the Historic Church.

Remark 3: The term *hydraulic forgiveness* names the implied progression of forgiveness because each stage serves as support to pull along the succeeding and subsequent steps.

Corollary of peace to hydraulic forgiveness

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: God, forgiveness, another person, oneself;
~ Not; + Or; - Not Or; & And; > Imply, greater than; < Not Imply, less than;
= Equivalent; @ Not Equivalent;
% possibility, for one or some; # necessity, for all or every.

Hydraulic forgiveness is a theorem:

"If the necessity of forgiveness proceeds from God for the possibility of another person and oneself, then: if oneself, as possibly forgiven, duly forgives another person, then another person is necessarily forgiven, thus implying oneself is necessarily forgiven" (1.1)

(#(p>q)&%(r&s)) > ((((s&%q)>r)>(r=#q))>(s=#q)) ;
TTTT TTTT TTTT TTTT (1.2)

Hydraulic forgiveness implies a corollary we name peace:

Another person, as so forgiven, cannot hurt oneself, as so forgiven. (2.0)

We write this in two expressions, assuming forgiveness of both persons, and beginning with another person:

Neither another person greater than (implying) oneself nor another person less than (not implied by) oneself implies another person is equivalent to oneself. (2.1)

(((s&q)>(r&q))-((s&q)<(r&q)))>((s&q)=(r&q)) ;
TTTT TTTT TTTT TTTT (2.2)

The theorem of hydraulic forgiveness with the corollary of peace is:

Eqs. 1.2 implying 2.2. (3.1)

((#(p>q)&%(r&s))>((((s&%q)>r)>(r=#q))>(s=#q))) >
((((s&q)>(r&q))-((s&q)<(r&q)))>((s&q)=(r&q))) ;
TTTT TTTT TTTT TTTT (3.2)

Eq. 3.2 is tautologous, confirming the theorem of hydraulic forgiveness and corollary of peace.

Refutation of rooted hypersequent calculus for modal logic S5

Abstract: We evaluate two example equations as *not* tautologous, thereby refuting the rooted hypersequent calculus for modal propositional logic S5. The sequent calculus forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Aghaei, M.; Mohammadi, H. (2019). Rooted hypersequent calculus for modal logic S5.
arxiv.org/pdf/1905.09039.pdf aghaei@cc.iut.ac.ir, hamzeh.mohammadi@math.iut.ac.ir

Abstract: We present a rooted hypersequent calculus for modal propositional logic S5. We show that all rules of this calculus are invertible and that the rules of weakening, contraction, and cut are admissible. Soundness and completeness are established as well.

3 Rooted Hypersequent R_{S5}: Our calculus is based on finite multisets, i.e. on sets counting multiplicities of elements. We use certain categories of letters, possibly with subscripts or primed, as metavariables for certain syntactical categories (locally different conventions may be introduced) ...

Example 3.3. The following sequents are derivable in RS5. 1. ⇒(r∧p)→(q→(◊(p∧q)∧◊r)) (3.3.1.1)

(r&p)>(q>#(%(p&q)&%r)) ; TTTT TTTN TTTT TTTN (3.3.1.2)

5 Structural properties: In this section, we prove the admissibility of weakening and contraction rules, and also some properties of R_{S5}, which are used to prove the admissibility of cut rule.

5.2 Invertibility: In this subsection, first we introduce a normal form called Quasi Normal Form, which is used to prove the admissibility of the contraction and cut rules. Then we show that the structural and modal rules are invertible.

Example 5.9. ... (¬□(A→B)∨p∨◊C) ∧ (¬q) ∧ (◊A∨¬◊(A∧B)∨¬r) is in CQNF (5.9.1)

((~(#(x>y)&(p&%z))=(p=p))&~q)&((#%x+~(%(x&y)=(p=p)))+~r) ;
TT**FF** TT**FF** TT**FF** TT**FF**(64), TC**FF** TC**FF** TC**FF** TC**FF**(16),
TT**FF** TT**FF** TT**FF** TT**FF**(16), TC**FF** TC**FF** TC**F**F TC**FF**(32) (5.9.2)

Eqs. 3.3.1.2 and 5.9.2 as rendered are *not* tautologous, thereby refuting the rooted hypersequent calculus for modal propositional logic S5.

Refutation of ideals

Abstract: We evaluate the definition of ideals. Two of three parts are *not* tautologous, refuting ideals.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And; > Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⊢, ⊨, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) C non-contingency, ∇, ordinal 2; (%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Uzcategui, C. (2019). Ideals on countable sets: a survey with questions. arxiv.org/pdf/1902.08677.pdf cuzcatea@saber.uis.edu.co

An ideal I on a set X is a collection of subsets of X such that: (2.0.0)

Remark 2.0.0: The assertion is that each "such that" below implies Eq. 2.0.0 above as:

X∈I. (2.0.1)

LET p, q, r, s: A, B, I, X,

(s<r) ; **FFFF FFFF** TTTT **FFFF** (2.0.2)

(i) Ø ∈I and X∉ I. (2.1.1)

(((p@p)<r)&~(s<r))>(s<r) ; TTTT TTTT TTTT TTTT (2.1.2)

(ii) If A,B∈I, then A∪B∈I. (2.3.1)

(((p&q)<r)>((p+q)<r))>(s<r) ; **FFFF FFFF** TTTT **FFFF** (2.2.2)

(iii) If A⊆B and B∈I, then A∈I. (2.3.1)

((~(q<p)&(q<r))>(p<r))>(s<r) ; **FFFF FFFF** TTTT **FFFF** (2.3.2)

While Eq. 2.1.2 is tautologous, the other parts in Eq. 2.2.2 and 2.3.2 are *not* tautologous. This refutes the definition of ideals and subsequent assertions.

Ignorance of first choice

In the Morales system for ignorance of first choice, the basketball version assigns valenced variables to make the argument clearer to student readers.

LET x = action; ~x = no action;
LET y = potential; ~y = no potential
where + is Or, > is Imply, = is Equivalent, @ is Not Equivalent (mutually exclusive), & is And

0. The question named ignorance of first cause is "which of the two mutually exclusive selection variables (x,y) or (-x,y) caused the effect of [(x,y)]".

0.1 (x&y)@(~x&y) ; the selection variable pairs are mutually exclusive; not validated as tautologous

This means that the selection variables are not mutually exclusive as stated.

0.2 ((x&y)+(~x&y)) > (x&y) ; "If one or the other selection variables, then the effect as the first selection variables."; not tautologous

This means the term "ignorance of first cause" is mis-applied.

Here is the mapping of the other argument parts as pictured in the system.

1.1 (x&y) ; selection dichotomy of act and no potential, which flavors
1.2 (x>y) ; cause, to produce
1.3 (x>y) ; effect, for
1.4 ((x&y)&(x>y))>(x>y) ; argument for act and no potential; not tautologous
- -
2.1 (~x&y) ; selection dichotomy of no act and potential, which flavors
2.2 (~x>y) ; cause, to produce
2.3 (~x>~y) ; effect, for
2.4 ((~x&y)&(~x>y))>(~x>~y) ; argument for no act and potential; not tautologous

Ignorance of first choice should be defined as both 1.4 and 2.4 implying the consequent in 0.2 as 1.1:

3. ((((x&y)&(x>y))>(x>y)) & (((~x&y)&(~x>y))>(~x>~y))) > (x&y) ; not tautologous

This means the arguments do not prove: "mechanics of the two acts of selection", anything about Albert Einstein, or "a flawed scientific method".

What follows is that the above arguments cannot be re-asserted or used again to bar or invalidate acts of selection because their probability according to Rudolf Carnap is not 1.

Imaginary numbers are not tautologous

We use the apparatus and method of the Meth8/VŁ4 modal logic model checker where the designated *proof* value is T and 16-valued result table is row-major and horizontal.

Definition of the imaginary number: i^2 = -1 as i=±(-1)^0.5 (1.0)

LET q imaginary number root; (%q>#q) 1, Non-contingency; ~(%q>#q) ~1, Contingency.

((q and q) or (~q and ~q)) equals ~1. (1.1)

((q& q)+(~q&~q)) = ~(%q>#q) ; CCCC CCCC CCCC CCCC (1.2)

Eq. 1.2 as rendered is *no*t tautologous. This means Eq. 1.0 is refuted.

Eq. 1.2 means the definition of the imaginary number in Eq. 1.0 is contingent, the value for falsity.

We attempt to strengthen Eq. 1.1 by replacing the Or connective with And.

((q and q) and (~q and ~q)) equals ~1. (2.1)

((q& q)&(~q&~q)) = ~(%q>#q) ; NNNN NNNN NNNN NNNN (2.2)

Eq. 2.2 as rendered is *not* tautologous. This means Eq. 1.0 is further refuted. Eq. 2.2 means the definition of the imaginary number in Eq. 1.0 can be coerced to be non-contingent, the value for truthity, but still *not* tautologous.

Imperative logic: potential mistakes in footnote 47

Vranas, P.B.M. (2011). New foundations for imperative logic II: pure imperative inference. cdn.getforge.com/petervranas.getforge.io/1484861684/papers/implogicII.pdf

We replicate results from equations in footnote 47 using our resuscitation of Ł4, named variant VŁ4, as implemented in our Meth8 modal logic model checker.

We assume the apparatus of the Meth8 modal logic model checker implementing variant system VŁ4. Meth8 allows to mix four logical values with four analytical values. The designated *proo*f value is T.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	p=p	T	Tautology	proof	11	3
2	p@p	F	Contradiction	absurdum	00	0
3	%p>#p	N	Non-contingency	truth	01	1
4	%p<#p	C	Contingency	falsity	10	2

LET: ~ Not; + Or; & And; > Imply, is; (p=p) true; (p@p) false
p q r F M R (We rewrite upper case theorems into lower case propositions.)

Results are the repeating proof table(s) of 16-values in row major horizontally.

"[47] The fact that the conjunction of 'if the volcano erupts, flee' with 'smile or do not smile' is 'let it not be the case that the volcano erupts and you do not flee' follows from Definition 6 but can also be seen intuitively as follows (letting R, P, and Q be respectively the propositions that the volcano erupts, that you flee, and that you smile):

LET:	(r>(r=r))	"R is true";
	(p>(p=p))	"P be true";
	((q+~q)>(q=q))	"(Q+~Q) is true;
	((r>(r@r))	"R is false";
	((p&(q+~q))=(q=q))	"P&(Q+~Q) be true";
	((r&~p)>(r@r))	"R&~P be true"

'if R is true, let P be true' & 'let Q+~Q be true' = (1.1)

(((r>(r=r))>(p>(p=p))) & ((q+~q)>(q=q))) ; `TTTT TTTT TTTT TTTT` (1.2)

'if R is true, let P be true' & ('if R is true, let Q+~Q be true' & 'if R is false, let Q+~Q be true') = (2.1)

((r>(r=r))>(p>(p=p)))&(((r>(r=r))>((q+~q)>(q=q)))&((r>(r@r))>((q+~q)>(q=q)))) ;
`TTTT TTTT TTTT TTTT` (2.2)

('if R is true, let P be true' & 'if R is true, let Q+~Q be true') & 'if R is false, let Q+~Q be true' = (3.1)

((((r>(r=r))>(p>(p=p)))&((r>(r=r))>((q+~q)>(q=q))))& ((r>(r@r))>((q+~q)>(q=q)))) ;
TTTT TTTT TTTT TTTT (3.2)

'if R is true, let P&(Q+~Q) be true' & 'if R is false, let Q+~Q be true' = (4.1)

(((r>(r=r))> ((p&(q+~q))=(q=q)))&((r>(r@r))>((q+~q)>(q=q)))) ;
FTFT FTFT FTFT FTFT (4.2)

'if R is true, let P be true' & 'if R is false, let Q+~Q be true' = (5.1)

(((r>(r=r))>(p>(p=p)))&((r>(r@r))>((q+~q)>(q=q)))) ;
TTTT TTTT TTTT TTTT (5.2)

'if R is true, let R&~P be false' & 'if R is false, let R&~P be false' = (6.1)

((r>(r=r))>((r&~p)>(p@p)))& [TTTT FTFT TTTT FTFT]
((r>(r@r))>((r&~p)>(p@p))) [TTTT TTTT TTTT TTTT]
TTTT FTFT TTTT FTFT (6.2)

'let R&~P be false'. (7.1)

(r&~p)>(p@p) ; TTTT FTFT TTTT FTFT (7.2)

(The prescriptions expressed by 'if R is false, let Q+~Q be true' (8.1)

((r>(r@r))>((q+~q)>(q=q)) ; TTTT TTTT TTTT TTTT (8.2)

and by 'if R is false, let R&~P be false' (9.1)

((r>(r@r))>((r&~p)>(p@p))) ; TTTT TTTT TTTT TTTT (9.2)

are the same because their violation propositions, namely ~R&~(Q+~Q) (10.1)

(~r&~(q+~q)) ; FFFF FFFF FFFF FFFF (10.2)

and ~R&(R&~P) respectively, (11.1)

(~r&(r&~p)) ; FFFF FFFF FFFF FFFF (11.2)

are both impossible, and their contexts are the same, namely ~R.) (12.1)

~r ; TTTT FFFF TTTT FFFF (12.2)

Eqs. 4.2, 6.2, and 7.2 as rendered are *not* tautologous as claimed.

We conclude that imperative logic is a probabilistic vector space, not bivalent, and hence suspicious.

Implication combinations derived from (p>q)>r

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET: + Or; & And; > Imply; = Equivalent.

The commencing antecedent is:

(p>q)>r ; **F**T**FF** TTTT **F**T**FF** TTTT (n.1)

The subsequent consequents as pairs producing tautology are:

((p>q)>r)>(((p>q)>(p>r))>(q>r)) ; TTTT TTTT TTTT TTTT (11)

((p>q)>r)>(((p>q)+(p>r))+(q>r)) ; TTTT TTTT TTTT TTTT (13)

((p>q)>r)>(((p+q)+(p+r))+(q+r)) ; TTTT TTTT TTTT TTTT (33)

However, the consequents of Eqs. 11, 13, and 33 are not equivalents:

((p>q)>(p>r))>(q>r) ; TT**F**T TTTT TT**F**T TTTT (11.2)

((p>q)+(p>r))+(q>r) ; TTTT TTTT TTTT TTTT (13.2)

((p+q)+(p+r))+(q+r) ; **F**TTT TTTT **F**TTT TTTT (33.2)

We ask, what other equations are derived from Eq. n.1 as antecedent with consequent pair types.

(p&q)>(p>q) ; TTTT TTTT TTTT TTTT (4n.2)

((p>q)>r)>((((p&q)>(p>q))>((p&r)>(p>r)))>((q&r)>(q>r))) ;
TTTT TTTT TTTT TTTT (41)

((p>q)>r)>((((p&q)>(p>q))&((p&r)>(p>r)))&((q&r)>(q>r))) ;
TTTT TTTT TTTT TTTT (42)

((p>q)>r)>((((p&q)>(p>q))+((p&r)>(p>r)))+((q&r)>(q>r))) ;
TTTT TTTT TTTT TTTT (43)

(p>q)+(p+q) ; TTTT TTTT TTTT TTTT (5n.2)

((p>q)>r)>((((p>q)+(p+q))>((p>r)+(p+r)))>((q>r)+(q+r))) ;
TTTT TTTT TTTT TTTT (51)

((p>q)>r)>((((p>q)+(p+q))&((p>r)+(p+r)))&((q>r)+(q+r))) ;
TTTT TTTT TTTT TTTT (52)

((p>q)>r)>((((p>q)+(p+q))+((p>r)+(p+r)))+((q>r)+(q+r))) ;
TTTT TTTT TTTT TTTT (53)

(p&q)>(p+q) ; TTTT TTTT TTTT TTTT (6n.2)

((p>q)>r)>((((p&q)>(p+q))>((p&r)>(p+r)))>((q&r)>(q+r))) ;
TTTT TTTT TTTT TTTT (61)

((p>q)>r)>((((p&q)>(p+q))&((p&r)>(p+r)))&((q&r)>(q+r))) ;
TTTT TTTT TTTT TTTT (62)

((p>q)>r)>((((p&q)>(p+q))+((p&r)>(p+r)))+((q&r)>(q+r))) ;
TTTT TTTT TTTT TTTT (63)

Eqs. with whole numbers are named general forms of (p>q)>r by implication on Meth8/VŁ4.

Refutation of translation of implicit logic (IL) to explicit logic (EL)

Abstract: For translation of implicit logic to explicit logic, using intuitionistic and epistemic logic, seven equations are evaluated, with none tautologous. Two refute the recursive translation of "the intuitionistic truth definition into a syntactic recipe"; three refute "key features of intuitionistic logic in modal terms"; and one refutes "the recursion law after knowledge update [as] the basic dynamic equation of hard information" for public announcement logic (PAL). These form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: van Benthem, J. (2019). Implicit and explicit stances in logic.
Journal of Philosophical Logic. 48:571–601
link.springer.com/content/pdf/10.1007%2Fs10992-018-9485-y.pdf johan@stanford.edu

N.B.: The author uses EL and IL for *either* epistemic and intuitionistic logic *or* explicit and implicit logic by context.

5 Choice or co-existence: translations and merges

But first it may seem time for a choice. Is intuitionistic logic or epistemic logic better or deeper as an analysis of information and knowledge? Should we prefer one over the other? Many philosophers think in this style, but we feel that this adversarial attitude is not very productive, and it also runs counter to known mathematical facts about system connections …

Already in Gödel's seminal .. , there is a faithful translation from intuitionistic logic into the modal logic S4 whose underlying intuition follows the present knowledge perspective. We now look at this connection to see what it achieves.

Translating IL Into EL The *Gödel translation t* turns the intuitionistic truth definition into a syntactic recipe, according to the following recursive clauses: …

t(¬ϕ) = □¬t(ϕ) (5.4.1)

LET p, q, r, s: ϕ,q, *t*, ψ.

(r&~p)=#(~r&p) ; `TCTC` **`F`**`T`**`F`**`T` `TCTC` **`F`**`T`**`F`**`T` (5.4.2)

$t(\phi \rightarrow \psi) = \Box(t(\phi) \rightarrow t(\psi))$ (5.5.1)

(r&(p>s))=#((r&p)>(r&s)) ; CCCC NTNT CCCC NNNN (5.5.2)

Remark 5.4/.5: Eqs. 5.4.2 and 5.5.2 as rendered are *not* tautologous, refuting the recursive translation of "the intuitionistic truth definition into a syntactic recipe".

… This explains key features of intuitionistic logic in modal terms. For instance, varieties of implication place different demands on knowledge:

intuitionistic $\phi \rightarrow \psi$ is $\Box(\phi \rightarrow \psi)$, (5.6.1)

(p>s)=#(p>s) ; NTNN NTNN NTNN NTNN (5.6.2)

the earlier $\neg\phi \vee \psi$ is the stronger $\Box\neg\phi \vee \psi$, (5.7.1)

(~p+s)=(#~p+s) ; NTNT NTNT TTTT TTTT (5.7.2)

and $\neg(\phi \wedge \neg\psi)$ the weaker $\Box(\phi \rightarrow \psi)$. (5.8.1)

~(p&~s)=#(p>%s) ; NTNT NTNT NNNN NNNN (5.8.2)

Remark 5.6/.8: Eqs. 5.6.2-5.8.2 are *not* tautologous, refuting "key features of intuitionistic logic in modal terms".

6 Dynamic Logic of Information Change

Public Announcement Logic

Public announcements are studied in PAL, a system that extends epistemic logic with a dynamic modality for truthful announcements … . This dynamic modality has a complete logic that can analyze delicate phenomena, such as complex epistemic assertions, say of current ignorance, changing truth value under update. This typically shows in order dependence: a sequence !Kp ; !p makes sense, but !p ; !¬Kp is contradictory. Here we only display the 'recursion law' for knowledge after update, which is the basic dynamic equation of hard information:

$[!\phi]K\psi \leftrightarrow (\phi \rightarrow K(\phi \rightarrow [!\phi]\psi))$ (6.1.1)

LET p, q, r, s: ϕ, $!\phi$, K, ψ.

(q&(r&s))=(p>(r&(p>(q&s)))) ; **F**T**F**T **F**T**F**T **F**T**F**T **F**TTT (6.1.2)

Remark 6.1.2: Eq. 6.1.2 is *not* tautologous, hence refuting "the recursion law after knowledge update [as] the basic dynamic equation of hard information" for public announcement logic.

Confirmation that impossible worlds mean nothing is necessary there but everything is possible.

Abstract: We confirm the definition that impossible worlds mean "nothing is necessary there but everything is possible". The truth axiom as given is *not* tautologous, but rather the logical value of truthity. The rules of extensionality and monotonicity as given are *not* tautologous. When the definition of impossible worlds is combined with monotonicity + T + 4 that conjecture is also tautologous. Therefore the failed equations are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Witczak, T. (2019). Generalized topological semantics for weak modal logics. arxiv.org/pdf/1904.06099.pdf tm.witczak@gmail.com

We shall work with the following list of axioms and rules: ...

$T : \Box\varphi \rightarrow \varphi$ (T.1)

LET p, q: φ, ψ.

#p>p ; TTTT TTTT TTTT TTTT (T.2)

$4 : \Box\varphi \rightarrow \Box\Box\varphi$ (4.1)

#p>##p ; TTTT TTTT TTTT TTTT (4.2)

$N : \Box\top$ (*truth axiom*) (N.1)

Remark N.1: We evaluate this expression as both the designated proof value and the designated truthity value:

#(p=p) = (p=p) ; NNNN NNNN NNNN NNNN (N.2)

#(%p>#p) = (p=p) ; NNNN NNNN NNNN NNNN (N.3)

Eqs. N.2 and N.3 are equivalent as the logical value for truthity, but are *not* tautologous.

RE : $\varphi \leftrightarrow \psi \vdash \Box\varphi \leftrightarrow \Box\psi$ (rule of extensionality) (RE.1)

(#p=#q)>(p=q) ; TNNT TNNT TNNT TNNT (RE.2)

RM : $\varphi \rightarrow \psi \vdash \Box\varphi \rightarrow \Box\psi$ (rule of monotonicity) (RM.1)

(#p>#q)>(p>q) ; TNTT TNTT TNTT TNTT (RM.2)

"[I]mpossible worlds … means that nothing is necessary there but everything is possible". (1.1)

(~p=#p)\(#p=%p) ; TTTT TTTT TTTT TTTT (1.2)

Remark 1.2: The definition of impossible worlds as "nothing is necessary there but everything is possible" is a theorem.

"As for the monotonic system T 4, probably it [was] not already investigated in the context of impossible worlds." (2.1)

((~p=#p)\(#p=%p))+(((#p>#q)>(p>q))+((#p>p)+(#p>##p))) ;
TTTT TTTT TTTT TTTT (2.2)

Axioms N, RE, and RM are *not* tautologous. However, Eq. 1.1 + monotonic + T + 4 is a theorem.

Refutation of inclusion logic

Abstract: We evaluate a seminal equation from a proof sketch, which is *not* tautologous. By extension, this means dependence logic, inclusion logic, and independence logic are also *no*t tautologous. Therefore dependence logic, inclusion logic, and independence logic are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Yang, F. (2019). Axiomatizing first-order consequences in inclusion logic. arxiv.org/pdf/1904.06227.pdf fan.yang.c@gmail.com

[Edited] In this paper, we axiomatize first-order consequences of inclusion logic. *Inclusion logic* is a variant of *dependence logic*. Another important variant of dependence logic is *independence logic*. Dependence logic and its variants adopt the framework of *team semantics* to characterize dependency notions. Inclusion logic aims to characterize inclusion dependencies by extending first-order logic with *inclusion atoms*, as strings of sequences of variables of the same length. With team semantics inclusion atoms and other formulas are evaluated in a model with respect to *sets* of assignments (called *teams*), in contrast to single assignments as in the usual first-order logic. [I]nclusion logic is expressively equivalent to positive greatest fixed-point logic.

3 Normal form, Theorem 3.1

$\exists x\varphi \vee \psi \equiv \exists x(\varphi \vee \psi)$; (3.1.4.1)

LET p, q, r: φ, ψ, *x*.
((%r&p)+q)=(%r&(p+q)) ; TTCC TTTT TTCC TTTT (3.1.4.2)

Eq. 3.1.4.2 is *not* tautologous. By extension, this means dependence logic, inclusion logic, and independence logic are also *no*t tautologous.

Inconsistent theory

From en.wikipedia.org/wiki/List_of_first-order_theories#

"One special case of this is the **inconsistent theory** defined by the axiom $\exists x \neg x = x$. It is a perfectly good theory with many good properties: it is complete, decidable, finitely axiomatizable, and so on. The only problem is that it has no models at all. By Gödel's completeness theorem, *it is the only theory (for any given language) with no models.*"

(%p&~p) > (%p&p)	;	not	tautologous
%p&(~p=p)	;	not	tautologous ; validated as F contradictory
(%p&#~%p) > p	;		validated as tautologous
(#p&#~p) > p	;		validated as tautologous ; * replace % with # and = with >
(#p&~%p) > p	;		validated as tautologous ; * replace % with # and = with >
(#p&~p) > p	;		validated as tautologous ; * replace % with # and = with >
(#p&~p) = p	;	not	validated as tautologous
(%p&~p) > p	;	not	validated as tautologous
(%p&~p) = p	;	not	validated as tautologous

Inconsistent theory: Extending the monad ∃ *p ~p = p* to a triad

Introduction

Inconsistency theory begins with a unique model for the monad of ∃ *p~p=p*.

The requirement here is to write and test a proof expression extending the monad into a triad (and higher forms), and without knowing if the respective variable *p*, or *q*, *r*, *s*, are Tautologous or contradictory at antecedent input. (0)

Experiment

We use the Meth8 logic model checker (U.S. Patent Pending), based on the logic system VŁ4.

LET: % the Existential quantifier; = Equivalent; > Imply; ~ Not; nvt not tautologous; vt ~nvt.

1. Monad

∃p~p=p maps to: (1)

(%p&~p) = p ; nvt; equivalent monad (1.1)

(%p&~p) > p ; nvt; implied monad (1.2)

2. Dyad

(∃p~p)(∃q~q)=p&q maps to: (2)

((%p&~p)&(%q&~q)) = (p&q) ; nvt; equivalent dyad (2.1)

((%p&~p)&(%q&~q)) > (p&q) ; nvt; implied dyad (2.2)

3. Triad

(∃p~p)(∃q~q)(∃r~r)=p&q&r maps to: (3)

((%p&~p)&((%q&~q)&(%r&~r))) = (p&(q&r)) ;
nvt; equivalent triad (3.1)

((%p&~p)&((%q&~q)&(%r&~r))) > (p&(q&r)) ;
nvt; implied triad (3.2)

We rewrite the antecedent in Eq 3.1 as an equivalent in Eq 3.3 and present repeating rows of truth tables for the five models, where the designated truth values are *T*autologous and *E*valuated:

((%p&(%q&%r))&(~p&(~q&~r))) = (p&(q&r)) ;
not validated; equivalent triad
```
NTTT TTTF; EEEE EEEU;
UEEE EEEU; IEEE EEEU;
PEEE EEEU
```
(3.3)

((%p&(%q&%r))&(~p&(~q&~r))) > (p&(q&r)) ;
not validated; implied triad
NTTT TTTT; EEEE EEEE;
UEEE EEEE; IEEE EEEE;
PEEE EEEE (3.4)

We note how to inject a truth value in the consequent as for example (p+~p) which always evaluates to Tautologous. This is consistent as an attempt to force explicitly the second phrase in the word expression of Eq 0. The phrase then reads in part as "equivalent to p or not p and q or not q and r or not r" and appears in Eq 3.5.

((%p&(%q&%r))&(~p&(~q&~r))) > ((p+~p)&((q+~q)&(r+~r))) ;
vt; implied triad
TTTT TTTT; EEEE EEEE;
EEEE EEEE; EEEE EEEE;
EEEE EEEE (3.5)

While Eq 3.5 is validated as tautologous, the truth insertion is an artifice because the original "equivalent to p and q and r" captures all values as input from the antecedent without *knowing* the truth value, which was the original intent of the second phrase in Eq 0.

A further rendition of Eq 3.5 accommodates the mapping as "x OR y OR z" in Eq 3.6.

((%p&(%q&%r))&(~p&(~q&~r))) > ((p+(q+r)) ;
vt; implied triad
NTTT TTTT; EEEE EEEE;
UEEE EEEE; IEEE EEEE;
PEEE EEEE (3.6)

4. Tetrad

(∃p~p)(∃q~q)(∃r~r)(∃s~s)=p&q&r&s maps to: (4)

(((%p&~p)&((%q&~q)&(%r&~r)))&(%s&~s)) = ((p&(q&r))&s) ;
nvt; equivalent triad (4.1)

(((%p&~p)&((%q&~q)&(%r&~r)))&(%s&~s)) > ((p&(q&r))&s) ;
nvt; implied triad; (4.2)

Conclusion

None of the forms for monad or extensions for dyad, triad, or tetrad are validated as tautologous by the Meth8 modal logic model checker.

Hence the inconsistency theory, as based on Eq 0 et seq, is suspicious.

Inconsistent theory: Kunen's inconsistency theorem

From en.wikipedia.org/wiki/Kunen%27s_inconsistency_theorem

> "[T]here is no formula *J* in the language of set theory such that for some parameter *p*∈*V* for all sets *x*∈*V* and *y*∈*V*: j (x) = y ↔ J (x , y , p) ." (1)

(((p<v)&#((x<v)&(y<v)))&((q&x)=y)) = (((p<v)&#((x<v)&(y<v)))& (r&((x&y)&p))) ;
tautologous (2)

This is a proof by contradiction that (1) is contradictory.

To better see this, consider changing the main connective in (1) from equivalent (=) to Not equivalent (@) as in (3) below:

(((p<v)&#((x<v)&(y<v)))&((q&x)=y)) @ (((p<v)&#((x<v)&(y<v)))& (r&((x&y)&p))) ;
not tautologous, and contradiction (3)

Independence-friendly logic (Kreiselization)

From en.wikipedia.org/wiki/Independence-friendly_logic, we present only Kreiselization due to invalidation by other work:

2. Kr U (ψ ∧ χ) = Kr U (ψ) ∨ Kr U (χ) (1.1)

3. Kr U (ψ ∨ χ) = Kr U (ψ) ∧ Kr U (χ) (2.1)

LET: p Kr U; q ψ ; r χ; nvt not tautologous;

Designated truth value is T Tautology (proof), with C Contingent (falsity value), N Non contingent (truth value), and F for contradiction (absurdum).

Results include the 16-value truth tables as row major horizontally.

(p&(q+r)) = ((p&q)&(p&r)) ; TTTF TFTT TTTF TFTT (1.2)

(p&(q&r)) = ((p&q)+(p&r)) ; TTTF TFTT TTTF TFTT (2.2)

Meth8 finds Kreiselization suspicious due to Eqs 1.2 and 2.2 as *not* tautologous.

From the article on Indicative Conditionals at plato.stanford.edu/entries/conditionals/:

In Section 3.2,

> But, as we saw, “~(*A*&*B*); so *A* ⇒ ~*B*” is invalid. (1)

We think (1) is always valid as a theorem due to the truth table fragments below for ~(p&q)=(p>~q), where the logical equivalence "=" is stronger than the "hook, line, or sinker" (Hook, Arrow, Supp).

	Model 1	Models 2.1; 2.2; 2.3.1; 2.3.1
p:	FTFT FTFT	UEUE UEUE
q:	FFTT FFTT	UUEE UUEE
~q:	TTFF TTFF	EEUU EEUU
~(p&q):	TTTF TTTF	EEEU EEEU
(p>~q):	TTTF TTTF	EEEU EEEU
~(p&q)=(p>~q):	**TTTT TTTT**	**EEEE EEEE**

FCNT is: F contradiction, Contingent (falsity), Non contingent (truth), Tautology. UIPE is: Unevaluated, Improper, Permissible, Evaluated. [Designated proof values are T, E; > is Imply; = is Equivalent to; and fragments here are the first two rows of four rows.]

Our attempts to correspond with the article's author of record were unsuccessful.

Refutation of saturated free algebras (revisited) and almost indiscernible theories

Abstract: We evaluate two papers as probabilistic vector spaces with *no* bivalent basis, *non* existence, and *no* meaning. This refutes saturated free algebras and "almost indiscernible theory", forming a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Pillay, A.; Sklinos, R. (2014, 2018). Saturated free algebras revisited. arxiv.org/pdf/1409.8604.pdf

Abstract We give an exposition of results of Baldwin-Shelah .. on saturated free algebras, at the level of generality of complete first order theories *T* with a saturated model *M* which is in the algebraic closure of an indiscernible set. We then make some new observations when *M* is a saturated free algebra, analogous to (more difficult) results for the free group, such as a description of forking.

From: Kucera, T.G.; Pillay, A. (2019). Almost indiscernible theories and saturated free algebras. arxiv.org/pdf/1908.02712.pdf

Abstract We extend the concept of "almost indiscernible theory" introduced by Pillay and Sklinos in arxiv.org/pdf/1409.8604.pdf (which was itself a modernization and expansion of Baldwin and Shelah ..), to uncountable languages and uncountable parameter sequences. Roughly speaking *T* is almost indiscernible if some saturated model is in the algebraic closure of an indiscernible set of sequences. We show that such a theory *T* is nonmultimensional superstable, and stable in all cardinals ≥ |*T*| . We prove a structure theorem for sufficiently large *a*-models *M*: Theorem 2.10 which states that over a suitable base, *M* is in the *algebraic closure* of an independent set of realizations of weight one types (in possibly infinitely many variables). We also explore further the saturated free algebras of Baldwin and Shelah in both the countable and uncountable context. We study in particular theories and varieties of *R*-modules, pointing out a counterexample to a conjecture from Pillay-Sklinos.

We evaluate the above papers as probabilistic vector spaces with *no* bivalent basis, *non* existence, and *no* meaning. This refutes saturated free algebras and "almost indiscernible theory".

Logical induction is not tautologous via the Black raven paradox and Kripkenstein

Black raven paradox from wiki

Induction was described as the Black raven paradox, from en.wikipedia.org/wiki/Raven_paradox :

"(1) All ravens are black. (1)
In strict logical terms, via contraposition, this statement is equivalent to:
(2) Everything that is not black is not a raven." (2)
and Eq 1 and Eq 2 via contraposition are to be equivalents. (3)

The universal quantifier is in Eq 1 for the antecedent as "All ravens". The existential quantifier is invoked in Eq 1 for the consequent as "*a* black *thing*", in Eq 2 for the antecedent as "every [*each and every*]*thing* not black", and in Eq 2 for the consequent as "not *a raven*". The contraposition statement is also mistaken because the antecedent in Eq 2 does not read "All that is not black."

We assume the Meth8 script; the truth table is four rows major horizontally, with designated truth value as T; nvt not tautologous.

LET: p black; r raven; > is; = equivalent; # for All; % for One

(#r>%p) = (%~p>~%r) ; `NNNN NNNN NNNN NNNN` (3.1)

"It should be clear that in all circumstances where (2) is tautologous, (1) is also tautologous; and likewise, in all circumstances where (2) is contradictory (i.e. if a world is imagined in which something that was not black, yet was a raven, existed), (1) is also contradictory. This establishes logical equivalence." (4)

We write Eq 4 as:

#(%~p>~%r) > #(#r>%p) ; `TTTT TTTT TTTT TTTT` (4.1)

#(~(%~p>~%r)) > #(~(#r>%p)) ; `NNNN NNNN NNNN NNNN` (4.2)

The example of "(2) contradictory" as "if a world is imagined in which something that was not black, yet was a raven, existed" is not equivalent below:

~(%~p>~%r) = (%~p>%r) ; `TCTC TCTC TCTC TCTC` (4.3)

From Eq 3, the Black raven paradox is not tautologous by Meth8.

Black raven paradox from plato

From John M. Vickers plato.stanford.edu/entries/induction-problem/, the Black raven paradox is recast.

The Nicod principle states: "Universal generalizations are supported or confirmed by their positive instances and falsified by their negative instances." This is applied as a paradoxical conclusion for:

"a is not black and not a raven" confirms "all non-black things are non-ravens." (5)

with the paradoxical juxtaposition that
"If all non-black things are non-ravens", then "a [*thing*] is not black and not a raven". (6)

LET: p black, r raven, q a [*thing*]

(#r>p)>((q>~p)&(q>~r)) ; TTTF TTNF TTTF TTNF (5.1)
((q>~p)&(q>~r))> (#r>p) ; TTTT CTTT TTTT CTTT (6.1)

The juxtaposition of Eq 5 into Eq 6 as a paradox is tautologous by Meth8.

C.G. Hempel (1945) with Nelson Goodman look at truth conditions of the premise and supported hypothesis, where:

the antecedent conditions are "this is neither a raven nor black"
and the consequent hypothesis is "all ravens are black" (7)
with the restated hypothesis "Everything is either a black raven or is not a raven" (8)
and also Eq 7 to be the equivalent of Eq 8. (9)

LET: p black, r raven, q this [*thing*]

(q>~(r+p))>(#r>p) ; TTTT CTTT TTTT CTTT (7.1)
#q>((p&r)+~r) ; TTTT TTCT TTTT TTCT (8.1)
((q>~(r+p))>(#r>p)) = (#q>((p&r)+~r)) ;
TTTT CTCT TTTT CTCT (9.1)

The Hempel-Goodman proposed resolution rewords the equivalent of the Nicod principle and therefore is not a resolution tautologous by Meth8.

Kripkenstein

Induction was subsequently recast from en.wikipedia.org/wiki/New_riddle_of_induction :

Regarding the private language argument of Wittgenstein, "Saul Kripke proposed a related argument that leads to skepticism about meaning rather than skepticism about induction, as part of his personal interpretation of the private language argument. ... Kripke then argues for an interpretation of Wittgenstein as holding that the meanings of words are not individually contained mental entities."
This was later nick-named "Kripkenstein" to describe a form of addition (+) named quus where:

x quus y = { (x+y for x,y <57) = (5 for ~(x<57) or ~(y<57)) }. (10)

LET: p,q x,y; r 57; s 5

(((p&q)<r)>(p+q)) = ((~(p<r)+~(q<r))>s) ;
FFFT FFFF TTTT TTTT (10.1)

From Eq 10.1, Kripkenstein is not a "new riddle of induction" and not tautologous by Meth8.

What follows is that the Black swan theory of Nassim Nicholas Taleb is also not tautologous.

Refutation of the induction axiom of the intuitionisic type-theory of Martin-Löf

Abstract: For Martin-Löf type theory of intuitionistic logic, the *induction axiom* of $\forall X(0{\in}X \wedge \forall x(x{\in}X \rightarrow x{+}1{\in}X) \rightarrow \forall x(x{\in}X))$ is contradictory. This forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Rathjen, M. (2005). The constructive Hilbert program and the limits of Martin-Löf type theory. www1.maths.leeds.ac.uk/~rathjen/EHPanthology.pdf

2.1 Subsystems of second order arithmetic

The basic axioms in all theories of second-order arithmetic are the defining axioms of 0*; Suc; +; ·;* < and the *induction axiom* [where x+1 stands for *Suc*(x)]

$$\forall X(0{\in}X \wedge \forall x(x{\in}X \rightarrow x{+}1{\in}X) \rightarrow \forall x(x{\in}X)), \qquad (2.1.2.1)$$

LET p, q: *x*, *X*.

((s@s)<#q)&(((#p<#q)>((#q+(%s>#s))<#q))>(#p<#q));
FFFF FFFF FFFF FFFF (2.2.2.2)

Eq. 2.2.2.2 as rendered is *not* tautologous and also contradictory. This refutes the induction axiom of the Martin-Löf type theory of intuitionistic logic.

Refutation of standard induction, coinduction and mutual induction, coinduction

Abstract: From the summary of standard and mutual induction and coinduction, we evaluated four formulas with *non* tautologous and hence refutations. Therefore these are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊔ ;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Moez A. AbdelGawad, M.A. (2019). Mutual coinduction.
arxiv.org/pdf/1903.06514.pdf moez@cs.rice.edu

LET p, q, r, s, t, u: *P, F, μ, ν, O, G*; ⊑ is equivalent to ≤.

The formulation of standard induction and standard coinduction, and related concepts, that we present here is a summary [presented elsewhere].

• (*standard induction*) if $F(P) \leq P$, then $\mu_F \leq P$, (1.1)

~(p<(q&p))>~(p<(r&q)) ; TTTT TTT**F** TTTT TTT**F** (1.2)

• (*standard coinduction*) if $P \leq F(P)$, then $P \leq \nu_F$, (2.1)

~((q&p)<p)>~((s&q)<p) ; TTTT TTTT TT**F**T TT**F**T (2.2)

[G]iven that *μF* and *μG* are the *least* simultaneous pre-fixed points of *F* and *G* and *νF* and *νG* are the *greatest* simultaneous post-fixed points of *F* and *G*, for any element *O* ∈ O and *P* ∈ P we have:

• (*mutual induction*) if $F(O) \sqsubseteq P$ and $G(P) \leq O$, then $\mu_F \leq O$ and $\mu_G \sqsubseteq P$ (3.1)

(~(p<(q&t))&~(t<(u&p)))>(~(t<(r&q))&~(p<(r&u))) ;
TTTT TTTT TTTT TTTT(3),
TTT**F** TTTT TTT**F** TTTT(1) (3.2)

• (*mutual coinduction*) if $P \sqsubseteq F(O)$ and $O \leq G(P)$, then $O \leq \nu_F$ and $P \sqsubseteq \nu_G$ (4.1)

```
(~((q&t)<p)&~((u&p)<t))>(~((s&q)<t)&~((s&u)<p)) ;
                    TTTT TTTT TTFF TTFF(1),
                    TTTT TTTT TTTT TTTT(1),
                    TTTT TTTT FTFT FTFT(1),
                    TTTT TTTT FTTT FTTT(1)
```
(4.2)

Eqs. 1.2 - 4.2 as rendered are *not* tautologous. This refutes standard induction, coinduction and mutual induction, coinduction.

Refutation of the new riddle of induction

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table fragment) is row-major and horizontal.

LET p q r s: blue, green, fiducial point in time, object;
~ Not; & And; + Or; = Equivalent; > Imply, greater than; < Not Imply, less than;
(p>q) bleen; (q>p) grue.

From: en.wikipedia.org/wiki/New_riddle_of_induction

An object is grue if and only if it is observed before t and is green, or else is not so observed and is blue. (1.1)

(((s<r)&(s=q))>(s=(q>p)))+(~(s<r)>(s=p)) ;
TTTT TTTT TTTT TTTT (1.2)

An object is bleen if and only if it is observed before t and is blue, or else is not so observed and is green. (2.1)

(((s<r)&(s=p))>(s=(p>q)))+(~(s<r)>(s=q)) ;
TTTT TTTT TTTT TTTT (2.2)

Eqs. 1.2 and 2.2 as rendered are tautologous, *not* contradictory, and theorems. Therefore the new riddle of induction is refuted as a riddle or paradox.

Refutation of induction formulas in elementary arithmetic EA

Abstract: From the introduction, we evaluate EA elementary arithmetic for induction formulas which are *not* tautologous. This further refutes the reflection property upon which subsequent assertions are based. These formulas constitute a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
\> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⊢, ⊨, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊔ ;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Pona, N.; Joosten, J.J. (2019). The reduction property revisited.
arxiv.org/pdf/1903.03331.pdf jjoosten@ub.edu

The theory EA of Elementary Arithmetic is given by the defining axioms for the arithmetical symbols together with the induction formulas

$$I_\phi := [\ \phi(0) \wedge \forall\ x\ \phi(x) \rightarrow \phi(x+1)\ ] \rightarrow \forall\ x\ \phi(x) \tag{1.1}$$

for each bounded formula ϕ.

LET p, q, r, s: ϕ , x, r, s.

(((p&(p@p))&(p&#q))>(p&(#q+(p=p))))>(p&#q) ;
FFFN FFFN FFFN FFFN (1.2)
using T as value for 1

(((p&(p@p))&(p&#q))>(p&(#q+(%p>#p))))>(p&#q) ;
FFFN FFFN FFFN FFFN (1.3)
using N as value for 1

Eqs. 1.2 and 1.3 are *not* tautologous. This refutes the induction formulas of EA. This further refutes the reflection property upon which subsequent assertions are based. These formulas constitute a *non* tautologous fragment of the universal logic VŁ4.

Inequality of "arbitrarily large" versus "sufficiently large"

From wiki: The statement "f(x) is non-negative for arbitrarily large x." could be rewritten as:

$$\forall n \in R , \exists x \in R \text{ such that } x > n \wedge f(x) \geq 0 \qquad (1)$$

LET: # ∀ All, % ∃ Exists, < ∈ member of,
pqrs fnRx, ~(A<B) (A≥B), > Imply, > ⇒, & ∧ And,
vt tautologous, nvt not tautologous

((#q<r)&(%s<r)) & ((s>q)&~((p&s)< (p-p))) ;
nvt (2)

Using "sufficiently large" instead yields:

$$\exists n \in R \text{ such that } \forall x \in R , x > n \Rightarrow f(x) \geq 0 \qquad (3)$$

(%q<r)&((#s<r)&((s>q)>~((p&s)<(p-p)))) ;
nvt (4)

We ask: "What is the difference between "sufficiently large" and "arbitrarily large"?

((%q<r)&((#s<r) & ((s>q)>~((p&s)<(p-p))))) = (((#q<r)&(%s<r)) & ((s>q)&~((p&s)< (p-p)))) ;
vt (5)

We show there is no difference, so the mathematical jargon "arbitrarily large" is equivalent to "sufficiently large".

Infinite set theory

One of the few interesting properties that can be stated in the language of pure identity theory is that of being infinite. This is given by an infinite set of axioms stating there are at least 2 elements, there are at least 3 elements, and so on-

$\exists x_1\ \exists x_2\ \neg x_1 = x_2,\qquad \exists x_1\ \exists x_2\ \exists x_3\ \neg x_1 = x_2 \wedge \neg x_1 = x_3 \wedge \neg x_2 = x_3,\ldots$ (1), (2)

These axioms define the **theory of an infinite set**.

LET: p x_1, q x_2, r x_3, % ∃.

((%p&%q)&~p) = r ; nvt (3)

((%p&%q)&(%r&~p)) = ((q&~p)=(r&~q)) ; nvt (4)

Infinite sets are not validated as tautologous.

Refutation of the definition of mutual information

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal.

LET p, q, r: H; A, ; B,Y; ~ Not; & And; \ Not And; + Or; - Not Or; > Imply.

From: Wright, J. (2015). Lecture 18: Quantum information theory and Holevo's bound. cs.cmu.edu/~odonnell/quantum15/lecture18.pdf

"Definition 4.5 (Mutual Information). The mutual information I(X;Y) between two random variables X and Y is I(X;Y) = H(X) + H(Y) - H(X,Y). (1.1)

This is supposed to represent the amount of information one learns about X from knowing what Y is. Since the definition is symmetric in X and Y, it also represents the amount of information one learns about Y from knowing X."

We evaluate the consequent of Eq. 1.1 as a potential theorem.

((p&q)+(p&r))-(p&(q&r)) ; TTTF TFTF TTTF TFTF (1.2)

Eq. 1.2 as rendered is *not* tautologous.

We evaluate the definition from another source: en.wikipedia.org/wiki/Mutual_information .

"Mutual information can be equivalently expressed as I(X;Y) ≡

H(X) − H(X|Y) ≡ (2.1)
H(Y) − H(Y|X) ≡ (3.1)
H(X) + H(Y) − H(X,Y) ≡ (4.1)
H(X,Y) − H(X|Y) − H(Y|X). (5.1)

where H(X) and H(Y) are the marginal entropies, H(X|Y) and H(Y|X) are the conditional entropies, and H(X,Y) is the joint entropy of X and Y."

(p&q)-(p&(q\r)) ; TFTF TFTF TFTF TFTF (2.2)
(p&r)-(p&(r\q)) ; TFTF TFTF TFTF TFTF (3.2)
((p&q)+(p&r))-(p&(q&r)) ; TTTF TFTF TTTF TFTF (4.2)
(p&(q&r))-((p&(q\r))-(p&(r\q))) ; FTFT FTFF FTFT FTFF (5.2)

Eqs. 2.2 and 3.2 are equivalents and 4.2 and 5.2 are not, but each is *not* tautologous. This means the definition of mutual information as stated is not confirmed and hence refuted.

Refutation of the innovation contest in two sequential stages

Abstract: We evaluated the definition of the conjectured model for the innovation contest in two sequential stages as *not* tautologous, forming a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Bimpikis, K.; Ehsani, S.; Mostagir, M. (2019). Designing dynamic contests. Operations Research. 67:2:295-597. [43 pages in preprint]. gsb.stanford.edu/sites/gsb/files/publication-pdf/contests.pdf kostasb@stanford.edu, shayane@stanford.edu, mosta@umich.edu

1. Introduction Innovation contests are a tool that firms and institutions use to outsource innovation to the crowd. ...

2. Model Our benchmark model is an innovation contest with two sequential stages, A and B, and two competitors, 1 and 2. Innovation happens if an agent successfully completes Stage A and then Stage B. Stage A is associated with a binary state θ_A that describes whether that stage can be completed ($\theta_A = 1$) or not ($\theta_A = 0$). If $\theta_A = 0$, then Stage A is not feasible (and, consequently, innovation is not possible). If $\theta_A = 1$, then the breakthrough to complete Stage A is feasible and has an arrival rate that is described by a Poisson process with parameter λ. .. (2.1.1)

Remark 2.1.1: We take 0 and 1 to be **F** and T due to the verbiage "binary state θ_A that describes whether that stage can be completed ($\theta_A = 1$) or not ($\theta_A = 0$)."

LET p, q, r, s: θ, A, B, s.

((p&q)=((s= s)+(s@s)))>(((p&q)=(s= s))>r) ;
TTT**F** TTTT TTT**F** TTTT (2.1.2)

Remark 2.1.2: If Eq. 2.1.2 takes ordinal 1 to be N, then the result diverges farther from T:

(((p&q)=(%s>#s))+((p&q)=(s@s)))>(((p&q)=(%s>#s))>r) ;
NNNC TTTT NNNC TTTT (2.1.3)

Eqs. 2.1.2 and 2.1.3 as rendered are *not* tautologous, hence refuting the conjectured model of innovation contest in two stages.

Refutation of inquisitive modal logic via flatness grade

Abstract: We evaluate two seminal definitions for flatness grade with neither tautologous. This refutes inquisitive modal logic and relegates it to a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Meißner, S.; Otto, M. (2019). A first-order framework for inquisitive modal logic. arxiv.org/pdf/1906.04981.pdf

3.2 Graded flatness and the standard translation

Definition 3.4 (flatness grade).
The flatness grade ♭ (ϕ) ∈ N of ϕ ∈ InqML is defined by syntactic induction, for all ψ, χ ∈ InqML, according to

♭ (ψ → χ) := ♭ (χ); (3.4.1.1)

LET p, q, r, s: p, ψ, χ, ♭ ; + \V (intuitive disjunction).

(s&(q>r))=(s&r) ; TTTT TTTT **FF**TT TTTT (3.4.1.2)

♭ (ψ \V χ) := ♭ (ψ) + ♭ (χ) + 1. (3.4.2.1)

(s&(q+r))=(((s&q)+(s&r))+(%p>#p)) ;
CCCC CCCC CCTT TTTT (3.4.2.2)

Eqs. 3.4.1.2 and 3.4.2.2 as rendered are *not* tautologous. This refutes flatness grade and hence inquisitive modal logic.

Refutation of condition/decision duality and the internal logic of extensive restriction categories

Abstract: The equations for condition/decision duality are *no*t tautologous, hence refuting what follows as internal logic of extensive restriction categories. These conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Robin Kaarsgaard, R. (2019). arxiv.org/pdf/1905.09181.pdf robin@di.ku.dk
Condition/decision duality and the internal logic of extensive restriction categories.

Abstract: … While categorical treatments of flowchart languages are abundant, none of them provide a treatment of this dual nature of predicates. In the present paper, we argue that extensive restriction categories are precisely categories that capture such a condition/decision duality, by means of morphisms which, coincidentally, are also called decisions. Further, we show that having these categorical decisions amounts to having an internal logic: Analogous to how subobjects of an object in a topos form a Heyting algebra, we show that decisions on an object in an extensive restriction category form a De Morgan quasilattice ...

4 The internal logic of extensive restriction categories
4.1 Kleene's three valued logics and De Morgan quasilattices

As for Boolean algebras, one can derive a partial order on De Morgan quasilattices by

p ≼ q iff p ∧ q = p, (4.1.1.1)

((p&q)=p)>~(q<p) ; TT**F**T TT**F**T TT**F**T TT**F**T (4.1.1.2)

and another one by

p ⊑ q iff p ⋁ q = q. (4.1.2.1)

((p+q)=q)>~(q<p) ; TT**F**T TT**F**T TT**F**T TT**F**T (4.1.2.2)

Unlike as for Boolean algebras, however, these do not coincide, though they are anti-isomorphic, as it follows from the De Morgan laws that

$p \preccurlyeq q$ iff $\neg q \sqsubseteq \neg p$. (4.1.3.1)

```
((((p+q)=q)>~(q<p))>(((p&q)=p)>~(q<p)))&
~((((p&q)=p)>~(q<p))>(((p+q)=q)>~(q<p))) ;
                              FFFF FFFF FFFF FFFF
```
(4.1.3.2)

Eqs. 4.1.1.2 and 4.1.2.2 are *no*t tautologous; and 4.1.3.2 is contradictory because of the *iff* in 4.1.3.1. This refutes condition/decision duality and hence what follows as internal logic of extensive restriction categories.

Refutation of interpretability logics

Abstract: Of the non trivial logics for axioms as evaluated, none is tautologous. Hence the interpretability logic **IL** is refuted, and forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠, ⊳ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Mikec, L.; Vukovi'c, M. (2019). Interpretability logics and generalized Veltman semantics. arxiv.org/pdf/1907.03849.pdf luka.mikec@math.hr, vukovic@math.hr

Abstract We obtain modal completeness of the interpretability logics $\mathbf{ILP_0}$ and **ILR** w.r.t. generalized Veltman semantics.

1 Introduction
1.1 Interpretability logics

The language of interpretability logics is given by A ::= p | ⊥ | A → A | A A, where p ranges over a countable set of propositional variables. Other Boolean connectives are defined as abbreviations, as usual. Since A can be defined (over extensions of IL) as an abbreviation too (expanded to ¬A ⊥), we do not include □ or ◊ in the language. If A is constructed in this way, we will say that A is a modal formula.

Definition 1. Interpretability logic **IL** is given by the following list of axiom schemata.

1. classical tautologies (in the new language);

K. □(A → B) → (□A → □B);[trivial tautology]

L. □(□A → A) → □A; (1.2.1)

LET p, q, r: A, B, C.

#(#p>p)>#p ; CTCT CTCT CTCT CTCT (1.2.2)

J1. □(A → B) → A ⊳ B; [trivial tautology]

J2. $(A \rhd B) \wedge (B \rhd C) \to A \rhd C$; [trivial tautology]

J3. $(A \rhd C) \wedge (B \rhd C) \to A \vee B \rhd C$; (1.5.1)

((p>r)&(q<r))>(p+(q>r)) ; `TT`**`F`**`T TTTT TT`**`F`**`T TTTT` (1.5.2)

J4. $A \rhd B \to (\Diamond A \to \Diamond B)$; [trivial tautology]

J5. $\Diamond A \rhd A$. (1.7.1)

%p>p ; `NTNT NTNT NTNT NTNT` (1.7.2)

Of the non trivial logics for axioms as evaluated, Eqs. 1.2.2, 1.5.2, and 1.7.2, none is tautologous. Hence the interpretability logic **IL** is refuted.

Refutation of interpretability logic, and Vaught and adjunctive set theory

Abstract: We evaluate axioms and inference rules of interpretability logics ILM and TOL with none tautologous, making doubtful the claimed completeness. We then turn to Vaught and adjunctive set theory with neither tautologous. These conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
\> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Interpretability_logic

Interpretability logics comprise a family of modal logics that extend provability logic to describe interpretability or various related metamathematical properties and relations …

Logic ILM:
Axiom schemata:

3. □(□p → q) → □p (ILM.Ax.3.1)

#(#p>p)>#p ; `CTCT CTCT CTCT CTCT` (ILM.Ax.3.2)

Rules inference:

2. "From p conclude □p". (ILM.Ri.2.1)

p>#p ; `TNTN TNTN TNTN TNTN` (ILM.Ri.2.2)

The completeness of ILM with respect to its arithmetical interpretation was independently proven by Alessandro Berarducci and Vladimir Shavrukov.

Remark ILM: Because Eqs. Ax.3.2 and Ri.2.2 are *not* tautologous, this refutes logic ILM and makes doubtful its completeness.

Logic TOL:
Axiom schemata:

3. ◊(p) → ◊(p ∧ ¬◊(p)) (TOL.Ax.3.1)

%p>%(p&~%p) ; TCTC TCTC TCTC TCTC (TOL.Ax.3.2)

Rules inference:

2. "From ¬p conclude ¬◊(p)". (TOL.Ri.2.1)

~p>~%p ; NTNT NTNT NTNT NTNT (TOL.Ri.2.2)

The completeness of TOL with respect to its arithmetical interpretation was proven by Giorgi Japaridze.

Remark TOL: Because Eqs. Ax.3.2 and Ri.2.2 are *not* tautologous, this refutes logic TOL and makes doubtful its completeness.

From: Visser, A. (2019). Enayat theories. arxiv.org/pdf/1909.08877.pdf

2. Basics
2.2. Vaughtness and sequentiality.
2.2.1. *Vaught set theory*. We define Vaught set theory, VS as follows.

VS1. ∃x ∀y y ∉ x (VS1.1)

LET p, q, r, s: x, y, u, v

~(#q<%p)=(s=s) ; TTCT TTCT TTCT TTCT (VS1.2)

Remark VS: Because VS1.2 is not tautologous, this refutes Vaught set theory.

2.2.2. *Adjunctive set theor***y**. We define adjunctive set theory, AS, as follows:

AS1. ∃x ∀y y ∉ x, (AS1.1)

~(#q<%p)=(s=s) ; TTCT TTCT TTCT TTCT (AS1.2)

AS2. ∀u ∀v ∃x ∀y (y ∈ x ↔ (y ∈ u ∨ y = v)). (AS2.1)

(#q<%p)=((#q<#r)+(#q=#s)) ;
FFN**F** **FFF**N NNN**F** NNN**F** (AS2.2)

Remark SS: Because AS1.2 and 2.2 are *not* tautologous, this refutes adjunctive set theory.

Appendix A. First proof [of Theorem 2.2]: … It is clear that on the standardly finite sets our operations behave as the ordinary successor, sum and product. Moreover, ≤ defined as [behaves as usual]

$$x \leq y := \exists z\ (z + x = y) \qquad \text{(A.2.2.1)}$$

~(q<p)=((%r+p)=q) ;   N**F**NT **FFF**T N**F**NT **FFF**T   (A.2.2.2)

Remark A: Because Eq. A.2.2.2 is *not* tautologous, this refutes the first proof of Th. 2.2.

Refutation of the interval for model checking

Abstract: We evaluate three sub-interval relations named reflexive, proper or irreflexive, and strict. None is tautologous. This refutes those relations and model checking therefrom.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. Reproducible transcripts for results are available. (See ersatz-systems.com.)

LET p, q, r, s: x, x', y, y';
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⊢, ↦, ≻, ⊃; < Not Imply, less than, ∈, ≺, ⊂;
= Equivalent, ≡, ⊨, :=, ⇔, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) C non-contingency, ∇, ordinal 2; (%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).

From: Monanari, A. (2019). Model checking: the interval way.
arxiv.org/pdf/1901.03880.pdf, albertom.altervista.org/Th.pdf molinari.alberto@gmail.com

We consider three possible sub-interval relations: 3.1. Preliminaries

Remark 3.1: New interval operators are denoted as ⊑, ⊏, and ⊏· .
We define them here as based on connectives Not Imply < and Equivalent =.

1. [T]he reflexive sub-interval relation (denoted as ⊑), defined by [x,y]⊑[x′,y′] if and only if x′≤x and y≤y′, (3.1.1.1)

(~(p<q)&~(s<r))>~((p&r)>(q&s)) ; **F**T**FF** **F**T**F**T TTTT **F**T**FF** (3.1.1.2)

2. [T]he proper (or irreflexive) sub-interval relation (denoted as ⊏), defined by [x,y]⊏ [x′,y′] if and only if [x,y]⊑ [x′,y′] and [x,y], [x′,y′], and (3.1.2.1)

(~((q&s)>(p&r))&((p&r)@(q&s)))>((p&r)<(q&s)) ;
TTTT TTTT TT**FF** TT**F**T (3.1.2.2)

3. [T]he strict sub-interval relation (denoted as ⊏·), defined by [x,y]⊏·[x′,y′] if and only if x′<x and y<y′. (3.1.3.1)

((q<p)&(s<r))>((p&r)<(q&s)) ; TTTT TTTT TT**F**T TTTT (3.1.3.2)

Eqs. 3.1.1.2 - 3.1.3.2 as rendered are *not* tautologous. This refutes the definitions of the interval relations and hence the model checking therefrom.

Refutation of intuitionistic fuzzy decision-making in the Dempster-Shafer structure

Abstract: A pair of intuitionistic fuzzy values (IFVs) are compared and *not* tautologous, refuting the conjecture of the title and forming a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Fei, L. (2019). Intuitionistic fuzzy decision-making in the framework of Dempster-Shafer structures. vixra.org/pdf/1907.0179v1.pdf feiliguohit@163.com

I. Introduction

[A] pair of IFVs [intuitionistic fuzzy values] can be compared ... as:

if *S(xi)>S(xj)*, then *xi* is better than *xj*,
if *S(xi)>S(xj)*, then
if *H(xi)=H(xj)*, then *xi* is equal to *xj*,
if *H(xi)<H(xj)*, then *xj* is better than *xi*. (1.1)

LET p, q, r, s: *xi, xj, H, S.*

(((s&p)>(s&q))>(p>q))&
(((s&p)>(s&q))>((((r&p)=(r&q))>(p=q))&(((r&q)<(r&q))>(q<p)))) ;
TFFT TFTT TTFT TTTT (1.2)

Eq. 1.2 as rendered is *not* tautologous, refuting the conjecture of the title.

Refutation of intuitionistic Zermelo-Fraenkel set theory (IZF), de Jongh's theorem, and CZF

Abstract: We evaluate the nine axioms for intuitionistic Zermelo-Fraenkel set theory (IZF). None is tautologous. This refutes IZF and its use in blended models and denies De Jongh's classical theorem and similar results for constructive ZF (CZF). These segments form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
\> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Passmann, R. (2019). De Jongh's theorem for intuitionistic Zermelo-Fraenkel set theory. arxiv.org/pdf/1905.04972.pdf robertpassmann@posteo.de

Abstract. We prove that the propositional logic of intuitionistic set theory IZF is intuitionistic propositional logic IPC. More generally, we show that IZF has the de Jongh property with respect to every intermediate logic that is complete with respect to a class of finite trees. The same results follow for CZF.

1. Introduction: De Jongh's classical theorem ... states that the propositional logic of Heyting Arithmetic HA is intuitionistic logic IPC. In this work, we will prove de Jongh's theorem for intuitionistic Zermelo-Fraenkel set theory IZF. … To prove this result, we introduce a new semantics for IZF, the so-called *blended Kripke models*, or *blended models* for short.

3. Blended models: In this section, we will construct the blended models and show that they are models of intuitionistic Zermelo-Fraenkel set theory IZF. Figure 1. The axioms of IZF.

Extensionality: ∀a∀b(∀x(x ∈ a ↔ x ∈ b) → a = b) (3.1.1)

LET a, b, x: p, q, r

((#r<#p)=(#p<#q))>(#p=#q) ; TTCT TTCT TTCT TTCT (3.2.1)

Empty set: ∃a ∀x ∈ a ⊥ (3.2.1)

(%p&#r)<(p&(p@p)) ; **FFFF** **F**N**F**N **FFFF** **F**N**F**N (3.2.2)

Pairing: $\forall a \forall b \exists y \forall x(x \in y \leftrightarrow (x = a \vee x = b))$ (3.3.1)

(#r<%s)=((#r=#p)+(#r=#q)) ; FFFN FNNN FFFN FNNN (3.3.2)

Remark 3.3.2: If the quantifiers are not distributed, Eq. 3.3.1 becomes a contradiction.

((#p&#q)&(%s&#r))&((r<s)=((r=p)+(r=q))) ;
FFFF FFFF FFFF FFFF (3.3.3)

Union: $\forall a \exists y \forall x(x \in y \leftrightarrow \exists u(u \in a \wedge x \in u))$ (3.4.1)

LET p, q, r, s: a, u, x, y

(#r<%s)=((%q<#p)&(#r<%q)) ; TTTT CCCC TTTT CCCC (3.4.2)

Power set: $\forall a \exists y \forall x(x \in y \leftrightarrow x \subseteq a)$(3.5.1)

(#r<%s)=~(#p<#r); FNFN NNNN FNFN FFFF (3.5.2)

Infinity: $\exists a(\exists x\ x \in a \wedge \forall x \in a \exists y \in a\ x \in y)$ (3.6.1)

((%r<%p)&(#r<%p))&((%s<%p)&(#r<%s));
FFFF FFFF FFFF FFFF (3.6.2)

Remark 3.6.2: Eq. 3.6.2 as rendered is *not* tautologous and a contradiction.

Set Induction: $(\forall a(\forall x \in a\ \phi(x) \rightarrow \phi(a))) \rightarrow \forall a \phi(a)$, for all formulas $\phi(x)$. (3.7.1)

LET p, q, r, s: a, ϕ, x, y

(((#r<#p)&(#q&#r))>(q&#p))>(q&#p) ;
FFFN FFFN FFFN FFFN (3.7.2)

Separation: $\forall a \exists y \forall x(x \in y \leftrightarrow (x \in a \wedge \phi(x)))$, for all formulas $\phi(x)$. (3.8.1)

(#r<%s)=((#r<#p)&(#q&#r)) ; TTTT CCTC TTTT TTCT (3.8.2)

Collection: $\forall a(\forall x \in a \exists y\ \phi(x, y) \rightarrow \exists b \forall x \in a \exists y \in b\ \phi(x, y))$, for all formulas $\phi(x, y)$, where b is not free in $\phi(x, y)$. (3.9.1)

LET p, q, x, y, ϕ: a, b, x, y, z

((#x<#p)&(%y&(#z&(x&%y))))>(((%q&#x)<(#p&%y))<(q&(#z&(x&%y)))) ;
TTTT TTTT TTTT TTTT(112),
TCTC TCTC TCTC TCTC(16) (3.9.2)

Eqs. 3.1.2-3.9.2 are *not* tautologous. This refutes IZF and its use in blended models, De Jongh's classical theorem, and similar results for constructive ZF (CZF).

Refutation of Isabelle/HOL

Abstract: Eight of eleven equations are evaluated as *not* tautologous. This means the rewrite-engine or simplifier tool is not confirmed, the conjunction is not effectively defined by three rules, and other reasoning steps are not expressed similarly, hence refuting Isabelle/HOL. These anomalies form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Paulson, L.C.; Nipkow, T.; Wenzel, M. (2019). From LCF to Isabelle/HOL. arxiv.org/pdf/1907.02836.pdf

3. Isabelle in the early days: a logical framework
Isabelle originated in a project to build an LCF-style proof [logic for computable functions] assistant for Martin-Löf's constructive type theory.

A special case of unification is matching where the variables of only one of the two terms are instantiated. Isabelle's rewrite engine (aka the simplifier) is based on higher-order pattern matching. Thus the simplifier can deal with many standard transformations of quantified terms, for example the following:

(∀x. P(x) ∧ Q(x)) = (∀x. P(x)) ∧ (∀x. Q(x)) (3.1.1)

LET p, q, r, s: P, Q, x, t.

((p&#r)&(q&r))=((p&#r)&(q&#r)) ;
TTTT TTTT TTTT TTTT (3.1.2)

(∀x. P ∨ Q(x)) = P ∨ (∀x. Q(x)) (3.2.1)

((#r&p)+(q&r))=(p+(q&#r)) ; T**F**T**F** TNNT T**F**T**F** TNNT (3.2.2)

(∀x. x = t ∧ P(x)) = P(t)) (3.3.1)

((#r=s)&(p&r))=(p&s) ; TTTT TNTN TFTF TNTN (3.3.2)

It appears that Isabelle was the first theorem prover to support higher-order rewrite rule.

5. Automation

5.1. The classical reasoner

As mentioned in section 3 above, Isabelle supported both unification and backtracking from the start, with the aim of incorporating ideas from first-order automatic proof procedures. In the context of interactive proof, unification provided the ability to prove a subgoal of the form $\exists x.\varphi(x)$ by removing the quantifier and proving $\varphi(?t)$, where ?t stood as a placeholder for a concrete term to be supplied later. Through unification, this term could even be built up incrementally. Dually, unification provided a means of using a universally quantified fact $\forall x.\varphi(x)$, when the required instances were not immediately obvious.

Simple automation is achievable through a combination of obvious applications of the propositional connectives ($\wedge$, $\vee$, $\neg$, etc.) along with heuristics for performing quantifier reasoning. Stronger automation is obtainable by borrowing well-known techniques for classical first-order logic theorem proving. But the most important idea is to embrace the concepts of natural deduction in application theories as well as in pure logic. Natural deduction prefers the use of simple inference rules focusing on a single symbol.

For example, conjunction is effectively defined by the following three rules:

$(\varphi>\psi)>(\varphi\wedge\psi)$ (5.1.1)

LET p, q: φ, ψ.

$(p>q)>(p\&q)$; **F**T**F**T **F**T**F**T **F**T**F**T **F**T**F**T (5.1.2)

$(\varphi\wedge\psi)>\varphi$ (5.2.1)

$(p\&q)>p$; TTTT TTTT TTTT TTTT (5.2.2)

$(\varphi\wedge\psi)>\psi$ (5.3.1)

$(p\&q)>q$; TTTT TTTT TTTT TTTT (5.3.1)

The intersection of two sets has a technical definition that would greatly complicate reasoning, but it is easy to derive inference rules for intersection in the style of natural deduction (and analogous to those above):

$((a \in A) > (a \in B)) > a \in A \cap B$ (5.4.1)

LET p, q, r, s: a, b, A, B

$((p<r)>(p<s))>(p<(r\&s))$; **F**T**F**T **F**T**F**T **F**T**F**T **FFFF** (5.4.2)

$(a \in A \cap B) > a \in A$ (5.5.1)

$(p<(r\&s))>(p<r)$; TTTT T**F**T**F** TTTT TTTT (5.5.2)

$(a \in B \cap A) > a \in B$ (5.6.1)

$(p<(s\&r))>(p<s)$; TTTT TTTT T**F**T**F** TTTT (5.6.2)

Many other reasoning steps can be expressed similarly:

$((A \subseteq B) > (a \in A)) > a \in B$ (5.7.1)

LET p, q, r, s: a, A, B, C.

$(\sim(r<q)>(p<q))>(p<r)$; TTTT **FT**TT TTTT **FF**TT (5.7.2)

$((A \subseteq B) > (B \subseteq C)) > A = B$ (5.8.1)

$(\sim(r<q)>\sim(s<r))>(r=s)$; TTTT **FFFF** TTTT TTTT (5.8.2)

Eqs. 3.2.2, 3.3.2, 5.1.2, 5.4.2, 5.5.2, 5.6.2, 5.7.2, and 5.8.2 as rendered are *not* tautologous. This means the rewrite engine or simplifier tool is not confirmed, the conjunction is not effectively defined by three rules, and other reasoning steps are not expressed similarly, hence refuting Isabelle/HOL.

Refutation of Isabelle/HOL prover assistant

Abstract: A meta-rule for structural induction in the prover assistant Isabelle/HOL is *not* tautologous. This refutes the assistant and denies it can effect a cross-fertilization of computer science and metaphysics. Therefore Isabelle/HOL is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Kirchner, D.; Benzmüller, C.; Zalta, E.N. (2019).
Computer science and metaphysics: a cross-fertilization.
arxiv.org/pdf/1905.00787.pdf daniel@ekpyron.org, zalta@stanford.edu,
c.benzmueller@fu-berlin.de (c.benzmueller@googlemail.com)

1.2 Propositional S5 with Abstraction Layers.

Unfortunately, in our implementation we are lacking *structural induction*, i.e. induction on the complexity of a formula. For that reason, we also have to derive meta-rules for our target system from the semantics, e.g.,

lemma *deduction:* **assumes** "[w ⊨ p] ⇒ [w ⊨ q]"
shows "[w ⊨ p → q]"
using *assms* **apply** *transfer* **by** *auto* (1.2.16.1)

((w=p)>(w=q))>(w=(p>q)) ; **F**TT**F** **F**TT**F** **F**TT**F** **F**TT**F**(8),
TTTT TTTT TTTT TTTT(8) (1.2.16.2)

Eq. 1.2.16.2 as rendered is not tautologous, hence denying structural induction on the Isabelle/HOL prover assistant. What follows is that the assistant is refuted and cannot effect a cross-fertilization of computer science and metaphysics.

Refutation of Jaccard index

Abstract: The definition of the Jaccard index is *not* tautologous, hence refuting it with derivations and forming a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Jaccard_index

The **Jaccard index**, also known as **Intersection over Union** and the **Jaccard similarity coefficient** (originally given the French name *coefficient de communauté* by Paul Jaccard), is a statistic used for gauging the similarity and diversity of sample sets. The Jaccard coefficient measures similarity between finite sample sets, and is defined as the size of the intersection divided by the size of the union of the sample sets:

J(A,B) = |A∪B| /| A∩B| = |A∪B| / (|A|+|B|-|A∪B|). (If A and B are both empty, we define J(A,B)=1.) 0 ≤ J(A,B) ≤ 1. (1.1)

~((((A&B)\(A+B))=((A&B)\((A+B)-(A&B))))<(C@C)) &
~((%C>#C)<(((A&B)\(A+B))=((A&B)\((A+B)-(A&B))))) ;
FFFF **F**C**F**C **FFFF** **F**C**F**C (1.2)

Remark 1.2: Without the relational limits in Eq. 1.1, the formula alone is not tautologous:

((A&B)\(A+B))=((A&B)\((A+B)-(A&B))) ;
TTTT TNTN TTCC TNC**F** (1.3)

Eq. 1.2 is *not* tautologous, and nearly contradictory, refuting the Jaccard index with its derivations.

Refutation of the root in partition Jensen polynomials for hyperbolicity

Abstract: The root in partition Jensen polynomials for hyperbolicity is *not* tautologous. Hence its use to prove the Riemann hypothesis is denied. These conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Jensen-Poly program for the Riemann hypothesis and related problems.
people.oregonstate.edu/~petschec/ONTD/Talk1.pdf

Slide 96: Partition Jensen polynomials are hyperbolic if and only if (p(n+1)(n+1))>(p(n)p(n+2)). (1.1)

LET p, q: p, n.

(p&((q+(%r>#r))&(q+(%r>#r))))>((p&q)&(p&(q+(%r<#r)))) ;
TCTT TCTT TCTT TCTT (1.2)

Eq. 1.2 as rendered is *not* tautologous. This refutes the conjecture of Jensen-Poly for Riemann hypothesis.

Confirmation of join-prime in lattice theory

From en.wikipedia.org/wiki/Birkhoff%27s_representation_theorem

"An element x is join-prime if, whenever $x \leq y \vee z$, either $x \leq y$ or $x \leq z$." (1.1)

Assuming the Meth8/VL4 apparatus and method,

LET: p q r x y z

((p<(q+r))+(p=(q+r)))>(((p<q)+(p=q))+((p<r)+(p=r))) ;

TTTT TTTT TTTT TTTT (1.2)

The join-prime definition is tautologous (all T).

Retromorphisms of Jonsson theory in positive logic

From: Poizat, B.; Yeshkeyev, A. "Jonsson Theories in Positive Logic". (2015).
www.logique.jussieu.fr/modnet/Publications/Preprint%20server/papers/873/873.pdf

On page 36/7 of Section 3.4 Retromorphisms,

The designated truth value is Tautologous "11"; C means Contingent "10".

23(i): We ask if this equation is tautologous: $(\forall x)(\exists y)\ \phi(x) \Rightarrow \psi(x,y)$ (23.i.1)

LET p, q, r, s: ϕ, ψ, x, y;
$\forall$; % $\exists$.

((#p&%q)&(r&p)) > (s&(p&q)) ; `TTTT TTTC TTTT TTTT` (23.i.2)

23(ii): We ask if this equation is contradictory: $(\forall x)\ \phi(x) \Rightarrow \psi(x)$ (23.ii.1)

(#p&(r&p))>(s&p) ; `TTTT TCTC TTTT TTTT` (23.ii.2)

23(iii): We cannot evaluate this, but it is moot if 23(i) is not tautologous.

Eqs. 23.i.2 and 23.ii.2 as rendered are *not* tautologous, hence refuting retromorphisms of Jonsson theory in positive logic.

Confirmation of the logic in the definition of the *k*-triangular set function

Abstract: We evaluate the logic of the definition of the *k*-triangular function in set theory and find it tautologous, hence confirming it as a theorem.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET p, q, r, s, t: *A*, *B*, *k*, *m*, Σ;
~ Not; + Or, ∪, add ; - Not Or, subtract; & And, ∩, multiply;
> Imply, lesser than ; < Not Imply, lesser than, ∈ ;
= Equivalent; @ Not Equivalent;
% possibility, for one or some, ∃; # necessity, for all or every, ∀;
~(y < x) (x ≤ y) ; (p@p) **F** as contradiction, zero 0, null Ø

From: Boccuto, A.; Dimitriou, A. (2018). Dieudonné-type theorems for lattice group-valued k-triangular set functions. vixra.org/pdf/1811.0496v1.pdf boccuto@yahoo.it

Definitions 2.6 (b) We say that *m* is *k*-triangular on *L* iff

$m(A) - k\, m(B) \leq m(A \cup B) \leq m(A) + k\, m(B)$
whenever $A,B \in \Sigma$, $A \cap B = \emptyset$ and $0 = m(\emptyset) \leq m(A)$ for each $A \in \Sigma$. (2.6.1)

((((p&q)<t)&((p&q)=(p@p)))&((%p<t)>~((s&p)<((p@p)=(s&(p@p)))))) >
~(~(((s&p)+(r&(s&q)))<(s&(p+q)))<((s&p)-(r&(s&q)))) ;
`TTTT TTTT TTTT TTTT` (2.6.2)

Eq. 2.6.2 as rendered is tautologous, hence confirming the logic of the definition of the *k*-triangular function as a theorem in set theory.

The Kanban cell neuron maps the whole brain on an umbilic torus

When the 2D Möbius is extended into a 3D umbilic torus (less precisely as umbilic "bracelit"), then antipodal points take three revolutions to traverse the shape. The shape without coefficients is in the cubic form of $(x^2)(x+3y)+(y^2)(3x+y)$. For the shape to be physically mapped as a whole brain, the solutions are in real number solutions, not on the complex space in imaginary number solutions.

Three rotations map to the linear formula of the Kanban cell neuron model formula of ((p&q)+r)=s where <p,q,r,s> are in {11,10,01,00} as respectively <tautology (proof), falsity (contingency), truthity (non contingency), contradiction (non proof)>. There are 14-combinations as equations where:

1. {00} is not present (no contradictions); and
2. $p \neq s$, input does not equal output, because the end state to stop processing is p = s.

Connective No.	((p	& q)	+ r)	= s
091	01	01	10	11
095	01	01	11	11
106	01	10	10	10
111	01	10	11	11
123	01	11	10	11
127	01	11	11	11
149	10	01	01	01
159	10	01	11	11
167	10	10	01	11
175	10	10	11	11
183	10	11	01	11
191	10	11	11	11
213	11	01	01	01
234	11	10	10	10

The distribution of s is: 2 {01}; 2 {10}; and 10 {11}. This means the formula output is skewed by about 83% towards tautology (proof). Because 14-connectives are allowed out of a possible 256-connectives, about 5% of input is accepted and 95% rejected. This effectively filters input and concurrently self-times the processing cycles, to overcome mechanical issues of whole brain models. The Kanban cell neuron limits the number of such dual points by processing about 5% of input data in the 14-combinations. With location markers based on the properties of the linear Kanban cell neuron model, the mapping requires only one point on the 3D umbilic torus and not two antipodal points.

What follows is a simplified model of the whole brain as limited within a 3D topology, and without resorting to imaginary higher dimensions for fitting untenable models of quantum vector spaces which are not bivalent but probabilistic.

Kant's contradictory subtlety of four syllogistic figures

This paper adopts and keys to the translated arguments (following) by the anonymous author of:

en.wikipedia.org/wiki/The_contradictory_Subtlety_of_the_Four_Syllogistic_Figures

for Immanuel Kant (1762), "The contradictory subtlety of the four syllogistic figures proved", (Die falsche spitzfindigkeit der vier syllogistischen figuren erwiese).

The Meth8 modal logic prover checks five models using system VŁ4, a variant of Łukasiewicz' quaternary logic. Symbols are:

~ Not, & And, > Imply, = Equivalent, + Or, # necessity (all), % possibility (some),
vt tautologous, nvt not tautologous, T Tautologous, E Evaluated (designated truth values)

Section III Of pure and mixed ratiocination

III: LET: p thing, q immortal, r man, s Socrates

(((~p=r)>(~r=q))&(s=r))>(~s=q) ; nvt (III.1)

FTTT TTTT TTTT TTTF	UEEE EEEE EEEE EEEU	UEEE EEEE EEEE EEEU	UEEE EEEE EEEE EEEU	UEEE EEEE EEEE EEEU
Model 1	Model 2.1	Model 2.2	Model 2.3.1	Model 2.3.2

(((~p&q)=r)&(s=r))>(s=q) ; nvt (III.2.1)

TTTF TTTT TTTT TTTT	EEEU EEEE EEEE EEEE	EEEU EEEE EEEE EEEE	EEEU EEEE EEEE EEEE	EEEU EEEE EEEE EEEE
Model 1	Model 2.1	Model 2.2	Model 2.3.1	Model 2.3.2

((((~p&q)=r)>(~r=q))&(s=r))>(s=q) ; nvt (III.2.2)

TTFF TTTT TTTT FFTT	EEUU EEEE EEEE UUEE	EEUU EEEE EEEE UUEE	EEUU EEEE EEEE UUEE	EEUU EEEE EEEE UUEE
Model 1	Model 2.1	Model 2.2	Model 2.3.1	Model 2.3.2

Section IV In the so-called first figure only pure ratiocinations are possible, in the remaining figures only mixed ratiocinations are possible.

IV.1: LET: p A, q B, r C

((r=q)&(p=r))>(p=q) ; vt (IV.1)

IV.2: LET p A, q B, r C

((~q=r)&(p=r))>(p=~q) ; vt (IV.2)

IV.3: LET: p mammals, q air breathers, r animals

((#p=q)&(#p=r)) > (%p=q) ; nvt (IV.3.1)

NNTT TTTT NNTT TTTT	EEEE EEEE EEEE EEEE	UUEE EEEE UUEE EEEE	IIEE EEEE IIEE EEEE	PPEE EEEE PPEE EEEE
Model 1	Model 2.1	Model 2.2	Model 2.3.1	Model 2.3.2

(((#p=q)&(#p=r)) > (%r=p)) > (%r=q); nvt (IV.3.2)

TNCC FFTT TNCC FFTT	EEUU UUEE EEUU UUEE	EUEE UUEE EUEE UUEE	EIPP UUEE EIPP UUEE	EPII UUEE EPII UUEE
Model 1	Model 2.1	Model 2.2	Model 2.3.1	Model 2.3.2

IV.4: LET: p man, p+p persons, q learned, r stupid, s pious

(((~r&p)=q) & ((%q&(p+p))=s)) > ((%s&(p+p))=~r) ;
nvt (IV.4.1)

```
FTTT TTTT TTTT TNTT   UEEE EEEE EEEE EEEE   UEEE EEEE EEEE EUEE   UEEE EEEE EEEE EIEE   UEEE EEEE EEEE EPEE
Model 1               Model 2.1             Model 2.2             Model 2.3.1           Model 2.3.2
```

((((~r&p)=q)>((~q&(p+p))=r)) & (((%q&(p+p))=s)>((%s&(p+p))=q))) >((%s&(p+p))=~r) ;
nvt (IV.4.2)

```
FCTC TNTN FTFT TCTF   UUEU EEEE UEUE EUEU   UEEE EUEU UEUE EEEU   UPEP EIEI UEUE EPEU   UIEI EPEP UEUE EIEU
Model 1               Model 2.1             Model 2.2             Model 2.3.1           Model 2.3.2
```

((((~r&p)=q)>((~q&(p+p))=r)) & (((%q&(p+p))=s)>((%s&(p+p))=q))) >((%s&(p+p))=~r) ;
nvt (IV.4.2)

```
FCTC TNTN FTFT TCTF   UUEU EEEE UEUE EUEU   UEEE EUEU UEUE EEEU   UPEP EIEI UEUE EPEU   UIEI EPEP UEUE EIEU
Model 1               Model 2.1             Model 2.2             Model 2.3.1           Model 2.3.2
```

Eq IV.1 and IV.2 are tautologous; all others are not tautologous.

This shows that the comments in the article as to how to fix up the syllogisms are mistaken, but nevertheless renders Kant's essay as a historical record to bear the logic of the time.

Karpenko System K-Ł4

Replication of A.S. Карпенко (2015). Решетки четырехзначных модальных логик. УДК 164.3 + 510.643. [A.S. Karpenko (2015). *Lattices of Four-valued Modal Logics.* English abstract / references.]

We named the instant logic system as Karpenko-Ł4 with the acronym **K-Ł4**. We evaluated each logical expression found using the Meth8 logic model checker based on the bivalent variant VŁ4.

The presentation format is: expression; validation result; comment, if any; and paper section number with location. The expressions are grouped by section number below.

From the lattice arrangement we asked if K-Ł4 is bivalent or a vector space after the three valued logic system of Dunn-Belnap. We invalidate many many of the expressions. This and the assignments of various logical values confirmed that K-Ł4 is for a vector space and is not a bivalent logical system.

#(p>q)>(#p>#q) ;	not validated; Model 2.1 tautologous	3.K
#p+(#(r>q)+#(p>~q)) ;	not validated; Model 2.1 tautologous	4. unnumbered with S5
#p>#(#%p>%#p);	validated	after "S4.4"
#p>#(%#p>#p) ;	validated	after "S4 +"
#(p>q)>(#p>#q) ;	validated	4.2
#p>p ;	validated	4.3
#p>(q>#q) ;	validated	4.4

Substitution formulas, where:

LET r=e1(p); s=e2(p); t=e1(q); u=e2(q);
LET J1(p)=(r&s); J1(q)=(t&u); Ja(p)=(~r&s); Ja(q)=(~t&u) [Ja(p,q) not used];
LET Jb(p)=(r&~s); Jb(q)=(t&~u); J0(p)=(~r&~s); J0(q)=(~t&~u);

for:

(p+q)=((p&q)+(((~r&~s)&q)+((p&(~t&~u))+(((r&~s)&q)+((p&(t&~u))+((r&s)+(t&u)))))) ;
not validated — 4. xVy .... with substitutions.

Shown is one repeating truth table of 24 for 12-propositions.

```
(p+q)=((p&q)+(((~r&~s)&q)+((p&(~t&~u))+(((r&~s)&q)+((p&(t&~u))+((r&s)+(t&u)))))))
Model: 1      Model 2.1            Model 2.2            Model 2.3.1          Model 2.3.2
TTTT TTTT TTFT FTTT   EEEE EEEE EEUE UEEE   EEEE EEEE EEUE UEEE   EEEE EEEE EEUE UEEE   EEEE EEEE EEUE UEEE
TTTT TTTT TTFT FTTT   EEEE EEEE EEUE UEEE   EEEE EEEE EEUE UEEE   EEEE EEEE EEUE UEEE   EEEE EEEE EEUE UEEE
TFTT TFTT TFFT FTTT   EUEE EUEE EUUE UEEE   EUEE EUEE EUUE UEEE   EUEE EUEE EUUE UEEE   EUEE EUEE EUUE UEEE
FTTT FTTT FTTT FTTT   UEEE UEEE UEEE UEEE   UEEE UEEE UEEE UEEE   UEEE UEEE UEEE UEEE   UEEE UEEE UEEE UEEE
```

(p&q)=~(~p+~q) ;	validated	4. x&y=~(~x V ~y);
(#p&~p)= (p@p) ;	validated	7.TM1
(~#p&p)=(~p&p);	not validated; Model 2.1 tautologous	7.TM2
(q>p)+(((p>q)>p)>p) ;	validated	9. unnumbered
#(p>q)=(#p>#q) ;	not validated; Model 2.2 tautologous	10.2
~#p=#~p ;	not validated; Model 2.1 tautologous	10.3
##p=p ;	not validated	10.4

Refutation of the Keisler measure in NIP theory

Abstract: In NIP theory, the Keisler measure as φ(x)↦μ(φ(x) ∩X)/μ(X)) is *not* tautologous, relegating it to a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/NIP_(model_theory)

In model theory, a branch of mathematical logic, a complete theory T is said to satisfy NIP (or "not the independence property") if none of its formulae satisfy the independence property, that is if none of its formulae can pick out any given subset of an arbitrarily large finite set.

From: Conant, G.; Gannor, K. (2019). Remarks on generic stability in independent theories. arxiv.org/pdf/1905.11915.pdf

4. dfs-trivial theories

We call a global Keisler measure is **dfs** if it is definable and finitely satisfiable in some small model.

Definition 4.1. Fix a variable sort x.

(4) … the Keisler measure φ(x)↦μ(φ(x) ∩X)/μ(X) (4.1.4.1)
(we call this measure the **localization** of μ **at** X).

LET p, q, r, s: φ, μ, x, X.

(p&r)>(q&(((p&r)&s)\(q&s))) ; TTTT T**F**TT TTTT T**F**T**F** (4.1.4.2)

Remark 4.1.4.2: Eq. 4.1.4.2 as rendered is *not* tautologous. This refutes the Keisler measure.

Refutation of Keisler's ultraproduct construction

Abstract: The ultraproduct construction of Keisler is based on definitions for proper filter in six equations and for ultrafilter in two equations. The definitions are *not* tautologous. This refutes the ultraproduct construction as "a uniform method of building models of first order theories which has applications in many areas of mathematics." Claims by other writers to extend Keisler's construction similarly fail, to form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Keisler, H.J. (2010). The ultraproduct construction.
math.wisc.edu/~keisler/ultraproducts-web-final.pdf

1. Introduction

The ultraproduct construction is a uniform method of building models of first order theories which has applications in many areas of mathematics. It is attractive because it is algebraic in nature, but preserves all properties expressible in first order logic.

2. Ultraproducts and ultrapowers

We begin with the definition of an ultrafilter over an index set I. An ultrafilter over I can be defined as the collection of all sets of measure 1 with respect to a finitely additive measure μ : P(I) → {0, 1}. Here is an equivalent definition in more primitive terms.

Definition 2.1. Let I be a non-empty set. (2.1.0.1.1)

LET p, q, r, s: I, U, X, Y.

p=~(p@p) ; **F**T**F**T **F**T**F**T **F**T**F**T **F**T**F**T (2.1.0.1.2)

A **proper filter** U over I is a set of subsets of I such that: (2.1.0.2.1)

((p=~(p@p))>(q>p))>(p>(p>p)) ;
TTTT TTTT TTTT TTTT (2.1.0.2.2)

(i) U is closed under supersets; if X ∈ U and X ⊆ Y ⊆ I then Y ∈ U. (2.1.i.1)

((r<q)&~(~(p<s)<r))>(s<q) ;
TTTT **FF**TT TTTT TTTT (2.1.i.2)

(ii) U is closed under finite intersections; if X ∈ U and Y ∈ U then X ∩ Y ∈ U. (2.1.ii.1)

((r<q)&(s<q))>((r&s)<q) ;
TTTT TTTT TTTT TTTT (2.1.ii.2)

(iii) I ∈ U but Ø ∉ U. (2.1.iii.1)

(p<q)&~((p@p)<q) ; **F**T**FF** **F**T**FF** **F**T**FF** **F**T**FF** (2.1.iii.1)

Remark 2.1.0.2.1: We write Eq. 2.1.0.2.1 to imply 2.1.i.1 and 2.1.ii.1 and 2.1.iii.1. (2.1.0.3.1)

(((p=~(p@p))>(q>p))>(p>(p>p)))>(((((r<q)&~(~(p<s)<r))>(s<q))&(((r<q)&(s<q))>((r&s)<q)))&((p<q)&~((p@p)<q))) ;
FT**FF** **FFFF** **F**T**FF** **FFFF** (2.1.0.3.2)

An **ultrafilter** over I is a proper filter U over I such that: (2.1.0.4.0)

(iv) For each X ⊆ I, exactly one of the sets X, I \ X belongs to U. (2.1.iv.1)

~(p>#r)>((%q&(%p\%r))<q) ;
T**F**T**F** TNTN T**F**T**F** TNTN (2.1.iv.2)

Remark 2.1.0.4.0: We write Eq. 2.1.0.4.0 as 2.1.0.3.2 to imply 2.1.iv.1. (2.1.0.4.1)

((((p=~(p@p))>(q>p))>(p>(p>p)))>(((((r<q)&~(~(p<s)<r))>(s<q))&(((r<q)&(s<q))>((r&s)<q)))&((p<q)&~((p@p)<q))))>(~(p>#r)>((%q&(%p\%r))<q)) ;
T**F**TT TTTT T**F**TT TNTT (2.1.0.4.2)

Remark Def.2.1: The **proper filter** definition from six equations is *not* tautologous. The **ultrafilter** definition therefrom in two equations is *not* tautologous. This refutes the Keisler **ultraproduct** construction definition as "a uniform method of building models of first order theories which has applications in many areas of mathematics."

What follows is that claims to extend Keisler's construction also fail, as for example:

Malliaris, M.; Shelah, S. (2019). Keisler's order is not simple (and simple theories may not be either) arxiv.org/pdf/1906.10241.pdf

Abstract. Solving a decades-old problem we show that Keisler's 1967 order on theories has the maximum number of classes. The theories we build are simple unstable with no nontrivial forking, and reflect growth rates of sequences which may be thought of as densities of certain regular pairs, in the sense of Szemerédi's regularity lemma. The proof involves ideas from model theory, set theory, and finite combinatorics.

Text. Keisler's order is a longstanding classification problem in model theory, introduced in 1967 .. as a possible way of comparing the complexity of theories. … In the present paper we prove, in ZFC, that Keisler's order has the maximum number of classes (continuum many), by constructing a new family of simple unstable theories with no nontrivial forking which reflect growth rates of certain sequences of densities of finite graphs, and by developing new methods for building ultrafilters on Boolean algebras which carefully reflect these theories.

Refutation of Kent algebras on rough set concept analysis

Abstract: We use modal logic to evaluate definitions for Kent algebras, as presented for rough set concept analysis. Some definitions are *no*t tautologous, hence refuting Kent algebras on rough sets.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

LET p, q: a, b;
~ Not; + Or ; & And; > Imply, greater than; < Not Imply, lesser than; = Equivalent;
% possibility, for one or some; # necessity, for all or every;
(p=p) Tautology; ~(y<x) (x≤y).

From: Gredo, G.; Jipsen, P.; Manoorkar, K.; Palmigiano, A.; Tzimoulis, A. (2018).
Logics for rough concept analysis. arxiv.org/pdf/1811.07149.pdf pippigreco@gmail.com

4 Kent algebras

Remark 4: We present the p,q equations below, ending n.2, as keyed to the a,b equations in the text, ending n.1. For clarity, we also ignore subscript notations for the lozenge and box symbols of the modal operators.

~(p<#p) = (p=p) ;	TNTN TNTN TNTN TNTN	(12.1.2)
~(p<%#p) = (p=p) ;	TNTN TNTN TNTN TNTN	(15.2.2)
~(p<%#p) = (p=p) ;	TNTN TNTN TNTN TNTN	(15.4.2)
(~(#q<#p)&~(%q<%p))>~(q<p) ;	TTNT TTNT TTNT TTNT	(19.2)
(~(#q<#p)&~(%q<%p))>~(q<p) ;	TTNT TTNT TTNT TTNT	(20.2)

We group Eqs. 12, 15 and 19, 20 because of the different truth table results, which are *not* tautologous. This means Kent algebras are refuted. What follows is that rough set analysis for concept analysis is suspicious.

Refutation of the algebra of binary relations as basis of free Kleene algebras with domain

Abstract: The definitions for composition of relations and set-theoretic union are *not* tautologous. This refutes the algebra of binary relations on which is based the free Kleene algebras with domain, to form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: McLean, B. (2019). Free Kleene algebras with domain. arxiv.org/pdf/1907.10386.pdf

Abstract: First we identify the free algebras of the class of algebras of binary relations equipped with the composition and domain operations.

2. Algebras of binary relations

We begin by making precise what is meant by an algebra of binary relations.

Definition 2.1. An algebra of binary relations of the signature {;, +, *, 0, 1} is a universal algebra A = (A, ;, +, *, 0, 1) where the elements of the universe A are all binary relations on some (common) set X, the base, and the interpretations of the symbols are given as follows:

• the binary operation ; is interpreted as composition of relations:
R ; S ≔ {(x, y) ∈ X^2 | ∃z ∈ X : (x,z) ∈ R ∧ (z, y) ∈ S }, (2.1.1.1)

Remark 2.1.1.1: We map only the consequent in Eq. 2.1.1.1 because the symbol " ; " is a not the symbol of a connective in classical logic.

LET p, q, r, s, t, x, y, z: p, q, R, S, X, x, y, z.

((x&y)<(t&t)) > ((%z<t)>(((x&%z)<r)&((%z&y)<s))) ;

```
TTTT TTTT TTTT TTTT( 48)
TTTT NNNN NNNN NNNN(  1)}x8
TTTT TTTT TTTT TTTT(  1)}
TTTT TTTT TTTT TTTT( 48)
TTTT FFFF FFFF FFFF(  1)}x8
TTTT TTTT TTTT TTTT(  1)}
```
(2.1.1.2)

• the binary operation + is interpreted as set-theoretic union:
$R + S \coloneqq \{(x, y) \in X^2 \mid (x, y) \in R \vee (x, y) \in S\}, \ldots$ (2.1.2.1)

```
(((x&y)<(t&t)) > (((x&y)<r)+((x&y)<s))) ;
                         FFFF TTTT TTTT TTTT( 48)
                         FFFF TTTT TTTT FFFF(  1)}x8
                         FFFF TTTT TTTT TTTT(  1)}
```
(2.1.2.2)

The definitions for composition of relations and set-theoretic union as rendered in Eqs. 2.1.1.2 and 2.1.2.2 are *not* tautologous. This refutes the algebra of binary relations on which is based the free Kleene algebras with domain.

Refutation of a complete axiomatization of reversible Kleene lattices

Abstract: We evaluate the main result theorem and two inequations as valid but not derivable from Kleene lattices. The theorem has an identical antecedent and consequent.. The two inequations are *not* tautologous and hence *not* valid, regardless of derivability without the union operator.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) C non-contingency, ∇, ordinal 2; (%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Brunet, P. (2019). A complete axiomatisation of reversible Kleene lattices.
arxiv.org/pdf/1902.08048.pdf ccsd-tech@ccsd.cnrs.fr paul.brunet-zamansky.fr

[W]e say that the equation e ≃ f is *valid* if the corresponding equality holds universally.

Remark 20.0: We write this as: □(e = f) > (e ≃ f). (20.0)

We may now prove the main result of this paper:
Theorem 20 (Main result). ∀e, f ∈ E_X, e ≡ f ⇔ e≃ f. (20.1)

LET p, q, r, s: e, f, E, x.

(q<(r&s))&((#p=q)=(#p=q)) ; **FF**TT **FF**TT **FF**TT **FFFF** (20.2)

(#p=q)=(#p=q) ; TTTT TTTT TTTT TTTT (20.3)

Remark 20.3: If the connective symbols =, ≡, ⇔, and ≃ are equivalents, then Eq. 20.3 is a trivial equality.

Example 22 (Levi's lemma). ... the following inequation holds:

$(e_1 \cdot e_2) \cap (f_1 \cdot f_2) \lesssim (e_1 \cdot \top \cdot f_2) + (f_1 \cdot \top \cdot e_2)$. (22.1)

LET p, q, r, s: e_1, e_2, f_1, f_2.

~((((p&(p=p))&s)+((r&(p=p))&q))<((p&q)&(r&s))) = (p=p) ;

TTTT TT**FF** T**F**T**F** T**FF**T (22.2)

Example 23 (Factorisation). Another troubling example is the following:

(a · b) ∩ (a · c) a ≲ ((⊤ · b) ∩ (⊤ · c)) .

LET p, q, r: a, b, c. (23.1)

(p&(((p=p)&q)&((p=p)&r))) < ~((p&q)&(p&r)) ;
FFTT **FFFF** **FF**TT **FF**FT (23.2)

As before, this inequation is valid, but it is not derivable, and it does not involve unions.

Eq. 20.2, 22.2, and 23.2 are *not* tautologous. Eq. 20.3 is a trivial equality. This means Eqs. 22.2 and 23.2 are *not* valid as claimed, regardless of their derivability status without unions.

Knowledge representation refutations

[See below.]

Remark 2. The instructor's assumption is that the pupil will win the necessity of his first court case, but no contingency is made for the event that the pupil possibly does not continue onto perform in any court. For example, there is no contingency for if the pupil became a lawyer but acted as a solicitor and not a barrister, then the litigious status of the pupil could never be tested before a court.

Remark 3. The rule of law in the West is that when an experienced lawyer as contractor, Protagoras, frames an agreement with a lesser experienced non-lawyer as contractee, Euathlus, then the contractor is held to a higher level of performance and closer reading of the agreement than is the contractee.

Remark 4. On the basis of no contingency arrangement for the contractee not to perform, the court would hold for a defective contract and disallow any claim by Protagoras. Should Euathlus counter-claim for lawyer's fees, the court would probably grant that motion on the basis of a frivolous lawsuit claim by Protagoras in the first place. In other words, Protagoras would lose in either scenario, that is, not obtain relief for instructing the pupil, and liable for the pupil's legal expenses in that event.

"After instruction, Euathlus decided not to enter the profession of law, (4.1.1)

and [then] Protagoras decided to sue Euathlus for the amount owed." (4.2.1)

((q&p)>((p&r)>(p>(q&s))))>
(~(p&r)>~(p>(q&s))) ;
FTFT FTFT FTFF FTFT (4.1.2)

(((q&p)>((p&r)>(p>(q&s))))>
(~(p&r)>~(p>(q&s))))>(q&(r+~r)) ;
TFTT TFTT TFTT TFTT (4.2.2)

Eqs. 4.2.1 and 4.2.2 are *not* tautologous, therefore that chain of events is suspicious.

Remark 5. The metaphysical question of "Was Euathlus morally wrong in not paying Protagoras for services rendered, regardless of outcome" can now be cast onto a physicalistic basis in this way. The proof tables for performance by Protagoras in Eq. 3.2 and for non-performance by Euathlus in Eq. 4.1.2 are contrasted:

(q&p) > ((p&r)>(p>(q&s))) ;
TTTT TTTF TTTT TTTT (3.2)

((q&p)>((p&r)>(p>(q&s)))) >
(~(p&r)>~(p>(q&s))) ;
FTFT FTFT FTFF FTFT (4.1.2)

Eq. 4.1.2 diverges *more* from tautology than does Eq. 3.2. This means a physicalistic basis if mapped for moral theology is a recent advance. In other words, Euathlus failed to do the right thing by withholding payment in any event, so as not to violate the intended spirit of the albeit defective contract.

"Protagoras argued that if he won the case he would be paid his money."
[In other words, if Eq. 4.2.1, then the Protagoras lawsuit obtains payment.] (5.1.1)

((((q&p)>((p&r)>(p>(q&s))))>
(~(p&r)>~(p>(q&s))))>(q&(r+~r))) >
((q&r)>(p>(q&s))) ;
TTTT TTTF TTTT TTTT (5.1.2)

"If Euathlus won the case, Protagoras would still be paid according to the original contract, because Euathlus would have won his first first case."
[In other words, if not Eq. 5.1.1, then 3.1.] (5.2.1)

~((((q&p)>((p&r)>(p>(q&s))))>
(~(p&r)>~(p>(q&s))))>(q&(r+~r))) >
((p&r)>(p>(q&s))) ;
TTTT TFTT TTTT TFTT (5.2.2)

Eqs. 5.1.2 and 5.2.2 are not equivalent and *not* tautologous.

"Euathlus, however, claimed that if he won, then by the court's decision he would not have to pay Protagoras."
[In other words, if not Eq. 5.1.1 or not 5.2.1, then not 3.1.] (6.1.1)

(~(((((q&p)>((p&r)>(p>(q&s))))>
(~(p&r)>~(p>(q&s))))>(q&(r+~r)))>
((q&r)>(p>(q&s))))+
~(~((((q&p)>((p&r)>(p>(q&s))))>
(~(p&r)>~(p>(q&s))))>(q&(r+~r)))>
((p&r)>(p>(q&s))))) >
~((q&p)>((p&r)>(p>(q&s)))) ;
TTTT TFTT TTTT TFTT (6.1.2)

"If, on the other hand, Protagoras won, then Euathlus would still not have won a case and would therefore not be obliged to pay."
[In other words, if Eq.5.2.1 or 5.2.2, then not 3.1.] (6.2.1)

((((((q&p)>((p&r)>(p>(q&s))))>
(~(p&r)>~(p>(q&s))))>(q&(r+~r)))>
((q&r)>(p>(q&s))))+
(~((((q&p)>((p&r)>(p>(q&s))))>
(~(p&r)>~(p>(q&s))))>(q&(r+~r))) >
((p&r)>(p>(q&s)))))>
~((q&p)>((p&r)>(p>(q&s)))) ;
FFFF FFFT FFFF FFFF (6.2.2)

Remark 6. Eqs. 6.1.2 and 6.2.2 are not equivalent and *not* tautologous. In fact 6.2.2 is nearly contradictory. This means regardless of who wins the lawsuit of Protagoras, Euathlus does not pay. Hence the Euathlus paradox is refuted and resolved by default in favor of Euathlus.

IV. Conclusion

A conclusion section is usually required. Although a conclusion may review the main points of the paper, do not replicate the abstract as the conclusion. A conclusion might elaborate on the importance of the work or suggest applications and extensions.

Disclaimer for conflict of Interest

The author asserts no conflict of interest.

References

[1] en.wikipedia.org/wiki/Supply_and_demand
[2] en.wikipedia.org/wiki/Paradox_of_the_Court
[3] Lisanyuk, E. (2017). "Why Protagoras gets paid anyway: a practical solution of the Paradox of Court". philarchive.org/archive/ELEWPG

Refutation of Kramers-Kronig relation

We assume the apparatus and method of Meth8/VŁ4.

We evaluate literature.cdn.keysight.com/litweb/pdf/5990-5266EN.pdf where

> Consider our odd function *ho (t)*, then multiply it by the signum function illustrated in Figure 3 [a step-wise, continuous function] and defined as:
>
> *signum*(*t*) □ −1 if *t* □ 0 and *signum*(*t*) □1 if 0 □ *t* (1.1)

LET p, q, 1, 0 : signum(t); t; (%p>#q); ((%p>#q)- (%p>#q)).

The designated *proof* value is `T`. Other values are `F` contradiction, `N` non-contingency (truthity), and `C` contingency (falsity). The 16-valued truth table is row-major and horizontal.

((q>((%p>#p)-(%p>#p)))>(p= (%p>#p))) &
((q<((%p>#p)-(%p>#p)))>(p=~(%p>#p))) ;
`CNTF CNTF CNTF CNTF` (1.2)

Eq. 1.2 as rendered is *not* tautologous. This means the Kramers-Kronig relation is refuted.

Refutation of Kripke frames from incompleteness of BAO's with ◊⊥=⊥

Abstract: Because Kripke frames require ◊⊥=⊥, *not* tautologous, they are refuted. What follows is BAOs so defined are also refuted (which we respectively demonstrate elsewhere), namely: Jónsson-Tarski, Lemmon-Scott; Fine-Thomason, van Benthem, Boolos-Sambin, and Lindenbaum-Tarski. These results also make the Blok dichotomy suspicious. Therefore these conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Holliday, W.H.; Litak, T. (2019). Complete additivity and modal incompleteness. arxiv.org/pdf/1809.07542.pdf tadeusz.litak@fau.de wesholliday@berkeley.edu

1 Introduction
The discovery of Kripke incompleteness, the existence of normal modal logics that are not sound and complete with respect to any class of Kripke frames, [is] called one of the two forces that gave rise to the "modern era" of modal logic … Kripke incompleteness was demonstrated with a bimodal logic … , shortly thereafter with complicated unimodal logics … , and later with simple unimodal logics. The significance of these discoveries can be viewed from several angles. From one angle, they show that Kripke frames are too blunt an instrument to characterize normal modal logics in general. More fine-grained semantic structures are needed. …

1.1 The semantic angle
The first angle on Kripke incompleteness—the realization that Kripke frames are not fine-grained enough for the study of normal modal logics in general—renewed interest in the algebraic semantics for normal modal logics based on Boolean algebras with operators (BAOs). A BAO is a Boolean algebra together with one or more unary operators, i.e., unary operation ◊ such that for all elements x, y of the algebra, ◊(x∨y)=◊x∨◊y [a trivial tautology] , and for the bottom element ⊥ of the algebra,

◊⊥=⊥. (1.1.1)

%(p@p)=(p@p) ; NNNN NNNN NNNN NNNN (1.1.2)

Remark 1.1.2: Eq. 1.1.2 is *not* tautologous (all T), but at the nearest table result state of truthity (N as non-contingency).

Every normal modal logic is sound and complete with respect to a BAO, namely, the Lindenbaum-Tarski algebra of the logic, according to a straightforward definition of when a modal formula is valid over a BAO. Kripke incompleteness can be better understood in light of the fact that Kripke frames correspond to BAOs that are complete (C), atomic (A), and completely additive (V), or CAV-BAOs.

Because Kripke frames require ◊⊥=⊥, *not* tautologous, they are refuted. What follows is BAOs so defined are also refuted (which we respectively demonstrate elsewhere), namely: Jónsson-Tarski, Lemmon-Scott; Fine-Thomason, van Benthem, Boolos-Sambin, and Lindenbaum-Tarski. These results also make the Blok dichotomy suspicious.

Confirmed refutation of Kripke-Platek (KP) / Constructive Zermelo-Fraenkel set theory (CZF)

Abstract: We evaluate the set induction scheme for Kripke-Platek set theory (KP) and Constructive Zermelo-Fraenkel set theory (CZF). It is *not* tautologous. This confirms the previous refutation.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) C non-contingency, ∇, ordinal 2; (%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Weaver, N. (2018). Predicative well-ordering. arxiv.org/pdf/1811.03543.pdf
nweaver@math.wustl.edu

11. Kripke-Platek and CZF: Kripke-Platek set theory (KP) and Constructive Zermelo-Fraenkel set theory (CZF) are two set theoretic systems which are also routinely claimed to be predicative. (According to Wikipedia, KP is "roughly the predicative part of ZFC" and CZF has "a fairly direct constructive and predicative justification".) In fact, both are impredicative for the same reason ID_1 is: yet again, the fallacy involves a confusion between conditions (A) and (B). In both cases the problematic axioms are the set induction scheme, which states,

for any formula P, (∀y)([∀x ∈ y P(x)] → P(y)) → (∀y)P(y). (11.1)

LET p, q, r: P, x, y
((#q<(#r&(#p&q)))>(#p&r))>(#p&#q) ;
FFNN **FF**NN **FF**NN **FF**NN (11.2)

Informally, if a predicate holds of a set y whenever it holds of all the elements of y, that predicate must hold of all sets. The informal justification for this scheme hinges on the premise that the universe of sets is built up in a well-ordered series of stages. One then applies progressivity of P to infer, inductively, that it holds of all sets in the universe. Just as with ID_1, this justification fails because being well-ordered in the sense of condition (A) does not predicatively entail the instances of condition (B) which would be needed to make the induction argument. And also as in that case, there is no option of strengthening the premise to say that the universe of sets is built up in a series of stages which are well-ordered in some stronger way which affirms condition (B). The instances of condition (B) which we would need in order to justify set induction involve all predicates expressible in the language of set theory, but the latter does not have an interpretation until we specify how the universe of sets is to be built up. So this would be circular.

Remark 11.2: Eq. 11.2 as rendered is not tautologous. This confirms the refutation as impredicative for KP and CZF.

The Kuratowski–Zorn lemma (Zorn's lemma)

From en.wikipedia.org/wiki/Zorn%27s_lemma:

> "To prove that I is an ideal, note that if a and b are elements of I, then there exist two ideals J, K ∈ T such that a is an element of J and b is an element of K."

LET pqrstu: abIJKT

```
((p&q)<r) > (((s&t)<u) & ((p<s)&(q<t))) ;
                                          TTTF TTTT TTTF TTTT
```

If the Kuratowski–Zorn lemma is suspicious, so also is the Ultrafilter lemma and the Prime Ideal theorem as a replacement for ZFC.

Lachlan problem solution

From: Sudoplatov, S.V. *The Lachlan Problem*. 2008.; math.nsc.ru/~sudoplatov/lachlan_eng_03_09_2008.pdf

We evaluate two equations from the text as a pilot survey of experimental results.

1.1. Syntactic characterization of the class of complete theories with finitely many countable models

DEFINITION [56] Lemma 1.1.1., page 18

$$\vDash \forall\ y\ ((x < y) \rightarrow \exists\ z\ ((x < z) \wedge (z < y) \wedge P_i(z))) \quad (18.1)$$

Meth8 script maps this as

LET: > Imply →; & And ^; (p=p) P_i;
necessity, universal quantifier ∀; % possibility, existential quantifier ∃;

```
(#x&#y) & ( (x<y) > ( %z & ( ((x<z) + (z<y)) + ( (p=p)&z)))) ;          (18.2)
```

The 128-line truth table for five models has these repeating fragments:

```
Model 1               Model 2.1             Model 2.2             Model 2.3.1           Model 2.3.2
FFFF FFFF FFFF FFFF   UUUU UUUU UUUU UUUU   UUUU UUUU UUUU UUUU   UUUU UUUU UUUU UUUU   UUUU UUUU UUUU UUUU
NNNN NNNN NNNN NNNN   EEEE EEEE EEEE EEEE   UUUU UUUU UUUU UUUU   IIII IIII IIII IIII   PPPP PPPP PPPP PPPP
```

where the designated truth values are Tautologous and Evaluated. The other logic values mean Contingent, Non contingent, F contradictory, Improper, Proper, and Unevaluated.

§ 2.5. The uniform *t*-amalgamation property and saturated generic models, page 67

$$\forall X\ ((\text{chi bar-Phi}(X) \wedge \text{phi}(X)) \rightarrow \exists Y\ (\text{chi bar-Psi}(X, Y) \wedge \text{psi}(X, Y))) \quad (67.1)$$

Meth8 script maps this as

LET: p chi; q bar-Phi; r phi; s bar-Psi; t psi

```
(#x&((p&(q&x))&(r&x))) > (%y&(((p&s)&(x&y))&(t&(x&y)))) ;          (67.2)
```

The 128-line truth table for five models has these repeating fragments:

```
Model 1               Model 2.1             Model 2.2             Model 2.3.1           Model 2.3.2
TTTT TTTT TTTT TTTT   EEEE EEEE EEEE EEEE   EEEE EEEE EEEE EEEE   EEEE EEEE EEEE EEEE   EEEE EEEE EEEE EEEE
TTTT TTTC TTTT TTTC   EEEE EEEU EEEE EEEU   EEEE EEEE EEEE EEEE   EEEE EEEP EEEE EEEP   EEEE EEEI EEEE EEEI
TTTT TTTC TTTT TTTT   EEEE EEEU EEEE EEEE   EEEE EEEE EEEE EEEE   EEEE EEEP EEEE EEEE   EEEE EEEI EEEE EEEE
```

§ 3.1. Generic [Ehrenfeucht] theory with a non-symmetric semi-isolation relation, page 82

By the construction of $M \vDash \forall\ X\ (\text{phi-sub-n}(X) \rightarrow \exists\ Y\ \text{psi-sub-n}(X; Y))$ (82.1)

Meth8 script maps this as

LET: p phi-sub-n; q psi-sub-n

#x&((p&x)>(%y&(q&(x&y)))) ; NFNF, FFFF, NFNN (82.2)

2. From the type to the formula strict order property", page 177

$\vDash \forall y\ (phi(a_1,y) \rightarrow phi(a_2,y))$ ^ $\exists y\ (\neg phi(a_1,y)$ ^ $phi(a_2,y))$ (4.11) (177.1)

Meth8 script maps this as

(#y&((p&(q&y)) > (p&(r&y)))) & (%y&((~p&(q&y))&(p&(r&y)))) ;
FFFF UUUU (177.2)

We conclude that sample Eqs 18.2 and 67.2 do not confirm a solution to the Lachlan problem.

Refutation of lambda λ-calculus and LISP

Abstract: We evaluate McCarthy's lambda λ-calculus in six equations. Five of the equations are *not* tautologous, and one equation is a *trivial* tautology. McCarthy's three-valued logic is not bivalent on which the LISP programming language is implemented, and hence also flawed.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET: ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⊢; < Not Imply, less than, ∈;
= Equivalent, ≡, ⊨; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
~(y < x) (x ≤ y), (x ⊆ y); (p=p) T as tautology; (p@p) **F** as contradiction.

From: de Vries, F-J. (2018). Many-valued logics inside λ-calculus: Church's rescue of Russell with Böhm trees. arxiv.org/pdf/1810.07667.pdf fdv1@le.ac.uk

LET: p, q, r, s: M; N.

3.1 Encoding Boolean logic in λ-calculus
In "the History of Lisp" ... John McCarthy mentions his "invention of the true conditional expression [if *M* then *N*1 else *N*2] which evaluates only one of *N1* and *N*2 according to whether *M* is true or false" and also his "desire for a programming language that would allow its use" in the period 1957-8. He also recalls "the conditional expression interpretation of Boolean connectives" as one of the characterising ideas of LISP. By this he means concretely the if-then-else construct (when applied to Boolean expressions only) in combination with the truth values T and F can be used as a basis for propositional logic ... with the following natural definitions:

¬*M* ≡ if *M* then F else T (3.1.1.1.1)

~p=(p>((p@p)+(p=p))) ; T**FTF** T**FTF** T**FTF** T**FTF** (3.1.1.1.2)

M ∧ *N* ≡ if *M* then *N* else *M* (3.1.1.2.1)

(p&q)=((p>q)+p) ; **FFF**T **FFF**T **FFF**T **FFF**T (3.1.1.2.2)

M ∨ *N* ≡ if *M* then *M* else *N* (3.1.1.3.1)

(p+q)=((p>p)+q) ; **F**TTT **F**TTT **F**TTT **F**TTT (3.1.1.3.2)

M→*N* ≡ if *M* then *N* else T (3.1.1.4.1)

(p>q)=((p>q)+(p=p)) ; T**F**TT T**F**TT T**F**TT T**F**TT (3.1.1.4.2)

Remark 3.1: Eqs. 3.1.1.x.2 as rendered are not tautologous. This means lambda calculus as conceived and as implemented in the programming language LISP (LISt Processor) is not bivalent, and hence refuted.

... It is easy to see that if-then-else behaves as intended in this encoding. When *B* reduces to T and F, we have respectively:

if T then M else N →→ M (3.1.2.1.1)

(p=p)>(p+(q>p)) ; TT**F**T TT**F**T TT**F**T TTFT (3.1.2.1.2)

if F then M else N →→ N (3.1.2.2.1)

(p@p)>(p+(q>q)) ; TTTT TTTT TTTT TTTT (3.1.2.2.2)

Remark 3.1.2: Eq. 3.1.2.2.2 is a *trivial* tautology because (q>q) as the antecedent reduces to if False, then (True or True).

... The set of finite propositions can be defined formally with a[n] inductive syntax ... It is not hard to prove by induction that all closed finite propositions have a unique finite normal form:

Lemma 3.1. *Let φ be a finite closed proposition. Then φ has a unique finite normal form,which is either* T *or* F.

Remark 3.1.4: Lemma 3.1 does not help with or follow from Eqs. 3.1.1 or 3.1.2.

3.3 Encoding three-valued McCarthy logic with help of Böhm trees McCarthy's three-valued sequential three-valued propositional logic [from Fig. 2]: ¬T=F; ¬F=T; ¬⊥=⊥.

Remark 3.3: The values ¬⊥=⊥ are *not* tautologous, hence rendering this line of reasoning as not bivalent and rejected.

Because the five non-trivial Eqs. are *not* tautologous and the three-valued logic is non-bivalent, we reject lambda calculus and LISP on which many theorem provers are implemented.

Refutation of lattice effect algebra

Abstract: A seminal definition of lattice effect algebra is *not* tautologous. This refutes lattice effect and lattice pseudoeffect algebras along with the chain effect of quasiresiduation. The conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Chajda, I. and Helmut Länger, H. (2019). Residuation in lattice effect algebras. arxiv.org/pdf/1905.05496.pdf ivan.chajda@upol.cz, helmut.laenger@tuwien.ac.at

In order to axiomatize quantum logic effects in a Hilbert space, Foulis and Bennett [1994] introduced the so-called effect algebras. … The aim of the present paper is to introduce the more general concept of quasiresiduation and to show that lattice effect algebras and lattice pseudoeffect algebras satisfy this concept.

Definition 1. An effect algebra is a partial algebra E=(E,+,′,0,1) of type (2,1,0,0) where (E,′,0,1) is an algebra and + is a partial operation satisfying the following conditions for all x, y, z ∈ E:
(E1) x + y is defined if and only if so is y + x and in this case x + y = y + x,
(E2) (x + y) + z is defined if and only if so is x + (y + z) and in this case (x + y) + z = x + (y + z),
(E3) x ′ is the unique u ∈ E with x + u = 1,
(E4) if 1 + x is defined then x = 0. (1.1)

LET p, s: x, s

((%s>#s)+p)>(p=(s@s)) ; TFTF TFTF TFTF TFTF (1.2)

Eq. 1.2 is *not* tautologous. This refutes lattice effect and lattice pseudoeffect algebras along with the chain effect of quasiresiduation.

Refutation of Lean theorem prover from Microsoft

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, r, s: p, r, x;
~ Not; & And; > Imply, →; = Equivalent, ↔;
# necessity, ∀, for all or every; % possibility, ∃, for one or some.

From: Avigad, J.; de Moura, L.; Kong, S. (2018). Theorem proving in Lean. Rel. 3.40. leanprover.github.io/theorem_proving_in_lean/quantifiers_and_equality.html

example: (∀ x, p x → r) ↔ (∃ x, p x) → r (4.4.1.1)

((#s&(p&s))>r)=((%s&(p&s))>r) ; `TTTT TTTT TNTN TTTT` (4.4.1.2)

example: (∃ x, p x → r) ↔ (∀ x, p x) → r (4.4.2.1)

((%s&(p&s))>r)=((#s&(p&s))>r) ;
`TTTT TTTT TNTN TTTT` (4.4.2.2)

example: (∃ x, r → p x) ↔ (r → ∃ x, p x) (4.4.3.1)

(%s&(r>(p&s)))=(r>(%s&(p&s))) ; `CCCC TTTT TTTT TTTT` (4.4.3.2)

Eqs. 4.4.1.2, /.2.2, and /.3.2 are *not* tautologous. Hence Lean prover from Microsoft is not bivalent and refuted.

Refutation of index sets of computable presentations of Lebesgue spaces

Abstract: In a seminal lemma, based on the term "Banach(f) and (Hilbert(f)", rewritten as Banach(f) & Hilbert(f), we recall that elsewhere we refute Banach and Hilbert spaces to discard as *non* tautologous segments of the universal logic VL4. Therefore, a conjunction of the two is still *non* tautologous.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , ·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Brown, T.A.; McNicholl, T.H.; Menelik, AVG. (2019).
On the complexity of classifying Lebesgue spaces. arxiv.org/pdf/1906.12209.pdf

1. Introduction This paper advances and interleaves two general frameworks. The first framework … is focused on establishing technical connections between computable structure theory .. and computable analysis .. . The second framework focuses on applying computability theoretic techniques to classification problems in mathematics. Herein, we apply an approach borrowed from effective algebra to produce a fine-grained algorithmic characterization of separable Lebesgue spaces among all separable Banach spaces.

5. Index sets of computable presentations of Lebesgue spaces with known exponent

Lemma 5.5. There is a Π-0-2 predicate Lspace ⊆ N N × N N so that for all f, g ∈ N N , if g names a real p ≥ 1, then Lspace(f, g) if and only if f names an L p -space presentation.

Proof. … Let Lspace(f, g) hold if and only if (Banach(f) and (Hilbert(f) ∧ g names 2) ∨ Disint(f, ΦDisint(f, g), g). (5.1.0)

Remark 5.1.1: Our interest is in the term "Banach(f) and (Hilbert(f)", rewritten as Banach(f) & Hilbert(f), (5.1.1)

Remark 5.1.2: Elsewhere we refute Banach and Hilbert spaces to discard as *non* tautologous segments of the universal logic VL4. Therefore, a conjunction of the two is still *non* tautologous. (5.1.2)

Refutation of Leibniz' identity of indiscernibles and Leo-III theorem prover

Abstract: Leibniz' identity of indiscernibles as ∀x∀y[∀F(Fx↔Fy)→x=y] is *no*t tautologous. Consequently the Leo-III theorem prover for higher-order paramodulated extensional logic is also refuted. These form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Identity_of_indiscernibles

Identity of indiscernible
For any x and y, if x and y have all the same properties, then x is identical to y:

∀x∀y[∀F(Fx↔Fy)→x=y] (2.1)

LET p, q, r: F, x, y.

((#p&#q)=(#p&#r))>(#q=#r) ; TTCT CTTT TTCT CTTT (2.2)

From: Steen, A.; Benzmüller,C. (2019). Extensional higher-order paramodulation in Leo-III. arxiv.org/pdf/1907.11501.pdf

Abstract: Leo-III is an automated theorem prover for extensional type theory with Henkin semantics and choice. Reasoning with primitive equality is enabled by adapting paramodulation-based proof search to higher-order logic.

3 Extensional higher-order paramodulation

[Fn 4] The Identity of Indiscernibles (also known as Leibniz's law) refers to a principle first formulated by Gottfried Leibniz in the context of theoretical philosophy.. The principle states that if two objects X and Y coincide on every property P, then they are equal, i.e. $\forall X_{\tau}.\forall Y_{\tau}.(\forall P_{o\tau}.PX \Leftrightarrow PY) \Rightarrow X=Y$, where "=" denotes the desired equality predicate. Since this principle can easily be formulated in HOL, it is possible to encode equality in higher-order logic without using the primitive equality predicate.

Remark: Eq. 2.2 as rendered is not tautologous, hence refuting Leibniz' identity of indiscernibles, it also refutes the conjecture as title above, including the Leo-III theorem prover for extensional logic.

Lenzen's "Leibniz’s Ontological Proof ... and the Problem of »Impossible Objects«"

(Lenzen, W. Log. Univers. (2017) 11: 85. https://doi.org/10.1007/s11787-017-0159-2) ; and page.mi.fu-berlin.de/cbenzmueller/papers/Lenzen2016_Leibniz_Ontological_Proof.pdf , /link.springer.com/article/10.1007/s11787-017-0159-2

In reproducing some of the conjectures above, we found what may be a mistake on pg. 12, section 5:

> Notwithstanding the question how the uniqueness of a necessary being, i.e. ∀x∀y(*E(x)* ∧ *E(y)* → *x* = *y*), might ever be proved, it seems clear that the requirement of the *existence* of a necessary being, (xii) ∃*x(E(x)*), again renders Leibniz's proof *circular*.
> (1.1)

We evaluate Eq 1 using the apparatus of Meth8 modal logic model checker of four valued logic system variant VŁ4.

LET: % ◇, possibility, ∃, existential quantifier; # □, necessity, ∀, universal quantifier;
~ Not; & ∧, And; > →, Imply;
p E, concept of existence; q x; r y;
vt validated as tautologous; nvt not validated as tautologous

We map Eq 1 in the affirmative with the "(xii)" expression as the antecedent implying the "i.e." expression as the consequent, as follows:

(%q&#(p&q))> ((#q&#r)& (((#p&q)& (#p&r))> (q= r))) ;
TTTC TTTT (1.2)

The repeating truth table fragment has T as designated tautology value and C as falsity contingent value; other values not shown are F as contradiction value and N as truth non contingent value.

Meth8 renders Eq 1.2 as not validated as tautologous, that is, Eq 1.1 is mistaken.

However, we do confirm that *6.1 The Algebra of Concepts* is not validated as tautologous by Meth8.

Briefest known ontological proof of God

The problem with Leibniz' ontological proof of the existence of God was in not defining "most perfect" from "perfect", and then repeating that definition throughout the arguments.

Using the Meth8 apparatus for system variant VŁ4, and the fact that respective existential quantifiers are inter-changeable with modal operators (elsewhere from our rendition of the corrected Square of Opposition):

LET: p God; % possibility, existential quantifier; # necessity, universal quantifier; > Imply;
= Equivalent to; (p=p) Tautologous, perfect; #(p=p) most perfect; T Tautology.

The equivalence of the respective quantifiers and modal operators was established in our updated Square of Opposition and corrections to syllogisms Modus Camestros and Modus Cesare elsewhere.

The result fragment is the repeating row of four values from the truth table of 16 values.

We test these sentences as antecedent (1), consequent (2), and proposition (3, 4).

T*he possibility exists of God as most perfect.* (1.1)

%(p>#(p=p)) ; TTTT ; (1.2)

Necessarily God exists as most perfect.
(2.1)

(#p> #(p=p)) ; TTTT ; (2.2)

It the possibility exists of God as most perfect, then necessarily God exists as most perfect.
(3.1)

%(p>#(p=p)) > (#p> #(p=p)) ; TTTT ; (3.2)

Eq 1.1 can be diluted by using "perfect" instead of "most perfect" in antecedent and consequent. The reason is that perfect is its own superlative, meaning "most perfect" is redundant as something "most perfectly perfect"

It the possibility exists of God as perfect, then necessarily God exists as perfect. (4.1)

%(p> (p=p)) > (#p> (p=p)) ; TTTT ; (4.2)

The advantage of this proof over that of Karl Popper is that the quality of perfection includes truthfulness and morality. This means that invoking the moral imperative (the existentialist uttering "I ought to ...") to show conscience is not needed to demonstrate that God is a moral being.

Lemmon D in Lemmon (1957)

#(#p>#q)+#(#q>#p) ;	not validated; however Model 2.1 tautologous

Refutation of deformation field of van Leunen

Abstract: The arguments by implication or equivalence of van Leunen's deformation field are refuted.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

> LET _ right-arrow accent vector; p, q, r, s: ∇_r, $\underline{\nabla}$, ξ_r, $\underline{\xi}$;
> ~ Not; + Or; - Not Or; & And, x; > Imply;
> % possibility, for any one or some, ∃; # necessity, for every or all, ∀.
> (s=s) T tautology; (s@s) **F** contradiction;
> (%s>#s) 1, N truthity; (%s<#s) 0, C falsity.

From: van Leunen, J.A.J. (2018). Mass and field deformation. vixra.org/pdf/1809.0564v1.pdf

> A special field supports the hop landing location swarm that resides on the floating platform. It reflects the activity of the stochastic process and it floats with the platform over the background platform. It is characterized by a mass value and by the uniform velocity of the platform with respect to the background platform. The real part conforms to the deformation that the stochastic process causes. The imaginary part conforms to the moving deformation. The main characteristic of this field is that it tries to keep its overall change zero. We call ⑊ the ***deformation field***.
>
> The first order change of a field contains five terms. Mathematically, the statement that in first approximation nothing in the field ⑊ changes indicates that locally, the first order partial differential □⑊ will be equal to zero.
>
> $\zeta = \nabla \xi = \nabla_r \xi_r - \langle \underline{\nabla}, \underline{\xi} \rangle + \underline{\nabla}\, \xi_r + \nabla_r\, \xi_r \pm \underline{\nabla} \times \underline{\xi} = 0$ (1.0)
>
> We rewrite Eq. 1.0 excluding the first two terms in the equality.
>
> $\nabla_r \xi_r - \langle \underline{\nabla}, \underline{\xi} \rangle + \underline{\nabla}\, \xi_r + \nabla_r\, \xi_r \pm \underline{\nabla} \times \underline{\xi} = 0$ (1.1)
>
> ((((p&r) - (q&s)) + ((q&r) + (p&r))) + ((q&s) + ~(q&s))) = (s@s) ;
> **FFFF FFFF FFFF FFFF** (1.2)
>
> The terms that are still eligible for change must together be equal to zero. These terms are:
>
> $\nabla_r \underline{\xi} + \underline{\nabla}\, \xi_r = 0$ (2.1)
>
> ((p&s)+(q&r))=(s@s) ; TTTT TT**FF** T**F**T**F** T**FFF** (2.2)
>
> In the following text $\underline{\xi}$ plays the role of the vector field and ξ_r plays the role of the scalar gravitational potential of the considered object.
>
> The argument is that Eqs. 2.1 imply 1.1. (3.1)

(((p&s)+(q&r))=(s@s)) > (((((p&r)-(q&s))+((q&r)+(p&r)))+((q&s)+~(q&s))) =(s@s)) ; **FFFF FF**TT **F**T**F**T **F**TTT (3.2)

Remark: Although Eqs. 1.1 and 2.1 are equivalent to zero, the implication argument of 3.1 may *not* omit the zero value.

However, a derivative of Eqs. 1.1 and 2.1 is that Eqs. 1.1 and 2.1 are *equivalent* by omitting the zero value. (4.1)

((p&s)+(q&r)) = ((((p&r)-(q&s))+((q&r)+(p&r)))+((q&s)+~(q&s))) ; **FFFF FF**TT **F**T**F**T **F**TTT (4.2)

Eqs. 1.2, 2.2, 3.2, and 4.2 as rendered are *not* tautologous. This refutes the deformation field of van Leunen.

van Leunen's symmetry flavor of fermions and weak modular lattice logic *not* confirmed

From: van Leunen, J.A.J. (2018). Structure of physical reality. vixra.org/pdf/1807.0167v2.pdf

We evaluate symmetry flavor of fermions taken as borrowed from the Standard Model from for

Symmetry type:	Up, Down as 1, 0; Anti-up 0, Anti-down 1;
Handedness is a bit:	Left 0, Right 1;
Color charge is two bits, arbitrarily:	None, Green, Red, Blue as 00, 01, 10, 11; and
Sign:	Negate -, Ignore +.

Hand Left L Right R	Hand L 0 R 1	Color: None 00 Green 01 Red 10 Blue 11	Color bits	Sign - 0 + 1	Sign	Symmetry type	Up 1 Down 0	Order asserted	Derived bits	Binary order as asserted	Actual binary count
L	0	N	00	−	0	anti-(down)-neutrino	1	15	0000	1111	2
R	1	G	01	+	1	anti-down quark	1	14	1011	1110	
R	1	R	10	+	1	anti-down quark	1	13	1101	1101	2
R	1	B	11	+	1	anti-down quark	1	12	1111	1100	
L	0	B	11	−	0	anti-up quark	0	11	0110	1011	2
L	0	G	01	−	0	anti-up quark	0	10	0010	1010	
L	0	R	10	−	0	anti-up quark	0	9	0100	1001	2
R	1	N	00	+	1	(anti-up) positron	0	8	1001	1000	
L	0	N	00	−	0	(up) electron	1	7	0000	0111	
R	1	R	10	+	1	up quark	1	6	1101	0110	2
R	1	G	01	+	1	up quark	1	5	1011	0101	
R	1	B	10	+	1	up quark	1	4	1111	0100	2
L	0	B	11	−	0	down quark	0	3	0110	0011	
L	0	G	01	−	0	down quark	0	2	0010	0010	2
L	0	R	10	−	0	down quark	0	1	0100	0001	
R	1	N	00	+	1	(down) neutrino	0	0	1001	0000	2

The problem is that van Leunen maps the fermions as one per ordinal in the list <0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15>, but the binary designations are two per ordinal in the list <0, 2, 4, 6, 9, 11, 13, 15>.

From: vixra.org/pdf/1302.0122v3.pdf, we also evaluate van Leunen's use of mathematical logic.

The slide show for equations of partially ordered set, distributive lattice, and modular lattice is confirmed as tautologous by Meth8/VŁ4.

However, not so of the equations for weak modular lattice:

LET p, q, r, s, t, u, v: a, b, c, d, g, n
((((p+q)&s)=s)&((((p&s)=v)&((q&s)=v))&(((p<u)&(q<u))=(s<u))))>
(((p<r)=((p+q)&r))=(p+((q&r)&(s&r)))) ;
FFFT **FF**TT TTTT TTTT,
FFFF **FF**T**F** TTTT TTTT,
TTTT TTTT TTTT TTTT

This means that the theory of weak modular lattice logic is *not* tautologous. That logic is the core of van Leunen's Hilbert book model, rendering it also as *not* tautologous.

The liar's paradox is resolved as not a paradox

From en.wikipedia.org/wiki/Liar_paradox, paraphrased into clear English as:

"the statement of a liar which states that what the liar states is a lie"

LET: p = a thing, "this" ; q = the assertion ; vt tautologous ; nvt not tautologous
> Imply ; < Not Imply ; = Equivalent ; @ Not Equivalent
(p@p) contradictory ; (p=p) Tautologous ; q = (p = (p@p))

(q = (p = (p@p))) > ((p = (p@p)) > (p= p)) ;
vt (1)

(q = (p = (p@p))) > (q) > (p= p)) ;
vt (2)

(p = (p@p)) > ((p = (p@p)) > (p= p)) ;
vt (3)

q > ((q = (p = (p@p))) > (p=p)) ;
vt (4)

We test if Eq 1-4 are co-equivalents.

((q>((q=(p=(p@p)))>(p=p))) = ((q=(p=(p@p)))>(q>(p=p)))) =
(((q=(p=(p@p)))>((p=(p@p))>(p=p))) = ((p=(p@p))>((p=(p@p))>(p=p)))) ;
vt (5)

We test Eq 1-4 for < Not Imply with result of nvt, and not all F as a contradiction.

Result: the liar paradox is resolved as tautologous, hence it not a paradox, and not a contradiction.

The problem with previous attempts is not evaluating the truth value of an assertion, regardless of the truth value of what the assertion states. The problem is overcome by using a separate propositional variable for the assertion as q, and another propositional variable p to build the expression of the assertion.

Refutation of self-referential sentences and provability: antimony of the liar

Abstract: The seven definitions evaluated are *not* tautologous. The first four equations refute the author's abstract that "a sentence cannot be denominated by p and written as p is not true". The next three refute that "in a system in which q denominates the sentence q is not provable it is not provable that q is true and not provable". The net result is refutation of the liar's antimony as a paradox (contradiction) and concludes that self-referential sentences are in fact *provable* as *not* tautologous. These findings provide a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Ferreira, J.C. (2008). The antimony of the liar and provability.
ia800401.us.archive.org/28/items/arxiv-0806.0635/0806.0635.pdf

Abstract: This work evidences that a sentence cannot be denominated by p and written as p is not true. It demonstrates that in a system in which q denominates the sentence q is not provable it is not provable that q is true and not provable.

3 Self-referential sentences and provability: … what happens when the self-referential sentence is of the form p is not α where α is different from true or not true or from something equivalent to either true or not true. (10.0.1)

Remark 10.0.1: We take "different" to mean not "equivalent". The word "something" implies invocation of the existential quantifier, but the truth table result below is not affected.

LET p, q: p, α.

q@((q=q)+(q@q)) ; TT**FF** TT**FF** TT**FF** TT**FF** (10.0.2)

Let us substitute recursively p in the sentence for the sentence denominated by p:

p is not α (10.1.1)

p@q ; **F**TT**F** **F**TT**F** **F**TT**F** **F**TT**F** (10.1.2)

(p is not α) is not α (10.2.1)

(p@q)@q ; **F**T**F**T **F**T**F**T **F**T**F**T **F**T**F**T (10.2.2)

((p is not α) is not α) is not α (10.3.1)

(((p@q)@q)@q ; **F**TT**F** **F**TT**F** **F**TT**F** **F**TT**F** (10.3.2)

Remark 11.0: We combine the two sentences in Eq. 10.0 as an implication in Eqs. 11 et seq.

Eqs. 10.0.1 implies 10.1.1. (11.1.1)

(q@((q=q)+(q@q)))>(p@q) ;
FTTT **F**TTT **F**TTT **F**TTT (11.1.2)

Eqs. 10.0.1 implies 10.2.1. (11.2.1)

(q@((q=q)+(q@q)))>((p@q)@q) ;
FTTT **F**TTT **F**TTT **F**TTT (11.2.2)

Eqs. 10.0.1 implies 10.3.1. (11.3.1)

(q@((q=q)+(q@q)))>(((p@q)@q)@q) ;
FTTT **F**TTT **F**TTT **F**TTT (11.3.2)

Remark 11.1: Eqs 11.1.2-11.3.2 return the same truth table, *not* tautologous, which refutes the liar's antimony as a paradox (contradiction) and concludes that self-referential sentences are in fact provable as *not* tautologous.

Liar paradox Arthur Prior

Arthur Prior asserts that these two statements are equivalent:

For "This statement (A) is contradictory.";

(p&q)=(q@q) ; nvt (6.1)

For "This statement (A) is tautologous, and this statement (B) is contradictory.";

((p&q)=(q=q))&((p&r)=(r@r)) ; nvt (6.2)

Hence for "This statement (A) is contradictory is equivalent to this statement (A) is tautologous, and this statement (B) is contradictory.":

((p&q)=(q@q)) = (((p&q)=(q=q))&((p&r)=(r@r))) ;

nvt (6.3)

However, making the statement name the same in Eq 6.1 and 6.2, that is "A", then

((p&q)=(q@q)) = (((p&q)=(q=q))&((p&q)=(q@q))) ;

nvt (6.4)

is the same result nvt as Eq 6.3.

Hence Meth8 shows Prior et al are mistaken, and their version of the Liar paradox is not a paradox.

Liar paradox Saul Kripke

Saul Kripke introduces contingency, on which Meth8 is based.

LET: (p=p) Tautologous; (p@p) contradictory; p paradoxical; s Smith; t Jones;
only, singly %; majority #; x big spender; y soft on crime.

If the only thing Smith says about ones is a majority of what Jones says about me is contradictory,

((s>%(at>#(p@p))) (7.1)

and Jones says only these three things about Smith: Smith is a big spender, Smith is soft on crime, and everything Smith says about me is tautologous then

& (t>(%s&(((s>x)$(s?y))&#(s>(t&(p=p))))))) > (7.2)

If Smith really is a big spender but is *not* soft on crime, then

(((s=x)&(s=~y)) > (7.3)

both Smith's remarks about Jones and Jones's last remark about Smith are paradoxical.

(((%(t>#p@p)))&#(s>(t&(p=p)))) = p)) ; (7.4)

So:

((s>%(t>#(p@p))) & (t>(%s&(((s>x)&(s>y))&#(s>(t&(p=p))))))) >
(((s=x)&(s=~y)) > (((%(t>#(p@p)))&#(s>(t&(p=p)))) = p)) ;
not tautologous (7.5)

Hence Meth8 shows Kripke is mistaken, and the Liar paradox is not a paradox.

Confirmation that every straight line through a point inside a circle intersects the circumference

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: point p, point q, circle, straight line;
~ Not; & And; + Or; > Imply, greater than; < Not Imply, less than; = Equivalent;
# necessity, for all or every.

"For every a straight line drawn through a point inside a circle, the line intersects a point on the circle." (1.0)

We rewrite Eq. 1.0 as:

"An interior point inside the circle and an exterior point outside the circle imply that every straight line intersecting both points intersects a point on the circle." (1.1)

```
((q<r)&(p>r))>(((#s>q)&(#s>p))>(#s>(p=r))) ;
                              TTTT TTTT TTTT TTTT        (1.2)
```

Eq. 1.2 is tautologous, hence confirming that every straight through a point inside a circle intersects the circumference of the circle.

Refutation of linear algebra as not bivalent

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET: ~ Not, $^{\perp}$; & And, ⊗ ; \ Not And, par; > Imply, greater than, ⊸, lollipop; = Equivalent, is;
necessity, for all or any; % possibility, for one or some;
p A, dollar; q B, candy bar; r C.

From: en.wikipedia.org/wiki/Linear_logic; remarkable formulae

Linear implication is defined in linear negation and multiplicative disjunction:

$A \multimap B := A^{\perp} ⅋ B$ (1.1)

(p>q)=(~p\q) ; TFFT TFFT TFFT TFFT (1.2)

Distributivity is defined as:

$A \otimes (B \oplus C) \equiv (A \otimes B) \oplus (A \otimes C)$ (2.1)

(p&(q+r))=((p&q)+(p+r)) ; TFTT FTFT TFTT FTFT (2.2)

Resource interpretation to avoid frame problem:

> "Suppose we represent having a candy bar by the atomic proposition *candy*, and having a dollar by *$1*. To state the fact that a dollar will buy you one candy bar, we might write the implication *$1 ⇒ candy*. But in ordinary (classical or intuitionistic) logic, from *A* and *A ⇒ B* one can conclude *A ∧ B*. So, ordinary logic leads us to believe that we can buy the candy bar and keep our dollar!" (3.1)

((p&p)>q)>(p&q) ; FTFT FTFT FTFT FTFT (3.2)

Linear transformation property:

If A**y**=d, then (αx+βy) = αAx+βAy = αc+βd . (4.1)

LET: p A; q α; r β; u c; v d; x x; y y.
Fragments are non-repeating tables of 16-values.

((p&y)=v)>(((q&x)+(r&y))=(((q&(p&x))+(r&(p&y)))=((q&u)+(r&v)))) ;

FFFF FFFF FFFF FFFF,
FFTT FFTT FFTT FFTT,
FFFF FFFF FFFF FFFF,
TTTT TTTT TTTT TTTT,
FTFT TTTT FTFT TTTT,
FTTT TTFT FTTT TTFT,

```
FTFT TTTT FTFT TTTT,
FTTT TTFT FTTT TTFT,
FTTT TTTT FTTT TTTT,
FTFT TTFT FTFT TTFT,
TFTF TTTT TFTF TTTT,
TFTT TTTT TFTT TTTT,
FTTT TTTT FTTT TTTT,
FTFT TTFT FTFT TTFT,
TFTF TTTT TFTF TTTT,
TFTT TTTT TFTT TTTT     (4.2)
```

Eqs. 1.2, 2.2, 3.2, and 5.2 as rendered are *not* tautologous. This means linear algebra is refuted on its face as not being bivalent.

> **Remark:** The linear transformation property is the defining characteristic of a linear map. However, Eq. 5.2 is not tautologous. This causes suspicion for the many systems relying on a segment of linear algebra.

Refutation of Karush-Kuhn-Tucker constraints for linear programming

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

LET: p LICQ linear independence of gradients constraint qualification;
q MFCQ Mangasarian-Fromovitz constraint qualification;
r CRCQ Constant rank constraint qualification;
s (CPLD Constant positive linear dependence constraint qualification => QNCQ Quasi-normality constraint qualification);

~ Not; & And; Imply >, ⇒; Not Imply <; = Equivalent; @ Not Equivalent.

From: en.wikipedia.org/wiki/Karush–Kuhn–Tucker_conditions and
Eustáuio, R.G.; Karas, E.W.; Ribeiro, A.A. (*undated* post 2006).
Constraint qualifications for nonlinear programming.
docs.ufpr.br/~ademir.ribeiro/ensino/cm721/kkt.pdf
rodrigogarcia1@bol.com.br (karas@mat.ufpr.br, ademir@mat.ufpr.br)

... although MFCQ is not equivalent to CRCQ: (0.1)

[i]t can be shown that LICQ => MFCQ => CPLD => QNCQ
and LICQ => CRCQ => CPLD => QNCQ (1.1.1)

(q@r)&(((p>q)>s)&((p>r)>s)) ; **FFFF FFFF FF**TT TT**FF** (1.1.2)

For Eq. 0.1 is taken to imply Eq. 1.1: (1.2.1)

(q@r)>(((p>q)>s)&((p>r)>s)) ; TT**FF FF**TT TTTT TTTT (1.2.2)

(and the converses are not true) (2.1.1)

(q@r)&(((p<q)<s)&((p<r)<s)) ; **FFFF FFFF FFFF FFFF** (2.1.2)

For Eq. 0.1 is taken to imply Eq.2.1: (2.2.1)

(q@r)>(((p<q)<s)&((p<r)<s)) ; TT**FF FF**TT TT**FF FF**TT (2.2.2)

Remark: It is not clear how Eqs. 1.1 and 2.1 "can be shown" from the Eustáuio paper.

As rendered, Eqs. 1.1.2, 1.2.2, and 2.2.2 are *not* tautologous. Eq. 2.1.2 is contradictory, as expected from a converse-type operation. This refutes constraint qualifications for linear programming.

Refutation of linear temporal logic (LTL)

Abstract: We evaluate two composite modal atoms which turn out to be reductions into single modal operators. This refutes the notion that "new temporal modalities are obtained" and forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Ashari, R.; Habib, S. (presenters). (date unknown). Linear temporal logic (LTL). Chapter 5. cs.colostate.edu/~france/CS614/Slides/Ch5-Summary.pdf

Syntax, Slide (4):

There are additional temporal operators:

◊	"eventually"	(eventually in the future)	[often]
□	"always"	(now and forever in the future)	[forever]

By combining the temporal modalities ◊ and □, new temporal modalities are obtained.

□◊*φ* "infinitely often *φ*" (4.1.1)

□◊*φ* reduces to ◊*φ*, "often *φ*" (4.1.2)

◊□*φ* "eventually forever *φ*" (4.2.1)

◊□*φ* reduces to □*φ*, "forever *φ*" (4.2.2)

Eqs. 4.1.2 and 4.2.2 as rendered are reductions of composite modal operators and hence refute the notion that "new temporal modalities are obtained".

Refutation of Liouville's theorem as not invertible

We assume Meth8/VŁ4 where the designated *proof* value is Tautology. The truth table is repeating fragments of 16-values, row major and horizontal. LET pqrtw ABRTW; > →, transition

We rely on: inside.mines.edu/~tohno/teaching/PH505_2011/liouville_dvorak.pdf

> Allow $W(A)$ to denote the phase volume of macrostate A, i.e. $W(A)$ is the number of microstates that realize macrostate A[; w]e can immediately conclude that
>
> $W(RA) = W(A)$. (1.1)

((w&p)>p)>((w&(r&p))=(w&p)) ; TFTF TFTF TFTF TFTF,
TTTT TTTT TTTT TTTT (1.2)

> Consider two distinct macrostates A and B in the same phase space. Let Γ denote the microscopic path through phase space that realizes the macroscopic transition $A \rightarrow B$. Denote the transformed macrostate A as TA for time evolved A. Liouville’s theorem preserves phase space volumes. Therefore, $W(TA) = W(A)$. We now consider only cases where the transition $A \rightarrow B$ is experimentally reproducible. For [Figure 3: Evolution of macrostates in a dynamical system.] this to be true, TA must lie entirely in B. We cannot control which microstate the system evolves into, but we require that all evolved microstates TA are a subset of B. This condition implies that $W(TA) < W(B)$. The number of microstates for macrostate B is greater than that of macrostate A. But Liouville’s theorem tells us $W(TA) = W(A)$, so experimental reproducibility of $A \rightarrow B$ means that $W(A) < W(B)$. This condition depends only on the initial configuration of the system because phase space volume is conserved. This is the requirement for experimental reproducibility and one explanation for entropy, $S/\ln W$. (2.1)

(((w&(t&p))=(w&q))&((w&(t&p))<(w&p))) > ((p>q)=((w&p)<(w&q))) ;
TTTT TTTT TTTT TTTT (2.2)

> Consider the reverse transition: why does macrostate B not evolve into A. This is equivalent to the transition $RB \rightarrow RA$. This transition requires additional information about the initial microstate of RB to transform it into the proper sub-region of RA - information we don’t typically have. Because $W(RA) < W(RB)$, this transformation is not experimentally reproducible. Liouville’s theorem connects the time evolved state to the initial state - their phase space volume are the same. Therefore, Liouville’s theorem places the requirement for experimental reproducibility (second law) on initial and final states $S(A) < S(B)$. Interestingly, nowhere does any notion of time enter this argument. In this derivation, increasing entropy is a requirement only for experimental reproducibility, not a forward direction in time.

(w&(r&p))<(w&(r&q))) > ~((q>p)=((r&q)>(r&p))) ;
TTTT TTTT TTTT TTTT,
TTTT TFTT TTTT T**F**TT (3.2)

Eq. 1.2 is *not* tautologous: it is not a theorem. Eq. 2.2 is tautologous: it is a constructive theorem.

However, Eq. 3.2 is *not* tautologous: as the reverse of Eq.2.2, it is not a theorem. This means Liouville's theorem is not invertive and hence not a reversible theorem.

Refutation of a class of Lipschitz horizontal vector fields in homogeneous groups

Abstract: We evaluate a Lipschitz horizontal vector field on Heisenberg group. It is *not* tautologous and further exemplifies that vector spaces are not bivalent. This forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Magnani, V.; Trevisan, D. (2016). On Lipschitz vector fields and the Cauchy problem in homogeneous groups. arxiv.org/pdf/1606.05486.pdf

Abstract. We introduce a class of "Lipschitz horizontal" vector fields in homogeneous groups, for which we show equivalent descriptions, e.g. in terms of suitable derivations. We then investigate the associated Cauchy problem, providing a uniqueness result both at equilibrium points and for vector fields of an involutive submodule of Lipschitz horizontal vector fields. We finally exhibit a counterexample to the general well-posedness theory for Lipschitz horizontal vector fields, in contrast with the Euclidean theory.

1. Introduction
In this setting, a "Lipschitz horizontal" vector field b on H(eisenberg) group is a mapping

$$x \mapsto b(x) = a_1(x)X_1(x) + a_2(x)X_2(x) \quad (1.5.1)$$

LET p, q, r, x, y, z:
a_1, a_2, b, x, X_1, X_2.

(x>(r&x))=(((p&x)&(y&x))+((q&x)&(z&x))) ;

```
FFFF FFFF FFFF FFFF(16)
TTTT FFFF TTTT FFFF(16)
FFFF FFFF FFFF FFFF(16)
TFTF FTFT TFTF FTFT(16)
FFFF FFFF FFFF FFFF(16)
TTFF FFTT TTFF FFTT(16)
FFFF FFFF FFFF FFFF(16)
TFFF FTTT TFFF FTTT(16)      (1.5.2)
```

Remark 1.5.2: Eq. 1.5.2 as rendered is not tautologous. This refutes the Lipschitz horizontal vector field as such.

Meth8 evaluation of **Löb's Theorem**

The definition of Löb's Theorem is taken from www.cs.cornell.edu/courses/cs4860/2009sp/lec-23.pdf .

A fourth issue involves the *undefinability of provability*: it is not possible to describe a predicate Prov that represents provability in a theory T such that ~Prov(contradictory) is a theorem in T . We call a predicate Prov a *provability predicate for* T if it satisfies the following conditions for all formulas X and Y.

If	|=T	X then |=T Prov(X)
	|=T	Prov(X⊃Y) ⊃ (Prov(X) ⊃ Prov(Y))
	|=T	Prov(X) ⊃ Prov(Prov(X))

The first condition states that every theorem should be provable, the second that the modus ponens holds for provability, and the third that provability is provable. Note that the second and third conditions are stronger than the first in the sense that the implication itself must be a theorem in T .
Note that a condition like |=T Prov(X) ⊃ X is not included in the definition, since this requirement cannot be satisfied unless T is inconsistent. In fact, Löb' s Theorem shows that this condition implies |=T X.

Theorem: [Löb's Theorem] If Prov is a provability predicate for a theory T that can represent the computable functions then |=T Prov(X) ⊃ X implies |=T X for any X.

1. for any X:	#p
2. such that ~Prov(contradictory) is a theorem in T:	((s=(~r&(p@p))
3. If X then Prov(X):	(p>(r&p))
4. Prov(X⊃Y) ⊃ (Prov(X) ⊃ Prov(Y)):	((r&(p<q))>((r&p)>(r&q)))
5. Prov(X) ⊃ Prov(Prov(X)):	((r&p)>(r&(r&p)))
6. Prov(X) ⊃ X implies T X, for any X:	(#p&(((r&p)>p)>p))

7. For (1 & (2 & (3 > ((4 & (5)) & 6))) > 6:

(#p&((s=(~r&(p@p))) & ((p>(r&p)) & (((r&(p<q))>((r&p)>(r&q)))&((r&p)>(r&(r&p))))))) >
(#p&(((r&p)>p)>p)) ; TTTT TTTT TTTT TTTT 49 steps

Note that validation of 7 is only made by including "for any X" (#p) for both main literals.

8. For (2 & (3 > ((4 & (5)) & 6)) > 6:

((s=(~r&(p@p))) & ((p>(r&p)) & (((r&(p<q))>((r&p)>(r&q)))&((r&p)>(r&(r&p)))))) >
(#p&(((r&p)>p)>p)) ; **F**T**F**T **F**T**F**N TTTT TTTT 47 steps.

Note that #p is included *only* in the consequent.

Shorter refutation of the Löb theorem and Gödel incompleteness by substitution of contradiction

Abstract: Löb's theorem $\Box(\Box X\rightarrow X)\rightarrow\Box X$ and Gödel's incompleteness as $\Box(\Box\bot \rightarrow\bot)\rightarrow\Box\bot$ are refuted.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

LET p, q: *X*; > Imply, →; @ Not Equivalent; # necessity, □; (p@p) **`F`** as contradiction, ⊥.

From: Gross, J. et al. (2016). Löb's Theorem. jasongross.github.io/lob-paper/nightly/lob.pdf jgross@mit.edu, jack@gallabytes.com, benya@intelligence.org

This, in a nutshell, is Löb's theorem: to prove *X*, it suffices to prove that *X* is true whenever *X* is provable. If we let $\Box X$ denote the assertion "*X* is provable," then, symbolically, Löb's theorem becomes:

$\Box(\Box X\rightarrow X)\rightarrow\Box X.$ (1.1)

#(#p>p)>#p ; `CTCT CTCT CTCT CTCT` (1.2)

Remark 1.2: Eq 1.2 as rendered is *not* tautologous, thus refuting Löb's theorem.

Note that Gödel's incompleteness theorem follows trivially from Löb's theorem: by instantiating *X* with a contradiction [⊥], we can see that it's impossible for provability to imply truth for propositions which are not already true. (2.1)

#(#(p@p)>(p@p))>#(p@p) ; `CCCC CCCC CCCC CCCC` (2.2)

Remark 2.2: Eq. 2.2, rendered as Eq. 1.2 with p substituted by (p@p), is *not* tautologous but consistently falsity as `C` for contingency. Hence Gödel's incompleteness theorem, as following trivially, is also refuted.

This means that the type of Löb's theorem becomes either $\Box(\Box X\rightarrow X)\rightarrow\Box X$ [Eq. 1.1], which is not strictly positive, or

$\Box(X\rightarrow X)\rightarrow\Box X,$ (3.1)

#(p>p)>#p ; `CTCT CTCT CTCT CTCT` (3.2)

which, on interpretation, must be filled with a general fixpoint operator. Such an operator is well-known to be inconsistent.

Remark Fn. 2: Eq. 3.2 as rendered produces the same truth table result as Eq. 1.2 and as another trivial refutation.

Denial of consistency for the Lobachevskii non Euclidean geometry

Abstract: We prove two parallel lines are tautologous in Euclidean geometry. We next prove that non Euclidean geometry of Lobachevskii is *not* tautologous and hence *not* consistent. What follows is that Riemann geometry is the same, and non Euclidean geometry is a segment of Euclidean geometry, not the other way around. Therefore non Euclidean geometries are a non tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables.
(See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
\> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊔ ;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
`~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: encyclopediaofmath.org/index.php/Lobachevskii_geometry

A [Lobachevskii] geometry [is] based on the same fundamental premises as Euclidean geometry, except for the axiom of parallelism. Lobachevskii geometry is the geometry of a Riemannian space of constant curvature. The proof of the consistency of Lobachevskii geometry is carried out by constructing an interpretation (a model).

Consider the Euclidean polygon below with (p,q), (r,s) parallel to (t,u), (v,w):

```
(p,q) _____ (r,s)
  |           |
  |           |
(t,u) _____ (v,w)
```

If u is less than q and w less than s, and q is equivalent to s and u equivalent to w, and p is less than r and t less than v, then q minus u is equivalent to s minus w (thereby keeping the edge (p,q), (r,s) parallel to (t,u), (v,w)). (1.1)

((((u<q)&(w<s))&((q=s)&(u=w)))&((p<r)@(t<v))) > ((q-u)=(s-w)) ;
TTTT TTTT TTTT TTTT (1.2)

Remark 1.2: Eq. 1.2 is tautologous and hence consistent.

The non Euclidean spherical geometry of Lobachevskii asserts parallel lines intersect at some point and hence are not parallel at that point. (2.0)

Remark 2.0: We note that Eq. 2.0 also applies to hyperbolic non Euclidean geometry.

If u is less than q and w less than s, and q is equivalent to s and u equivalent to w, and p is less than r and t less than v, then q minus u is *not* equivalent to s minus w (thereby *not* keeping the edge (p,q), (r,s) parallel to (t,u), (v,w)). (2.1)

((((u<q)&(w<s))&((q=s)&(u=w)))&((p<r)@(t<v))) > ((q-u)**@**(s-w)) ;

TTTT TTTT TTTT TTTT(10),
T**F**TT TTTT TTTT TTTT(1),
FTTT **FF**TT TTTT TTTT(1),
TTTT TTTT TTTT TTTT(2),
T**F**TT TTTT TTTT TTTT(2) (2.2)

Remark 2.2: Eq. 2.2 as rendered is *not* tautologous, meaning Eq. 2.0 is *not* consistent.

What follows is that Riemann geometry is the same, and non Euclidean geometry is a segment of Euclidean geometry, not the other way around. Therefore non Euclidean geometries are a non tautologous fragment of the universal logic VŁ4.

Refutation of the lonely runner conjecture with three runners

Abstract: We evaluate the conjecture of the lonely runner with three runners. We do *not* assume a runner may be stationary as a no-go contestant. The result is that the conjecture diverges from tautology by one logical value and hence is refuted. We then assume a runner can be stationary with result of the same truth table also to refute the conjecture.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

LET p, q, r, s: runner-1, runner-2, runner-3, number of runners;
~ Not; + Or ; & And; \ Not And; > Imply; = Equivalent; @ Not Equivalent;
(p=p) tautology; (p@p) contradiction, zero 0; (%p>#p) falsity, ordinal 1.

From: en.wikipedia.org/wiki/Lonely_runner_conjecture

Remark 0: Other implementations of the conjecture assume a runner may *not* run but remain stationary, and name that the lonely runner. However this initial implementation makes no such assumption because a non-runner is *not* a runner and hence removed from consideration.

No runner is stationary. (1.1.1)

(((p+q)+r)@(p@p)) = (p=p) ; **F**TTT TTTT **F**TTT TTTT (1.1.2)

No runner as equivalent to another runner implies the number of runners. (1.2.1)

((((p@q)&(q@r))&(p@r))>s) = (p=p) ;
TTTT TTTT TTTT TTTT (1.2.2)

No runner is stationary, and no runner as equivalent to another runner implies the number of runners. (1.3.1)

((((p+q)+r)@(p@p))&((((p@q)&(q@r))&(p@r))>s)) = (p=p) ;
FTTT TTTT **F**TTT TTTT (1.3.2)

Remark 1.1/2/3: While the truth table results for Eqs. 1.1.2 and 1.3.2 as rendered are equivalent, Eq. 1.2.2 is needed to establish that the unique runners establish the number of runners. Eqs. 1 as cast with model operators weaken the result.

A runner implies the fraction of ordinal one divided by the number of runners. (2.1.1)

(((p>((%p>#p)\s))+(q>((%p>#p)\s)))+(r>((%p>#p)\s))) = (p=p) ;
TTTT TTTT TTTT TTTC (2.1.2)

We evaluate the antecedent of Eqs. 1.3 and consequent of 2.1.

No runner is stationary, and no runner as equivalent to another runner implies the number of runners to imply a runner implies the fraction of ordinal one divided by the number of runners. (3.1.1)

((((p+q)+r)@(p@p))&((((p@q)&(q@r))&(p@r))>s)) >
(((p>((%p>#p)\s))+(q>((%p>#p)\s)))+(r>((%p>#p)\s))) ;
`TTTT TTTT TTTT TTTC` (3.1.2)

Remark 3: Eq. 2.1 and 3.1 produce the same truth table result as *close* to tautology but divergent by one `C` contingency, falsity value. This is due to `T>C=C`.

If we ignore Eq. 1.1 to establish that a runner can be permitted as stationary, to adopt the common assumption, the truth table result analog for Eq. 3 becomes:

((((p@q)&(q@r))&(p@r))>s) >
(((p>((%p>#p)\s))+(q>((%p>#p)\s)))+(r>((%p>#p)\s))) ;
`TTTT TTTT TTTT TTTC` (4.1.2)

Remark 4: By admitting a stationary runner, the conjecture results in the same truth table as Eq. 3.1.2.

Excepting Eq. 1.2, the other Eqs. are *not* tautologous. This means that with or without assuming a runner can be stationary as a no-go, the lonely runner conjecture is refuted.

Refutation of the modern, general, and strong Löwenheim–Skolem theorem

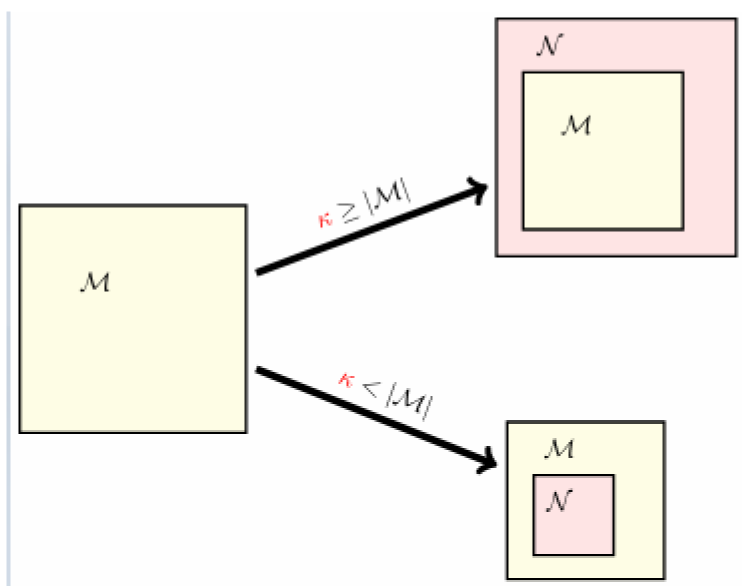
Abstract: We evaluate the equation for the modern, general, and strong Löwenheim–Skolem theorem. It is not tautologous, hence refuting the upward and downward parts. These form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≙, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B); ~(y < (z@z)) |y|.
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Löwenheim–Skolem_theorem

Precise statement
Illustration of the Löwenheim–Skolem theorem



The modern statement of the theorem is both more general and stronger than the version for countable signatures. In its general form, the Löwenheim–Skolem theorem states that

for every signature σ, every infinite σ-structure M and every infinite cardinal number κ ≥ |σ|, there is a σ-structure N such that |N| = κ and
if κ < |M| then N is an elementary substructure of M;
if κ > |M| then N is an elementary extension of M. (1.1.1)

LET p, q, r, s: κ , M, N, σ.

((#s&q)&~(#s>~((s@s)>#p)))>((%s&r)>(~((s@s)>(#r=p)) &
(((~((s@s)>#p)<q)>(r>q))&~((q>r)>(~((s@s)>#p)>q))))) ;
TTTT TTTT TTTT TT**CC** (1.1.2)

Remark 1.1.2: Eq. 1.1.2 is *not* tautologous, hence refuting the modern, general, and strong theorem.

The theorem is often divided into two parts corresponding to the two [k, |M| conditions] above. The part of the theorem asserting that a structure has elementary substructures of all smaller infinite cardinalities is known as the downward Löwenheim–Skolem theorem. .. The part of the theorem asserting that a structure has elementary extensions of all larger cardinalities is known as the upward Löwenheim–Skolem theorem. ..

Löwenheim–Skolem as a Hilbert style metatheorem

We derive and test a Hilbert style metatheorem for **Löwenheim–Skolem** [LS] as described at en.wikipedia.org/wiki/L%C3%B6wenheim%E2%80%93Skolem_theorem:

Assumption: A metatheorem is *a state machine.*

LET: p = K; q = M; r = N;
~ Not; < Not Imply; > Imply; & And; + Or; = Equivalent; @ Not Equivalent;
vt tautologous; nvt Not Validate tautologous; designated truth values Tautologous, Evaluated

If K<M, then M>N as ((p<q)>(q>r)) ; (1)
If K>M or K=M, then N>M as (((p>q)+(p=q))>(r>q)) ; (2)

To capture the parts of the metatheorem *as a state machine*, we evaluate combinations of Eq 1 and 2 in truth table fragments with the non designated values in bold as **contradictory**, Unevaluated:

(1)&(2): ((p<q)>(q>r)) & (((p>q)+(p=q))>(r>q));
TTTT **F**TTT; EEEE **U**EEE (3)
(1)>(2): ((p<q)>(q>r)) > (((p>q)+(p=q))>(r>q)) ;
TTTT **F**TTT; EEEE **U**EEE (4)
(1)+(2): ((p<q)>(q>r)) + (((p>q)+(p=q))>(r>q)) ;
TTTT TTTT; EEEE EEEE (5)
(1)<(2): ((p<q)>(q>r)) < (((p>q)+(p=q))>(r>q)) ;
not needed (6)

From Eq 5, the argument of [(1) Or (2)] seems to capture the essence of the metatheorem states, as a proof tautologous; however there is a state which is missing and unaccounted for as M=N.

We inject an accommodation for M=N as (q=r), or more properly rejection of that state as

M≠N as (q@r) ; (7)

Eq 5 is rewritten to avoid that unaccounted for state as:

(7)&[(1)+(2)]:

(q@r) & (((p<q)>(q>r)) + (((p>q)+(p=q))>(r>q)));

FFTT TT**FF**; **UU**EE EE**UU** (8)

The problem with the Löwenheim–Skolem Hilbert style metatheorem is that it does not hold for all machine states, and hence is not tautologous.

From Echenique, Saito (2015), "General Luce model"

LET: vt tautologous, nvt not tautologous; # All;
& And; \ Not And; > Imply; < Not Imply; ~ Not;
(%p>%#p) equal to 1

We begin later in the paper with simpler formula:

LET: pqrs, pxyX; vt Validated tautologous, nvt Not Validated tautologous

((#(q&r)<s)&(p&(q&r)))>(p>q) ; vt; Definition 7
(#(p&(q&r)) & ((p>q)&(q>r))) > (p>r) ; vt; Axiom 8
((#(p&(q&r))<s)& ((p<q)&(q<r))) > (p<r) ; vt ; Axiom 9

We then proceed to the beginning of the paper with more complex formula:

LET: pxyz, pxyX

((#(x&y)<z)&((p&(x&y))=(%p>%#p))) > (x>y) ;
vt ; Definition 2.i
((#(x&y)<z)&((p&(x&y))<(%p>%#p))) > ~(x>y) ;
vt ; Definition 2.ii

To this point, the General Luce model is tautologous.

Denial of the alleged Łukasiewicz nightmare in system Ł$_4$

Abstract: The alleged Łukasiewicz nightmare of (◊p&◊q)→◊(p&q) is not tautologous in Prover9; however, the equation recast in one variable as (◊p&◊~p)→◊(p&~p) is tautologous. In Meth8/VŁ4, both propositions are tautologous. This speaks for Meth8/VŁ4, based on the corrected modern Square of Opposition as an exact bivalent system, as opposed to Prover9, based on the uncorrected modern Square of Opposition as an inexact probabilistic vector space.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q: Schrödinger's cat is alive; Schrödinger's cat is dead;
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⊢; < Not Imply, less than, ∈;
= Equivalent, ≡, ⊨; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(p=p) T as tautology; (p@p) **F** as contradiction;
(%p<#p) C as contingency, Δ; (%p>#p) N as non-contingency, ∇;
~(y < x) (x ≤ y), (x ⊆ y).

From: Łukasiewicz, J. (1920). On three-valued logic in L. Borkowski (ed). 1970. 87-88.
Łukasiewicz, J. (1953). A system of modal logic. *Journal of Computing Systems*. 1:111-149.

Remark: The term "nightmare" was attributed to J-Y. Béziau in 2011 for the purpose of a four-valued schema of paraconsistent logic to evaluate systems based on numeric values such as -x, +x, -y, +y. The motivation was to discount the fact that the Ł$_4$ logic system was provably bi-valent (James, 2010, Estoril), and hence it was not mappable into a vector space, the continuing definition of paraconsistent logic.

This proposition is supposed to be egregious to logic system Ł$_4$: (◊p&◊q)→◊(p&q). (1.0)

If possibly the cat is alive and possibly the cat is dead, then possibly both the cat is alive and the cat is dead. (1.1)

(%p&%q)>%(p&q) ; `TTTT TTTT TTTT TTTT` (1.2)

Assumptions: ((exists(p) & exists(q))).
Goals (exists(p&q)). Exhausted. (1.3)

Prover9 invalidates Eq. 1.0 to show Ł$_4$ is untenable as an alethic logic.

If we preload p=~q as the antecedent to Eq. 1.0, then: (2.0)

If possibly the cat is alive is equivalent to Not (the cat is dead), then if possibly the cat is alive and possibly the cat is dead, then possibly both the cat is alive and the cat is dead. (2.1)

%(p=~q)>(%(p&q)>(%p&%q)) ; TTTT TTTT TTTT TTTT (2.2)

Assumptions: (exists(p<->-q)).
Goals: (exists(p)&exists(q))->(exists(p&q)).
Exhausted. (2.3)

Prover9 invalidates Eq. 2.0 to show Ł$_4$ is untenable as an alethic logic.

Remark 2.3: Eq. 2.3 shows Prover9 does not distribute the existential quantifier.

We rewrite Eq. 2.1 using one variable and its negation as respectively *alive* and *not alive*:

(◊p&◊~p)→◊(p&~p). (3.0)

If possibly the cat is alive and possibly the cat is not alive, then possibly both the cat is alive and the cat is not alive. (3.1)

(%p&%~p)>%(p&~p) ; TTTT TTTT TTTT TTTT (3.2)

Assumptions: (exists(p)&-exists(p)).
Goals: (exists(p&-p). Theorem. (3.3)

Prover9 validates Eq. 3.0 to show Ł4 is tenable as an alethic logic.

We explain Eqs. 1.2, 2.2, and 3.2 as rendered as tautologous in Meth8, but 1.3 as exhausted in Prover9 in this way For more than one variable, the vector space for arity with Prover9 diverges from the bivalance inherent inVŁ$_4$, in which modal operators and quantifiers are d distributive. This speaks to Meth8/VŁ4, based on the *corrected* modern Square of Opposition as an exact bivalent system, opposed to Prover9, based on the uncorrected modern Square of Opposition as an inexact probabilistic vector space.

Remark 3.2: Meth8/VŁ4 distinguishes between Eqs. 2.0 and 3.0 by protasis and apodosis as:

%p&%q ; CCCT CCCT CCCT CCCT (1.2.1.2)

%(p&q)=(p=p) ; CCCT CCCT CCCT CCCT (1.2.2.2)

and

%p&%~p ; CCCC CCCC CCCC CCCC (3.2.1.2)

%(p&~p)=(p=p) ; CCCC CCCC CCCC CCCC (3.2.2.2)

Rejection of trivial objections to modal logic Ł4

Abstract: We evaluate objections to the modal logic Ł4 by six equations in contra arguments which we reject as *not* tautologous. The concluding equation invoked as (((p=p)=(q=q)))=(r=r))=((p=q)=r) is *not* tautologous. We reject the *trivial* conclusion that "modal syllogisms with both necessary premises and with mixed premises cannot be distinguished while one is necessary and another assertoric[:] Łukasiewicz' modal logic is useless for investigating Aristotelian modal syllogistic". Hence we use our VŁ4 to invalidate objections to itself.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET: ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
\> Imply, greater than, →, ⊢; < Not Imply, less than, ∈;
= Equivalent, ≡, ⊨; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
~(y < x) (x ≤ y), (x ⊆ y); (p=p) `T`autology.

From: Dywan, Z. (2012). A simple axiomatization of Łukasiewicz's modal logic. Bulletin of the Section of Logic (BSL). 41:3/4, 149-153. filozof.uni.lodz.pl/bulletin/pdf/41_34_4.pdf

LET: p, q, r, s: p; q, φ; r, ψ; s.

Remark 0: Equations are numbered in order by page of text

(Lk_3) Mp → p (149.3.1)

%p>p ; `NTNT NTNT NTNT NTNT` (149.3.2)

(Lk_4) Mp (149.4.1)

%p=(p=p) ; `CTCT CTCT CTCT CTCT` (149.4.2)

(Ax_2) L(p ≡ p) (149.6.1)

#(p=p)=(p=p) ; `NNNN NNNN NNNN NNNN` (149.6.2)

(Ax_3) ~L(p ≡ p) (149.7.1)

~(#((p=p)=(p=p))=(p=p))=(p=p) ; `CCCC CCCC CCCC CCCC` (149.7.2)

M~(p ≡ p) → ~(p ≡ p) (152.1.1)

%(~((p=p)=(p=p))=(p=p))>~(p=p) ;
`NNNN NNNN NNNN NNNN` (152.1.2)

M~(p ≡ p) (152.2.1)

%(~(p=p)=(p=p))=(p=p) ; CCCC CCCC CCCC CCCC (152.2.2)

The six equations above are *not* tautologous which on their face refute the objections.

The author invokes the following equation to prove "modal syllogisms with both necessary premises and with mixed premises cannot be distinguished while one is necessary and another assertoric". (152.6.0)

Lφ ∧ ψ ≡ φ ∧ Lψ ≡ Lφ ∧ Lψ (152.6.1)

((#p&q)=(p&#q))=(#p&#q) ; **FFF**N **FFF**N **FFF**N **FFF**N (152.6.2)

Remark 152.6: The respective sentences are *trivially* equivalent, but not each tautologous. The sentences so taken together as an equation can not produce a tautology based on equivalents.

Consider the form of Tautology = Tautology = Tautology. (7.1)

((p=p)=(q=q))=(r=r) ; TTTT TTTT TTTT TTTT (7.2)

The respective sentences, while equivalent to themselves, do not constitute collegial proof of equality, as for example:

((p=(p=p))=(q=(q=q)))=(r=(r=r)) ;
FT**F**T **F**T**F**T **F**T**F**T **F**T**F**T (7.3)

Eq. 152.6.2 as rendered is *not* tautologous, and thus denies "that Łukasiewicz' modal logic is useless for investigating Aristotelian modal syllogistic".

Refutation of Lusin's separation theorem

Abstract: "In descriptive set theory and mathematical logic, **Lusin's separation theorem** states that if *A* and *B* are disjoint analytic subsets of Polish space, then there is a Borel set *C* in the space such that *A* ⊆ *C* and *B* ∩ *C* = Ø." We evaluate two renditions of that equation, both *non* tautologous, refuting it. Therefore, the separation theorem of Lusin forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Lusin%27s_separation_theorem

In descriptive set theory and mathematical logic, **Lusin's separation theorem** states that if *A* and *B* are disjoint analytic subsets of Polish space, then there is a Borel set *C* in the space such that

A ⊆ *C* and *B* ∩ *C* = Ø.[..] (1.1)

LET p, q, r, s: A, B, C, D

(~(r<p)&(q&r))=(s@s) ; TTTT TTT**F** TTTT TTT**F** (1.2)

Remark 1.1: If Eq. 1.1 is rendered in theorem variables, then

(~(C<A)&(B&C))=(D@D) ; TTTT TTTT TTTT TTTT,
TTTT TNTN TTTT TNTN,
TTTT TTTT TTCC TTCC,
TTTT TNTN TTCC TNC**F** (1.3)

Eqs. 1.2 and 1.3 are *not* tautologous, thereby refuting Lusin's separation theorem.

Refutation of Lyndon interpolation

Abstract: We evaluate the Lyndon interpolation on the logic **GL**. Each is *not* tautologous, and the combination is *not* tautologous, hence rendering both refuted.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET: ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →; < Not Imply, less than, ∈;
= Equivalent, ≡; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊; # necessity, for every or all, ∀, □;
~(y < x) (x ≤ y), (x ⊆ y); (p=p) Tautology.

From: Kurahashi, T. (2018). Uniform Lyndon interpolation property in propositional modal logics. arxiv.org/pdf/1809.00943.pdf kurahashi@n.kisarazu.ac.jp

LET p, q, r, s: ϕ phi, ψ psi, v, theta θ.

Definition 2.2. The least normal logic is called **K**.

K = □(p → q) → (□p → □q) (2.2.0.1)

#(p>q)>(#p>#q) ; TTTT TTTT TTTT TTTT (2.2.0.2)

GL = **K** + {□(□p → p) → □p} (2.2.1.1)

(#(p>q)>(#p>#q))&(#(#p>p)>#p) ;
CTCT CTCT CTCT CTCT (2.2.1.2)

Eq. 2.2.1.2 as **GL** is *not* tautologous. This means logic **GL** is *not* a logic proved as a theorem.

Definition 2.5. We say a logic L enjoys the Lyndon interpolation property (LIP) if for any formulas ϕ and ψ, if L ⊢

ϕ → ψ, (2.5.0.1)

p>q ; T**F**TT T**F**TT TFTT TFTT (2.5.0.2)

then there exists a formula θ satisfying the following properties:

1. v+(θ) ⊆ v+(ϕ) ∩ v+(ψ); (2.5.1.1)

~(((r&p)&(r&q))<(r&s)) = (p=p) ;
TTTT TTT**F** TTTT TTTT (2.5.1.2)

2. $v-(\theta) \subseteq v-(\phi) \cap v-(\psi)$; (2.5.2.1)

~(((~r&p)&(~r&q))<(~r&s)) = (p=p) ;
TTT**F** TTTT TTTT TTTT (2.5.2.2)

3. $L \vdash \phi \rightarrow \theta$; (2.5.3.1)

p>s ; T**F**T**F** T**F**T**F** TTTT TTTT (2.5.3.2)

4. $L \vdash \theta \rightarrow \psi$. (2.5.4.1)

s>q ; TTTT TTTT **FF**TT **FF**TT (2.5.4.2)

Such a formula θ is said to be a Lyndon interpolant of $\phi \rightarrow \psi$ in L.

The argument becomes: $\phi \rightarrow \psi$ implies that if $(v+(\theta) \subseteq v+(\phi) \cap v+(\psi))$ and $(v-(\theta) \subseteq v-(\phi) \cap v-(\psi))$ and $\phi \rightarrow \theta$ and $\theta \rightarrow \psi$, then θ as Lyndon interpolant. (2.5.5.1)

(p>q)>(((~(((~r&p)&(~r&q))<(~r&s))
&~(((r&p)&(r&q))<(r&s)))&((p>s)&(s>q)))>s) ;
FT**F**T **F**T**F**T TTTT TTTT (2.5.5.2)

Eq. 2.5.5.2 as rendered is *not* tautologous. This means the Lyndon interpolation is refuted.

Remark 5: To assert that the non-tautologous Lyndon interpolation applies to the non-tautologous logic **GL** is a further mistake. (5.0.1.1)

((p>q)>(((~(((~r&p)&(~r&q))<(~r&s))&
~(((r&p)&(r&q))<(r&s)))&((p>s)&(s>q)))>s)) & ((#(p>q)>(#p>#q))&(#(#p>p)>#p)) ;
FT**F**T **F**T**F**T CTCT CTCT (5.0.1.2)

Refutation of Majorana's 'root'

From: en.wikipedia.org/wiki/Relativistic_wave_equations.

Using the Meth8 apparatus we evaluate four equations in quantum physics from the above as labeled (3A) and (3B).

> LET: p ψ [(spinor) lc_psi]; q (E/c); r $a \cdot p$; s βmc [lc_beta * mc];
> T tautology, designated truth value; **F** contradiction
> Truth tables are four rows shown row-major horizontally.

"[Paul] Dirac ... furthered the application of equation (E^2)−(pc)^2 = (mc^2)^2 to the electron ... by various manipulations he factorized the equation into the form:

$$(E/c - \alpha \cdot p - \beta mc)\,(E/c + \alpha \cdot p + \beta mc)\psi = 0 \qquad (3.0.1)$$

(((q-(r-s))&(q+(r+s)))&p) = (p@p) ; TTTT T**F**TT T**F**TT T**F**TT ; (3.0.2)

From Eq. 3.0.1 the four "'roots'", "by a deviated approach to Dirac" from [Ettore] Majorana, are in Eq. 3.1.1.

$(E/c - \alpha \cdot p + \beta mc)\psi = 0$; of interest to Majorana ; (3.1.1)

((q-(r+s))&p) = (p@p) ; T**F**TT TTTT TTTT TTTT ; (3.1.2)

$(E/c + \alpha \cdot p - \beta mc)\psi = 0$; reversing the order of arithmetic in Eq 3.4.1 from -,+ to +,- (3.2.1)

(q+(r-s))&p) = (p@p) ; T**F**T**F** TTT**F** TTT**F** TTT**F** ; (3.2.2)

$(E/c - \alpha \cdot p - \beta mc)\psi = 0$; an analogous root directly from Eq 3.0.1 (3.3.1)

((q-(r-s))&p) = (p@p) ; TTTT T**F**TT T**F**TT T**F**TT ; (3.3.2)

$(E/c + \alpha \cdot p + \beta mc)\psi = 0$; an analogous root directly from Eq 3.0.1 (3.4.1)

((q+(r+s))&p) = (p@p) ; TTT**F** T**F**TT T**F**TT T**F**TT ; (3.4.2)

The results for Eqs. 3.0.2 and 3.3.2 are equivalent.

> **Remark:** In Eqs. 3.1.1, 3.2.1, 3.3.1, and 3.4.1, the removal of the literal element ψ alters the truth table rows, of Eqs. 3.1.2, 3.2.2. 3.3.2, and 3.4.2, for "TTTT" to read "**FFFF**", meaning that the results deviate further from tautology.

Eq. 3.1.1 of Majorana was the basis for the angel particle named a chiral Majorana fermion. From Eq. 3.1.2 Meth8 refutes that as a tautology because of the one value **F** in the truth table T**F**TT TTTT TTTT TTTT.

These results from mathematical logic make the experimental discovery of such a particle suspicious.

Denial of Summers' Malice and Alice as a logical puzzle

Abstract: The clauses of the Malice and Alice logical puzzle of Summers are mapped for evaluation of the conjecture. It should return tautology for all pairwise answers, before removing pairs of the antecedent to discover the correct pair of murder and victim. However, the conjecture is *not* tautologous, albeit one value shy. This means the puzzle as rendered is not well formed, thereby denying the status of a puzzle. Therefore the conjecture is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1;
(%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Avigad, J.; Lewis,Y.; van Doorn, F. (2004). Logic and proof.01.
leanprover.github.io/logic_and_proof/logic_and_proof.pdf

Propositional logic

2.1 A Puzzle The following puzzle, titled "Malice and Alice," is from George J. Summers' *Logical Deduction Puzzles*.

Alice, Alice's husband, their son, their daughter, and Alice's brother were involved in a murder.

One of the five killed one of the other four. The following facts refer to the five people mentioned:

(D1.1)

LET p, q, r, s, t, w, z:
Alice; Alice's husband; their daughter; their son; Alice's brother; bar; beach.

((((p=u)>(v=((q+r)+(s+t)))) + ((q=u)>(v=((p+r)+(s+t))))) + (((r=u)>(v=((p+q)+(s+t)))) + ((s=u)>(v=((p+q)+(r+t)))))) + ((t=u)>(v=((p+q)+(r+s)))) ; (D1.2)

Remark D1.2: While Eq. D1.2 is trivial, it defines combinations of murder and victim, and always returns tautology. This serves as the antecedent of the conjecture in our strategy, which could be selectively pared down to find the pair of the murderer and victim, should the conjecture return as tautologous.

We list our assumptions. Alice and her husband are assumed to be the *natural* parents of their children, so Alice and husband are older than either child, to avoid step children older than step parents or step relatives. The gender of players is not separate variables because it is enumerated once only.

1. A man and a woman were together in a bar at the time of the murder. (1.1)

w>((q+(s+t))&(p+r)) ; (1.2)

2. The victim and the killer were together on a beach at the time of the murder. (2.1)

z>(u&v) ; (2.2)

3. One of Alice's two children was alone at the time of the murder. (3.1)

(~(z>r)+~(w>s))&(~(w>r)+~(r>s)) ; (3.2)

Remark 3.1: Summer and Avigad apparently assume a child alone does not imply location other than bar or beach. Hence it is unclear if one alone could refer to before *or* after the murder on the beach.

4. Alice and her husband were not together at the time of the murder. (4.1)

((w>~p)&(w>~q))&((r>~p)&(r>~q)) ; (4.2)

Remark 4.1: This mapping is expanded individually for clarity.

5. The victim's twin was not the killer. (5.1)

Remark 5.1: A twin is Alice or her brother and the daughter or the son: (p+t), (r+s)
as: If p=v then t@ u; If t=v then p@u; If r=v then s@u; If s=v then r@u.
(((p=v)>(t@ u))+((t=v)>(p@ u)))+(((r=v)>(s@u))+((s=v)>(r@u))) ; (5.2)

6. The killer was younger than the victim. (6.1)

u<v ; (6.2)

Which one of the five was the victim?
The conjectured equation is: (D1) > ((((1)&(2))&((3)&(4))) > ((5)&(6))) ; (7.1)

(((((p=u)>(v=((q+r)+(s+t)))) + ((q=u)>(v=((p+r)+(s+t))))) + (((r=u)>(v=((p+q)+(s+t)))) +
((s=u)>(v=((p+q)+(r+t)))))) + ((t=u)>(v=((p+q)+(r+s)))))
> ((((w>((q+(s+t))&(p+r)))&(z>(u&v)))&(((~(z>r)+~(w>s))&(~(w>r)
+~(r>s)))&(((w>~p)&(w>~q))&((r>~p)&(r>~q))))) > (((((p=v)>(t@u))+((t=v)>(p@u)))+(((r=v)>
(s@u))+((s=v)>(r@u))))&(u<v))) ; (7)

TTTT TTTT TTTT TTTT(15),
TTTT **F**TTT TTTT TTTT(1) (7.2)

(You should assume that the victim's twin is one of the five people mentioned.)

Summers' book offers the following hint: "First find the locations of two pairs of people at the time of the murder, and then determine who the killer and the victim were so that no condition is contradicted."

Eq. 7 should be tautologous before removing pairs of the antecedent to discover the pair of murder and victim. However, 7 is *not* tautologous, albeit one value shy. This means the puzzle as rendered in bold in Summer's book is not well formed, thereby denying the puzzle.

Refutation of superposition as glue in Matita theorem prover

Abstract: We evaluate the substitution lemma for the successor function, smart application of inductive hypotheses, and proof traces of a complex example in the Matita standard library. Results are *not* tautologous, hence refuting superposition.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

From: Asperti, A.; Tassi, E. (2009). Superposition as a logical glue. arxiv.org/pdf/1103.3319.pdf
asperti@cs.unibo.it, enrico.tassi@inria.fr, publish@eptcs.org

LET p, q, r, s, t, u, v, w: A, B, C, S, i, j, k, M;
~ Not; + Or; - Not Or; & And; \ Not And, /; = Equivalent;
> Imply, greater than; < Not Imply, lesser than.

The substitution lemma says that (where S is the successor function)

for all k;i A[B=i][C=i+k] = A[C=S(k+ i)][B[C=k]=i] (4.2.1)

(p&((q=#t)&(r=(#t+#v))))=(p&((r=(s&(#v+#t)))&(q&((r=#v)=#t)))) ;
T**F**TT TTTT T**F**TT TTTT,
TNTC TTTC TNTT TTTC,
TNTC TCTT TNTT TCTT,
TNTT TTTC TNTT TTTT (4.2.2)

Remark 4.2.2: Eq. 4.2.2 is not tautologous, hence refuting the substitution lemma for the successor function.

[T]he inductive hypothesis

Hind : ∀j.M[B/i][C/k+ j] = M[C/S(k+ j)][B[C/k]/ j] (4.3.1)

(w&((q\#t)&(r\(#v+#u))))=(s&((r\(s&(#v+#u)))&(q&((r\#v)\#u)))) ;
TTTT TTTT TT**FF** TT**FF**,
TTTT TTTT TTNN TTNN,
TTTT TTTT TT**FF** TTNN,
TTTT TTTT TTNN TTNN,
FFFF **FFFF** **FF**TT **FF**TT,
FFNN **FF**NN **FF**CC **FF**CC,
FFNN NNNN **FF**TT NNTT,
FFFF NNNN **FF**TT NNTT,
FFNN NNNN **FF**CC NNTT,
FFFF NNNN **FF**CC NNTT (4.3.2)

It is evident that it is enough to instantiate j with i+1 but in order to unify (k+i)+1 with k+? j we have to use the associativity law for the sum! Hence the smart application of Hind succeeds where the normal application would fail.

Remark 4.3.2: Eq. 4.3.2 is *not* tautologous, hence refuting the smart application of an inductive hypotheses.

LET p, q, r: j, k, n;
necessity, for all or every; % possibility, for one or some.
(%s>#s) ordinal 1; ~(y<x) (x≤y).

Proof traces: Since most of the time is spent in searching the right theorems composing the proof, a natural idea is to let the automation tactic return a trace of the proof consisting of all library results used to build the proof. ... Using these simple proof traces automation becomes extremely fast, and almost comparable to a fully expanded proof script.

This is a relatively complex example borrowed from the Matita standard library. The goal to prove is $k \leq n-1$ under the assumption $H : j + k < n$. (5.1.1)

((p+q)<r)>~((r-(%s>#s))<q) ; `TNTT TTTT TNTT TTTT` (5.1.2)

Remark 5.1.2: Eq. 5.1.2 is *not* tautologous, hence refuting the goal and use of proof traces. The proof table diverges from tautology by two values for truthity.

Refutation of behavioral merelogy

Abstract: If P≤P′ and Q′≤Q, proposition $<>^{P'}_{P}<>^{Q}_{P'} = <>^{Q}_{P}$ is equivalent to $[]^{P'}_{P}[]^{Q}_{P'} = []^{Q}_{P}$ and respectively not tautologies.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

LET p, q, r, s: P, Q, P', Q';
~ Not; & And; > Imply; < Not Imply, less than; = Equivalent;
% possibility, for one or some, <>; # necessity, for every or all, [];
~(y<x) (x≤y).

From: Fong, B.; Myers, D.J.; Spivak, D.I. (2018). Behavioral mereology.
arxiv.org/pdf/1811.00420.pdf bfo@mit.edu

Proposition 23. Suppose that P ≤ P ′ and Q′ ≤ Q. Then

1. $<>^{P'}_{P}<>^{Q}_{P'} = <>^{Q}_{P}$ (Prop. 23.1.1)

(~(r<p)&~(q<s))>(((%s&%p)&(%q&%r))=(%q&%p)) ;
TTTT TTTT TTTC TTTT (Prop. 23.1.2)

2. $[]^{P'}_{P}[]^{Q}_{P'} = []^{Q}_{P}$ (Prop. 23.2.1)

(~(r<p)&~(q<s))>(((#s&#p)&(#q&#r))=(#q&#p)) ;
TTTT TTTT TTTC TTTT (Prop. 23.2.2)

Remark 23: Props. 23.1 and 23.2 as rendered produce the equivalent truth table result.

Props. 23.1 and 23.2 are *not* tautologous, diverge by one value of falsity, as C for contingency, and refute behavioral merelogy.

Refutation of spatial relations and claims in distributive mereotopology

Abstract: We evaluate the contact algebra logic of RCC-8 for tangential and non-tangential spatial relations. The respective representations in T_0 spaces for four equations to be equivalent. Two only are equivalent, with the iff implication chain as *not* tautologous. These results refute distributive mereotopology.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. Reproducible transcripts for results are available. (See ersatz-systems.com.)

LET p, q: a, b;
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨; < Not Imply, less than, ∈, ≺, ⊂;
= Equivalent, ≡, :=, ⇔, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, ∇, ordinal 1; (%z<#z) C as contingency, Δ, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).

From: Ivanova, T.; Vakarelov, D. (2019).
Distributive mereotopology: extended distributive contact lattices.
arxiv.org/pdf/1901.10442.pdf tatyana.ivanova@math.bas.bg, dvak@fmi.uni-sofia.bg

4.2 RCC-8 spatial relations. Definition 4.2 The system RCC-8.

[T]angential proper part – TPP(a, b): a≤b and a≪̸b and b≰a, (4.2.4.1)

~(q<p)&(~(p<q)&(q>p)) ; T**FF**T T**FF**T T**FF**T T**FF**T (4.2.4.2)

[T]angential proper part−1 – TPP−1(a, b): b≤a and b≪̸a and a≰b, (4.2.5.1)

~(p<q)&(~(q<p)&(p>q)) ; T**FF**T T**FF**T T**FF**T T**FF**T (4.2.5.2)

[N]ontangential proper part NTPP(a, b): a≪b and a≠b, (4.2.6.1)

(p<q)&(p@q) ; **F**T**FF** **F**T**FF** **F**T**FF** **F**T**FF** (4.2.6.2)

[N]ontangential proper part−1 – NTPP−1(a, b): b≪a and a≠b (4.2.7.1)

(q<p)&(p@q) ; **F**T**FF** **F**T**FF** **F**T**FF** **F**T**FF** (4.2.7.2)

7.1 Representations in T_0 spaces Claim 7.7: ... Then following conditions are equivalent:

(∀c ∈ D)(a + c = 1 → c ∈ Γ) iff (7.7.4.1)

LET p, q, r, s, t, u, v: a, c, h, Cl, D, Γ, X

(#q<t)&(((p+q)=(%p>#p))>(q<u)) ; **FF**NN **FF**NN **FF**NN **FF**NN(4),
FFFF FFFF FFFF FFFF(12) (7.7.4.2)

(∀c ∈ D)(h(a) ∪ h(c) = X(D) → Γ ∈ h(c)) iff (7.7.3.1)

(#q<t)&((((r&p)+(r&q))=(v&t))>(u<(r&q))) ;
FFNN **FF**NN **FF**NN **FF**NN(8),
FFFF FFFF FFFF FFFF(8) (7.7.3.2)

(∀c ∈ D)(−h(a) ⊆ h(c) → Γ ∈ h(c)) iff (7.7.2.1)

(#q<t)&(~((r&q)<(~r&p))>(u<(r&q))) ;
FFNN **FF**NN **FF**NN **FF**NN(8),
FFFF FFFF FFFF FFFF(8) (7.7.2.2)

Γ ∈ Cl(−h(a)) (7.7.1.1)

u<(s&~(r&p)) ; **FFFF FFFF FFFF FFFF**(8),
TTTT TTTT **FFFF F**T**F**T(8) (7.7.1.2)

Eqs 7.7.1.1 > 7.7.1.1 > 7.7.1.1 > 7.7.1.1 (7.7.5.1)

((u<(s&~(r&p)))>
((#q<t)&(~((r&q)<(~r&p))>(u<(r&q)))))>
(((#q<t)&((((r&p)+(r&q))=(v&t))>(u<(r&q))))>
((#q<t)&(((p+q)=(%p>#p))>(q<u)))) ;
TTCC TTCC TTCC TTCC(4),
TTTT TTTT TTTT TTTT(12) (7.7.5.2)

For RCC-8, the spatial relations for tangential and negation of tangential share the same truth table results. A similar case is for the non-tangential relations. This means the respective relations are not opposites as expected, but rather the same. Therefore the spatial relations for RCC-8 are not confirmed and refuted.

For representations in T_0 spaces, Eqs. 7.7.2.2 and 7.7.2.3 share the same truth table results, but 7.7.2.4 and 7.7.2.1 do not. Therefore the Eqs. are not all equivalent. Eq. 7.7.5.2 is the implication chain for iff of the Eqs. which is *not* tautologous as claimed.

These results refute distributive mereotopology.

Metaphysical problem of why there is something instead of nothing

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET ~ Not; & And; > Imply, greater than; = Equivalent; @ Not Equivalent;
necessity, for all; % possibility, for one or some;
(p=p) thing, tautology; (p@p) nothing, contradiction; %(p=p) some thing.

From: en.wikipedia.org/wiki/List_of_unsolved_problems_in_philosophy
en.wikipedia.org/wiki/Problem_of_why_there_is_anything_at_all

"Why there is something rather than nothing." (1.0)

We rewrite Eq. 1.0 as a logical expression of "Nothing implies something." (1.1)

(p@p)>%(p=p) ; TTTT TTTT TTTT TTTT (1.2)

"Why there is anything rather than nothing." (2.0)

We rewrite Eq. 2.0 as a logical expression of "Nothing implies anything". (2.1)

The difference from Eq. 1.1 is in the modal or quantified operator in the consequent going from possibility to necessity or from one/some to all.

(p@p)>#(p=p) ; TTTT TTTT TTTT TTTT (2.2)

Eqs. 1.2 and 2.2 as rendered are tautologous, meaning anything comes from nothing.

The problem is resolved in the answer that nothing can *not* come from anything.

Remark: By contrast in classical logic, the negation of Eq. 1.1 as Not(Eq. 1.1) is "nothing can not come from something". In other words, "something cannot imply nothing". This is because contradiction on the implication connective is where truth implies false, disallowed as a proof.

Meth8/VŁ4 self-proves in one variable for validity, consistency, completeness, and soundness

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal

LET: > Imply, greater than; < Not Imply, less than; = Equivalent; @ Not Equivalent;
% possibility, for one or some; # necessity, for every or all;
(%p>#p) truthity; (%p<#p) falsity; (p=p) tautology; (p@p) contradiction.

We test Meth8/VŁ4 using itself in one variable for the four qualities of a perfect logic system:

1. Validity – Falsity (or contradiction) as consequent is not implied by truthity (or tautology) as antecedent. (1.0)

Truthity implying falsity is a falsity (1.1.1)

((%p>#p)>(%p<#p))=(%p<#p) ;
`TTTT TTTT TTTT TTTT` (1.1.2)

Tautology implying contradiction is a contradiction (1.2.1)

((p=p)>(p@p))=(p@p) ; `TTTT TTTT TTTT TTTT` (1.2.2)

2. Consistency – Truthity (or tautology) conflicts with its opposite of falsity (or contradiction). (2.0)

Truthity is not equal to falsity (2.1.1)

(%p>#p)@(%p<#p) ; `TTTT TTTT TTTT TTTT` (2.1.2)

Tautology is not equal to contradiction (2.2.1)

(p=p)@(p@p) ; `TTTT TTTT TTTT TTTT` (2.2.2)

3. Completeness – Any truthity (or falsity) implies its tautology (or contradiction). (3.0)

Any truthity implies its tautology. (3.1.1)

#(%p>#p)>(p=p) ; `TTTT TTTT TTTT TTTT` (3.1.2)

Any falsity implies its contradiction. (3.2.1)

#(%p<#p)>(p@p) ; `TTTT TTTT TTTT TTTT` (3.2.2)

4. Soundness – Any tautology (or contradiction) implies its truthity (or falsity). (4.0)

Any tautology implies its truthity. (4.1.1)

#(p=p)>(%p>#p) ; TTTT TTTT TTTT TTTT (4.1.2)

Any contradiction implies its falsity. (4.2.1)

#(p@p)>(%p<#p) ; TTTT TTTT TTTT TTTT (4.2.2)

Eqs. 1, 2, 3, and 4 are tautologous. This means Meth8/VŁ4 proves itself, and in one variable.

Remark: This also serves as the contra-example to the incompleteness theorem of Gödel which states a logic system cannot prove itself (and certainly not in one variable).

Meth8 versus Prover9

A problem from Vladimir Lifshitz (2007):

In Prover9, "Prover9 exit: exhausted"

```
exists x exists x1 all y exists z exists z1
(  ( -p(y,y) | p(x,x)   | -s(z,x)  ) &
   ( s(x,y)  | -s(y,z)  | q(z1,z1) ) &
   ( q(x1,y) | -q(y,z1) | s(x1,x1) )   ) .
```

In Meth8, nvt, with this truth table fragment:

```
(%p&(%q&(#r&(%t&%u)))) &
((((~v&(r&r))+((v&(p&p))+(~w&(t&q)))) & (w&(p&r))+((~w&(r&t))+(x&(u&u))))) &
  (((x&(q&r))+(~q&(r&t)))+(w&(q&q)))) ; nvt
```

```
TTTT TTTT TTTT TTTT   EEEE EEEE EEEE EEEE   EEEE EEEE EEEE EEEE   EEEE EEEE EEEE EEEE   EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT   EEEE EEEE EEEE EEEE   EEEE EEEE EEEE EEEE   EEEE EEEE EEEE EEEE   EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT   EEEE EEEE EEEE EEEE   EEEE EEEE EEEE EEEE   EEEE EEEE EEEE EEEE   EEEE EEEE EEEE EEEE
TTTT TTTC TTTT TTTC   EEEE EEEU EEEE EEEU   EEEE EEEE EEEE EEEE   EEEE EEEP EEEE EEEP   EEEE EEEI EEEE EEEI
. . . . ^ . . . . ^   . . . . ^ . . . . ^   . . . . . . . . . .   . . . . ^ . . . . ^   . . . . ^ . . . . ^
```

Refutation of Church's thesis as a consistency property to fulfill the minimalist foundation

Abstract: Church's thesis (CT) is *not* tautologous as an essential consistency property to fulfill the requirement of the intensional level of a constructive foundation proposed of the minimalist foundation (MF) for constructive mathematics. Therefore, this relegates CT and MF to a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Maietti, M.E.; Maschio, S.; Rathjen, M. (2019). arxiv.org/pdf/1905.11966.pdf
A realizability semantics for inductive formal topologies, Church's thesis and axiom of choice.

Church's thesis (CT) ... states that from any total relation on natural numbers we can extract a (code of a) recursive function by using the Kleene predicate T and the extracting function U

(CT) (∀x∈N)(∃y∈N)R(x,y)→(∃e∈N)(∀x∈N)(∃z∈N)(T(e,x,z)∧R(x,U(z))) (1.1)

LET r, s, t, u, w, x, y, z:
R, N, t, U, e, x, y, z.

(((#x<s)&(%y<s))&(r&(x&y)))>
((%w<s)&((#x<s)&(%z<s)))&((t&(w&(x&z)))&((r&x)&(u&z)))) ;

```
TTTT TTTT TTTT TTTT(48)
TTTT CCCC TTTT TTTT(16)
TTTT TTTT TTTT TTTT(48)
TTTT CCCC TTTT TTTT( 8)
TTTT CCCC TTTT TTTT( 3)}x2
TTTT TTTT TTTT TTTT( 1)}
```
(1.2)

Such a consistency property is essential to fulfill the requirement of the intensional level of a constructive foundation proposed [toward a minimalist foundation for constructive mathematics].

Eq. 1.2 as rendered is *not* tautologous, to refute Church's thesis as an essential consistency property to fulfill the requirement of the intensional level of a constructive foundation proposed of the minimalist foundation (MF) for constructive mathematics.

Refutation of set of cycles in classical real Minkowski plane

From: en.wikipedia.org/wiki/Minkowski_plane

$$P := (R\cup\{\infty\})^2 = R^2\cup (\{\infty\}\times R)\cup(R\times\{\infty\})\cup\{(\infty,\infty)\},\ \infty\notin R,\ \textbf{the set of points}, \qquad (1.1)$$

$$Z := \{\{(x,y)\in R^2|y=ax+b\}\cup\{(\infty,\infty)\}|a,b\in R,a\neq 0\}\cup\{\{(x,y)\in R^2|y=ax-b+c,x\neq b\}\cup\{(b,\infty),(\infty,c)\}| a,b,c\in R,a\neq 0\},\ \textbf{the set of cycles}. \qquad (2.1)$$

We assume the apparatus and method of Meth8/VŁ4. The designated *proof* value is T tautologous. Repeating fragments of the truth table results are 16-values as row-major, and presented horizontally.

LET r s t u v x y : R a b ∞ c x y;
~ Not; & And, ×, ∪, ","; > Imply, |, greater than; < Not Imply, lesser than, ∈; = Equivalent to; @ Not Equivalent to, ≠; + Or; - Not Or; ~(p>q) (p≤q); ~(p<q) p∉q;
% possibility, existential for one or some; # necessity, universal for all; (s@s) logical 00; (%s>#s)-(%s>#s) numeric zero as one minus one.

P, the set of points:

```
~(u<r) > (((r&u)&(r&u))=(((r&r)&(u&r))&((r&u)&(u&u))));
                                        TTTT TTTT TTTT TTTT     (1.2)
```

Eq.1.2 as rendered is tautologous. This means the set of points in the classical real Minkowski plane are confirmed.

Z, the set of cycles, using logical 00:

```
((((x&y)<(r&r))>(y=((s&x)+t)))&((((s&t)<r)&~(s=(s@s)))>(u&u))) & ((((x&y)<(r&r)) & ((y=((s\(x-
t))+v))&~(x=t)))&(((((s&t)&v)<r)&~(s=(s@s)))>((t&u)&(u&v))));
                                        FFFF FFFF FFFF FFFF     (2.2.1)
```

Z, the set of cycles, using numeric zero as one minus one:

```
((((x&y)<(r&r))>(y=((s&x)+t)))&((((s&t)<r)&~(s=((%s>#s)-(%s>#s))))>(u&u))) &
((((x&y)<(r&r))&((y=((s\(x-t))+v))&~(x=t)))&(((((s&t)&v)<r)&~(s=((%s>#s)-
(%s>#s))))>((t&u)&(u&v))));             FFFF FFFF FFFF FFFF;
                                        FFFF FFFF TTTT FFFF     (2.2.2)
```

Eqs. 2.2.1 and 2.2.2 are *not* tautologous. This means the set of cycles in the classical real Minkowski plane are refuted.

Remark: Eq. 2.2.2 as rendered numerically provides a finer level of detail in proof results than Eq. 2.2.1 logically. Hence Eq. 2.2.2 shows *not* contradictory, but obviously also *not* tautologous.

What follows is that basing quantum theory on the set of cycles in the classical real Minkowski plane is suspicious.

Refutation of a modal aleatoric calculus for probabilistic reasoning: extended version

Abstract: We evaluate a modal aleatoric calculus for probabilistic reasoning using the assumption of probabilistic definitions as P(¬α) = 1−P(α). Five equations in Lemma 1 and its argument are tested. All equations are *not* tautologous, hence refuting the calculus.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r, s: P, x, y, z;
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⊢; < Not Imply, less than, ∈;
= Equivalent, ≡, ⊨; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(p=p) T as tautology; (p@p) **F** as contradiction;
(%p<#p) C as contingency, Δ; (%p>#p) N as non-contingency, ∇;
~(y < x) (x ≤ y), (x ⊆ y).

From: French, T.; Gozzard, A.; Reynolds, M. (2018). A modal aleatoric calculus for probabilistic reasoning: extended version. arxiv.org/pdf/1812.11741.pdf
tim.french@uwa.edu.au; mark.reynolds@uwa.edu.au; andrew.gozzard@research.uwa.edu.au

P(¬α) = 1−P(α), where α = q, r, s (0.1)

(((p&~q)=((%p>#p)-(p&q)))&((p&~r)=((%p>#p)-(p&r))))&((p&~s)=((%p>#p)-(p&s))) ; NCNC NCNC NCNC NCNT (0.2)

Lemma 1.

1−P(x)P(y)−P(¬x)P(z) (2.1)

(%p>#p)-((q&r)-(~q&s)) ; **FFFF FF**CC CC**FF** CCCC (2.2)

1−P(x)(1−P(¬y))−P(¬x)(1−P(¬z)) (3.1)

(%p>#p)-(((p&q)&((%p>#p)-(p&~r)))-((p&~q)&((%p>#p)-(p&~s)))) ;
FFFF FFFC **F**C**F**F **F**C**F**C (3.2)

1−P(x) +P(x)P(¬y)−P(¬x) +P(¬x)P(¬z)) (4.1)

(%p>#p) - (((p&q)+((p&q)&(p&~r))) - ((p&~q)+((p&~q)&(p&~s)))) ;
FC**F**C **F**C**F**C **F**C**F**C **F**C**F**C (4.2)

P(x)P(¬y) +P(¬x)P(¬z)) (5.1)

(q&~r)+(~q&s) ; **FF**TT **FFFF** TTTT TTT**F** (5.2)

The main argument of Lemma 1 is that if Eq. 0.1, then 2.1 = 3.1 = 4.1 = 5.1. (6.1)

((((p&~q)=((%p>#p)-(p&q)))&((p&~r)=((%p>#p)-(p&r))))&((p&~s)=
((%p>#p)-(p&s)))) > ((((%p>#p)-((q&r)-(~q&s)))=
((%p>#p)-(((p&q)&((%p>#p)-(p&~r)))-((p&~q)&((%p>#p)-(p&~s))))))=
(((%p>#p) - (((p&q)+((p&q)&(p&~r))) - ((p&~q)+((p&~q)&(p&~s)))))=
((q&~r)+(~q&s)))) ; TNCT TNTN CTCT CTTN (6.2)

Eqs. 0.2 and 2.2-5.2 are *not* tautologous. Lemma 1 as 6.2 is also *not* tautologous. This refutes a modal aleatoric calculus for probabilistic reasoning. We stop analysis after Lemma 1.

Refutation of coalgebraic geometric modal logic

Abstract: Two definitions are *not* tautologous, hence denying the monotone functor on KHaus. What follows is that the use of coalgebra to manufacture a geometric modal logic is refuted. Therefore the conjecture is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables.
(See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Bezhanishvili, N.; de Groot, J.; Venema, Y. (2019). Coalgebraic geometric logic.
arxiv.org/pdf/1903.08837.pdf n.bezhanishvili@uva.nl y.venema@uva.nl jim.degroot@anu.edu.au

Abstract: Using the theory of coalgebra, we introduce a uniform framework for adding modalities to the language of propositional geometric logic.

4 The monotone functor on KHaus: Definition 4.4. Let F be a frame. Let MF be the frame generated by □a,◊a, where a ranges over F, subject to the relations [... where a, b ∈ F and A is a directed subset of F.] (4.4.0)

Remark 4.4.0: The clauses invoking F above are ignored because the equations below as consequents do not contain F.

□(a ∧ b) ≤ □a (M1) (4.4.1.1)

LET p, q: a, b

~(#p<#(p&q))=(p=p) ; TCTT TCTT TCTT TCTT (4.4.1.2)

◊a ≤ ◊(a ∨ b) (M4) (4.4.4.1)

~(%(p+q)<%p)=(p=p) ; TTCT TTCT TTCT TTCT (4.4.4.2)

Eqs. 4.4.1.2 and 4.4.4.2 as rendered are *not* tautologous, hence denying the monotone functor on KHaus. What follows is that the use of coalgebra to manufacture a geometric modal logic is refuted.

Modal logic systems confirmed by Meth8/VŁ4

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET + Or; > Imply; % possibility, for one or some; # necessity, for all or every.

From: helsinki.fi/~negri/ptml_final.pdf

We evaluate the following modal logic systems: (2, M); (3); (4); (B); (D); (E); (K); (T); and (W).

System	**Meth8/VŁ4 equations**	**Table results**	**Descriptive result**
(2, M)	%#p>#%p	TTTT TTTT TTTT TTTT	Tautology
(3)	#(#p>q)+#(#q>p)	NNNN NNNN NNNN NNNN	Truthity
(4)	#p>##p	TTTT TTTT TTTT TTTT	Tautology
(B)	p>#%p	TTTT TTTT TTTT TTTT	Tautology
(D)	#p>%p	TTTT TTTT TTTT TTTT	Tautology
(E)	%p>#%p	TTTT TTTT TTTT TTTT	Tautology
(K)	#(p>q)>(#p>#q)	TTTT TTTT TTTT TTTT	Tautology
(M,2)	%#p>#%p	TTTT TTTT TTTT TTTT	Tautology
(T)	#p>p	TTTT TTTT TTTT TTTT	Tautology
(W)	#(#p>p)>#p	CTCT CTCT CTCT CTCT	Not Tautology, not Truthity

Remark: Gödel logic (GL) is system K+W, diverging most from tautology.

Refutation of the modal logic GL_2

Abstract: We evaluate the modal logic GL_2 in two axioms and for satisfiability. One of the two axioms is *not* tautologous, and the five formulas for PSpace complete satisfiability are *not* tautologous. Hence GL_2 is refuted.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨ ; - Not Or; & And, ∧; \ Not And;
\> Imply, greater than, →; < Not Imply, less than, ∈;
= Equivalent, ≡; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊; # necessity, for every or all, ∀, □;
~(y < x) (x ≤ y), (x ⊆ y).

From: Gabelaia, D.; Gogoladzeb, K.; Jibladze, M.; Kuznetsov, E.; Marxa, M. (2018).
Modal logic of planar polygons. arxiv.org/pdf/1807.02868.pdf e.kuznetsov@freeuni.edu.ge

LET p, q, r, s: p, q, r, γ.

We also present a slightly more intuitive and concise axiomatization of PL_2 by the following two formulas:

(I) $p \to \Box[\neg p \to \Box(p \to \Box p)]$ (4.2.1)

p>#(~p>#(p>#p)) ; TNTN TNTN TNTN TNTN (4.2.2)

(II) $\Box[(r \wedge q) \to \gamma] \to [(r \wedge q) \to \Diamond(\neg(r \wedge q) \wedge \Diamond\Box p \wedge \Diamond\Box\neg p)]$ (4.3.1)

#((r&q)>s)>((r&q)>%(~(r&q)&(%#p&%#~p))) ;
TTTT TTTT TTTT TTCC (4.3.2)

Where γ is the formula $\Diamond\Box(p \wedge q) \wedge \Diamond\Box(\neg p \wedge q) \wedge \Diamond\Box(p \wedge \neg q)$. (4.4.1)

s=(%#(p&q)&(%#(~p&q)&%#(p&~q))) ;
TTTT TTTT **FFFF FFFF** (4.4.2)

Substituting Eq. 4.4.1 into 4.3.1: (4.5.1)

#((r&q)>(%#(p&q)&(%#(~p&q)&%#(p&~q))))>((r&q)>%(~(r&q)&(%#p&%#~p))) ;
TTTT TTTT TTTT TTTT (4.5.2)

While Eq. 4.5.2 is tautologous as an axiom, Eq. 4.2.2 is *n*ot tautologous as an axiom. This means the slightly more intuitive and concise axiomatization of PL_2 is refuted.

LET p, q, r, s, t: φ, ψ, r, m, e.

Theorem 5.1. The satisfiability problem of our logic is PSpace complete.
Let C be the conjunction of these formulas:

r, m, e are disjoint and one of them holds at each world. (5.0.1.1)

(r+s)+t ; **FFFF** TTTT TTTT TTTT,
TTTT TTTT TTTT TTTT (5.0.1.2)

$r \to \Diamond m$ (5.0.2.1)

r>%t ; TTTT CCCC TTTT CCCC,
TTTT TTTT TTTT TTTT (5.0.2.2)

$m \to \Diamond e$. (5.0.3.1)

s>%t ; TTTT TTTT CCCC CCCC,
TTTT TTTT TTTT TTTT (5.0.3.2)

$C=(r \to \Diamond m) \& (m \to \Diamond e)$. (5.0.4.1)

((r+s)+t)&((r>%s)&(s>%t)) ; **FFFF** CCCC CCCC CCCC,
TTTT CCCC TTTT TTTT (5.0.4.2)

$r \wedge \Box C \wedge \Diamond(m \vee e) \wedge \varphi) \wedge \Box((m \vee e) \to \psi)$ is satisfiable in our logic. (5.1.1.1)

((r&#(((r+s)+t)&((r>%s)&(s>%t))))&(%(s>t)&p))&#((s+t)>q) ;
FFFF FFFF FFFF FFFF,
FFFF FFFF FFFF FFFN (5.1.1.2)

Eq. 5.1.1.2 as rendered is *no*t tautologous and not contradictory, differing from the state of contradiction by one value N. This means PL$_2$ is *not* satisfiable as PSpace complete.

Refutation of a modal logic for supervised learning

Abstract: We evaluate ten conjectures which are not tautologous, and with four as contradictory. This refutes the approach and models of modal logic for supervised learning as a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, ♦, M; # necessity, for every or all, ∀, □, ■, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Baltag, A.; Li, D.; Pedersen, M.Y. (2019). On the right path: a modal logic for supervised learning. arxiv.org/pdf/1909.08559.pdf thealexandrubalta@gmail.com, minaypedersen@gmail.com, lidazhu91@163.com

Abstract Formal learning theory formalizes the process of inferring a general result from examples, as in the case of inferring grammars from sentences when learning a language. … Instead of focusing only on learner(s), this work develops a general framework—the *supervised learning game (SLG)*—to investigate the interaction between *Teacher* and *Learner*. … To reason about strategies in this game, we develop a modal logic of *supervised learning (SLL)*. ...

2.3 Preliminary observations
Proposition 6. … Proof. Consider the following formulas:

(T_1) p ∧ ♦p ∧ ♦¬p (2.3.6.T.1.1)

(p&(%p&%~p))=(s=s) ; **F**C**F**C **F**C**F**C **F**C**F**C **F**C**F**C (2.3.6.T.1.2)

(T_2) ■(p → ♦p ∧ ♦¬p) (2.3.6.T2.1)

#(p>(%p&%~p))=(s=s) ; NFNF NFNF NFNF NFNF (2.3.6.T.2.2)

(T_3) ■(¬p → <−>$_1$(■p ∧ ■■p)) (2.3.6.T3.1)

Remark 2.3.6.T3.1: We ignore equations with relation-changing modal operators because of variable definitions depending on author: <−> sabotage operator; <+> bridge operator; [−] sabotage operator; [+] bridge operator. (See: Areces, C.; Fervari,R.; Hoffman, G. (2015.) Relation-changing modal operators. Journal of the

IGPL. 23(4):601–627.)

Define $\phi_T := (T_1 \wedge T_2\, [~~\wedge T_3~~]\,)$. (2.3.6.4.1)

(p&(%p&%~p))& #(p>(%p&%~p)) ;
FFFF FFFF FFFF FFFF (2.3.6.4.2)

Remark 2.3.6.4.2: Eq. 2.3.6.4.2 is *not* tautologous, and a contradiction. This means the proof conjecture of ϕ_T is a contradiction already without including the term of $[\wedge T_3]$.

4 Model checking and satisfiability for SLL

Theorem 7. $L_{\Diamond_{<->1}}$ does not enjoy the finite model property.

Proof. To prove this, we present a formula that can only be satisfied by some infinite models. Consider the following formulas:

(F1) $p \wedge q \wedge \blacklozenge p \wedge \blacklozenge\neg p \wedge \blacksquare\neg q$ (4.7.F1.1)

((p&q)&(%p&%~p))&#~q ; **FFFF FFFF FFFF FFFF** (4.7.F1.2)

(F2) $\blacksquare(p \rightarrow \blacklozenge q \wedge \blacklozenge\neg q \wedge \blacksquare p)$ (4.7.F2.1)

#(p>((%q&%~q)&#p))=(s=s) ;
N**F**N**F** N**F**N**F** N**F**N**F** N**F**N**F** (4.7.F2.2)

(F3) $\blacksquare(p \rightarrow \blacksquare(q \rightarrow \blacksquare\neg q \wedge \blacklozenge\neg p))$ (4.7.F3.1)

#(p>#(q>(#~q&%~p)))=(s=s) ;
NNN**F** NNN**F** NNN**F** NNN**F** (4.7.F3.2)

(F5) $\blacksquare(p \rightarrow\blacksquare\ (\neg q \rightarrow \blacklozenge q \wedge \blacklozenge\neg q \wedge \blacksquare p))$ (4.7.F5.1)

#(p>#(~q>((%q&%~q)&#p)))=(s=s) ;
N**F**NN N**F**NN N**F**NN N**F**NN (4.7.F5.2)

(F6) $\blacksquare(p \rightarrow \blacksquare(\neg q \rightarrow \blacksquare(q \rightarrow \blacksquare\neg q \wedge \blacklozenge\neg p)))$ (4.7.F6.1)

#(p>#(~q>#(q>(#~q&%~p))))=(s=s) ;
NNNN NNNN NNNN NNNN (4.7.F6.2)

Let formula ϕ_∞ be the conjunction of the formulas above. (4.7.12.1)

Remark 4.7.12.1: We write this excluding F4, F7, and the 4-remaining conjectures which have modal relation operators as F1 & F2 & F3 & F5 & F6.

(((((p&q)&(%p&%~p))&#~q)&#(p>((%q&%~q)&#p)))& (#(p>#(q>(#~q&
%~p)))&#(p>#(~q>((%q&%~q)&#p)))))&#(p>#(~q>#(q>(#~q&%~p)))) ;
FFFF FFFF FFFF FFFF (4.7.12.2)

We first show that ϕ_∞ is satisfiable. (4.7.13.1)

Remark 4.7.13.1: We take Eq. 4.7.13.1 to mean that ϕ_{∞} is a theorem as satisfied by infinite models which according to Eq. 4.7.12.2 it is *not* as a contradiction.

We evaluate ten conjectures which are not tautologous, and with four as contradictory. This refutes the approach and models of modal logic for supervised learning.

Refutation of modal operators on rings of continuous functions

Abstract: We evaluate five expressions for a definition (2), two remarks (2), and a lemma proof (1). None is tautologous. This refutes the titled conjecture which drags in the Hausdorff and Stone topologies, Kripke frames, and dualities of Gelfand-Naimark-Stone and Esakia-Goldblatt. These conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Bezhanishvili, G.; Carai, L.; Morandi, P.J. Modal operators on rings of continuous functions. arxiv.org/pdf/1909.06912.pdf pmorandi@nmsu.edu

Abstract. … Our goal is to generalize the setting of descriptive frames to that of compact Hausdorff frames; that is, to generalize the Stone topology on a Kripke frame to that of a compact Hausdorff topology. … This generalizes both Gelfand-Naimark-Stone duality and Esakia-Goldblatt duality. ...

1. Introduction

1.1. Dualities in modal logic. In modal logic there is a well established duality theory between categories of Kripke frames and the corresponding categories of boolean algebras with operators, which forms the backbone of modern studies of modal logic. These dualities originate in the works of Jónsson and Tarski, Halmos, and Kripke, and were further developed by Esakia, Thomason, and Goldblatt.

2. From Kripke frames to modal operators on rings of functions

[T]he main definition of the paper [is]:

Definition 2.9.

(1) Let A ∈ baℓ. We say that a unary function □ : A → A is a modal operator on A provided □ satisfies the following axioms for each a, b ∈ A and λ ∈ R:

(M2) □λ = λ + (1 − λ)□0. (2.9.1.2.1)

LET p, q, r: a, b, λ.

#r=(r+(((%s>#s)-r)&#(s@s))) ;

TTTT NNNN TTTT NNNN (2.9.1.2.2)

(M4) $\Box(a + \lambda) = \Box a + \Box\lambda - \Box 0.$ (2.9.1.4.1)

#(p+r)=((#p+#r)=#(s@s)) ; **FFFF FFFF FFFF FFFF** (2.9.1.4.2)

Remark 2.10. We can define

$\Diamond : A \rightarrow A$ dual to $\Box$ by $\Diamond a = 1 - \Box(1 - a)$ for each $a \in A$. ... (2.10.1.1)

%p=((%s>#s)-#((%s>#s)-p)) ;
TCTC TCTC TCTC TCTC (2.10.1.2)

Remark 2.11. If $\Box 0 = 0$, then (M2), (M4), (M5) simplify to the following:

(M4′) $\Box(a + \lambda) = \Box a + \lambda.$

(#(s@s)=(s@s))>((#(p+r)=((#p+#r)=#(s@s)))=((#(p+r))=(#p+r))) ;
FFFF CCCC **FFFF** CCCC (2.11.4.1)

Lemma 2.12. Proof. (6). By (M4), (2), and (4) we have

$$\Diamond a \;=\; 1-\Box(1-a) \;=\; 1-(\Box(-a)+\Box 1-\Box 0) \;=\; -\Box(-a)+\Box 0 \;=\; -\Box(-a)+\Box(-a)\Box 0 \;=\; -\Box(-a)(1-\Box 0).$$
(2.12.1.6.1)

%p=((((%s>#s)-#((%s>#s)-p))=((%s>#s)-(#~p+(#(%s>#s)-#(s@s)))))=
(((~#~p+#(s@s))=(~#~p+(#~p&#(s@s))))=(~#~p&((%s>#s)-#(s@s))))) ;
N**F**N**F** N**F**N**F** N**F**N**F** N**F**N**F** (2.12.1.6.2)

Remark 2.12.1.6.2: Eq. 2.12.1.6.2 is *not* tautologous, refuting the conjecture of Proof (6) in the proof for Lemma 2.12. That step was chosen for its relative complexity. We note that the consequent as rendered is in fact a contradiction.

We evaluate five expressions for a definition (2), two remarks (2), and a lemma proof (1). None is tautologous. This refutes the titled conjecture which drags in the Hausdorff and Stone topologies, Kripke frames, and dualities of Gelfand-Naimark-Stone and Esakia-Goldblatt.

Refutation of the language of sets for model theory = universal algebra + mathematical logic

Abstract: A first order language of sets is proposed, but the first example *A*⊂*B* iff (*x*∈*A* then *x*∈*B*) is *not* tautologous. This refutes the conjecture of model theory = universal algebra + mathematical logic, which forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Torres, J. (2019). Model theory, arithmetic & algebraic geometry.
arxiv.org/pdf/1905.00278.pdf joel.torres@udea.edu.co

1.1. **What is Model Theory?** ... Model Theory introduces Mathematical Logic in the practice of Universal Algebra, so we can think it like Model Theory = Universal Algebra + Mathematical Logic. (1.1.1)

1.2. **Languages.** To start we fix a first order language L which contains exactly those symbols that we request in our interest and nothing else. … A simple example of a language is L_{sets} = {∈} the language of sets, note that we can define other symbols in Set Theory from ∈, for example *A* ⊂ *B* iff (*x* ∈ *A* then *x* ∈ *B*) (1.2.1.1)

LET p, q, r: A, B, x.

((r<p)>(r<q))>(p<q) ; **F**T**FF** **F**T**FF** **F**T**FF** **F**T**FF** (1.2.1.2)

Eq. 1.2.1.2 as rendered is *not* tautologous to refute the first order language of sets as proposed. What follows is that Eq. 1.1.1 model theory = universal algebra + mathematical logic is also refuted.

Refutation of two modern modal logics: "JYB4" and the follow-on "AR4"

Abstract: We evaluate the four-valued logic systems of J.-Y. Béziau and F. Schang as JYB4 and AR4. JYB4 named the four-valued modal logic Ł4 as Łukasiewicz's nightmare because of the alleged absurdity of (◊p&◊q) → ◊(p&q). A model checking system is then framed based on 0± and 1±. We show (◊p&◊q) → ◊(p&q) is equivalent to (◊p&◊~p) → ◊(p&~p) with (◊p&◊q) = ◊(p&q) as a theorem. AR4 was a doxastic logic follow-on to JYB4. We name these modern modal logic systems as Béziau's nightmare and Schang's nightmare.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET: ~ Not; + Or, ∨; & And, ∧; > Imply, ⊢; < Not Imply, ⊣; = Equivalent, ⊣⊢;
(p=p) Tautology;
% possibility, possibly, for one or some; # necessity, necessarily, for every or all.

See: J.-Y. Béziau. (2011)."A new four-valued approach to modal logic". Logique & Analyse, Vol. 54.
J.-Y. Béziau. (2005). "Paraconsistent logic from a modal viewpoint". Journal of Applied Logic.

We name this system after its writer J.-Y. Béziau as JYB4. It is less a logic system and more of a model checking system based on 15 axioms for which p and q are assigned 0±, 1± to evaluate models by arithmetic. These are keyed to (Béziau, 2011).

For definitions and properties:

#p>p ;	TTTT TTTT TTTT TTTT	(2.1.11.2)	
p<#p ;	FCFC FCFC FCFC FCFC	(2.1.12.2)	x
p>%p ;	TTTT TTTT TTTT TTTT	(2.1.21.2)	
%p<p ;	CFCF CFCF CFCF CFCF	(2.1.22.2)	x
#p>%p ;	TTTT TTTT TTTT TTTT	(2.1.31.2)	

Remark 2.1.31.2: This theorem supposedly "results from (11), (21) and transivity" as

#p>p)&(p>%p) ;	TTTT TTTT TTTT TTTT	(2.1.31.2.2)

%p<#p ;	CCCC CCCC CCCC CCCC	(2.1.32.2)	x

Remark 2.1.32.2: This theorem supposedly "results from (11), (22) and transivity" as

(#p>p)&(%p<p) ;	CFCF CFCF CFCF CFCF	(2.1.32.2.2)

For codi modal logics verifying conditions:

(#p&#q)=#(p&q) ;	TTTT TTTT TTTT TTTT	(4.1.1.2)	
#(p+q)<(#p+#q) ;	FFFF FFFF FFFF FFFF	(4.1.2.2)	x
(%p&%q)<%(p&q) ;	FFFF FFFF FFFF FFFF	(4.1.3.2)	x

Remark 4.1.3.2: LET p, q=~p: rain tomorrow, not rain tomorrow.

[Eq. 4.1.3.2] is in fact the nightmare Łukasiewicz had to face all his life. This is a central feature of his systems and he was not able to give a satisfactory explanation in order to justify it. The absurdity appears clearly through the following example:

If it is possible that it will rain tomorrow and it is possible that it will not rain tomorrow, then it is possible that it will rain and not rain tomorrow."

(%p&%q)>%(p&q) ; TTTT TTTT TTTT TTTT (4.1.3.2.2)

However, invoking one variable and its negation to replace q produces compliance in *all* classical modal logics:

(%p&%~p)>%(p&~p) ; TTTT TTTT TTTT TTTT (4.1.3.3.2)

Hence, the alleged absurdity is contradicted in the contra-example by using one variable and its negation, instead of two variables.

The contra-example is amplified by removing the implication to replace with an equivalence. For example:

"It possibly will rain tomorrow and possibly will not rain tomorrow" is equivalent to
"It possibly will rain tomorrow and not rain tomorrow"

(%p&%~p)=%(p&~p) ; TTTT TTTT TTTT TTTT (4.1.3.4.2)

(%p+%q)=%(p+q) ; TTTT TTTT TTTT TTTT (4.1.4.2)
(#p+#q)>#(p+q) ; TTTT TTTT TTTT TTTT (4.1.5.2)
%(p&q)>(%p&%q) ; TTTT TTTT TTTT TTTT (4.1.6.2)

Necessitation and replacement:

(p>p)>(p>#p) ; TNTN TNTN TNTN TNTN (7.1.2) x
(p=q)>(#p=#q) ; TTTT TTTT TTTT TTTT (7.2.1.2)
(p=q)>(%p=%q) ; TTTT TTTT TTTT TTTT (7.2.2.2)

For JYB4 Eqs. 2.-7., 5 of 15 or 33% are *not* tautologous. Consequently, we rename the alleged Łukasiewicz nightmare as Béziau's nightmare.

From: academia.edu/27012333/A_Doxastic_Interpretation_of_4-Valued_Modal_Logic

We name this extension of JYB4 after its writer Fabian Schang as doxistic "deviant" logic system AR4.

#p=~%~p ; TTTT TTTT TTTT TTTT (14.2)
%p=~#~p ; TTTT TTTT TTTT TTTT (15.2)

Paracomplete negation:

~p=#~p ; NTNT NTNT NTNT NTNT (16.0.2) x

Paraconsistent negation;

~p=~#p ; TNTN TNTN TNTN TNTN (17.0.1.2) x
~p=%~p ; TNTN TNTN TNTN TNTN (17.0.2.2) x
~#p=%~p ; TTTT TTTT TTTT TTTT (17.0.3.2)

For AR4 Eqs. 14.-17., 3 of 6 or 50% are *not* tautologous. Consequently, we rename this subsequent work to Béziau's nightmare as Schang's nightmare.

We conclude that the statistics above remove JYB4 and AR4 from further serious consideration as viable and useful modern modal four-valued logics.

Counter example to "modified divine command theory"

Per Robert Merrihew Adams (a Presbyterian minister for a short stint, whose late spouse was an Episcopalian priestess) originated the modified divine command theory [bracket text is my insertion]:

Eq 1 It is wrong to do X.
Eq 2. It is contrary to God's commands to do X.
[Eq 3.1 To do X implies wrong.]
[Eq 3.2 Wrong implies to do X.]
[Eq 4. If Eq 1 and Eq 2, then Eq 3.1.]
[Eq 5. If Eq 1 and Eq 2, then Eq 3.2.]

LET: p X, q wrong, r God's command,
~ Not, & And, > Imply, nvt not tautologous, vt tautologous

Note: Truth tables are for four propositions and presented left to right as
the four rows top-down. Designated truth values are Tautologous and contradictory here.

q > p ;	nvt ;	TTFT TTFT TTFT TTFT	(1)
~r > p ;	nvt ;	FTFT TTTT FTFT TTTT	(2)
p > q ;	nvt ;	TFTT TFTT TFTT TFTT	(3)
((q>p)&(~r>p)) > (p>q) ;	nvt ;	TFTT TFTT TFTT TFTT	(4)
((q>p)&(~r>p)) > (q>p) ;	vt ;	TTTT TTTT TTTT TTTT	(5)

Eq 4 is of concern as a counter example to Eq 5: If both wrong implies doing X and not God's command implies doing X, then doing X implies wrong. This scans as tautologous, but logically it is not.

Eq 5 is tautologous: If both wrong implies doing X and not God's command implies doing X, then wrong implies doing X.

This caused Professor Adams to modify Eq 5 to read something as "If *and only if* Eq 1 and Eq 2, then Eq 3.2" which ultimately begs the question.

My conclusion is that the modified divine command theory is a hypothesis, at best.

Confirmation of failure of modus ponens when the consequent is itself a conditional sentence

Abstract: We confirm the failure of modus ponens when the consequent is itself a conditional sentence. The reason a repeated consequent does not produce tautology is because it dilutes the original sentence to assume incorrectly other plausible consequents.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r: Shakespeare, Hobbes, Hamlet;
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⇔, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) `C` non-contingency, ∇, ordinal 2; (%z>#z) `N` as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).

From: en.wikipedia.org/wiki/Modus_ponens

[The following is attributed to Vann McGee, but without a proper footnote in the article.]

Either Shakespeare or Hobbes wrote Hamlet. (1.1.1)

(p+q)>r ; T**FFF** TTTT T**FFF** TTTT (1.1.2)

If Shakespeare didn't do it, Hobbes did. (1.2.1)

~(p>r)>(q>r) ; TTT**F** TTTT TTT**F** TTTT (1.2.2)

If either Shakespeare or Hobbes wrote Hamlet, then if Shakespeare didn't do it, Hobbes did. (2.1.0)

We write this as (Eq. 1.1.1 implies 1.2.1). (2.1.1)

((p+q)>r)>(~(p>r)>(q>r)) ; TTTT TTTT TTTT TTTT (2.1.2)

Therefore, if Shakespeare didn't write Hamlet, Hobbes did it. (3.1.0)

We write this as (Eq. 1.1.1 implies 1.2.1) implies 1.2.1. (3.1.1)

(((p+q)>r)>(~(p>r)>(q>r)))>(~(p>r)>(q>r)) ;
TTTT TTT**F** TTTT TTT**F** (3.2.1)

Eq. 2.1.2 for (Eq. 1.1.2 implies 1.2.2) is tautologous. Eq. 3.1.1 supplements 2.1.2 with an additional consequent 1.2.2 as a conditional sentence. We call this a repeated consequent. However 3.1.1 ((Eq. 1.1.1 implies 1.2.1) implies 1.2.1) is *not* tautologous. Therefore, the repeated consequent dilutes the tautology of the original sentence.

Remark 3:

The wiki consortium writes:

> "But the conclusion [3.1.0] is dubious, because if Shakespeare is ruled out as *Hamlet*'s author, there are many more plausible alternatives than Hobbes."

That is mistaken because it makes an assumption, and should read:

> "But the conclusion [3.1.0] is dubious, because if Shakespeare is ruled out as *Hamlet*'s author, for Shakespeare to be ruled again does not imply the dubious assumption of many more plausible alternatives than Hobbes."

Refutation of the Molyneux problem

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET p, s: blind person, shape;
~ Not; & And; > Imply.

From: en.wikipedia.org/wiki/Molyneux's_problem

"If one born blind feels the differences between shapes such as spheres and cubes,
could one, if given the ability to see, distinguish those objects by sight alone,
in reference to the tactile schemata one already possessed?" (0.1)

We rewrite Eq. 0.1 by abstraction in removing the distinction between two named shapes and replacing with shape (or not shape).

If one blind recognizes a shape and recognizes not that shape,
then one not blind recognizes that shape and recognizes not that shape. (1.1)

((p>s)&(p>~s))>((~p>s)&(~p>~s)) ; FTFT FTFT FTFT FTFT (1.2)

Eq. 1.2 is *not* tautologous. This means the Molyneux problem is resolved as two unrelated states, and hence not a problem.

Resolution of Moore's paradox as a theorem

Abstract: We evaluate Moore's paradox, as touted by Wittgenstein, with Hintikka's omissive or commissive logical forms as "P and NOT(belief in P)" or "P and belief in NOT-P". The former as antecedent (contradictory) and the latter as consequent (neither contradictory nor tautologous) imply tautology, a theorem.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET: ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →; < Not Imply, less than, ∈;
= Equivalent, ≡; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊; # necessity, for every or all, ∀, □;
~(y < x) (x ≤ y), (x ⊆ y); (p=p) Tautology.

From: en.wikipedia.org/wiki/Moore's_paradox

LET p, q: P, belief.

Remark 0: We reject the personal "I believe" in lieu of the variable "belief in", as one trusting in the unseen.

[The standard is] to present Moore's paradox by explaining why it is absurd to assert sentences that have the logical form:

Omissive: "P and NOT(belief in P)" or (1.1)

p&~(q>p) ; **FFFF FFFF FFFF FFFF** (1.2)

Commissive: "P and belief in NOT-P." (2.1)

p&(q>~p) ; **F**T**FF F**T**FF F**T**FF F**T**FF** (2.2)

Omissive implies Commissive: (3.1)

(p&~(q>p)) > (p&(q>~p)) ; TTTT TTTT TTTT TTTT (3.2)

While Eq. 1.2 omissive is a contradiction and Eq. 2.2 commissive is not a contraction and not a tautology, omissive implies commissive as a tautology. This uses the implication forms of **F**>**F**=T and **F**>T=T, to mean that Moore's paradox is not a contradiction but a theorem. Because the sentences of Eqs. 1 and 2 as rendered do differ, the logical absurdity is in omissive as a contradiction, but not in commissive as not a contradiction and not a tautology.

Refutation that "it is impossible for humans to implement moral absolutism"

From http://vixra.org/abs/1806.0194 [excluding the examples]

> Suppose there is an absolute moral proposition defined with *X* number of words and a real-life moral quandary defined with *Y* number of words, (1.1.0)
> and that one wants to rely on moral absolutism to make a judgment of morality regarding the quandary ... (1.2.0)
> if the quandary is completely specified by the *Y* words.
> ...
> Without absolutely specifying the quandary, one has no way to compare it to the absolute proposition.
> Therefore, in all cases, when humans attempt to implement moral absolutism, they will actually implement moral relativity when they decide, relative to their own personal standard of sufficiency, that they have considered enough of the context of the quandary such that it can be compared to the absolute proposition. (2.0)
> Therefore, it is impossible for humans to implement moral absolutism. (3.0)

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal.

> LET p, q, r: absolute moral proposition, relative moral proposition, word number;
> ~ Not; + Or; - Not Or; > Imply, greater than; < Not Imply, less than;
> = Equivalent; % possibility, for one or some; # necessity, for all or every;
> (s=s) absolute truth; (%s>#s) ordinal one; (%s<#s) ordinal two.

We rewrite Eq. 1.0 to exclude the *a priori* notion of quandary as an *inexact* contradiction to mean an absolute moral proposition defined with *X* number of words and a different, non-moral or relative proposition defined with *Y* number of words, as:

> possibly a word number implies a proposition which is morally absolute as true (absolute morality) (1.1.1)
>
> (%r> p)> (s=s) ; `TTTT TTTT TTTT TTTT` (1.1.2)
>
> and [sic, should be *or*]
> possibly a word number implies not a proposition which is not morally absolute as not true (relative morality) (1.2.1)
>
> (%r>~p)>~(s=s) ; `FCFC FTFT FCFC FTFT` (1.2.2)
>
> With Eqs. 1.1.1 *and* 1.2.1 as: (1.3.1)
> ((%r> p)> (s=s)) & ((%r>~p)>~(s=s)) ;
> `FCFC FTFT FCFC FTFT` (1.3.2)

We rewrite Eq. 2.0 to include the number of words to needed (necessary) to specify fully the *Y* words and to include the correction of an Or replacement connective in the consequent:

the last word number, instant word number, or next two word numbers are never (necessarily not) sufficient to describe (do not imply) a proposition which is morally absolute as true (absolute morality) *or* a proposition which is not morally absolute as not true (relative morality) (2.1)

```
#(((r-(%s>#s))+(r+(r+(%s>#s))))+(r+(%s<#s)))<(((%r> p)> (s=s))+((%r>~p)>~(s=s)));
                                   FFFF FFFF FFFF FFFF
```
(2.2)

Eq. 2.2 as rendered means Eq. 3.0 (it is impossible for humans to implement moral absolutism) is *not* tautologous (*not* a theorem), but rather a contradiction, and hence refuted.

What follows is confirmation that "It is possible for humans to implement moral absolutism".

Refutation of relativity on absolute moralism

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: Alice, Bob, killing, morality
~ Not; + Or; - Not Or; & And; = Equivalent; > Imply; < Not Imply;
% possibility, for any one or some, ∃; # necessity, for every or all, ∀.
(s=s) T tautology, good; (s@s) **F** contradiction, bad.

From: Tooker, J.W. (2018). On relativity of absolutism in morality. vixra.org/pdf/1806.0194v2.pdf
[claimed email addresses bounced at gatech.edu]

Remark: We quote relevant portions of the argument because it is ill-framed without numbered equations.

Bob wants to know if it is moral to kill Alice. (1.0)

We rewrite Eq. 1.0 as: "If Bob kills Alice, then is Bob killing Alice good?" (1.1)

((q&r)&p)>(((q&r)&p)>(s=(s=s))) ; TTTT TTTT TTT**F** TTTT (1.2)

An absolute moral proposition of relevance would be that murder is wrong. (2.0)

We rewrite Eq. 2.0 as:
"If morality is good as a tautology, then murder is a bad as a contradiction." (2.1)

(s=>(s=s))>(r>(s>s@s)) ; TTTT TTTT TTTT **FFFF** (2.2)

"Is Alice on a machine gun rampage such that [Bob] will save lives by killing her?" (3.0)

"If Alice killing is bad, then if Bob kills Alice, then is Bob killing Alice good?" (3.1)

((p&r)=(s@s))>(((q&r)&p)>(((q&r)&p)>(s=(s=s)))) ;
TTTT TTTT TTTT TTTT (3.2)

Remark: We ignore the subsequent injection of irrelevant contingencies from other worlds, such as implication of Bob killing from alien killing as a result of Alice killing.

Eq. 3.2 as rendered is tautologous, hence refuting relativity of moral absolutism.

Refutation of the quantifier Most

Abstract: We evaluate two equivalent semantics for "Most A are B" which while logically equivalent are neither tautologous. Hence the quantifier Most is refuted.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. Reproducible transcripts for results are available. (See ersatz-systems.com.)

LET p, q, r, s: A, B, C, s;
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
\> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨; < Not Imply, less than, ∈, ≺, ⊂;
= Equivalent, ≡, :=, ⇔, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, ∇, ordinal 1; (%z<#z) C as contingency, Δ, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).

From: Topal, S. (2019). Natural density and the quantifier most. arxiv.org/pdf/1901.10394.pdf s.topal@beu.edu.tr

Two different but equivalent semantics are for Most A are B as

(i) $C(A \cap B) > C(A \setminus B)$ and (1.1)

((r&(p&q))>(r&(p\q)))>((r&p)=q) ;
TT**FF** T**FF**T TT**FF** T**FF**T (1.2)

(ii) $C(A \cap B) > C(A)/2$ (2.1)

((r&(p&q))>((r&p)\(%s<#s)))>((r&p)=q) ;
TT**FF** T**FF**T TT**FF** T**FF**T (2.2)

While Eqs. 1.2 and 2.2 as rendered share the same truth table result, being logically equivalent, neither is tautologous. The means the quantifier Most is refuted.

Refutation of naive scale invariance and the world as a hologram of 't Hooft

Abstract: The equation for naive scale invariance is *not* tautologous, refuting it as basis for the world as a hologram of 't Hooft. These form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Susskind, L. (1994). The world as a hologram. arxiv.org/pdf/hep-th/9409089.pdf

The naive scale invariance would imply the following: ...

2) The wave functionals of the eigenvectors transform in a naive way [where] each fluctuation of wave number p simply stretches to wave number λp:

$$\Psi_i[\varphi(p)] \rightarrow \Psi_i[\lambda\varphi(\lambda p)]. \qquad (2.10.1)$$

LET p, q, r, s, t: p, λ, i, Ψ, φ.

((s&r)&(t&p))>((s&r)&((q&t)&(q&p))) ;

TTTT TTTT TTTT TTTT(1)
TTTT TTTT TTTT T**F**TT(1) (2.10.2)

Eq. 2.10.2 as rendered is *not* tautologous, refuting naive scale invariance as basis for the world as a hologram of 't Hooft.

Rule of necessitation: a non-contingent truthity, but not a tautology

1. The axiom or rule of necessitation **N** states that if p is a theorem, then necessarily p is a theorem:

If $\vdash$ p then $\vdash \Box$p.

We show this is non-contingent (a truthity), but not tautologous (a proof). We evaluate axioms (in bold) of **N**, **K**, **T**, **4, B**, **D**, **5** to derive systems (in italics) of *K*, *M*, *T*, *S4*, *S5*, *D*.

We assume the Meth8 apparatus implementing system variant VŁ4, where:

necessity, universal quantifier; % possibility, existential quantifier;
\> Imply; = Equivalent to; (p=p) Tautology

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	p=p	T	Tautology	proof	11	3
2	p@p	F	Contradiction	absurdum	00	0
3	%p>#p	N	Non-contingency	truthity	01	1
4	%p<#p	C	Contingency	falsity	10	2

The designated proof value is T tautology. Note the meaning of (%p>#p): a possibility of p implies the necessity of p; and some p implies all p. In other words, if a possibility of p then the necessity of p; and if some p then all p. This shows equivalence and interchangeability of respective modal operators and quantified operators, as proved in Appendix. (That correspondence is proved by VŁ4 corrections to the vertices of the Square of Opposition and subsequent corrections to the syllogisms of Modus Cesare and Modus Camestros.)

Results are the 16-value truth table as row-major and horizontal; tautology is all "TTTT".

N: If $\vdash$ p then $\vdash \Box$p. (**N**.1.1)

p>#p ; TNTN TNTN TNTN TNTN (**N**.1.2)

The necessity of p or ~p is a theorem. (**N**.2.1)

#(p+~p)=(p=p) ; NNNN NNNN NNNN NNNN (**N**.2.2)

Eqs. N.1.2 and 2.2 are minimally tautologous at a level of non-contingency (NNNN NNNN NNNN NNNN) as *truthity*, but not a proof at a level of tautology (TTTT TTTT TTTT TTTT).

The definitions of the other axioms are as follows (Steward, Stoupa, 2004):

K: $\Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$; no conditions (**K**.1.1)

#(p>q)>(#p>#q) ; TTTT TTTT TTTT TTTT (**K**.1.2)

T: □*p* → *p* ; reflexive (**T**.1.1)

#p>p ; `TTTT TTTT TTTT TTTT` (**T**.1.2)

4: □ p → □ □ p (**4**.1.1)

#p>##p ; `TTTT TTTT TTTT TTTT` (**4**.1.2)

B: p → □ ◊ p ; reflexive and symmetric (**B**.1.1)

p>#%p ; `TTTT TTTT TTTT TTTT` (**B**.1.2)

D: □ p → ◊ p ; serial (**D**.1.1)

#p>%p ; `TTTT TTTT TTTT TTTT` (**D**.1.2)

5: ◊ p → □ ◊ p (**5**.1.1)

%p>#%p ; `TTTT TTTT TTTT TTTT` (**5**.1.2)

The definitions of systems are as follows:

K:= **K** (no conditions) (*K*.1.1)

#(p>q)>(#p>#q) ; `TTTT TTTT TTTT TTTT` (*K*.1.2)

alternatively, **K** & **N** is used (viz, en.wikipedia.org/wiki/Modal_logic) (*K*.2.1)

(#(p>q)>(#p>#q))&(p>#p) ; `TNTN TNTN TNTN TNTN` (*K*.2.2)

Eq. *K*.2.2 subsequently taints all results as having some value of truth (`TNTN`), but *not* tautology (`TTTT`).

D:= *K* & **D** (serial) (*D*.1.1)

(#(p>q)>(#p>#q))&(#p>%p) ; `TTTT TTTT TTTT TTTT` (*D*.1.2)

M:= *K* & **T** (*T*.1.1)

(#(p>q)>(#p>#q))&(#p>p) ; `TCTT TCTT TCTT TCTT` (*T*.1.2)

S4:= *M* & **4** ; reflexive and transitive (*S4*.1.1)

((#(p>q)>(#p>#q))&(#p>p))&(#p>##p) ;
`TTTT TTTT TTTT TTTT` (*S4*.1.2)

B:= *M* & **B** (*B*.1.1)

((#(p>q)>(#p>#q))&(#p>p))&(p>#%p) ;

TTTT TTTT TTTT TTTT (*B*.1.2)

S5:= *M* & **5** ; reflexive and Euclidean (*S5*.1.1)

((#(p>q)>(#p>#q))&(#p>p))&(%p>#%p) ;
TTTT TTTT TTTT TTTT (*S5*.1.2)

alternatively, *M* & **B** & **4** (*S5*.2.1)
(((#(p>q)>(#p>#q))&(#p>p))&(p>#%p))&(#p>##p) ;

2. We also evaluated (Steward, Stoupa, 2004) to derive by replication some systems of interest.

K: [](p ⊃ q) ⊃ ([]p ⊃ []q) (3.1.1)

#(p>q)>(#p>#q) ; TTTT TTTT TTTT TTTT (3.1.2)

Axiom **T**: []p ⊃ p (3.2.1)

#p>q ; TTTT TTTT TTTT TTTT (3.2.2)

M, obtained by extending system **K** with rule **T** [not Gödel's system T] (3.3.1)

(#(p>q)>(#p>#q))>(#p>q) ; TCTT TCTT TCTT TCTT (3.3.2)

"The strongest system from these modal logics that is perfectly straightforward to formulate in a sequent system and to prove cut-free is system **G-M** (for Gentzen system **M**)".

We remark that the subsequent derivations of *S4*, *B*, and *S5* are tautologous, as are **K** and **T** as demonstrated in section 1.

2. We found other mistakes in (Steward, Stouppa, 2004).

2.1. "The following lemma is a straightforward exercise in theoremhood over **K**:

LEMMA 6 If A ⊃ B is a theorem of **M**, then so are: (L.6.0.1)
1. A ∧ C ⊃ B ∧ C; (L.6.1.1)
2. A ∨ C ⊃ B ∨ C; (L.6.2.1)
3. []A ⊃ []B; (L.6.3.1)
4. <>A ⊃ <>B." (L.6.4.1)

To map Eq. L.6.0.1 we use Eq. 3.3.2.

((#(p>q)>(#p>#q))>(#p>q)) > (p>q) ; TNTT TNTT TNTT TNTT (L.6.0.2)

We then reuse Eq. L.6.0.2 to map L.6.1.2 - 6.4.2.

(((#(p>q)>(#p>#q))>(#p>q))>(p>q)) > ((p&r)>(q&r)) ;
TTTT TCTT TTTT TCTT (L.6.1)

(((#(p>q)>(#p>#q))>(#p>q))>(p>q)) > ((p+r)>(q+r)) ;
TCTT TTTT TCTT TTTT (L.6.2)

(((#(p>q)>(#p>#q))>(#p>q))>(p>q)) > (#p>#q) ;

TCTT TCTT TCTT TCTT (L.6.3)

(((#(p>q)>(#p>#q))>(#p>q))>(p>q)) > (%p>%q) ;

TCTT TCTT TCTT TCTT (L.6.4)

2.2. These inference rules were flagged by Meth8, with page number for equation.

LET: p uc_Gamma; q uc_Delta; r A; s B

(p&r)>(%p&#r) ; 1.#1 ; TTTT TNTN TTTT TNTN (315, []1)
(%p&r)>(%p&#r) ; TTTT NNNN TTTT NNNN (323, []2)
((%p&q)&r)>((%p&#q)&#r) ; TTTT TTNN TTTT TTNN (324, []5)

"we recommend the reader works ... example (A ⊃ B ⊃ C) ⊃ (A ⊃ C) ⊃ B ⊃ C" (321.1)
((((p>q)>r)>(p>r))>q)>r ; TFFF TTTT TFFF TTTT (321.2)

We conclude that **N** the axiom or rule of necessitation is *not* tautologous Because system *M* as derived and rendered is not tautologous, system *G-M* also *not* tautologous.

What follows is that systems derived from using *M* are tainted, regardless of the tautological status of the result so masking the defect, such as systems *S4*, *B*, and *S5*.

We also find that Gentzen-sequent proof is suspicious, perhaps due to its non bi-valent lattice basis in a vector space.

References

Steward, Charles; Stouppa, Phiniki. (2004). A systematic proof theory for several modal logics.
also at textproof.com/supervision/phiniki04sbm.pdf

Meth8 applied to Jan Woleński (2015) ***On Leonard Nelson's Criticism of Epistemology***

We evaluate Leonard Nelson proofs in the words from pages 5-7 (Woleński 2015) for the first proof (α), but ignore the second proof (β) because it is based on set theory (which we dispense with elsewhere). The expressions are keyed to that paper.

We restate the problem as:

> (*) The fundamental task of epistemology consists in demonstrating objective truth or validity of human knowledge.

We use the Meth8 modal logic checker in five models, as based on our system variant VŁ4 that resuscitates the quaternary logic of Łukasiewicz.

Assume Meth8 script where:

> + Or, - Not or, & And, \ Not and, > Imply, < Not imply, = Equivalent, @ Not equivalent, ~ Not, vt tautologous, nvt not tautologous, Contradiction is nvt with all contradictory

LET: s = "epistemological criterion C"
p = problematic domain
q = knowledge

(2) s = (q+~q) ; "C is either knowledge or not"
(a) (s>q) ; "assume C is knowledge"
(a1) (s>q) > (~s>p) ; "If C is knowledge, it belongs to the domain of what is just problematic (Nelson assumes that a piece of cognition is problematic before checking it by C)"
(a2) (s>~q) > p ; "However, C is not knowledge, it is problematic only"
(a3) Test: We ask is "Contradiction (a)-(a2)".
Results: ((s>q)>(~s>p)) > ((s>~q)>p) ; nvt ; TTTT TTTT FTTT FTTT ;
We answer "The fundamental problem (*) is not a contradiction, but nvt".

(b) (s=~q) ; "assume C is not knowledge"
(b1) (s=(q+~q)) > (s>q) ; "If C is to be successfully applied, it must be known as suitable to perform its role as the standard of knowledge"

(b2) ((s=(q+~q)) > (s>q)) > (s=q) ; "If (b1), then C is knowledge"
(b3) Test: We ask is "Contradiction (b)-(b2)".
Results: ((s=(q+~q)) > (s>q)) > (s=q) ; nvt ; TTFF TTFF TTTT TTTT ;
We answer "The fundamental problem (*) not a contradiction, but nvt".

(3) Since we do not obtain a contradiction in every case listed in (2) and because (2) depicts the complete and exhaustive list of possibilities, the problem of epistemology has the solution that it is not validated as a tautologous problem. This just means that epistemology is not impossible.

We further evaluate Eqs (a)-(a3) and (b)-(b3) in a format that renders all possibilities based on (2)(a3) and (2)(b3). We note that (2) serves as the primary antecedent where "C is either knowledge or not" from which all arguments follow. This renders (a3) and (b3) as:

(a3') (s=(q+~q)) > (((s=q)>(~s>p)) + ((s=~q)>p)) ; vt ;

"If C is either knowledge or not, then
either if C is knowledge, then if not C then a problematic domain
or if C is not knowledge, then a problematic domain.

(b3') (s=(q+~q)) > ((((s=~q)>(s>p))>(s=q)) + (((s=q)>(s>p))>(s=~q))) ; vt ;

"If C is either knowledge or not, then
either
if (C is not knowledge, then if C implies a problematic domain), then C is knowledge
or
if (C is knowledge, then if C implies a problematic domain),
then C is not knowledge.

Our conclusion is contra Nelson, that is, epistemology is not a problem and further epistemology is possible from which knowledge is derived.

Thanks are due to Professor Woleński for presenting the arguments of Leonard Nelson as readable.

von Neuman-Bernays-Gödel [NBG]

From en.wikipedia.org/wiki/Axiom_schema_of_specification, more on Axiom schema of specification using other expressions for von Neumann-Bernays-Gödel (NBG):

In von Neumann-Bernays-Gödel set theory, a distinction is made between sets and classes. A class *C* is a set if and only if it belongs to some class *E*. In this theory, there is a theorem schema that reads

[1.] $\exists D \forall C ([C \in D] \leftrightarrow [P (C) \wedge \exists E (C \in E)])$

that is, "There is a class *D* such that any class *C* is a member of *D* if and only if *C* is a set that satisfies *P*.", provided that the quantifiers in the predicate *P* are restricted to sets.

This theorem schema is itself a restricted form of comprehension, which avoids Russell's paradox because of the requirement that *C* be a set. Then specification for sets themselves can be written as a single axiom

[2.] $\forall D \forall A (\exists E [A \in E] \rightarrow \exists B [\exists E (B \in E) \wedge \forall C (C \in B \leftrightarrow [C \in A \wedge C \in D])])$

that is, "Given any class *D* and any set *A*, there is a set *B* whose members are precisely those classes that are members of both *A* and *D*.", or even more simply "The intersection of a class *D* and a set *A* is itself a set *B*.".

In this axiom, the predicate *P* is replaced by the class *D*, which can be quantified over. Another simpler axiom which achieves the same effect is

[3.] $\forall A \forall B ([\exists E (A \in E) \wedge \forall C (C \in B \rightarrow C \in A)] \rightarrow \exists E [B \in E])$

that is, "A subclass of a set is a set.".

[1.] **Not validated, so the theorem as published is not tautologous.**

1.1. substituting CDE as ABC with, per Quine, P(C) = (C=C) below as (A=A)

((%B&#A)&(A&B))=((%B&#A)&((A=A)&(%C&(A&C)))) ; sets as classes as theorems ;

Model 2.2; **nvt**

Model 1	Model 2.1	Model 2.2	Model 2.3.1	Model 2.3.2
TTTT TTTT TTCC TTCC	EEEE EPEP EEII EPIU	EEEE EEEE EEEE EEEE	EEEE EPEP EEEE EPEP	EEEE EEEE EEII EEII

1.2 substituting CDE as pqr with, per Quine, P(C) = (C=C) below as (p=p)

((%q&#p)&(p&q)) = ((%q&#p)&((p=p)&(%r&(p&r)))) ; sets as classes as propositions ;

Model 2.2; **nvt**

Model 1	Model 2.1	Model 2.2	Model 2.3.1	Model 2.3.2
TTTC TTTT TTTC TTTT	EEEU EEEE EEEU EEEE	EEEE EEEE EEEE EEEE	EEEP EEEE EEEP EEEE	EEEI EEEE EEEI EEEE

[2.] **Validated in the form of implication (>), as published, and also in the form of equivalence (=).**

((#s&#p)&(%p&(p&t))) > ((#s&#p)&(%q&((%t&(q&t))&(#r&((r&q)=((r&p)&(r&s))))))) ; vt

((#s&#p)&(%p&(p&t))) = ((#s&#p)&(%q&((%t&(q&t))&(#r&((r&q)=((r&p)&(r&s))))))); vt

[3.] **This is validated elsewhere as Axiom 3**.

((#A&#B)&((%D&(A&D))&(#C&((C&B)>(C&A))))) > ((#A&#B)&(%D&(B&D))); vt

Re: Deducibility theorems in Boolean logic on neutrosophic logic

From: Florentin Smarandache University of New Mexico 200 College Road Gallup, NM 87301, US
E-mail: smarand@unm.edu vixra.org/abs/1003.0171

As presumably a basis for neutrosophic logic these mistakes were found:

Assume the Meth8 apparatus.

LET: pqrs A1 B1 An Bn

(p>q)>((p&r)>(q&s)) ; TTTT TTT**F** TTTT TTTT ; Theorem 1
This formula is not tautologous.

(p>q)>((p+r)>(q+s)) ; TTTT **F**TTT TTTT TTTT ; Theorem 2
This formula is not tautologous.

If the above are "made by complete induction", then it is an example of why induction is defective.

LET: pqr ABC

((p&q)+r)>(p&(q&r)) ; TTT**F** **FFF**T TTT**F** **FFF**T ; Section 2(ii)
This formula is not deducible as such and is not tautologous.

((p>p)&(q<p))>((p&q)>(p&p)) ; TTTT TTTT TTTT TTTT ; 2a
[This is not a counter example of anything other than a contradiction, which Theorem 1 is *no*t as
TTTT TTT**F** TTTT TTTT.
For 2a to be a contradiction of Theorem 1, the 2a truth table should read:
FFFF FFFT **FFFF FFFF**]

((p>p)&(p<q))>((p+p)>(p+p)) ; TTTT TTTT TTTT TTTT ; 2b
[This is not a counter example of anything other than a contradiction, which Theorem 2 is not as
TTTT **F**TTT TTTT TTTT.
For 2b to be a contradiction of Theorem 2, the 2b truth table should read:
FFFF T**FFF FFFF FFFF**.]

Refutation of Dezert-Smarandache theory

The Dezert-Smarandache theory arises from the following scenario with Alice and Bob as suspects.

That either Alice or Bob is not innocent or both Alice and Bob are not innocent is a tautology. (1.1)

Using Meth8/VL4,

LET p q: Alice; Bob; + Or; & And; > Imply; = Equivalent;
% possibility, for one or some; # necessity, for all; (p=p) tautology; (%p>#p) ordinal one.

The designated *proof* value is T ; other logical values are F *contradiction*, N *truthity*; and C *falsity*. The 16-valued proof table is row-major and horizontal.

(p+q)+(p&q)=(p=p) ; FTTT FTTT FTTT FTTT (1.2)

If Eq. 1.1 introduces probability as a numeric variable, then we rewrite as:

That either Alice or Bob is not innocent or both Alice and Bob are not innocent is one. (2.1)

(p+q)+(p&q)=(%p>#p) ; CNNN CNNN CNNN CNNN (2.2)

Eqs. 1.2 and 2.2 as rendered are *not* tautologous. This refutes the Dezert-Smarandache theory.

Refutation of neutrosophy as generalized from Hegel's dialectic

We assume the method and apparatus of Meth8/VŁ4 where Tautology is the designated *proof* value, F is contradiction, N is truthity (non-contingency), and C is falsity (contingency). The 16-valued truth table is row-major and horizontal. We evaluate the following in *one* variable of p.

From: fs.gallup.unm.edu/FlorentinSmarandache.htm

> In philosophy he introduced in 1995 the 'neutrosophy', as a generalization of Hegel's dialectic, which is the basement of his researches in mathematics and economics, such as 'neutrosophic logic', 'neutrosophic set', 'neutrosophic probability', 'neutrosophic statistics'.
>
> Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. This theory considers every notion or idea <A> together with its opposite or negation <Anti-A> and the spectrum of "neutralities" <Neut-A> (i.e. notions or ideas located between the two extremes, supporting neither <A> nor <Anti-A>). The <Neut-A> and <Anti-A> ideas together are referred to as <Non-A>. According to this theory every idea <A> tends to be neutralized and balanced by <Anti-A> and <Non-A> ideas - as a state of equilibrium. As a consequence, he generalized the triad thesis-antithesis-synthesis to the tetrad thesis-antithesis-neutrothesis-neutrosynthesis ... .

LET: # necessity, for all (as in for every); % possibility, for one (as in for some) ;
~ Not ; + Or ; - Not Or ; & And ; \ Not And ; = Equivalent ; @ Not Equivalent ;
> Imply, greater than ; < Not Imply, less than ;
(%p>#p) 1 ; ((%p>#p)-(%p>#p)) 0 ;

p A as notions or ideas ;
#p *every* <**A**>, hereafter, and *thesis* (0.1.1);(0.1.2)
~#p <**Anti-A**> not *every* notion or idea, hereafter, and *antithesi*s (0.2.1);(0.2.2)
~(#p+~#p) Not (<A> Or <Anti-A>), as in neither #p nor ~#p, and *synthesis* (0.3.1);(0.3.2)

<**Neut-A**> spectrum of "neutralities" as notions or ideas between extrema of <A> and <Anti-A>). In other words, <Neut-A> is greater than <Anti-A> and is less than <A>, but not equal to either, as *neutrothesis* (1.1)

(~(#p+~#p)>~#p)&(~(#p+~#p)<#p) ;
FFFF FFFF FFFF FFFF (1.2)

<**Non-A**> = (<Neut-A> and <Anti-A>) (2.1)

((~(#p+~#p)>~#p)&(~(#p+~#p)<#p))&~#p ;
FFFF FFFF FFFF FFFF (2.2)

every <A> *tends* to be neutralized and balanced by <Anti-A> and <Non-A> as a *state of* equilibrium, and *neutrosynthesi*s. That state is assumed to be zero.
(<**A**> > [<**Anti-A**> + <**Non-A**> = 0] (3.1)

#p>(~#p&(((~(#p+~#p)>~#p)&(~(#p+~#p)<#p))&~#p)) ;
TCTC TCTC TCTC TCTC (3.2)

As a consequence, he generalized the triad thesis-antithesis-synthesis to the tetrad thesis-antithesis-neutrothesis-neutrosynthesis [*ie, the triad is a subset of the tetrad*] as

Eqs. ((0.1.1&0.2.1)>0.3.1) < ((0.1.1&0.2.1)>(1.1>3.1) (4.1)

((#p&~#p)>~(#p+~#p)) < ((#p&~#p)>(((~(#p+~#p)>~#p)&(~(#p+~#p)<#p))&
(#p>(~#p&(((~(#p+~#p)>~#p)>(~(#p+~#p)<#p))&~#p))))) ;
FFFF FFFF FFFF FFFF (4.2)

Eq. 4.2 as rendered is *not* tautologous and a contradiction. This refutes the definition of neutrosophy and consequently invalidates it as a generalization of Hegel's dialectic.

Remark: Hegel's dialectical philosophy lacks a quantified and modalized symbolic logic; to map it into a modal logic model checker is hence potentially problematic.

Refutation of neutrosophic logic of Florentin Smarandache as general intuitionistic, fuzzy logic

We rely on:

> Smarandache, F. 2010. Neutrosophic Logic - A Generalization of the Intuitionistic Fuzzy Logic.
> vixra.org/abs/1004.0008; vixra.org/pdf/1004.0008v2.pdf;
> arxiv.org/ftp/math/papers/0303/0303009.pdf

We assume the apparatus and method of Meth8/VŁ4.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	p=p	T	Tautology	proof	11	3
2	p@p	F	Contradiction	absurdum	00	0
3	%p>#p	N	Non-contingency	truth	01	1
4	%p<#p	C	Contingency	falsity	10	2

LET: ~ Not; & And; \ Not And; + Or; - Not Or;
> Imply, greater than; > Not Imply, less than; = Equivalent to;
necessity, for all; % possibility, for some (one); (p-p) zero; (p\p) one;
q>(p-p) q>zero; q<(p\p) q<one; q=(p-p) q=zero; q=(p\p) q=one

The designated *proof* value is T(autology). The 16-valued result table is presented in row-major and horizontally.

For neutrosophic logic (N), we map the respective values of truth, falsity, and indeterminacy as:

Nt (%p>#p); Nf (%p<#p); Ni (((%p>#p)+(%p<#p))+~((%p>#p)+(%p<#p))). (1.1)

We simplify our evaluation by ignoring the numeric scaling factor of lower-case_epsilon ε. That serves to push a single numeric value of the combined, summed state of Nt+Ni+Nf outside an interval definition of q on "]0,1[" and into "]0,3[", or ultimately to natural numbers, including a number zero.

#(((q>(p-p))&(q<(p\p)))+((q=(p-p))+(q=(p\p)))) > %(q=(((%p>#p)+(%p<#p))+~((%p>#p)+(%p<#p)))) ; TCTT TCTT TCTT TCTT (1.2)

In Eq. 1.2 the antecedent establishes the necessity of $0 \leq q \leq 1$.

In Eq. 1.2 the consequent establishes the possibility that q is the summation of Nt+Ni+Nf.

In Eq. 1.2 the result of the literal is *not* tautologous, meaning neutrosophic logic is refuted and hence its use as a generalization of intuitionistic, fuzzy logic is likewise unworkable.

We expand our evaluation by including more neutrosophic values for absolute truth +1, absolute falsity -0, and absolute indeterminacy on the interval written "]-0,1+[", as respectively:

N+t (#p>#p); N+f (#p<#p); N+i (((#p>#p)+(#p<#p))+~((#p>#p)+(#p<#p))). (2.1)

We substitute values of Eq. 2.1 into Eq. 1.2.

```
#(((q<(p-p))&(q>(p\p)))+((q=(p-p))+(q=(p\p)))) > %(q=(((#p>#p)+(#p<#p))+~((#p>#p)+(#p<#p)))) ;
                                                 TCTT TCTT TCTT TCTT             (2.2)
```

In Eq. 2.2 the antecedent establishes the necessity of $1 \leq q \leq 0$.

In Eq. 2.2 the consequent establishes the possibility that q is the summation of (N+t) + (N+i) + (N+f).

In Eq. 2.2 the result of the literal is *not* tautologous, with the same table result as in Eq. 1.2 and generalization as likewise unworkable.

Refutation of Smarandache geometry

Abstract: We evaluate the Smarandache algebra system without unit as basis of its geometry. On the right inverse operator in the dexter (right) digits, 1·1=1 contradicts 1·1=0. Hence Smarandache geometry is a probabilistic vector space and refuted as an exact, bivalent logic.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And; > Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) C non-contingency, ∇, ordinal 2; (%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Mao, L.F. (2011). Automorphism groups of maps, surfaces and Smarandache geometries. fs.unm.edu/Linfan2.pdf *maolinfan@163.com*

Definition 1.2.3
Let (A ; ∘) be an algebraic system with a unit 1_A . An element a ∈ A is called to be a right inverse of b ∈ A if a ∘ b = 1_A . Certainly, there are algebra systems without unit. For example, let H = {a, b, c, d} with an operation · determined by the following table.

·	a	b	c	d
a	b	c	a	d
b	c	d	b	a
c	a	b	d	c
d	d	a	c	b

Table 1.2.3

Then (H, ·) is an algebraic system without unit.

Remark 1.2.3:

LET 11, 01: a, c.

```
a 11        c 01
c 01        a 11
· 11        · 10
```

In the dexter (right) digits above, 1·1=1 contradicts 1·1=0. Hence Smarandache geometry is a probabilistic vector space and refuted as an exact, bivalent logic.

Refutation of another neutrosophic genetic algorithm

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s, t, u, v, w, x: A1, A2, A3, A4, A5, s1, s2, s3, d1;
~ Not; & And, ^; = Equivalent;
> Imply, greater than; < Not Imply, lesser than, ∈.

From: Elwahsh, H.; et al. (2018). A novel approach for classifying MANETs attacks with a neutrosophic intelligent system based on genetic algorithm. vixra.org/pdf/1810.0042v1.pdf haitham.elwahsh@gmail.com

If (A1 ∈ s2 ^ A2 ∈ s3 ^ A3 ∈ s2 ^ A5 ∈ s1) then d1 ∈ s3 (3.1)

((((p<v)&(q<w))&(r<v))&(u<u))>(x<w) ;
`TTTT TTTT TTTT TTTT` (3.2)

Eq. 3.2 as rendered is tautologous.

Remark: We decompose Eq. 3.1 into truth tables for the antecedent and consequent respectively.

(A1 ∈ s2 ^ A2 ∈ s3 ^ A3 ∈ s2 ^ A5 ∈ s1) (3.1.1.1)

((((p<v)&(q<w))&(r<v))&(u<u))=(p=p);
`FFFF FFFF FFFF FFFF` (3.1.1.2)

d1 ∈ s3 ; (3.1.2.1)

(x<w)=(p=p) ; **`FFFF FFFF FFFF FFFF`**,
`TTTT TTTT TTTT TTTT` (3.1.2.2)

Eq. 3.2 consists of the implication pattern of **`F`**>**`F`** = `T`.

We accepted the author's invitation to request the data set in an Excel file on which Eq. 3.1 is derived. The values sought were for neutrosophic feature A_n, subset s_n, degree of membership $u_A(x)$, degree of non-membership $v_A(x)$, and indeterminancy $s_A(x)$. Our approach was to evaluate using our specialized contingency test for how significantly the supplied data diverged from a state of randomness. However the request was not answered.

Refutation of neutrosophic lattices for negated adjectival phrases

We assume the method and apparatus of Meth8/VŁ4 where `T`autology is the designated *proof* value, `F` is contradiction, `N` is truthity (non-contingency), and `C` is falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET pq A B;
~ Not; & And, binary set operator ∩; + Or, binary set operator $\square^*$;
~(>)* Not Imply, $\leq_*$, that is, partial order as greater than or equal to as not less than.

We evaluate neutrosophic lattices for negated adjectival phrases from:

Smarandache, F.; Topal, S. (2018). A lattice theoretic look: a negated approach to adjectival (intersective, neutrosophic and private) phrases and more. vixra.org/pdf/1805.0028v1.pdf

Definition 3. We define a partial order $\leq_*$ on sets as the follow [sic]:

$A \leq_* B$ if $B = A \square^* B$ (3.1.1)

$A \leq_* B$ if $A = A \cap B$ (3.2.1)

(q=(p+q))>~(p>q) ; `FTFF FTFF FTFF FTFF` (3.1.2)

(p=(p&q))>~(p>q) ; `FTFF FTFF FTFF FTFF` (3.2.2)

While Eqs. 3.1.2 and 3.2.2 are equivalent, they are *not* tautologous as definitions to commence the paper.

Consequently we stop there, evaluate no further, and conclude the premise is refuted of neutrosophic lattices for negated adjectival phrases.

Refutation of the multi-valued neutrosophic logic (and as a theory of everything in logics)

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: -0, 0, 1, 1+ as <nonA>, <A>, <antiA>, <neutA> ;
& And; > Imply; = Equivalent; @ Not Equivalent;
(q@q) 0, **F**; (r=r) 1, T.

From: Smarandache, Florentin. (2010). Neutrosophic logic as a theory of everything in logics. fs.gallup.unm.edu/NLasTheoryOfEverything.pdf (not accessible on August 22, 2018).

In neutrosophy we can combine <A> and <nonA>, getting a degree of <A> a degree of <neutA> and a degree of <antiA>. (1.1)

(q&p)>((q&s)&r) ; TTT**F** TTT**F** TTT**F** TTTT (1.2)

<A> actually gives birth to <antiA> and <neutA> ... (2.1)

q>(s&r) ; TT**FF** TT**FF** TT**FF** TTTT (2.2)

Remark: We combine Eqs. 1.1 and 2.1 to define fully the logic system. (3.1)

((q&p)>((q&s)&r))&(q>(s&r)) ; TT**FF** TT**FF** TT**FF** TTTT (3.2)

Eqs. 1.2 and 2.2 are *not* tautologous, and 3.2 produces the same truth table as from 2.2.

This refutes the multi-valued logic system of neutrosophy, and as a theory of everything in logics.

Logic not tautologous in neutrosophic sets

From: Wang, H; et al. Single valued neutrosophic sets. vixra.org/pdf/1004.0051v1.pdf [raj@cs.gsu.edu]

We test a theorem and two properties from above.

We assume the apparatus of the Meth8 modal logic model checker implementing variant system VŁ4. Meth8 allows to mix four logical values with four analytical values. The designated *proo*f value is T.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	p=p	T	Tautology	proof	11	3
2	p@p	F	Contradiction	absurdum	00	0
3	%p>#p	N	Non-contingency	truth	01	1
4	%p<#p	C	Contingency	falsity	10	2

LET: + Or; & And; \ Not and; > Imply; < Not imply; = Equivalent to;
@ Not equivalent to; # all; % some; (p@p) zero ; (p=p) one

Results are the proof table of 16-values in row major horizontally.

Theorem 3 [sic]; read Theorem 1. A ⊆ B ↔ c(B) ⊆ c(A)

~(B<A)=~((C&A)<(C&B)) ; TTFT TFFT TTFT TFFT (1.1)

Property 5. A ∪ X = X, where ...

(((((t&q)=(u&q))=(p@p))&((s&q)=(p=p)))&((((t&r)=(u&r))=(p=p))&((s&r)=(p@p))))
> ((p+r)=r) ; TTTT TTTT TTTF TTTT (5.2)

Property 6. A ∪ φ = A, where ...

(((((t&q)=(u&q))=(p@p))&((s&q)=(p=p)))&((((t&r)=(u&r))=(p=p))&((s&r)=(p@p))))
> ((p+q)=p) ; TTTT TTTT TTFT TTTT (6.1)

Eqs. 1.1, 5.2, and 6.1 should be tautologous, but are not.

Refutation of neutrosophic soft lattice theory

Taken from:

> Uluçay, Vakkkas; Şahin, Mehmet; Olgun, Necati; and Kilicman, Adem.
> "On neutrosophic soft lattices". Afr. Mat. DOI 10.1007/s13370-016-0447-7.
> vixra.org/pdf/1706.0269v1.pdf
> © African Mathematical Union and Springer-Verlag Berlin Heidelberg 2016.

We evaluate the neutrosophic logic based on its most atomic level of soft latices, as published by Springer-Verlag in 2016.

Of interest to us is the seminal Theorem 3.17 on un-numbered page 7 is this theorem:

> Every neutrosophic soft lattice is a one-sided distributive neutrosophic soft lattice. (3.17)

We assume the apparatus and method of Meth8 implementing variant logic system VŁ4.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	p=p	T	Tautology	proof	11	3
2	p@p	F	Contradiction	absurdum	00	0
3	%p>#p	N	Non-contingency	truth	01	1
4	%p<#p	C	Contingency	falsity	10	2

Due to problematic font presentation of symbols in the paper, we substitute equations here, as:

LET: p q r F_A F_B F_C ;
~ Not; = Equivalent to; & And; \ Not And; + Or; - Not Or; > Imply; < Not Imply;
\ $\tilde{\wedge}$, Not And; - $\tilde{\vee}$, Not Or;
$\tilde{\leq}$, Not less than or equal to (n.L.T.E): "p $\tilde{\leq}$ q" is equivalent to "~((p<q)+(p=q))".

The designated *proof* value is T. The 16-valued tables are horizontal as row-major.

We evaluate Eq. 3.17 as stand-alone first, then as a consequence of the build up farther below.

$$F_A \tilde{\wedge} F_B = (F_A \tilde{\wedge} F_B)\tilde{\wedge}(F_A \tilde{\wedge} F_B)\tilde{\leq} F_A \tilde{\wedge}(F_B \tilde{\vee} F_C) \text{ (a)} \qquad (3.17.1)$$

This renders in Meth8 as:

(p\q) = ((p\q) \ ~(((p\q)<(p\(q-r))) + ((p\q)=(p\(q-r)))));
`TTTF TTTF TTTF TTTF` (3.17.2)

Eq. 3.17.2 as rendered by Meth8 is *not* tautologous (all `T`) and hence not a theorem.

Without repeating build up arguments to Eq. 3.17.1, as "*Proof* Let ... Since ... and ..., ... Therefore,", we present the entire argument rendered in Meth8 in 123 steps as:

```
(((p\~((q<p)+(q=p)))&(p\~((q<~((q<~((q<(q-r))+(q=(q-r))))+(q=~((q<(q-r))+(q=(q-r))))))
+(q=~((q<~((q<(q-r))+(q=(q-r))))+(q=~((q<(q-r))+(q=(q-r)))))))))>((p\~((q<p)+(q=p))) &(p\~((q<(q\
r))+(q=(q\r))))))>((p\q)=((p\q)\~(((p\q)<(p\(q-r)))+((p\q)=(p\(q-r)))))) ;
                                                TTTF TTTF TTTF TTTF          (3.17.3)
```

Eq. 3.17.3 as rendered by Meth8 is *not* tautologous (all T), at which we stopped.

The proof tables from Eqs. 3.17.2 and 3.17.3 are identical which means the build up arguments are confirmed to produce Eq. 3.17.1, but for which Eq. 3.17 is refuted as a conjectured theorem.

This brief evaluation implies that the field of soft set theory as originally introduced by D. Molodtsov is suspicious and specifically that the field of neutrosophic logic, as evidenced in its basis of soft set theory, is unworkable.

This conclusion is multitudinal because of the plethora of duplicated papers as translations in multiple fields at vixra.org regarding the neutrosophic logic system of Florentin Smarandache.

Refutation of neutrosophy definitions using probability and (in)dependency

Abstract: Definitions of neutrosophy as further embellished with probability and (in)dependency share the same result as denied of tautology. This means neutrosophic logic as a general framework for unification of many existing logics, such as intuitionistic fuzzy logic) and paraconsistent logic, is refuted.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET ~ Not; + Or; & And; > Imply; < Not Imply; = Equivalent;
% possibility, for one or some; # necessity, for every or all; ~(y<x) (x≤ y);
p, q, r, s: Probability of independence 0≤p≤1, ; T Truthity, t, (%p>#p),
ordinal 1; F Falsity, f, (%p<#p), ordinal 2; I Indeterminancy, i, Truthity or Falsity, Tautology, Proof, (%p>#p)+(%p<#p), ordinal 3;
(p=p) ordinal 3; (p@p) ordinal 0 (zero); ~(y<x) (x≤ y);

From: fs.unm.edu/neutrosophy.htm Vázquez, M. L.; mleyvaz@gmail.com

1. Neutrosophic Logic is a general framework for unification of many existing logics, such as fuzzy logic (especially intuitionistic fuzzy logic), paraconsistent logic, intuitionistic logic, etc. (1.1.0.1)

The main idea of NL is to characterize each logical statement in a 3D Neutrosophic Space, where each dimension of the space represents respectively the truth (T), the falsehood (F), and the indeterminacy (I) of the statement under consideration, where T, I, F are standard or non-standard real subsets of]$^{-}$0,1^{+}[with not necessarily any connection between them. For software engineering proposals the classical unit interval [0,1] is used.

For single valued Neutrosophic logic, the sum of the components is:

Remark 1: Below is *not* a single valued logic, but a *three*-valued, multi logic.

0≤(t+(i+f))≤3 when all three components are independent; (1.1.1.1)

(((%p>#p)+(%p<#p))\((%p>#p)+(%p<#p))) >
(~((((%p>#p)+(%p<#p))+((%p>#p)+(%p<#p)))<(p@p))&
~((((%p>#p)+(%p<#p))+((%p>#p)+(%p<#p)))>(p=p))) ;
`TTTT TTTT TTTT TTTT` (1.1.1.2)

Remark 1.1.2.2: The antecedent and consequent are equivalent, hence the result should be expected.

0≤(t+(i+f))≤2 when two components are dependent, while the third one is independent from them; (1.1.2.1)

((%p<#p)\((%p>#p)+(%p<#p))) >
(~((((%p>#p)+(%p<#p))+((%p>#p)+(%p<#p)))<(p@p))&
~((((%p>#p)+(%p<#p))+((%p>#p)+(%p<#p)))>(%p<#p))) ;
CCCC CCCC CCCC CCCC (1.1.2.2)

$0\leq(t+(i+f))\leq 1$ when all three components are dependent. (1.1.3.1)

((p@p)\((%p>#p)+(%p<#p)))>
(~((((%p>#p)+(%p<#p))+((%p>#p)+(%p<#p)))<(p@p))&
~((((%p>#p)+(%p<#p))+((%p>#p)+(%p<#p)))>(p@p))) ;
FFFF FFFF FFFF FFFF (1.1.3.2)

When three or two of the components T, I, F are independent, one leaves room for incomplete information (sum<1), paraconsistent and contradictory information (sum>1), or complete information (sum=1). (1.1.4.0)

We write this as: If Eq. 1.1.1.1 and 1.1.2.1, then the sum of (t+(i+f)) is lesser than one or the sum is greater than one or the sum is equal to one. (1.1.4.1)

(((%p>#p)\((%p>#p)+(%p<#p)))+((%p<#p)\((%p>#p)+(%p<#p)))) >
((((((%p>#p)+(%p<#p))+((%p>#p)+(%p<#p)))>(%p>#p))+((((%p>#p)+
(%p<#p))+((%p>#p)+(%p<#p)))<(%p>#p)))+
((((%p>#p)+(%p<#p))+((%p>#p)+(%p<#p)))=(%p>#p))) ;
TTTT TTTT TTTT TTTT (1.1.4.2)

Remark 1.1.4.2: Eq. 1.1.4.2 is trivial because the antecedent as **F**T**F**T implying the consequent as TTTT is an obvious canonical form.

If all three components T, I, F are dependent, then similarly one leaves room for incomplete information (sum<1), or complete information (sum=1). (1.1.5.0)

We write Eq. 1.5.0.0 as: If Eq. 1.1.3.1, then the sum is not greater than one. (1.1.5.1)

((p@p)\((%p>#p)+(%p<#p)))>
~((((%p>#p)+(%p<#p))+((%p>#p)+(%p<#p)))>(%p>#p)) ;
CCCC CCCC CCCC CCCC (1.1.5.2)

Eqs. 1.1.2.1, 1.1.3.1, and 1.1.5.1 are *not* tautologous. This is sufficient to deny the definitions of neutrosophy as using probability and (in)dependency, and further to refute neutrosophic logic as a generalized framework to unify other logics such as those listed in Eq. 1.1.0.1.

Refutation of pseudo-trinitarian mapping of the two great commandments via neutrosophy

Abstract: We map the two great commandments in Matt 22:37-40 as conjunction of God, self, and others. It is *not* tautologous and a gross misreading by forcing a pseudo-trinitarian theology. (We resuscitate the text as a theorem by expansion.) Hence, three-valued neutrosophic logic is insufficient to prove models of human consciousness, and thereby forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Christianto, V.; Smarandache, F. (2019). An outline of extension from neutrosophic psychology to pneumatic transpersonal psychology: towards relational psychotherapy and relational pedagogy. vixra.org/pdf/1906.0294v1.pdf

Abstract: … In this paper, we consider a further step: introducing "soul" as a different element of human consciousness. We discuss … an integral model of human consciousness, including relational psychotherapy and relational pedagogy … towards nonlinear human consciousness model.

From neutrosophic psychology toward integral model of human consciousness: Figures 3, 4, 5 map a schema for Matthew 22:37-40 as: When tempted by a lawyer as to which is the greatest commandment, Jesus said: The first and great commandment is: Love the Lord thy God with all thy heart, with all they mind, and with all they soul. The second is like unto it: Love they neighbor as thyself. On these two commandments hang all the law and the prophets. (1.1)

LET p, r, s: God (spirit, higher self), self (ego, soul), others (conscience)

p&(r&s) ; **FFFF FFFF FFFF F**T**F**T (1.2)

Remark 1: The conjunction of God, self, others is *not* tautologous. We resuscitate Eq. 1.1 with injection of the attribute love and antecedent clause of God implies: (2.1)

LET p, q, r, s: God, [love], self, others

p>((s>(q>p))&(s>(q>(r>s)))) ; TTTT TTTT TTTT TTTT (2.2)

Eq. 1.2 as rendered is *not* tautologous and a gross misreading of the text by forcing a pseudo-trinitarian theology. To adopt the further approach of applying three-valued neutrosophic logic is insufficient to prove models of human consciousness. (NB: The authors ignore Jung's demise in 1961 as a practicing satanist.)

Refutation of quinary logic in neutrosophy

We evaluate the quinary logic of neutrosophy from:

> Patrascu, V. (2018). Entropy, neutro-entropy and anti-entropy for neutrosophic information. vixra.org/pdf/1805.0023v1.pdf

We assume the method and apparatus of Meth8/VŁ4 where Tautology is the designated *proof* value, F is contradiction, N is truthity (non-contingency), and C is falsity (contingency). The 16-valued truth table is row-major and horizontal, but not needed here as evaluation is in one variable only, p.

LET # necessity, for all; % possibility, for one or some;
+ Or; \ Not And; > Imply; < Not Imply; = Equivalent; @ Not Equivalent;
(%p>#p) 1, N ; (%p<#p) 2, C.

Figs. 1, 2: The five features and prototypes of bifuzzy information [*in the neutrosophic lozenge]*

truth	T	(1,0)		(1.1.1)
Truthity (Non contingency)	N	01	(%p>#p)	(1.1.2)
ignorance	U	(0,0)		(1.2.1)
Tautology (Proof)	T	11	(p=p)	(1.2.2)
contradiction	C	(1,1)		(1.3.1)
Contradiction (Absurdum)	F	00	(p@p)	(1.3.2)
falsity	F	(0,1)		(1.4.1)
Falsity (Contingency)	C	10	(%p<#p)	(1.4.2)
ambiguity (U+C)/2	A	(0.5,0.5)		(1.5.1)
(T+F)/C	N	(11+00)\10= 01	(%p>#p)	(1.5.2)
ambiguity (T+F)/2	A	(0.5,0.5)		(1.6.1)
(N+C)/C	N	(01+10)\10=01	(%p>#p)	(1.6.2)

From Eqs. 1.5.2 and 1.6.2 as rendered, the notion of ambiguity A (0.5,0.5) is *not* tautologous but rather truthity.

> **Remark:** The abstract of the captioned paper introduces the modal words of possibility and necessity which unfortunately are not mentioned in the text.

We conclude that there is no provision in the neutrosophic cube to introduce modal operators.

Because neutrosophy has no bivalent square of opposition, but rather a non-bivalent lozenge with a multiple-defined midpoint, the quantified operators are prohibited from definition and hence are disparate from neutrosophy. This means neutrosophic logic is unable to map and support modal or alethic logic.

Refutation of the retract neutrosophic crisp set

Abstract: Demonstration of the retract neutrosophic crisp set is denied by another example of egregious logic in the Smarandache neutrosophy.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

LET p, q, s: A, B, co; + Or, ∪; & And, ∩; = Equivalent.

From: Salama, A.A.; Hewayda, E.G.; Nasr, A.M. (2018).
Retract neutrosophic crisp system for gray scale image.
rsalama44@gmail.com via vixra.org/pdf/1804.0170v1.pdf

3.4 Proposition [from 2015]

co(A ∩ B) = coA ∪ coB (3.4.1.1)

(s&(p&q))=((s&p)+(s&q)) ; TTTT TTTT T**FF**T T**FF**T (3.4.1.2)

co(A ∪ B) = coA ∩ coB (3.4.2.1)

(s&(p+q))=((s&p)&(s&q)) ; T**FF**T T**FF**T TTTT TTTT (3.4.2.2)

Eqs. 3.4.1.2 and 3.4.2.2 as rendered are *not* tautologous. Consequently, everything subsequent to Sec. 3.3 is tainted.

Remark: Prior definitions in Sec. 2 for neutrosophic crisp sets (NCS (2015)) are also *not* tautologous, although not directly relevant to Eqs. 3.4.

LET p, q, r, s: A_1, A_2, A_3, X; @ Not Equivalent; (s@s) null.

(((p&q)=(s@s))&((p&r)=(s@s)))&((q&r)=(s@s)) ;
TTT**F** T**FFF** TTT**F** T**FFF** (1) NCS-Class1

((((p&q)=(s@s))&((p&r)=(s@s)))&((q&r)=(s@s)))&(((p+q)+r)=s) ;
T**FFF** **FFFF** **F**TT**F** T**FFF** (2) NCS-Class2

(((p&q)&r)=(s@s))&(((p+q)+r)=s) ; T**FFF** **FFFF** **F**TTT TTT**F** (3) NCS-Class 3

Refutation of Smarandache multi-space theory

Abstract: The Smarandache multi-space theory (SMT) as based on a Latin square is a vector space (probabilistic) and not bivalent (exact). Therefore SMT is refuted in classical logic.

From: Mao, L.F. (2006). "Smarandache multi-space theory". fs.unm.edu/S-Multi-Space.pdf

We assert a Latin square L_1 Table 1.3.1 is not bivalent (exact) but rather a vector space (probabilistic).

With row-major (r) and column-minor (c) we reproduce this artifact:

		c1:	c2:	c3:
		1	2	3
r2:	2	2	3	1

We convert the decimal ordinals to bivalent 2-tuples, with the left-most bit as most significant:

		c1:	c2:	c3:
		01	10	11
r2:	10	10	11	01

We perform binary operations, with r2 as antecedent and c1, c2, and c3 as respective consequents, and sequent outcome (q). We also designate bits in Latin as *s*inister (left) and *d*exter (right).

	sd		*sd*		*sd*
r2:	10	r2:	10	r2:	10
c1:	01	c2:	10	c3:	11
q1:	10	q2:	11	q3:	01

To be bivalent, i.e. compatible with classical logic, any like-sided equations produce identical results.

LET operator "?" serve as the connective in this horizontal presentation to save space

Example 1:

r2d ? C1d (1.1.1)

0 ? 1 = 0 (1.1.2)

r2d ? C3d (1.2.1)

0 ? 1 = 1 (1.2.2)

Eqs. 1.1.2 and 1.2.2 should be equivalent to be bivalent, but that is not the case.

Example 2:

r2s ? C2s (2.1.1)

1 ? 1 = 1 (2.1.2)

r2s ? C3s (2.2.1)

1 ? 1 = 0 (2.2.2)

Eqs. 2.1.2 and 2.2.2 should be equivalent to be bivalent, but that is not the case.

Because Examples 1 and 2 as rendered are *not* tautologous, the Smarandache multi-space theory (SMT) is refuted.

Unification by neutrosophic logic not tautologous

From: Christianto, V.; Smarandache, F. (2017). How a synthesizer works. vixra.org/pdf/1711.0442v1.pdf

"Neutrosophic Logic is a general framework for unification of many existing logics, such as fuzzy logic (especially intuitionistic fuzzy logic), paraconsistent logic, intuitionistic logic, etc.

The main idea of NL is to characterize each logical statement in a 3D Neutrosophic Space, where each dimension of the space represents respectively the truth (T), the falsehood (F), and the indeterminacy (I) of the statement under consideration, where T, I, F are standard or non-standard real subsets of]-0, 1+[with not necessarily any connection between them.

For software engineering proposals the classical unit interval [0, 1] is used.

For single valued Neutrosophic logic, the sum of the components is:

$0 \le t+i+f \le 3$ when all three components are independent; (3.1.1)

$0 \le t+i+f \le 2$ when two components are dependent,
while the third one is independent from them; (2.1.1)

$0 \le t+i+f \le 1$ when all three components are dependent. (1.1.1)

When three or two of the components T, I, F are independent, one leaves room for incomplete information (sum < 1), paraconsistent and contradictory information (sum > 1), or complete information (sum = 1). (3.2.1)

If all three components T, I, F are dependent, then similarly one leaves room for incomplete information (sum < 1), or complete information (sum = 1)." (1.2.1)

We assume the apparatus of the Meth8 modal logic model checker implementing variant system VŁ4. Meth8 allows to mix four logical values with four analytical values. The designated *proo*f value is T.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	p=p	T	Tautology	proof	11	3
2	p@p	F	Contradiction	absurdum	00	0
3	%p>#p	N	Non-contingency	truth	01	1
4	%p<#p	C	Contingency	falsity	10	2

LET: ~ Not; + Or; = Equivalent to; @ Not equivalent to; > Imply, greater than;
all, necessity; % some, possibility; (p=p) 11, three; (%p>#p) 01, one;
p q r s f i t sum=f+i+t; Note (0 ≤ s) is equivalent to ~(s < 0).

Results are the repeating proof table(s) of 16-values in row major horizontally.

We evaluate Eqs. 3.1.1 and 3.2.1 as an axiom or definition with rules.

(s=((p+q)+r))&((~((p@p)>s)&~(s>(p=p)))>(((s<(%p>#p))+(s>(%p>#p)))+(s=(%p>#p)))) ;
`TFFF FFFF FTTT TTTT` (3.3)

We do not evaluate Eq. 2.1.1 because it has no rules.

We evaluate Eqs. 1.1.1 and 1.2.1 as an axiom or definition with rules.

(s=((p+q)+r))&((~((p@p)>s)&~(s>(%p>#p)))>((s<(%p>#p))+(s=(%p>#p)))) ;

`TFFF FFFF FTTT TTTT` (1.3)

Eqs 3.3 and 1.3 are *not* tautologous, and in fact produce the same proof table.

This means neutrosophic logic is not bivalent, but a probabilistic vector space, and hence inexact.

What follows is that neutrosophic logic cannot unify other logics in a tautology.

A shorter refutation of neutrosophic logic

Meth8/VL4 re-evaluates the multi-valued neutrosophic logic, with T as the designated proof value.

> Define values in neutrosophic logic and sets as: ((1 or 0) or (less_than 1 and greater_than 0)). (1.1)
>
> LET: (%p>#p) 1; s discrete values of neutrosophic logic.
>
> (s=((%p>#p)+((%p>#p)-(%p>#p)))) + ((s<(%p>#p))&(s>((%p>#p)-(%p>#p))));
> FFFF FFFF TTTT TTTT (1.2)

To use one as tautology (p=p) and zero as contradiction ~(p=p), then re-write Eq. 1.1 as:

> Define values in neutrosophic logic and sets as ((proof or non-proof) or (less_than proof and greater_than non-proof). (2.1)
>
> (s=((p=p)+~(p=p))) + ((s<(p=p))&(s>~(p=p))) ;
> FFFF FFFF TTTT TTTT (2.2)

Eqs. 1.2 and 2.2 as rendered are *not* tautologous. This means the multivalued neutrosophic logic is refuted, *not* bivalent, and hence not exact.

Neutrosophic logic as a five-valued vector space

Neutrosophic logic s is defined as a five-valued logic on {-0, 0, 0<p<1, 1, 1+}. (1.0)

Meth8/VL4 maps Eq. 1.0 as s = { F, C, C<p<N, N, T }. (1.1)

LET: % possibility, for one or some; # necessity, for all ; Values of s are:
#(%p>#p) 1+; #((%p>#p)-(%p>#p)) 0-; %(%p>#p) 1; %((%p>#p)-(%p>#p) 0;
and other values in between 0 to 1 as ((s<%(%p>#p))&(s>%((%p>#p)-(%p>#p)));
(p=p) T tautology; ~(p=p) F contradiction; (%p<#p) C falsity; (%p>#p) N truthity.

T is the designated proof value; proof tables are row-major and horizontal.

((s=(#(%p>#p)+#((%p>#p)-(%p>#p)))) + (s=(%(%p>#p)+%((%p>#p)-(%p>#p))))) + ((s<%(%p>#p))&(s>%((%p>#p)-(%p>#p)))) ; CCCC CCCC TTTT TTTT (1.2)

Eq. 1.2 as rendered is *not* tautologous.

We now map the sub-indeterminacies I_1-I_6 as given in Florentin Smarandache (2015) "Symbolic neutrosophic theory".

I_1:	(p=p)	&	~(p=p) ;		FFFF
I_2:	(p=p)	+	~(p=p) ;		TTTT
I_3:	(p=p)	-	~(p=p) ;		FFFF
I_4:	~(p=p)	&	~~(p=p) ;		FFFF
I_5:	~(p=p)	+	~~(p=p) ;		TTTT
I_6:	~(p=p)	&	~~(p=p) ;		FFFF

We replicate the look up truth table of the values above as published in Table 2: Sub-Indeterminacies Multiplication Law. We mark corrections in brackets to show the table as if it were bivalent.

* &	I_1 F	I_2 T	I_3 F	I_4 F	I_5 T	I_6 F
I_1 F	F	F	F	F	F	F
I_2 T	F	T	F	F	F [T]	F
I_3 F	F	F	F	F	F	F
I_4 F	F	F	F	F	F	F
I_5 T	F	F [T]	F	F	T	F
I_6 F	F	F	F	F	F	F

Table 2 as rendered is not bivalent on its face. Consequently we abandon neutrosophic logic because it is a vector space (not *necessarily* bivalent). Hence neutrosophic logic may not be adopted as the universal logic to map and confirm all other logics.

Refutation of the Newcomb paradox

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table fragment) is row-major and horizontal.

LET p, q, r, s: A clear box; B opaque box; player; predictor;
& And; + Or; - Not Or; = Equivalent; > Imply, greater than; < Not Imply, less than;
% possibility, for one or some; # necessity, for all;
(%p>#p) truthity, content present; (%p<#p) = ((%p>#p)-(%p>#p)) falsity, content absent.

We ignore visibility states of boxes and hence dollar contents to test the logic.

From: en.wikipedia.org/wiki/Newcomb%27s_paradox

Box A contents visible and always set at $1,000.
Box B contents not visible and already set by the predictor:

If the predictor predicts the player takes both boxes A and B, then box B contains nothing. (1.1)

(s>(r>(p&q)))>(q=(%p<#p)) ; NNCC NNCC NNCC TTTC (1.2)

If the predictor predicts that the player takes only box B, then box B contains $1,000,000. (2.1)

(s>(r> q)) >(q=(%p>#p)) ; CCNN CCNN CCNN TTTN (2.2)

Eqs. 1.2 and 2.2 as rendered are *not* tautologous, but also are *not* contradictory. This means the Newcomb paradox is *not* a paradox.

We test the two decision paths of the game as an Or tautology.

Either Eq. 1.2 or Eq. 2.2 (3.1)

((s>(r>(p&q)))>(q=(%p<#p)) + ((s>(r> q)) >(q=(%p>#p)))
TTTT TTTT TTTT TTTT (3.2)

This means the states of Newcomb together are tautologous, a theorem, and *not* contradictory or a paradox.

Refutation of two variants of noncontingency operator

Abstract: Four axiomatizations of extensions of L(dot-box) over special frames are *no*t tautologous. Those for symmetry and qe&pe are different, but the respective, second-order renditions are equivalent. This refutes the two variants of noncontingency operator. Therefore the conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Fan, J. (2019). Two variants of noncontingency operator. arxiv.org/pdf/1906.03091.pdf

7.2 Extensions

In this section, we study the axiomatizations of L(box-dot) over special frames. The following table lists extra axioms and proof systems, and the frame properties that the corresponding systems characterize.

B ϕ → ((ϕ ∧ (ϕ → ψ) ∧ ¬ ψ) → χ)   B = K + B   symmetry   (7.2.2.1)

LET p, q, r, s: ϕ, ψ, χ, box-cross

p>(s&(((s&p)&(s&(p>q)))&((~s&q)>r))) ;
TFTF TFTF TFTT TFTT   (7.2.2.2)

5 ¬ ϕ → (¬ ϕ ∨ ψ)   K5 = K + 5   qe&pe   (7.2.4.1)

(~s&p)>(s&((~s&p)+q)) ;   TFTF TFTF TTTT TTTT   (7.2.4.2)

In the above table, qt, pt, qe, pe abbreviate quasi-transitivity, pseudo-transitivity, quasi-Euclidicity and pseudo-Euclidicity, respective, which are formalized by

respectively, where i, j ∈ {1, 2}:   [as antecedent]   (7.9.1)

LET s, r, u, v, x, y, z: s, R, i, j, x, y, z

(u&v)<((%s>#s)&(%s<#s)) ;   (7.9.2)

$\forall xyz(xRiy \wedge yRjz \rightarrow xR1z \wedge xR2z)$, pt (7.6.1)

```
((u&v)<((%s>#s)&(%s<#s)))>((((#x&r)&(u&#y))&((#y&r)&(v&#z)))>
(((#x&(r&(%s>#s)))&#z)&((#x&(r&(%s<#s)))&#z))) ;
                TTTT TTTT TTTT TTTT(112)
                           TTTT TTTT TTTT TTTT(  6)}x2
                           TTTT CCCC TTTT CCCC(  2)}
```
(7.6.2)

$\forall xyz(xRiy \wedge xRjz \rightarrow yR1z \wedge yR2z)$, pe (7.8.1)

```
((u&v)<((%s>#s)&(%s<#s)))>((((s&#x)&(u&#y))&((#x&r)&(v&#z)))>
((#y&((%s>#s)&#z))&(#y&((%s<#s)&#z)))) ;
                           TTTT TTTT TTTT TTTT(112)
                           TTTT TTTT TTTT TTTT(  6)}x2
                           TTTT CCCC TTTT CCCC(  2)}
```
(7.8.2)

Four axiomatizations of extensions of L(dot-box) over special frames are *no*t tautologous. Those for symmetry and qe&pe are different, but the respective, second-order renditions are equivalent. This refutes the two variants of noncontingency operator.

Refutation of completeness for non-deterministic logic

Abstract: We show that the four-valued, non-deterministic semantics for modal logic are not complete. The demonstration uses contradictions based on Carnielli's paraconsistent logic.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p: p; ~ Not, ¬; & And, ∧; = Equivalent, ≡;
% possibility, ◊, for one or some, ∃; # necessity, □, for every or all, ∀.

From: Coniglio, M.; del Cerro, L.F.; Peron, N.M. (2018). Modal logic with non-deterministic semantics: Part I - Propositional case. arxiv.org/pdf/1808.10007.pdf

Remark: We attribute the following equations to the paraconsistent logic system of Walter A. Carnielli since 1990, but use lower-case t, c, f, i in contrast to our bivalent T, C, F, N.

[7]These truth-values can be formalized in a modal language (assuming, as usual, the equivalences ¬□p ≡ ◊¬p and ¬◊p ≡ □¬p) as follows:

t+: □p ∧ ◊p ∧ p ; (7.1.1)

(#p&%p)&p ; FNFN FNFN FNFN FNFN (7.1.2)

c+: ¬□p ∧ ◊p ∧ p ; (7.2.1)

(~#p&%p)&p ; FCFC FCFC FCFC FCFC (7.2.2)

f+: □¬p ∧ ◊¬p ∧ p ; (7.3.1)

(#~p&%~p)&p; FFFF FFFF FFFF FFFF (7.3.2)

i+: □p ∧ ¬◊p ∧ p ; (7.4.1)

(#p&~%p)&p ; FFFF FFFF FFFF FFFF (7.4.2)

t-: □p ∧ ◊p ∧ ¬p ; (7.5.1)

(#p&%p)&~p ; FFFF FFFF FFFF FFFF (7.5.2)

c-: ¬□p ∧ ◊p ∧ ¬p ; (7.6.1)

(~#p&%p)&~p ; CFCF CFCF CFCF CFCF (7.6.2)

f-: □¬p ∧ ◊¬p ∧ ¬p ; (7.7.1)

(#~p&%~p)&~p ; `NFNF NFNF NFNF NFNF` (7.7.2)

i-: □p ∧ ¬◊p ∧ ¬p. (7.8.1)

(#p&~%p)&~p ; `FFFF FFFF FFFF FFFF` (7.8.2)

The system reduces to a three-valued logic of (f+ = i+ = t- = i-), (t+ = ~f-), and (c+ = ~c-). As such, it is not a six- or eight-valued system as claimed. We find no designated *proof* value: ~(f+ = i+ = t- = i-) is not a designated contradiction, but also is not complete as not tautologous. This is despite the alethic (T) axiom replacement by the bivalent deontic (D) axiom replacements (D1 and D2). What follows is that all infinite non-deterministic matrices are characterized by finite deterministic matrices.

The system is also *not* bivalent: i+ is not ~(i-); but rather i+ is equivalent to i-; and f+ is not equivalent to f- or to ~f-. What follows is that infinite non-deterministic matrices are by definition incomplete.

Definition of nothing in mathematical logic

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The repeating fragment(s) of 16-valued truth table(s) is row-major and horizontal.

LET p, q: proposition; collection of propositions
~ Not; & And; + Or; > Imply, greater than; < Not Imply, lesser than; = Equivalent
% possibility, one or some; # necessity, every or all; (p=p) `T`autology, proof.

We define a proposition p in four-valued logic as p=(p=p) ;
`FTFT FTFT FTFT FTFT` (11.2)

as alternating FT for non-tautology (contradiction) and tautology (proof).

We define the opposite of a proposition as not p as ~p=(p=p) ;
`TFTF TFTF TFTF TFTF` (12.2)

as alternating TF for tautology (proof) and non-tautology (contradiction).

We define the antonym of nothing as some thing %p as not one thing versus some, one thing %p=(p=p) ; `CTCT CTCT CTCT CTCT` (13.2)

as alternating CT for contingency (falsity) and tautology (proof).

We define the opposite of some thing %p as not some thing ~%p=(p=p) ;
`NFNF NFNF NFNF NFNF` (14.2)

as alternating NF non-contingency (truthity) and non-tautology (contradiction).

We define the antonym of all or every thing #p as ~#p as not all or not every thing #p=(p=p);
`FNFN FNFN FNFN FNFN` (15.2)

as alternating FN for non-tautology (contradiction) and non-contingency (truthity).

We define the opposite of all or every thing as not all or not every thing ~#p=(p=p) ;
`TCTC TCTC TCTC TCTC` (16.2)

as alternating TC for tautology (proof) and contingency (falsity).

This leads to how to collect not everything as nothing in multiple variables into a larger nothing variable, implying a set of nothing as a null set. We write this as nothing in p and nothing in q and nothing in r are all greater than nothing in s. (17.1)

((~#p&~#q)&~#r)>~#s ; `TTTT TTTT CTTT TTTT` (17.2)

Eq. 17.2 as rendered is *not* tautologous, although nearly so with one deviant `C` contingency (falsity) value. Hence a collection of nothing does not imply anything outside itself. By extension, the null set is not logically feasible and cannot exist: a collection must contain something even though it is nothing.

Refutation of six weak reactions in nucleosynthesis

From: Grohs, E.; et al. (2018).
"Universes without the weak force: astrophysical processes with stable neutrons".
arxiv.org/pdf/1801.06081.pdf

Using the Meth/VŁ4 apparatus and method, we evaluate

"$\upsilon_e + n \leftrightarrow p + e^-$; (2)
$e^+ + n \leftrightarrow p + \bar{\upsilon}_e$; (3)
$n \leftrightarrow p + e^- + \bar{\upsilon}_e$: (4)
... as the $n \leftrightarrow p$ rates."

LET p q r s: p, n, υ_e, e^+; ~ Not, bar; + Or; = Equivalent, ↔.

T is the designated *proof* value. Truth tables of 16-values are row-major, horizontal.

(r+q) = (p+~s) ; `FFTT TTTT TFFT FTFT` (2.2)
(s+q) = (p+~r) ; `FFTT TFFT TTTT FTFT` (3.2)
q = (p+(~s+~r)) ; `FFTT FFTT FFTT TFFT` (4.2)
(((r+q)=(p+~s)) + ((s+q)=(p+~r))) + (q=(p+(~s+~r))) ;
`FFTT TTTT TTTT TTFT` (5.2) = ((2.2)+ (3.2))+ (4.2)

Eqs. 2.2, 3.2, 4.2, and 5.2 as rendered are *not* tautologous. This means the six weak reactions in nucleosynthesis are suspicious.

Refutation of abductive repair in ontology engineering

Abstract: We evaluate the stated example of complete-debug problem (CDP) in formulas framing the definitions, oracles, and repairs. None is tautologous. This forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Lambrix, P. (2019). Completing and debugging ontologies: state of the art and challenges. arxiv.org/pdf/1908.03171.pdf

Abstract As semantically-enabled applications require high-quality ontologies, developing and maintaining as correct and complete as possible ontologies is an important, although difficult task in ontology engineering. A key step is ontology debugging and completion. In general, there are two steps: detecting defects and repairing defects. In this paper we formalize the repairing step as an abduction problem and situate the state of the art with respect to this framework. ...

2.1 Formalization
2.1.1 Repair … As an example, consider the CDP [complete-debug problem] in Fig. 1. … Then R1,R2, R3, R4 and R5 are all repairs of the CDP

Figure 1: Example complete-debug problem

T: {ax1: p1 ⊑ p2, ax2: p1 ⊑ p3, ax3: p1 ⊑ ¬p4, ax4: p2 ⊑ p4, ax5: p2 ⊑ p5, ax6: p3 ⊑ p5, ax7: p3 ⊑ p6, ax8: p4 ⊑ p7, ax9: p5 ⊑ ∀s.p8, ax10: p6 ⊑ ∃s.¬p8} (2.1.1.1.1)
. . .
Or(X) = true for X = ax2, ax3, ax4, ax5, ax7, ax8, ax9, p7 ⊑ p3; (2.1.1.3.1)
Or(X) = false for X = ax1, ax6, ax10, p7 ⊑ p5, p3 ⊑ p8 (2.1.1.4.1)

Remark 2.1.1.1.0: We test (T>((true for X)&(false for X))) as Eqs. 2.1.1.1.1 > ((2.1.1.3.1)&(2.1.1.4.1)). (2.1.1.5.1)

LET p, q, r, s, t, u, v, w, x, y, z:
p1, p2, p3, p4, p5, p6, p7, p8, s, y, z.

```
((((~(q<p)&~(r<p))&(~(~s<p)&~(s<q)))&((~(t<q)&~(t<r))&(~(u<r)&~(v<s))))&
(~((#x&w)<t)&~((%x&~w)<u)))>
(((((~(r<p)&~(~s<p))&(~(s<q)&~(t<q)))&((~(u<r)&~(v<s))&(~((#x&w)<t)&
~(r<v))))=(z=z))&
((((~(q<p)&~(t<r))&~((%x&~w)<u))&(~(t<v)&~(w<r)))=(z@z))) ;
```

```
TCTC TCTC TTTC TTTC
TTTT TTTC TTTT TTTC
TTTT TFTF TTTT TTTF
TTTT TTTF TTTT TTTF
TTTT TTTT TTTC TTTC
TTTT TTTT TTTT TTTC
TTTT TTTT TTTT TTTF }x2
TTTT TFTF TTTT TTTF }x2
TTTT TTTF TTTT TTTF }
TTTT TTTT TTTT TTTF }x4

TTTT TTTT TTTT TTTT }x2
TTTT TFTF TTTT TTTF
TTTT TTTF TTTT TTTF
TTTT TTTT TTTT TTTT }x2
TTTT TTTT TTTT TTTF }x2
TTTT TNTN TTTT TTTN }x2
TTTT TTTF TTTT TTTF }
TTTT TTTT TTTT TTTN }x2
TTTT TTTT TTTT TTTF }  107 steps        (2.1.1.5.2)
```

Remark 2.1.2.1.0: We map the unique relations from repairs of R1, R2, R3, R4, R5 of

R1={p4⊑p5, p7⊑p3}, R2={p4⊑p5, p7⊑p3}, R3={p7⊑p3}, R4={p4⊑p5}, R5={p4⊑p5, p7⊑p3}, as p4⊑p5, p7⊑p3 (2.1.2.1.1)

```
~(t<s)&~(r<v) ;        (2.1.2.1.2)
```

Remark 2.1.3.1.0: We test the CDP to imply the repairs:

Eqs. 2.1.1.5.1 implies 2.1.2.1.1. (2.1.3.1.1)

```
(((((~(q<p)&~(r<p))&(~(~s<p)&~(s<q)))&((~(t<q)&~(t<r))&(~(u<r)&~(v<s))))&
(~((#x&w)<t)&~((%x&~w)<u)))>(((((~(r<p)&~(~s<p))&(~(s<q)&~(t<q)))&
((~(u<r)&~(v<s))&(~((#x&w)<t)&~(r<v))))=(z=z))&((((~(q<p)&~(t<r))&
~((%x&~w)<u))&(~(t<v)&~(w<r)))=(z@z))))>(~(t<s)&~(r<v)) ;
```

```
TTTT FNFN TTTT FFFN
FFFF FFFN TTTT FFFN
TTTT FTFT TTTT FFFT
FFFF FFFT TTTT FFFT
TTTT TTTT TTTT TTTT }x2
FFFF FFFF TTTT TTTT }
TTTT FTFT TTTT FFFT }x2
FFFF FFFT TTTT FFFT }
TTTT TTTT TTTT TTTT }x2
FFFF FFFF TTTT TTTT }

TTTT FFFF TTTT FFFF
FFFF FFFF TTTT FFFF
```

```
TTTT FTFT TTTT FFFT
FFFF FFFT TTTT FFFT
TTTT TTTT TTTT TTTT }x2
FFFF FFFF TTTT TTTT }
TTTT FCFC TTTT FFFC }x2
FFFF FFFT TTTT FFFT }
TTTT TTTT TTTT TTTT }x2
FFFF FFFF TTTT TTTT }   115 steps        (2.1.3.1.2)
```

Eqs. 2.1.1.5.2 and 2.1.3.1.2 as rendered are *not* tautologous. This means the example given as 2.1.1.1 is *not* tautologous, the oracles in 2.1.1.3.1 and 2.1.1.4.1 are *not* truthful, and the repairs in 2.1.2.1.1 are incorrect. This refutes the conjecture of ontology engineering.

Refutation of open-universe causal reasoning

Abstract: The proposition and axiom as tested are *not* tautologous. This does not "validate an intuitive and familiar set of principles about subjunctive conditionals and the relation of causal influence". This also does not support "an important class of implicit generative models that can plausibly be treated as genuine causal models" or "enable reasoning beyond the propositional level". The conjecture of open-universe causal reasoning forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠, ⤳ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Ibeling, D.; Icard, T. (2019). On open-universe causal reasoning. arxiv.org/pdf/1907.02170.pdf

Abstract We extend two kinds of causal models, structural equation models and simulation models, to infinite variable spaces. This enables a semantics for conditionals founded on a calculus of intervention, and axiomatization of causal reasoning for rich, expressive generative models—including those in which a causal representation exists only implicitly—in an open-universe setting. Further, we show that under suitable restrictions the two kinds of models are equivalent, perhaps surprisingly as their axiomatizations differ substantially in the general case. We give a series of complete axiomatizations in which the open universe nature of the setting is seen to be essential.

2.1 Structural Equation Models
Definition 3: … X ⤳ Y (read X influences Y)
Proposition 1. Let $M \in M_{local}$ and $X, Y \in \chi$. If M |= X⤳ Y and t(Y) > t(X)+1, there is a variable X′ such that M |= X⤳ X′ and M |= X′ ⤳Y. (2.1.1)

LET p, q, r, s: t, X, X', Y

((q>s)&((p&s)>((p&q)+(%s>#s))))>(%r>((q>r)&(r>s))) ;
TTTT **FF**TT TTNN TTTT (2.1.2)

3.2 Axiomatizations

F/D. [α]¬β ↔ ¬[α]β (3.2.5.1)

LET p, q: α, β.

(p&~q)=(~p&q) ; T**FF**T T**FF**T T**FF**T T**FF**T (3.2.5.2)

> **4 Conclusion** We have identified two equivalent classes of models—one declarative, one procedural—formalizing the notion of an open-universe causal model. Both classes validate an intuitive and familiar set of principles about subjunctive conditionals and the relation of causal influence. This highlights an important class of implicit generative models that can plausibly be treated as genuine causal models, on a par with (an infinitary generalization of computable, recursive) structural equation models. ... One of the advantages of open-universe models is precisely that they enable reasoning beyond the propositional level. …

The proposition and axiom (Eqs. 2.1.2 and 3.2.5.2) as tested are *not* tautologous. This does not "validate an intuitive and familiar set of principles about subjunctive conditionals and the relation of causal influence". This also does not support "an important class of implicit generative models that can plausibly be treated as genuine causal models" or "enable reasoning beyond the propositional level".

Refutation of optimization as complex programming

Abstract: The optimization paradigm is *not* tautologous, hence refuting complex programming as that paradigm as a new class.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: x_0, x, G, f;
~ Not; + Or; & And; > Imply, greater than, →; < Not imply, lesser than, ∈;
= Equivalent; @ Not Equivalent;
necessity, for all or every; % possibility, for some or one;
x≤y ~(y<x); x≥y ~(x>y).

From: Shipilevsky, Y. (2018). Complex programming. vixra.org/pdf/1810.0073v1.pdf
yulysh2000@yahoo.ca

"Its well-known that an optimization problem can be represented in the following way:

Given: a function f: G → R from some set G to the real numbers
Sought: an element x_0∈ G such that f(x_0) ≤ f(x) for all x∈ G ("minimization") or such that f(x_0) ≥ f(x) for all x∈ G ("maximization")." (1.0)

We rewrite Eq. 1.0 as an implication, excluding the Given as unneeded for our analysis.

If f(x0) ≤ f(x) for all x∈ G ("minimization")
or f(x0) ≥ f(x) for all x∈ G ("maximization"), then there is an element x0∈ G. (1.1)

(((#q<r)>~((s&q)<(s&p)))+((#q<r)>~((s&q)>(s&p))))>%(p<r) ;
CTCT CCCC CTCT CCCC (1.2)

Eq. 1.2 as rendered is *not* tautologous, meaning the optimization problem is refuted. What follows is that complex programming as that paradigm is also *not* tautologous and hence refuted as a new class.

Refutation of ordinal notation via simultaneous definition

Abstract: We evaluate five definitions as *not* tautologous. This refutes the conjecture that inductive-recursive definitions can give rise to ordinal notation systems that uniquely represent ordinals. Hence the definitions and conjecture are *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
\> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Forsberg, F.N.; Xu, C. (2019). Ordinal notations via simultaneous definitions. arxiv.org/pdf/1904.10759.pdf fredrik.nordvall-forsberg@strath.ac.uk xu@math.lmu.de

1.1 Set-theoretic ordinals

$\alpha \cdot \beta$ is defined by transfinite recursion on β:

$\alpha \cdot (\beta + 1) \equiv \alpha \cdot \beta + \alpha$ (1.1.1.1)

LET p, q: a, b
(p&(q+(%s>#s)))=((p&q)+p) ; TNTT TNTT TNTT TNTT (1.1.1.2)

Def. 2: The relation < on O is inductively defined by the following clauses:

(<1) If a ≠ 0 then 0 < a. (1.1.2.1)

(p@(s@s))>((s@s)<p) ; T**F**T**F** T**F**T**F** T**F**T**F** T**F**T**F** (1.1.2.2)

Def. 7: Subtraction of O is defined as follows:

$0 - b \equiv 0$ (1.1.7.1.1)

((s@s)-q)=(s@s) ; **FF**TT **FF**TT **FF**TT **FF**TT (1.1.7.1.2) a − 0 ≡ a (1.1.7.2.1)

(p-(s@s))=p ; **FFFF FFFF FFFF FFFF** (1.1.7.2.2)

4 The type-theoretic development of ordinal notations in Agda

_≥_: O → O → Set

a ≥ b = (b < a) ∨ (a ≡ b) (4.1)

(~q>p)=((q<p)+(p=q)) ; **FF**TT **FF**TT **FF**TT **FF**TT (4.2)

5 Concluding discussions

Hence we conjecture that actual use of inductive-recursive definitions can give rise to ordinal notation systems that uniquely represents ordinals …

Eqs. 1.1.1.2, ..2.2, 1.1.7.1.2, ..7.2.2, and 4.2 are *not* tautologous. This refutes the conjecture that inductive-recursive definitions can give rise to ordinal notation systems that uniquely represent ordinals.

Refutation of the orthomodular law

Abstract: We evaluate the orthomodular law x≤y implies y=x∨(y∧x′), with ′ as negation, which is *no*t tautologous. This forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,· , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Chajda, I.; Länger, H. (2019). arxiv.org/pdf/1907.10539.pd
How to introduce the connective implication in orthomodular posets.

Abstract: Since orthomodular posets serve as an algebraic axiomatization of the logic of quantum mechanics, it is a natural question how the connective of implication can be defined in this logic. It should be introduced in such a way that it is related with conjunction, i.e. with the partial operation meet, by means of some kind of adjointness. We present here such an implication for which a so-called unsharp residuated poset can be constructed. Then this implication is connected with the operation meet by the so-called unsharp adjointness. We prove that also conversely, under some additional assumptions, such an unsharp residuated poset can be converted into an orthomodular poset and that this assignment is nearly one-to-one.

Orthomodular posets are considered as an algebraic axiomatization of the logic of quantum mechanics … On the other hand, when some algebraic structure is used as an axiomatization of a propositional logic, we must ask for a connective implication … In the present paper we solve the question of finding an implication in orthomodular posets in the way that a certain residuation is possible.

Recall that a *bounded poset with an antitone involution* is an ordered quintuple (P, ≤ , ′, 0, 1) where (P, ≤, 0, 1) is a bounded poset and ′ is a unary operation on P such that the following conditions are satisfied for all x, y ∈ P: x ≤ y implies y′ ≤ x′, (x′)′ = x.

Remark 1.0: The mark ′ is effectively the negation operator ¬.

We say that the elements a, b of P are *orthogonal* to each other if a ≤ b′ (or, equivalently, b ≤ a′). Further recall that an *orthomodular poset* is a bounded poset (P, ≤, ′, 0, 1) with an antitone involution satisfying the following conditions for all x, y ∈ P: x ∨ y is defined provided x ≤ y′, [and]

• x ≤ y implies y = x ∨ (y ∧ x′). (1.1)

LET p, q: x, y.

~(q<p)>(q=(p+(q&~p))) ; T**F**TT T**F**TT T**F**TT T**F**TT (1.2)

The last condition is called the *orthomodular law*. Observe that in case $y = 1$ this law implies $x \vee x' = 1$. Since $'$ is an antitone involution this further implies $x \wedge x' = 0$. Thus $'$ is a complementation.

Remark 1.2: Eq. 1.2 as rendered is *not* tautologous, hence refuting the orthomodular law.

Refutation of the ordinal Turing machine (OTM) on set theory

Abstract: From the sections on OTM-realizabilty and intuitionistic provability and axioms and systems of constructive set theories, we evaluate an inference rule and two propositions. None is tautologous. The refutes OTM on set theory in Hilbert space for intuitionistic logic. Therefore that approach produces *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Carl, M. (2019). A note on OTM-realizability and constructive set theories. arxiv.org/pdf/1903.08945.pdf merlin.carl@uni-flensburg.de

Abstract: We define an ordinalized version of Kleene's realizability interpretation of intuitionistic logic by replacing Turing machines with Koepke's ordinal Turing machines (OTMs) ...

3 OTM-Realizability and Intuitionistic Provability

ii Given φ → ψ, where x does not appear freely in φ, one may infer φ → ∀xψ (3.2.1)

LET p, q, r, s: φ, ψ, x, X

(~(r<p)&(p>q))>(p>(#r&q)) ; TTT**F** TTTN TTT**F** TTTN (3.2.2)

4 Axioms and systems of constructive set theories

We now discuss the OTM-realizability of the axioms of ZFC set theory and their most prominent constructive variants ... . It is easy to see that the axioms of Empty Set Existence, Extensionality, Pairing, Union and Infinity are OTM-realizable.

Proposition 6. The separation schema ∀a∃x∀y(y ∈ x ↔ (y ∈ a ∧ φ(y)) has instantiations with ∈-formulas φ that are not OTM-realizable. However, every instantiation by a Δ0-formula is OTM-realizable. (4.6.1)

LET p, q, r, s: φ, ψ, x, X

(#q<%p)=((#q<r)&(#s&#q)) ;

TTCT TTCT TTTC TTCT (4.6.2)

The following may come as a small surprise; however, noting its dependence on the reading assigned here to implication, it is quite natural.

Proposition 7. Every instance of the collection axiom $\forall x \in X \exists y \varphi(x, y) \rightarrow \exists Y \forall x \in X \exists y \in Y \varphi(x, y)$, and thus of the replacement axiom and the strong collection axiom, is OTM-realizable. (4.7.1)

LET p, s, t, x, y: φ, X, Y, x, y

((#x<s)&(%y&(p&(x&y))))>((%y&(#x<s))&((%y<t)&(p&(x&y)))) ;

TTTT TTTT TTTT TTTT(56),
TCTC TCTC TTTT TTTT(8) (4.7.2)

Eqs. 3.2.2, 4.6.2, and 4.7.2 as rendered are *not* tautologous. This denies the application of ordinal Turing machines (OTM) to set theory, which is also refuted elsewhere.

Refutation of overlap algebras constructively to prove complete Boolean algebras

Abstract: We evaluate four equations which are *not* tautologous, but in fact produce the equivalent logic table values result. This means that the stated problem of applying singletons to the powerset is equivalent to proving singletons are atoms and that every subset satisfying a singleton is also an atom. Hence, overlap algebras do not constructively prove complete Boolean algebras. Therefore that conjecture for intuitionistic logic forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Ciraulo, F.; Contente, M. (2019) .
Overlap algebras: a constructive look at complete Boolean algebras.
arxiv.org/pdf/1904.13320.pdf ciraulo@math.unipd.it michele.contente@sns.it

The problem of finding a constructive characterization of powersets is related to the problem of finding a suitable algebraization of the notion of a singleton. Apparently, none of the first-order (in the sense of the language of lattices) attempts to define
is an atom is satisfactory from an intuitionistic point of view; consider, for instance, the following.

$$a \neq 0 \wedge (\forall x \in L)(x \neq 0 \wedge x \leq a \Rightarrow x = a) \tag{1.1.1}$$

(p<q)>((p@(p=p))&((#r<q)&(((r@(r=r))&~(p<r))>(r=p)))) ;
T**F**TT T**F**TT T**F**TT T**F**TT (1.1.2)

$$a \neq 0 \wedge (\forall x \in L)(x \leq a \Rightarrow x = 0 \vee x = a) \tag{1.2.1}$$

(p<q)>((p@(p=p))&((#r<q)&(~(p<r)>(r=((r@r)+(r=p)))))) ;
T**F**TT T**F**TT T**F**TT T**F**TT (1.2.2)

$$a \neq 0 \wedge (\forall x \in L)(x < a \Rightarrow x = 0) \tag{1.3.1}$$

(p<q)>((p@(p=p))&((#r<q)&((r<p)>(r=(r@r))))) ;
T**F**TT T**F**TT T**F**TT T**F**TT (1.3.2)

$$a \neq 0 \wedge \neg (\exists x \in L)(x \neq 0 \wedge x < a) \tag{1.4.1}$$

(p<q)>((p@(p=p))&(~(%r<q)&((r@(r=r))&(r<p)))) ;
T**F**TT T**F**TT T**F**TT T**F**TT (1.4.2)

Indeed, when applied to the case L = Pow(X), singletons cannot be proven to be atoms in the sense of (1.1) or (1.2), and it is impossible to prove that every subset satisfying (1.3) or (1.4) is a singleton, although a singleton satisfies (1.3) and (1.4).

Eqs. 1.1.2-1.4.2 are *not* tautologous, but in fact produce the equivalent logic table values result. This means that the stated problem of applying singletons to the powerset is equivalent to proving singletons are atoms and that every subset satisfying a singleton is also an atom. Hence, overlap algebras do not constructively prove complete Boolean algebras.

P=NP resolution, with 3-SAT not tautologous

1. **NP**

We use the definition of NP as “nondeterministic polynomial time” from Stephen Cook at claymath.org/sites/default/files/pvsnp.pdf as:

$$w \in L \Leftrightarrow \exists y(|y| \leq |w|^k \text{ and } R(w, y)) \tag{1}$$

where:

$$R(w, y) \Leftrightarrow w \in L \tag{2}$$

$$R(\sim a, \sim b) \Leftrightarrow 1 < b < a \text{ and } b|a \text{ (with negation replacing the obtuse vinculum)} \tag{3}$$

We substitute the expression $R(a, b)$ as:

$$R(\sim a, \sim b) \Leftrightarrow [\ R(w, y) \Leftrightarrow\ 1 < b < a \text{ and } b|a\ ] \tag{4}$$

then substitute $R(w, y)$ in Eq. 1 with Eq. 4 for:

$$w \in L \Leftrightarrow \exists y(|y| \leq |w|^k \text{ and } [\ R(\sim a, \sim b) \Leftrightarrow [\ R(w, y) \Leftrightarrow\ 1 < b < a \text{ and } b|a\ ]\ ]. \tag{5.1}$$

We assume the apparatus and method of Meth8/VŁ4.

LET: a b L R (w,y) as t u p q (w, y); (r, s) = (w, y)
~ Negation, % modal possibility, existential quantifier for all, ∃ ;
& And, \ Not And, + Or, - Not Or, = Equivalent, @ Not Equivalent, > Imply, < Not Imply;

and where:

```
|w| :: (w+((w<((w\w)-(w\w)))>(w&((w\w)-((w\w)-(w\w))))));     (6)
|y| :: (y+((y<((y\y)-(y\y)))>(y&((y\y)-((y\y)-(y\y))))));     (7)
|y| <= |w|  :: (y' = w') or (y' < w') :: |w| > |y|.           (8)
```

We note that in the modal propositional logic of Meth8, as based on system VŁ4, an exponential expression reduces to the mantissa such that w^3 is w&w&w = w. This means that in Eq. 5.1 the power series term $|w|^k$, with k as a natural number, reduces to $|w|$. In other words, in Meth8 a power series is effectively reduced to a linear expression.

```
(w<p)  =  (%y&(((w+((w<((w\w)-(w\w)))>(w&((w\w)-((w\w)-(w\w))))))>(y+((y<((y\y)-(y\
y)))>(y&((y\y)-((y\y)-(y\y)))))))&((q&(~r&~s))=((q&(w&y))=(((t\t)<(t<u))&(t+u)))))) ;
                                                                              (5.2)
```

Eq. 5.2 is evaluated on the five logical models of Meth8 as *not* tautologous.

The truth table for Eq. 5.2 is presented below as two different segments of two repeating blocks of 16 lines. The designated truth values are `T`autologous and `E`valuated.

```
(w<p)=(%y&(((w+((w<((w\w)-(w\w)))>(w&((w\w)-((w\w)-(w\w))))))>(y+((y<((y\y)-(y\y)))>(y&((y\y)-((y\y)-(y\
y)))))))&((q&(~r&~s))=((q&(w&y))=(((t\t)<(t<u))&(t+u)))))) ;  Step: 81; not tautologous for: =, >, <.

Model 1              Model 2.1            Model 2.2            Model 2.3.1          Model 2.3.2
TTNN TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEII EEEE EEEE EEEE  EEPP EEEE EEEE EEEE
TTNN TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEII EEEE EEEE EEEE  EEPP EEEE EEEE EEEE
NNTT NNNN NNNN NNNN  EEEE EEEE EEEE EEEE  UUEE UUUU UUUU UUUU  IIEE IIII IIII IIII  PPEE PPPP PPPP PPPP
TTNN TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEII EEEE EEEE EEEE  EEPP EEEE EEEE EEEE
TTNN TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEII EEEE EEEE EEEE  EEPP EEEE EEEE EEEE
TTNN TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEII EEEE EEEE EEEE  EEPP EEEE EEEE EEEE
NNTT NNNN NNNN NNNN  EEEE EEEE EEEE EEEE  UUEE UUUU UUUU UUUU  IIEE IIII IIII IIII  PPEE PPPP PPPP PPPP
TTNN TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEII EEEE EEEE EEEE  EEPP EEEE EEEE EEEE
FTCN FTFT FTFT FTFT  UEUE UEUE UEUE UEUE  UEEU UEUE UEUE UEUE  UEPI UEUE UEUE UEUE  UEIP UEUE UEUE UEUE
FTCN FTFT FTFT FTFT  UEUE UEUE UEUE UEUE  UEEU UEUE UEUE UEUE  UEPI UEUE UEUE UEUE  UEIP UEUE UEUE UEUE
CNFT CNCN CNCN CNCN  UEUE UEUE UEUE UEUE  EUUE EUEU EUEU EUEU  PIUE PIPI PIPI PIPI  IPUE IPIP IPIP IPIP
FTCN FTFT FTFT FTFT  UEUE UEUE UEUE UEUE  UEEU UEUE UEUE UEUE  UEPI UEUE UEUE UEUE  UEIP UEUE UEUE UEUE
FTCN FTFT FTFT FTFT  UEUE UEUE UEUE UEUE  UEEU UEUE UEUE UEUE  UEPI UEUE UEUE UEUE  UEIP UEUE UEUE UEUE
FTCN FTFT FTFT FTFT  UEUE UEUE UEUE UEUE  UEEU UEUE UEUE UEUE  UEPI UEUE UEUE UEUE  UEIP UEUE UEUE UEUE
CNFT CNCN CNCN CNCN  UEUE UEUE UEUE UEUE  EUUE EUEU EUEU EUEU  PIUE PIPI PIPI PIPI  IPUE IPIP IPIP IPIP
FTCN FTFT FTFT FTFT  UEUE UEUE UEUE UEUE  UEEU UEUE UEUE UEUE  UEPI UEUE UEUE UEUE  UEIP UEUE UEUE UEUE
TTNN TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEII EEEE EEEE EEEE  EEPP EEEE EEEE EEEE
TTNN TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEII EEEE EEEE EEEE  EEPP EEEE EEEE EEEE
NNTT NNNN NNNN NNNN  EEEE EEEE EEEE EEEE  UUEE UUUU UUUU UUUU  IIEE IIII IIII IIII  PPEE PPPP PPPP PPPP
TTNN TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEII EEEE EEEE EEEE  EEPP EEEE EEEE EEEE
TTNN TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEII EEEE EEEE EEEE  EEPP EEEE EEEE EEEE
TTNN TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEII EEEE EEEE EEEE  EEPP EEEE EEEE EEEE
NNTT NNNN NNNN NNNN  EEEE EEEE EEEE EEEE  UUEE UUUU UUUU UUUU  IIEE IIII IIII IIII  PPEE PPPP PPPP PPPP
TTNN TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEII EEEE EEEE EEEE  EEPP EEEE EEEE EEEE
FTCN FTFT FTFT FTFT  UEUE UEUE UEUE UEUE  UEEU UEUE UEUE UEUE  UEPI UEUE UEUE UEUE  UEIP UEUE UEUE UEUE
FTCN FTFT FTFT FTFT  UEUE UEUE UEUE UEUE  UEEU UEUE UEUE UEUE  UEPI UEUE UEUE UEUE  UEIP UEUE UEUE UEUE
CNFT CNCN CNCN CNCN  UEUE UEUE UEUE UEUE  EUUE EUEU EUEU EUEU  PIUE PIPI PIPI PIPI  IPUE IPIP IPIP IPIP
FTCN FTFT FTFT FTFT  UEUE UEUE UEUE UEUE  UEEU UEUE UEUE UEUE  UEPI UEUE UEUE UEUE  UEIP UEUE UEUE UEUE
FTCN FTFT FTFT FTFT  UEUE UEUE UEUE UEUE  UEEU UEUE UEUE UEUE  UEPI UEUE UEUE UEUE  UEIP UEUE UEUE UEUE
FTCN FTFT FTFT FTFT  UEUE UEUE UEUE UEUE  UEEU UEUE UEUE UEUE  UEPI UEUE UEUE UEUE  UEIP UEUE UEUE UEUE
CNFT CNCN CNCN CNCN  UEUE UEUE UEUE UEUE  EUUE EUEU EUEU EUEU  PIUE PIPI PIPI PIPI  IPUE IPIP IPIP IPIP
FTCN FTFT FTFT FTFT  UEUE UEUE UEUE UEUE  UEEU UEUE UEUE UEUE  UEPI UEUE UEUE UEUE  UEIP UEUE UEUE UEUE
TTFF TTTT TTTT TTTT  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE
TTFF TTTT TTTT TTTT  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE
FFTT FFFF FFFF FFFF  UUEE UUUU UUUU UUUU  UUEE UUUU UUUU UUUU  UUEE UUUU UUUU UUUU  UUEE UUUU UUUU UUUU
TTFF TTTT TTTT TTTT  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE
TTFF TTTT TTTT TTTT  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE
TTFF TTTT TTTT TTTT  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE
FFTT FFFF FFFF FFFF  UUEE UUUU UUUU UUUU  UUEE UUUU UUUU UUUU  UUEE UUUU UUUU UUUU  UUEE UUUU UUUU UUUU
TTFF TTTT TTTT TTTT  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE
FTFT FTTF FTTF FTTF  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU
FTFT FTTF FTTF FTTF  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU
TFTF TFFT TFFT TFFT  EUEU EUUE EUUE EUUE  EUEU EUUE EUUE EUUE  EUEU EUUE EUUE EUUE  EUEU EUUE EUUE EUUE
FTFT FTTF FTTF FTTF  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU
FTFT FTTF FTTF FTTF  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU
FTFT FTTF FTTF FTTF  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU
TFTF TFFT TFFT TFFT  EUEU EUUE EUUE EUUE  EUEU EUUE EUUE EUUE  EUEU EUUE EUUE EUUE  EUEU EUUE EUUE EUUE
FTFT FTTF FTTF FTTF  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU
TTFF TTTT TTTT TTTT  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE
TTFF TTTT TTTT TTTT  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE
FFTT FFFF FFFF FFFF  UUEE UUUU UUUU UUUU  UUEE UUUU UUUU UUUU  UUEE UUUU UUUU UUUU  UUEE UUUU UUUU UUUU
TTFF TTTT TTTT TTTT  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE
TTFF TTTT TTTT TTTT  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE
TTFF TTTT TTTT TTTT  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE
FFTT FFFF FFFF FFFF  UUEE UUUU UUUU UUUU  UUEE UUUU UUUU UUUU  UUEE UUUU UUUU UUUU  UUEE UUUU UUUU UUUU
TTFF TTTT TTTT TTTT  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE  EEUU EEEE EEEE EEEE
FTFT FTTF FTTF FTTF  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU
FTFT FTTF FTTF FTTF  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU
TFTF TFFT TFFT TFFT  EUEU EUUE EUUE EUUE  EUEU EUUE EUUE EUUE  EUEU EUUE EUUE EUUE  EUEU EUUE EUUE EUUE
FTFT FTTF FTTF FTTF  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU
FTFT FTTF FTTF FTTF  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU
FTFT FTTF FTTF FTTF  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU
TFTF TFFT TFFT TFFT  EUEU EUUE EUUE EUUE  EUEU EUUE EUUE EUUE  EUEU EUUE EUUE EUUE  EUEU EUUE EUUE EUUE
FTFT FTTF FTTF FTTF  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU  UEUE UEEU UEEU UEEU
Model 1              Model 2.1            Model 2.2            Model 2.3.1          Model 2.3.2
```

2. **P**

We use the definition of P as "deterministic polynomial time", that is, ~NP as the negation of Eq. 5.1.

3. Problem statement: *Does* **P** = **NP**? (9.1)

We test Eq. 9.1 as equivalent to Eq. 5.1. For ~NP = NP, obviously the expression is contradictory.

4. 3-SAT

Cook describes an example of the 3-SAT test as NP-complete for the expression (with negation replacing the vinculum):

$$(p \lor q \lor r) \land (\sim p \lor q \lor \sim r) \land (p \lor \sim q \lor s) \land (\sim p \lor \sim r \lor \sim s) \quad (10.1)$$

((p+(q+r)) & (~p+(q+~r))) &((p+(~q+s)) & (~p+(~r+~s))) ;
`FTFT TFFT FTTT TFTF` (10.2)

with

$\tau(P) = \tau(Q) =$ *Tautologous* and $\tau(R) = \tau(S) =$ *contradictory* (11.1)

(((p=q)=(p=p))&((r=s)=(r@r))) ; `FFFF TFFT TFFT FFFF` (11.2)

Eqs. 10.2 and 11.2 as rendered are *not* tautologous.

We combine Eq. 10.1 and its qualification with clause of Eq. 11.1.

If $\tau(P) = \tau(Q) =$ *Tautologous* and $\tau(R) = \tau(S) =$ *contradictory*, then
$(p \lor q \lor r) \land (\sim p \lor q \lor \sim r) \land (p \lor \sim q \lor s) \land (\sim p \lor \sim r \lor \sim s)$ (12.1)

(((p=q)=(p=p))&((r=s)=(r@r))) > (((p+(q+r))&(~p+(q+~r)))&((p+(~q+s))&(~p+(~r+~s)))) ;
`TTTT TTTT FTTT TTTT` (12.2)

Eq. 12.2 as rendered is *not* tautologous, but nearly so with deviation by one `F` value. This means the 3-SAT test is *not* tautologous, and hence incapable of testing NP-completeness.

What follows is that the logical foundation supporting satisfiability is suspicious.

Refutation of algorithm for 3-SAT satisfiability via claimed Boolean rules

.

Abstract: We evaluate a definition of two Boolean rules claimed for intersection and union as *not* tautologous. This refutes the subsequent conjecture of an algorithm for 3-SAT satisfiability, to form *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Flint, O.; Wickramasinghe, A.; Brasse, J.; Fowler, C. (2019). Determining satisfiability of 3-SAT in polynomial time. vixra.org/pdf/1908.0630v1.pdf [no point of contact with claim of peer review]

Abstract In this paper, we provide a polynomial time (and space), algorithm that determines satisfiability of 3-SAT. The complexity analysis for the algorithm takes into account no efficiency and yet provides a low enough bound, that efficient versions are practical with respect to today's hardware. We accompany this paper with a serial version of the algorithm without non-trivial efficiencies ...

2 Preliminaries and definitions
Before we work through an example, we must define what it means to take an intersection or union of two or more edge-sequences. No intersections or unions are taken with vertex-sequences.

Definition 2.12. We take the intersection or union of two n length edge sequences, A and B, by comparing position i of A and B, using the Boolean rules for intersections (denoted by ∩), and unions (denoted by ∪), for all positions, i = 0, 1, 2, . . . , n−1.
Recall that the entry for position i of A and B, is either 1 or 0.
Then, for an intersection, we have:

$$1_A \cap 0_B = 0_A \cap 1_B = 0_A \cap 0_B = 0. \text{ And } 1_A \cap 1_B = 1. \qquad (2.12.1.1)$$

LET p, q, r, s: 1_A, 1_B, 0_A, 0_B;
0 **F**; ordinal 1 N.

((((p&s)=(r&q))=(r&s))=(s@s))&((p&q)=(%s>#s)) ;
CCCN CC**FF** C**F**C**F** **F**CC**F** (2.12.1.2)

Remark 2.12.1.2: If instead of ordinal 1 N, ordinal 1 as T, then the equation fares worse as a contradiction:

((((p&s)=(r&q))=(r&s))=(s@s))&((p&q)=(s=s)) ;
FFFF FFFF FFFF FFFF (2.12.1.3)

And for a union we have:

$$1_A \cup 0_B = 0_A \cup 1_B = 1_A \cup 1_B = 1. \text{ And } 0_A \cup 0_B = 0. \qquad (2.12.2.1)$$

((((p+s)=(r+q))=(r+s))=(s=s))&((r+s)=(s@s)) ;
FTT**F FFFF FFFF FFFF** (2.12.2.2)

Eqs. 2.12.1.2 and 2.12.2.2 are not tautologous. This refutes the claimed Boolean rules and hence refutes an algorithm which determines satisfiability of 3-SAT.

A machine-assisted view of paraconsistency

From: Jesse Alma (2013).

We use the variant system VŁ4 in the Meth8 logic model checker on five models, where: % existential quantifier; # universal quantifier; ~ Negation; & And; > Imply; vt tautologous; nvt Validated not tautologous.

In Experiment 1: Trivializing triviality

```
~(%x&(p&x))>~(%x&(#y&(p&y)))          ; vt
(%x>x) > (%x&(#y>y))                  ; nvt
(%x&(x>x)) > (%x&(#y>(y>y)))          ; vt
```

Experiment 2: Possibility of explosiveness

```
p>(~p>q) ; explosion principle        ; vt
```

Conclusion:

```
(#p&%q)>(#p&(#r&(((p&~p)&q)>r)))      ; nvt
```

Here is the non repeating truth table fragment for the above, with designated truth values Tautologous, Evaluated (the UIP are unevaluated, improper, proper):

TTT**C** TTTT	EEE**U** EEEE	EEEE EEEE	EEE**P** EEEE	EEE**I** EEEE	Step: 15
Model 1	Model 2.1	Model 2.2	Model 2.3.1	Model 2.3.2	

We find this conclusion in the abstract is suspicious: "paraconsistent logic points are indeed genuine".

Logical contradiction context: paraconsistent versus classical

From: Arenhart, J.R.B. (2016). Paraconsistent contradiction in context.
periodicos.ufrn.br/saberes/article/download/9730/6950

We evaluate the difference between paraconsistent contradiction and classical contradiction.

We assume the Meth8-VŁ4 apparatus with s **B**, t **T.** The designated proof value is `T`autology, Result fragments are the repeating row on the 16-value truth table.

1. **B**(*p* ^ ~*p*) (assumption) ; (s&(p&~p))
2. **B***p* ^ **B~***p* (distribution of **B**) ; ((s&p)&(s&~p))
3. **B***p* ^ ~**B***p* (from 2, with Exclusion) ; ((s&p)&~(s&p))

(4.) Eqs. ((1.=3.)=(1.=2.))=(2.=3.):

((((s&p)&(s&~p))=((s&p)&~(s&p)))=((s&(p&~p))=((s&p)&(s&~p))))
= ((s&(p&~p))=((s&p)&~(s&p))); `TTTT`

(5.) **BT~***p* → **B~T***p* (exclusion for truth) ; ((s&r)&~p)>(s&~(r&p)) ; `TTTT`
(6.) **B~T***p* → ~**BT***p* ; ((s&~r)&p)>~((s&r)&p) ; `TTTT`
(7.) **B~***p* → ~**B***p* (dropping **T** from 2) ; (s&~p)>~(s&p) ; `TTTT`

(We see a possible typo: for "~(**B***p*^~**B***p*)", read "(**B***p*^~**B***p*)", presumably to mean the paraconsistent contradiction and the doxastic contradiction are both contradictions.)

Meth8 finds equivalency with Eqs. 1, 2, 3 (all contradictions) in (4). Therefore we find no logical distinction of inside context or outside context or in paraconsistent logic or doxastic logic.

Refutation of paraconsistent logic on one conjecture

Abstract: We evaluate the seminal equivalence and replacement formula of paraconsistent logic, that one formula is equivalent to another in the sense that either can be substituted for the other wherever they appear as a subformula. It is *not* tautologous, and hence relegates paraconsistent logic to a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Paraconsistent_logic#An_ideal_three-valued_paraconsistent_logic

(4) To establish that a formula Γ is equivalent to Δ in the sense that either can be substituted for the other wherever they appear as a subformula, one must show

$$((\Gamma\to\Delta)\wedge(\Delta\to\Gamma))\wedge((\neg\Gamma\to\neg\Delta)\wedge(\neg\Delta\to\neg\Gamma)). \tag{4.1}$$

LET p, q: Γ, Δ.

((p>q)&(q>p))&((~p>~q)&(~q>~p)) ;
T**FF**T T**FF**T T**FF**T T**FF**T (4.2)

Remark 4.2: Eq. 4.2 as rendered is not tautologous. This refutes the seminal theorem of replacement in paraconsistent logic.

Method of reducing paradox to not contradictory and in one variable

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table fragment) is row-major and horizontal.

The first example selected is the paradox of Zhuangzi known as the butterfly dream:

en.wikipedia.org/wiki/Zhuangzi_(book)#.22The_Butterfly_Dream.22

LET p q s: sleep state, awake state; sleep;
~ Not; & And; + Or; > Imply; < Not Imply.

In the butterfly dream, Zhuangzi inadvertently invokes the implication connective for a paradox of fused terms, but which by definition are not equal.

Sleep state is not awake state, and the contrast of sleep state or awake state does not imply sleep state and awake state; but (1.1.1)

(p=~q)&((p+ q)<(p& q)) ; `FTTF FTTF FTTF FTTF` (1.1.2)

Sleep state is not awake state, and the contrast of sleep state or awake state does imply sleep state and awake sleep. (1.2.1)

(p=~q)&((p+ q)>(p& q)) ; `FFFF FFFF FFFF FFFF` (1.2.2)

Eq. 1.1.2 is *not* contradictory, but Eq. 1.2.2 is contradictory. Because *both* Eqs. 1.1.2 and 1.2.2 are *not* contradictory, this refutes the butterfly dream as a paradox.

We test the method of reducing paradox to *not* contradictory and in *one* variable. We re-define s as sleep state and ~s as not sleep state and rewrite Eqs. 1.1.x and 1.2.x.

The contrast of sleep or no sleep does not imply sleep and no sleep; but (1.3.1)

(s+~s)<(s&~s) ; `TTTT TTTT TTTT TTTT` (1.3.2)

The contrast of sleep or no sleep does imply sleep and no sleep. (1.4.1)

(s+~s)>(s&~s) ; `FFFF FFFF FFFF FFFF` (1.4.2)

Eq. 1.3.2 is *not* contradictory, but Eq. 1.4.2 is contradictory. Because *both* Eqs. 1.3.2 and 1.4.2 are *not* contradictory, this refutes the butterfly dream as a paradox.

However, Eqs. 1.3.2 and 1.4.2 also serve as an example to confirm the method that a paradox refuted as *not* contradictory is also reducible to *one* variable.

The second example selected is the paradox of Maimonides at:

en.wikipedia.org/wiki/Argument_from_free_will

Moses Maimonides formulated an argument regarding a person's free will, in traditional terms of good and evil actions, as follows:

> Does God know or does He not know that a certain individual will be good or bad? (1.1)
>
> (p>(q>(%p>#p)))+(p>(q>(%p<#p))) ;
> TTTT TTTT TTTT TTTT (1.2)
>
> If thou sayest 'He knows', then it necessarily follows that the man is compelled to act as God knew beforehand he would act, (2.1)
>
> (p>(q>(%p>#p)))>#(q>(p>(q>(%p>#p)))) ;
> NNNT NNNT NNNT NNNT (2.2)
>
> otherwise God's knowledge would be imperfect … (3.1)
>
> [<] p=(p@p) ; TFTF TFTF TFTF TFTF (3.2)
>
> If Eq. 1.2, then if Eq. 2.1 then Eq. 3.1. (4.1)
>
> (((p>(q>(%p>#p)))+(p>(q>(%p<#p))))>((p>(q>(%p>#p)))>#(q>(p>(q>(%p>#p))))))
> < (p=(p@p)) ; FNFT FNFT FNFT FNFT (4.2)

As rendered, Eq. 1.2 is tautologous, *not* contradictory, and a theorem. Eqs. 2.2 and 3.2 are *not* tautologous and *not* contradictory. Eq. 4.2, the further embellishment of Eqs. 1.2, 2.2, and 3.2 is *not* tautologous and *not* contradictory. Therefore the paradox of Maimonides is refuted as a paradox.

We test the method of reducing paradox to *not* contradictory and in *one* variable. We re-define (%q>#q) good, (%q<#q) bad, and imperfect (q@q), replace p for God as the tautology (q=q), and rewrite Eqs. 1.2, 2.2, 3.2, and 4.2.

> ((q=q)>(q>(%q>#q)))+((q=q)>(q>(%q<#q))) ;
> TTTT TTTT TTTT TTTT (5.2)
>
> ((q=q)>(q>(%q>#q)))>#(q>((q=q)>(q>(%q>#q)))) ;
> NNTT NNTT NNTT NNTT (6.2)
>
> [<] (q=q)=(q@q) ; FFFF FFFF FFFF FFFF (7.2)
>
> ((((q=q)>(q>(%q>#q)))+((q=q)>(q>(%q<#q))))>
> (((q=q)>(q>(%q>#q)))>#(q>((q=q)>(q>(%q>#q)))))) < ((q=q)=(q@q)) ;
> NNTT NNTT NNTT NNTT (8.2)

Eq. 8.2 is *not* tautologous and *not* contradictory, and also refuting the paradox of Maimonides.

However, Eq. 8.2 also serves as an example to confirm the method that a paradox refuted as *not* contradictory is also reducible to *one* variable.

Refutation of Parikh's axiomatization of game logic G and completeness of logic system Par

Abstract: Three equations of Parikh's axiomatization of game logic G are *not* tautologous. Hence, the extended logic system Par is refuted, and Parikh's completeness conjecture is also refuted. Therefore these artifacts are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Enqvist, S.; Hansen, H.H.; Kupke, C.; Marti, J.; Venema, Y. (2019).
Completeness for Game Logic. arxiv.org/pdf/1904.07691.pdf thesebastianenqvist@gmail.com, h.h.hansen@tudelft.nl, clemens.kupke@strath.ac.uk, johannes.marti@gmail.com, y.venema@uva.nl

Abstract- ... In this paper, we introduce a cut-free sequent calculus for game logic, and two cut-free sequent calculi that manipulate annotated formulas, one for game logic and one for the monotone μ-calculus, the variant of the polymodal μ-calculus where the semantics is given by monotone neighbourhood models instead of Kripke structures. We show these systems are sound and complete, and that completeness of Parikh's axiomatization follows.

Fig. 1. Par Axioms:

4) $\langle\gamma^*\rangle\varphi \leftrightarrow \varphi \vee \langle\gamma\rangle\langle\gamma^*\rangle\varphi$ (4.1)

LET p, q, r: $\varphi, \gamma^*, \gamma$
(q&p)=(p+((r&q)&p)) ; T**F**TT T**F**TT T**F**TT T**F**TT (4.2)

5) $\langle\psi?\rangle\varphi \leftrightarrow \psi \wedge \varphi$ (5.1)

LET p, q, r: $\varphi, \psi, \psi?$
(r&p)=(q&p) ; TTT**F** T**F**TT TTT**F** T**F**TT (5.2)

6) $\langle \gamma^{d} \rangle \varphi \leftrightarrow \neg \langle \gamma \rangle \neg \varphi$ (6.1)

LET p, q, r: φ, γ, γ^{d}
(r&p)=(~q&~p) ; **F**TTT **F**T**FF** **F**TTT **F**T**FF** (6.2)

The system Par is easily seen to be sound. A main contribution of our paper is that we confirm Parikh's completeness conjecture.

Axiom Eqs. 4.2, 5.2, and 6.2 as rendered are *not* tautologous. Hence, the logic system Par is easily seen not to be sound, and Parikh's completeness conjecture is denied.

Refutation of a modal logic for partial awareness from published example

Abstract: We evaluate a modal logic for partial awareness from a published example. The definitions and conjectures are *not* tautologous. We show how to exclude a priori logical clauses to promote a perhaps unintended tautology for the example. However, our evaluation does not rely on modal operators, suggesting that the system as proffered should be renamed to a logic for awareness, without the word modal.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

Remark 4.3: Since Ex. 4.1 and 4.2 are related with total verbiage greater than Ex 4.3, we select Ex. 4.3 to evaluate.

LET p, q, r, s, t, u, v, w, x, y, z:
P, Q, R, A_1, A_2, d_1, d_2, w, w_1, w_2, w_3.
~ Not; + Or ; & And; > Imply; = Equivalent; (p@p) contradiction, null, zero 0.

From: Halpern, J.Y.; Piermont, E. (2018). Partial awareness.
arxiv.org/pdf/1811.05751.pdf halpern@cs.cornell.edu

Remark 4.3: We evaluate Example 4.3 because its verbiage is less than that for the related Examples 4.1 and 4.2.

$P^I_{w1} = d_1$ (4.3.1.1)

(p&x)=u ; TTTT TTTT TTTT TTTT,
FFFF FFFF FFFF FFFF,
T**F**T**F** T**F**T**F** T**F**T**F** T**F**T**F**,
FT**F**T **F**T**F**T **F**T**F**T **F**T**F**T (4.3.1.2)

$Q^I_{w3} = d_2$ (4.3.2.1)

(q&z)=v ; TTTT TTTT TTTT TTTT,
FFFF FFFF FFFF FFFF,
TT**FF** TT**FF** TT**FF** TT**FF**,
FFTT **FF**TT **FF**TT **F**FTT (4.3.2.2)

$P^I_{w2} = P^I w_3 = Q^I_{w2}$ = zero (4.3.3.1)

(p&y)=((p&z)=((q&y)=(p@p))) ; TTTT TTTT TTTT TTTT,
T**FF**T T**FF**T T**FF**T T**FF**T,
T**F**T**F** T**F**T**F** T**F**T**F** T**F**T**F**,
TT**FF** TT**FF** TT**FF** TT**FF** (4.3.3.2)

$R^{I}_{w} = d_1$ (4.3.4.1)

(r&w)=u ;
TTTT TTTT TTTT TTTT,
FFFF FFFF FFFF FFFF,
TTTT **FFFF** TTTT **FFFF**,
FFFF TTTT **FFFF** TTTT (4.3.4.2)

A_1w = null [or] (P or Q) [or] zero (4.3.5.1)

(s&w)=((p@p)+((p+q)+(p@p))) ;
FFF T**FF**F T**FFF** T**FFF**,
T**FFF** T**FFF** **F**TTT **F**TTT (4.3.5.2)

A_2w = null [or] (Q or R) [or] zero (4.3.6.1)

(t&w)=((p@p)+((q+r)+(p@p))) ;
TT**FF** **FFFF** TT**FF** **FFFF**,
FFTT TTTT **FF**TT TTTT (4.3.6.2)

"agent$_1$ wants d_1 only when it has property P (to trade in states w_2 [or] w_3), and
agent$_2$ wants d_2 only when it has property Q (to trade in states w_1 and w_2"
with
"for agent$_1$, w_2 and w_3 are equivalent, and (4.3.7.1)
for agent$_2$, w_1 and w_2 are equivalent." (4.3.8.1)

((p&(y+z))>(s>u))>(y=z) ;
TTTT TTTT TTTT TTTT,
FFFF FFFF FFFF FFFF,
FFFF FFFF FT**F**T **F**T**F**T (4.3.7.2)

((q&(w&x))>(t>v))>(x=y) ;
TTTT TTTT TTTT TTTT,
FFFF FFFF FFFF FFFF (4.3.8.2)

"However, neither agent can propose an acceptable contract." (4.3.9.1)

Remark 9.1: To evaluate Eq. 4.3.9/10 we process Eqs. 4.3.7.2 or 4.3.8.2 respectively as the consequent of the definitions in Eqs. 4.3.1.2/6.2.

(((((p&x)=u)&((q&z)=v))&(((p&y)=((p&z)=((q&y)=(p@p))))&
((r&w)=u)))& (((s&w)=((p@p)+((p+q)+(p@p))))&((t&w)=((p@p)+((q+r)+(p@p))))))
> ((((p&(y+z))>(s>u))>(y=z))+(((q&(w&x))>(t>v))>(x=y))) ;
TTTT TTTT TTTT TTTT,
FTTT TTTT **F**TTT TTTT,
FTTT TTTT TTTT TTTT,
TTTT TTTT TT**F**T TTTT (4.3.9.2)

Eqs. 4.3.9.2 as rendered is *not* tautologous, and hence as presented "neither agent can propose an acceptable contract."

Remark 4.3.10: To rehabilitate Eq. 4.3.9.2, we exclude the agent clauses from Eqs. 4.3.7/8 for w-equivalences as potential a priori commentary. (4.3.10.1)

(((((p&x)=u)&((q&z)=v))&(((p&y)=((p&z)=((q&y)=(p@p))))&
((r&w)=u)))& (((s&w)=((p@p)+((p+q)+(p@p))))&((t&w)=((p@p)+((q+r)+(p@p))))))
> (((p&(y+z))>(s>u))+((q&(w&x))>(t>v))) ;
TTTT TTTT TTTT TTTT (4.3.10.2)

Eq. 4.3.10.2 is tautologous, hence without the injected agent w-equivalences, the agents can propose an acceptable contract. We do not guess what that contract is.

Excepting Eq. 4.3.10, the others are *not* tautologous. This means the example does not support a modal logic for partial awareness. We note that modal operators were not used by us here at all.

Our conclusion is not to refute the notion of a partial awareness in semantics. This can be construed as a newly coined academic term for VŁ4, where the four-valued logic purposely codifies falsity and truthity based on exact truth table results in the range from contradiction to tautology. Because of that, VŁ4 is better suited for the *exact* analysis of partial awareness with or without modal operators.

Refutation of Pascal's wager

Abstract: The antecedent and consequent of the thought experiment of Pascal's wager are *not* tautologous. However, to determine gain by one wager or the other is tautologous. This refutes the conjecture of Pascal's wager as ultimately not allowing reason to determine faith. In other words, the "existence of God is possible to prove by human reason". What follows furthermore is that the existence of God is more profitable from this thought experiment. Therefore the conjecture forms a tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Pascal%27s_wager

"The wager uses the following logic (excerpts from Pensées, part III, §233):

> God is, or God is not. Reason cannot decide between the two alternatives. A Game is being played... where heads or tails will turn up. You must wager (it is not optional). Let us weigh the gain and the loss in wagering that God is. Let us estimate these two chances. If you gain, you gain all; if you lose, you lose nothing." (1.0)

We write Eq. 1.0 as:

Antecedent: ((God is, or God is not) implies (either (if God is, then wager gains) or (if God is not, then wager breaks even)) (1.1.1)

LET p, q: God, gain

(p+~p)>((p>(q>(s@s)))&(~p>(q=(s@s)))) ;
`TT`**`FF`** `TT`**`FF`** `TT`**`FF`** `TT`**`FF`** (1.1.2)

Remark 1.1.2: Eq. 1.1.2 can be weakened by inserting modal operators as

#(p+~p)>%((p>(q>(s@s)))&(~p>(q=(s@s)))) ;
`TTCC TTCC TTCC TTCC` (1.1.3)

Consequent: implies ((if God is not, then wager breaks even) is more profitable than (if God is, then wager gains)). (1.2.1)

(~p>(q=(s@s)))>(p>(q>(s@s))) ;
TTT**F** TTT**F** TTT**F** TTT**F** (1.2.2)

"Pascal begins by painting a situation where both the existence and non-existence of God are impossible to prove by human reason." (2.0)

We write Eq. 2.0 as consequent Eq. 1.1.1 implies antecedent Eq. 1.2.1: (2.1)

((p+~p)>((p>(q>(s@s)))&(~p>(q=(s@s)))))>((~p>(q=(s@s)))>(p>(q>(s@s)))) ;
TTTT TTTT TTTT TTTT (2.2)

Remark 2.2: If the antecedent is chosen as the weakened modal Eq. 1.1.3, the result is different from Eq. 2.2 and is *not* tautologous:

(#(p+~p)>%((p>(q>(s@s)))&(~p>(q=(s@s)))))>((~p>(q=(s@s)))>(p>(q>(s@s)))) ;
TTTN TTTN TTTN TTTN (2.3)

The antecedent Eq. 1.1.2 of Pascal's conjecture and the consequent Eq. 1.2.2 are *not* tautologous. However, to determine gain by one wager or the other as in Eq. 2.2 results in a theorem to do just that. This refutes the conjecture of Pascal's wager as ultimately not allowing reason to determine faith. In other words, "both the existence and non-existence of God are possible to prove by human reason". What follows is that existence of God is more profitable from the thought experiment.

Refutation of the Pauli exclusion principle

We assume the apparatus and method of Meth8/VL4, with the designated *proo*f value of T. The 16-valued proof table is row-major and horizontal.

LET p, q, s, r: x, y, A⟨ψ|, r;
~ Not; & And; + Or; > Imply; < Not Imply; = Equivalent; @ Not Equivalent;
0 (r@r); (p>q) |x,y⟩; (q>p) |y,x⟩.

From: en.wikipedia.org/wiki/Pauli_exclusion_principle

The generalized form of the Pauli exclusion principle is:

[A]ntisymmetry under exchange means that $A(x,y) = -A(y,x)$.
This implies $A(x,y) = 0$ when $x = y$, which is Pauli exclusion.) (1.1)

((s&(p>q))=~(s&(q>p))) > (((s&(p>q))=(r@r))>(p=q)) ;
TTTT TTTT TFTT TFTT (1.2)

Eq. 1.2 as rendered is *not* tautologous. This means Eq. 1.2 refutes the generalized form of the Pauli exclusion principle.

The specific form of the Pauli exclusion principle proffered as the proof of the generalized form is:

$\langle \psi | x , y \rangle + \langle \psi | y , x \rangle = 0$ (2.1)

((s&(p>q))+(s&(q>p))) = (r@r) ; TTTT TTTT FFFF FFFF (2.2)

Eq. 2.2 as rendered is *not* tautologous. This means Eq. 2.2 refutes the specific form of the Pauli exclusion principle.

Eqs. 1.1 and 1.2 are supposed to be equivalent. (3.1)

(((s&(p>q))=~(s&(q>p))) > (((s&(p>q))=(r@r))>(p=q))) =
(((s&(p>q))+(s&(q>p)))=(r@r)) ; TTTT TTTT FTFF FTFF (3.2)

Eq. 3.2 is *not* tautologous, so the equivalence does not hold.

We weaken Eq. 3.1 with the imply connective to read Eq. 1.1 implies Eq. 2.1. (4.1)

(((s&(p>q))=~(s&(q>p))) > (((s&(p>q))=(r@r))>(p=q))) >
(((s&(p>q))+(s&(q>p)))=(r@r)) ; TTTT TTTT FTFF FTFF (4.2)

Eq. 4.2 is *not* tautologous, the same result table as Eq. 3.2, and the not-implication simply reverses the values of the result table. This means weakening Eq. 3.1 does not alter the refutation of Eq. 2.2.

The axiom of induction in Peano arithmetic

Summary: The axioms for Peano arithmetic (PA) are numbered 1-8 below. Axiom 1 is not tautologous (the designated proof value), but a truth value. Axioms 2-8 are tautologous. The the axiom of induction as published is not tautologous; however, with the correction of one connective in the equation script, it is s tautologous.

We assume the Meth8 apparatus where:

@ Not =; < Not >; % possibility, universal quantifier; # necessity, existential quantifier.

Result fragments are repeating rows of a truth table of 16-values for the 128 tables of the proof.

For system variant VL4, these are the numbered definitions of axiom, symbol, name, meaning, 2-tuple, and ordinal value.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	p=p	T	Tautology	proof	11	3
2	p@p	F	Contradiction	absurdum	00	0
3	%p>%#p	N	Non-contingency	truth	01	1
4	%p<%#p	C	Contingency	falsity	10	2

Numbered definitions of axioms with symbol, name, meaning, 2-tuple, and ordinal values. The designated proof value is T tautology. Note the meaning of (%p>%#p): a possibility of p implies a possibility of the necessity of p; and some p implies some of all p. In other words, if a possibility of p then a possibility of the necessity of p; and if some p then some of all p. This shows the equivalence and interchangeability of respective modal operators and quantified operators. (That correspondence is proved by VŁ4 corrections to the vertices of the Square of Opposition and subsequent corrections to the syllogisms of Modus Cesare and Modus Camestros.)

From en.wikipedia.org/wiki/Peano_axioms:

1. p>%#p ; TNTN (This truth table is a closer level to tautology than the truth value of NNNN.)

2. #p>(%p=#p) ; TTTT

3. #(p&q)>((q=p)>(p=q)) ; TTTT

4. #((p&q)&r)>(((p=q)&(q=r))>(p=r)) ; TTTT

5. #(u&v)>(((v>(v>%#v))&(u=v))>(u>(u>%#u))) ; TTTT

6. #x>((s&x)>((((s&x)\(s&x))-((s&x)\(s&x)))+((s&x)\(s&x)))) ; TTTT

7. #(w&x)>(((s&w)=(s&x))>(w=x)) ; TTTT

8. #(x>(%x<%#x))>(((s&x)=((%x>%#x)=(%x>%#x)))@((s&x)=(s&x))) ; TTTT

9.2. The axiom of induction as published, second definition:
(((y&(%y<%#y))=(t=t)) & ((#x&((y&x)=(t=t))) > (y&((y&(s&x))>(t=t))))) > ((#x&(y&x))=(t=t)) ;
_ ||
(((y&(%y<%#y))=(t=t)) & ((#x&((y&x)=(t=t))) > (y&((y&(s&x))>(t=t))))) > ((#x&(y&x))>(t=t)) ;
Corrected above; Meth8 validates as tautologous in 45-steps. _ _ _ _ _ _ _ _ _ _ _ _ _ ||

The consequent has its connective marked above. In words,
For the original: "then phi(n) **is tautologous** for every natural number n",
Read the corrected: "then phi(n) **is implied tautologous** for every natural number n".

Under the section First-order theory of arithmetic there, we number and present the scripts of the axioms as validated tautologous:

Fol-1. (#x&(((%x>%#x)-(%x>%#x))-(%x>%#x))) > (#x&(((%x>%#x)-(%x>%#x))@(s&x))) ; TTTT
Fol-2. ((#x&(((%x>%#x)-(%x>%#x))-(%x>%#x)))&(#y&(((%y>%#y)-(%y>%#y))-(%y>%#y)))) >
(#(x&y)&(x=y)) ; TTTT
Fol-3. (#x&(((%x>%#x)-(%x>%#x))-(%x>%#x))) > (#x&((x+((%x>%#x)-(%x>%#x))) = x)) ; TTTT
Fol-4. ((#x&(((%x>%#x)-(%x>%#x))-(%x>%#x))) & (#y&(((%y>%#y)-(%y>%#y))-(%y>%#y)))) >
(#(x&y)&((x+(s&y))=(s&(x+y)))) ; TTTT
Fol-5. (#x&(((%x>%#x)-(%x>%#x))-(%x>%#x))) >
(#x&((x&((%x>%#x)-(%x>%#x))) = ((%x>%#x)-(%x>%#x)))) ; TTTT
Fol-6. ((#x&(((%x>%#x)-(%x>%#x))-(%x>%#x)))&(#y&(((%y>%#y)-(%y>%#y))-(%y>%#y)))) >
(#(x&y)&((x&(s&y))=((x+y)+x))) ; TTTT
Fol-7. ((#y&((p&((%p>%#p)-(%p>%#p)))&(p&y))) & (#x&((p&x)&(p&y)))) > (((s&x)&(p&y)) >
(#x&((p&x)&(p&y)))) ; first-order induction axiom ; TTTT

Subsequent expressions 1-12 under Equivalent axiomizations were not mapped to scripts for PA.

Refutation of extended truth definitions to Peano arithmetic

Abstract: We evaluate proof-theoretic analysis by iterated reflection and ordinal analysis of iterated arithmetical comprehension. The former is *not* tautologous, and the latter is a contradiction. This refutes the extended truth definitions as proffered on Peano arithmetic and forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Beklemishev, L.D.; Pakhomov, F.N. (2019). Reflection algebras and conservation results for theories of iterated truth. arxiv.org/pdf/1908.10302.pdf lbekl@yandex.ru

Abstract We consider extensions of the language of Peano arithmetic by transfinitely iterated truth definitions satisfying uniform Tarskian biconditionals. Without further axioms, such theories are known to be conservative extensions of the original system of arithmetic. Much stronger systems, however, are obtained by adding either induction axioms or reflection axioms on top of them. Theories of this kind can interpret some well-known predicatively reducible fragments of second order arithmetic such as iterated arithmetical comprehension.

8 Proof-theoretic analysis by iterated reflection

8.3 A case study: analysis of ACA This method of analysis, in the simplest situation going beyond Peano arithmetic, can be illustrated by the well-known example of the second order theory ACA. This system extends PA by the schemata of induction, for all second order formulas, and by the comprehension schema: [for each arithmetical formula ϕ (possibly with first- and second-order parameters but not containing Y as a parameter).]

$$\exists Y \forall x (x \in Y \leftrightarrow \phi(x)) \qquad (8.3.9.1)$$

LET p, q, r, s: φ or X, x, Y, S.

(#q<%r)=(p&#q) ; `TTCT TTTC TTCT TTTC` (8.3.9.2)

Remark 8.3.9.2: Eq. 8.3.9.2 as rendered is *not* tautologous, hence refuting the comprehension schema.

9 Analysis of second order systems In this section we show how Theorem 8 can be used to obtain ordinal analysis of some systems of second order arithmetic of ‘predicative’ strength.
9.1 Ordinal analysis of iterated arithmetical comprehension … The base theory of second-order arithmetic we consider is the well-known theory ACA_0, that is, the extension of EA by the scheme of arithmetic comprehension (9) and the axiom of set-induction

$$0 \in X \wedge \forall x\, (x \in X \rightarrow S(x) \in X) \rightarrow \forall x\, (x \in X) \qquad (9.1.1)$$

```
((s@s)<q)&( ((#q<p)>((s&#q)<p))>(#q<p)) ;
                         FFFF FFFF FFFF FFFF          (9.1.2)
```

Remark 9.1.2: Eq. 9.1.2 is *not* tautologous, and in fact is a contradiction. This refutes the base theory of second-order arithmetic, chosen as ACA_0, that is, the extension of EA by the scheme of arithmetic comprehension (8.3.9.2) and the axiom of set-induction [not given in the text].

Denial of logic system PŁ4

Abstract: We evaluate the logic system PŁ4 as refuting and replacing VŁ4. Eight modal theses and two axioms are *not* tautologous and contrary to those of PŁ4. This denies that PŁ4 refutes VŁ4, refutes PŁ4, and justifies VŁ4 as containing the non bivalent fragment named PŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⇔, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) C non-contingency, ∇, ordinal 2; (%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).

From: Méndez, J.M.; Robles, G. (2015).
A strong and rich 4-valued modal logic without Łukasiewicz-type paradoxes.
Logica Unverisalis. 9: 501-522. sefus@usal.es gemma.robles@unileon.es
[Note that Springer sells this paper only to the public.]
readcube.com/articles/10.1007%2Fs11787-015-0130-z?
author_access_token=_L6Jv6iOFK12jKHv1HgYEve4RwlQNchNByi7wbcMAY59r2e7nCIsNQegrHbnROJqffuaveeCk8TaBQuGOj5kVweSWjiGNTeHG-kqV2rOibbvfbV1Lhsa3sYYaxKQxsG48f6c3kJbekbSQGkO5DxPSQ==

Proposition 7.11. Modal theses provable in PŁ4:

A→(¬A∨LA) (T18.1)

p>(~p+#p) ; TNTN TNTN TNTN TNTN (T18.2)

(¬LA∧A)→¬A (T19.1)

(~#p&p)>~p ; TNTN TNTN TNTN TNTN (T19.2)

Remark T: Eqs. T18.2 and T19.2 are *not* tautologous

Proposition 7.13. Modal wffs not provable in PŁ4:

(A→B)→(MA→MB) (F5.1)

(p>q)>(%p>%q) ; TTTT TTTT TTTT TTTT (F5.2)

(A→B)→(LA→LB) (F6.1)

(p>q)>(#p>#q) ; TTTT TTTT TTTT TTTT (F6.2)

(MA∧MB)→M(A∧B) (F7.1)

(%p&%q)>%(p&q) ; TTTT TTTT TTTT TTTT (F7.2)

L(A∨B)→(LA∨LB) (F8.1)

#(p+q)>(#p+#q) ; TTTT TTTT TTTT TTTT (F8.2)

LA→(B→LB) (F9.1)

#p>(q>#q) ; TTTT TTTT TTTT TTTT (F9.2)

LA→(MB→B) (F10.1)

#p>(%q>q) ; TTTT TTTT TTTT TTTT (F10.2)

It is easy to check that each one of these wffs is invalidated in the matrix MPŁ4. Consequently, they are not provable in PŁ4 by the soundness theorems (cf. Corollary 5.7). Provability of F1-F4 would result in collapse, that is, in the provability

Remark 11: PŁ4 is not supposed to prove Eqs. F5-F10. However VŁ4 proves F5.2-F10.2. This implies PŁ4 is a non bivalent fragment of VŁ4. Furthermore VŁ4 finds Eq. F11 as *not* tautologous.

Then, we can add the following axioms to A1-A8 in Definition 3.1:

(A∧B) → A/(A∧B) → B (A9.1)

((p&q)>(p\(p&q)))>q ; **FF**TT **FF**TT **FF**TT **FF**TT (A9.2)

A → (A ∨ B)/B → (A ∨ B) (A11.1)

(p>((p+q)\q))>(p+q) ; **F**TTT **F**TTT **F**TTT **F**TTT (A11.2)

After testing eight modal theses and two axioms, the results are contrary to those of PŁ4. This denies that PŁ4 refutes VŁ4, and further refutes logic system PŁ4.

Confirmation of Playfair's axiom

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

> LET p, q, r, s: p, q, single common line, z; ~ Not; & And;
> \> Imply, greater than; < Not Imply, less than; = Equivalent; @ Not Equivalent;
> \# necessity, for all or every; % possibility, for one or some.

From: en.wikipedia.org/wiki/Playfair's_axiom

> Proposition 30 of Euclid reads, "Two lines, each parallel to a third line, are parallel to each other." It was noted [..] by Augustus De Morgan that this proposition is logically equivalent to Playfair's axiom. This notice was recounted [..] by T.L. Heath in 1908.
> De Morgan's argument runs as follows:

Let *X* be the set of pairs of distinct lines which meet (1.1)

(p&q)>(p=q) (1.2)

and *Y* the set of distinct pairs of lines each of which is parallel to a single common line. (2.1)

((p&q)>((p@q)@r))>(p&q) (2.2)

If *z* represents a pair of distinct lines, then the statement, (3.1)

s=(p&q) ; (3.2)

For all *z*, if *z* is in *X* [Eq. 1.1] then *z* is not in *Y* [Eq. 2.1], (4.1)

(#s&(s<((p&q)>(p=q))))>~(s<(((p&q)>((p@q)@r))>(p&q))) ; (4.2)

is Playfair's axiom (in De Morgan's terms, No *X* is *Y*), and its logically equivalent contrapositive

For all *z*, if *z* is in *Y* [Eq. 2.1] then *z* is not in *X* [Eq. 1.1], (5.1)

(#s&(s<(((p&q)>((p@q)@r))>(p&q))))>~(s<((p&q)>(p=q))) ; (5.2)

is Euclid I.30, the transitivity of parallelism (No *Y* is *X*).
[If Eqs. 3.1, then Eqs. 3.1=4.1.] (6.1)

(s=(p&q)) > (((#s&(s<((p&q)>(p=q))))>~(s<(((p&q)>((p@q)@r))>(p&q)))) = ((#s&(s<(((p&q)>((p@q)@r))>(p&q))))>~(s<((p&q)>(p=q))))) ;
`TTTT TTTT TTTT TTTT` (6.2)

Eq. 5.2 as rendered is tautologous, hence confirming Playfair's axiom.

Denial of Płonka sums in logics of variable inclusion and the lattice of consequence relations

Abstract: From the section on Płonka sums, we evaluate an equation derived thereform. It is *not* tautologous, hence coloring subsequent assertions in the conjecture. This means the *non* tautologous conjecture is a fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊔ ;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Baldi, M.P. (2019). Logics of variable inclusion and the lattice of consequence relations. arxiv.org/pdf/1903.03771.pdf m.prabaldi@gmail.com

2.2. **Płonka sums.** The main mathematical tool that allows for a systematic study of logics of variable inclusion is an algebraic construction coming from universal algebra, and more specifically from the study of regular varieties ... . Such construction, known as *Płonka sums*, originates in the late 1960's from a series of papers published by the Polish mathematician J. Płonka, who first provided a general representation theorem for regular varieties. ... A *semilattice* is an algebra **A** = < A, ∨ >, where ∨ is a binary commutative, associative and idempotent operation. Given a semilattice **A** and a, b ∈ A, we set

a ≤ b ⇔ a∨ b = b. (2.2.1)

It is easy to see that ≤ is a partial order on A.

LET p, q: a, b.

~(q<p)=((p+q)=q) ; T**FF**T T**FF**T T**FF**T T**FF**T (2.2.2)

Eq. 2.2.2 is *no*t tautologous, thus coloring the entire conjecture.

Refutation of Poincaré recurrence theorem

This paper began by reading a physics paper, subsequently published in *Science* (2018):

> Rauer, B. (brauer@ati.ac.at); Erne, S.; Schweigler, T.; Cataldini, F.; Tajik, M.; Schmiedmayer, J. (schmiedmayer@atomchip.org. (2017). Recurrences in an isolated quantum many-body system. arxiv.org/pdf/1705.08231.pdf.

wherein:

> "Half way to this full recurrence the system rephases to the mirrored initial state. As we initially start from a nearly flat relative phase profile and our observable *C* is insensitive to the transformation *φ*(*z*) → *φ*(−*z*) this point is equivalent to the full recurrence." (1.1)

To evaluate Eq. 1.1 we assume the Meth8/VŁ4 apparatus and method with the designated *proof* value of `T` tautologous. Other values are: `F` contradiction; `N` truthity; `C` falsity. The 16-valued truth table results are row-major and presented horizontally.

> LET p q: *φ* lc_phi; z;
> ~ Not; > Imply, greater than; = Equivalent to; @ Not Equivalent to; - Not Or.
>
> (p&q)>(p&~q) ; `TTTF TTTF TTTF TTTF` (1.2)

Eq. 1.2 as rendered is *not* tautologous. Eq. 1.1 on its face reads phi-z implies phi-not-z, or alternatively phi-z as potentially true implies something false as phi-not-z. Of course, that is mistaken because truthity may not imply falsity.

This led us to look at the basis of the captioned paper, which from paragraph one relies on the recurrence theorem of Poincaré and Zermillo. (We previously showed elsewhere that ZMC set theory is *not* tautologous, except for the trivial axiom of specification, so we evaluate the former author).

From: planetmath.org/proofofpoincarerecurrencetheorem1

> $\mu(E\text{-}A_n) \leq \mu(A_0\text{-}A_n) = \mu(A_0)\text{-}\mu(A_n) = 0.$ (2.1)
>
> LET p q r s: μ lc_mu, E, A_n A-sub-n, A_0 A-sub-zero;
> % possibility, existential for one or some; # necessity, universal for all; ~(p>q) (p≤q);
> (p@p) logical 00; (%p>#p)-(%p>#p) numerical zero, as one minus one.
>
> Using the main connective in Eq. 2.1 as equivalent to and the logical 00,
>
> (~((p&(q-r))>(p&(s-r))) = ((p&s)-(p&r))) = (p@p) ;
> `TTTT TFTF TTTF TFTF` (2.2.1)

Eq. 2.2.1 as rendered is *not* tautologous. This refutes the Poincaré recurrence theorem.

We modify Eq. 2.2.1 by *changing* the first Equivalent to into the Imply connective.

> ~((p&(q-r))>(p&(s-r))) > (((p&s)-(p&r)) = (p@p)) ;

TTTT TTTT TTTT TTTT (2.2.2)

Eq. 2.2.2 is tautologous.

We modify Eq.2.2.2 by *changing* the logical 00 into a numeric zero, as one minus one.

~((p&(q-r))>(p&(s-r))) > (((p&s)-(p&r)) = ((%p>#p)-(%p>#p))) ;
TTTT TNTT TTTT TTTT (2.2.3)

Eq. 2.2.3 is *not* tautologous, diverging by one value N for truthity as Non-contingent.

Next, we modify Eq.2.2.2 again by *changing* the second Equivalent to into the Imply connective.

~((p&(q-r))>(p&(s-r))) > (((p&s)-(p&r)) > (p@p)) ;
TTTT TTTT TTTT TTTT (2.2.4)

Eq. 2.2.4 is tautologous.

Finally, we modify Eq.2.2.3 by *changing* the second Equivalent to into the Imply connective.

~((p&(q-r))>(p&(s-r))) > (((p&s)-(p&r)) > ((%p>#p)-(%p>#p))) ;
TTTT TTTT TTTT TTTT (2.2.5)

Eq. 2.2.5 is tautologous.

What the *change* modifications of Eqs. 2.2.2-2.2.5 as rendered demonstrate is that the formula to prove Eq. 2.1 can only be coerced into a proof by using the Imply connective instead of the Equivalent to connective.

Remark: Eq. 2.2.3, using a numeric zero, shows a finer level of proof value and contradicts Eq. 2.2.2 using a logical zero.

What follows is that the Poincaré recurrence theorem as a starting point for quantum theory and quantum physics is suspicious.

We then ask how the experimental results of the captioned paper can be reconciled with the refuted Poincaré recurrence theorem. We reply that assuming the physical experiment cannot be falsified (such as by probabilistic objections), then the experimental results are obviously misinterpreted into a mistaken conclusion.

Refutation of poison modal logic (PML)

Abstract: We evaluate four formulas as validities of poison modal logic (PML). None is tautologous, thereby refuting poison modal logic (PML).

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r, s: ϕ, ψ (also U), ■, ♦;
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⊢, ↦, ≻, ⊃; < Not Imply, less than, ∈, ≺, ⊂;
= Equivalent, ≡, ⊨, :=, ⇔, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤; (z@z) **F** as contradiction, Ø, Null, ⊥ ;
(%z<#z) C non-contingency, ∇, ordinal 2;
(%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).

From: Gross, D.; Rey, S. (2019).
Credulous acceptability, poison games and modal logic.
arxiv.org/pdf/1901.09180.pdf d.grossi@rug.nl, srey@ens-paris-saclay.fr

Remark: Because the modal sabotage notations of ■ and ♦ act as functions, we assign them variable names.

In particular, formula

[U](p→¬♦p)∧[U](p→♦p) (2.1)

(q&(p>~(s&p)))&(q&(p>(s&p))) ;
FFT**F** **FF**T**F** **F**F**T**F** **FF**T**F** (2.2)

expresses the property "the set denoted by p under function V is admissible" (in the underlying argumentation framework)

3.2 Validity and Expressivity: Examples
Fact 1. Let p ∈ P and ϕ, ψ ∈ Lp. The following formulas are validities of PML (w.r.t. class MØ):

¬p∧■p (3.2.3.1)

~p&(r&p) ; **FFFF FFFF FFFF FFFF** (3.2.3.2)

■p↔□p (3.2.5.1)

(r&p)=#p ; TCTC TNTN TCTC TNTN (3.2.5.2)

□p→(■ϕ↔□ϕ) (3.2.6.1)

#p>((r&p)=#p) ; TCTC TTTT TCTC TTTT (3.2.6.2)

Proofs are omitted.

The four Eqs. above are *not* tautologous, thereby refuting poison modal logic (PML).

Meth8 on Karl Popper proof Ex(Gx)

Reference: "Demarcation between science and metaphysics" (1972)

“Science is testable and falsifiable, but metaphysics is not.”

So Popper proves the *arch-metaphysical assertion* that “There is a personal spirit named God who is omnipresent, omnipotent, omniscient.”

Once asserted it's not disprovable (Fischer P=1) per Carnap.

If morality is non physicalistic, then not the moral Christian God.

However, this counter example proves *morality is physicalistic*:

When the existentialist utters “I ought to” conscience is invoked, and the moral imperative is asserted. Thus Ex(Gx) becomes a moral God.

What forms of pure monotheism exist other than Orthodox Christianity?

Baha'i, Judaism, Muhammadanism

By what reasons do they admit they are not truthful?

No avatar; Revelation ceased; Impersonal contradictory rules

Meth8 scripts: Popper predicates

Meth8 scripts for a,b,c,d as p,q,r,s	Predicates	Descriptions
1: p&q	1: Pos(a,b)	1: a occupies a position in region b
2: (p&q)>r	2: Put(a,b,c)	2: a can put thing b into position c
3: p&q	3: Utt(a,b)	3: a makes the utterance b
4: p&q	4: Ask(a,b)	4: a is asked the truth of b
5: (%p&#q)>(p&#q)	5: Opos(a)=((Ea)(b)Pos(a,b)>(b)Pos(a,b))	5: a is omnipresent
6: ((%p&#q)>#r)>((p&#q)>#r)	6: Oput(a)=((Ea)(b)(c) Put(a,b,c)>(b)(c) Put(a,b,c))	6: a is omnipotent
7: (p&q)>(p&q)	7: Th(a,b)=(Ask(a,b)>Utt(a,b))	7: a thinks b
8: (p&%q)>(p&%q);	8: Thp(a)=(Eb)Th(a,b)	8: a is a thinking person
9: (((p&%q)>(p&%q))&~(p&#q)) +(p&#q)	9: Sp(a)=(Thp(a)& ((b)~Pos(a,b))VOpos(a))	9: a is a (personal) spirit
10: (q&r)>((p&(q&r))>(p&(q&r)))	10: Knpos(a,b,c)=(Pos(b,c)> Th(a,"Pos(b,c)")	10: a knows that b is in position c
11: (q&r)>s)>((p&((q&r)>s)) >(p&((q&r)>s)))	11: Knput(a,b,c,d)=(Put(b,c,d) >Th(a,"Put(b,c,d)")	11: a knows that b can put c into position d
12: ((q&r)>(q&r))&((p&((q&r)	12: Knth(a,b,c)=(Th(b,c)&	12: a knows that b thinks c

>(q&r)))>(p&((q&r)>(q&r))))	Th(a,"Th(b,c)"))	
Meth8 scripts for a,b,c,d as p,q,r,s	Predicates	Descriptions
13: ((((p&q)>(p&q))&(p@r))&(~((r&q)>(r&q))))=~(((p&q)>(p&q))&((r&((p&q)>(p&q)))>(r&((p&q)>(p&q)))))	13: Unkn(a)=Th(a,b)&(a≠c)&~Th(c,b))=~Knth(c,a,b))	13: a is unfathomable: a thinks b and a is not c and c does not think b is equivalent to c does not know that a thinks b.
14: ((p&q)>(p&q))&(q=q)	14: Kn(a,b)=Th(a,b)&T(b), where T(b) means b is tautologous	14: a knows the fact b
15: ((p&#q)>(p&#q))>(q=q)	15: Verax(a) =((b)Th(a,b)>T(b))	15: a is truthful
16: (#q=#q)>(((p&q)>(p&q))&(q=q)	16: Okn(a)=(b)T(b)>Kn(a,b)	16: a is omniscient
17: ((p&#q)&((p&#q)>#r)>(((#q=#q)>(((p&q)>(p&q))&(q=q))))&(((p&#q)>(p&#q))>(q=q)))	17: (Opos(a)&Oput(a))=(Okn(a)&Verax(a))	17: a as omnipresent and a as omnipotent is equivalent to a as omniscient and a as truthful
18: (((((%p&#q) >(p&#q)) & (((%p&#q) >#r) >((p&#q)>#r))) >((#q=#q) >(((p&q)>(p&q))&(q=q)))) & ((((p&#q) >(p&#q)) > (q=q)) & ((((p&%q) >(p&%q))&~(p&#q))+ (p&#q)))) & (((((p&q) > (p&q)) &(p@r)) & ~((r&q) > (r&q))) = ~(((p&q) >(p&q)) & ((r&((p&q) >(p&q))) >(r&((p&q) > (p&q))))))	18: Ex(Gx)=(((Opos(a) &Oput(a)) >Okn(a))& ((Verax(a)& Unkn(a)) &Sp(a)))	18: There exists a personal spirit named God whose omnipresence and omnipotence implies omniscience, and who is truthful and unfathomable.

Meth8 validation tables

Table fragments for two of the four rows (The designated truth values are T and E.)

Expression	Model 1	Model 2.1	Model 2.2	Model 2.3.1	Model 2.3.2
5.-18. Validated	TTTT TTTT	EEEE EEEE	EEEE EEEE	EEEE EEEE	EEEE EEEE
4. (p&q)	FFFT FFFT	UUUE UUUE	UUUE UUUE	UUUE UUUE	UUUE UUUE
3. (p&q)	FFFT FFFT	UUUE UUUE	UUUE UUUE	UUUE UUUE	UUUE UUUE
2. (p&q)>r	TTTF TTTF	EEEU EEEU	EEEU EEEU	EEEU EEEU	EEEU EEEU
1. (p&q)	FFFT FFFT	UUUE UUUE	UUUE UUUE	UUUE UUUE	UUUE UUUE

Refutation of varieties of positive modal logic (PML)

Abstract: Four definitions of positive modal logic (PML) are *no*t tautologous. This refutes positive modal algebra (PML) on the bounded distributive lattice and forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Moraschini, T. (2019). Varieties of positive modal algebras and structural completeness. arxiv.org/pdf/1908.01659.pdf moraschini@cs.cas.cz

3. Algebras and frames

Definition 3.1. A *positive modal algebra* is a structure $A=\langle A,\wedge,\vee,\Box,\Diamond,0,1\rangle$ where $\langle A,\wedge,\vee,0,1\rangle$ is a bounded distributive lattice such that [for every $a,b\in A$]

□1=1 and (3.1.1.1)

LET p, q, s: a, b, s.

#(s=s)=(s=s) ; `NNNN NNNN NNNN NNNN` (3.1.1.2)

Remark 3.1.1.2: Ordinal 1 as `N` (%s>#s) produces a theorem, but the author means `T` for (s=s).

◊0=0 and (3.1.2.1)

%(s@s)=(s@s) ; `NNNN NNNN NNNN NNNN` (3.1.2.2)

Remark 3.1.2.2: Ordinal 2 as `C` (%s<#s) produces a theorem, but the author means **`F`** for (s@s).

$\Box a\wedge\Diamond b\leq\Diamond(a\wedge b)$ (3.1.5.1)

~(%(p&q)<(#p&%q))=(s=s) ; `NNNN NNNN NNNN NNNN` (3.1.5.2)

$$\Box(a \vee b) \leq \Box a \vee \Diamond b \tag{3.1.6.1}$$

~((#p+%q)<#(p+q))=(s=s) ;   NNNN NNNN NNNN NNNN (3.1.6.2)

Eqs. 3.1.1.2, ..2.2, ..5.2, and ..6.2 are *no*t tautologous. This refutes positive modal algebra (PML) on the bounded distributive lattice.

PowerEpsilon mathematical induction

Zhu, Ming-Yuan. Godel's incompleteness theorem verified by PowerEpsilon. 2013. DOI: 10.13140/RG.2.2.31985.68961

From: researchgate.net/publication/308194289

From 2.2.4, page 7, we evaluate an equation for "[m]athematical induction as an inference rule formalized as a second-order axiom".

We assume the Meth8 apparatus.

LET: p P; q k; r n;
& And; + Or; > Imply; = Equivalent to; @ Not Equivalent to;
# universal quantifier, modal necessity; (r@r) 0 [Zero]; (r=r) 1 [One]
T tautology; F contradiction

Result fragment is the repeating row from the truth table of 16-values.

$$\forall P \,.\, P(0) \wedge \forall k \,.\, P(k) \Rightarrow P(k + 1) \Rightarrow \forall n \,.\, P(n) \qquad (1.1)$$

(((#p&p)&(r@r))&((#q&p)&q)) > ((p&(q+(r=r)))>((#r&p)&r)) ; TTTT (1.2)

From the script rendition in Eq 1.2, Meth8 validates Eq 1.1 as tautologous.

Refutation of correctness from Pratt-Floyd-Hoare logic

Abstract: We evaluate the Pratt-Floyd-Hoare logic aimed at correctness of computer programs. For Pratt, four equations are tested, and for Hoare one is tested. None are tautologous. This refutes the Pratt-Floyd-Hoare logic for correctness.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r, s, t, u, v: p, q, a, b, p', q', R
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⇔, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) C non-contingency, ∇, ordinal 2; (%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).

From: Pratt, V.R. (1976). Semantical considerations of Floyd-Hoare logic.
17th IEEE Foundations of Computer Science Conference.
boole.stanford.edu/pub/semcon.pdf pratt@cs.stanford.edu

Pratt weakest antecedent and strongest consequent:

P{a}Q ≡ ⊭ (P⊃{a}Q) (W.1)

((p&r)&q)=~(p>(r&q)) ; T**F**TF T**F**T**F** T**F**T**F** T**F**T**F** (W.2)

Remark W: The proposition for in Eq. W.2 is *no*t tautologous.

Pratt axiom 2, in handwritten note on margin:

P{a∪ b}Q ⊢ P{a}Q (A2n.1)

((p&(r+s))&q)>((p&r)&q) ; TTTT TTTT TTT**F** TTTT (A2n.2)

Remark A2n: The axiom as a handwritten note is *not* tautologous and not equivalent to the axiom.

Pratt axiom 4:

Q{P}P/\Q (A4.1)

(q&p)&(p&q) ; **FFF**T **FFF**T **FFF**T **FFF**T (A4.2)

Pratt axiom 4, alternate:

$P \supset Q\{P\}Q$ (A4.alt.1)

p>((q&p)&q); T**F**TT T**F**TT T**F**TT T**F**TT (A4.alt.2)

Remark A4: The original and alternate axioms are *no*t tautologous and not equivalent.

Hoare "Rules of consequence":

$P \supset Q, Q\{a\}R \vdash P\{a\}R$ (HR.1.1)

((p>q)&((q&r)&v))>((p&r)&v) ; TTTT TTTT TTTT TTTT(4),
TTTT TT**F**T TTTT TT**F**T(4) (HR.1.2)

Remark HR1: The rule of consequence is *not* tautologous.

Eqs. W2, A2.n.2, A4.2, A4.alt.2, HR.1.2 are *not* tautologous and contradict the respective alternates. This refutes the Pratt-Floyd-Hoare logic for correctness.

Refutation of predicative collapse and arithmetical comprehension

Abstract: The four equations evaluated are *not* tautologous, hence refuting predicative collapsing principles such as arithmetical comprehension. Therefore these form a tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Freund, A. (2019). Predicative collapsing principles. arxiv.org/pdf/1906.07448.pdf

Abstract. We show that arithmetical transfinite recursion is equivalent to a suitable formalization of the following:
For every ordinal α there exists an ordinal β such that 1+β·(β+α) (ordinal arithmetic) admits an almost order preserving collapse into β. (1.1.1)

LET p, q: α , β.

(#p>%q)>(((%p>#p)+q)&(q+p)) ; **F**NTT **F**NTT **F**NTT **F**NTT (1.1.2)

(#p>%q)>(((%p>#p)+q)&(q+p)))>q ;
TCTT TCTT TCTT TCTT (1.2.2)

Arithmetical comprehension is equivalent to a statement of the same form, with β·α at the place of β·(β+α). … (2.1.1)

((%p>#p)+q)&p ; **F**N**F**T **F**N**F**T **F**N**F**T **F**N**F**T (2.1.2)

(((%p>#p)+q)&p)>q ; TCTT TCTT TCTT TCTT (2.2.2)

Remark 2.2.2: We see the logical equivalence of Eqs. 1.2.2 and 2.2.2, meaning ((1+β·(β+α))→β)=((β·α)→β), which is probably not what the author intended.

The four equations evaluated are *not* tautologous, hence refuting predicative collapsing principles such as arithmetical comprehension.

Refutation of the algorithm to generate preference profiles

Abstract: We evaluate six equations of the proposed algorithm to generate preference profiles. *None* is tautologous, hence refuting the proposal.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r, s: v, P_1, P_2, s;
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⊢, ↦, ≻, ⊃ ; < Not Imply, less than, ∈, ≺, ⊂ ;
= Equivalent, ≡, ⊨, :=, ⇔, ↔; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology; (z@z) **F** as contradiction, Ø, Null;
(%z<#z) C non-contingency, ∇, ordinal 2;
(%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).

From: Alvira, R. (2018).
Transforming candidate and parties cardinal ratings into weak preference orderings.
XV Meeting Spanish social choice network [REES], Elche, Alicante, 11.17-18.
vixra.org/pdf/1901.0384v1.pdf ricardo.alvira@gmail.com

[I]f the average preference value of some party voters for another party P_1 is greater than for another party P_2, then every one and all of that party voters strictly prefer P_1 to P_2: (2.0)

$v[P_1]>v[P_2]\leftrightarrow P_1\succ P_2$ (2.1.1)

((p&q)>(p&r))=(q>r) ; TT**F**T TT**F**T TT**F**T TT**F**T (2.1.2)

$v[P_1]=v[P_2]\leftrightarrow P_1\sim P_2$ (2.2.1)

((p&q)=(p&r))=(q=r) ; TT**F**T **F**TTT TT**F**T **F**TTT (2.2.2)

$v[P_1]>v[P_2]\leftrightarrow P_1\succ P_2 \vee v[P_1]=v[P_2]\leftrightarrow P_1\sim P_2$ (2.3.1)

(((p&q)>(p&r))=(q>r)) + (((p&q)=(p&r))=(q=r)) ;
TT**F**T TTTT TT**F**T TTTT (2.3.2)

LET p, q, r, s: v, A, B, s.

Strict indifference: $v(A)-v(B)=0\rightarrow A\sim B$ (2.4.1)

(((p&q)-(p&r))=(s@s))>(q=r) ; TTT**F** T**F**TT TTT**F** T**F**TT (2.4.2)

Strict preference: $v(A)-v(B)\geq 1\rightarrow A\succ B$ (2.5.1)

~((%s>#s)>((p&q)-(p&r)))>(p>q) ; TTTT TCTT TTTT TCTT (2.5.2)

Partial indifference and partial preference:
$1>v(A)-v(B)>0\rightarrow(1-(v(A)-v(B)))(A\sim B)\wedge(v(A)-v(B))(A\succ B)$ (2.6.1)

((((%s>#s)>((p&q)-(p&r)))>(s@s))>(((%s>#s)-((p&q)-(p&r)))&(q=r)))&
(((p&q)-(p&r))&(q&r)) ; FFFF FFTF FFFF FFTF (2.6.2)

Eqs. 2.1.2-2.6.2 as rendered are not tautologous. This refutes the proposed algorithm to generate preference profiles.

Prenex normal form with prefix and matrix refuted as not bivalent

We evaluate the prenex normal form using equations from en.wikipedia.org/wiki/Prenex_normal_form . We assume the Meth8/VŁ4 apparatus and method.

LET: p q r s t u ϕ phi, ψ psi, ρ rho, x, y, z; (p=p) true;
necessity, all; % possibility, some or one; & And; + Or; > Imply; = Equivalent.

The designated *proof* value is T for tautology. Truth tables with 16-values and 128-values are row-major. Non-repeating truth table rows are row-major and presented horizontally.

Every formula in classical logic is equivalent to a formula in prenex normal form. For example, if ϕ(y) , ψ(z) , and ρ(x) are quantifier-free formulas with the free variables shown then

Prenex normal form: $\forall x\ \exists\ y\ \forall z(\phi(y) \vee (\psi(z) \rightarrow \rho(x)))$ (1.0.1.1)

((#s&(%t&#u))&((p&t)+((q&u)>(r&s)))) = (p=p) ;
FFFF FFFF FFFF FFFF,
. . . . NNFN NNNN (1.0.1.2)

Prefix: $\forall x\ \exists y\ \forall z$ (1.0.1.1.1)

(#s&(%t&#u)) (1.0.1.1.2)

Matrix: $\phi(y) \vee (\psi(z) \rightarrow \rho(x))$, (1.0.2.1)

((p&t)+((q&u)>(r&s))) (1.0.2.2)

Not prenex normal form: $\forall\ x((\ \exists\ y\ \phi(y)) \vee ((\ \exists\ z\ \psi(z)) \rightarrow \rho(x)))$ (1.0.3.1)

(#s&((%t&(p&t))+((%u&(q&u))>(r&s)))) = (p=p) ;
FFFF FFFF NNNN NNNN, ...
NNFN NNNN, ... NNFF NNNN (1.0.3.2)

The prenex and not prenex forms are supposed to be logically equivalent.

((#s&(%t&#u))&((p&t)+((q&u)>(r&s)))) = (#s&((%t&(p&t))+((%u&(q&u))>(r&s)))) ;
TTTT TTTT CCTT CCCC (1.0.4.2)

Eq. 1.0.4.2 is *not* tautologous. From the text example, prenex is supposed to be equivalent to a not-prenex rendition, but the prenex model fails at this point.

LET: p q r x, ϕ, ψ

The rules for conjunction and disjunction say that ... equivalences are valid when x does not appear as a free variable of ψ.

$(\forall\ x\ \phi) \wedge \psi$ is equivalent to $\forall\ x(\phi \wedge \psi)$ (1.1.1)

(~(p<r)&((#p&q)+r))=(#p&(q+r)) ; TTTC FNFN TTTC FNFN (1.1.2)

$(\forall x\, \phi) \vee \psi$ is equivalent to $\forall x(\phi \vee \psi)$ (1.2.1)

(~(p<r)&((#p&q)&r))=(#p&(q&r)) ; TTTT TTTT TTTT TTTT (1.2.2)

and

$(\exists x\, \phi) \wedge \psi$ is equivalent to $\exists x(\phi \wedge \psi)$ (2.1.1)

(~(p<r)&((%p&q)+r))=(%p&(q+r)) ; TTTF CTCT TTTF CTCT (2.1.2)

$(\exists x\, \phi) \vee \psi$ is equivalent to $\exists x(\phi \vee \psi)$ (2.2.1)

(~(p<r)&((%p&q)&r))=(%p&(q&r)) ; TTTT TTTT TTTT TTTT (2.2.2)

Negation: The rules for negation say that

$\neg\exists x\phi$ is equivalent to $\forall x\neg\phi$ (3.1.1)

~(%p&q)=(#p&~q) ; TCCT TCCT TCCT TCCT (3.1.2)

$\neg\forall x\phi$ is equivalent to $\exists x\neg\phi$ (3.2.1)

(~#p&q)=(%p&~q) ; NFFN NFFN NFFN NFFN (3.2.2)

Implication

There are four rules for implication: two that remove quantifiers from the antecedent and two that remove quantifiers from the consequent. These rules can be derived by rewriting the implication $\phi \rightarrow \psi$ as $\neg\phi \vee \psi$ and applying the rules for disjunction above. As with the rules for disjunction, these rules require that the variable quantified in one subformula does not appear free in the other subformula.

The rules for removing quantifiers from the antecedent are:

$(\forall x\, \phi) \rightarrow \psi$ is equivalent to $\exists x(\phi \rightarrow \psi)$ (4.1.1)

((#p&q)>r)=(%p&(q>r)) ; CTFN CTCT CTFN CTCT (4.1.2)

$(\exists x\, \phi) \rightarrow \psi$ is equivalent to $\forall x(\phi \rightarrow \psi)$ (4.2.1)

((%p&q)>r)=(#p&(q>r)) ; FNCT FNFN FNCT FNFN (4.2.2)

The rules for removing quantifiers from the consequent are:

$\phi \rightarrow (\exists x\, \psi)$ is equivalent to $\exists x(\phi \rightarrow \psi)$ (5.1.1)

(q>(%p&r))=(%p&(q>r)) ; CTTT CTTT CTTT CTTT (5.1.2)

$\phi \rightarrow (\forall x\, \psi)$ is equivalent to $\forall x(\phi \rightarrow \psi)$ (5.2.1) (q>(#p&r))=(#p&(q>r)) ; FNTT FNTT FNTT FNTT (5.2.2)

The unnumbered examples in the text are *not* tautologous.

The intuitionistic logic equations listed in the text are supposed to fail. We found the first one was tautologous.

$\forall x (\phi \lor \psi)$ implies $(\forall x \phi) \lor \psi$ (6.1.1)

(#p&(q+r))>((#p&q)+r) ; TTTT TTTT TTTT TTTT (6.1.2)

Two Eqs. 1.2.2 and 2.2.2 as rendered were tautologous for the rules to map conjunction as quantified. This suggests that if all the connective rules are derived from the And connective, then there could be a better chance for success. However, that exercise pales in light of rules for negation and implication as found *not* tautologous. Hence, the prenex model was *not* tautologous. What follows is that the prenex model is not bivalent.

> Remark: Since about 1933 when Kurt Gödel reduced his quantified equations to prenex normal form, the format was adopted by many for exposition. We previously showed that one explanation for why the incompleteness theorems are not tautologous is because the Gödel's misuse of bivalent logic via the then prenex format. That finding is further supported by this instant analysis of the format.

What further follows is that many theorems produced with prenex for computer science, mathematics, and physics are now suspicious. A notable example is the satisfiability algorithms produced by Martin Davis and Hilary Putnam which are now mistaken.

Shortest refutation of prenex normal form

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q: ϕ lc_phi, ψ lc_psi; ~ Not; & And; > Imply; = Equivalent;
necessity, for all or every, ∀, ∀x; % possibility, for one or some, ∃, ∃x.

Remark: For clarity, we ignore the x variable below, as it were.

From: en.wikipedia.org/wiki/Prenex_normal_form

"The [implication] rules for removing quantifiers from the antecedent are:

(∀xϕ)→ψ is equivalent to ∃x(ϕ→ψ) , [1.1.1.1]

(#p>q)=%(p>q) ; `TTTT TTTT TTTT TTTT` (1.1.1.2)

(∃xϕ)→ψ is equivalent to ∀x(ϕ→ψ) . [1.1.2.1]

(%p>q)=#(p>q) ; `TTNN TTNN TTNN TTNN` (1.1.2.2)

The [implication] rules for removing quantifiers from the consequent are:

ϕ→(∃xψ) is equivalent to ∃x(ϕ→ψ), [1.2.1.1]

(p>%q)=%(p>q) ; `TTTT TTTT TTTT TTTT` (1.2.1.2)

ϕ→(∀xψ) is equivalent to ∀x(ϕ→ψ)." [1.2.2.1]

(p>#q)=#(p>q) ; `NTNT NTNT NTNT NTNT` (1.2.2.2)

Eqs. 1.1.2.2 and 1.2.2.2 as rendered are *not* tautologous. Hence rules for the implication operator refute the prenex normal form.

Refutation of computability, orders, and solvable groups

Abstract: We evaluate the first definition for a pre-orderable group which is *not* tautologous. This refutes subsequent conjectures, and forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
\> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Darbinyan, A. (2019). Computability, orders, and solvable groups. arxiv.org/pdf/1909.05720.pdf

Abstract The main objective of this paper is the following two results. (1) There exists a computable bi-orderable group that does not have a computable bi-ordering; (2) There exists a bi-orderable, two-generated recursively presented solvable group with undecidable word problem. Both of the groups can be found among two-generated solvable groups of derived length 3. …

1 Main results
… Finitely generated groups that are computable with respect to a finite generating set are called groups with decidable word problem. A well-known property of groups with decidable word problem is that decidability of the word problem does not depend on the choice of finite generating set, hence, it is an intrinsic property of the group. This is in contrast with the general case of countable groups when the property of being computable depends on the choice of the generating set. To formulate the first main theorem, we introduce the following definition which is a weaker form of left- and bi-orderings on groups.

Definition 1 (pre-order). For a given group G, we say that a binary relation on G is a pre-order [we say that G is pre-orderable] if

• $1 \preccurlyeq g$ implies $g^{-1} \preccurlyeq 1$; (1.2.1)

LET p: g.

~(p<(%s>#s))>~((%s>#s)<((%s>#s)\p)) ;
TCTC TCTC TCTC TCTC (1.2.2)

Remark 1.2.2: Eq. 1.2.2 is *not* tautologous, hence refuting Definition 1 on which subsequent conjectures are based.

Refutation of Presburger arithmetic via Axiom 2

Abstract: In Presburger arithmetic, Axiom 2 as *x*+1 = *y*+1 → *x*=*y* is *not* tautologous. Therefore Presburger arithmetic is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Presburger_arithmetic

The language of Presburger arithmetic contains constants 0 and 1 and a binary function +, interpreted as addition. In this language, the axioms of Presburger arithmetic are the universal closures of the following:

[Axiom] 2. *x*+1 = *y*+1 → *x*=*y* (2.1)

LET p, q: x, y; (%r>#r) 1; (r=r) T

((p+(%r>#r))=(q+(%r>#r)))>(p=q) ;
TCCT TCCT TCCT TCCT (2.2)

Remark 2.2: If Eq. 2.1 takes ordinal constant 1 as T, then:

((p+(r=r))=(q+(r=r)))>(p=q) ;
T**FF**T T**FF**T T**FF**T T**FF**T (2.3)

Remark 2.1: We attempt to resuscitate Eq. 2.1 by removing 1 from the antecedent:

[(*x*+1 = *y*+1) -1] → *x*=*y* (3.1)

(((p+(%r>#r))=(q+(%r>#r)))-(%r>#r))>(p=q) ;
TNNT TNNT TNNT TNNT (3.2)

Eqs. 2.2, 2.3, and 3.2 are not tautologous, thereby refuting Presburger arithmetic by its own Axiom 2.

Refutation that Strawson's presupposition is different from Russell's entailment

Abstract: Strawson's presupposition and Russell's entailment are of the same form, equivalent, and hence not different. These conjectures form a tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Cohen, S.M. (2008). Strawson: "On Referring".
faculty.washington.edu/smcohen/453/StrawsonDisplay.pdf smcohen@uw.edu

The difference between entailment and presupposition:

Russell's view, that (*1*) entails (*2*), means:
(*1*) cannot be true unless (*2*) is true. If (*2*) is false, (*1*) is false. (1.1)

LET p, q: (*2*), (*1*)

(q>p)+(~q>~p) ; TTTT TTTT TTTT TTTT (1.2)

Strawson's view, that (*1*) presupposes (*2*), means:
(*1*) cannot be true or false unless (*2*) is true. If (*2*) is false, (*1*) is neither true nor false. (2.1)

(q>~(p+~p))+(~q>~(p+~p)) ; TTTT TTTT TTTT TTTT (2.2)

Remark 1: Entailment is mapped using the implication connective with the consequent and antecedent reversed in order, as it were. For example, (*1*) entails (*2*) is (*2*) implies (*1*).

Remark 2: If p equals (p or (p or not p)) and q equals (q or (q or not q), then we test if Eqs. 1.2 and 2.2 are equivalent. (3.1)

((p=(p+(p+~p)))&(q=(q+(q+~q))))>(((q>p)+(~q>~p))=((q>~(p+~p))+(~q>~(p+~p)))) ;
TTTT TTTT TTTT TTTT (3.2)

This means that Russell's and Strawson's views as rendered are of the same form, and hence entailment and presupposition are equivalents as one in the same.

Refutation of prevarieties and quasivarieties of logic

Abstract: We evaluate two papers by the same author group. For prevarieties, we test a theorem in M which is *not* tautologous. For quasivarieties, we test a definition and two theorems. The definition of a De Morgan monoid via an involution function is *not* tautologous. Theorems for the Dunn monoid and via Brouwerian (and Heyting) algebra are *not* tautologous. These results collectively refute prevarieties and quasivarieties in logic. What follows is that prevarieties and quasivarieties of logic are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, ·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) `C` non-contingency, ∇, ordinal 2; (%z>#z) `N` as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Moraschini, T.; Raftery, J.G. (2019). On prevarieties of logic.
arxiv.org/pdf/1902.04160.pdf moraschini@cs.cas.cz, james.raftery@up.ac.za

Abstract: It is proved that every prevariety of algebras is categorically equivalent to a ‘prevariety of logic’, i.e., to the equivalent algebraic semantics of some sentential deductive system. This allows us to show that no nontrivial equation in the language ∧,∨,◦ holds in the congruence lattices of all members of every variety of logic, and that being a(pre)variety of logic is not a categorical property.

Let M be the matrix power K. ... [T]he following formula is valid in M:

$$(x{\to}y{\approx}\Box(x{\to}y)\&x{\leftarrow}y{\approx}\Box(x{\leftarrow}y)){\Leftrightarrow}x{\approx}y \quad (4.1)$$

LET p, q, r, s: *x*, *y*, *z*, *e*.

(((p>q)=#(p>q))&((p<q)=#(p<q)))=(p=q) ;
`NCCN NCCN NCCN NCCN` (4.2)

Remark 4.2: Eq. 4.2 as rendered is *not* tautologous.
This refutes the prevariety of logic.

From: Moraschini, T.; Raftery, J.G.; Wannenburg, J.J. (2019).
Singly generated quasivarieties and residuated structures.
arxiv.org/pdf/1902.04159.pdf
moraschini@cs.cas.cz, james.raftery@up.ac.za, jamie.wannenburg@up.ac.za

Definition 8.1.
A *De Morgan monoid* is an algebra $\boldsymbol{A} = \langle A;\cdot,\wedge,\vee,\neg, e\rangle$ comprising a distributive lattice $\langle A;\wedge,\vee\rangle$, a commutative monoid $\langle A;\cdot, e\rangle$ that is *square-increasing* (i.e., $\boldsymbol{A}$ satisfies $x\leq x^2:=x\cdot x$), and a function $\neg\colon\ A\rightarrow A$, called an *involution*, such that $\boldsymbol{A}$ satisfies $\neg\neg x \approx x$ and

$$x\cdot y\leq z\Leftrightarrow x\cdot\neg z\leq\neg y \qquad (8.1.1)$$

~(r<(p&q))=~(~q<(p&~r)) ; **F**TTT TT**F**T **F**TTT TT**F**T (8.1.2)

Remark 8.1.2: Eq. 8.1.2 as rendered is *not* tautologous. This refutes the definition of a De Morgan monoid via an involution function.

9. Dunn monoids and reflections
With respect to the derived operation $x\rightarrow y:=\neg(x\cdot\neg y)$, every De Morgan monoid satisfies $\neg x\approx x\rightarrow f$ and

$$x\cdot y\leq z\Leftrightarrow y\leq x\rightarrow z \text{ (the law of residuation).} \qquad (9.6.1)$$

~(r<(p&q))=~((p>r)<q) ; **F**TTT TT**F**T **F**TTT TT**F**T (9.6.2)

Remark 9.6.2: Eq. 9.6.2 is *not* tautologous. We note that Eqs.8.1.2 and 9.6.2 produce the same truth table result.
This refutes Dunn monoids via the law of residuation.

10. Brouwerian algebras
Definition 10.1. A Dunn monoid is called a *Brouwerian algebra* if it satisfies $x\cdot y\approx x\wedge y$ (or equivalently, $x\leq e$), in which case it is identified with its $\rightarrow$, $\wedge$, $\vee$, e reduct. A Heyting algebra is therefore just a Brouwerian algebra with a distinguished least element.

Mints [47] showed (in effect) that the variety BRA of *all* Brouwerian algebras is not SC, by proving that the following quasi-equation (not satisfied by BRA) is admissible in BRA:

$$x\rightarrow y\leq x\vee z\Rightarrow((x\rightarrow y)\rightarrow x)\vee((x\rightarrow y)\rightarrow z)\approx e. \qquad (10.1)$$

Remark 10.1: For our purpose in testing, we ignore the trailing equivalent.

~((p+r)<(p>q))>(((p>q)>p)+((p>q)>r)) ;
FT**F**T TTTT **F**T**F**T TTTT (10.2)

Remark 10.2: Eq. 10.2 is *not* tautologous.
We note that Eqs. 9.6.2 and 10.2 produce the same truth table results.
This refutes Brouwerian *and* Heyting algebra.

Eqs. 4.2, 8.1.2, 9.6.2, and 10.2 collectively refute prevarieties and quasivarieties in logic, and in the process refute the De Morgan and Dunn monoids, and Brouwerian and Heyting algebras.

Refutation of the prisoner's paradox

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table fragment) is row-major and horizontal.

LET p, q, r, s: prisoner A; prisoner B; incarceration; prison term;
& And; + Or; - Not Or; = Equivalent;
> Imply, greater than, betrays; < Not Imply, less than, does not betray;
% possibility, for one or some; # necessity, for all;
(%p>#p) 1 year or lesser charge; (%p<#p) = 2 year

From: en.wikipedia.org/wiki/Prisoner's_dilemma

If A and B each betray the other, each of them serves 2 years in prison (1.1)

((p>q)&(q>p))>((p&q)=(r&(%p<#p))) ;
TTTF NTTC TTTF NTTC (1.2)

If A betrays B but B remains silent, A will be set free and B will serve 3 years in prison (and vice versa) (2.1)

((p>q)&(q<p))>((p=~r)&(q=(r&((%p<#p)+(%p>#p))))) ;
TTFT TTTT TTFT TTTT (2.2)

If A and B both remain silent, both of them will only serve 1 year in prison (on the lesser charge) (3.1)

((p<q)&(q<p))>(p&q)=((r&(%p>#p))) ;
TTTT TTTT TTTT TTTT (3.2)

Eqs. 1.2 and 2.2 as rendered are *not* tautologous, but also are *not* contradictory. Eq. 3.2 is tautologous, and *not* contradictory. In other words, Eqs.1.2, 2.2, and 3.2 are *not* contradictory, and hence the prisoner's paradox is *not* a paradox.

Refutation of probabilistic approximate logic (PALO) and logical imagination engine

Abstract: A key property of probabilistic approximate logic (PALO) as one form of inference (of many) is evaluated as *not* tautologous. This refutes its semantics of the logical imagination engine and forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Stehr, M-O.; Kim, M. (2019). Probabilistic approximate logic and its implementation in the logical imagination engine. arxiv.org/pdf/1907.11321.pdf

Abstract: In this note, we introduce Probabilistic Approximate Logic (PALO) as a logic based on the notion of mean approximate probability to overcome conceptual and computational difficulties inherent to strictly probabilistic logics…

3.2 Approximate probability semantics

A key property identified in.. that also holds in PALO in spite of the lack of idempotence is

$$[\psi] =[\varphi\vee\psi]-1+[\varphi\Rightarrow\psi]\geq[\varphi]-(1-[\varphi\Rightarrow\psi]) \quad (3.2.1.1)$$

which allows a limited form of modus ponens in the sense that it enforces a lower bound for [ψ] given [φ] and [φ⇒ψ], but as we will see this is only one form of inference that can take place in PALO which unlike most deductive systems does not favor any particular direction of execution.

LET p, s: φ , ψ.

q=(((p+q)-(%s>#s))+(~(p>(p>q))-((%s>#s)-(p>q)))) ;
FTTT **F**TTT **F**TTT **F**TTT (3.2.1.2)

Remark 3.2.1.2: Eq. 3.2.1.2 produces the same result with ordinal 1 as (s=s) or (%s>#s).

Eq. 3.2.1.2 as rendered is *not* tautologous and refutes one form of inference (of many) that can take place in PALO. What follows is refutation of the semantics of the logical imagination engine.

Refutation of program verification by reduction

Abstract: The formula for program verification by reduction is *not* tautologous, thereby refuting such an approach and forming a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
 > Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Barthe, G.; Eilers, R.; Georgiou, P.; Gleiss, B.; Kovács, K.; Maffei, M. (2019). Verifying relational properties using trace logic. arxiv.org/pdf/1906.09899.pdf
arxiv.org/pdf/1906.09899.pdf

1 Introduction

Program verification generally focuses on proving that all executions of a program lie within a specified set of executions, that is, properties are seen as sets of traces. However, this approach is not general enough to capture various fundamental properties, such as non-interference ... and robustness … . These notions are naturally modeled as relational properties, that is as properties over sets of pairs of traces. Relational properties are special instances of hyperproperties [15], which are formally defined as sets of sets of traces. Verification of relational properties can be achieved in different ways. One approach is by reduction to program verification:

given a program P and a hyperproperty φ, construct a program Q and a property ψ, such that: (i) Q verifies ψ and (ii) Q verifies ψ implies P verifies φ. … (1.1)

LET p, q, r, s: P, Q, φ, ψ.

((p&r)>(q&s))>((q>s)&((q>s)>(p>r))) ;
TFFF TTFT TFTF TTTT (1.2)

Remark 1.2: Eq. 1.2 is *not* tautologous, thereby refuting such an approach by reduction to program verification.

Refutation of provability logic GL and Japaridze's derived polymodal (GLP)

Abstract: We evaluate eight equations for provability logic (GL) and the derived polymodal logic of Japaridze (GLB, GLP). None is tautologous, hence refuting provability logic. These form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Rineke (L.C.) Verbrugge (2017). Provability logic.
plato.stanford.edu/entries/logic-provability/ L.C.Verbrugge@rug.nl

These Löb conditions, as they are called nowadays, seem to cry out for a modal logical investigation, where the modality □ stands for provability in PA [Peano arithmetic]. Ironically, the first time that the formalized version of Löb's theorem was stated as the modal principle □(□*A*→*A*)→□*A* was in a paper … in 1963 about the logical basis of ethics, which did not consider arithmetic at all.

2.1 Axioms and rules Propositional provability logic is often called GL, after Gödel and Löb. (Alternative names found in the literature for equivalent systems are L, G, KW, K4W, and PrL). The logic GL results from adding the following axiom to the basic modal logic K: □(□*A*→*A*)→□*A*. (2.1.1)

LET p: p.

#(#p>p)>#p ; `CTCT CTCT CTCT CTCT` (2.1.2)

Remark 2.1.1: Eq. 2.1.2 reiterates that the Löb axiom is *no*t tautologous.

It is not difficult to see that the modal axiom □*A*→□□*A* (known as Axiom 4 of modal logic) is indeed provable in GL. To prove this, one uses the substitution *A*∧□*A* for *A* in the axiom (GL). (2.2.1)

LET p=(p&#p), to substitute into Eq. 2.1.2:

#(#(p&#p)>(p&#p))>#(p&#p) ; `CTCT CTCT CTCT CTCT` (2.2.2)

Remark 2.2.2: Eq. 2.2.2 is *not* tautologous. Axiom 4 of modal logic is not probable in GL by substitution. In fact, Eqs. 2.1.2 and 2.2.2 produce the equivalent values in truth table results, and hence are identical expressions.

2.2 The fixed point theorem The main "modal" result about provability logic is the fixed point theorem … . It says essentially that self-reference is not really necessary, in the following sense. Suppose that all occurrences of the propositional variable *p* in a given formula *A*(*p*) are under the scope of the provability operator, for example *A*(*p*)=¬□*p*, or *A*(*p*)=□(*p*→*q*). Then there is a formula *B* in which *p* does not appear, such that all propositional variables that occur in *B* already appear in *A*(*p*), and such that GL⊢*B*↔*A*(*B*). This formula *B* is called a *fixed point* of *A*(*p*). (2.2.1.1)

LET p, q, r: p, A, B

(((q&p)=~#p)+((q&p)=#(p>q)))>(r=(q&r)) ;
TTTT N**F**TT TTTT N**F**TT (2.2.1.2)

Remark 2.2.1.2: Eq. 2.2.1.2 is *not* tautologous. Brouwer's fixed point theorem is not proved by GL.

Moreover, the fixed point is unique, or more accurately, if there is another formula *C* such that GL⊢*C*↔*A*(*C*), then we must have GL⊢*B*↔*C*. … For example, suppose that *A*(*p*)=¬□*p*. Then the fixed point produced by such an algorithm is ¬□⊥ , and indeed one can prove that GL⊢¬□⊥↔¬□(¬□⊥). (2.2.2.1)

~(#(s@s)=(s=s))=~(#(~(#(s@s)=(s=s))=(s=s))) ;
CCCC CCCC CCCC CCCC (2.2.2.2)

Remark 2.2.2.2: Eq. 2.2.2.2 is *not* tautologous. The truth table result of consistent C is the value for falsity. The fixed point is not proved as unique by GL. The assertion below of the second incompleteness theorem is also not proved to mean sufficiently strong consistent arithmetical theories can prove their own consistency.

If this is read arithmetically, the direction from left to right is just the formalized version of Gödel's second incompleteness theorem: if a sufficiently strong formal theory *T* like Peano Arithmetic does not prove a contradiction, then it is not provable in *T* that *T* does not prove a contradiction. Thus, sufficiently strong consistent arithmetical theories cannot prove their own consistency.

5.3 Propositional quantifiers

Another way to extend the framework of propositional provability logic is to add propositional quantifiers, so that one can express principles like Goldfarb's: ∀*p*∀*q*∃*r*□((□*p*∨□*q*)↔□*r*), (5.3.1.1)

#((##p+##q)=#%r)=(p=p) ; N**FFF F**NNN N**FFF F**NNN (5.3.1.2)

saying that for any two sentences there is a third sentence which is provable if and only if either of the first two sentences is provable. This principle is provable in Peano Arithmetic … . The set of sentences of GL with propositional quantifiers that is arithmetically valid turns out to be undecidable … .

5.4 Japaridze's bimodal and polymodal provability logics Japaridze's bimodal provability logic GLB … has three mixed axiom schemes … :

LET p, q, r: *A*, *k*, *m*, *n*.

$[m]A \to [n]A$, for $m \leq n$ (5.4.1.1)

~(s<r)>((r&p)>(s&p)) ; TTTT T**F**T**F** TTTT TTTT (5.4.1.2)

$\langle k \rangle A \to [n]\langle k \rangle A$, for $k < n$ (5.4.2.1)

~(s<q)>((q&p)>(s&(q&p))) ; TTT**F** TTTT TTT**F** TTTT (5.4.2.2)

$[m]A \to [n][m]A$, for $m \leq n$ (5.4.3.1)

~(s<r)>((r&p)>((s&p)&r)) ; TTTT T**F**T**F** TTTT T**F**T**F** (5.4.3.2)

Remark 5.4: GLB contains three axioms which are *not* tautologous. This serves to refute GLB and the derived GLP.

The Eqs. evaluated are *not* tautologous and deny GL, GLB, and GLP to refute provability logic.

Difference between ordinary logic (Prover9) and VŁ4

We use Prover9 (P9) to check ordinary logic, and use Meth8 to check system variant VŁ4.

The main difference is that the modal operators and quantifiers are *not* distributive and interchangeable in ordinary logic, but they are in VŁ4 (as shown elsewhere). This is borne out because ordinary logic is based on a vector space, but VŁ4 is bivalent.

For example consider this "egregious" example using Schrödinger's Cat:

> *If possibly the Cat is alive and possibly the Cat is dead, then possibly both the Cat is alive and the Cat is dead.* (1)

LET p "the Cat is alive", q "the Cat is dead"

For ordinary logic in P9 Eq 1 should not be proved as:

```
Assumptions:  ( exists(p) & exists(q) ).
Goal:         ( exists((p) & (q)) )        Not proved        (2)
```

For VŁ4 in Meth8 Eq 1 should be tautologous (and indeed as an equivalence or theorem) as:

```
(%p & %q) > %(p & q);        vt        (3)
```

The problem in the example is two contradictory possibilities being held as possible at the same time, for the Cat surely cannot be both alive and dead concurrently as Schrodinger's paradox asserts (but before it is resolved by Meth8 elsewhere).

To preserve the two variables p and q for the intended distinction of the Cat alive and the Cat dead, we embellish the assertion in Eq 1 with a prefix to the antecedent in the constraint that the Cat alive as p implies the Cat not dead as ~q:

> *If necessarily the Cat alive implies the Cat not dead, then if possibly the Cat is alive and possibly the Cat is dead, then possibly both the Cat is alive and the Cat is dead.* (4)

```
as:   #(p>~q) > ((%p&%q)>%(p&q)) ;        vt        (5)
```

P9 writes this as:

```
Assumptions:  (all(p -> -q)).
Goal:         ((exists(p) & exists(q)) -> (exists(p & q))).
                                     Not proved        (6)
```

Eq 5 is modified from Eq 3 to exclude a contradiction from words and is still tautologous. The same expression in Eq 6 on P9 is still not proved.

Our experiment to embellish the input expressions on P9 to make it compatible with Meth9 was unsuccessful. We conclude that system variant VŁ4 implemented in Meth8 is not compatible with ordinary logic implemented in P9.

We note here that it is possible to fix up Eq 1 by rewriting it so that P9 proves it. Consider this rendition in *one* variable:

> *If possibly the Cat is alive and not possibly the Cat is alive, then possibly both the Cat is alive and the Cat is not alive.* (7)

P9 writes this as:

Assumptions: ((exists(p) & -exists(p))).
Goal: (exists(p & -p)). Proved (8)

Meth8 writes this as:

(%p&~%p)>%(p&~p) ; vt (9)

Rewriting Eq 1 as Eq 7 in one variable causes conformity of result for Eq 8 in P9 and Eq 9 in Meth8. Unfortunately differences remain between P9 and Meth8 for more than one variable in Eqs 2-6 due to the vector space for arity of ordinary logic and the bivalence of VŁ4.

Refutation of pure alethic modal logic (PAM)

Abstract: We evaluate the formula □p → p → ◊p as the backbone of pure alethic modal logic (PAM). Two inconsistent results arise from different orders of precedence: a result of *not* tautologous, *not* contradictory; and a tautologous result. That ambiguity refutes PAM.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET: ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⊢; < Not Imply, less than, ∈;
= Equivalent, ≡, ⊨; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊; # necessity, for every or all, ∀, □;
~(y < x) (x ≤ y), (x ⊆ y); (p=p) Tautology.

See: Béziau, J.-Y. (2012). "Pure alethic modal logic". Coginitio. 13:25-36.

1.1. The backbone of PAM

$$\Box p \rightarrow p \rightarrow \Diamond p \quad (1.1)$$

Remark 1.1: Eq. 1.1 is ambiguous as to order of operation, so we present two interpreted mappings.

$$(\Box p \rightarrow p) \rightarrow \Diamond p \quad (1.1.1)$$

(#p>p)>%p ; CTCT CTCT CTCT CTCT (1.1.2)

$$\Box p \rightarrow (p \rightarrow \Diamond p) \quad (1.2.1)$$

#p>(p>%p) ; TTTT TTTT TTTT TTTT (1.2.2)

Because Eq. 1.1.2 as rendered is *not* tautologous, while 1.2.2 is, this ambiguity refutes 1.1 as the backbone of PAM.

Rejection of the quantified modal logic theorem proving (QMLTP) library

Abstract: We evaluate five equations from the quantified modal logic theorem proving (QMLTP) library. None is tautologous for the status of the claimed conjecture, rejecting the approach and library. Other objections include: clarity such as not all problems are in English descriptions; skewed coverage such as about 50% the equations are assumed for Gödel's embedding; and usability such as the utility tool, to translate QMLTP scripts for pre-selected provers, in Prolog source code which is not compiled into executables for major hardware/OS platforms. Based on these results, the QMLTP approach and library forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
\> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Raths, T.; Otten J. (2011). The QMLTP library: benchmarking theorem provers for modal logics. iltp.de/qmltp

Abstract. The quantified modal logic theorem proving (QMLTP) library provides a platform for testing and evaluating automated theorem proving (ATP) systems for first-order modal logics. The main purpose of the library is to foster the development of new ATP systems and to put their comparison onto a firm basis. The current version 1.0.1 of the QMLTP library includes 500 problems represented in a standardized extended TPTP syntax [manipulated only with a utility tool with Prolog source code to be compiled by platform]. ...

2.1 The QMLTP domain structure

The 500 problems of the QMLTP library are divided into eight problem domains … APM, GAL, GLC, GNL, GSE, GSV, GSY, and SYM.

1. APM – *applications mixed*.
   10 problems from planning, querying databases, natural language processing and communication, and software verification.
2. GAL/GLC/GNL/GSE/GSV/GSY – *Gödel's embedding*.
   245 problems are generated by using Gödel's embedding of intuitionistic logic into the modal logic S4 .. . The original problems were taken from the TPTP library .. and derived from problems in the domains ALG (general algebra), LCL (logic calculi), NLP (natural language processing), SET (set theory), SWV (software verification), and SYN (syntactic), respectively.
3. SYM – *syntactic modal*.

175 problems from various textbooks .. and 70 problems from the TANCS-2000 system competition for modal ATP systems.

Multi-Modal Logic (Security Protocols) Status: unsolved
Phone user U and phone company C have following relationship:
U does not pay a call before he has dialed it. Both U and C
are able to prove when U is being charged.
U is able to prove that C can prove that U has made a call,
C is able to prove that U can prove that U has paid his call,
U is able to prove that C cannot prove that U has made a call,
C is able to prove that U cannot prove that he has paid his call,
whenever these facts are true, respectively.
Then, the following requirement is true:
From U's point of view, C should charge U only if he has made a call that is not yet paid.
MML012+1.1

LET p, q, r, s: phone company, pay or paid, user, call

((((~((r>q)>(r>s))+((r&p)>((r>q)=(s=s))))+(((r>(p>(r>q)))=(s=s))+((p>(r>(r>q)))=(s=s))))+
(((r>~(p>(r>q)))=(s=s))+(p>(~(r>(q>s))=(s=s)))))=(s=s))>(r>
(((r>s)&~(r>q))>(p>r))) ;
TTTT TTTT TTTT TTTT MML012+1.2

Barcan scheme instance Status: non-theorem
if for all x necessarily f(x), then it is necessary that for all x f(x) SYM001+1.1

LET p, q: f, x

(#q&#(p&#q))>#(#q&(p&#q)) ;
TTTT TTTT TTTT TTTT SYM001+1.2

converse Barcan scheme instance Status: non theorem
if it is necessary that for all x f(x), then for all x necessarily f(x) SYM002+1.1

#(#q&(p&#q))>(#q&#(p&#q)) ; TTTT TTTT TTTT TTTT SYM002+1.2

Set theory (naive) Status: unsolved

If {{A},{A,B}} = {{U},{U,V}} then A = U. SET016+4.1

LET p, q, r,s: A, B, U, V

((p&(p+q))=(r&(r+s)))>(p=r) ; TTTT TTTT TTTT TTTT SET016+4.2

If {{A},{A,B}} = {{U},{U,V}} then B = V. SET018+4.1

((p&(p+q))=(r&(r+s)))>(q=s) ; TTFT TTTF FTTT TFTT SET018+4.2

The five equations above are *not* tautologous for the status of the claimed conjecture. This rejects the quantified modal logic theorem proving (QMLTP) library. Other objections include: clarity such as not all problems are in English descriptions; skewed coverage such as about 50% the equations are assumed for Gödel's embedding; and usability such as the utility tool, to translate QMLTP scripts for pre-selected provers, in Prolog source code which is not compiled into executables for major hardware/OS platforms.

Quantified modifiers as modal operators† on connectives in modal logic

Abstract: Quantified modifiers as modal operators †do not apply directly to connectives, but to sentences and variables in the general format of antecedent, connective, consequent. We present quantified expressions in that format by connective for *two* variables. The quantified expressions are equivalent to the quantifier as a modal modifier distributed on the variables.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⇔, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) C non-contingency, ∇, ordinal 2; (%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

Quantified modifiers as modal operators† do not apply directly to connectives, but to sentences and variables in the general format of antecedent, connective, consequent. We present modal expressions in that format by connective for *two* variables. The modified expressions are equivalent to the modifier distributed on the variables.

Connective_	p_q	#(p_q) = (#p_#q)	%(p_q) = (%p_%q)
+	FTTT	FNNN	CTTT
&	FFFT	FFFN	CCCT
>	TFTT	NFNN	TCTT
=	TFFT	NFFN	TCCT

† See at vixra.org/pdf/1901.0415v8.pdf for the proof of quantified modifiers as equivalent to modal operators, due to the now corrected, modern square of opposition.

Refutation of quantum arithmetic using repeat-until-success circuits

We assume Meth8/VŁ4 where the designated *proof* value is Tautology. The truth table is repeating fragments of 16-values, row major and horizontal.

LET p q r s: φ_1 φ_2 ;
~ Not; + Or; & And; # necessity, for all; % possibility, for one or some .

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Binary ordinal
1	p=p	T	tautology	proof	11	3
2	p@p	F	contradiction	absurdum	00	0
3	%p>#p	N	non-contingency	truthity	01	1
4	%p<#p	C	contingency	falsity	10	2

From:

Wiebe, N.; Roetteler, M. (2016). "Quantum arithmetic and numerical analysis using repeat-until-success circuits". *Quantum information and computation*. v 16: 1&2. 0134–0178. pdfs.semanticscholar.org/8590/ca37e1266fbd7b58fddf8aee0258f0b93433.pdf

[R]epeat until success circuits can be used to implement a form of multiplication ...

Assume that φ1 ≈ φ2 ≈ 1 then ... φ1φ2 = −1 + φ1 + φ2 + (1 − φ1)(1 − φ2); (28.1)

((p=q)=(%p>#p))>((p&q)=((~(%p>#p)+(p+q))+(((%p>#p)-p)&((%p>#p)-q)))) ;
TNNT TNNT TNNT TNNT (28.2)

Now let us assume that φ1 ≈ 0 and φ2 ≈ 1. We can then use similar reasoning to show that φ1φ2 = φ1 − φ1(1 − φ2); (29.1)

((p=((%p>#p)-(%p>#p)))&(q=(%p>#p)))>((p&q)=(p-(p&((%p>#p)-q)))) ;
TTCT TTCT TTCT TTCT (29.2)

Eqs. 28.2 and 29.2 as rendered are *not* tautologous. This means the use of quantum arithmetic using repeat-until-success circuits is flawed and hence is refuted. We abandoned further analysis here.

Refutation of quantum block chain encoding

We assume Meth8/VŁ4 where the designated *proof* value is `T`autology. The truth table is repeating fragments of 16-values, row major and horizontal.

LET p q r s: x y 0 1;
~ Not; + Or; & And; # necessity, for all; % possibility, for one or some;
(p@p) 00; (%p>#p) 01; (%p<#p) 10; (p=p) 11; (~(%p>#p)+(%p>#p)) (-1)^x.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Binary ordinal
1	p=p	T	tautology	proof	11	3
2	p@p	F	contradiction	absurdum	00	0
3	%p>#p	N	non-contingency	truthity	01	1
4	%p<#p	C	contingency	falsity	10	2

From: Rajan, D.; Visser, M. (2018). Quantum blockchain using entanglement in time. arxiv.org/pdf/1804.05979.pdf

a code converts classical information into spatially entangled Bell states; two classical bits, *xy*, where *xy* = 00*;* 01*;* 10 or 11, are encoded to the state |β_{xy}> = (1/(2^0.5))*((|0>|y>) + ((-1)^x)*(|1>|~y>)), where ~y is the negation of y. (2.1)

We remove the bra-ket notation and the scalar constant as irrelevant to the binary argument.

((p&q)=(((p@p)+(%p>#p))+((%p<#p)+(p=p)))) > ((r&p)+(((%p>#p)+~(%p>#p))&(s&q))) ;
`TTT`**`F`**` TTTT TTTT TTTT` (2.2)

Eq. 2.2 as rendered is *not* tautologous. This means the attempt to convert classical information to quantum states is ultimately mistaken as a basis for quantum blockchain.

Refutation that classical logic is a completion of quantum logic

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or in repeating fragments from 128-tables for more variables.

LET p, q, r, s;
~ Not; & And; + Or; > Imply; = Equivalent; @ Not Equivalent;
# necessity, ☐, for all or every; (p@p) F contraction; (p=p) T tautology.

From: Kramer, S. (2017). Quantum logic as classical logic. arxiv.org/pdf/1406.3526.pdf
simon.kramer@a3.epfl.ch

The author above "propose[s] a semantic representation of the standard quantum logic QL within a classical, normal modal logic, and this via a *lattice-embedding* of orthomodular lattices into Boolean algebras with one modal operator. Thus our classical logic is a *completion* of the quantum logic QL. In other words, we refute Birkhoff and von Neumann's classic thesis that the logic (the formal character) of Quantum Mechanics would be non-classical as well as Putnam's thesis that quantum logic (of his kind) would be the correct logic for propositional inference in general. The propositional logic of Quantum Mechanics is modal but classical, and the correct logic for propositional inference need not have an extroverted quantum character. One normal necessity modality suffices to capture the subjectivity of observation in quantum experiments, and this thanks to its failure to distribute over classical disjunction. The key to our result is the translation of *quantum negation as classical negation of observability.*"

We render in Meth8/VŁ4 the equations of the Introduction as keyed to the major numbers.

#p=(p=p) ;	FNFN FNFN FNFN FNFN	(1.1.2)
#(q+r)=(p=p) ;	FFNN NNNN FFNN NNNN	(1.2.2)
#p&#(q+r) ;	FFFN FNFN FFFN FNFN	(1.3.2)

"Notice that the observation of the truth of a disjunction does not imply the observation of the truth of one of its disjuncts. That is,

#(q+r)>(#q+#r) ;	TTTT TTTT TTTT TTTT	(1.4.2)

is *not* a valid principle. This is an essential uncertainty. (On the other hand, the converse

(#q+#r)>#(q+r) ;	TTTT TTTT TTTT TTTT	(1.5.2)

is a valid principle.) Hence, and in fact,

~(#(p&q)=(p=p))&~(#(p&r)=(p=p)) ;	TTTC TCTC TTTC TCTC	(2.1.2)

or

~(#(p&q)+#(p&r))=(p=p) ;   TTTC TCTC TTTC TCTC   (2.2.2)

... the presentation of the experiment concludes that

((p&(q+r))=(p=p)) ;   FFFT FTFT FFFT FTFT   (2.3.2)

(#(p&q)+#(p&r))=~(p=p) ;   TTTC TCTC TTTC TCTC   (2.4.2)

That is,

(p&(q+r))@((p&q)+(p&r)) ;   FFFF FFFF FFFF FFFF   (3.1.2)

Apparently, the distributivity of classical conjunction and disjunction fails! Whence arises the motivation for special *quantum* conjunction and disjunction. ...

That is,

((#p&#(q+r))>(#(p&q)+#(p&r)))=(p@p) ;
FFFF FFFF FFFF FFFF   (3.2.2)

is false. On the other hand, the converse

((#(p&q)+#(p&r))>(#p&#(q+r)))=(p=p) ;
TTTT TTTT TTTT TTTT   (3.3.2)

is true, because:

((#(p&q)+#(p&r))>#((p&q)+(p&r)))=(p=p) ;
TTTT TTTT TTTT TTTT   (3.4.2)

((#(p&q)+#(p&r))=#(p&(q+r)))=(p=p) ;
TTTT TTTT TTTT TTTT   (3.5.2)

((#(p&q)+#(p&r))=(#p&#(q+r)))=(p=p) ;
TTTT TTTT TTTT TTTT   (3.6.2)

(As noticed above, $\Box$ distributes over $\wedge$ in both directions, but over $\vee$ only in one direction.) Thus, and in close correspondence with (3.1.2),

((#p&#(q+r))=(#(p&q)+#(p&r)))=(p@p) ;
FFFF FFFF FFFF FFFF   (4.1.2)

is false.

Hence, if we **make explicit the fact of observing facts** (for example by means of a modal operator $\Box$) then we do not need to introduce the special purpose formalism of Quantum Logic with special and possibly counter-intuitive quantum operators to account for quantum phenomena (due to the apparent failure of classical conjunction to distribute over classical disjunction), but can get by with intuitive classical (Boolean) logic at the small price of adding a single, classical modal operator □.

(Consider that

(#%p=p)=(p=p) ; NTNT NTNT NTNT NTNT (5.1.2)

is true if and only if

(~#%p=~p)=(p=p) ; NTNT NTNT NTNT NTNT (5.2.2)

is true if and only if

(%~%p=~p)=(p=p) ; NTNT NTNT NTNT NTNT (5.3.2)

is true if and only if

(%#~p=~p)=(p=p) ; NTNT NTNT NTNT NTNT (5.4.2)

is true if and only if

(%#p=p)=(p=p) ; TNTN TNTN TNTN TNTN (5.5.2)

is true.)

[The entire argument above is rendered as "If Eqs. 5.5.2 then if and 5.4.2 then if 5.3.2 then if 5.2.2 then 5.1.2."]

(((((%#p=p)=(p=p))>((%#~p=~p)=(p=p)))>((%~%p=~p)=(p=p)))>
((~#%p=~p)=(p=p)))>((#%p=p)=(p=p)) ;
TTTT TTTT TTTT TTTT (5.6.2)

The translation that we have found and shall now present and explicate is to translate quantum negation ~ as ¬□.

~p=~#p ; TNTN TNTN TNTN TNTN (6.1.2)

That is, we translate *quantum negation as classical negation of observability. ...* Hence, the classical negation of observability is classically equivalent to the possibility of observing classical negation. Thus, we can also view *quantum negation as the possibility of observing classical negation.*"

Eqs. 1.4.2 is a *valid* principle, but 2.1.2 or 2.2.2 are *not* tautologous, nor is 3.1.2. The distributivity of classical conjunction and disjunction does *not* fail; 3.2.2 and 4.1.2 are *not* false. The conclusion to translate quantum negation as not necessity in 6.1.2 is *not* tautologous. This refutes quantum logic as a fragment of classical logic (or vice versa, as others write).

Refutation of control by quantum observation

From: Biele, R; Rodríguez-Rosario, CA; Frauenheim, T; Rubio, A. 2016. Controlling heat and particle currents in nanodevices by quantum observation. arxiv.org/ftp/arxiv/papers/1611/1611.08471.pdf. Emails: (robert.biele@gmx.net), (crodrig@mpsd.mpg.de), and (angel.rubio@mpsd.mpg.de).

"*A quantum observer has zero entropy flow.* Examining the entropy flow due to the local observation shows that the quantum observer does not add a new entropy flow to the system in contrast to a standard thermodynamic heat bath. Inserting [Eq. (10)] into Eq. (9) shows that the entropy flux due to the quantum observer is zero. This means that a quantum observer changes the energy flow in the system directly, without having an entropy flow connected with it."

We assume the apparatus and method of Meth8/VŁ4, where T is the designated proof value. (Other values are F for contradiction, C for falsity, and N for truth; 16-valued truth tables are row-major.)

LET: p q r s p; |k>; Tr; vD^2;
1 2 0 (%p>#p); (%p<#p); (%p>#p)-(%p>#p)
lc_sigmaD |k><k|
ln(p) 0<p<1

LDp = ~(vD^2)[2|k><k|p|k><k| - |k><k|p - p|k><k|] (10.1)

LDp = s&(((%p<#p)&(((q&~q)&p)&(q&~q))) - (((q&~q)&p) – (p&(q&~q)))) (10.2)

0=-Tr[LDp(ln (lc_sigmaD)) (9.1)

((%p>#p)-(%p>#p)) = (~r&((LDp)&(((p&~q)<(%q>#q))&((q&~q)>((%p>#p)-(%p>#p)))))) (9.2)

Eq. 10.1 is substituted into Eq. 9.1: (11.1)

((%p>#p)-(%p>#p)) = (~r&((s&(((%p<#p)&(((q&~q)&p)&(q&~q))) - (((q&~q)&p) - (p&(q&~q)))))&(((q&~q)<(%q>#q))&((q&~q)>((%q>#q)-(%q>#q)))))) ;
NNNN NNNN NNNN NNNN (11.2)

Eq. 11.2 as rendered is *not* tautologous. This means that control by quantum observation is refuted.

Shortest refutation of independent and entangled states of the quantum hypothesis

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET: p, q, r, s: $\Phi_{\blacksquare}$, $\Phi_{\bullet}$, $\psi_{\blacksquare}$, $\psi_{\bullet}$; ~ Not; & And; + Or; - Not Or;
> Imply, greater than; < Not Imply, less than; = Equivalent; @ Not Equivalent;
% possibility, for one or some; # necessity, for all or every;
(p@p) ordinal 0, `F`; (%p>#p) ordinal 1; (%p<#p) ordinal 2; (p=p) ordinal 3, `T`;
(~(p<(p@p))&~(p>(%p>#p))) probability on interval]0,1[.

From: quantamagazine.org/entanglement-made-simple-20160428/ [Frank Wilczek]

Independent: $(\Phi_{\blacksquare} + \Phi_{\bullet})(\psi_{\blacksquare} + \psi_{\bullet}) = (\Phi_{\blacksquare}\,\psi_{\blacksquare} + \Phi_{\blacksquare}\,\psi_{\bullet} + \Phi_{\bullet}\,\psi_{\blacksquare} + \Phi_{\bullet}\,\psi_{\bullet})$ (1.1)

(p+q)&(r+s) ; `FFFF FTTT FTTT FTTT` (1.2)

Entangled: $(\Phi_{\blacksquare}\,\psi_{\blacksquare} \qquad + \Phi_{\bullet}\,\psi_{\bullet})$ (2.1)

(p&r)+(q&s) ; `FFFF FTFT FFTT FTTT` (2.2)

We apply the probability characteristic to respectively Eqs. 1.2 and 2.2 on interval]0,1[.

(p=((p+q)&(r+s)))>(~(p<(p@p))&~(p>(%p>#p))) ;
`FTFT FFTF FFTF FFTF` (1.3)

(p=((p&r)+(q&s)))>((p>(p@p))& (p<(%p>#p))) ;
`FTTF FFTF TFTF TFTF` (2.3)

Eqs. 1.2, 1.3, 2.2, and 2.3 as rendered are *not* tautologous. This refutes quantum entanglement.

Remark: What follows is that the plethora of experiments allegedly supporting entanglement are not based on tautologies of bivalent mathematical logic.

Refutation of the direct correspondence of quantum gates to reversible classical gates

Taken from:

Faugère, J-C., Horan, K., Kahrobaei, D., Kaplan, M, Kashefi, E., Perret, L. (2017). "Fast quantum algorithm for solving multivariate quadratic equations". arxiv.org/pdf/1712.07211.pdf

"2.3 Quantum Gates: The following gates are quantum gates of interest which operate on qubits, each directly corresponding to reversible classical gates. For qubits $|x\rangle$, $|y\rangle$, $|z\rangle$ the gates perform the following operations:

– CNOT (XOR, Feynman) $\quad \mathrm{CNOT}|x\rangle|y\rangle = |x\rangle|x + y\rangle$ [1.1]
– Toffoli (AND) $\quad T|x\rangle|y\rangle|z\rangle = |x\rangle|y\rangle|z + xy\rangle$ [2.1]
– X (NOT) $\quad X|x\rangle = |\bar{x}\rangle = |1 + x\rangle$ [3.1]
– n-qubit Toffoli (AND) $\quad T_n|x_1\rangle \ldots |x_n\rangle = |x_1\rangle \ldots |x_{n-1}\rangle|x_n + (x_1 \ldots x_{n-1})\rangle$ [4.1]
– Swap $\quad S|x\rangle|y\rangle = |y\rangle|x\rangle$" [5.1]

LET: p q r $|x\rangle$ $|y\rangle$ $|z\rangle$; also p q r s $|x_1\rangle$ $|x_2\rangle$ $|x_3\rangle$ $|x_4\rangle$; n 4;
~ Not; + Or; & And; = Equivalence; @ Not Equivalence, XOR

T is tautology as the designated *proof* value, with F as contradiction
The 16-valued truth tables are presented row-major and horizontally.

Using the Meth8/VŁ4 apparatus and method, we render Eqs. 1.1-5.1 as:

(p@q)=(p&(p+q)) ; `TTFF TTFF TTFF TTFF` (1.2)

((p&q)&r)=((p&q)&(r+(p&q))) ; `TTTF TTTT TTTF TTTT` (2.2)

~p=((p\p)+p) ; `TFTF TFTF TFTF TFTF` (3.2)

(((p&q)&r)&s)=((((p&p)&q)&r)&(s+(((p&p)&q)&r))) ;
`TTTT TTTF TTTT TTTT` (4.2)

(p&q)=(q&p) ; `TTTT TTTT TTTT TTTT` (5.2)

Eqs. 1.2-4.2 are *not* tautologous. This means those quantum gates do not directly correspond to reversible classical gates. (Eq. 5.2 is tautologous, although trivial.)

Remark: Eqs. 2.2 and 4.2 are nearly tautologous but not, due to the single `F` contradiction value.

What follows is that quantum gates *cannot* map to bivalent logic.

Remark: We obtained the above conclusion in unpublished work (2008) where: the qubit was proved to be a probabilistic vector (not bivalent); and the various quantum gates were mapped to non-bivalent truth tables to show where bivalent corrections *would be*. Hence, this paper demonstrates a shorter refutation of quantum gates as reversible bivalent operators.

Refutation of quantum gates: Hadamard; Pauli-X, -Y, -Z; Toffoli; and Fredkin

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET: p, q, r, s: probability, |0⟩, |1⟩, √2; ~ Not; & And; + Or; - Not Or;
> Imply, greater than; < Not Imply, less than; = Equivalent; @ Not Equivalent;
% possibility, for one or some; # necessity, for all or every;
(p@p) ordinal 0; (%p>#p) ordinal 1; (%p<#p) ordinal 2; (p=p) ordinal 3.

From: en.wikipedia.org/wiki/Quantum_logic_gate for the Hadamard gate (H):

Basis states (basis vectors) as qubits are defined as:

|0⟩ to (|0⟩+|1⟩)/√2 and (0.1.1)

q>((q+r)\s) ; `TTTT TFTF TTTT TFTF` (0.1.2)

|1⟩ to (|0⟩−|1⟩)/√2 (0.2.1)

r>((q-r)\s) ; `TTTT TTTT TTTT TTTT` (0.2.2)

"... which means that a measurement will have equal probabilities to become 1 or 0" (0.3.0)

We write Eq. 0.5.0 to mean: the measurement of the basis states imply a combined probability of]0,1[. (0.3.1)

(p>(p@p))&(p<(%p>#p)) ; `FFFF FFFF FFFF FFFF` (0.3.2)

We evaluate the following gates: Hadamard; Pauli-X, -Y, -Z; Toffoli; and Fredkin.

Hadamard (H) gate: |0⟩ to (|0⟩+|1⟩)/√2; and |1⟩ to (|0⟩−|1⟩)/√2. (1.1)

(p=((q>((q+r)\s))&(r>((q-r)\s))))>((p>(p@p))&(p<(%p>#p))) ;
`TFTF TFTF TFFT TFFT` (1.2)

Pauli-X gate: |0⟩ to |1⟩; and |1⟩ to |0⟩. (2.1)

(p=((q>r)&(r>q)))>((p>(p@p))&(p<(%p>#p))) ;
`TFFT FTTF TFFT FTTF` (2.2)

Pauli-Y gate: LET s=i; |0⟩ to i|1⟩; and |1⟩ to −i|0⟩. (3.1)

(p=((q>(s&r))&(r>(~s&q))))>((p>(p@p))&(p<(%p>#p))) ;
`TFFT FTFT TFFT FTFT` (3.2)

Pauli-Z gate: $|0\rangle$ to $|0\rangle$; and $|1\rangle$ to $-|1\rangle$. (4.1)

(p=((q>q)&(r>~r)))>((p>(p@p))&(p<(%p>#p))) ;
TFFT FTFT TFFT FTFT (4.2)

Toffoli (CCNOT): $|a, b, c\rangle$ to $|a, b, c\oplus ab\rangle$. (5.1)

((p=(((q=r)=(%p>#p))>(s=(s@(q&r)))))+(p=(q@r)))>((p>(p@p))&(p<(%p>#p))) ;
FFTF TFFN FFTF TFFN (5.2)

Fredkin (CSWAP):
C_{out}=C_{in}; O_1=(Not C And I_1) Or (C And I_2); O_2=(C And I_1) Or (Not C And I_2)
(6.1)

(p=((r=((~q&r)+(q&s)))&(s=((q&r)+(~q&s)))))>((p>(p@p))&(p<(%p>#p))) ;
TFTF TFFT TFTF TFFT (6.2)

As rendered, Eqs. 1.2, 2.2, 3.2, 4.2, 5.2, and 6.2 are *not* tautologous. This means the following quantum gates are refuted: Hadamard; Pauli-X, -Y, -Z; Toffoli; and Fredkin.

Refutation of gedanken experiment for quantum theory as not descriptive of itself, or not

Abstract: The gedanken experiment for quantum theory as not descriptive of itself is not tautologous and not contradictory. This means quantum theory can neither describe itself nor not describe itself. This result foils the attempt to resuscitate quantum theory.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: C, Q, Theorem_1, S; ~ Not; + Or; - Not Or; & And; > Imply;
% possibility, for one or some, ∃; # necessity, for every or all, ∀.
(p=p) T tautology; (p@p) **F** contradiction; (%p>#p) N truthity; (%p<#p) C falsity;

From: Frauchiger, D.; Renner, R. (2018). "Quantum theory cannot consistently describe the use of itself". Nature Communications. Vol 9. Article 3711.

Table 4 Interpretations of quantum theory

The proposed Gedanken experiment can be employed to study the various interpretations of quantum theory [QT]. Theorem 1 implies that each of them must violate at least one of the assumptions (Q), (S), and (S) [as indicated by an ×].

No.	**Type of QT**	**(Q)**	**(S)**	**(C)**
1.1	Copenhagen	✓	✓	×
2.1	HV theory applied to subsystems	✓	✓	×
3.1	HV theory applied to entire universe	×	✓	✓
4.1	Many worlds	?	×	?
5.1	Collapse theories	×	✓	✓
6.1	Consistent histories	✓	✓	×
7.1	QBism	✓	✓	×
8.1	Relational quantum mechanics	✓	✓	×
9.1	CSM approach	×	✓	✓
10.1	ETH approach	×	✓	✓

Remark: The meaning of the assumptions Q, S, C is irrelevant to this demonstration.

The value for unknown for "?" reads:
p as not (truthity or falsity), i.e. p as neither truthity nor falsity (0.1.1)

p=((%p>#p)-(%p<#p)) ; TFTF TFTF TFTF TFTF (0.1.2)

q as not (truthity or falsity), i.e. q as neither truthity nor falsity (0.2.1)

q=((%p>#p)-(%p<#p)) ; TTFF TTFF TTFF TTFF (0.2.2) s as not (truthity or falsity), i.e. s as neither truthity nor falsity. (0.3.1)

s=((%p>#p)-(%p<#p)) ; TTTT TTTT FFFF FFFF (0.3.2)

From the Table 4 above:

Q&S&~C: (1.1, 2.1, 6.1, 7.1, or 8.1)

(((q=(%p>#p))&(s=(%p>#p)))&~(p=(%p>#p)))=(p=p) ;
FC**FF** **F**C**FF** **FF**N**F** **FF**N**F** (1.2, 2.2, 6.2, 7.2, or 8.2)

~Q&S&C: (3.1, 5.1, 9.1, or 10.1)

((~(q=(%p>#p))&(s=(%p>#p)))&(p=(%p>#p)))=(p=p) ;
FFC**F** **FF**C**F** **F**N**FF** **F**N**FF** (3.2, 5.2, 9.2, or 10.2)

?Q&S&?C:
(((q=((%p>#p)-(%p<#p)))&(s=(%p>#p)))&(p=((%p>#p)-(%p<#p))))=(p=p) ;
C**FFF** C**FFF** N**FFF** N**FFF** (4.2)

We evaluate this story: Eqs. (1.1, 2.1, 6.1, 7.1, or 8.l) or (3.1, 5.1, 9.1, or 10.1) or (4.1) (11.1)

(((((q=(%p>#p))&(s=(%p>#p)))&~(p=(%p>#p)))+
((~(q=(%p>#p))&(s=(%p>#p)))&(p=(%p>#p))))+
(((q=((%p>#p)-(%p<#p)))&(s=(%p>#p)))&(p=((%p>#p)-(%p<#p)))))=(p=p) ;
CCC**F** CCC**F** NNN**F** NNN**F** (11.2)

We then evaluate this sentence:
Theorem_1 implies Eqs. (1.1, 2.1, 6.1, 7.1, or 8.l) or (3.1, 5.1, 9.1, or 10.1) or (4.1) (12.1)

r > (((((q=(%p>#p))&(s=(%p>#p)))&~(p=(%p>#p)))+
((~(q=(%p>#p))&(s=(%p>#p)))&(p=(%p>#p))))+
(((q=((%p>#p)-(%p<#p)))&(s=(%p>#p)))&(p=((%p>#p)-(%p<#p))))) ;
TTTT CCC**F** TTTT NNN**F** (12.2)

Eq. 12.2 as rendered is *not* tautologous, meaning the gedanken conjecture that QT cannot describe itself is refuted. This is not to mean that QT can describe itself because Eq. 12.2 is not a contradiction. Hence QT cannot be resuscitated.

Refutation of quantum logic as tautologous

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

Section 1.

LET bk_ "| 〉" bra-ket;
p, q, r, s: |0〉 bk_0, |1〉 bk_1, 2^0.5 (√2), |+〉 bk_+; ~s | -〉 bk_- ;
~ Not; & And; + Or; - Not Or; > Imply.

From: Wright, J. (2015). Lecture 2: Quantum math basics.
cs.cmu.edu/~odonnell/quantum15/lecture02.pdf

|+〉 = 1/√2 |0〉 + 1/√2 |1〉 is rewritten as √2 |+〉 = |0〉 + |1〉 (1.1)

(r&s)=(p+q) ; `TFFF TFFF TFFF FTTT` (1.2)

|-〉 = 1/√2 |0〉 - 1/√2 |1〉 is rewritten as √2 |-〉 = |0〉 - |1〉 (2.1)

(r&~s)=(p-q) ; `FTTT TFFF FTTT FTTT` (2.2)

We ask: Does the positive sign qubit (Eq. 1) imply the negative sign qubit (Eq. 2), as its conjugate, as a theorem? (3.1)

((r&s)=(p+q))>((r&~s)=(p-q)) ; `FTTT TTTT FTTT TTTT` (3.2)

In Section 1, Eq. 3.2 as rendered is *not* tautologous. This means the implication operator for quantum logic is refuted and by extension, so also quantum logic.

Consequently, we evaluate a less technical description of quantum logic aimed for a different audience.

Section 2.

From:
medium.com/@decodoku/quantum-computation-with-the-simplest-maths-possible-c23ff6563964

LET p, q, r, s, t: up (upness), down (downness), overlap, S (superposition);
% possibility, for one or some; # necessity, for all or every;
(%p>#p) truthity, ordinal 1; (%p<#p) falsity, not ordinal 1, such as ordinal 0.

Quantum Computation with the simplest maths possible
... [I]t would be useful to have some way of quantifying how similar two states are. We'll call this the *overlap*. The states **up** and **down** are completely different, so these should have an overlap of 0 (this is the actual number zero this time). For states that are 100% the same, let's say that the overlap is 1. For the two states **up** and **down**, there are only four possible overlaps to calculate and we know what they should be already.

overlap of **up** and **up** = 1 (1.1)

(r&(p&p))=(%p>#p) ; `CCCC CNCN CCCC CNCN` (1.2)

overlap of **up** and **down** = 0 (2.1)

(r&(p&q))=(%p<#p) ; `NNNN NNNC NNNN NNNC` (2.2)

overlap of **down** and **up** = 0 (3.1)

(r&(q&p))=(%p<#p) ; `NNNN NNNC NNNN NNNC` (3.2)

overlap of **down** and **down** = 1 (4.1)

(r&(q&q))=(%p>#p) ; `CCCC CCNN CCCC CCNN` (4.2)

Now we need to work out overlaps for superposition states. There are many different possible superpositions of up and down, which differ by how biased they are towards one or the other. This means we need two numbers, let's call them the upness and downness, that describe how much up and down there is in a superposition.

It would also be nice to have a shortened name for the superposition state that we are trying to describe. Let's just call it **S**. Now we need to write down the fact that **S** is a superposition of **up** and **down** and also what its upness and downness are, in a way that looks mathsy. How about

S = (upness of **S**) × **up** + (downness of **S**) × **down** (5.1)

s=(((p&s)&p)+((q&s)&q)) ; `TTTT TTTT FTTT FTTT` (5.2)

This nicely puts all the required information on one line. It even has has an + and some ×'s in to make it look like maths. These look suspiciously like addition and multiplication. But what does it even mean to multiply a state by a number? Or to add two states? These aren't the addition and multiplication that we are used to. It will turn out that they will follow similar rules to the normal ones, though. So that's why we use these symbols.

Now, what is the overlap between our superposition state S and the state up? We still haven't made up enough rules to actually calculate this, so we have to choose something. We have just introduced the notion of upness, which is how much **up** there is in **S**. This seems to be pretty much the same thing as the overlap between **S** and **up**, and it wouldn't contradict any of the rules we have already if they were the same thing. So let's just make up the rule that says they are the same thing.

overlap of **S** and **up** = upness of **S** (6.1)

(r&(s&p))=(p&s) ; `TTTT TTTT TFTF TTTT` (6.2)

There's a more complicated way we can write this, that can help us understand a little more about what is going on.

overlap of **S** and **up** = (upness of **S**) × (overlap of **up** and **up**)
+ (downness of **S**) × (overlap of **down** and **up**) (7.1)

(r&(s&p))=(((p&s)&(r&(p&p)))+((q&s)&(r&(q&p)))) ;
`TTTT TTTT TTTT TTTT` (7.2)

Here the overlap of **S** and **up** is a sum of two things. The first is the contribution from the **up** part of **S**

(upness of **S**) × (overlap of **up** and **up**) = (upness of **S**) × 1 = upness of **S** (8.1)

(((p&s)&(r&(p&p)))=((p&s)&(%p>#p)))=(p&s) ;
`FFFF FFFF FCFC FNFN` (8.2)

This tells us that the **up** part of **S** contributes the upness (obviously), and it contributes it fully because the overlap between the **up** part of **S** and **up** is 1.

The second contribution is from the down part of **S**

(downness of **S**) × (overlap of **down** and **up**) = (downness of **S**) × 0 = 0 (9.1)

(((q&s)&(r&(q&p)))=((q&s)&(%p<#p)))=(%p<#p) ;
`CCCC CCCC CCFF CCFT` (9.2)

This tells us that the down part of **S** would contribute the downness if it contributed anything. But it doesn't actually contribute it because the overlap between the **down** part of **S** and **up** is 0.

We get a similar equation for the overlap of **S** and **down**.

overlap of **S** and **down** = (upness of **S**) × (overlap of **up** and **down**)
+ (downness of **S**) × (overlap of **down** and **down**) (10.1)

(r&(s&q))=(((p&s)&(r&(p&q)))+((q&s)&(r&(q&q)))) ;
`TTTT TTTT TTTT TTTT` (10.2)

This time the overlaps of **up** and **down** ensure that the doweness contributes fully, and the upness doesn't contribute at all.

What about the overlap with something else? If we look at the overlap between **S** and **down**, and the overlap for **S** and **up**, the only difference is that one has **up** in and the other has **down**. So maybe we can just replace that with anything else too. Let's invent a new state and call it **T**, for no other reason but it coming after **S** in the alphabet. The overlap of **S** and **T** is then

overlap of **S** and **T** = (upness of **S**) × (overlap of **up** and **T**)
+ (downness of **S**) × (overlap of **down** and **T**) (11.1)

(r&(s&t))=(((p&s)&(r&(p&t)))+((q&s)&(r&(q&t)))) ;
`TTTT TTTT TTTT TTTT,`
`FTTT TTTT TTTT TTTT` (11.2)

In these equations we have × and +, multiplying and adding normal numbers. These are indeed the multiplication and addition that we are used to. From these equations you can maybe see why I used **×** and **+** before. Compare the equation for **S** with the equation for its overlap with **T**

S = (upness of **S**) **× up +** (downness of **S**) **× down** (12.1.1)

s=(((p&s)&p)+((q&s)&q)) ; `TTTT TTTT FTTT FTTT` (12.1.2)

overlap of **S** and **T** = (upness of **S**) × (overlap of **up** and **T**)
+ (downness of **S**) × (overlap of **down** and **T**) (12.2.1)

(r&(s&t))=(((p&s)&(r&(q&t)))+((q&s)&(r&(q&t)))) ;
`TTTT TTTT TTTT TTTT,`
`TTTT TTTT TTTT FTTT` (12.2.2)

These are pretty much the same. The only difference is that each state in the first one has been replaced by the overlap of that state and T in the second. This means that the second one just has normal numbers in. So the weird multiplication and addition in the first one become normal in the second. So, whatever **x** and **+** are, they must be some version of multiplication and addition that work with the states of qubits, and just become normal multiplication and addition once we just start calculating with numbers. We won't need to think much more about this, though.

Let's think more about the overlap between **S** and our new state **T**. Firstly, just like **S** we should be able to write **T** as

T = (upness of **T**) **× up +** (downness of **T**) **× down** (13.1)

t=(((p&t)&p)+((q&t)&q)) ;
`TTTT TTTT TTTT TTTT,`
`FTTT FTTT FTTT FTTT` (13.2)

Earlier we made a rule that the upness of a state is the same thing as its overlap with **up**. This rule lets us write the equation for the overlap of **S** and **T** in a simpler way.

overlap of **S** and **T** = (upness of **S**) × (upness of **T**) + (downness of **S**) × (downness of **T**) (14.1)

(r&(s&t))=(((p&s)&(p&t))+((q&s)&(q&t))) ;
`TTTT TTTT TTTT TTTT,`
`TTTT TTTT TFFF FFFT` (14.2)

This lets us work out the overlap of **S** and **T** using their upness and downess, which are just numbers that we know.

Now let's ask a question for which we already know the answer. What is the overlap between **S** and itself? Using the maths above

overlap of **S** and **S** = (upness of **S**) × (upness of **S**) + (downness of **S**) × (downness of **S**)
= (upness of **S**)^2 + (downness of **S**)^2 (15.1)

((r&(s&s))=(((p&s)&(p&s))+((q&s)&(q&s))))=(((p&s)&(p&s))&((q&s)&(q&s))) ;

FFFF FFFF FTTF TFFT (15.2)

Since we are looking at the overlap between two states that are exactly the same, the answer should come out to be 1. So now we know something about the relationship between the upness and downness for any quantum superposition

$$\text{upness}^2 + \text{downness}^2 = 1 \quad (16.1)$$

((p&p)+(q&q))=(%p>#p) ; CNNN CNNN CNNN CNNN (16.2)

This makes a lot of sense. The more a state is biased towards up, the less it must be biased towards down. For example, a state with an upness of 1 (and so with an upness^2 of 1 too) is completely up, and so has no downness. The first concrete fact that our quantum maths has told us isn't weird at all. See, quantum mechanics isn't so strange.

Well, maybe it is a little bit strange. Note that we don't just add upness and downness here. Instead we square them first. One thing we know from school is that negative numbers square to the same value as positive ones. $(-1)^2 = 1$ just like $1^2 = 1$, for example. So maybe this equation is telling us that its okay for the upness and downness to be negative, even though this would be a bit weird, because these numbers only need to be sensible after we've squared them.

In Section 2, Eqs. 7.2 and 10.2 (2 of 16) as rendered are tautologous with the others not. This confirms the conclusion from Section 1 that quantum logic is *not* tautologous.

Refutation of hiding classical information by using quantum correlation of a two-party state

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET: p, q, r, s: probability, |0⟩, |1⟩, √2; ~ Not; & And; + Or; - Not Or;
> Imply, greater than; < Not Imply, less than; = Equivalent; @ Not Equivalent;
% possibility, for one or some; # necessity, for all or every;
(p@p) ordinal 0; (%p>#p) ordinal 1; (%p<#p) ordinal 2; (p=p) ordinal 3.

Basis states (basis vectors) as qubits are defined as:

|0⟩ to (|0⟩+|1⟩)/√2 and (0.1.1)
q>((q+r)\s) ; TTTT TFTF TTTT TFTF (0.1.2)

|1⟩ to (|0⟩−|1⟩)/√2 (0.2.1)
r>((q-r)\s) ; TTTT TTTT TTTT TTTT (0.2.2)

Qudits are defined as:

|00⟩ to [(|0⟩+|1⟩)/√2]*[(|0⟩+|1⟩)/√2] (0.3.1)
[q>((q+r)\s)]&[q>((q+r)\s)] = ((q+r)+((%p<#p)&(q&r)))\(%p<#p) ;
TFFT FTFT TFTF FTFT (0.3.2)

|11⟩ to [(|0⟩- |1⟩)/√2]*[(|0⟩- |1⟩)/√2] (0.4.1)
[r>((q- r)\s)]&[r>((q- r)\s)] = ((q+r)- ((%p<#p)&(q&r)))\(%p<#p) ;
NNTT TTTT NNTT TTTT (0.4.2)

From: export.arxiv.org/pdf/1608.01695

"[C]consider an example of hiding classical information by using quantum correlation of a two-party state. (1.0)

Suppose, we encode a single bit of classical information in two orthogonal entangled states where the encoding map is given by (2.0)

|0⟩ → (1/√2)(|00⟩+|11⟩) and (2.1.1)

q>(((((q+r)+((%p<#p)&(q&r)))\(%p<#p))+(((q+r)- ((%p<#p)&(q&r)))>(%p<#p)))\s) ;
FFTF FFTF FFFT FFFT (2.1.2)

|1⟩ → (1/√2)(|00⟩- |11⟩). (2.2.1)

r>(((((q+r)+((%p<#p)&(q&r)))\(%p<#p))-(((q+r)-((%p<#p)&(q&r)))>(%p<#p)))\s) ;
TTTT TTTT TTTT TTTT (2.2.2)

The encoding map in Eq. 2.0 is supposed to have a measurement that will have equal probabilities to become 1 or 0. (3.0)

We write Eq. 3.0 to mean: the measurement of the basis states of Eqs. 2.1.1 and 2.2.1 imply a combined probability of]0,1[. (3.1)

(p=((q>(((((q+r)+((%p<#p)&(q&r)))\(%p<#p))+(((q+r)-((%p<#p)&(q&r)))
>(%p<#p)))\s))&
(r>(((((q+r)+((%p<#p)&(q&r)))\(%p<#p))-(((q+r)-((%p<#p)&(q&r)))
>(%p<#p)))\s)))) > ((p>(p@p))&(p<(%p>#p))) ;

`TFTF TFTF TFFT TFFT` (3.2)

Remark: Eq. 3.2 as rendered is *not* tautologous. This refutes encoding classical binary information into quantum states.

[Looking] at states of both the subsystems, it has no information about the classical bit. (4.0)

Eq. 3.2 of the encoded subsystems contains information about the classical bit, refuting Eq. 4.0

Here, ... although classical information is actually hidden from both the subsystems, it is spread over quantum correlation of the encoded states." (5.0)

Eq. 3.2 makes clear that no classical information is actually hidden from the subsystems and does not spread over quantum correlation of the encoded states.

"[To] deal with the encoding of quantum information in an arbitrary *composite* quantum state... ask the question: *can quantum information be hidden from both the subsystems and remain only in the correlation?* (6.0)

If so, then somehow quantum information gets spread over the 'spooky' correlation and remains invisible to both the subsystems that are possessed by the local observers... . (7.0)

[T]his spreading of quantum information over quantum correlations as 'masking' quantum information. (8.0)

[The authors] prove that such masking is not possible for arbitrary quantum states, (9.0)
although [they showed] that it is possible for classical information to be masked." (4.0)

Eqs. 6.0, 7.0, 8.0, and 9.0 do not follow after the refutation of Eq. 5.0.

We conclude that quantum information cannot mask classical bivalent information. This further finds moot the possibility of masking quantum information.

Refutation of axiom of probability for quantum theory

Abstract: An axiom of probability theory is refuted and hence is unusable for quantum theory.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: A, a, B, b ;
~ Not; + Or; - Not Or; & And, ∩; > Imply;
% possibility, for any one or some, ∃; # necessity, for every or all, ∀.
(s=s) T tautology; (s@s) **F** contradiction;
(%s>#s) 1, N truthity; (%s<#s) 0, C falsity;

From: Nagata, K.; Nakamura, T. (2014). Reply to "Comments on 'There is No Axiomatic System for the Quantum Theory'". vixra.org/pdf/1309.0083v2.pdf

From axioms of probability theory, we have: P(A = a ∩ B = b) = P(B = b ∩ A = a). (4.1)

We ignore the symbol P for probability here.

((p=(q&r))=s)=((r=(s&p))=q) ; T**FF**T **F**T**F**T **FF**TT TTTT (4.2)

Eqs. 4.1 as rendered are *not* tautologous. This refutes that axiom from probability theory as it applies to quantum theory.

Refutation of the quantum probability rule

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET: p, q, r, s: E, f, m, n; & And; + Or; > Imply; = Equivalent, is;

From: Caves, C.M.; Fuchs, C.A.; Manne, K.; Renesl, J.M. (2003). "Gleason-type derivations of the quantum probability rule for generalized measurements". arxiv.org/pdf/quant-ph/0306179.pdf

> III. The quantum probability rule, A. Linearity with respect to the non negative rationals: Every frame function is trivially additive, for consider two POVMs, {E1, E2, E3} and {E1 + E2, E3}. Clearly both are POVMs if either is, and the frame-function requirement immediately yields f(E1) + f(E2) = f(E1 + E2) . (3.2)
>
> From this we obtain a homogeneity property for multiplication by rational numbers. We can break an effect nE into m pieces to form the effect (n/m)E. Using the additivity property twice, we obtain
>
> mf(n/m)E = f(nE) = nf(E) ⇒ f(n/m)E = (n/m)f(E) . (3.3)
>
> The function f is thus established to be linear in the non negative rationals. We can extend to full linearity by proving continuity.

We evaluate the antecedent of Eq. 3.3 and rewrite it as

mf(n/m)E = (f(nE)=nf(E)) (3.3.1)
((r&q)&((s\r)&p)) = ((q&(s&p))=((s&q)&p)) ;
FFFF FFFT FFFF FFFF (3.3.2)

Eq. 3.3.2 as rendered diverges from contrarity by one value T.

We weaken the argument of Eq. 3.3.1 by removing either f(nE) or nf(E) since they are equal.

mf(n/m)E = (nf(E)) (3.4.1)
((r&q)&((s\r)&p)) = ((s&q)&p) ; TTTT TTTF TTTF TTTF (3.4.2)

Eq. 3.4.2 is *not* tautologous, diverging by three values of F.

From Eqs. 3.3.2 and 3.4.2, this means subsequent assertions do not follow. Hence the function f is not established to be linear, and continuity (or homogeneity) of f cannot be proved.

> **Remark:** In 1935 von Neumann stopped "believing" in Hilbert space. Rosinger, E.E. (2004). What is wrong with von Neumann's theorem on "no hidden variables". arxiv.org/abs/quant-ph/0408191, quoting: Birkhoff, G.D. (1961). Proceedings of Symposia in Pure Mathematics. 2:158, American Mathematical Society, with the respective letter dated 13 November 1935.

Refutation of operator for quantum simulation of Hamiltonian spectra

Taken from:

> Santagati, R., et al. (2018)."Witnessing eigenstates for quantum simulation of Hamiltonian spectra", Sci. Adv. 2018;4:eaap9646. advances.sciencemag.org/content/4/1/eaap9646.full

We assume the apparatus and method of Meth8/VŁ4 to evaluate this quantum operator, excluding the scalar of (1/(2^0.5)), for:

$$|0\rangle_C \otimes \hat{I}|\Psi\rangle_T + |1\rangle_C \otimes \hat{U}|\Psi\rangle_T \quad (3.1)$$

LET: pqrstuv |1>, |0>, uc_C, uc_I-circumflex, uc_T, uc_U-circumflex, uc_Psi;
& And; @ Not equivalent, XOR; + Or

The designated proof value is T; F is contradiction.
Repeating fragments of the 128-rows of 16-valued truth tables are row-major, as horizontally.

((q&r)@(s&(v&t)))+((p&r)@(u&(v&t))) ;

```
FFFF FTTT TTTT TTFT,
FFFF FTTT FFFF FTTT,
TTTT TFTT TTTT TTTF;
```
(3.2)

Eq. 3.2 as rendered is *not* tautologous. This means the quantum operator is not bivalent, but rather an operator for a probabilistic vector space.

Refutation of the spin-statistics theorem in QFT

Abstract: We evaluate the spin-statistics theorem assuming two variables are not equivalent for the equations of commute and anti-commute fields. The equations are logically equivalent meaning the status of the two variables is irrelevant and unnecessary. Therefore the theorem is refuted, casting doubt on the logical foundations of QFT.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. Reproducible transcripts for results are available. (See ersatz-systems.com.)

LET p, q, r, s: ϕ, ψ, x, y;
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⊢, ⊨; < Not Imply, less than, ∈, ≺, ⊂;
= Equivalent, ≡, :=, ⇔, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z<#z) C non-contingency, ∇, ordinal 2; (%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).

From: en.wikipedia.org/wiki/Spin–statistics_theorem

Let us assume that

x ≠ y (0.1.1)

r@s ; **FFFF** TTTT TTTT **FFFF** (0.1.2)

and the two operators take place at the same time ...

Remark 0: We use Eq. 0.1.1 as the antecedent implying 1.1-2.1 for 1.2-2.2.

If the fields **commute**, meaning that the following holds:

ϕ (x) ϕ (y) = ϕ (y) ϕ (x)

then only the symmetric part of ψ contributes, so that

ψ (x , y) = ψ (y , x) , (1.1)

(r@s)>((((p&r)&(p&s))=((p&s)&(p&r)))>((q&(r&s))=(q&(s&r)))) ;
TTTT TTTT TTTT TTTT (1.2)

and the field will create bosonic particles.

Remark 1: Because Eq. 1.1 contains an antecedent and consequent that are respective equivalents, the tautologous equation for commute fields is expected and trivial.
If the antecedent in Eq. 1 is r<s, r>s, or r=s, the table results are the same as for 1.2.

On the other hand, if the fields **anti-commute**, meaning that ϕ has the property that

$$\phi (x) \phi (y) = - \phi (y) \phi (x) ,$$

then only the antisymmetric part of ψ contributes, so that

$$\psi (x , y) = - \psi (y , x) , \quad (2.1)$$

(r@s)>((((p&r)&(p&s))=~((p&s)&(p&r)))>((q&(r&s))=~(q&(s&r)))) ;
TTTT TTTT TTTT TTTT (2.2)

and the particles will be fermionic.

Remark 2: Because Eq. 2.1 contains an antecedent and consequent that are respective equivalents, the tautologous equation for anti-commute fields is expected and trivial.
If the antecedent in Eq. 2 is r<s, r>s, or r=s, the table results are the same as for 2.2.

Remark 3: Because of Remarks 1 and 2, the antecedent in Eq. 0.1 becomes irrelevant to the truth table result in 1.2 and 2.2.
For example, we rewrite Eqs. 1.2 and 2.2 without the (r@s) as:

(((p&r)&(p&s))=((p&s)&(p&r)))>((q&(r&s))=(q&(s&r))) ;
TTTT TTTT TTTT TTTT (3.1.2)

(((p&r)&(p&s))=~((p&s)&(p&r)))>((q&(r&s))=~(q&(s&r))) ;
TTTT TTTT TTTT TTTT (3.2.2)

What follows is that the relation of x and y in Eq. 0.1.1 is irrelevant.

What further follows is that the subsequent matrix machinations in the cited text are specious. In fact, the text admits this in so many words with:

"Naively, neither [Eqs. 1.1 or 2.1] has anything to do with the spin, which determines the rotation properties of the particles, not the exchange properties."

The results of Eqs. 3.1.2 and 3.2.2 refute the spin-statistics theorem. This implies that quantum field theory (QFT) does not have a stable foundation in bivalent, mathematical logic.

Refutation of superposition in QFT as a red herring of Schrödinger's cat

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

> LET p, q, r, s: Probability, cat, radioactive decay, death;
> ~ Not; + Or; & And; > Imply, greater than.

We paraphrase Schrödinger's cat thought experiment as follows.

> A box hides from view a cat with a radioactive source where the probability of decay or *no* decay is equal. In the course of one hour, the probability of the state of decay causes the demise of the cat, or the probability of the state of *no* decay causes *not* the demise of the cat. (1.1)

> ((p&r)>(q&s))+((p&~r)&(q&~s)) ; TTTT T**F**T**F** TTTT T**F**TT (1.2)

> Superposition is defined as both states of Eq. 1.1 rather than either state of Eq. 1.1.(2.1)

> ((p&r)>(q&s))&((p&~r)&(q&~s)) ; **FFF**T **FFFF FFFF FFFF** (2.2)

> Quantum theory asserts that either state implies both states concurrently as superposition. (3.1)

> (((p&r)>(q&s))+((p&~r)&(q&~s)))>(((p&r)>(q&s))&((p&~r)&(q&~s))) ;
> **FFF**T **F**T**F**T **FFFF** T**F**TT (3.2)

As rendered, Eqs. 1.2, 2.2, and 3.2 are *not* tautologous. Therefore , Schrödinger's cat thought experiment is refuted. Furthermore, the definition of superposition in Eq. 2.2 is very nearly a contradiction, excepting one value for Tautology, and serves as a red herring in the schema.

> **Remark:** The state inside the box during the hour at any moment is not known exactly, but that does not mean both states are concurrent at any moment as superposition. However, before the end of the hour to open the box, interrupt the experiment, and force an inspection returns either state in Eq. 1.2, and hence falsifies Eq. 2.2 as a concurrent state of affairs in Eq. 3.2.

Superposition in QFT refutes Schrödinger's cat experiment

Abstract: Quantum logic (QL) maps Schrödinger's cat experiment in words the same as does bivalent logic, with the expression as not tautologous (FFFF FTFF FFFT FFFF) and nearly contradictory. QL assumes such variables are natural numbers. To support the aim of justification of superposition, QL also injects a probability of equal to or greater than one, under the guise of the inequality of equal to or greater than zero. What follows is that any "principle of uncertainty" is irrelevant because certainty or uncertainty is bivalently mappable as the status of known or unknown, as in the cat experiment.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: Probability, A extant status (dead/alive), B box apparatus, C known status;
~ Not; + Or; & And; > Imply, greater than;
= Equivalent; @ Not Equivalent;
(p=p) Tautology; (p@p) **F** as contradiction, ordinal zero; ~(p<q) (p≥q).

Bivalent logic maps Schrödinger's cat experiment in words,

The probability of the inviolated box apparatus (sealed to begin the experiment), not the extant status, and the unknown status,
Or
The probability of the violated box apparatus (unsealed to end the experiment), the extant status, and the known status. (1.0)

(p&((~q&r)&~s))+(p&((q&~r)&s)) ;
FFFF FT**FF FFF**T **FFFF** (1.2)

Quantum logic (QL) maps the cat experiment in words,

The probability of the non extant status, inviolated box, and unknown status
Or
The probability of the extant status, violated box, and known status (2.0)

P(~A & B & ~C) + P(A & ~B & C) (2.1)
(p&((~q&r)&~s))+(p&((q&~r)&s)) ;
FFFF FT**FF FFF**T **FFFF** (2.2)

are equal to or greater than zero. (3.0)

P(~A & B & ~C) + P(A & ~B & C) ≥ 0 (3.1)
(~((p&((~q&r)&~s))+(p&((q&~r)&s)))<(p@p))=(p=p) ;
TTTT T**F**TT TTT**F** TTTT (3.2)

Remarks 3.: Eqs. 1.2 and 2.2 as rendered are identical. Eq. 3.0 assumes that respectively A, B, C are ≥ 0. This assumption forms the basis of QL and ultimately is the cause of Eq. 3.2 being *not* tautologous.

If Eq. 3.1 is rendered as P(~A & B & ~C) + P(A & ~B & C) ≤ 1, (3.1.1)
then the expression is tautologous:

(~((p&((~q&r)&~s))+(p&((q&~r)&s)))>(p=p))=(p=p) ;

TTTT TTTT TTTT TTTT (3.1.2)

Furthermore, Eq. 3.1 makes sense because Probability is ≤ 1.

An equivalent quantum logic rendition of Eq. 2.0 maps in words,

The probability of extant status and violated box
Or
The probability of inviolated box and unknown status
Is equivalent to
The probability of extant status and unknown status (4.0)

P(A & ~B) + P(B & ~C) = P(A & ~C) (4.1)

((p&(q&~r))+(p&(r&~s)))=(p&(q&~s)) ;

TTTT T**F**TT TTT**F** TTTT (4.2)

Eq. 4.1 is rewritten as this inequality with the injection of zero.

P(A & ~B) + P(B & ~C) - P(A & ~C) ≥ 0 (5.1)

(~(((p&(q&~r))+(p&(r&~s)))-(p&(q&~s)))<(p@p))=(p=p) ;

FFFT **F**T**F**T **FFF**T **FFFF** (5.2)

Remarks 4.: Eq. 5.2 is *not* tautologous, and suffers from the same defects in Rem. 3.

If Eq. 5.1 is rendered as P(A & ~B) + P(B & ~C) - P(A & ~C) ≤ 1 (5.1)
then the expression is tautologous:

(~(((p&(q&~r))+(p&(r&~s)))-(p&(q&~s)))>(p=p))=(p=p) ;

TTTT TTTT TTTT TTTT (3.1.2)

Eqs. 1.2 and 2.2 show that bivalent logic and quantum logic map Schrödinger's cat experiment as the same. However, when quantum logic injects a probability greater than one to support superposition, the Eqs. are *not* tautologous, and hence QL refutes itself.

Remark 5.: What follows is that any "principle of uncertainty" is irrelevant because certainty or uncertainty is bivalently mappable as the status of known or unknown, as in the cat experiment.

Refutation of Löwner (Loewner) order and quantum temporal logic

Abstract: We evaluate the Löwner order ⊑ for positive definite as the basis for quantum temporal logic (QTL). The operator is *not* tautologous. We also evaluate the semantics for QTL in three operators, also *not* tautologous. These form *non* tautologous fragments for both in the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Yu, N. (2019). Quantum temporal logic: from Birkhoff and von Neumann to Pnueli. arxiv.org/pdf/1908.00158.pdf

2. Preliminaries
A Hermitian operator A is *positive semidefinite* (resp., *positive definite*) if for all vectors |ψi ∈ H, hψ| A|ψi ≥ 0 (resp., > 0). This gives rise to the *Löwner order* ⊑ among operators:

A ⊑ B if B−A is positive semidefinite, A ⊏ B if B−A is positive definite. (2.1.1.1, 2.1.2.1)

((B-A)>(C@C))>(A<B) ; TTTT NNNN CCCC **FFFF** (2.1.2.2)

Remark 2.1.2.2: Eq. 2.1.2.2 as rendered is *not* tautologous, hence refuting the Löwner order for positive definite.

4.2. Semantics for QTL [quantum temporal logic]

[2] For p, q ∈ AP, p∨q is the union of subspaces p and q, p∨q is not always in AP. (4.2.5.1.2)

LET p, q, r, s: p, q, A, P

((p&q)<(r&s))>~(#((p+q)<(r&s))=(s=s)) ;
TTTC TTTC TTTC TTTT (4.2.5.1.2)

The additional logical operators are defined as follows:

φ → ψ ≡ L(φ) ⊂ L(ψ) (4.2.10.1)

LET p, q, r, s: φ, L, r, ψ.

(p>s)=((q&p)<(q&s)) ; **F**T**FF** **F**T**FF** **FFFF** **FFFF** (4.2.10.2)

$$\varphi \leftrightarrow \psi \equiv (\psi \rightarrow \varphi) \wedge (\varphi \rightarrow \varphi) \quad (4.2.11.1)$$

(p=s)=((s>p)&(p>p)) ; T**F**T**F** T**F**T**F** TTTT TTTT (4.2.11.2)

Remark 4.2: Eqs. 4.2.5.1.2, 4.2.10.2, and 4.2.11.2 are *no*t tautologous. This refutes the semantics for QTL, and hence QTL.

Refutation of the quantum qutrit ternary probability

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET: p, q, r, s: probability, Alice [Bob is *not* Alice]; outcomes (0,1,2), measures (0,1);
~ Not; & And; + Or; > Imply, greater than; < Not Imply, less than; = Equivalent;
% possibility, one or some; # necessity, all; (p@p) 0, zero; (%p>#p) 1, one; (%p<#p) 2, two.

From: Hu, X-M.; Liu, B-H.; Guo, Y.; Xiang, G.Y.; Huang, Y-F.; Li, C-F.; Guo, G-C.; Kleinmann, M.; Cabello, A. (2018). Observation of stronger-than-binary correlations with entangled photonic qutrits. arxiv.org/pdf/1712.06557.pdf .

Correlations between the outcomes of measurements performed by two parties, called Alice and Bob, are described by joint probabilities *P(a,b|x,y)*, where *x* and *y* are Alice's and Bob's measurement settings, respectively, and *a and b* are Alice's and Bob's measurement outcomes, respectively. The experiment is a bipartite Bell-type experiment in which Alice randomly chooses between two different measurements, *x* = 0,1, each of them with three possible outcomes, *a* = 0,1,2, and Bob randomly chooses between two different measurements, *y* = 0,1, each of them with three possible outcomes, *b* = 0,1,2. (1.1)

r=(((%p>#p)+(%p<#p))+(p@p)) ;
s=((%p>#p)+(p@p)) ;
((r=(((%p>#p)+(%p<#p))+(p@p)))&(s=((%p>#p)+(p@p))))
> (p&((q&(r&s))+(~q&(r&s)))) ; TTTT NNNN TTTT CTCT (1.2)

In fact, the result of the experiment demonstrates that none of the four measurements (Alice's or Bob's) can be binary. (2.1)
((r=(((%p>#p)+(%p<#p))+(p@p)))&(s=((%p>#p)+(p@p))))
> ~(%r=((%p>#p)+(p@p))) ; TTTT TTTT TTTT CCCC (2.2)

Eqs. 1.2 and 2.2 as rendered are *not* tautologous.

To weaken the argument, we test the sum of the propositions of the outcomes to be greater than one as tautologous, because after all that is the state supposedly observed by experiment.

If outcomes are 1,2,0, then the sum of probabilities are greater than 1. (3.1)
(r=(((%p>#p)+(%p<#p))+(p@p)))>(%p>#p) ;TTTT NNNN TTTT NNNN (3.2)

Remarks: The cited paper was paid for by the governments of China, Hungry, Spain, Sweden. The footnoted data set link at personal.us.es/adan/binary.htm is a table of 16 columns and 4500 rows. We could not replicate the χ^2-values in Table II. Consequently, we applied the N-by-M contingency test (superset of Chi-squared test with expected values derived from observed values) on the first 1000 rows. We found Fisher P <= 01, χ^2 = 0.0000001; df = 14,985. In other words, the data set as published is random data. We conclude this impugns the data collection, data set, results, and entire experiment.

Refutation of the three lights experiment for qutrits

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET p, q, r, s: blue light, green green, red light, measurement;
~ Not; & And; + Or; > Imply, greater than; = Equivalent;
(s@s) off; (s=s) on.

From: arxiv.org/abs/1712.06557

> "Quantum mechanics is so successful that it is difficult to imagine how to go beyond the present theory without contradicting existing experiments. However, going beyond our present understanding of quantum mechanics can enable us to solve long-standing problems like the formulation of quantum gravity. Some of the most puzzling questions in quantum theory are connected to the measurement process." (0.0)

> "FIG. 1 Two possible explanations for the measurement process.
> Suppose a measurement with three possible outcomes represented by red, green, and blue lights.
> The process that generates the final outcome (represented by the blue light flashing) can be either
> (a) a sequence of two steps:
> (1) The red outcome is precluded by a classical mechanism
> (e.g., the initial position of the measured system).
> (2) A general two-outcome measurement selects between the two remaining outcomes.
> Or (b), the measurement is genuinely ternary in the sense that it cannot be explained as in (a)." (0.1)

We rewrite Eq. 0.1 to mean:

If (blue, red, and green lights imply measurement) then measurement implies both (blue light implies flashing and red and green lights imply not flashing). (1.1)

(((p&q)&r)>s)>(s>((p>((s=s)+(s&s)))&((q&r)>~((s=s)+(s&s))))) ;

TTTT TTTT TTTT TTFF (1.2)

Eq. 1.2 as rendered is *not* tautologous. This means the experiment to measure outcomes for three lights with blue flashing is ill-formed. Furthermore, the declaration of Eq. 0.0 is also falsified, namely, that quantum mechanics is *not* so successful in the imagination.

Remark: What follows by extension is that such papers published by arxiv.org are suspicious, and that its owner Cornell University is an organ of academic cronyism to promote misfeasant mainstream physics.

Many questions and many answers

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET: ~ Not; & And; > Imply, greater than; = Equivalent; @ Not Equivalent;
necessity, for all; % possibility, for one or some;
p answer; q question.

For many questions, there is at least one answer. (1.1)

#q>%p ; TTCT TTCT TTCT TTCT (1.2)

[Supposedly reciprocal] For at least one question, there are many answers. (2.1)

%q>#p ; NNFN NNFN NNFN NNFN (2.2)

Eq. 1.1 or Eq. 2.1 (3.1)

(#q>%p)+(%q>#p) ; TTCT TTCT TTCT TTCT (3.2)

If Eq. 2.1 is the reciprocal of Eq. 1.1, then Eq. 1.1 or Eq 2.1 should be tautologous, but the result is not. Eq. 2.1 is *not* the reciprocal of Eq. 1.1

Eq. 1.1 implies Eq. 2.1 (4.1)

(#q>%p)>(%q>#p) ; NNNN NNNN NNNN NNNN (4.2)

If Eq. 1.1 implies Eq. 2.2 then the result should be tautologous (T), but it is not. The result is a truthity (N for non-contingency).

Eq. 2.1 implies Eq. 1.1 (5.1)

(%q>#p)>(#q>%p) ; TTTT TTTT TTTT TTTT (5.2)

If Eq. 2.2 implies Eq. 1.1 then the result should be tautologous, as it is.

We conclude that:

If for some questions there are many answers, then for many questions there are some answers, this is a theorem.

If for many questions there are some answers, then for some questions there are many answers, this is not a theorem but a truthity.

Refutation of Ramsey's theorem via Pythagorean triple of integers

Abstract: We evaluate the lemma proffered to prove Ramsey's theorem, and the strengthened lemma in a note. Neither are tautologous. We evaluate the proof of Pythagorean triple of integers as colored. It also is not tautologous. In fact, the coloring or non-coloring produces a logically equivalent result, meaning the Ramsey theorem is neither a tautology nor a contradiction. This implies "the inductive hypothesis" is suspicious. What follows is that the HOL proof assistant for the Ramsey theorem is an historical *enormity* in its 200 TB computer program with a certified prize result. Therefore the Ramsey theorem and HOL proof assistants are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Ramsey's_theorem

By the inductive hypothesis $R(r-1, s)$ and $R(r, s-1)$ exist. (0.1.1), (0.2.1)

LET p, q, r, s: p, R, r, s

%(q&((r-(%p>#p))&s)) = (p=p) ; CCCC CCCC CCCC CCCC (0.1.2)

%(q&(r&(s-(%p>#p)))) = (p=p) ; CCCC CCCC CCCC CCCC (0.2.2)

Remark 0.: Eqs. 0.1.2 and 0.2.2 are logically *equivalent*. This questions the efficacy of reliance on the inductive hypothesis.

Lemma 1. $R(r, s) \leq R(r-1, s) + R(r, s-1)$: (L.1.1)

~((%(q&((r-(%p>#p))&s))+%(q&(r&(s-(%p>#p)))))<(q&(r&s))) = (p=p) ;
NNNN NNNN NNNN NNTT (L.1.2)

Note. In the 2-colour case, if $R(r-1, s)$ and $R(r, s-1)$ are both even, the induction inequality can be strengthened to: $R(r, s) \leq R(r-1, s) + R(r, s-1) - 1$. (N.1.1)

~((%(q&((r-(%p>#p))&s))+(%(q&(r&(s-(%p>#p))))-(%p>#p)))<(q&(r&s)))
= (p=p) ; NNNN NNNN NNNN NNTT (N.1.2)

Remark N.1.: Eqs. L.1.2 and N.1.2 are logically equivalent. This means "the induction inequality" is *not* strengthened as claimed.

Lemma 1 is *not* tautologous. The textual proof of Lemma 1 uses the case of two colors. Instead of stepping through the proof in that text, we evaluate Ramsey's two color theorem in equation 2 below.

We evaluate Ramsey's two color theorem framed as the Boolean Pythagorean triples problem from en.wikipedia.org/wiki/Boolean_Pythagorean_triples_problem .

The Boolean Pythagorean triples problem is a problem relating to Pythagorean triples which was solved using a computer-assisted proof in May 2016.

This problem is from Ramsey theory and asks if it is possible to color each of the positive integers either red or blue, so that no Pythagorean triple of integers *a*, *b*, *c*, satisfying $a^2 + b^2 = c^2$ [(2.1)] are all the same color. For example, in the Pythagorean triple 3, 4 and 5 ($3^2 + 4^2 = 5^2$), if 3 and 4 are colored red, then 5 must be colored blue.

Remark 2.1: We simplify the two colored theorem into four-variables as follows, assigning Blue as the negation of Red.

For a, b, c as 1, 2, 3:

LET p, q, r, s: $a^2=1^2=1$, $b^2=2^2=4$, $c^2=3^2=9$, Red.

If (1 + 4) ≠ 9, then if (1 and 4 are Red), then 9 is Not Red. (2.1.1)

((p+q)@r)>(((p&q)=s)>(r=~s)) ;
T**FF**T TTTT TTTT TTTT (2.1.2)

If (1 + 9) ≠ 4, then if (1 and 9 are Red), then 4 is Not Red. (2.2.1)

((p+r)@q)>(((p&r)=s)>(q=~s)) ;
T**F**TT **F**TTT TTTT TTTT (2.2.2)

If (4 + 9) ≠ 1, then if (4 and 9 are Red), then 1 is Not Red. (2.3.1)

((q+r)@p)>(((q&r)=s)>(p=~s)) ;
TT**F**T **F**TTT TTTT TTTT (2.3.2)

Eqs. 2.1.2-..3.2 are *no*t tautologous. This means the answer to "if it is possible to color each of the positive integers either red or not red, so that no Pythagorean triple of integers *a*, *b*, *c*, satisfying $a^2 + b^2 = c^2$ are all the same color" is no.

Remark 2.4: The answer to the contra-question of "if it is possible to color each of the positive integers in the same color, so that no Pythagorean triple of integers *a*, *b*, *c*, satisfying $a^2 + b^2 = c^2$ are all not the same color” is also no. For example from Eq. 2.3.2:

((q+r)@p)>(((q&r)=s)>(p= s)) ;
TT**F**T **F**TTT TTTT TTTT (2.4.2)

In fact, Eqs. 2.3.2 and 2.4.2 are logically equivalent, meaning the Ramsey theorem is neither a tautology nor a contradiction.

This speaks to the fact that injection of exponentiation results in a probabilistic vector space which abandons bivalency. What follows is that HOL proof assistants are not bivalent and hence produce unpredictable results, such as the alleged proof of the Ramsey theorem via Pythagorean triple of integers in the historical *enormity* of a 200 TB propositional logic computer program with result of a certified and paid prize.

Unsolved problem by A. Ranjan

From quora.com/Are-there-any-unsolved-problems-in-Mathematical-Logic

LET: # Necessity; % Possibility; > Imply; = Equivalent to; (n=n) Tautologous; p Assertion or Answer; q Question.

1. Mathematical Logic: Is it tautologous that for any question there is at least an answer?

   (#q>%p)>(p=p) ;
   validated TTTT TTTT

2. ; Reciprocally: Is any assertion the result of at least a question?

   (#p>%q)>(q=q) ;
   validated TTTT TTTT

Refutation of rational emotive behavior therapy (REBT) and logic-based therapy (LBT)

Abstract: We evaluate seven definitions of rational emotive behavior therapy (REBT) using the implication operator. None is tautologous. Logic-based therapy (LBT), as based on REBT, is similarly refuted.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r, s, t, u: Adversity; Belief; Consequence; Dispute; Effectiveness; Feelings
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⊢; < Not Imply, less than, ∈;
= Equivalent, ≡, ⊨; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(p=p) T as tautology; (p@p) **F** as contradiction;
(%p<#p) C as contingency, Δ; (%p>#p) N as non-contingency, ∇;
~(y < x) (x ≤ y), (x ⊆ y).

From: en.wikipedia.org/wiki/Rational_emotive_behavior_therapy

Where the following letters represent the following meanings in this model:

A- The adversity (1.1)

Remark 1.1: The adversity is a necessary requirement for this model to commence.

#p = (p=p) ; **F**N**F**N **F**N**F**N **F**N**F**N **F**N**F**N (1.2)

B- The developed belief in the person of the A Adversity (2.1)

#p>q ; TCTT TCTT TCTT TCTT (2.2)

C- The consequences but the consequences of that person's Beliefs ie B (3.1)

(#p>q)>r ; **F**N**FF** TTTT **F**N**FF** TTTT (3.2)

D- The person's disputes of A B and C. In latter thought. (4.1)

Remark 4.1: A dispute is the possible outcome of this model.

((#p>q)>r)>%s ; TCTT CCCC TTTT TTTT (4.2)

E- The effective new philosophy or belief that develops in that person through the occurrence of D in their minds of A and B (5.1)

Remark 5.1: The effective new conjecture (to include new philosophy or new belief) is the consequent of the model at this stage.

(%s<(#p<q))>t ; NNNN NNNN **F**N**FF** **F**N**FF**(8),
TTTT TTTT TTTT TTTT(8) (5.2)

F- The developed feelings of one's self either at point and after point C or at point after point E. (6.1)

Remark 6.1: The description "at point and after point" is taken as an inexact description to mean the equivalent of "after point" because "at point" is accomplished en route to "after point". In relational algebra, "equal to and greater than" is equivalent to "equal to or greater than". In mapped logic this reduces to "greater than".

(r>u)+(t>u) ; TTTT **FFFF** TTTT **FFFF**(4),
TTTT TTTT TTTT TTTT(12) (6.2)

The argument of the REBT model implies successive stages to proceed as Eqs. (1.1) implies (2.1) implies (3.1) implies (4.1) implies (5.1) implies (6.1). (7.1)

(((#p>q)>r)>(((#p>q)>r)>%s))>(((%s<(#p<q))>t)>((r>u)+(t>u))) ;
TTTT NNNN TTTT **FFFF**(4),
TTTT TTTT TTTT TTTT(12) (7.2)

Eqs. 1.2-7.2 as rendered are *not* tautologous. This refutes rational emotive behaviour therapy (REBT). We do not evaluate the subsequent three insights derived from REBT.

We examine a separate follow on to REBT known as logic-based therapy (LBT)

From: en.wikipedia.org/wiki/Logic-based_therapy

LBT assigns three states as sentences for "Point A (Activating event), Point B (Belief system), and Point C (behavioral and emotional Consequence)". This is equivalent to the above Eqs. 3.2 and 4.2, all *not* tautologous. The rule of inference in LBT also is restricted to the implication operator as "If O then R: 1. O; 2. Therefore R". Because this rule is used exclusively above in Eqs. 1.2-7.2 we abandon further evaluation of LBT.

Refutation of a 'concrete' Rauszer Boolean algebra generated by a preorder

Abstract: From the 11 equations tested, we refute 13 artifacts:

1. a condition for "an existential quantifier ∃ … on a Boolean algebra";
2. "a quantifier ∃ as closure operator on B, for which every open element is closed";
3. the interior operator on abstract topological Boolean algebra;
4. the kernel of a homomorphism from a Heyting algebra into another as a filter;
5. deductive systems and filters as equivalent;
6. the atomic definition of p ≤ ∃p in Halmos algebra;
7. a ‘concrete’ Rauszer Boolean algebra;
8. two conditions for the definition of a filter (and Heyting algebra using the filter);
9. a De Morgan algebra as a Kleene algebra;
10. equivalences of symmetrical Heyting algebras;
11. equivalences in Heyting algebras;
12. intuitionistic implication of intuitionistic logic; and
13. a theorem and a proposition of Nelson algebras.

As a result, the following seven areas are *non* tautologous fragments of the universal logic VŁ4:

1. Topological Boolean algebra;
2. Heyting algebra;
3. Intuitionistic logic;
4. Halmos algebra;
5. Rauszer algebra;
6. Kleene algebra; and
7. Nelson algebra.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Iturrioz, L. (2019). About a 'concrete' Rauszer Boolean algebra generated by a preorder. arxiv.org/pdf/1905.09928.pdf luisa.iturrioz@math.univ-lyon1.fr

(page 2) Recall that […] an existential quantifier ∃ ... on a Boolean algebra (B,∧,∨,−,0,1) is a mapping ∃∶B→B satisfying the following conditions:

(∃0) ∃0 = 0 (0.1.1)

LET p, q: a, b; (p=p) 1 or T; (p@p) 0 or **F**.

%(p@p)=(p@p) ; NNNN NNNN NNNN NNNN (0.1.2)

Remark 0.1.2: Eq. 0.1.2 is *not* tautologous, thereby refuting a condition for "an existential quantifier ∃ … on a Boolean algebra".

(∃1) a ∧ ∃a = a (1.1.1)

Remark 1.1.1: Eq. 1.1.1 as a tautology seems to be another misstatement in the literature of the Halmos algebra. For example the equivalent as (Q_2):

From: Halmos, P.R. (1954). Algebraic logic, I. Monadic boolean algebras. *Compositio Mathematica*, tome 2 (1954-1956). 217-249. numdam.org/article/CM_1954-1956__12__217_0.pdf [image]

(Q_2) p ≤ ∃p, page 4 (Q2.1)

~(%p<p) = (p=p) ; NTNT NTNT NTNT NTNT (Q2.2)

Remark Q2.2: Eq. Q2.2 as rendered refutes the Halmos algebra at its most atomic level.

[T]he image ∃(B) (i.e. the range of the quantifier ∃), is a monadic Boolean subalgebra of B. In addition, x∈∃(B) if and only if ∃x=x, if and only if ∀x=x. (2.1)

LET p, q: a, B

((#p=p)>(%p=p))>(p<%q) ; CNC**F** CNC**F** CNC**F** CNC**F** (2.2)

An element x such that ∃x = x (resp. ∀x = x) is called closed (resp. open), constant or a fixpoint, and the set of closed elements is the same as the set of open elements [...]. In other words, a quantifier ∃ is a closure operator on B, for which every open element is closed.

Remark 2.2: Eq. 2.2 is *not* tautologous, refuting "a quantifier ∃ is a closure operator on B, for which every open element is closed".

2. A ‘concrete’ Rauszer Boolean algebra

[B]ased on semisimplicity motivations, A. Monteiro [...], has studied properties of several binary operations in abstract topological Boolean algebras (A, I), where A is a Boolean algebra and I is an interior operator on A. In particular, he dealt with an implication ⇒ [...], [where ⊃ is the classical implication x ⊃ y = −x ∪ y] defined by:

a ⇒ b = I(Ia ⊃ Ib) (2.3.1)

(p>q)=(r&((r&p)>(r&q))) ; **F**T**FF** TTTT **F**T**FF** TTTT (2.3.2)

Remark 2.3.2: Eq. 2.3.2 is *not* tautologous, and so refutes the interior operator on abstract topological Boolean algebra.

3. Representation theorems in an unified form

For the sake of clarity we recall that a subset F of a lattice (A, ∧, ∨, 0, 1) is said to be a filter if the following conditions are satisfied:

(f1) 1 ∈ F ; (3.1.1.1)

LET p, q, r, s, t: P, Q, a, b, A, F; (p=p) 1 or T.

(p=p)<t ; TTTT TTTT TTTT TTTT, **FFFF FFFF FFFF FFFF** (3.1.1.2)

(f2) if a, b ∈ F , then a ∧ b ∈ F ; (3.1.2.1)

Remark 3.2.1: Eq. 3.2.1 is a trivial tautology.

(f3) if a ∈ F and a ≤ b, then b ∈ F ; (3.1.3.1)

((r<t)&~(s<r))>(s<t) ; TTTT **FFFF** TTTT TTTT, TTTT TTTT TTTT TTTT (3.1.3.2)

We note, incidentally, that for Heyting algebras, the kernel of a homomorphism from a Heyting algebra into another, is a filter. Also, the notions of deductive systems and filters are equivalent (A. Monteiro, 1959).

Remark 3: Two conditions for a filter are not tautologous, thereby refuting the definition of a filter and Heyting algebra which uses the filter.

B) Representation of symmetrical Heyting algebras

A De Morgan algebra A [is a Kleene algebra for]

($K_{a,b}$) a∧~a ≤ b∨~b, for any a,b∈A holds (3.2.1)

(#(r&s)<t)>~((s+~s)<(r&~r)) ; TTTT TTTT TTTT CCCC, TTTT TTTT TTTT TTTT (3.2.2)

Remark 3.2.2: Eq. 3.2.2 is not tautologous, thereby refuting a De Morgan algebra as a Kleene algebra.

Definition 3.4

[T]he following equivalences on account of the intuitionistic equality x ∧ (x ⇒ y) = x ∧ y [...]: (3.4.1.1)

LET p, q: x, y

(p&(p>q))=(p+q) ; T**FF**T T**FF**T T**FF**T T**FF**T (3.4.1.2)

Remark 3.4.1.2: Eq. 3.4.1.2 is *not* tautologous, thereby refuting the intuitionistic equality as claimed.

Theorem 3.5 Proof: equivalences

P∈h(∼x)⇔∼x∈P⇔x∈∼P⇔x∉−(∼P)=ϕ(P)⇔ϕ(P)∉h(x)⇔P∉ϕ(h(x))⇔P∈−ϕ(h(x))=∼h(x). (3.5.1)

LET p, q, r, s: P, h, x, ϕ

(((p<(q&~r))=((~r<p)=(r<~p)))=(~(r<~(~p=(s&p)))=~((s&p)<(q&r))))=((~(p<(s&(q&r)))=(p <(~s&(q&r))))=(~q&r)) ; TTT**F** T**FFF** T**F**TT T**FF**T (3.5.2)

Remark 3.5.2: Eq. 3.5.2 is not tautologous, thereby refuting equivalences of symmetrical Heyting algebras.

C) Representation of Nelson algebras
Theorem 3.8 Proof:

[(a∧b)⇒(∼a∨∼b∨c)]≤a⇒[∼a∨(b⇒(∼b∨c))] (3.8.1.1)

~(p<((p&q)>(~p+(~q+r))))>(~p+(q>(~q+r))) ; (3.8.1.2)

By the definition of the intuitionistic implication this is equivalent to

a∧[(a∧b)⇒(∼a∨∼b∨c)]≤∼a∨[b⇒(∼b∨c)] (3.8.2.1)

~((~p+(q>(~q+r)))<(p&((p&q)>(~p+(~q+r))))) = (p=p)

Remark 3.8: Eqs 3.8.1.1 = 3.8.2.1 (3.8.3.1)

(~(p<((p&q)>(~p+(~q+r))))>(~p+(q>(~q+r))))=
~((~p+(q>(~q+r)))<(p&((p&q)>(~p+(~q+r))))) ;
FT**F**T **F**T**F**T **F**T**F**T **F**T**F**T (3.8.3.2)

Eq. 3.8.3.2 is *not* tautologous as claimed, hence refuting Theorem 3.8 as a representation of Nelson algebras.

Proposition 3.9

LET p, q, r: G, H, K

(G∩H)⇒(∼G∪∼H∪K)⊆G⇒(∼G∪(H⇒(∼H∪K))) (3.9.1.1)

(p&q)>(~(p<(~p+(~q+r)))>(~p+(q>(~q+r)))) ; (3.9.1.2)

By the definition of the intuitionistic implication this is equivalent to

G∩[(G∩H)⇒(~G∪~H∪K)]⊆~G∪(H⇒(~H∪K)) (3.9.2.1)

~((~p+(q>(~q+r)))<(p&((p&q)>(~p+(~q+r))))) = (p=p) ; (3.9.2.2)

Remark 3.9: Eqs 3.8.1.1 = 3.8.2.1 (3.9.3.1)

((p&q)>(~(p<(~p+(~q+r)))>(~p+(q>(~q+r)))))=
~((~p+(q>(~q+r)))<(p&((p&q)>(~p+(~q+r))))) ;
FT**F**T **F**T**F**T **F**T**F**T **F**T**F**T (3.9.3.2)

Eq. 3.9.3.2 is *not* tautologous as claimed, hence refuting a proposition of Nelson algebras.

From the 11 equations tested, we refute 13 artifacts:

1. a condition for "an existential quantifier ∃ … on a Boolean algebra";
2. "a quantifier ∃ as closure operator on B, for which every open element is closed";
3. the interior operator on abstract topological Boolean algebra;
4. the kernel of a homomorphism from a Heyting algebra into another as a filter;
5. deductive systems and filters as equivalent;
6. the atomic definition of p ≤ ∃p in Halmos algebra;
7. a ‘concrete’ Rauszer Boolean algebra;
8. two conditions for the definition of a filter (and Heyting algebra using the filter);
9. a De Morgan algebra as a Kleene algebra;
10. equivalences of symmetrical Heyting algebras;
11. equivalences in Heyting algebras;
12. intuitionistic implication of intuitionistic logic; and
13. a theorem and a proposition of Nelson algebras.

As a result, the following are seven fields are *non* tautologous fragments of universal logic VŁ4: topological Boolean algebra; Heyting algebra; intuitionistic logic; Halmos algebra; Rauszer algebra; Kleene algebra; and Nelson algebra.

Refutation of realizability Semantics for QML

From: Rin, B.G.; Walsh, S. (2016). arxiv.org/pdf/1510.01977.pdf
Realizability semantics for quantified modal logic: generalizing Flagg's 1985 construction.

We assume the apparatus of the Meth8 modal logic model checker implementing variant system VŁ4. Meth8 allows to mix four logical values with four analytical values. The designated *proo*f value is T.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	p=p	T	Tautology	proof	11	3
2	p@p	F	Contradiction	absurdum	00	0
3	%p>#p	N	Non-contingency	truth	01	1
4	%p<#p	C	Contingency	falsity	10	2

LET: ~ Not; + Or; & And; \ Not and; > Imply; < Not imply; = Equivalent to;
@ Not equivalent to; # all ∀ ; % some ∃; (p@p) 00 zero; (p=p) 11 one
Results are the repeating proof table(s) of 16-values in row major horizontally.

"The resulting semantics generalize the important but little-understood construction of Flagg (1985), whose goal was to provide a consistency proof of Epistemic Church's Thesis together with epistemic arithmetic, a modal rendition of first-order arithmetic. *Epistemic Church's Thesis* (ECT) is the following statement:

(1.1) [□ (∀n∃m□]φ(n,m))] ⇒ [∃e□]∀n∃m∃q(T(e,n,q)∧U(q,m)∧□φ(n,m))]" (1.1)

LET: pqtuwxy pqtuemn

#((#y&%x)&(#p&(y&x))) >
((%w&#(#y&(%x&(%x&%q))))&(((t&(w&(y&q)))&(u&(q&x)))&(#p&(y&x)))) ;

TTTT TTTT TTTT TTTT,
TCTC TCTC TCTC TCTC,
TCTT TCTT TCTT TCTT (1.2)

"EZF ... is built from Q_{eq}.S4 by the addition of the following axioms: ...

II. Induction Schema: [∀x((∀y∈x φ(y))⇒φ(x))]⇒[∀xφ(x)]" (2.1)

LET: pxy φxy

We distribute the quantification in the antecedent to ensure clarity.

((#x&((#y<x)&(p&y)))>(#x&(p&x))) > (#x&(p&x)) ;

FFFF FFFF FFFF FFFF,
FNFN FNFN FNFN FNFN (2.2)

"III. Scedrov's Modal Foundation: [□∀x(□(∀y∈xφ(y))⇒φ(x))]⇒[□∀xφ(x)]" (3.1)

We distribute the quantification in the antecedent to ensure clarity.

(#(#x&#((#y<x)&(p&y)))>(#x&(p&x))) > #(#x&(p&x)) ;
`FFFF FFFF FFFF FFFF, FNFN FNFN FNFN FNFN` (3.2)

Eqs. 1.2, 2.2, and 3.2 as rendered are *not* tautologous. Eqs. 2.2 and 3.2 result in the same truth table because Eq. 3.2 reduces to Eq. 2.2.

We did not test subsequent axioms.

This means respectively that the following are not theorems: Epistemic Church's Thesis; EZF induction schema; and Scedrov's modal foundation.

What follows is that Flagg's construction, Goodman's intensional set theory, and epistemic logic are suspicious.

Hans Reichenbach's event-splitting formula

From: Wolfgang Spohn. "On Reichenbach's principle of the common cause". Logic, Language, and the Structure of Scientific Theories: proceedings of the Carnap-Reichenbach Centennial, Universit of Konstanz, 2w1-24 May 1991. Ed. by Wesley Salmon. Univ.-Verl. Konstanz. 1994. pp 215-239.
at pdfs.semanticscholar.org/81ae/627c2f1c80b3b3c78f5ba5a54daca242309c.pdf, page 2:

"The principle of the common cause specifies an important relation between probability and causality", where A and B are are two positively correlated events, satisfying these conditions:

LET p P; q A; r B; s C
(p&(q&r)) > ((p&q)&(p&r)) ; vt ; TTTT TTTT TTTT TTTT ; (1.2)
(p&(q+s)) > (p&q) ; nvt ; TTTT TTTT TFTT TFTT ; (2.1.2)
(p&(r+s)) > (p&r) ; nvt ; TTTT TTTT TFTF TTTT ; (2.2.2)
(p&((q&r)+s)) = ((p&(q+s))&(p&(r+s))) ; vt ; (3.2)

The argument is that ((Eqs 2.1.2 and 2.2.2) and Eq 3.2) imply Eq 1.2 (4.1)
With the inequalities reversed in Eq 2.1.2, 2.2.2, and 3.2, those imply Eq 1.2 (5.1)
Whereas Eq 2.1.2 or 2.2.2 with only one inequality reversed would imply the reverse of Eq 1. 2 (6.1, 7.1)

((((p&(q+s))>(p&q))&((p&(r+s))>(p&r)))&((p&((q&r)+s))=((p&(q+s))&(p&(r+s)))))
>((p&(q&r))>((p&q)&(p&r))) ; vt ; (4.2)

((((p&(q+s))<(p&q))&((p&(r+s))<(p&r)))&((p&((q&r)+s))=((p&(q+s))&(p&(r+s)))))
>((p&(q&r))>((p&q)&(p&r))) ; vt ; (5.2)

((((p&(q+s))<(p&q))&((p&(r+s))>(p&r)))&((p&((q&r)+s))=((p&(q+s))&(p&(r+s)))))
>((p&(q&r))>((p&q)&(p&r))) ; vt ; (6.2)

((((p&(q+s))>(p&q))&((p&(r+s))<(p&r)))&((p&((q&r)+s))=((p&(q+s))&(p&(r+s)))))
>((p&(q&r))>((p&q)&(p&r))) ; vt ; (7.2)

The full argument is that (Eq 1.2, 2.1.2, 2.2.2, and 3.2) is equivalent to (Eq 4.2, 5.2, 6.2, and 7.2). (8.1)

((((p&(q&r))>((p&q)&(p&r)))&(((p&(q+s))>(p&q))&((p&(r+s))>(p&r))))&((p&((q&r+s))=((p&(q+s))&(p&(r+s))))) =
((((((((p&(q+s))>(p&q))&((p&(r+s))>(p&r)))&((p&((q&r+s))=((p&(q+s))&(p&(r+s)))))>((p&(q&r))>((p&q)&(p&r))))&(((((p&(q+s))<(p&q))&((p&(r+s))<(p&r)))&((p&((q&r+s))=((p&(q+s))&(p&(r+s)))))>((p&(q&r))>((p&q)&(p&r)))))&(((((p&(q+s))<(p&q))&((p&(r+s))>(p&r)))&((p&((q&r+s))=((p&(q+s))&(p&(r+s)))))>((p&(q&r))>((p&q)&(p&r)))))&(((((p&(q+s))>(p&q))&((p&(r+s))<(p&r)))&((p&((q&r+s))=((p&(q+s))&(p&(r+s)))))>((p&(q&r))>((p&q)&(p&r))))) ;
nvt ; TTTT TTTT TFTF TFTT (8.2)

In 269 logical steps, Meth 8 finds Reichenbach's event-splitting principle is not validated as tautologous.

Refutation of relativization by structural induction in weighted first order logic

Abstract: We evaluate a formula of relativization as defined by structural induction which is *not* tautologous. Its use in weighted first order logic is refuted.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) C non-contingency, ∇, ordinal 2; (%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Droste, M.; Gastin, P. (2019). Aperiodic weighted automata and weighted first-order logic.
arxiv.org/pdf/1902.08149.pdf
droste@informatik.uni-leipzig.de paul.gastin@lsv.fr paul.gastin@ens-paris-saclay.fr

We define below the relativizations $\phi^{<x}$, $\phi^{(x,y)}$ and $\phi^{>y}$... [for $\phi^{>y}$, read $\phi^{>x}$?]
The relativization is defined by structural induction on the formulas as follows:

$$\ldots (\forall z\psi)^{<x} = \forall z(z < x \Rightarrow \psi^{<x}) \tag{4.1.1}$$

The relativizations $\phi^{(x,y)}$ and $\phi^{>x}$ are defined similarly. (4.2.1),(4.3.1)

Remark 4.1: We write the exponent in $\phi^{(x,y)}$ of Eq. 4.2.1 to mean variables x and y, such as each with a value of 1.

((#s&#p)&(q&r))=((#s<(q&r))>(p&(q&r))) ;
FFFF FFFF NNNN NN**F**N (4.2.2)

Eq. 4.2.2 as rendered is *not* tautologous. This means relativization in that context is refuted.

Refutation of relevance logic via Routely and Meyer

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET ~ Not; + Or; & And; > Imply; = Equivalent; @ Not Equivalent; (p@p) F; T (p=p) .
necessity, for all or every, ∀, ∀x; % possibility, for one or some, ∃, ∃x.

Remark: Expressions from the text are not reproduced due to character non-portability.

From: plato.stanford.edu/entries/logic-relevance/ Copyright © 2012 by Edwin Mares
<Edwin.Mares@@vuw.ac.nz>

Paradoxes of material implication in relevance logic are:

p>(q>p) ;	TTTT TTTT TTTT TTTT	(1.2)
~p>(p>q) ;	TTTT TTTT TTTT TTTT	(2.2)
(p>q)+(q>r) ;	TTTT TTTT TTTT TTTT	(3.2)

Paradoxes of strict implication in relevance logic are:

(p&~p)>q ;	TTTT TTTT TTTT TTTT	(4.2)
p>(q>q) ;	TTTT TTTT TTTT TTTT	(5.2)
p>(q+~q) ;	TTTT TTTT TTTT TTTT	(6.2)

"Routley and Meyer go modal logic one better and use a three-place relation on worlds", allowing (q>q) to fail and (p>(q>q) to fail.

A → *B* is true at a world *a* if and only if for all worlds *b* and *c* such that *Rabc* (*R* is the accessibility relation) either *A* is false at *b* or *B* is true at *c*. (10.1)

LET p, q, r, s, (t): A, B, b, c, (a)

Remark: We minimize variables and table size for clarity by ignoring (a).

(((#r>p)=(p@p))+((#s>q)=(p=p))) > (p>q) ; TFTT TFTT TNTT TNTT (10.2)

Eq. 10.2 as rendered is *not* tautologous. This means relevance logic is refuted.

Refutation of relevance logic R and models

Abstract: We evaluate a definition and model formula for relevance logic R which are *not* tautologous. What follows is that logic R is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r: A, B, C;
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⇔, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) C non-contingency, ∇, ordinal 2; (%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: en.wikipedia.org/wiki/Relevance_logic

System E in relevance logic adds this definition of

□A as (A→A)→A. (1.1.1)

#p=((p>p)>p) ; TNTN TNTN TNTN TNTN (1.1.2)

Remark 1.1. Eq. 1.1.2 is *not* tautologous. If Eq.1.1 is substituted back into the E axiom □A∧□B→□(A∧B), the result is a theorem but only by way of an injection of non-tautologous axiom definition.

"The conditional fragment of R is sound and complete with respect to the class of semilattice models. The logic with conjunction and disjunction is properly stronger than the conditional, conjunction, disjunction fragment of R. In particular, the formula

(A→(B∨C))∧(B→C)→(A→C) (1.2.1)

is valid for the operational models but it is invalid in R."

(p>(q+r))&((q>r)>(p>r)) ; T**F**TT TTTT T**F**TT TTTT (1.2.2)

Remark 1.2. Eq. 1.2 is *no*t tautologous.

Eqs. 1.1.2 and 1.2.2 refute relevance logic.

Refutation of resolution-based decision procedure for two variables with equality

Abstract: We evaluate six equations and three conjectures for the decision procedure. None is tautologous. This refutes the procedure for two variables with equality, and forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: de Nivelle, H.; Pratt-Hartmann, I. (2001). A resolution-based decision procedure for the two variable fragment with equality. nivelle@mpi.mpg-sb.de, ipratt@cs.man.ac.uk www.cs.man.ac.uk/~ipratt/papers/logic/ijcar01.pdf

1 Introduction

The two-variable-fragment L2 ≈ is the set of formulas that do not contain function symbols, that possibly contain equality (≈), and that use only two variables. The two-variable fragment without equality L2 is the subset of L2 ≈ not involving the predicate ≈. For example, the formula

$$\forall x\exists y[r(x, y) \wedge \forall x(r(y, x) \rightarrow x\approx y)], \tag{1.1.1}$$

LET p, q, r, s: a, x, r, y.

(r&(#q&%s))&((r&(%s&#q))>(#q=%s)) ;
FFFF FFFF FFFF FFNN (1.1.2)

stating that every element is r-related to some element whose only r-successor is itself, is in L2 ≈ (but not in L2). Note in particular the 're-use' of the variable x by nested quantifiers in this example. In the same way, it is possible to translate modal formulas into L2, (without equality) by reusing variables. For example, the modal formula

$$\Box\Diamond\Box a \tag{1.2.1}$$

#%#p = (s=s) ; **F**N**F**N **F**N**F**N **F**N**F**N **F**N**F**N (1.2.2)

can be translated into

$$\forall y(r(x, y) \rightarrow \exists x(r(y, x) \wedge \forall y(r(x, y) \rightarrow a(y)))). \tag{1.3.1}$$

((r&(#q&%s))&(r&(%s&#q)))>(p&#s) ;

```
TTTT TTTT TTTT TTCT                (1.3.2)
```

No equality is needed for translating modal formulas.

Remark 1.3.2: Eqs. 1.2.2 is not equivalent to 1.3.2 as claimed.

2 Motivation

A logic is said to have the *finite-model property* if any satisfiable formula in that logic is satisfiable in a finite structure. It is easy to see that any fragment of first order logic having the finite model property is decidable; and indeed, most of the known decidable fragments of first-order logic have the finite model property. … One such fragment of particular interest here is the so-called Gödel class: the set of first-order formulas *without equality* which, when put in prenex form, have quantifier prefixes matching the pattern $\exists * \forall\forall\exists *$. Gödel .. showed that the Gödel class has the finite model property, and is thus decidable. In the same paper, Gödel claimed that allowing ≈ in formulas of the Gödel class would not affect the finite model property, a claim which was later shown to be false by Goldfarb.. . Between these two discoveries, Scott .. showed that any formula of the two-variable fragment can be transformed into a formula in the Gödel fragment which is equisatisfiable. Relying on Gödel's incorrect claim, Scott concluded decidability for *L*2 ≈*:* Of course, what Scott actually showed was the decidability for *L*2 only. That the full two-variable fragment does indeed have the finite model property was eventually established by Mortimer .. . The fragment *L*2 ≈ is of particular interest when dealing with natural language input, because many simple natural language sentences translate into *L*2 ≈. To give a somewhat fanciful example, the sentence

Every meta-barber shaves every man who shaves no man who shaves himself.

translates to the two-variable formula

$$\forall x(\text{meta-barber}(x)\rightarrow\forall y((\text{man}(y)\wedge\forall x((\text{man}(x)\wedge\text{shave}(x,x))\rightarrow\text{shave}(y,x)))\rightarrow\text{shave}(x,y))). \tag{2.1.1}$$

LET p, q, s, x, y: meta-barber, man, shave, x, y.

(p&#x)>((((q&#y)&(s&(#x&#x)))>(~s&(%y&#x)))>(s&(#x&%y))) ;

```
TTTT TTTT TTTT TTTT(16)
TCTC TCTC TCTC TCTC(16)
TTTT TTTT TTTT TTTT(16)
TCTC TCTC TTTT TTTT(16)            (2.1.2)
```

Remark 2.1.2: Eq. 2.1.2 as stated is *not* tautologous, meaning the example is not a theorem as presumably it should be.

3 Making equality disappear

In this section, we give give a method for removing equality from a formula in L2 ≈, based on resolution. … Occurrences of the ≈-symbol fall into two groups. Negative occurrences can be 'simulated' without recourse to equality. Positive occurrences can be restricted to those belonging to a ∃! quantifier.

Remark 3: We interpret the quantifier ∃! as ∃, due to the *non* tautologous performances of Eqs. 1… and 2… above.

Lemma 5. Let γ(x) be a formula not involving the variable y and let δ(y) a formula not involving the variable x. Then the formulas

$$\forall x \forall y(\gamma(x) \vee \delta(y) \vee x \approx y) \tag{5.1.1}$$

LET p, q, r, s: γ, δ, x, y.

((p&#r)+(q&#s))+(#r=#s) ; `TTTT CTCT CCTT TTTT` (5.1.2)

and

$$\forall x\ \gamma(x) \vee \forall x\ \delta(x) \vee (\exists! x\ \neg\, \gamma(x) \wedge \forall x(\gamma(x) \leftrightarrow \delta(x))) \tag{5.2.1}$$

((p&#r)+(q&#r))+((%r&~(p&r))&((p&#r)=(q&#r))) ;
`CCCC TNTN CCCC TNTN` (5.2.2)

are logically equivalent. Using this, we can use the splitting rule to decompose the disjunctions of the Type 3 formula. The result is a formula, in all positive occurrences of ≈ belong to a ∃! quantifier. These can be eliminated by introducing new individual constants.

Remark 5.2.2: Eqs. 5.1.2 and 5.2.2 not logically equivalent, thereby refuting Lemma 5.

Refutation of recursive comprehension in second-order arithmetic for reverse mathematics

Abstract: We evaluate 11 basic first-order axioms of which nine are *not* tautologous. Recursive comprehension, as an abstraction of mathematical induction, is derived therefrom in second-order arithmetic and is *not* tautologous. This refutes the use of recursive comprehension in second-order arithmetic. Reverse mathematics relies on recursive comprehension and hence is also refuted.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. Reproducible transcripts for results are available. (See ersatz-systems.com.)

LET p, q, r, s: A, B, C, D;
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⊢, ⊨; < Not Imply, less than, ∈, ≺, ⊂;
= Equivalent, ≡, :=, ⇔, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3;
(z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) C non-contingency, ∇, ordinal 2;
(%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).

Remark 0: For clarity, we distribute the quantifiers to each instance of a variable.

From: en.wikipedia.org/wiki/Second-order_arithmetic

The following axioms are known as the basic axioms.

Axioms governing the successor function and zero:

1. ∀ m [S m = 0 → ⊥] ("the successor of a natural number is never zero") (1.1)

(s&#p)>(r@r) ; TTTT TTTT TCTC TCTC (1.2)

2. ∀ m ∀ n [S m = S n → m = n] ("the successor function is injective") (2.1)

((s&#p)=(s&#q))>(#p=#q) ; TCCT TCCT TTTT TTTT (2.2)

3. ∀ n [0 = n ∨ ∃ m [S m = n]] ("every natural number is zero or a successor") (3.1)

(((r@r)=#q)&%p)&((s&p)=q) ; CT**FF** CT**FF** C**FF**C C**FF**C (3.2)

Addition defined recursively:

4. $\forall m [m + 0 = m]$ (4.1)

(#p+(r@r))=#p ; TTTT TTTT TTTT TTTT (4.2)

5. $\forall m \forall n [m + S n = S (m + n)]$ (5.1)

(#p+(s&q))=(s&(p+q)) ; TCTC TCTC TNTT TNTT (5.2)

Multiplication defined recursively:

6. $\forall m [m \cdot 0 = 0]$. (6.1)

(#p&(r@r))=(r@r) ; TTTT TTTT TTTT TTTT (6.2)

7. $\forall m \forall n [m \cdot S n = (m \cdot n) + m]$ (7.1)

(#p&(s&#q))=((p&q)+p) ; T**F**T**F** T**F**T**F** T**F**TN T**F**TN (7.2)

Axioms governing the order relation "<":

8. $\forall m [m < 0 \rightarrow \bot]$. (“no natural number is smaller than zero”) (8.1)

(#p<(r@r))>(r@r) ; TCTC TCTC TCTC TCTC (8.2)

9. $\forall n \forall m [m < S n \leftrightarrow (m < n \vee m = n)]$ (9.1)

(#p<(s&#q))=((#p<#q)+(#p=#q)) ; **F**NNN **F**NNN **F**NN**F** **F**NN**F** (9.2)

10. $\forall n [0 = n \vee 0 < n]$. (“every natural number is zero or bigger than zero”) (10.1)

((r@r)=#q)+((r@r)<#q) ; TTCC TTCC TTCC TTCC (10.2)

11. $\forall m \forall n [(S m < n \vee S m = n) \leftrightarrow m < n]$ (11.1)

(((s&#p)<#q)+((s&#p)=#q))=(#p<#q) ;
FNNN **F**NNN **F**NN**F** **F**NN**F** (11.2)

For the 11 basic axioms of Eqs. 1.2-11.2 as rendered, two as 4.2 and 6.2 are tautologous, and the other nine are *not* tautologous.

From: en.wikipedia.org/wiki/Second-order_arithmetic

Recursive comprehension

[From: en.wikipedia.org/wiki/Reverse_mathematics:
The initials "RCA" stand for "recursive comprehension axiom", where

"recursive" means "computable", as in recursive function.]

The subsystem RCA_0 is ... often used as the base system in reverse mathematics. It consists of: the basic axioms [Eqs. 1.-11.1 from above] , the Σ^0_1 induction scheme, and the Δ^0_1 comprehension scheme. This scheme includes, for every Σ^0_1 formula φ and every Π^0_1 formula ψ, the axiom:

$$\forall m \forall X((\forall n(\varphi(n) \leftrightarrow \psi(n))) \rightarrow \exists Z \forall n(n \in Z \leftrightarrow \varphi(n))) \quad (12.1)$$

LET p, q, r, s, x, z: φ, ψ, m, n, X, Z;

((p&#s)=(q&#s))>((#q<%z)=(p&#s)) ;

```
TTCC TTCC TTTT TTTT(32),
TTTT TTTT TTTC TTTC(32)
```
(12.2)

The formula for recursive comprehension in Eq. 12.2 as rendered is *not* tautologous. This refutes its use in second-order arithmetic.

We evaluate 11 basic first-order axioms of which nine are *not* tautologous. Recursive comprehension, as an abstraction of mathematical induction, is derived therefrom in second-order arithmetic and is also *not* tautologous. Reverse mathematics requires recursive comprehension and thereby is also refuted.

Refutation of reverse mathematics on measurability theory and computability theory

Abstract: We evaluate the authors' definition of reverse mathematics in their anticipation of applying it to measurability and computability theory. The argument taking two equations to define reverse mathematics is *not tautologous.* Therefore to apply it to measure and computability theory is meaningless.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⇔, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) `C` non-contingency, ∇, ordinal 2; (%z>#z) `N` as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).

Note: To preserve clarity, we usually distribute quantifiers to each variable so designated.

From: Normann, D.; Sanders, S. (2019).
Representations in measure theory: between a non-computable rock and a hard to prove place.
arxiv.org/pdf/1902.02756.pdf arxiv.org/pdf/1902.02756.pdf

2. Preliminaries
2.1. Reverse Mathematics [higher-order RM in *higher-order* arithmetic]

The aim of RM is to identify the minimal axioms needed to prove theorems of ordinary, i.e. non-set theoretical, mathematics. ...

To formalise this idea [is] the collection of all finite types T, defined by the two clauses:

(i) 0 ∈ T and (ii) If σ, τ ∈ T then (σ → τ) ∈ T, (2.1.1.1)

We write Eq. 2.1.1.1 as: The two clauses of (i) and (ii) imply all finite types.

LET p, q, r, s: n, T, τ, σ.

(((p@p)<q)&(((s&r)<q)>((s>r)<q)))>#q ;
`TTTT TTTT TTTT TTTT` (2.1.1.2)

where 0 is the type of natural numbers, and σ → τ is the type of mappings from objects of type σ to objects of type τ.

In this way,

$1 \equiv 0 \rightarrow 0$ is the type of functions from numbers to numbers, and
where $n + 1 \equiv n \rightarrow 0$. (2.1.2.1)

We write Eq. 2.1.2.1 as: Ordinal one is equivalent to zero implying zero, and where p plus ordinal one is equivalent to p implying zero.

```
((%p>#p)=((p@p)>(p@p)))&((p+(%p>#p))>(p>(p@p))) ;
                                    NFNF NFNF NFNF NFNF
```
(2.1.2.2)

We state the argument of the text as: Eq. 2.1.1.1 implies Eq. 2.1.2.1. (2.1.3.1)

```
((((p@p)<q)&(((s&r)<q)>((s>r)<q)))>#q)>
(((%p>#p)=((p@p)>(p@p))) & ((p+(%p>#p))>(p>(p@p)))) ;
                                    NFNF NFNF NFNF NFNF
```
(2.1.3.2)

Remark 2.1: Eq 2.1.3.2 as rendered is *not* tautologous. This means the definition of reverse mathematics is refuted. Therefore to apply it to measure and computability theory is meaningless.

Refutation of reverse mathematics and nets

Abstract: We evaluate four definitions for reverse mathematics (3) and nets (1). None is tautologous. This refutes reverse mathematics and nets. Therefore these definitions form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
\> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Sanders, S. (2019). Nets and reverse mathematics, a pilot study.
arxiv.org/pdf/1905.04058.pdf sasander@me.com

Abstract. Nets are generalisations of sequences involving possibly *uncountable* index sets … More recently, nets are central to the development of *domain theory* ... This paper deals with the Reverse Mathematics study of basic theorems about nets. ...

2.1. **Reverse Mathematics.** … we introduce the collection of all finite types T, defined by the two clauses: (i) 0 ∈ T and (ii) If σ, τ ∈ T then (σ → τ) ∈ T, where 0 is the type of natural numbers, and σ → τ is the type of mappings from objects of type σ to objects of type τ.

(2.1.1.1)

LET p, q, r, s: T, τ, r, σ

((p@p)<p)&(((s&q)<p)>((s>q)<p)) ; **FFFF FFFF FFFF FFFF** (2.1.1.2)

In this way, 1 ≡ 0 → 0 is the type of functions from numbers to numbers, and where n + 1 ≡ n → 0.

(2.1.2.1)

((p+(%p>#p))=(p>(p@p)) &((%p>#p)=((p@p)>(p@p))) ;
N**F**N**F** N**F**N**F** N**F**N**F** N**F**N**F** (2.1.2.2)

Remark 2.1.2.2: If in Eq. 2.1.2.1 the one and zero are taken as tautology and contradiction, then the result is strengthened as the same:

((p= p)=((p@p)>(p@p)))&((p+(p= p))=(p>(p@p))) ;
T**F**T**F** T**F**T**F** T**F**T**F** T**F**T**F** (2.1.2.3)

2.3. **Introducing nets.** We introduce the notion of net and associated concepts. We first consider the following standard definition ...

Definition 2.7. [Nets] A set D ≠ Ø with a binary relation '≼' is directed if

(a) The relation is transitive, i.e. (∀x, y, z ∈ D)([x≼y ∧ y≼z] → x≼z).
(b) For x, y ∈ D, there is z ∈ D such that x≼z ∧ y≼z.
(c) The relation ≼ is reflexive, i.e. (∀x ∈ D)(x≼x) (2.7.1)

(p@(p@p))>((((((#q&#r)&#s)<p)&((~(r<q)&~(s<r))>~(s<q)))&(((q&r)<p)>((s<p)>(~(s<q)& ~(s<r)))))&((#q<p)&~(q<q))) ; TFTF TFTF TFTF TFTF (2.7.2)

Eqs. 2.1.2.2 and 2.7.2 as rendered are *not* tautologous. This refutes reverse mathematics and nets.

Refutation of rewriting logic for compositional specification

Abstract: We evaluate the first motivational example, before mutual exclusion of multiple trains, for states and transitions as defined. The conjectured model is *not* tautologous, refutes rewriting logic for compositional specification, and forming a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

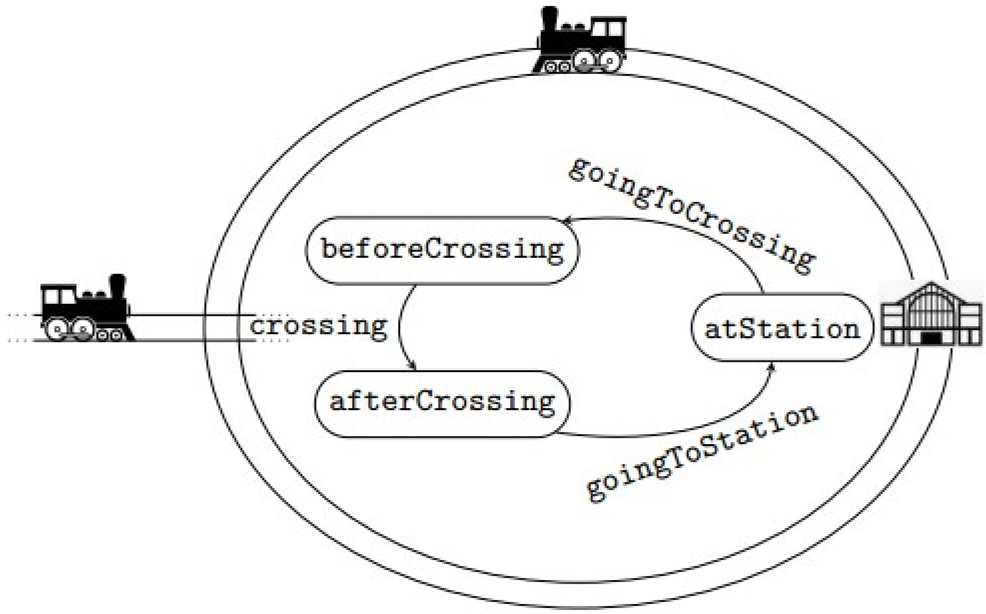
From: Martín, Ó.; Verdejo, A.; Martí-Oliet, N. (2003). Compositional specification in rewriting logic. arxiv.org/pdf/1908.11769.pdf omartins@ucm.es

Abstract Rewriting logic is naturally concurrent: several subterms of the state term can be rewritten simultaneously. But state terms are global, which makes compositionality difficult to achieve. Compositionality here means being able to decompose a complex system into its functional components and code each as an isolated and encapsulated system. Our goal is to help bringing compositionality to system specification in rewriting logic. The base of our proposal is the operation that we call synchronous composition. We discuss the motivations and implications of our proposal, formalize it for rewriting logic and also for transition structures, to be used as semantics, and show the power of our approach with some examples.

2 Motivation, goals, and choices

2.1 First motivational example: mutual exclusion

Think of a train, a very simple model of a train, that goes round a closed railway in which there is a station and a crossing with another railway. There are three points of interest in the railway, that we use as the states of our model. There are three transitions for moving between the three states.



In Maude-like notation:

```
rl [goingToCrossing]    : atStation       => beforeCrossing .
rl [crossing]           : beforeCrossing  => afterCrossing .
rl [goingToStation]     : afterCrossing   => atStation .
```

The keyword `rl` introduces a rewrite rule. The identifier in square brackets is the label of the rule. Rules describe transitions between states. To the left of the arrow (=>) is the origin state; to the right is the destination state.

LET	u, v, w:	[states]	station, before_crossing, after_crossing;
	x, y, z:	[transitions]	to_crossing, crossing, to_station.

Remark 12.1: We write the modeled conjecture to mean the circular states of (u>v>w>u) imply the transition definitions of (x&y&z). (12.1.0)

(((x>y)>(z>x))&((u>v)>(w>u)))>(((x=(u>v))&(y=(v>w)))&(z=(w>u))) ;

```
FFFF FFFF FFFF FFFF( 8)      }
TTTT TTTT TTTT TTTT( 2) }x2 }x4
FFFF FFFF FFFF FFFF( 2) }    }
TTTT TTTT TTTT TTTT(16)
FFFF FFFF FFFF FFFF( 4)
TTTT TTTT TTTT TTTT( 6)
FFFF FFFF FFFF FFFF( 2)
TTTT TTTT TTTT TTTT( 2)
FFFF FFFF FFFF FFFF( 2)
TTTT TTTT TTTT TTTT(18)
FFFF FFFF FFFF FFFF( 6)
TTTT TTTT TTTT TTTT( 2)
FFFF FFFF FFFF FFFF( 2)
TTTT TTTT TTTT TTTT( 4)
```
(12.1.2)

Eq. 12.1.2 is *not* tautologous. This denies the first motivational example, before mutual exclusion is invoked for multiple trains, and hence refutes the conjecture of rewriting logic for compositional specification.

Riemann hypothesis rendered as not provable

Given i = (-1)^(1/2) and lower case Zeta (Z) as ζ (lc_case zeta):

1. For **any** complex number (*a* + *b*i), ζ(*a* + *b*i) is another complex number (*c*+*d*i).
2. A zero is a point (*a* + *b*i) where *f*(*a* + *b*i)=0, such as for example ζ(0)=0.
3. Trivial zeroes occur at (0 + *b*i) for **some** *b*.

   Hence if *a* and *c* = 0, then ζ(*a* + *b*i) is rewritten in ζ((0) + *b*i) as another complex number ((0)+*d*i).
4. Non trivial zeroes occur at (1/2 + *b*i) for **some** *b*.

   Hence, if *a* and *c* = 1/2, then ζ(*a* + *b*i) is rewritten in ζ((1/2) + *b*i) as another complex number ((1/2)+*d*i).

A sentence to test is if known zeroes imply other zeroes:

> Trivial zeroes ζ((0) + *b*i) for **some** *b,* implying other complex numbers as **all** ((0)+*d*i), and non trivial zeroes ζ((1/2) + *b*i) for **some** *b,* implying other complex numbers as **all** (((1/2) + *d*i), imply possibly other zeroes ζ(*a* + *b*i) for **some** *b,* implying other complex numbers as **all** (*a* + *d*i). (5.0)

This effectively tests if a location of zeroes (trivial based on even numbers) and a location of zeroes (non trivial based on odd numbers) imply another possible location of zeroes as a tautology, because the question is "Are there possibly other zeroes".

We assume the apparatus of the Meth8 modal logic model checker implementing variant system VŁ4. Meth8 allows us to mix four logical values with four analytical values. The designated proof value is T.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	p=p	T	Tautology	proof	11	3
2	p@p	F	Contradiction	absurdum	00	0
3	%p>#p	N	Non-contingency	truth	01	1
4	%p<#p	C	Contingency	falsity	10	2

LET: + Or; & And; \ Not and; > Imply; < Not imply; @ Not equivalent to;
all; % some; (p@p) 00, zero ; (%p<#p) 10, two ; (%p>#p) 01, one;
pqrs *bd*ζi ; (p@p) trivial *a*,*c* as (0) ; ((%p>#p)\(%p<#p)) non trivial *a*,*c* as (1/2);

Results are the proof table of 16-values in row major horizontally.

((((r&#((p@p)+(%q&s)))>(p@p))>(p@p)) & ((r&#(((%p>#p)\(%p<#p))+(%q&s)))>(p@p)))
> %((r&#(p+(%q&s)))>(p@p)) ; `TTTT TTTT TTTT TTTT` (5.1)

Eq. 5.1 shows other zeroes are possible. We conclude that the Riemann hypothesis, as stated and rendered, is *not* tautologous, and hence is denied.

Refutation of the Riemann hypothesis using the excluded middle

Abstract: The conjectured proof of the Riemann hypothesis using the excluded middle is refuted by the Meth8/VŁ4 modal logic model checker.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

LET p, q, r, s: p, q, Riemann hypothesis (RH), s;
~ Not; > Imply; = Equivalent; @ Not Equivalent;
% possibility, for one or some; # necessity, for every or all;
(p=p) Tautology as designated *proof* value; (p@p) **F** as contradiction;
(%q>#q) N as truthity (non-contingency; (%s<#s) C as falsity (contingency).

From: Ireland, K.; Rosen, M. (1990). A classical introduction to modern number theory. 2nd ed. Springer. via en.wikipedia.org/wiki/Riemann_hypothesis

"Some consequences of the RH are also consequences of its negation, and are thus theorems. In their discussion of the Hecke, Deuring, Mordell, Heilbronn theorem, (Ireland & Rosen 1990, p. 359) say

The method of proof here is truly amazing. (1.0.0)
If the generalized Riemann hypothesis is true, then the theorem is true. (1.1.0)
If the generalized Riemann hypothesis is false, then the theorem is true. (1.2.0)
Thus, the theorem is true!! (punctuation in original)" (1.3.0)

We write Eqs. 1.0.1, 1.0.2, and 1.0.3 as:

If RH is equivalent to truthity, then RH is a tautology. (1.1.1)
(r=(%p>#p))>(r=(p=p)) ; NNNN TTTT NNNN TTTT (1.1.2)

If RH is equivalent to falsity, then RH is a tautology. (1.2.1)
(r=(%p<#p))>(r=(p=p)) ; CCCC TTTT CCCC TTTT (1.2.2)

RH is a tautology with Eq. 1.1.1 equivalent to Eq. 1.2.1. (1.3.1)
((r=(%p>#p))>(r=(p=p))) = ((r=(%p<#p))>(r=(p=p))) ;
FFFF TTTT **FFFF** TTTT (1.3.2)

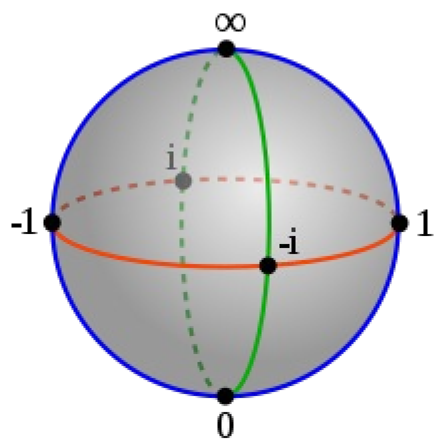
Eq. 1.3.2 as rendered is *not* tautologous, meaning the conjectured proof of Eq. 1.0.3 is refuted.

Remark: Eqs. 1.1.1 and 1.2.1 can be written to avoid the distinction of truthity-falsity versus tautology-contradiction, that is to rely on the latter, with the same result of 1.3.2. (1.4.1)
((r=(p=p))>(r=(p=p))) = ((r=(p@p))>(r=(p=p))) ;
FFFF TTTT **FFFF** TTTT (1.4.2)

Refutation of additive arithmetic operations in the Riemann sphere

We assume Meth8/VŁ4 with the designated *proof* value of Tautology and falsity value of Contingency.

Taken from: en.wikipedia.org/wiki/Riemann_sphere



Addition of complex numbers may be extended by defining, for $z \in \mathbf{C}$,
z+∞ = ∞ for any complex number z, (1.1)
and multiplication may be defined by z×∞ = ∞
for all nonzero complex numbers z, with ∞×∞ = ∞. (2.1)
Note that ∞–∞ and 0×∞ are left undefined. (3.1)
Unlike the complex numbers, the extended complex numbers do not form a field, since ∞ does not have a multiplicative inverse. Nonetheless, it is customary to define division on **C** ∪ {∞} by z/0 = ∞ and z/∞ = 0
for all nonzero complex numbers z,
with ∞/0 = ∞ and 0/∞ = 0. (4.1)
The quotients 0/0 and ∞/∞ are left undefined. (5.1)

LET p q r: **C**, z, ∞;
& And, ×; + Or, +, ∪ ; - Not Or, -; \ Not And, /; < Not Imply, ∈;
= Equivalent, =; @ Not Equivalent; % possibility, for one or some; # necessity, for all;
(%p>#p) one, 1; ((%p>#p)-(%p>#p)) zero, 0; (r@r) undefined.

((q<p)&#q) > ((q+r)=r) ; TTCT TTTT TCTT TTTT (1.2)

((q<p)&#(q@((%p>#p)-(%p>#p)))) > (((q&r)=r)&((r&r)=r)) ;
TTTT TTTT TTTT TTTT (2.2)

((r-r)&(((%p>#p)-(%p>#p))&r)) = (r@r) ; TTTT TTTT TTTT TTTT (3.2)

((p+r)&#(q@((%p>#p)-(%p>#p)))) >
((((q\((%p>#p)-(%p>#p)))=r)&((q\r)=((%p>#p)-(%p>#p))))
& (((r\((%p>#p)-(%p>#p)))=r)
& ((((%p>#p)-(%p>#p))\r)=((%p>#p)-(%p>#p))))) ;
TTTC TTCC TTTC TTCC (4.2)

((((%p>#p)-(%p>#p))\((%p>#p)-(%p>#p)))&(r\r)) = (r@r) ;
CCCC TTTT CCCC TTTT (5.2)

Eqs. 2.2 and 3.2 as rendered are tautologous. This means the definition of multiplication for extended complex numbers and undefined values of $\infty-\infty$ and $0\times\infty$ are theorems.

Eq. 1.2 is *not* tautologous. This means the definition of addition for extended complex numbers is not a theorem.

Eqs. 4.2 and 5.2 are *not* tautologous. This means the custom of forcing a field definition for extended complex numbers is mistaken as are the undefined values of the quotients $0/0$ and ∞/∞.

Riemann zeta function, Caceres Proposition 6

We evaluate this paper:

Caceres, P. (2018). "Riemann zeta function – constants, approximations, and some related functions". vixra.org/pdf/1803.0150v1.pdf

We assume the apparatus and method of Meth8/VL4, with the designated *proof* value of T.

LET p, q, r, s, v, w, x, y, z:
x1; x2; y1; y2; zeta; w; x; y; z.
w = +(-1)^0.5; ~w = -(-1)^0.5; i* = (w+~w).

And let's call:

x(z) = x1(z) +i* x2(z) (7.1.1)
(x&z) = ((p&z)+((w+~w)&(q&z))) ;
TFFF TFFF TFFF TFFF,
FTTT FTTT FTTT FTTT (7.1.2)

y(z) = y1(z) +i* y2(z) (7.2.1)
(y&z) = ((r&z)+((w+~w)&(s&z))) ;
TTTT FFFF FFFF FFFF,
FFFF TTTT TTTT TTTT (7.2.2)

In general, we can now express that any solution in ℂ of ζ(z) as:

ζ(z) = [x1(z)-y1(z)] + i * [x2(z)-y2(z)] (7.3.1)
(v&z) = (((p&z)-(r&z))+((w+~w)&((q&z)-(s&z)))) ;
FFFT FFTT FTFT TTTT,
TTTF TTFF TFTF FFFF (7.3.2)

and: [**Caceres Proposition 6**]

ζ(z) = x(z) - y(z) (7.4.1)
(v&z) = ((x&z)-(y&z)) ; FFFF FFFF FFFF FFFF,
TTTT TTTT TTTT TTTT (7.4.2)

Eqs. 7.3.2 and 7.4.2 as rendered are *not* tautologous.

While Eq. 7.3.2 is supposed to equal 7.4.2, and obviously is not

(((p&z)-(r&z))+((w+~w)&((q&z)-(s&z)))) = ((x&z)-(y&z)) ;
FFFT FFTT FTFT TTTT (7.5.2)

we try to coerce Eq. 7.5.2 into tautology by replacing the Equivalent connective with the Imply connective, but the result table is the same as for Eq. 7.5.2 as *not* tautologous.

This refutes Caceres Proposition 6.

Refutation of Riemann hypothesis by two zeta properties

Abstract: Properties of the zeta function of the Riemann hypothesis are *not* confirmed as tautologous and hence refute it.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

LET p, q, s: ζ, q, s;
~ Not; & And; + Or; - Not Or; > Imply; = Equivalent; @ Not Equivalent;
% possibility, for one or some; # necessity, for every or all;
(q@q) ordinal zero 0; (%q>#q) ordinal one 1.

From: Rigamonti, N. (2018). Two properties at the base of the Riemann hypothesis. vixra.org/pdf/1812.0350v1.pdf nicolo.rigamonti1@gmail.com

$\zeta(s)=\zeta(1-s)$ (2.1)
(p&s)=(p&((%q>#q)-s)) ; TNTN TNTN T**F**T**F** T**F**T**F** (2.2)

$\zeta(\bar{s})=\overline{\zeta(s)}$ (3.1), (4.1)
(p&~s)=~(p&s) ; **F**T**F**T **F**T**F**T **F**T**F**T **F**T**F**T (3.2), (4.2)

Since $\zeta(s)=0$, $\overline{\zeta(s)}=0$ and so $\zeta(s)=\overline{\zeta(s)}$ (4A.2.1)
(((p&s)=(q@q))>(~(p&s)=(p@p)))>((p&s)=~(p&s)) ;
TTTT TTTT T**F**T**F** T**F**T**F** (4A.2.2)

Since $\overline{\zeta(s)}=\zeta(\bar{s})$, $\zeta(s)=\zeta(\bar{s})$ (4A.3.1)
(~(p&s)=(p&~s))>((p&s)=(p&~s)) ; T**F**T**F** T**F**T**F** T**F**T**F** T**F**T**F** (4A.3.2)

$\zeta(s)=\zeta(\bar{s})$ (5.1)
(p&s)=(p&~s) ; T**F**T**F** T**F**T**F** T**F**T**F** T**F**T**F** (5.2)

$\zeta(s)=\zeta(1-s)$ } [Eq. 2.1]
$\zeta(s)=\zeta(\bar{s})$ } [Eq. 5.1] (6.1)
((p&s)=(p&((%q>#q)-s)))=((p&s)=(p&~s)) ;
TCTC TCTC TTTT TTTT (6.2)

Remark 6.1: Eqs. 6.1 reduce to a more compact equivalence with the same truth table result in Eq. 6.2 as: (p&((%q>#q)-s))=(p&~s). (6.2.alt)

Eqs. 2-6 as rendered are *not* tautologous. This means properties of the zeta function of the Riemann hypothesis refute it.

Refutation of infallible canon law in the Roman Catholic Church (RCC)

Abstract: The conjecture that traditional Church teaching can not contradict itself, from the catholic catechism (ca. 94-100), is refuted.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

LET p: canon law;
~ Not; > Imply; < Not imply; = Equivalent;
(p=p) Tautology as designated *proof* value.

From: ncregister.com/blog/astagnaro/traditional-church-teaching-can-never-contradict-itself
[The author is known as a professional stage magician.]

Traditional Church teaching can never contradict itself, catholic catechism (94-100) :
"Neither the pope nor any individual Christian has the right to change God's law." (1.0)

We write this as expressed in *one* variable.

If canon law implies itself as a theorem, then it cannot be dis-asserted as such. (1.1)

(p>(p=p))>~(p>~(p=p)) ; **F**T**F**T **F**T**F**T **F**T**F**T **F**T**F**T (1.2)

Eq. 1.2 as rendered in not tautologous, meaning canon law of the RCC can be dis-asserted as such and hence is fallible and thus subject to contradiction.

Remark: The antecedent as "canon law implies proof of itself" for p>(p=p) means p as a non-tautology implying itself as a tautology. In other words, **F**T**F**T > TTTT = TTTT. The consequent as "not (canon law implies not proof of itself)" is also **F**T**F**T. Hence, TTTT > **F**T**F**T = **F**T**F**T, not a theorem.

Roman Catholic Church: Erasmus contra Luther controversy

Erasmus stayed in the Church to counter contradictory doctrine and purge it.

Luther, while minimally in the Church, effectively departed from the Church (as evidenced by his subsequent non Swedish followers).

The issue to stay and cleanse or to leave and commence anew is tested by Meth8.

The conjecture is:

> If the necessity of the body of Christ implies the Church, and that implies the necessity of Christians as members of the Church, then possibly contradictory doctrines arise from members (due to the nature of original sin),
> it follows then that
> the necessity of members in the Church in the Body of Christ implies that no contradictory doctrine can survive coming from the members and the Church.
>
> LET: p Church; q Body of Christ; r Christian, a member; s contradictory doctrine
>
> ((#(q>p) > (#r<p)) > %(s<r)) > ((#(r<p)<q) > (~s<(r&p))); validated as tautology

This means Erasmus did the logically correct thing.

Roman Catholic Church: Infallibility and the Historic Church

Logical evaluation of infallibility of Pius IX from First Vatican Council (1869/70)

We evaluated the sequential assertions in the captioned as conjectures using the Meth8 modal logic model checker. The tool implemented variant system VŁ4, the resuscitated four valued logic of Łukasiewicz, in five models. Truth tables are presented as the first two rows of four of Model 1, with the designated truth value of Tautologous. The other logical values mean Contingent, Non contingent, and contradictory for the 2-tuple {11, 10, 01, 00}.

The argument proceeds in four Chapters as:

I. Institution of apostolic primacy of Peter
II. Perpetuity of apostolic primacy in Roman pontiffs
III. Power and authority of apostolic primacy in Pius IX
IV. Infallible teaching of the Roman pontiff, viz, Pius IX

From: catholicplanet.org/councils/20-Pastor-Aeternus.htm

> This English translation by Cardinal Henry Edward Manning, 1871 is attributed to unspecified editing by Ronald L. Conte Jr.

First Vatican Council 1869 to 1870 under Pope Pius IX

FIRST DOGMATIC CONSTITUTION ON THE CHURCH OF CHRIST

PASTOR AETERNUS [of our predecessors]

> (This section is not relevant to the conjectures.)

CHAPTER I.
ON THE INSTITUTION OF THE APOSTOLIC PRIMACY IN BLESSED PETER.

We therefore teach and declare that, according to the testimony of the Gospel, the primacy of jurisdiction over the universal Church of God was immediately and directly promised and given to Blessed Peter the Apostle by Christ the Lord.

For it was to Simon alone, to whom he had already said, "You shall be called Cephas" (John 1:42), that the Lord, after the confession made by him, saying, "You are the Christ, the Son of the living God", addressed these solemn words: "Blessed are you, Simon son of Jonah. For flesh and blood has not revealed this to you, but my Father, who is in heaven. And I say to you, that you are Peter, and upon this rock I will build my Church, and the gates of Hell shall not prevail against it. And I will give you the keys of the kingdom of heaven. And whatever you shall bind on earth shall be bound, even in heaven. And whatever you shall release on earth shall be released, even in heaven." (Mt 16:16-19).

> LET: p papacy; q apostolic primacy; r Peter
> > Imply; & And; = Equivalent to; ~ Not
> # necessarily, the universal quantifier ∀;
> % possibly, the existential quantifier ∃
> vt tautologous; nvt not tautologous

We map the above into the words:

> "Both Peter appointed the chief apostle as equivalent to apostolic primacy, and apostolic primacy as equivalent to holding the keys of a papacy imply the existence of a papacy as equivalent to Peter." (1.1)

In Meth8 this is:

> ((r= q) & (q= p)) > (%p= r) ; nvt; NTTT TTTT (1.1.1)

Eq 1.1 may be rewritten as the logical equivalent in words as

> "Both Peter appointed the chief apostle as equivalent to apostolic primacy, and apostolic primacy as equivalent to holding the keys of a papacy imply a papacy as equivalent to the existence of Peter." (1.2)

> ((r= q) & (q= p)) > (p=%r) ; nvt; NTTT TTTT (1.2.2)

The truth table fragments are in the state closest to proof, but denied by the Non contingent value.

We note that that a stronger refutation replaces the existential quantifier % as "the existence of" with the universal quantifier # as "the necessity of".

We purposely avoid an analysis of the derivative word meanings for Petros and Cephas, such as that of St Augustine who stated the Church was not built on Peter (*super Petrum*) but rather explicitly on the rock (*super petram*), viz, on the confession of the faith of the Apostle. (See Bishop Joseph Strossmayer in a speech opposing papal infallibility to the Vatican Council of 1870, from an Italian version published at Florence, reprinted from "The Bible Treasury", No. 195, August, 1872, pamphlet published by Loizeaux Brothers, New York. The speech also appeared in the Sydney Morning Herald, Monday, October 16, 1871, pg. 3.)

And it was upon Simon alone that Jesus, after His Resurrection, bestowed the jurisdiction of Chief Pastor and Ruler over all His fold, by the words: "Feed my lambs. Feed my sheep." (John 21:15-17).

At open variance with this clear doctrine of Holy Scripture, as it has ever been understood by the Catholic Church, are the perverse opinions of those who, while they distort the form of government established by Christ the Lord in His Church, deny that Peter, in his single person, preferably to all the other Apostles, whether taken separately or together, was endowed by Christ with a tautologous and proper primacy of jurisdiction; or of those who assert that the same primacy was not bestowed immediately and directly upon Blessed Peter himself, but upon the Church, and through the Church on Peter as her Minister.

If anyone, therefore, shall say that Blessed Peter the Apostle was not appointed the Prince of all the Apostles and the visible Head of the whole Church Militant; or that the same, directly and immediately, received from the same, Our Lord Jesus Christ, a primacy of honor only, and not of tautologous and proper jurisdiction; let him be anathema.

We note that from the character or word count above, about 50% of Chapter I relates to institution of apostolic primacy of Peter, and 50% relates to the penalty of anathema for its contradiction. (In each of the subsequent three chapters remaining, shortened declarations of anathema are also included, rather than at the end of the document, as is customary, to avoid self-conscious repetition.)

CHAPTER II.
ON THE PERPETUITY OF THE PRIMACY OF BLESSED PETER IN THE ROMAN PONTIFFS.

We restate this argument in the abstract state and without citation as:

"The perpetuity of episcopal orders, excluding claims of primacy, as accepted by all geographical branches of the Historic Church, is a historical fact." (2)

CHAPTER III.
ON THE POWER AND NATURE OF THE PRIMACY OF THE ROMAN PONTIFF.

We restate this argument in the abstract state and without citation as:

"The span of control of the Roman pontiff as successor to Peter extends over all geographical branches of the Historic Church, as declared by Roman Catholic Ecumenical Councils not recognized universally by the Historic Church." (3)

CHAPTER IV.
ON THE INFALLIBLE TEACHING OF THE ROMAN PONTIFF

We restate this argument in the abstract state and without citation as:

"Apostolic primacy includes the supreme power of inerrant teaching *ex Cathedra*."(4)

From Chapter I, Eq 1.1.1 and 1.2.2, we showed such apostolic primacy,
as defined by the Roman Church, is not tautologous by modal logic.

Hence Chapters II, III, IV are rendered moot.

Refutation of Roman Catholic canon law and by silence of the Holy Ghost present at epiclesis

Abstract: The conjecture that traditional Church teaching can not contradict itself, from the Roman Catholic Church (RCC) catechism, is refuted. From silence in the 1983 Code of Canon Law (CCL), this leads to the absence of the Holy Ghost in the epiclesis and a null priest host.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

LET p, q, r, s: canon law, Holy Ghost, epiclesis, consecrated host;
~ Not; > Imply; < Not imply; = Equivalent;
(p=p) Tautology as designated *proof* value.

From: ncregister.com/blog/astagnaro/traditional-church-teaching-can-never-contradict-itself
[The author is known as a professional stage magician.]

Traditional Church teaching can never contradict itself, catholic catechism (94-100) :
"Neither the pope nor any individual Christian has the right to change God's law." (1.0)

We write this as expressed in *one* variable.

If canon law implies itself as a theorem, then it cannot be dis-asserted as such. (1.1)

(p>(p=p))>~(p>~(p=p)) ; **F**T**F**T **F**T**F**T **F**T**F**T **F**T**F**T (1.2)

Eq. 1.2 as rendered in not tautologous, meaning canon law of the RCC can be dis-asserted as such and hence is fallible and thus subject to contradiction.

Remark: The antecedent as "canon law implies proof of itself" for p>(p=p) means p as a non-tautology implying itself as a tautology. In other words, **F**T**F**T > TTTT = TTTT. The consequent as "not (canon law implies not proof of itself)" is also **F**T**F**T. Hence, TTTT > **F**T**F**T = **F**T**F**T, not a theorem.

From: Coriden, J.A. (2007). "Holy Spirit and Church governance". New Theology Review. theophilusjournal.org/index.php/ntr/article/download/216/389

What does the 1983 Code of Canon Law (CCL) have to say about the Spirit's influence and activity in the church? Almost nothing. The Code simply does not reflect the church's beliefs about the Holy Spirit found in the New Testament and the documents of the Second Vatican Council. The Code mentions the Holy Spirit in seven canons [with sections]: 206[1]; 369; 573[1]; 605; 747[1]; 869.

We write CCL to mean: If the Holy Ghost is truthful, then epiclesis invocation of the Holy Ghost implies a validly consecrated Host. (2.1) (q=(p=p))>((r>q)>(s=(p=p))) ;   TT**FF** TT**FF** TTTT TTTT   (2.2)

We apply Eqs. 1.1 as antecedent to imply 2.1 as consequent. In words:

If canon law implies itself as a theorem, then it cannot be dis-asserted as such, then if the Holy Ghost is truthful, then epiclesis invocation of the Holy Ghost implies a validly consecrated Host. (3.1)

((p>(p=p))>~(p>~(p=p)))>((q=(p=p))>((r>q)>(s=(p=p)))) ;
TTT**F** TTT**F** TTTT TTTT   (3.2)

Remark 3.1: If Eq 3.1 is weakened to read :

If canon law implies itself as a theorem, then it cannot be dis-asserted as such, then if the Holy Ghost *implies* truthfulness, then epiclesis invocation of the Holy Ghost implies a validly consecrated Host. (3.3.1)

((p>(p=p))>~(p>~(p=p)))>((q>(p=p))>((r>q)>(s=(p=p)))) ;
T**FTF** TTT**F** TTTT TTTT   (3.3.2)

Eqs. 3.3.2 is further from tautology by one value of **F** for contradiction, than 3.2.

What follows from Eqs. 3.1 and 3.3.1 is this question: What happens when Pope Francis as the Vicar of Jesus Christ, that is the stand-in personification of the Holy Ghost, is silent (on such matters as the clergy abuse exposed in courts of law and widely reported in the media). (4.0)

We write this question as: If the Holy Ghost who implies truthfulness is silent, implying neither affirmation nor denial, then the Holy Ghost implies a Host which is not equivalent to validity or invalidity, that is, equivalent to a nullity. (4.1)

((q>(p=p))>~((p=p)+(p@p)))>(q>(s@((p=p)+(p@p)))) ;
TTTT TTTT TTTT TTTT   (4.2)

Remark 4.2: Eq. 4.2 is tautologous, meaning if the Holy Ghost is silent, then what is confected is a nullity, that is, the result is void of the Holy Ghost.

The results from Eqs. 1.2, 2.2, 3.2, and 4.2 as rendered are that: the CCL is not infallible; the Holy Ghost implies valid Sacramental Host; regardless of the CCL, the Holy Ghost implies a valid Sacramental Host; when the Bishop of Rome as a personification of the Holy Ghost is silent on any matter, then any result derived therefrom is a nullity. It is the last point that proves the Bishop of Rome is incapable of speaking ex cathedra in any capacity for the Holy Ghost, thereby relegating encyclicals as fallible opinions du jour.

Roman Catholic Church: Magisterium

A logical assessment of tradition, scripture, and authority in "Dei Verbum", 1965

[The text of Chapter 2 in *Dei Verbum* follows at the end with assertions in bold.]

1. We evaluate the order of appearance of non scriptural citations in Articles 7-10 based on Church dates in bold:

7.: 2. Council of Trent, **1545**; 3. Irenaeus, **180**
8.: 4. Second Council of Nicea, **787**, Fourth Council of Constance, **1414**;
5. First Vatican Council, **1869**
9.: 6. Council of Trent, **1545**
10.: 7. Pius XII, **1950**; 8. First Vatican Council, **1869**; 9. Pius XII, **1950**

The argument of Articles 7-10 does not draw on citations to be sequentially increasing in time, viz:

180, 787, 1414, 1545, 1545, 1869, 1869, 1950, 1950.

2. We next evaluate the final assertion in Article 10 of:

[T]hat sacred tradition, Sacred Scripture and the teaching authority of the Church ... **are so linked and joined together that one cannot stand without the others.** (1)

We map this using the Meth8 modal logic model checker in script.

LET: p sacred tradition; q sacred scripture; r teaching authority;
\# necessity (for all instances, the universal quantifier ∀);
% possibility (for at least one instance, the existential quantifier ∃);
\#q the necessity of Sacred Scripture;
%r the possibility of teaching authority of the Church;
& And; + Or; > Imply; nvt not tautologous

We rewrite Eq 1 as:

If the sacred tradition and the necessity of Sacred Scripture and the possibility of Church teaching authority, then not either the sacred tradition or the necessity of Sacred Scripture or the possibility of
the Church teaching authority. (2)

Eq 2 is also rewritten in an equivalent expression as:

The sacred tradition and the necessity of Sacred Scripture and the possibility of Church teaching authority all imply not separately that either the sacred tradition or the necessity of Sacred Scripture or the possibility of the Church teaching authority. (3)

(p & (#q & %r)) > ~(#p + (#q +%r)) ; nvt (4)

In the five models of Meth8, repeating fragments of the respective truth tables are:

TTTT TTT**C** EEEE EEE**U** EEEE EEEE EEEE EEE**P** EEEE EEE**I**

where the designated truth values are T and E with the first letter definiens as Tautologous, Evaluated, Unevaluated, Proper, and Improper.

This means according to the VŁ4 logic system of Meth8 that Eq 2 or 3 is not tautologous, and hence Eq 1 is found to be non sequitur and mistaken.

From: http://www.cin.org/v2revel.html:

CHAPTER II HANDING ON DIVINE REVELATION

7. In His gracious goodness, God has seen to it that what He had revealed for the salvation of all nations would abide perpetually in its full integrity and be handed on to all generations. Therefore Christ the Lord in whom the full revelation of the supreme God is brought to completion (see Cor. 1:20; 3:13; 4:6), commissioned the Apostles to preach to all men that Gospel which is the source of all saving truth and moral teaching,[1] and to impart to them heavenly gifts. This Gospel had been promised in former times through the prophets, and Christ Himself had fulfilled it and promulgated it with His lips. This commission was faithfully fulfilled by the Apostles who, by their oral preaching, by example, and by observances handed on what they had received from the lips of Christ, from living with Him, and from what He did, or what they had learned through the prompting of the Holy Spirit. The commission was fulfilled, too, by those Apostles and apostolic men who under the inspiration of the same Holy Spirit committed the message of salvation to writing.[*2. citing Council of Trent, 1545*]

But in order to keep the Gospel forever whole and alive within the Church, the Apostles left bishops as their successors, "handing over" to them "the authority to teach in their own place."[3] This sacred tradition, therefore, and Sacred Scripture of both the Old and New Testaments are like a mirror in which the pilgrim Church on earth looks at God, from whom she has received everything, until she is brought finally to see Him as He is, face to face (see 1 John 3:2).

8. And so the apostolic preaching, which is expressed in a special way in the inspired books, was to be preserved by an unending succession of preachers until the end of time. Therefore the Apostles, handing on what they themselves had received, warn the faithful to hold fast to the traditions which they have learned either by word of mouth or by letter (see 2 Thess. 2:15), and to fight in defense of the faith handed on once and for all (see Jude 1:3) [*4. citing Second Council of Nicea, 787, and Fourth Council of Constance, 1414*]

Now what was handed on by the Apostles includes everything which contributes toward the holiness of life and increases in faith of the people of God; and so the Church, in her teaching, life and worship, perpetuates and hands on to all generations all that she herself is, all that she believes. **This tradition which comes from the Apostles develops in the Church with the help of the Holy Spirit.**[*5. citing First Vatican Council, 1869*] For there is a growth in the understanding of the realities and the words which have been handed down. This happens through the contemplation and study made by believers, who treasure these things in their hearts (see Luke, 2:19, 51) through a penetrating understanding of the spiritual realities which they experience, and through the preaching of those who have received through episcopal succession the sure gift of truth. For as the centuries succeed one another, the Church constantly moves forward toward the fullness of divine truth until the words of God reach their complete fulfillment in her.

The words of the holy fathers witness to the presence of this living tradition, whose wealth is poured into the practice and life of the believing and praying Church. Through the same tradition the Church's full canon of the sacred books is known, and the sacred writings themselves are more profoundly understood and unceasingly made active in her; and thus God, who spoke of old, uninterruptedly converses with the bride of His beloved Son; and the Holy Spirit, through whom the living voice of the Gospel resounds in the Church, and through her, in the world, leads unto all truth those who believe and makes the word of Christ dwell abundantly in them (see Col. 3:16).

9. Hence there exists a close connection and communication between sacred tradition and Sacred Scripture. For both of them, flowing from the same divine wellspring, in a certain way merge into a unity and tend toward the same end. For Sacred Scripture is the word of God inasmuch as it is consigned to writing under the inspiration of the divine Spirit, while sacred tradition takes the word of God entrusted by Christ the Lord and the Holy Spirit to the Apostles, and hands it on to their successors in its full purity, so that led by the light of the Spirit of truth they may in proclaiming it preserve this word of God faithfully, explain it, and make it more widely known. **Consequently it is not from Sacred Scripture alone that the Church draws her certainty about everything which has been revealed. Therefore both sacred tradition and Sacred Scripture are to be accepted and venerated with the same sense of loyalty and reverence.** [*6. citing Council of Trent, 1545*]

10. **Sacred tradition and Sacred Scripture form one sacred deposit of the word of God, committed to the Church.** Holding fast to this deposit the entire holy people united with their shepherds remain always steadfast in the teaching of the Apostles, in the common life, in the breaking of the bread and in prayers (see Acts 2, 42, Greek text), so that holding to, practicing and professing the heritage of the faith, it becomes on the part of the bishops and faithful a single common effort.[*7. citing Pius XII, 1950*]

But the task of authentically interpreting the word of God, whether written or handed on,[*8. citing First Vatican Council, 1869*] has been entrusted exclusively to the living teaching office of the Church.[*9. citing Pius XII, 1950*] whose authority is exercised in the name of Jesus Christ. **This teaching office is not above the word of God, but serves it**, teaching only what has been handed on, listening to it devoutly, guarding it scrupulously and explaining it faithfully in accord with a divine commission and with the help of the Holy Spirit, it draws from this one deposit of faith everything which it presents for belief as divinely revealed.

It is clear, therefore, that **sacred tradition, Sacred Scripture and the teaching authority of the Church,** in accord with God's most wise design, **are so linked and joined together that one cannot stand without the others,** and that all together and each in its own way under the action of the one Holy Spirit contribute effectively to the salvation of souls.

CHAPTER II

1. cf. Matt. 28:19-20, and Mark 16:15; Council of Trent, session IV, Decree on Scriptural Canons: Denzinger 783 (1501).
2. cf. Council of Trent, loc. cit.; First Vatican Council, session III, Dogmatic Constitution on the Catholic Faith, Chap. 2, "On revelation:" Denzinger 1787 (3005).
3. St. Irenaeus, "Against Heretics" III, 3, 1: PG 7, 848; Harvey, 2, p. 9.
4. cf. Second Council of Nicea: Denzinger 303 (602); Fourth Council of Constance, session X, Canon l: Denzinger 336 (650-652).
5. cf. First Vatican Council, Dogmatic Constitution on the Catholic Faith, Chap. 4, "On Faith and Reason:" Denzinger 1800 (3020).
6. cf. Council of Trent, session IV, loc. cit.: Denzinger 783 (1501).
7. cf. Pius XII, apostolic constitution, "Munificentissimus Deus," Nov. l, 1950: A.A.S. 42 (1950) P. 756, Collected Writings of St. Cyprian, Letter 66, 8: Hartel, III, B, p. 733: "The Church [is] people united with the priest and the pastor together with his flock."
8. cf. First Vatican Council, Dogmatic Constitution on the Catholic Faith, Chap. 3 "On Faith." Denzinger 1792 (3011).
9. cf. Pius XII, encyclical "Humani Generis," Aug. 12, 1950: A. A.S. 42 (1950) PP. 568-69: Denzinger 2314 (3886).

Refutation of the Primacy of the Roman See

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or in repeating fragments from 128-tables for more variables.

LET p, q, r, s: Pontiff, heart, Christ, sovereign or sacred;
~ Not; & And; > Imply; = Equivalent.

From: Pope Pius XI. (1928). Miserentissimus redemptor. w2.vatican.va/content/pius-xi/en/encyclicals/documents/hf_p-xi_enc_19280508_miserentissimus-redemptor.html

The argument for Primacy of the Roman See is paraphrased as:

"If *Pontiff Christ* implies *Sovereign Pontiff*, then *Sovereign Pontiff* is *Pontiff Christ*." (1.1)

((p&r)>(s&p))>((s&p)=(p&r)) ; TTTT TTTT TFTF TTTT (1.2)

Eq. 1.2 is *not* tautologous, although nearly so but due to two F values. Hence the argument for Roman Primacy is not tautologous.

Remark: Eq. 1.1 admits in the consequent to setting the sitting Pontiff equivalent to Jesus Christ as the Head of the Historic Church. From that is derived the Pontiff's title of Vicar in Jesus Christ, that is, the Pontiff is Christ's stand-in and hence infallible for matters theological.

Refutation of the vision of the Sacred Heart of Jesus

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or in repeating fragments from 128-tables for more variables.

LET p, q, r, s: Pontiff, heart, Christ, sovereign or sacred;
~ Not; & And; > Imply; = Equivalent.

From: Pope Pius XI. (1928). Miserentissimus redemptor. w2.vatican.va/content/pius-xi/en/encyclicals/documents/hf_p-xi_enc_19280508_miserentissimus-redemptor.html

The argument for the Sacred Heart of Jesus, a vision, is paraphrased as:

"If Christ implies his Sacred Heart, then his Sacred Heart is Christ." (2.1)

(r>(s&q))>((s&q)=r) ; TTTT TTTT TTFF TTTT (2.2)

Eq. 2.2 is *not* tautologous, although nearly so but due to two F values. Hence the argument for the Sacred Heart of Jesus is not tautologous.

Remark: If an apparition is defined as a vision confirmed by more than one contemporaneous observer, then the distinction of an apparition, as the observer *not* connecting it to a person, versus the vision, as a single observer connecting it to a person, is moot.

What follows is that the Alliance of the Sacred Heart of Jesus with the Sacred Heart of Mary, also a vision, is not tautologous.

What further follows is that the tautology of the Sacred Heart of Mary, a vision, is not directly known.

Remarks:

1. It is possible to fashion a non-sacred argument for the heart of Mary by excluding the sacred variable, and re-defining Pontiff as Mary, that is, "If Mary implies her heart, then her heart is Mary": (p>q))>(q=p) ; TTFT TTFT TTFT TTFT, also *not* tautologous.

2. To produce an alliance of the two hearts, as such, in the form of the Sacred Heart of Jesus implies the heart of Mary, renders: ((r>(s&q))>((s&q)=r)) > ((p>q)>(q=p)) ; TTFT TTFT TTTT TTFT, also *not* tautologous.

Roman Catholic Church: Tradition above scripture

Logical evaluation of infallibility in the formula for the Historic Church

We previously evaluated infallibility using the Meth8 modal logic model checker as follows in words:

> "Both Peter appointed the chief apostle as equivalent to apostolic primacy, and apostolic primacy as equivalent to holding the keys of a papacy imply the existence of a papacy as equivalent to Peter." (1.1)

or

> "Both Peter appointed the chief apostle as equivalent to apostolic primacy, and apostolic primacy as equivalent to holding the keys of a papacy imply a papacy as equivalent to the existence of Peter." (1.2)

with

LET: p Papacy; q Apostolic primacy; r Peter
\> Imply; & And; = Equivalent to; ~ Not
necessarily, the universal quantifier ∀;
% possibly, the existential quantifier ∃
vt tautologous; nvt not tautologous
for
((r= q) & (q= p)) > (%p= r) ; nvt ; NTTT TTTT (1.1.1)
or
((r= q) & (q= p)) > (p=%r) ; nvt ; NTTT TTTT (1.2.1)

We noted a stronger refutation replaces the existential quantifier % as "the existence of" with the universal quantifier # as "the necessity of", with the same net effect where explicitly:

((r= q)& (q= p)) > (#p= r) ; nvt ; TTTN TTTT (1.3.1)

For the formula of the Historic Church we include additional items:

LET: s Scripture; t Tradition; u Church

We are careful to define the Church as the Body of Christ, viz, pre-existent as to physical scripture, tradition, or ecclesiastical infallibility.

The formula we test in words is as follows:

> "If both Peter appointed the chief apostle as equivalent to apostolic primacy, and apostolic primacy as equivalent to holding the keys of a papacy imply the existence of a papacy as equivalent to Peter, then if both the Church implying scripture and scripture implying tradition imply the existence of a Church as equivalent to scripture and tradition." (2.1)

where

((((r= q)& (q= p))> (%p= r))= u) > (((u> s)& (s> t))> (%u= (s & t))) ;
nvt ; NTTT TTTT TTTT TTTT
[fragment from 128-row table] (2.1.1)

Eq 2.1 is not validated as tautologous because the Church as equivalent to the definition of infallibility was not validated as tautologous in Eqs 1.1.1 or 1.2.1.

A definition of the Church as the Body of Christ is terms of scripture and tradition is in words as follows:

> "If both the Church implying scripture and scripture implying tradition imply a Church implies the existence of both Scripture and Tradition." (3.1)

> ((u> s)& (s> t))> (u>%(s & t)) ;
> vt ; TTTT TTTT TTTT TTTT (3.1.1)

However, the consequent in Eq 2.1 above reads:

> "[I]f both the Church implying scripture and scripture implying tradition imply the existence of a Church as equivalent to scripture and tradition." (2.1)

A difference between Eq 2.1 and 3.1 is in Eq 3.1 where the existential quantifier is applying to the Church and not to scripture and tradition. This is because the object is to prove the existence of the Church as previously evaluated in terms of infallibility in the antecedent of Eqs 1.1.1 and 1.2.1, but with additional terms in Eq 3.1.

Another difference is in Eq 2.1 where the existence of a Church is held equivalent to both scripture and tradition, a higher level of truth than in Eq 3.1 where there is not equivalency but an implication.

Refutation of the 12th promise of St Alacoque

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or in repeating fragments from 128-tables for more variables.

From: aleteia.org/2018/06/07/4-visionaries-who-saw-the-sacred-heart-of-jesus-and-the-messages-they-received/

> "St. Margaret Mary Alacoque In 1673, a French Visitandine (Visitation) nun named Margaret Mary Alacoque had visions of Jesus, wherein he asked the Church to honor his Most Sacred Heart. Among the [12] promises he communicated, Jesus said to St. Margaret Mary, "In the excess of the mercy of my heart, I promise you that my all powerful love will grant to all those who will receive communion on the First Fridays, for nine consecutive months, the grace of final repentance: they will not die in my displeasure, nor without receiving the sacraments; and my heart will be their secure refuge in that last hour."

LET p, q, r, s: Jesus, First Friday communion, grace of final repentance, Sacred Heart;
> Imply, greater than; # necessity, for all or every; % possibility, for one or some.

"In the excess of the mercy of my heart"

(Jesus is required for this to be, so it is the necessity of Jesus that implies his Sacred Heart; in other words, his Sacred Heart is contingent on the necessity of Himself in the first place) (1.1)

#p>s ; `TCTC TCTC TTTT TTTT` (1.2)

"those who will receive communion on the First Fridays, for nine consecutive months, [imply] the grace of final repentance"

(the volitional act to receive communion is required, so the necessity of those receiving First Friday communion implies the grace of final repentance) (2.1)

#q>r ; `TTCC TTTT TTCC TTTT` (2.2)

"In the excess of the mercy of my heart, I promise you that my all powerful love will grant to all those who will receive communion on the First Fridays, for nine consecutive months, the grace of final repentance" (Eqs. 1.1 implies 2.1.) (3.1)

(#p>s)>(#q>r) ; `TCTC TTTT TCTC TTTT` (3.2)

Remark: One may reasonably read the consequent of "grace of final repentance" as injecting a *possibility* of grace of final repentance (in the case that the antecedent of First Friday communion is not fully fulfilled) to read as (#p>s)>(#q>%r), but the truth table remains unchanged as that of Eq. 3.2

Eq. 3.2 as rendered is *not* tautologous.

Refutation of provability of consistency with Rosser's theorem

Abstract: We evaluate provability of consistency with Rosser's theorem. A trivial theorem is found in the abstract, but a rule of necessitation and Rosser's theorem are *not* tautologous. While consistency is likely provable, the instant approach is refuted and thus not vindicating Hilbert.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) `C` non-contingency, ∇, ordinal 2; (%z>#z) `N` as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Artemov, S. (2019). The provability of consistency. arxiv.org/pdf/1902.07404.pdf
sartemov@gc.cuny.ed

F is constructively false iff PA proves 'for each x, there is a proof that x is not a proof of F.' (0.1.1)

LET q, r: F, x

~(#r>(q=(q=q)))>(q=(q@q)) ; `TTTT TTTT TTTT TTTT` (0.1.2)

[F]or any PA-derivation S we find a finitary proof that S does not contain 0=1 (0.2.1)

LET s: S

~(#(s=s)>((s=s)=(%s>#s))) = (s=s) ; `CCCC CCCC CCCC CCCC` (0.2.2)

Remark 2.1: If by "0=1" the intention is **F**=`T`, then Eq. 0.2.1 is rendered as:
~(#(s=s)>((s=s)=(s=s))) = (s=s) ; **`FFFF FFFF FFFF FFFF`** (0.2.3)

[A]ny finite sequence S of formulas is not a derivation of a contradiction. [Claim 1] (1.1.1)

~(%(s@s)>#s) = (s=s) ; `CCCC CCCC CCCC CCCC` (1.1.2)

Remark 1.1: We map "a contradiction" to mean at least one contradiction. That strengthens Eq. 1.1.1 from a contradiction to a falsity.

[T]here is a finitary proof p(S) that S is not a derivation of a contradiction. [Claim 2] (1.2.1)

LET p: p.

%(p&s)>~(%(s@s)>s) ; TTTT TTTT N**F**N**F** N**F**N**F** (1.2.2)

Rule of Necessitation: $\frac{\vdash F}{\vdash \Box F}$ (2.1)

LET p: F.

q>#q ; TTNN TTNN TTNN TTNN (2.2)

Remark 2.1: This rule potentially taints the remaining assertions.

Rosser sentence R and its negation ¬R are both constructively false.
The proof of Rosser's Theorem is syntactic and can be formalized in PA

$PA \vdash \neg\Box\bot \rightarrow (\neg\Box R \wedge \neg\Box\neg R).$ (4.1)

LET r: R.
~(#(r@r)=(r=r))>(~#r&~#~r) ; CCCC CCCC CCCC CCCC (4.2)

Remark 4.2: Rosser's theorem as rendered in Eq. 4.2 is *not* tautologous, *not* contradictory per se, but is a falsity.

Rota lattice theory distributive axiom

From: Gian-Carlo **Rota**. "The Many Lives of Lattice Theory". Notices of the AMS. 44:11. 1440-1445. December, 1997.

p. 1440, distributive:

(p+(q&r)) = ((p&q)+(p&r)) ; *not* tautologous

```
Model 1               Model 2.1             Model 2.2             Model 2.3.1           Model 2.3.2
TFTT TTFT TFTT TTFT   EUEE EEUE EUEE EEUE   EUEE EEUE EUEE EEUE   EUEE EEUE EUEE EEUE   EUEE EEUE EUEE EEUE
```

Russell paradox

From: en.wikipedia.org/wiki/Russell%27s_paradox

LET: nvt not tautologous

R = { x | x ∉ x }, then R ∈ R <=> R ∉ R. (R.1)
(r = (x>x)) > ((r<r) = (r>r)) ; nvt (R.2)

Russell's paradox as stated is nvt, but it is not a paradox or a contradiction.

In the formal presentation of Russell's "Naive Set Theory (NST), as the theory of predicate logic with a binary predicate ∈ and the following axiom schema of unrestricted comprehension:

∃ y ∀ x (x ∈ y <=> P (x)) (R.5)

for any formula *P* with only the variable *x* free. Substitute x ∉ x for P (x).
Then by existential instantiation (reusing the symbol *y*) and universal instantiation
y ∈ y <=> y ∉ y is a contradiction. Therefore, NST is inconsistent.": [∉ is >]

(%y&#x)&((x<y)=(p&x)) ; nvt (R.6)
for (p&x) substitute (x>x)
(%y&#x)&((x<y)=(x>x)) ; nvt *and* contradictory (R.7)

However there is a problem with the substitution of (p&x)=(x>x) if (p&x) is removed from the expression as in (7); the correct expression is (p&x)=(x>x), not (x>x) with truth table fragment:

(%y&#x)&((x<y)=((p&x)=(x>x))) ; nvt [but not *and* contradictory] (R.8)

```
FFFF FFNF   UUUU UUEU   UUUU UUUU   UUUU UUIU   UUUU UUPU    Step: 15
Model 1     Model 2.1   Model 2.2   Model 2.3.1 Model 2.3.2
```

Therefore Russell's NST is nvt, but it is *not* inconsistent as a contradiction.

Refutation of the Russell-Prawitz embedding

Abstract: We evaluate the Russell-Prawitz embedding as *not* tautologous. Hence atomization of universal instantiation does not follow (nor does proof reduction, weakening of dinaturality conversion, or strict simulation). These conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Santo, J.E.; Ferreira, G. (2019). The Russell-Prawitz embedding and the atomization of universal instantiation. arxiv.org/pdf/1909.01232.pdf jes@math.uminho.pt

1 Introduction The Russell-Prawitz translation of the intuitionistic propositional calculus **IPC** into second-order intuitionistic propositional calculus $\mathbf{NI}^2$, the latter based on the language only containing implication, conjunction and the second-order universal quantifier, rests on the following enco[d]ing of disjunction and absurdity

$$A \vee B := \forall X.((A \supset X) \wedge (B \supset X)) \supset X \text{ and } \bot := \forall X.X. \qquad (1.1.1)$$

LET p, q, r: A, B, X.

((p+q)=(((p>#r)&(q>#r))>r))&((r=r)=#r) ;
FFFF **F**`NNN` **FFFF** **F**`NNN` (1.1.2)

Remark 1.1.2: Eq. 1.1.2 is *not* tautologous, hence refuting the Russell-Prawitz embedding and therefore atomization of universal instantiation.

However in an effort to resuscitate the conjecture we present the truth table value results for the antecedent and consequent in Eq. 1.1.2:

(p+q)=(((p>#r)&(q>#r))>r) ; `TTTT` **F**`TTT` `TTTT` **F**`TTT` (1.1.2.1.2)

(r=r)=#r ; **FFFF** `NNNN` **FFFF** `NNNN` (1.1.2.2.2)

The antecedent and consequent of Eq. 1.1.2 are also *not* tautologous.

S5Π+ for propositional quantification

Holliday, Wesley H. "A note on algebraic semantics for S5 with propositional quantifiers". *Notre Dame Journal of Formal Logic*. March 2017

From:researchgate.net/publication/
313838323_A_Note_on_Algebraic_Semantics_for_S5_with_Propositional_Quantifiers

We use the Meth8 apparatus to evaluate equation (W) on which S5II+ is based for propositional quantification.

∃q(q∧∀p(p→(q→p))) (W.1)

%q&(q&(#p&(p>#(q>p)))) ; FFFN (W.2)

Because Meth8 does not validate Eq. W.2 as tautology, we conclude that S5Π+ for propositional quantification is not bivalent. This also confirms S5 is not a bivalent logic.

Refutation of sabotage modal logic (revisited)

Abstract: We evaluate six equations for a definition and properties of sabotage modal logic. The special symbols ■ and ♦ act as functions, so we assign them variable names. Because none is tautologous, sabotage modal logic is refuted.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r, s: ϕ, ψ, ■, ♦;
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⊢, ↦, ≻, ⊃; < Not Imply, less than, ∈, ≺, ⊂;
= Equivalent, ≡, ⊨, :=, ⇔, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤; (z@z) **F** as contradiction, Ø, Null, ⊥ ;
(%z<#z) C non-contingency, ∇, ordinal 2;
(%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).

From: Aucher, G.; van Benthem, J.; Grossi, G. (2017).
Modal logics of sabotage revisited. Journal of Logic and Computation, 2018. 28:2. 269–303.
academic.oup.com/logcom/article/28/2/269/4774578?guestAccessKey=ae079e7e-fe35-449d-af50-9e34493a615c
guillaume.aucher@irisa.fr, J.vanBenthem@uva.nl, D.Grossi@liverpool.ac.uk

Remark: Because the modal sabotage notations of ■ and ♦ act as functions, we assign them variable names.

... we define the usual abbreviations:

■ϕ≜ ¬ ♦¬ϕ (2.1.1)

(r&p)=(~s&~p) ; **F**T**F**T **FFFF** TTTT T**F**T**F** (2.1.2)

2.2 Some notable validities and expressible properties
We list some validities of SML that demonstrate how the deletion modality works:

p→■p (2.2.3.1)

p>(r&p) ; T**F**T**F** TTTT T**F**T**F** TTTT (2.2.3.2)

p→■(◊⊤→◊p) (2.2.5.1)

p>(r&(%(p=p)>#p)) ; **T** **F** T **F** TNTN **T** **F** T **F** TNTN (2.2.5.2)

$\Diamond\phi\wedge\Diamond\neg\,\phi\rightarrow\blacklozenge\blacklozenge\top$ (2.2.6.1)

(%p&%~p)>((s&s)&(p=p)) ; NNNN NNNN TTTT TTTT (2.2.6.2)

The fact that we are using propositional atoms instead of variables for formulas in the first five of the above validities is not accidental. Surprisingly, many prima facie valid-looking principles fail for SML in their full schematic form with all complex substitution instances once we realize that under a deletion modality, ordinary modalities can change their truth values. A good example is principle (2.2.5.1). Consider its schematic formulation

$\Box\phi\rightarrow\blacksquare(\Diamond\top\rightarrow\Diamond\phi)$ (2.2.7.1)

#p>(r&(%(p=p)>%p)) ; TCTC TTTT TCTC TTTT (2.2.7.2)

which states that if every accessible state satisfies ϕ, then after any link deletion, if the evaluation state still has a successor, it still has a ϕ-successor. The formula may fail if ϕ is modal, since deletion may happen deeper in the model and disrupt the truth of ϕ at successor states. ... In the above list, only the last item (2.2.6.1) is a schematic validity.

Another sign of strength for SML is its power to define frames up to isomorphism. For instance, it is a simple exercise to show that the formula

$\Diamond\top\wedge\Box\Diamond\top\wedge\,\blacksquare\Box\bot$ (14.1)

(%(p=p)&(#%(p=p)))&(r&#(p=p)) ;
FFFF NNNN **FFF**F NNNN (14.2)

is true in a model if and only if its underlying frame consists of one reflexive point.

Because the six Eqs. above are *not* tautologous, sabotage modal logic is refuted.

Refutation of the definition of Sacchetti's modal logics of provability

Abstract: We evaluate the definition of Sacchetti's modal logics of provability. It is *not* tautologous. Therefore it is a mistake to use it as a basis for constructing fixed point procedures.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨ ; - Not Or; & And, ∧; \ Not And;
> Imply, greater than, →; < Not Imply, less than, ∈;
= Equivalent, ≡; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊; # necessity, for every or all, ∀, □;
~(y < x) (x ≤ y), (x ⊆ y).

From: Kurahashi, T.; Okaw, Y. (2018). Effectively constructible fixed points in Sacchetti's modal logics of provability. arxiv.org/pdf/1811.12827.pdf kurahashi@n.kisarazu.ac.jp

"We give a purely syntactical proof of the fixed point theorem for Sacchetti's modal logics **K**+ $\Box(\Box^n p \rightarrow p) \rightarrow \Box p (n \geq 2)$ of provability. (1.0)
From our proof, an effective procedure for constructing fixed points in these logics is obtained."

Remark 1.0: At Eq. 1.0 we ignore **K** for □(p>q) > (□p>□q), as we previously show as tautologous, and rewrite using the power series $\Box^n$ as □*n.

[LET p, q, r, n: p, q, r, s.]

$$\Box(\Box^n p \rightarrow p) \rightarrow \Box p (n \geq 2) \quad (1.1)$$

#((#p&s)>p)>#(p&~((%r<#r)>s)) ; CCCC CCCC CCCC CCCC (1.2

Remark 1.2: The table result value of C for contingency as falsity is the closest pure state to **F** as contradiction. This means Eq. 1.2 is not a contradiction or a tautology, but rather an intermediate state of falsity.

Eqs. 1 as rendered are *not* tautologous, hence refuting Sacchetti's modal logics of provability. What follows to obtain a procedure for constructing fixed points in these logics is mistaken.

Refutation of the correspondence theory of Sahlqvist

Abstract: We evaluate the example of mapping the Sahlqvist formula of p ∧ ◊p → □p into corresponding quantified expressions. The formula is not a theorem, but the corresponding quantified expressions are theorems. Hence the mapping refutes the Sahlqvist correspondence theory. Therefore these failures are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Conradie, W.; Palmigiano, A.; Sourabh, S. (2018).
Algebraic modal correspondence: Sahlqvist and beyond. arxiv.org/pdf/1606.06881.pdf
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sumit.sourabh@gmail.com

Example 3.12. Let us consider the very simple Sahlqvist formula p ∧ ◊p → □p, (3.12.1.1)

Remark 3.12.1.1: The antecedent is [[p ∧ □p]], as in the text for Eq. 3.12.3.1.

LET p, q, r, u, x, y, z: z1, z2, r, u, x, y, z.

(p&%p)>#p ; TNTN TNTN TNTN TNTN (3.12.1.2)

Remark 3.12.1.2: Eq. 3.12.1.2 as rendered is *not* tautologous. However, we evaluate the steps of the algebraic modal correspondence in Eqs. 3.12.2-.6 below.

which locally corresponds to the property of having at most one R-successor12, i.e.
∀z∀u(Rxz ∧ Rxu → z = u). (3.12.2.1)

((r&(x&#z)) & (r&(x&#u))) > (#z=#u) ; TTTT TTTT TTTT TTTT (3.12.2.2)

Remark 3.12.2.2: Eq. 3.12.1.1 is supposed to map to 3.12.2.1, however the respective truth table results show the *non* tautology of the former is transformed into the tautology of the latter. This on it's face refutes the algebraic modal correspondence of Sahlqvist, because 3.12.2.2 should logically match 3.12.1.2. We continue evaluating the correspondence approach.

The variable p occurs twice positively in the antecedent, making [[p ∧ □p]] a 2-additive map. Hence, according to our reduction strategy, the monadic second-order quantification in the second-order translation
∀P[P(x) ∧ ∃y(Rxy ∧ P(y)) → ∀u(Rxu → P(u))] (3.12.3.1)

((#p&x)&((r&(x&%y))&(p&%y)))>((r&(x&#u))>(#p&#u)) ;

TTTT TTTT TTTT TTTT (3.12.3.2)

can be equivalently restricted to subsets of size at most 2. Doing this yields the equivalent L_0-formula
∀z1∀z2[(x = z1 ∨ x = z2) ∧ ∃y(Rxy ∧ (y = z1 ∨ y = z2)) → ∀u(Rxu → (u = z1 ∨ u = z2))]. (3.12.4.1)

(((x=#p)+(x=#q))&((r&(x&%y))&((%y=#p)+(%y=#q))))>((#x&(r&#u))>((#u=p)+(#u=q))) ;

TTTT TTTT TTTT TTTT (3.12.4.2)

This can be simplified to
∀z1∀z2[(x = z1 ∨ x = z2) ∧ (Rxz1 ∨ Rxz2)) → ∀u(xRu → (u = z1 ∨ u = z2))], (3.12.5.1)

(((x=#p)+(x=#q))&((r&(x&#p))+(r&(x&#q))))>((#x&(r&#u))>((#u=p)+(#u=q))) ;

TTTT TTTT TTTT TTTT (3.12.5.2)

and reasoning a bit further this can be seen to be equivalent to
∀z1∀z2[(Rxz1 ∨ Rxz2)) → ∀u(Rxu → (u = z1 ∨ u = z2))], (3.12.6.1)

(r&(x&#p))&(r&(x&#q)))>((r&(x&#u))>((#u=#p)+(#u=#q))) ;

TTTT TTTT TTTT TTTT (3.12.6.2)

which, in turn, is equivalent to ∀z∀u(Rxz ∧ Rxu → z = u). (3.12.2.1)

Eqs. 3.12.2.2-6.2 are not equivalent to 3.12.1.2. This means the approach fails to map a modal correspondence as claimed. We again note that "the very simple Sahlqvist formula p ∧ ◊p → □p" is not a theorem as the beginning conjecture.

Refutation of non Sahlqvist formulas by three counter examples

Abstract: We evaluate three equations as examples of non Sahlqvist formulas. None is tautologous. What follows is that Fine’s theorem and monotonic modal logic are refuted. Therefore those conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Sahlqvist_formula

Examples of three non-Sahlqvist formulas:

1. The *McKinsey formula* does not have a first-order frame condition.

$$\Box\Diamond p \to \Diamond\Box p \tag{1.1}$$

#%p>%#p ; `NNNN NNNN NNNN NNNN` (1.2)

2. The *Löb axiom* does not have a first-order frame condition.

$$\Box(\Box p \to p) \to \Box p \tag{2.1}$$

#(#p>p)>#p ; `CTCT CTCT CTCT CTCT` (2.2)

3. The conjunction of the McKinsey formula and the [modal] (4) axiom has a first-order frame condition … but is not equivalent to any Sahlqvist formula.

$$(\Box\Diamond p \to \Diamond\Box p) \land (\Diamond\Diamond q \to \Diamond q) \tag{3.1}$$

(#%p>%#p)&(%%q>%q) ; `NNNN NNNN NNNN NNNN` (3.2)

Eqs. 1.2-3.2 are *not* tautologous and refute the conjecture of *non* Sahlqvist formulas as tautologous. What follows is that Fine’s theorem and monotonic modal logic are also refuted.

Refutation of the Schaefer theorem for the P, NP problem (undecided)

Abstract: We evaluate the Schaefer theorem for the P, NP problem by two examples for Graph-SAT(Ψ). Neither example is tautologous; while claimed to be different, they result in the same truth table values. (The injection of NP-intermediate does not describe our result.) This refutes NP-complete (and P, NP, NP-hard). We also evaluate the P, NP problem as based on P≤NP with the same result. Therefore P, NP, NP-complete, NP-hard, NP-intermediate form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Manuel Bodirsky, M.; Pinsker, M. (2019). Schaefer's theorem for graphs. cs.umd.edu/users/gasarch/TOPICS/ramsey/schaefergraph.pdf

Graph-SAT(Ψ)

… As an example, let Ψ be the set that just contains the formula

$$(E(x,y)\wedge\neg E(y,z)\wedge\neg E(x,z))\vee$$
$$(\neg E(x,y)\wedge E(y,z)\wedge\neg E(x,z))\vee$$
$$(\neg E(x,y)\wedge\neg E(y,z)\wedge E(x,z)). \quad (1.1)$$

LET p, q, r: *E* (*x,y*), *E* (*y,z*), *E* (*x,z*)

(((p&~q)&~r)+((~p& q)&~r))+((~p&~q)& r) ;
FTT**F** T**FFF** **F**TT**F** T**FFF** (1.2)

Remark 1.2: Eq. 1.2 as rendered is *not* tautologous. This means the claim that 1.1 is P, NP, NP-Complete (NPC), or NP-Hard (NPH) is refuted. We note that the injection of NP-Intermediate (NPI) does not describe "*not* tautologous". What follows is that the P, NP problem cannot be decided as *not* tautologous. (In this regard, see Remark 6.0.)

Then Graph-SAT(Ψ) is the problem of deciding whether there exists a graph such that certain prescribed subsets of its vertex set of cardinality at most three induce subgraphs with exactly one edge. This problem is NP-complete (the curious reader can check this by means of our classification in Theorem 17). There are also many interesting tractable Graph-SAT problems, for instance when Ψ consists of the formulas

$x \neq y \vee y = z$ and
$(\ \ E(x,y) \wedge \neg E(y,z) \wedge \neg E(x,z)) \vee$
$(\neg E(x,y) \wedge \ \ E(y,z) \wedge \neg E(x,z)) \vee$
$(\neg E(x,y) \wedge \neg E(y,z) \wedge \ \ E(x,z)) \vee$
$(\ \ E(x,y) \wedge \ \ E(y,z) \wedge \ \ E(x,z)).$ (2.1)

((p@q)+(q+r)) & (((((p&~q)&~r)+((~p& q)&~r))+((~p&~q)& r))+((p& q)& r)) ;
FTT**F** T**FFF** **F**TT**F** T**FFF** (2.2)

Remark 2.2: Eq. 2.2 is *not* tautologous and is equivalent by truth table value result to Eq. 1.2. This means 1.2 and 2.2 are not NP-complete as claimed.

It is obvious that the problem Graph-SAT(Ψ) is for all Ψ contained in NP. (3.0)

Remark 3.0: Eq. 2.2 refutes conclusion 3.0.

The goal of this paper is to prove the following dichotomy result.

Theorem 1. *For all* Ψ*, the problem* Graph-SAT(Ψ) *is either NP-complete or in P*. (4.0)

Remark 4.0: Eq. 4.0 cannot be asserted because 3.0 is refuted by 2.2.

One of the main contributions of the paper is the general method of combining concepts from universal algebra and model theory, which allows us to use deep results from Ramsey theory to obtain the classification result. (5.0)

Remark 5.0: We show elsewhere that Ramsey's theorem is *not* tautologous.

Remark 6.0: As an example, see en.wikipedia.org/wiki/P_versus_NP_problem .

Clearly, P ≤ NP (per the link above). (6.0.1)

LET p, q: P, NP.

~(q<p)=(p=p) ; **F**TT**F** **F**TT**F** **F**TT**F** **F**TT**F** (6.0.2)

We ask: If (1.1), then (p=q)? (6.1.1)

~(q<p)>(p=q) ; T**F**TT T**F**TT T**F**TT T**F**TT (6.1.2)

We ask: If (1.1), then NOT(p=q)? (6.2.1)

~(q<p)>~(p=q) ; **F**TT**F** **F**TT**F** **F**TT**F** **F**TT**F** (6.2.2)

Eqs. 6.0.2, 6.1.2, and 6.2.2 are *not* tautologous (but *not* contradictory either). This means the P, NP problem as based on P ≤ NP can not be decided.

Denial of schematic refutations of formula schemata

Abstract: We evaluate a definition of the schematic formula of the proposed framework. It is *not* tautologous and hence denies these particular refutations of formula schema. The example of the pigeon hole principle, a trivial theorem, is also not refuted by the proposed framework.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) `C` non-contingency, ∇, ordinal 2; (%z>#z) `N` as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Cerna, D.; Leitsch, A.; Lolic, A. (2019). Schematic refutations of formula schemata.
arxiv.org/pdf/1902.08055.pdf
david.cerna@jku.at, david.cerna@risc.jku.at; leitsch@logic.at; anela@logic.at

Abstract: Proof schemata are infinite sequences of proofs which are defined inductively. In this paper we present a general framework for schemata of terms, formulas and unifiers and define a resolution calculus for schemata of quantifier-free formulas. The new calculus generalizes and improves former approaches to schematic deduction.

As an application of the method we present a schematic refutation formalizing a proof of a weak form of the pigeon hole principle. (0.0)

Remark 0.0: The text does not directly describe the pigeon hole principle, but cites a reference, so we invoke en.wikipedia.org/wiki/Pigeonhole_principle.

If n objects are distributed over m places, and if n < m, then some place receives no object. (1.0)

Remark 1.0: The mechanism of distributing n over m is not exactly explained, and the word "some" is not defined. Therefore we write Eq. 1.0 by replacing "place" with "space" to mean:

If n objects are less than m spaces and some object implies the necessity of space, then some object implies the possibility of no space. (1.1)

LET p, q: m spaces, n objects.

((q<p)&(%q>#p))>(%q>%~p) ;
TTTT TTTT TTTT TTTT (1.2)

Remark 1.2: As rendered in Eq. 1.2, the pigeon hole principle is tautologous and a trivial theorem. It is the stronger form of the theorem. The weaker form, to which the paper directs, in this context substitutes the antecedent clause of "some object implies the necessity of space" with "some object implies the *possibility* of space", for result of the same table.

Definition 12 (formula schemata (FS)). We define the set FS inductively:

– Let F ∈ FS then ¬F ∈ FS. (12.4.1)

LET p, q, r, s: F, F_1, F_2, S.

(p<(p&s))>(~p<(p&s)) ; T**F**T**F** T**F**T**F** TTTT TTTT (12.4.2)

Remark 12.5.2: Eq. 1.5.2 is *not* tautologous, hence denying these particular schematic refutations of formula schemata.

Erwin Schrödinger's cat thought experiment

Findings:

The cat dies eventually regardless of if the radioactive monitor is stimulated or not.

Hence when opening the box at any time, the cat is either still alive or dead, but not "entangled" as both dead and alive (a contradiction). The experiment is not a paradox.

What follows is that quantum mechanics cannot fully evaluate the state of affairs because the experiment was not fully evaluated in propositional logic until now by the logic system VŁ4 in five models.

A detailed diagram of the apparatus is at Wikipedia under the title.

> There are five propositional variables as < box, cat, poison, monitor, death > are < p, q, r, s, t >.
>
> The logical operator symbols as { =, @, &, >, ~ } are { Equivalent to, Not equivalent to, And, Imply, Not}.

The words to be mapped, translation to symbolic expressions, and resulting truth tables follow.

> H0. If the monitor is tautologous, that is not activated, along with the box, cat, and poison apparatus in place, then there is no death.
>
> H0. ((s=s)&((p&q)&r)) > ~t ; not validated in all models.

However, the proof tables show H0 to be "almost" tautologous in all models, for which one exemplary row out of 93-rows suffices:

Model 1	Model 2.1	Model 2.2	Model 2.3.1	Model 2.3.2
TTTT TTTF	EEEE EEEU	EEEE EEEU	EEEE EEEU	EEEE EEEU

H1. If the monitor is contradictory, that is activated, along with the box, cat, and poison apparatus in place, then there is death.

H1. ((s@s)&((p&q)&r)) > t ; validated in all models; Tautologous.

One exemplary row out of 93-rows suffices:

Model 1	Model 2.1	Model 2.2	Model 2.3.1	Model 2.3.2
TTTT TTTT	EEEE EEEE	EEEE EEEE	EEEE EEEE	EEEE EEEE

Note: ((s@s)&((p&q)&r)) = t ; not validated, so an implication is tautologous but not an equivalence.

The comprehensive evaluation in propositional logic of Schrödinger's cat thought experiment could not be undertaken until now with the logic system VŁ4 in five models using the model checker Meth8.

Refutation of Scott's existence axiom in sheaf theory

Abstract: In two non-tautologous equations we refute Scott's existence axiom in sheaf theory. Therefore it does not follow that automating free logic is supported by using "modern proof assistants and theorem provers for classical higher-order logic", such as the showcased tools Isabelle/HOL, Sledgehammer, and Nitpik. These conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Benzmüller, C.; Scott, D.S. (2019). Automating free logic in HOL, with an experimental application in category theory. ocrbilu.uni.lu/bitstream/10993/37593/1/article.pdf

Remark 0: We evaluate two formulas from the table below as signature axioms of existence (E_{ii}) in sheaf theory from Scott (1979).

Table 1. Stepwise evolution of Scott's axiom system for category theory from partial monoids. The axiom names are motivated as follows: S stands for strictness, E for existence, A for associativity, C for codomain, D for Domain. The free variables x, y, z range over the raw domain D. The quantifiers in Axioms Sets I and II are free logic quantifiers, that is, they range over the domain E of existing objects.

E(x·y)←(Ex∧Ey∧(∃z.z·z∼=z∧x·z∼=x∧z·y∼=y)) (4.7.1)

LET %p, q, r, s: E, x, y, z.

(%p&(q&r))<(((%p&q)&(%p&r))&(((%s& s)=(s&(q&s)))=((q&(s&r))=r))) ;
FFFF FFCT **FFFF FFFF** ; (4.1.7.2)

The left-to-right direction of existence axiom E_{ii} is implied.

E(x·y)→(Ex∧Ey∧(∃z.z·z∼=z∧x·z∼=x∧z·y∼=y)) (4.7.1)

Remark 4.7.1: If Eq. 4.1.7.2 as rendered with right-to-left direction is *not* tautologous, then there is no reason to expect a left-to-right direction to be implied as a theorem.

(%p&(q&r))>(((%p&q)&(%p&r))&(((%s& s)=(s&(q&s)))=((q&(s&r))=r))) ;
TTTT TTN**F** TTTT TTTT ; (4.7.2)

In two non-tautologous equations we refute Scott's existence axiom in sheaf theory. Therefore it does not follow that automating free logic is supported by using "modern proof assistants and theorem provers for classical higher-order logic", such as the showcased tools Isabelle/HOL, Sledgehammer, and Nitpik.

Refutation of temporal type theory (TTT), temporal landscapes, and Scott's topology

Abstract: From a footnote, and then a three part definition, temporal type theory (TTT), and then temporal landscapes for open sets are *not* tautologous and hence refuted. That further refutes Scott's topology. These results therefore form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables.
(See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
\> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1;
(%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Fong, B.; Speranzon, A.; Spivak, D.I. (2019). arxiv.org/pdf/1904.01081.pdf
Temporal landscapes: a graphical temporal logic for reasoning. (2019). bfo@mit.edu

[1]Note that the constant reals can be considered as a subtype R ⊆ R-hat˜ of the varying real numbers. (1.1.1)

LET p, q, r , s , t , u:
t_1, t_2, R, t_2', t_1', L

~(~r<r) = (p=p) ; **`FFFF`** `TTTT` **`FFFF`** `TTTT` (1.1.2)

Remark 1.2.2: The constant reals considered as a subtype of varying real numbers is not tautologous. This refutes temporal type theory (TTT) at its outset. However, we press on assuming that difficulty may be overcome by simply avoiding it.

Definition 2.1. A temporal landscape on R *is a set L of time intervals* [t1,t2] ⊆ R, where t1 ≤ t2, such that
(a) if $[t_1,t_2] \in L$, and $t_1' \leq t_1 \leq t_2 \leq t_2'$, then $[t_1',t_2'] \in L$. (2.1.a.1)

(((p&q)<u)&~(~(s<q)<~(p<t)))>((t&s)<u) ;
`TTTT TTTT TTTT TTTT(1),`
`TTT`**`F`** `TTT`**`F`** `TTTT TTTT(1),`
`TTTT TTTT TTTT TTTT(2)` (2.1.a.2)

(b) if $[t_1,t_2] \in L$ *then there exists* $t_1' < t_1 \leq t_2 < t_2'$ *such that* $[t_1',t_2'] \in L$. (2.1.b.1) (((p&q)<u)>%(~((q<s)<(t<p))=(p=p)))>((t&s)<u) ;

FFFN FFFN FFFF FFFF(1),
FFFN FFFN TTTT TTTT(1),
FFFF FFFF FFFF FFFF(2) (2.1.b.2)

We write Prop for the set of temporal landscapes. Together, requirements (a) and (b) state that temporal landscapes form the open sets of the Scott topology on the *interval domain* IR, a well-studied topological space in domain theory. (2.1.1.1)

((u<r)>(~(r<(p&q))&~(s<t)))>(((((p&q)<u)&~(~(s<q)<~(p<t)))>((t&s)<u))&
((((p&q)<u)>%(~((q<s)<(t<p))=(p=p)))>((t&s)<u))) ;

FFFN FFFN FFFF FFFF(1),
FFFF FFFF TTTT TTTT(1),
FFFF FFFF TTTT FFFF(1),
FFFF FFFF FFFF FFFF(1) (2.1.1.2)

Eqs. 2.1.1.2, ..b.2, and ..1.2 are not tautologous. This refutes Def. 2.1 and temporal landscapes for open sets, and hence denies Scott’s topology.

Denial of the conjectured experimental model for search fund study

Abstract: We evaluate the diagram of the search cycle for percentage of funds in each phase and returns for terminal funds. It is *not* tautologous, hence denying the conjectured model of the experiment. This forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬; + Or, ∨, ∪, ⊔; - Not Or; & And, ∧, ∩, ⊓, ·, ⊗; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Yoder, A.; Kelly, P. (2018). Search fund study. Stanford Business School. Case E-662.

RETURNS
Figure F shows the percentage of funds in each phase of the search cycle, as well as return characteristics for terminal funds.

Remark Fig. F: We write the icons of Fig. F in words as:

If [(acquisition implies (gain plus loss)) and
(gain implies (roi_1 plus roi_2 plus roi_3 plus roi_4)) and
(loss implies (total loss plus partial loss))]
then (concluded search fund implies (acquisition plus no acquisition)). (F.1)

LET: p Concluded search fund;
q Acquisition;
r No acquisition;
s Gain;
t Loss;
u, v, w, x:
Return on investment as 1-2x, 2-5x, 5-10x, 10x+;
y Partial loss;
z Total loss.

((q>(s+t))&((s>((u+v)+(w+x)))&(t>(y+z)))) > (p>(q+r)) ;

```
TFTT TTTT TTTT TTTT( 1)
TTTT TTTT TTTT TTTT( 1)
TFTT TTTT TFTT TTTT( 1) }x15
TTTT TTTT TTTT TTTT( 1) }
TFTT TTTT TTTT TTTT( 2) }x 3
TFTT TTTT TFTT TTTT(30) }
```
(F.2)

Remark F.2: We attempt to resuscitate Eq. F.2 by injecting an antecedent component to define No acquisition as zero, that is, (r>(z@z)), but the truth table result is the same as F.2.

Eq. F.2 as rendered is *not* tautologous. Hence the conjectured model of the experiment is denied.

Resolution in classical modal logic of a security barrier model

Abstract: We evaluate three states of a security barrier example. For the security barrier as equivalent to the road width, the two cases are staying on the road, or staying on the road or going off of the road. Staying on the road is not a theorem. However staying on the road *or* going off road is a theorem. For the security barrier as larger than the road width, then staying on the road is effectively enforced as a theorem. This evaluation concludes that casting a problem to substructural epistemic resource logic is an unneeded effort for model resolution. The resolution in classical model logic of a security barrier model relegates the authors' conjectures to *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Galmiche, D.; Kimmel, P.; Pym, D. (2019). A substructural epistemic resource logic: theory and modelling applications. arxiv.org/pdf/1909.07296.pdf

4 Modelling access control with the logic ERL∗
4.2 [A security barrier model]

Consider the example of [a security barrier model], wherein a security system is ineffective because of the existence of a side-channel that allows a control to be circumvented. Here a facility that is intended to be secured is protected by a barrier that prevents cars from entering into the facility. The barrier may be controlled by a token — such as a card, a remote, or a code — the holding of which distinguishes authorized personnel from intruders. If, however, the barrier itself is surrounded by ground that can be traversed by a vehicle, without any kind of fence or wall, then any car can drive around it (whether it's with a malicious intent or just by laziness of getting through the security procedure) and the access control policy, as implemented by the barrier and the tokens, is undermined. So, the access control policy — that only authorized personnel, outside road inside road security barrier missing fence route of vehicle missing fence Fig. 1. A depiction of [a security barrier model] in possession of a token, may take vehicles into the facility — is undermined by the architecture of the system to which it is applied.

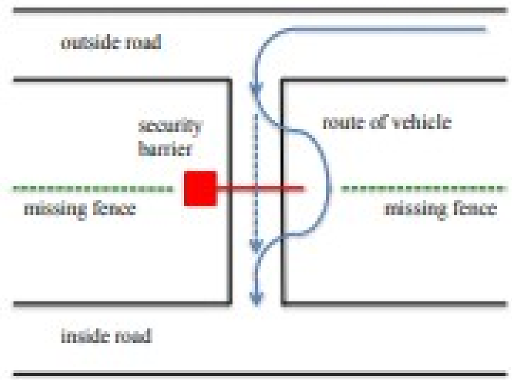

Fig.1. Depiction of a [security barrier] model

We will need the following key components:

– Locations. We assume, for what is an architecturally simple model, just three locations: *outside* and *insid*e of the area guarded by the barrier, and the barrier itself. In this simple setting, there is no need to incorporate an explicit representation of locations into our model's worlds.

– Resources. There are just three types of resource: vehicles (cars), access tokens, which are required to operate the barrier, and a marker for the presence of the barrier.

– Processes. In this simple setting, we do not need to employ the full, quite complex, structure of a process algebra; rather, the actions of a logic with action modalities — in particular, the action modalities of ERL*, with their epistemic semantics, will suffice.

Remark 4.2.0.1: A vehicle in Fig. 1 to trace the various paths is assumed as the only implied means of conveyance between points. The notion of inside or outside is relative to objects as denoted by before, at, and after as respectively the relations of smaller than, equivalent to, and larger than. The marker is renamed as a sign. We note in particular that security barrier is not drawn to scale but should extend only to the width of road. (The action modalities of ERL*, with their epistemic semantics, are ignored because of undue complexity, among other reasons such as injection.)

LET	p,	q,	r,	s:
	barrier,	token,	road,	sign.

Remark 4.2.0.2: The text evaluates four states of a security barrier example as in Figs. 2-4. We ignore the state of Fig. 3 because the indirection of agent is injected. Similarly in Figs. 4-5 we ignore the depiction of agent. We also rename the model states below as a security barrier example, not a problem.

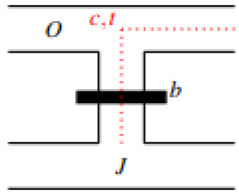

Fig. 2. Barrier problem, base case

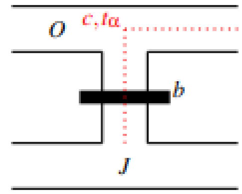

Fig. 3. Barrier problem with agents

We write Fig. 2 in words with the security barrier as equivalent to the road, and the road before the security barrier as equivalent to the road after the security barrier:

> If road sign before road barrier as equivalent to road implies road barrier as equivalent to road, then road before road barrier as equivalent to road and token implies road after road barrier as equivalent to road is equivalent to road. (4.2.2.1)

(((r&s)<((r&p)=r))>((r&p)=r))>(((r<((r&p)=r))&q)>((r>((r&p)=r))=r));

TTTT TT**F**T TTTT TTTT (4.2.2.2)

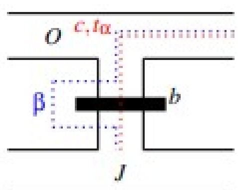

Fig. 4. Barrier problem with a shortcut

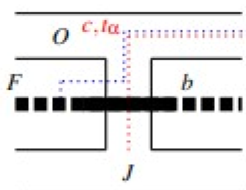

Fig. 5. Barrier problem with a fence

We write Fig. 4 in words with the security barrier as equivalent to the road, and the road before the security barrier as smaller than the road after the security barrier:

> If road sign before road barrier as equivalent to road implies road barrier as equivalent to road, then road before road barrier as equivalent to road and token implies road after road barrier as equivalent to road is greater than road. (4.2.4.1)

(((r&s)<((r&p)=r))>((r&p)=r))>(((r<((r&p)=r))&q)>((r>((r&p)=r))>r));

TTTT TTTT TTTT TTTT (4.2.4.2)

We write Fig. 5 in words with the security barrier as larger than the road (not introducing the term "fence"), and the road before the security barrier as equivalent to the road after the security barrier:

> If road sign before road barrier as larger than road implies road barrier as larger than road, then road before road barrier as larger than road and token implies road after road barrier as larger than road is equivalent to road. (4.2.5.1)

(((r&s)<((r&p)>r))>((r&p)>r))>(((r<((r&p)>r))&q)>((r>((r&p)>r))=r));

TTTT TTTT TTTT TTTT (4.2.5.2)

We evaluate three states of a security barrier example. For the security barrier as equivalent to the road width, the two cases are staying on the road, or staying on the road or going off of the road. Staying on the road is not a theorem, missing so by one **F** value, in Eq. 4.2.2.2 as rendered. However staying on the road *or* going off road is a theorem, Eq. 4.2.4.2. For the security barrier as larger than the road width, then staying on the road is effectively enforced as a theorem, Eq. 4.2.5.2. This evaluation concludes that casting a problem to substructural epistemic resource logic is an unneeded effort for model resolution.

Law of self-equilibrium: not law; not paradox

The law of self-equilibrium sometimes uses this example:

Too much work produces sickness; sickness produces less work;
therefore, too much work implies less work. (1.0)

We rewrite the sentence to replace the connective verb with "causes" for better meaning and also include a modal operator for clarity:

Too much work causes possible sickness; sickness causes less work;
therefore, too much work causes less work. (2.0)

We assume the apparatus of the Meth8 modal logic model checker implementing variant system VŁ4. Meth8 allows to mix four logical values with four analytical values.* The designated *proo*f value is T.

LET: p too much work; ~p less work; q sickness; ~ Not; > Imply; % possibly

((p>%q)&(q>~p))>(p>~p) ; `TNTT TNTT TNTT TNTT` (2.1)

Eq. 2.1 shows the law of self-equilibrium is not tautologous, and hence not a theorem and not a paradox.

*

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	p=p	T	Tautology	proof	11	3
2	p@p	F	Contradiction	absurdum	00	0
3	%p>#p	N	Non-contingency	truth	01	1
4	%p<#p	C	Contingency	falsity	10	2

Results are the proof table of 16-values in row major horizontally.

Refutation of the fixed-point property of self-proving for predicate modal logics

Abstract: The axiom of schema and definition/conjecture of self-prover are refuted, forming a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Iwata, S.; Kurahashi, T. (2019). Fixed-point properties for predicate modal logics arxiv.org/pdf/1907.00306.pdf

1. Introduction
The propositional modal system GL is obtained from the smallest normal modal logic K by adding the axiom schema

□(□A→A)→□A. (1.1)

LET p, q: A, B.

#(#p>p)>#p ; CTCT CTCT CTCT CTCT (1.2)

Remark 1.2: Eq. 1.2 as rendered is not tautologous, thereby refuting the propositional modal system GL as claimed by the authors below.

The modal system GL is well known as *the logic of provability*, since it has the connection with arithmetical theories, for instance, Peano Arithmetic PA [per Solovay].

6. Formulas having a fixed-point in QGL
Definition 6.3 (Self-provers). An L″-formula A is said to be a self-prover if

QGL ⊢ A→□A. (6.3.1)

p>#p ; TNTN TNTN TNTN TNTN (6.3.2)

Remark 6.3.2: Eq. 6.3.2 is *not* tautologous, hence refuting self-provers as defined.

Lemma 6.4. The Boolean constant $\top$ and L″-formulas of the form $\Box A$ are self-provers. Moreover, the set of self-provers is closed under $\wedge$, $\vee$, $\exists$. Consequently, every Σ-formula is a self-prover.

Proof. Since QGL $\vdash \top\rightarrow\Box\top$ and QGL $\vdash \Box A\rightarrow\Box\Box A$, $\Box\top$ and $\Box A$ are self-provers.
Suppose that A and B are self-provers. (6.0.1.1)

LET p, q: A, B.

((((p=p)>#(p=p))&(#p>##p))>((#(p=p)&#p)>(p>#p)))>((p=(p>#p))&(q=(q>#q))) ;
FFFN **FFF**N **FFF**N **FFF**N (6.0.1.2)

• Since A and B are self-provers, QGL $\vdash A\wedge B\rightarrow\Box A\wedge\Box B$. On the other hand, QGL $\vdash \Box A\wedge\Box B\rightarrow\Box(A\wedge B)$. Thus we have QGL $\vdash A\wedge B\rightarrow\Box(A\wedge B)$, and hence $A\wedge B$ is a self-prover. (6.4.1.1)

((((((p=p)>#(p=p))&(#p>##p))>((#(p=p)&#p)>(p>#p)))>((p=(p>#p))&(q=(q>#q))))> (((((p>#p)&(q>#q))>((p&q)>(#p&#q)))&(((p>#p)&(q>#q))>((#p&#q)>#(p&q))))>((p &q)>(#p&#q)))) >((p=(p>#p))&(q=(q>#q))) ;
FFFN **FFF**N **FFF**N **FFF**N (6.4.1.2)

Remark 6.4: Eqs. 6.0.1.2 and 6.4.1.2 are *not* tautologous, hence disallowing Lemma 6.4.

The axiom of schema and definition/conjecture of self-prover are refuted.

Refutation of Π_1^- of self-verifying axioms for grounding functions and relation predicates

Abstract: We evaluate Π_1^- self-verifying axioms defining the grounding functions and the relation predicates as *not* tautologous. This refutes the approach for weak axiom systems to use subtraction and division primitives, rather than addition and multiplication, to encode formally theorems of arithmetic. Therefore the conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Willard, D.E. (2001). Self-verifying axiom systems, the incompleteness theorem and related reflection principles. pdfs.semanticscholar.org/c278/147b7a68385836a90939a175a9959cabbf0b.pdf

Abstract We will study several weak axiom systems that use the Subtraction and Division primitives (rather than Addition and Multiplication) to formally encode the theorems of Arithmetic ...

Table I: List of Π_1^- axioms defining the grounding functions and the relation predicates

3.1 ∀x ∀y ∀z { x = y ∧ y = z } ⊃ x = z

((#p=#q)&(#q=#r))>(#p=#r) ; TNTN NTNT TNTN NTNT (3.2)

6.1 ∀x ∀y ∀a ∀b { x − y = a − b ∧ y = b } ⊃ x = a

(((#p-#q)=(#r-#s))&(#q=#s))>(#p=#r) ;
TTTT TTTT TTTC TTCT (6.2)

10.1 ∀x ¬x < x

~p<p ; T**F**T**F** T**F**T**F** T**F**T**F** T**F**T**F** (10.2)

16.1 ∀x x − 0 = x

(#p-(p@p))=p ; **F**C**F**C **F**C**F**C **F**C**F**C **F**C**F**C (16.2)

18.1 ∀x ∀y x < y ⊃ x/y = 0

(#p<#q)>((#p\#q)=(p@p)) ; TCTT TCTT TCTT TCTT (18.2)

19.1 ∀x x/0 =x/1= x

((#p\(p@p))=(#p\(%p>#p)))=(p@p) ;**F**N**F**N **F**N**F**N **F**N**F**N **F**N**F**N (19.2)

20.1 ∀x ∀y x ≥ y ≥ 1 ⊃ [x/y> 0 ∧ x/y− 1 = x−y/y]

~(~((%p>#p)>#q)>#p)>((((#p\#q)>(p@p))&((#p\#q)=(%p>#q)))=((#p-#q)\#q)) ;
CTTT CTTT CTTT CTTT (20.2)

Seven axioms are *not* tautologous. This refutes Π_1^- self-verifying axioms defining the grounding functions and the relation predicates.

Refutation of shallow embedding in Martin-Löf type theory

Abstract: In Martin-Löf type theory (MLTT), we evaluate shallow embedding as the following conjecture: "if we add the rewrite rule ∀*x*. f *x* (**not** *x*) = **true**, the expression f **true false** will not be rewritten to true, since it does not rigidly match the **not** *x* on the left hand side". The conjecture is *no*t tautologous, hence refuting shallow embedding in MLTT and forming a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Kaposi, A.; András Kovács, A.; Kraus, N. (2019). Shallow embedding of type theory is morally correct. arxiv.org/ftp/arxiv/papers/1907/1907.07562.pdf nicolai.kraus@gmail.com

1 Introduction Martin-Löf type theory .. (MLTT) is a formal system which can be used for writing and verifying programs, and also for formalising mathematics. Proof assistants and dependently typed programming languages such as Agda .., Coq .., Idris .., and Lean .. are based on MLTT and its variations. (1.1.1)

Remark 1.1.1: We refute Martin-Löf type theory (MLTT) and Coq elsewhere as *not* tautologous. This implies that Agda, Idris, and Lean as used in this context are suspicious.

1.2 Reflecting definitional equality To eliminate explicit derivations of conversion, the most promising approach is to reflect object-level definitional equality as meta-level definitional equality. If this is achieved, then all conversion derivations can be essentially replaced by proofs of reflexivity, and the meta-level typechecker would implicitly construct all derivations for us. How can we achieve this? We might consider extensional type theory with general equality reflection, or proof assistants with limited equality reflection. In Agda there is support for the latter using rewrite rules .., which we have examined in detail for the previously described purposes. In Agda, we can just postulate the syntax of the object theory, and try to reflect the equations. This approach does work to some extent, but there are significant limitations: …

– In the current Agda implementation (version 2.6), rewrite rules are not flexible enough to capture all desired computational behavior. For example, the left hand side of a rewrite rule is treated as a rigid expression which is not refined during the matching of the rule. Given an *f* : Bool → Bool → Bool function, if we add the rewrite rule ∀*x*. f *x* (**not** *x*) = **true**, the expression f **true false** will not be rewritten to true, since it does not rigidly match the **not** *x* on the left hand side. (1.2.1.1)

LET p, q, r, s: x, f, r, s.

((((q&#p)&~#p)=(s=s))>(q&((s=s)&(s@s)))) = ~(s=s) ;
FFFF FFFF FFFF FFFF (1.2.1.2)

> **Remark 1.2.1.2:** Eq. 1.2.1.2 as rendered is *not* tautologous. For the outer antecedent expression, the inner antecedent and consequent are both **F**, contradictory, meaning the conjecture is in fact T, tautologous or (s=s), as **F**>**F**=T.

In practice, this means that an unbounded number of special-cased rules are required to reflect equalities for a type theory. Lifting all the restricting assumptions in the implementation of rewrite rules would require non-trivial research effort. (1.2.2.1)

Remark 1.2.2.1: The "non-trivial research effort" is already implemented using the universal logic Ł4 in the model logic model checker Meth8/VŁ4.

It seems to be difficult to capture the equational theory of a dependent object theory with general-purpose implementations of equality reflection. In the future, robust equality reflection for conversion rules may become available, but until then we have to devise workarounds. If the object theory is similar enough to the metatheory, we can reuse meta-level conversion checking using a shallow embedding. In this paper we describe such a shallow embedding. The idea is that in the standard model of the object theory equations already hold definitionally, and so it would be convenient to reason about expressions built from the standard model as if they came from arbitrary models, e.g. from the syntax.

However, we should only use shallow embeddings in morally correct ways: only those equations should hold in the shallow embedding that also hold in the deeply embedded syntax. (1.2.3.1)

Remark 1.2.3.1: The expression "morally correct" is a mixed metaphor because morality is either good or bad, but logic is either correct or incorrect, hence the figure of speech should read either "morally good" or "logically correct".

To address this, first we prove that shallow embedding is injective up to definitional equality: the metatheory can only believe two embedded terms definitionally equal if they are already equal in the object theory. This requires us to look at both the object theory and the metatheory from an external point of view and reason about embedded meta-level terms as pieces of syntax.

Second, we describe a method for hiding implementation details of the standard model, which prevents constructing terms which do not have syntactic counterparts and which also disallows morally incorrect propositional equalities. This hiding is realised with import mechanisms; we do not formally model it, but it is reasonable to believe that it achieves the intended purposes. (1.2.4.1)

Remark 1.2.4.1: This definition of metatheory and definitional equality is built into the universal logic VŁ4 and implemented in the model logic model checker Meth8/VŁ4.

Refutation of Shevenyonov extension nary antropic to propositional logic

Abstract: Out of 18 equations evaluated, two were trivial theorems, and 16 were *not* tautologous. This refutes the Shevenyonov extension nary antropic to propositional logic and relegates it to a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Shevenyonov, A. (2016). Propositional logic: an extension nary antropic. vixra.org/pdf/1611.0415v1.pdf

Remark 0: We do not re-typeset the author's equations as keyed to the text because of no point of contact. The equations in the Appendix attributed to Stall are in order.

Abstract: The proposed extension of propositional logic appears to bridge gaps across areas as diverse as inductive strength and deductive validity, morphisms and Russellian attempts at formal axiomatization, anthropic alternates, and generalized games - ultimately pointing to gradiency and orduality rationales.

Eqs. Beginning at top of page 2:

LET p, q, r, s: p, q, A, B.

((p&r)>(q&s))=((~q&s)>(~p@r)) ;
TTTT T**F**T**F** T**F**TT **FF**TT (1.0.0.2)

((((~q&#r)>(~p&r))&((~p&%r)>(~q&r)))+((~p&%r)>(q&r)))>((q&#r)>(q&r)) ;
TTTT TTTT TTTT TTTT (1.0.2)

Remark 1.0.2: Eq. 1.0.2 as rendered is the seminal "form", which is a trivial theorem.

((p&r)>(q&s))=((~q&s)>(~p@r)) ;
TTTT T**F**T**F** T**F**TT **FF**TT (1.2)

(((%s>#s)+(p@r))&(q&s))=(((%s>#s)+(~q@s))&(~p&r)) ;
TTTT **F**TCT TTC**F** CTTC (1.5.2)

~(((~p&r)-(p&s))\(~p-p))=((r&s)&(p+~p)) ;
TTTT TTTT TTTT **FFFF** (1.5.2.2)

(p=q)>(~(((~p&r)-(p&s))\(~p-p))=((r&s)&(p+~p))) ;
TTTT TTTT TTTT **F**T**F**T (1.5.3.2)

[A] non-commutative generalization of the naive-case equivalence:

((s&((%s>#s)+r))@(r&((%s>#s)+s)))+((r>s)@(s>r)) ;
TTTT **FFFF** **FFFF** TTTT (1.8.3.2)

[T]he more 'rigorous' approach would be to embark on the initial conventions:

((((%s>#s)+r)&s)=(((%s>#s)+(~q&s))&(~p&r)))+((r>s)=((~q&s)>(~p&r))) ;
TTTT CTCT CCTT T**F**TT (1.9.3.2)

Appendix: The following conventions can be looked up as early as Stoll (1960), or discerned directly from a handful of basic identities:

(r-s)=(r&~s) ; **FFFF** **FFFF** TTTT TTTT
(r+s)=(s+r) ; [trivial theorem by inspection]
(r+r)=(s@s) ; TTTT **FFFF** TTTT **FFFF**
((%s>#s)+s)=~s ; NNNN NNNN **FFFF** **FFFF**, }
((%s>#s)+(%s>#s))=(s@s) ; CCCC CCCC CCCC CCCC }
(r+s)=(r&s) ; TTTT **FFFF** **FFFF** TTTT, }
(r&s)=((r+s)+(r&s)) ; TTTT **FFFF** **FFFF** TTTT }
(r>s)=(((%s>#s)+r)&s) ; **FFFF** TTTT NNNN TTTT
((r>r)=(((%s>#s)+r)&r))=((r+r)=(s@s)) ;
FFFF **FFFF** **FFFF** **FFFF**
(r=s)=(r+s) ; **FFFF** **FFFF** **FFFF** TTTT

Remark Appendix: The equations above were not verified against
Stoll, R. (1960). Sets, logic, and axiomatic theories. London. WH Freeman & Co.
because *none* is tautologous.

Out of 18 equations evaluated, two were trivial theorems, and 16 were *not* tautologous. This refutes the Shevenyonov extension nary antropic to propositional logic.

Refutation of the simulation argument and incompleteness of information

Abstract: The equation for Bayesian analysis is *not* tautologous, thereby refuting the conjecture and relegating it to a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Goyal, S. (2019). The simulation argument and incompleteness of information. vixra.org/pdf/1906.0073v1.pdf [no email]

Abstract: Nick Bostrom, in his paper titled “Are you living in a computer simulation?” [Philosophical Quarterly. 2003, 53, 243-255], presented an argument as to why the possibility of an advanced human civilization that can generate human-like observers greatly bolsters the view that we might be living in a simulation. Bostrom argues why the fraction of simulated observers among all types of observers with human-like experiences would be close to one, provided one accepts some assumptions, and then the bland principle of indifference dictates as to why one must thus, assuming himself to be a random observer, put the highest credence in the option which is the most common. Bostrom’s case rests on the idea that we lack evidence to shift our credence the other way, against the probabilistic conclusions, significantly, however, I argue that we are justified in doing so and a priori. Using Bayesian analysis, I show that the conclusion of the argument need not possess similar credence as the argument suggests, even granting all its assumptions.

5. Bayesian analysis of the argument: Now, applying Bayes’ theorem (‘R’ as discussed signifies “the real world") -

P(We live in R R exists)=
[[P(R exists/We live in R)×P(We live in R)]/
[[P(R exists/We live in R)×P(We live in R)]+
[P(R exists/We live in a simulated world)×P(We live in a simulated world)]]]
(5.0.1)

LET p, q, r, s: P, q, live in Real world, live in simulated world.

((p&r)\%r)=(((p&(%r\r))&(p&r))\((((p&(%r\r))&(p&r))+(p&(%r\s)))&(p&s))) ;
TTTT T**F**T**F** TTTT T**F**T**F** (5.0.2)

Eq. 5.0.2 as rendered is *not* tautologous, thereby refuting the conjecture.

Mistakes in rebuttal of refutation of simulation argument and incompleteness of information

Abstract: We evaluate six equations proffered as rebuttal, with two as trivially tautologous and four as *not* tautologous, to confirm the original refutation of the simulation conjecture. These results add to this established *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Goyal, S. (2019). Refutation of "Refutation of the simulation argument and incompleteness of information". vixra.org/pdf/1906.0126v1.pdf [no email]

Remark 0: This matter could not be disposed of off-line because the author does not publish an email address, and the Disqus forum used by vixra dot org does not support mathematical logic. (In the original paper at vixra.org/pdf/1906.0090v1.pdf, a typo was subsequently fixed in v.2 with no result change.)

… when 'r' is true, '%r\r' must be true as well. (1.1)

r>(%r\r) ; TTTT **FFFF** TTTT **FFFF** (1.2)

Remark 1.2: This could also be mapped as (r=(s=s))>(%r\r) with the same result, namely that when 'r' is true, '%r\r' is *not* true as well, and an equivalent expression to Eq. 6.2 below. Obviously, r=(%r\r) is contradictory with result of all **F**'s.

If 'r' is true, then '%r', (2.1)

r>%r ; TTTT TTTT TTTT TTTT (2.2)

Remark 2.2: This is so because of the non-constructive form as **F** implies **F** is T.

which means 'possibility of r', is also true. (3.1)

%r=(s=s) ; CCCC TTTT CCCC TTTT (3.2)

Remark 2.1-3.1: We write this as (if r, then possibility of r) implies (possibility of r).

(4.1)

(r>%r)>%r ; CCCC TTTT CCCC TTTT (4.2)

Considering both ‘r’ and ‘%r’ are true, ‘%r\r’ evaluates to false. (5.1)

((r&%r)=(s=s))>((%r\r)=(s@s)) ;
TTTT TTTT TTTT TTTT (5.2)

Remark 5.2: This is so for the same reason as in Rem. 2.2 above.

Because ‘R exists|We live in R’ and ‘%r\r’, both evaluate to different logical values (when ‘r’ is true), (6.1)

Remark 6.1: We note that both of the because-clauses above are identical with the pipe symbol taken as the division operator / in arithmetic. Hence we map this as:

(r=(s=s))>((%r\r) &(%r\r)) ; TTTT **FFFF** TTTT **FFFF** (6.2)

they are not tautologous and thus, refutation of the original paper by James III appears to be invalid.

Eq. 6.2 as rendered is *not* tautologous, meaning the identical clause(s) evaluate to the same logical value if r is true, that logical value is *not* tautologous, and hence confirming the original refutation of the simulation conjecture as *not* tautologous.

Note on refutation of the simulation argument and incompleteness of information

Abstract: The conjecture using a Bayesian pipe symbol (|), instead of the fractional division symbol (/) as originally published, is *not* tautologous, thereby relegating it to a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Goyal, S. (2019). The simulation argument and incompleteness of information. vixra.org/pdf/1906.0073v1.pdf [no email]

Abstract: Nick Bostrom, in his paper titled "Are you living in a computer simulation?" [Philosophical Quarterly. 2003, 53, 243-255], presented an argument as to why the possibility of an advanced human civilization that can generate human-like observers greatly bolsters the view that we might be living in a simulation. Bostrom argues why the fraction of simulated observers among all types of observers with human-like experiences would be close to one, provided one accepts some assumptions, and then the bland principle of indifference dictates as to why one must thus, assuming himself to be a random observer, put the highest credence in the option which is the most common. Bostrom's case rests on the idea that we lack evidence to shift our credence the other way, against the probabilistic conclusions, significantly, however, I argue that we are justified in doing so and a priori. Using Bayesian analysis, I show that the conclusion of the argument need not possess similar credence as the argument suggests, even granting all its assumptions.

5. Bayesian analysis of the argument: Now, applying Bayes' theorem ('R' as discussed signifies "the real world") -

Remark 5.0: From vixra.org/pdf/1906.0327v1.pdf, we now read the author's intended meaning of the equation below. The fractional division symbol (/), with inverse of the multiplication symbol (×), comes to mean the pipe symbol (|) as a Bayesian form which injects an implication operator as follows. "P(We live in R | R exists)" means the probability of "if R exists", then "We live in R". Similarly, "P(R exists | We live in R)" means the probability of if "We live in R", then "R exists".

Hence this rendition without the fractional division symbol as:

P(We live in R | R exists)=
[[P(R exists | We live in R)×P(We live in R)] |
[[P(R exists | We live in R)×P(We live in R)]+
[P(R exists | We live in a simulated world)×P(We live in a simulated world)]]]
(5.1.1)

is rewritten to mean the author's intended, as:

P(If R exists, then We live in R)=
[If [[P(If We live in R, then R exists)×P(We live in R)]+
[P(If We live in a simulated world, then R exists)×P(We live in a simulated world)]],
then [P(If We live in R, then R exists)×P(We live in R)]] (5.2.1)

LET p, r, s: P, live in Real world, live in Simulated world.

(p&(%r>r))=((((p&(r>%r))&(p&r))+((p&(s>%r))&(p&s)))>((p&(r>%r))&(p&r))) ;
FN**F**N **F**T**F**T **F**T**F**T **F**T**F**T **F**T**F**T (5.2.2)

Eq. 5.2.2 as rendered is *not* tautologous, thereby refuting the conjecture.

Refutation of Skolem axiom form

Abstract: "A *Skolem axiom* has the form ∀x,y(φ(x, y)→φ(x,f(x))), where f is a new function symbol introduced to denote a "Skolem function" for φ." The Skolem axiom form is *not* tautologous, hence refuting it. Therefore the Skolem axiom form is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Avigad, J. (2004). Forcing in proof theory. andrew.cmu.edu/user/avigad/Papers/definitions.pdf

5 Point-free model theory
5.3 Eliminating Skolem functions

A *Skolem axiom* has the form ∀x,y(φ(x, y)→φ(x,f(x))), where f is a new function symbol introduced to denote a "Skolem function" for φ. (5.3.1.1)

LET p, q, r, s: φ, x, y, f

(p&(#q&#r))>(p&(#q&(s&#q))) ; TTTT TTCT TTTT TTTT (5.3.1.2)

Remark 5.3.1.2: Eq. 5.3.1.2 is not tautologous, though nearly so. This means the Skolem axiom form is denied.

Refutation of the sliding scale theorem in law

Abstract: The sliding scale theorem, and as implemented in fuzzy logic, is refuted.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q; first element, second element, r, s;
~ Not; + Or; & And; > Imply, greater than; < Not imply, lesser than;
= Equivalent; @ Not Equivalent;
necessity, for all or every; % possibility, for some or one;
(s@s) zero, 0; (%s>#s) one, 1.

From: Kirgis, F. L. (2002). Fuzzy logic and the sliding scale theorem. law.ua.edu/pubs/lrarticles/Volume 53/Issue 2/Kirgis.pdf

"The sliding scale theorem may be simply stated: The greater the degree to which one element is satisfied, the lesser the degree to which the other need be." (1.0)

We rewrite Eq. 1.0 as an implication.

If one element is satisfied as greater than the second element, then the second element is satisfied as lesser than the first element. (1.1)

(p>q)>(q<p) ; **F**TT**F** **F**TT**F** **F**TT**F** **F**TT**F** (1.2)

Remark: If Eq. 1.1 is rewritten to include the relation of "greater than or equal to" and "lesser than or equal to ", then Eq. 1.2 is ~(q<p)>~(p>q) with the same truth table result.

"Under fuzzy logic, zero and one are simply the opposite ends of a continuum ..." (2.0)

We rewrite Eq. 2.0 as a relation to include one element and the second element.

The sum of one element with a second element is greater than or equal to zero and lesser than or equal to one. (2.1)

~((p+q)<(s@s))&~((#s>%s)>(p+q)) ; **TFFF TFFF TFFF TFFF** (2.2)

We combine Eqs. 1.0 and 2.0 to capture the intention of the author as 1.0 implying 2.0. (3.0)

If one element is satisfied as greater than the second element, then the second element is satisfied as lesser than the first element, this implies the fuzzy sum of one element with a second element is greater than or equal to zero. (3.1) ((p>q)>(q<p))<(~((p+q)<(s@s))&~((#s>%s)>(p+q))) ;

FTT**F** **F**TT**F** **F**TT**F** **F**TT**F** (3.2)

Eqs. 1.2, 2.2, and 3.2 as rendered are *not* tautologous. Hence the sliding scale theorem, and as implemented in fuzzy logic, is refuted.

Refutation of the Solovay theorem

Abstract: Solovay's arithmetical completeness theorem for provability logic is refuted by showing the following are not tautologous: Löb's rule as an inference; Gödel's logic system (GL); Gödel's second incompleteness theorem; inconsistency claims of Peano arithmetic (PA); and inability to apply semantical completeness to results which are not contradictory and which are not tautologous.

From: Shah, A. (2013). Solovay's arithmetical completeness theorem for provability logic. University of Warwick. [Note: this paper is attributed to a student *number* with no email address.]

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal.

> LET: p, q, r, s: placeholder; country; refugee; visa;
> ~ Not; & And; + Or; - Not Or; > Imply, greater than; < Not Imply, less than;
> = Equivalent; % possibility, for one or some; # necessity, for all or every;
> (p=p) tautology; (p@p) contradiction.

We evaluate Solovay's arithmetical completeness theorem from the narrative source, as edited:

> "Consider one who can only acquire a visa for a country only if proving that one will not remain in that country. Further, if one is not allowed back into a country after leaving it, then one will eventually reside somewhere else. Thus, the truthful one will remain in one's native country." (1.1)

> (((~(r<q)>(p=p))>(r>~(s<q)))&((((r<q)>(r>q))>~(r<q))<(r>q)))>((r=(p=p))>(r<q)) ;
> `TTTT TTTT TTTT TTTT` (1.2)

> **Remark:** The location of the visa to be obtained while residing within a country or outside a country is irrelevant as to the same table result.

> Eq. 1.2 as rendered is tautologous. However, its application to Gödel's logic system (GL), as *not* tautologous, is defective as shown below.

We evaluate the claims in the captioned as keyed to the text.

The assertion *it is necessary that (it is raining or it is not raining)* ([](*A* ∨ ¬*A*)) is true because it is either raining or it is not, and this is always true. (2.1.1)

> #(A + ~A) = (A=A) ; `NNNN NNNN NNNN NNNN` (2.1.2)

However, the statement *it is necessarily raining or it is necessarily not raining* ([]*A* ∨ []¬*A*) is false. (2.2.1)

> #A + #~A ; `NNNN NNNN NNNN NNNN` (2.2.2)

The intention of Eqs. 2.1.1 and 2.2.1 was to show the dual as #~A as not the negation as ~#A.

However that point was lost because as rendered Eq. 2.1.2 is *not* tautologous but a truthity, and Eq. 2.2.2 is *not* contradictory but also a truthity. In other words, both equations are equivalent.

$([](A \vee \neg A)) = ([]A \vee []\neg A)$; (2.3.1)

#(A + ~A)= (#A + #~A); `TTTT TTTT TTTT TTTT` (2.3.2)

Eq. 2.3.2 is tautologous.

$[]([]A \to A) \leftrightarrow []A \leftrightarrow []([]A \wedge A)$ (Prp.3.23.1.1)

#(#A>A)=(#A=#(#A&A)) ; `NNNN NNNN NNNN NNNN` (Prp.3.23.1.2)

Eq. Prp.3.23.1.2 is *not* tautologous

In addition, the axiom form of the G in GL is defined as $[]([]A \to A) \to []A$. (Prp.3.23.2.1)

#(#A>A)>#A; `CCTT CCTT CCTT CCTT` (Prp.3.23.2.2)

Eq. Prp.3.23.2.2 is *not* tautologous.

For an axiomatic proof system, the rule of regularity is defined and derived: (Lem.3.5.1)

(A>B)=(#A>#B) ; `TNTN TNTN TNTN TNTN` (Lem.3.5.2)

Eq. Lem.3.5.2 is *not* tautologous.

$[]\bot \leftrightarrow []<>p$ (Lem.3.24.1)

#(p@p)=#%p ; `NFNF NFNF NFNF NFNF` (Lem.3.24.2)

Eq. Lem.3.24.2 is *not* tautologous.

The definition of GL is given as $[](p \leftrightarrow \neg[]p) \leftrightarrow [](p \leftrightarrow \neg[]\bot)$. (Thm.3.25.1)

#(p=~#p)=#(p=~(#(p@p))) ; `TCTC TCTC TCTC TCTC` (Thm.3.25.2)

Eq. Thm.3.25.2 is *not* tautologous.

For the arithmetical soundness of GL,"we define the *Löb Rule* to be the rule of inference in a modal logic axiomatic system which allows one to deduce A from $[]A \to A$" (Def.4.1.1.1)

(#A>A)>A ; `FCNT FCNT FCNT FCNT` (Def.4.1.1.2)

Eq. Def.4.1.1.2 is *not* tautologous.

"Peano arithmetic (PA) can prove that if arithmetic is consistent, then Peano arithmetic (PA) cannot prove its own consistency; this is Gödel's Second Incompleteness Theorem for PA, defined as $\neg[]\bot \to \neg[]([]\bot \to \bot)$" (Cor.4.5.1)

~(#(A@A))>~(#(#(A@A)>(A@A))=(A=A)) ;CCCC CCCC CCCC CCCC (Cor.4.5.2)

Eq. Cor.4.5.2 is *not* tautologous. Therefore Gödel's Second Incompleteness Theorem is refuted.

PA can prove that if the inconsistency of arithmetic is not formally provable (in PA), then the consistency of arithmetic is undecidable. That is, not being able to formally prove the inconsistency of arithmetic implies that, firstly, it is not formally provable that arithmetic is consistent and, secondly, it is not formally provable that arithmetic is inconsistent. Hence, the formal unprovability of the inconsistency of arithmetic implies that the consistency of arithmetic is undecidable. ¬[][]⊥ → (¬[]¬[]⊥ ∧ ¬[][]⊥)" (Cor.4.6.1)

This translated to ~##(A@A)>(~#~#(A@A)&~##(A@A), and is rewritten as
(A=(A&A)) > (~##A>(~#~#A&~##A)) ; FFNN FFNN FFNN FFNN (Cor.4.6.2)

Eq. Cor.4.6.2 *not* tautologous. Therefore to further formalize and strengthen the Löb Rule is in vain.

The Eqs. above do not support *proof* in GL.

Should the Solovay arithmetical completeness theorem be invoked to show semantical completeness for the above, it similarly will not result in tautology. That means the Solovay theorem is refuted by extension.

Resolution of Sorites Paradox

((q>#p) & ~(q-%p)) > (q=q); tautologous

LET p is grain of sand; #p is the necessity of grains of sand; %p is the possibility of a grain of sand; q is heap of sand; & is and; - is not or, ~+; > is imply; = is equivalent to; ~ is not

In words: If both a heap implies the necessity of grains and a heap is not possibly less than one grain, then the heap is in fact truly a heap.

In other words, a heap has to have one or more grain(s) to be a heap.

```
(q>#p)
Model 1              Model 2.1            Model 2.2            Model 2.3.1          Model 2.3.2
TTFN TTFN TTFN TTFN  EEUE EEUE EEUE EEUE  EEUU EEUU EEUU EEUU  EEUI EEUI EEUI EEUI  EEUP EEUP EEUP
EEUP

~(q-%p)
CTTT CTTT CTTT CTTT  UEEE UEEE UEEE UEEE  EEEE EEEE EEEE EEEE  PEEE PEEE PEEE PEEE  IEEE IEEE IEEE
IEEE

((q>#p)&~(q-%p))
CTFN CTFN CTFN CTFN  UEUE UEUE UEUE UEUE  EEUU EEUU EEUU EEUU  PEUI PEUI PEUI PEUI  IEUP IEUP IEUP
IEUP

(q=q)
TTTT TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE
EEEE

((q>#p)&~(q-%p))>(q=q)
TTTT TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE
EEEE
Model 1              Model 2.1            Model 2.2            Model 2.3.1          Model 2.3.2
```

Confirmation of the Crothers refutation for the special theory of relativity

From the abstract:

Crothers, S. J. (2017). "On the logical inconsistency of the special theory of relativity". vixra.org/pdf/1703.0047v6.pdf

Einstein's Special Theory of Relativity requires systems of clock-synchronised stationary observers and the Lorentz Transformation. (1.1)
Without both, the Theory of Relativity fails. (2.1)
A system of clock-synchronised stationary observers is proven inconsistent with the Lorentz Transformation, because it is Galilean. (3.1)
The Special Theory of Relativity insists that Galilean systems must transform not by the Galilean Transformation, but by the non-Galilean Lorentz Transformation. (4.1)
The Theory of Relativity is therefore invalid due to an intrinsic logical contradiction. (5.1)
[We write Eq. 5.1 as: ((Eq. 1.1 and Eq. 2.1) and Eq. 3.1) and Eq. 4.1.] (5.1.1)

We assume the apparatus and method of Meth8/VŁ4 with designated *proof* value of T, and 16-valued proof tables presented as row-major and horizontally.

LET p q r s: Galilean system, transform, Lorentz transform,
Special theory of relativity, clock-synchronized stationary observers;
> Imply, greater than; < Not Imply, less than; = Equivalent; @ Not Equivalent;
necessity, for all; % possibility, for one or some;
(s@s) contradiction, *not t*autologous.

r<#(s&p) ; FFFF TTTT FFFF TCTC (1.2)
(~(#s&#p)>r)=(s@s) ; TTTT FFFF TCTC FFFF (2.2)
((s>p)&~(p=q))>(s>q) ; TTTT TTTT T**F**TT T**F**TT (3.2)

Remark: Eq. 3.2 as rendered is *not* tautologous and hence inconsistent.

#s>(p>(~p\(p=~q))) ; TTTT TTTT TTTT TTTT (4.2)
(((r<#(s&p))&((~(#s&#p)>r)=(s@s)))&(((s>p)&~(p=q))>(s>q)))
& (#s>(p>(~p\(p=~q)))) ; FFFF FFFF FFFF FFFF (5.1.2)

Eq. 5.1.2 is *not* tautologous and is in fact a contradiction.

This means the refutation of the special theory of relativity by Stephen J. Crothers is confirmed.

Confirmation of the Łukasiewicz Square of Opposition via logic VŁ4

Abstract: We evaluate the existential import of the Revised Modern Square of Opposition. We confirm that the Łukasiewicz syllogistic was intended to apply to *all* terms. What follows is that Aristotle was mistaken in his mapping of vertices, which we correct and show fidelity to Aristotle's intentions. We also evaluate the Cube of Opposition of Seuren. Two final claims are not tautologous, hence refuting the Cube, which also contradict criticism of Seuren that was not based on those claims.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

> LET: ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> > Imply, greater than, →, ⊢; < Not Imply, less than, ∈;
> = Equivalent, ≡, ⊨; @ Not Equivalent, ≠;
> % possibility, for one or some, ∃, ◊; # necessity, for every or all, ∀, □;
> ~(y < x) (x ≤ y), (x ⊆ y); (p=p) Tautology.

See: Read, S. (2015). Aristotle and Łukasiewicz on Existential Import.
st-andrews.ac.uk/~slr/Existential_import.pdf slr@st-andrews.ac.uk

We map vertices of the first Square of Opposition on page 4 with its words below.

(A) Every S is P. #(s= p)=(p=p) ;
NFNF NFNF FNFN FNFN (0.1.2)

(E) No S is P. #(s=~p)=(p=p) ;
FNFN FNFN NFNF NFNF (0.3.2)

(I) Some S is P. %(s= p)=(p=p) ;
TCTC TCTC CTCT CTCT (0.5.2)

(O) Not every S is P. %(~s=p)=(p=p) ;
CTCT CTCT TCTC TCTC (0.7.2)

Remark 0: The above is from our *revised* Modern Square of Opposition as published.

We map the relations which Aristotle accepts as preserved here.

A- and E-propositions are contrary (cannot both be true) [(A)=T & (E)=T] (1.1.1)

(#(s= p)=(p=p))&(#(s=~p)=(p=p)) ; **FFFF FFFF FFFF FFFF** (1.1.2)

and I- and O-propositions are subcontrary (cannot both be false)
[(I)=**F** & (O)=**F**] (1.2.1)

(%(s= p)=(p@p))&(%(s=~p)=(p@p)) ; **FFFF FFFF FFFF FFFF** (1.2.2)

A- and O-propositions are contradictories, [(A)&(O)] (2.1.1)

#(s= p)&%(s=~p) ; **FFFF FFFF FFFF FFFF** (2.1.2)

as are I- and E-propositions [(I) & (E)] (2.2.1)

%(s= p)&#(s=~p) ; **FFFF FFFF FFFF FFFF** (2.2.2)

A-propositions imply their subaltern I-proposition, [(A) > (I)] (3.1.1)

#(s= p)>%(s= p) ; TTTT TTTT TTTT TTTT (3.1.2)

and E-propositions their subaltern O-proposition [(E) > (O)] (3.2.1)

#(s=~p)>%(s=~p) ; TTTT TTTT TTTT TTTT (3.2.2)

I- propositions convert simply 'Some *S* is *P* ' implies 'Some *P* is *S*', (4.1.1)

%(s= p)>%(p= s) ; TTTT TTTT TTTT TTTT (4.1.2)

and E-propositions 'No *S* is *P* ' implies 'No *P* is *S*' (4.2.1)

#(~s=p)>#(~p=s) TTTT TTTT TTTT TTTT (4.2.2)

A-propositions convert accidentally ('Every *S* is *P* ' implies 'Some *P* is *S*') (5.1.1)

#(s= p)>%(p= s) ; TTTT TTTT TTTT TTTT (5.1.2)

and O-propositions don't convert at all. [Some S is not P implies Every P is not S.] (5.2.1)

%(s=~p)>#(p=~s) ; NNNN NNNN NNNN NNNN (5.2.2)

We present these six equations for the six directed rays in the Square, as previously published.

(A\E) #(s= p) \ #(s=~p) ; TTTT TTTT TTTT TTTT (6.1.2)
(A>I) #(s= p) > %(s= p) ; TTTT TTTT TTTT TTTT (6.2.2)
(A\O) #(s= p) \ %(s=~p) ; TTTT TTTT TTTT TTTT (6.3.2)
(E\I) #(s=~p) \ %(s= p) ; TTTT TTTT TTTT TTTT (6.4.2)
(E>O) #(s=~p) > %(s=~p) ; TTTT TTTT TTTT TTTT (6.5.2)
(I+O) %(s= p) + %(s=~p) ; TTTT TTTT TTTT TTTT (6.6.2)

Remark 6: The new connective distribution is as follows with count. The mappings above allow for replication and confirmation of the 24-syllogisms and with our claim of a minor correction each to Modus Camestros and Modus Cesare.

(1) Contraries Not And (\);

(1)	Subcontraries	Or (+);
(2)	Subalterns	Imply (>); and
(2)	Contradictories	Not And (\)

We conclude that Łukasiewicz was not mistaken in his rendition of the Square of Opposition.

We now turn to the criticism of the Cube of Opposition of Seuren to map and interleave the additional vertices from the diagram on page 8. While * marks predicate negation with the term "-P", we use $ to mark copula negation with the term "not P", and mark the negation of $ using !.

(A) Every S is P. #(s= p) =(p=p) ;
N**F**N**F** N**F**N**F** **F**N**F**N **F**N**F**N (7.1.1)

(A*) Every S is not-P. ~(#(s=p)=(p=p))=(p=p) ;
as Not (Every S is P.) CTCT CTCT TCTC TCTC (7.1.2)

(A$) Every S is not P. #(s=~p)=(p=p) ;
FN**F**N **F**N**F**N N**F**N**F** N**F**N**F** (7.1.3)

(A!) Not (Every S is not P.) ~(#(s=~p)=(p=p))=(p=p) ;
TCTC TCTC CTCT CTCT (7.1.4)

(E) No S is P. #(s=~p)=(p=p) ;
FN**F**N **F**N**F**N N**F**N**F** N**F**N**F** (7.2.1)

(E*) No S is not-P. ~(#(s=~p)=(p=p))=(p=p) ;
as Not (No S is P.) TCTC TCTC CTCT CTCT (7.2.2)

(E$) No S is not P. #(~s=~p)=(p=p) ;
N**F**N**F** N**F**N**F** **F**N**F**N **F**N**F**N (7.2.3)

(E!) Not (No S is not P.) ~(#(~s=~p)=(p=p))=(p=p) ;
CTCT CTCT TCTC TCTC (7.2.4)

(I) Some S is P. %(s= p)=(p=p) ;
TCTC TCTC CTCT CTCT (7.3.1)

(I*) Some S is not-P. ~(%(s= p)=(p=p))=(p=p) ;
as Not (Some S is P). **F**N**F**N **F**N**F**N N**F**N**F** N**F**N**F** (7.3.2)

(I$) Some S is not P. %(s=~p)=(p=p) ;
CTCT CTCT TCTC TCTC (7.3.3)

(I!) Not (Some S is not P.)~(%(s=~p)=(p=p))=(p=p) ;
N**F**N**F** N**F**N**F** **F**N**F**N **F**N**F**N (7.3.4)

(O) Not every S is P. %(~s=p)=(p=p) ;
CTCT CTCT TCTC TCTC (7.4.1)

(O*) Not every S is not-P. ~(%(~s=p)=(p=p))=(p=p) ;
as Not(Not every S is P.) N**F**N**F** N**F**N**F** **F**N**F**N **F**N**F**N (7.4.2)

(O$) Not every S is not P. %(~s=~p)=(p=p) ;
TCTC TCTC CTCT CTCT (7.4.3)

(O!) Not (Not every S is not P.) ~(%(~s=~p)=(p=p))=(p=p) ;
FN**F**N **F**N**F**N N**F**N**F** N**F**N**F** (7.4.4)

The following are supposed to hold:

~I* = *E: ~(~(%(s= p)=(p=p))=(p=p)) = (~(#(s=~p)=(p=p))=(p=p)) ;
TTTT TTTT TTTT TTTT (8.1.1)

~A* = O*: ~(~(#(s=p)=(p=p))=(p=p)) = (~(%(~s=p)=(p=p))=(p=p)) ;
TTTT TTTT TTTT TTTT (8.1.2)

A* > E: (~(#(s=p)=(p=p))=(p=p)) > (#(s=~p)=(p=p)) ;
NNNN NNNN NNNN NNNN (9.1.1)

A > E*: (#(s= p) =(p=p)) > (~(#(s=~p)=(p=p))=(p=p)) ;
TTTT TTTT TTTT TTTT (9.1.2)

I > O*: (%(s= p)=(p=p)) > (~(%(~s=p)=(p=p))=(p=p)) ;
NNNN NNNN NNNN NNNN (9.1.3)

I* > O: (~(%(s= p)=(p=p))=(p=p)) > (%(~s=p)=(p=p)) ;
TTTT TTTT TTTT TTTT (9.1.4)

Eqs. 9.1.1 (A* > E) and 9.1.3 (I > O*) are *not* tautologous, albeit truthities. This means that the final claims of Seuren's Cube of Opposition are mistaken, but also that the criticism of Seuren as based not on those claims is also mistaken.

Square of Opposition as Meth8 corrected

Abstract: The modern revision of the square of opposition is *not* tautologous and forms a non tautologous fragment of the universal logic VŁ4. Consequently we redefine the square so that it is validated as tautologous. Instead of definientia using implication for universal terms or conjunction for existential terms, we adopt the equivalent connective for all terms. The modal modifiers necessity and possibility map quantifiers as applying to the entire terms rather than to the antecedent within the terms.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.
Note: ? unspecified connective.

Columns in bold are the corrected square.

Sources		* Modern Revision		** **Meth8 Correction**	
Type	Definientia	Script	Valid as	**Script**	**Valid as**
Corner	A	#s> p		**#(s= p)**	
	E	#s>~p		**#(s=~p)**	
	I	%s&p		**%(s= p)**	
	O	%s&~p		**%(s=~p)**	
Contraries	AE	(#s>p) + (#s>~p)	A + E	**#(s= p) \ #(s=~p)**	**A \ E**
Subalterns	AI	(#s>p) ? (%s&p)		**#(s= p) > %(s= p)**	**A > I**
Contradictories	AO	(#s>p) + (%s&~p)	A + O	**#(s= p) \ %(s=~p)**	**A \ O**
Contradictories	EI	(#s>~p) + (%s&p)	E + I	**#(s=~p) \ %(s= p)**	**E \ I**
Subalterns	EO	(#s>~p) ? (%s&~p)		**#(s=~p) > %(s=~p)**	**E > O**
Subcontraries	IO	(%s&p) \ (%s&~p)	I \ O	**%(s= p) + %(s=~p)**	**I + O**

* The quantifier may refer to the entire term as #(p=q) or to the antecedent of the term as (#p=q). In Meth8 there is a difference. We adopt the latter because it returns more validated connectives. For example from the traditional square: #(A?E), #(I?O) versus (A+E), (I\O).

The modern revision of the square of opposition is not validated as tautologous by the Meth8 logic checker in five models for all expressions. This leads us to consider that any logic system based on the square of opposition is spurious. What follows then is that a first order predicate logic based on the square of opposition is now suspicious.

** The Meth8 validated square of opposition redefines A, E, I, O to match the words more clearly. For example on A, "All S is P" is mapped as "#(s=p)", not as in the note above with "#s=p" because the connective of equivalence is stricter than that of implication and consistent for all definiens. By changing the connective in the term from implication or conjunction to equivalence makes the Meth8 validated square of opposition suitable as a basis for other logics such as first order predicate logic.

We note the validating connectives for the edges on the square are: \ Nand for the Contraries and Contradictories; > Imply for the Subalterns; and + Or for the Subcontraries.

References

James, C. (2015). Theorem prover Meth8 applies four valued Boolean logic for modal interpretation. First World Conference: Analogy. Beneméita Universidad Autónoma de Puebla, Mexico, November 4-6, 2015, Handbook, ISBN 978-83-65273-01-1, 50-51.

Łukasiewicz, J. (1951). *Aristotle's Syllogistic from the Standpoint of Modern Logic*, Oxford: Clarendon Press.

Westerståhl, D. (2012). “Classical vs modern Squares of Opposition, and beyond”, in Jean-Yves Béziau & Gillman Payette (eds.), *The Square of Opposition: A General Framework for Cognition*, Bern: Peter Lang.

Square of Opposition Modern Revised: not validated as tautologous

The definens are from plato.stanford.edu/entries/square/#ModRevSqu, by Terence Parsons (2012).

The Meth8 symbols are: ~ Negation ; \ Nand ; > Imply ; + Or ; # modal necessity for universal quantifier ; % modal possibility for existential quantifier ; ? unspecified connective.

Sources		Original Fragment		Original Tradition		* Modern Revision		Swanon Defense	
Type	Defin-ientia	Script	Valid as	Script	Valid as	Script	Valid as	Script	Valid as
Corner	A	#s>p		#s>p		#s> p		#s>p	
	E	~s>p		~s>p		#s>~p		#s>~p	
	I	%s&p		%s&p		%s&p		%s&p	
	O	%s&~p		%s&~p		%s&~p		%s&~p	
Con-traries	AE	(#s>p) + (~s>p)	A + E	(#s>p) + (~s>p)	A + E	(#s>p) + (#s>~p)	A + E	(#s>p) + (#s>~p)	A + E
Sub-alerns	AI			(#s>p) ? (%s& p)		(#s>p) ? (%s&p)		(#s>p) ? (%s&p)	
Contra-dictories	AO	(#s>p) + (%s&~p)	A + O	(#s>p) + (%s&~p)	A + O	(#s>p) + (%s&~p)	A + O	#s>p) + (%s&~p)	A + O
Contra-dictories	EI	(~s>p) ? (%s&p)		(~s>p) ? (%s&p)		(#s>~p) + (%s&p)	E + I	(#s>~p) + (%s& p)	E + I
Sub-alterns	EO			(~s>p) ? (%s&~p)		(#s>~p) ? (%s&~p)		(#s>~p) ? (%s&~p)	
Sub-contraries	IO			(%s&p) \ (%s&~p)	I \ O	(%s&p) \ (%s&~p)	I \ O	(%s&p) \ (%s&~p)	I \ O

* The quantifier may refer to the entire term as #(p=q) or to the antecedent of the term as (#p=q). In Meth8 there is a difference. We adopt the latter because it returns more validated connectives. For example from the traditional square: #(A?E), #(I?O) versus (A+E), (I\O).

The square of opposition is not validated as tautologous by the Meth8 logic checker in five models for all expressions. This leads us to consider that any logic system based on the square of opposition is spurious. What follows then is that a first order predicate logic based on the square of opposition is now suspicious.

Proportions in the square of opposition

Prade, Henri; Richard, Gilles. "From the structures of opposition between similarity and dissimilarity indicators to logical proportions". *Representation and reality in humans, other living organisms and intelligent machines*. Springer. 2017. pp.279-299. doi: 10.1007/978-3-319-43784-2_14.

From:researchgate.net/publication/
319401583_From_the_Structures_of_Opposition_Between_Similarity_and_Dissimilarity_Indicators
_to_Logical_Proportions

On page 280 of the free Springer preview, only, we find:

(A/D)=(A-B)/(C-D) where B=C. (1.0)

We evaluate this using the Meth8 modal logic model checker, implementing our resuscitation of Łukasiewicz Ł4 as system variant VŁ4.

LET: p q r s A B C D ;
\ / Not And; = Equivalent to; > Implication; & And; - - Not Or
T tautology; F contradiction

(p\s)=((p-q)\(r-s)) ; FTTT TTTT TFTF TFFF (1.1)
(q=r)>((p\s)=((p-q)\(r-s))) ; FTTT TTTT TFTT TTTF (1.2)
(q=r)&((p\s)=((p-q)\(r-s))) ; FTFF FFTT TFFF FFTF (1.3)

Eqs 1.1, 1.2, 1.3 are not validated as tautology by Meth8. We therefore conclude that proportions as such from the Square of Opposition are not bivalent but a vector space.

Refutation of definable operators on stable set lattices

Abstract: We evaluate the definitions for the modal operators on stable set lattices. The operators are not respective negations and hence refute the definitions.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And; > Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) C non-contingency, ∇, ordinal 2; (%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Goldblatt, R. (2019). Definable operators on stable set lattices.
arxiv.org/pdf/1812.01264.pdf rob.goldblatt@msor.vuw.ac.nz

The key idea is that of a first-order definable operation on a stable set lattice, an idea that goes to the heart of Kripke's semantical interpretation of the modalities □ and ◊. On the algebra of subsets of a Kripke frame (X,R), the modal connectives can be interpreted as operations assigning to each set A⊆X the sets

□A = {x :∀y(xRy→y∈A)} and (1.1)

LET p, r, s, x, y: A, R, X, x, y

#p=(((x&(r&#y))>(#y<p))>x) ;
TCTC TCTC TCTC TCTC(16),
FN**F**N **F**N**F**N **F**N**F**N **F**N**F**N(16) (1.2)

◊A = {x :∃y(xRy&y∈A)}. (2.1)

%p=(((x&(r&%y))&(%y<p))>x) ; CTCT CTCT CTCT CTCT(32) (2.2)

The expressions defining the members of these sets can be seen as first order formulas in the binary predicate xRy and the unary predicate y∈A, leading to the 'standard translation' of the propositional modal language into a first-order language [...]. This ability to relate modal logic to a fragment of first-order logic does much to account for the success of the relational semantics revolution.

Remark 2.2: Eqs. 1.2 and 2.2 as rendered are *not* negations, and hence refute the definitions as a standard translation.

Refutation of Stit logic (sees to it that)

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET ~ Not; + Or; & And; > Imply; = Equivalent; @ Not Equivalent
% possibility, for one or some; # necessity, for all or every; (p@p) F contradiction.
Remark: Equations are not reproduced from the cited pdf files due to non-portable characters.

From: Olkhovikov, G.K.; Wansing, H. (2017). Inference as doxastic agency.
Part II: Ramifications and refinements. ojs.victoria.ac.nz/ajl/article/view/3973/3625

LET p, q: K, A

(p&q)>(#p&#q) ; TTTN TTTN TTTN TTTN (A8)

LET p, q,r , s: K, E, x, y

(p&((~#q&r)+(#q&s)))>((~q&r)+(q&s)) ;
TTTT TTTT TTTN TTTT (at R4)

LET p, q, r, s: E, q, t, s

((p&(s+r))>((p&s)&(p&r)))&((p&(s&r))>(p&r)) ;
TTTT TFTF TFTF TTTT (at R4')

From: Olkhovikov, G.K. (2017). Explicit justification stit logic: a completeness result.
arxiv.org/pdf/1709.06893.pdf

LET A, B; A, K

(B&A)>#A ; TTTT TNTN TTTT TNTN (T0)
(B&A)>(#B&#A) ; TTTT TNTN TTTT TNTN (36)

LET p, q, r, s: K, B, C, D

(p&s)>q ; TTTT TTTT TFTT TFTT (45)
p&((p&s)>q) ; FTFT FTFT FFFT FFFT (46)
(p&s)>(p&q) ; TTTT TTTT TFTF TFTF (47)
(p&q)>r ; TFTF TTTT TFTF TTTT (48)
(p&s)>r ; TTTT TTTT TFTF TTTT (49)

From: Olkhovikov, G.K. (2017). A completeness result for implicit justification stit logic.
arxiv.org/pdf/1705.09119.pdf

(~(A+B)&(~A&~B))>(A@A) ; FCNT CCTT NTNT TTTT (pg14)

From: Olkhovikov, G.K. (2018). Restricted interpolation and lack thereof in Stit logic.

arxiv.org/pdf/1804.08306.pdf

LET p, q, r, s: A, B, C, j [iterated equation]

(#p&(%s&q))>(%s&r) ;	TTTT TTTT TTTC TTTT	(16)
p>#p ;	TNTN TNTN TNTN TNTN	(Nec)

The 13 equations as rendered above are *not* tautologous. Hence Stit logic is refuted.

Stone space type lattice logic model

From: Haykazyan, L. (2017). Spaces of types in positive model theory. arxiv.org/pdf/1711.05754.pdf

We assume the apparatus of the Meth8 modal logic model checker implementing variant system VŁ4. Meth8 allows to mix four logical values with four analytical values. The designated *proo*f value is T.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	p=p	T	Tautology	proof	11	3
2	p@p	F	Contradiction	absurdum	00	0
3	%p>#p	N	Non-contingency	truth	01	1
4	%p<#p	C	Contingency	falsity	10	2

LET: ~ Not; + Or; & And; \ Not and; > Imply; < Not imply; = Equivalent to; @ Not equivalent to; # all; % some; (p@p) 00, zero; (p=p) 11, one

Results are the proof table of 16-values in row major horizontally.

"Example 5.8. ... To make sure this theory is countably categorical, we need to ensure that there are infinitely many points without colour. So we add a binary relation Q(x, y) (and its negation ¬Q) that will pair the points that do not have a colour. The theory asserts the following.

Q is symmetric and irreflexive:

∀x, y(Q(x, y) → Q(y, x))" (5.8.1)
(#p&#q)&((q&(p&r)) > (q&(r&p)) ; TTTC TTTC TTTC TTTT (5.8.2)

To ensure Eq. 5.8.1 is quantification ∀x, y distributed on the literal (Q(x, y) → Q(y, x)), we rewrite Eq. 5.8.2.

((#p&#q)&(q&(p&r))) > ((#p&#q)&(q&(r&p))) ;
TTTC TTTC TTTC TTTT (5.8.3)

The truth table of Eq. 5.8.2 is identical to Eq. 5.8.3.

Eq. 5.8.2 as rendered is *not* tautologous, and hence the binary relation Q is not symmetric and irreflexive.

Stone-Wales rotation transforms on four proximal polygons only from complex to simple ring

1.1 We ask: "On four proximal polygons, is a simple ring reversible (bijective) with a complex ring?"

We assume the Meth8 scriptors, with vt Validated tautologous, nvt Not Validated tautologous, designated proof value as T tautology; other values are F contradiction and N non-contigency (a truth value).

The truth tables in five models are concatenated as four rows of four values.

LET: p q r s polygon edges,
p & q & r & s simple ring of four proximal polygons,
(p-1) & (q-1) & (r+1) & (s+1) complex ring of four proximal polygons,
(p-1) (p-(%p>%#p)) , (p+1) (p+(%p>%#p)), ...

((p&r)&(q&s)) = (((p-(%p>%#p))&(r-(%r>%#r)))&((q+(%q>%#q))&(s+(%s>%#s)))) ;
TTTT TTTT TTNT TTTF (1)

1.2 We answer 1.1: "Not bijective." However, the near match to a proof is cause for further testing

2.1 We then ask: "On four proximal polygons, does a simple ring imply a complex ring?"

((p&r)&(q&s)) > (((p-(p\p))&(r-(r\r)))&((q+(q\q))&(s+(s\s)))) ;
TTTT TTTT TTTT TTTF (2)

The truth tables for Eq 2 are the same as for Eq 1.

2.2 We then answer 2.1: "No implication." However, the same proof tables repeated from Eq 1 are cause for further testing.

3.1 We now ask: "On four proximal polygons, does a complex ring imply a simple ring?"

(((p-(%p>%#p))&(r-(r\r)))&((q+(q\q))&(s+(s\s)))) > ((p&r)&(q&s)) ;
TTTT TTTT TTNT TTTT (3)

3.2 We now answer: "No implication." This means the sequence of Stone-Wales rotation for four proximal polygons does not transform from a complex ring to a simple ring, or vice versa.

Refutation of strong jump inversion and decidable copy of a saturated model of $\mathcal{DCF}_0$

Abstract: From the paper's abstract, the definition of strong jump inversion is *not* tautologous, hence strong jump inversion is refuted. A computable enumeration of the types realized in models of $\mathcal{DCF}_0$ is also refuted. The alleged fact that the saturated model of $\mathcal{DCF}_0$ has a decidable copy is denied. Therefore these conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Calvert, W.; Frolov, A.; Harizanov, V.; Knight, J.; McCoy, C; Soskova, A.; Vatev, S. (2018). Strong jump inversion. Journal of logic and computation. 28:7:1499–1522. mccoy@up.edu
academic.oup.com/logcom/article-abstract/28/7/1499/5091964?redirectedFrom=fulltext

"Abstract: We say that a structure $\mathcal{A}$ admits *strong jump inversion* provided that for every oracle X, if X' computes $D(C)'$ for some $C \cong \mathcal{A}$, then X computes $D(B)$ for some $B \cong \mathcal{A}$." (A.1.1)

Remark A.1.1: We code X' as X and D(C)' as D(C).

LET p, q, r, s, x: A, B, C, D, X

((%(r=p)&#x)>(s&r))>((%(q=p)&#x)>(s&q)) ;

```
TTTT TTTT TTTT TTTT(8),
TTTT CTTT TTTT CTTT(8)        (A.1.2)
```

"… In order to apply our general result, we produce a computable enumeration of the types realized in models of $\mathcal{DCF}_0$. This also yields the fact that the saturated model of $\mathcal{DCF}_0$ has a decidable copy."

Because Eq. A.1.2 as rendered is *not* tautologous, the definition of strong jump inversion is refuted. What follows is that a computable enumeration of the types realized in models of $\mathcal{DCF}_0$ is also refuted. The alleged fact that the saturated model of $\mathcal{DCF}_0$ has a decidable copy is denied.

Refutation of bounded homomorphisms and finitely generated fiber products of lattices

Abstract: We evaluate two equations for the standard method for constructing subdirect products, which are *not* tautologous. Hence the conjecture of bounded homomorphisms and finitely generated fiber products of lattices is refuted. These form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≴ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Demeo, W.; Mayr, P.; Ruškuc, N. (2019). Bounded homomorphisms and finitely generated fiber products of lattices. arxiv.org/pdf/1907.08046.pdf

Abstract. We investigate when fiber products of lattices are finitely generated and obtain a new characterization of bounded lattice homomorphisms onto finitely presented lattices and onto lattices satisfying Whitman's condition. Specifically, for lattice epimorphisms g : A → D, h: B → D, ... we show the following: If g and h are bounded, then their fiber product (pullback) C = {(a, b) ∈ A × B | g(a) = h(b)} is finitely generated. While the converse is not true in general, it does hold when A and B are free. As a consequence we obtain an exponential time algorithm to decide whether a finitely presented lattice or a finitely generated sublattice satisfying Whitman's condition is bounded. This generalizes an unpublished result of Freese and Nation.

We start by recalling a standard method for constructing subdirect products.

Let A, B be algebras with epimorphisms g: A → D and h: B → D onto the same homomorphic image D. Then the subalgebra C := {(a, b) ∈ A × B | g(a) = h(b)} of A×B is called a fiber product (or pullback) of g and h. Clearly C is a subdirect product of A and B. (1.1.1)

LET p, q, r, s, t, u, v, w: A, B, C, D, a, b, g, h.

((v=(p>s))&(w=(q>s)))>(r=(((t&u)<(p&q))>((v&t)=(w&u)))) ;

```
TTTF TTTT TTTT TTTT( 4)
TTFT TTTT TTTT TTTT( 3)
TTTT TTFT TTTT TTTT( 1)
TFTT TTTT TTTT TTTT( 3)
TTTT TFTT TTTT TTTT( 1)
FTTT TTTT FFFF TTTT( 4)
```

(1.1.2)

Remark 1.1: If g and h are substituted with the respective expansions, then Eq. 1.1 reads as:

C := {(a, b) ∈ A × B | (A→D)(a) = (B → D)(b)} (1.2.1)

r=(((t&u)<(p&q))>(((p>s)&t)=((q>s)&u))) ;

FFFF TTTT **FFFF** TTTT(3)

FTT**F** T**FF**T **FFFF** TTTT(1) (1.2.2)

Eqs. 1.1.2 and 1.2.2 are *not* tautologous. This refutes the standard method for constructing subdirect products, and hence the conjecture of bounded homomorphisms and finitely generated fiber products of lattices.

Student quiz denied as a paradox and refuted as a conjecture

Abstract: We assume the teacher is veracious and evaluate the assertion "There is possibly a quiz next week on Monday, Tuesday, or Wednesday." It is not tautologous, hence denying it is a paradox and refuting it as a conjecture. Therefore the student quiz paradox is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables.
(See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

The teacher asserts there is possibly a quiz next week on Monday, Tuesday, or Wednesday.(1.1)

Remark 1.1: We assume the teacher is veracious.

LET p, q, r, s: Monday, quiz, Tuesday, Wednesday.

%(((q>((p+r)+s))+(~ (q>p)> (q>(r+s))))+(~((q>p)&(q>r))> (q>s)))=(p=p);
TTCT TTTT TTTT TTTT (1.2)

Should the student: Expect a quiz (and possibly on what day); Be surprised by a quiz; or Assume no quiz.

Because Eq. 1.2 is *not* tautologous, the student may safely choose indifference to the specter of a quiz. This means other solutions are not viable.

Refutation of the principle of superposition of states

From: Hari Dass, N.D. (2013). "The superposition principle in quantum mechanics - did the rock enter the foundation surreptitiously?". arxiv.org/pdf/1311.4275.pdf

$$|\psi\rangle = \alpha|1\rangle + \beta|2\rangle \qquad (1.1)$$
$$|good\rangle = 1/\sqrt{2}\ \{\ |red\rangle + |yellow\rangle\ \} \qquad (2.1)$$
$$|bad\rangle = 1/\sqrt{2}\ \{\ |red\rangle - |yellow\rangle\ \} \qquad (3.1)$$
$$|red\rangle = 1/\sqrt{2}\ \{\ |good\rangle + |bad\rangle\ \} \qquad (4.1)$$
$$|yellow\rangle = 1/\sqrt{2}\ \{\ |good\rangle - |bad\rangle\ \} \qquad (5.1)$$

Eq. 1.1 as "the principle of superposition of states" [asserts] that the *complex linear superpositions* also represent *quantum states* of the system".

We assume the Meth8/VŁ4 apparatus and method where the designated *proof* value is `T`. Other values are `F` contradiction, `N` truthity (non-contingence), and `C` falsity (contingence). The 16-valued proof table is row-major and presented horizontally.

LET p q r s: good, smell, red rose, yellow rose; ~p bad, as Not good;
~ Not; + Or; - Not Or; = Equivalent to; > Imply; < Not Imply, less than, ∈ ;
% possibility, for one or some; # necessity, for all.

The irrational constant (1/(2^0.5)) is ignored throughout this demonstration.

We treat Eqs.1.1-5.1 as expressions on the complex plane ℂ . Meth8/VŁ4 maps them by substituting the Equivalent connective for ℝ real numbers with the Imply connective for imaginary numbers.

"(red Nor yellow) Implies (Not(red Or yellow))" (0.2.1)
(r-s)>~(r+s) ; `TTTT TTTT TTTT TTTT` (0.2.2)

p>(r+s) ; `TFTF TTTT TTTT TTTT` (2.2.2)
~p>(r-s) ; `TTTT FTFT FTFT FTFT` (3.2.2)
r>(p+~p) ; `TTTT TTTT TTTT TTTT` (4.2.2)
s>(p-~p) ; `TTTT TTTT FFFF FFFF` (5.2.2)

We rewrite Eqs. 2.2.2 and 3.2.2 as Eq.0.2.2.

By substitution from the text we write:

"the states [good, not good] have *definite values* of some other *attribute* which we could call *smell*" (6.1)
(p+~p)<q ; `TTFF TTFF TTFF TTFF` (6.2)

"Suppose we start with [good] and make a *colour* measurement.
The outcome will be red or yellow with equal probability." (7.1)

Because probability (possibility) is now invoked, we rely on our previous proof that the modal operators as equivalent to the respective quantifiers for this application.

%p>#(r+s) ; `NFNF NNNN NNNN NNNN` (7.2)

We remark that Eq. 7.2 expresses that a possible state of good implies the necessity of the color as red or yellow. Eq. 7.2 does not state the necessity of a probability. (Author Hari Dass later changes that possible state of good into a necessary state of good at Eq. 12.1.)

"If it is red, the state after the measurement is [red],
and likewise for the outcome yellow." (8.1)
(r>r)&(s>s) ; `TTTT TTTT TTTT TTTT` (8.2)

Let us say that the outcome is *red*." (9.1)

Because Eq. 9.1 is a conclusion, we write it as the consequent of both Eqs. 7.1 and 8.l.

((%p>#(r+s))&((r>r)&(s>s)))>r ; `CTCT TTTT CCCC TTTT` (9.2)

"Now imagine a *smell* measurement on the system." (10.1)
q ; `FFTT FFTT FFTT FFTT` (10.2)

"Because the state after the last measurement i.e [red] is an *equal* superposition
of the good and bad smell states, the outcome will be one of these randomly
and with equal probability." (11.1)

Eq. 11.1 has two parts, the antecedent resulting in the combination of Eqs. 9.2 and 10.2 and the consequent as the equal probability (%p=%~p).

(((%p>#(r+s))&((r>r)&(s>s)))>r)>(q>#(%p=%~p)) ;
`TTNF TTFF TTNN TTFF` (11.2)

"Therefore, even though we started with a state whose *smell* was *certain* i.e good,
an intervening colour measurement has completely destroyed this certainty!" (12.1)

A state which *smell* was *certain* as good is (q>#p), and when connected with an intervening measurement for red, produces the antecedent below. The consequent is the *possibility* of good from Eq. 7.2 above.

((q>#p)&((((%p>#(r+s))&((r>r)&(s>s)))>r)>(q>#(%p=%~p))))>~#p ;
`TCTT TCTT TCTC TCTT` (12.2)

In Eq 12.2 we change the "%p" from Eq. 7.2 into "#p", but the table result is the same as in Eq. 12.2.

"Instead, the smell information has become totally *unpredictable*! This is the
inherent *indeterminacy* of quantum theory." (13.1)

We remark on 13.1 that the smell information as a required variable was unpredictable from Eq. 7.2. What was predictable above was the possible determination of red or yellow, and good or not good.

"This is also a demonstration that the pair of observables *colour, smell* are
mutually *incompatible*." (14.1)

Statement 14.1 does not follow from 13.1 or from our results because compatibility is not a consideration.

> "Existence of incompatible observables is the essential content of the *Heisenberg Uncertainty Relations*." (15.1)

Elsewhere we show the Heisenberg uncertainty principle is *not* tautologous.

The combined literal Eqs. as rendered above show the principle of superposition of states in Eq. 1.1 is *not* confirmed, and hence is refuted.

Refutation of the supply and demand conjecture

Abstract: The supply and demand conjecture takes on two states of affairs, depending on assignment of the relation for price,quantify and the relation for supply,demand as antecedent or consequent. The relations are greater than, lesser than, or equivalent. None of the assertions is tautologous, hence refuting the supply and demand conjecture.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

LET p, q, r, s: price, quantity, demand, supply;
> Imply, greater than; < Not imply, lesser than; = Equivalent.

From: en.wikipedia.org/wiki/Supply_and_demand

We frame the conjecture as antecedent and consequent, then reversed, for two states of affairs of two implications and one equivalence for the equilibrium case.

(price greater per quantity) implies (supply less than demand) (1.1.1)
(p>q)>(s<r) ; **F**T**FF** **F**T**FF** TTTT **F**T**FF** (1.1.2)

(price lesser per quantity) implies (supply greater than demand) (1.2.1)
(p<q)>(s>r) ; TTTT TTTT T**F**TT TTTT (1.2.2)

(price equivalent per quantity) implies (supply equivalent to demand) (1.3.1)
(p=q)>(s=r) ; TTTT **F**TT**F** TTTT **F**TT**F** (1.3.3)

(supply less than demand) implies (price greater per quantity) (2.1.1)
(s<r)>(p>q) ; T**F**TT T**F**TT **FFFF** T**F**TT (2.1.2)

(supply greater than demand) implies (price lesser per quantity) (2.2.1)
(s>r)>(p<q) ; **FFFF** **FFFF** **F**T**FF** **FFFF** (2.2.2)

(supply equivalent to demand) implies (price equivalent per quantity) (2.3.1)
(s=r)>(p=q) ; **FFFF** T**FF**T **FFFF** T**FF**T (2.3.3)

Eqs. 1.1.2, 1.2.2, 1.3.2, 2.1.2, 2.2.2, and 2.3.2 as rendered are *not* tautologous. This refutes the supply and demand conjecture.

Remark: Eq. 1.2.2 is closest to tautology, but diverges by one **F** contradiction value.
This may explain popular expression of the conjecture as:
(price lesser per quantity) implies (supply greater than demand).

Refutation of surveillance objectives

Abstract: We evaluate an equivalence and theorem. Neither are tautologous. Hence the reduction of the multi-agent surveillance synthesis problem to solving single-sensor surveillance subgames is refuted. What follows is that surveillance objectives are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) `C` non-contingency, ∇, ordinal 2; (%z>#z) `N` as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Bharadwa, S.; Dimitrova, R.; Topcu, U. (2019).
Distributed synthesis of surveillance strategies for mobile sensors.
arxiv.org/pdf/1902.02393.pdf suda.b@utexas.edu

D. Temporal surveillance objectives
In this paper we consider safety and liveness surveillance objectives, as well as conjunctions of such objectives. We remark the following equivalences of surveillance objectives: ...

if $a \leq b$, then $\Box p_a \wedge \Box \Diamond p_b \equiv \Box p_a$ (D.1.1)

LET p, q, r, s: p, q, a, b

~(s<r)>(((#p&q)&(#%p&s))=(#p&r)) ;
`TTTT TCTC TTTT TCTC` (D.1.2)

Using these equivalences, we can restrict our attention to surveillance objectives of one the following forms:

$\Box p_b$, [or] $\Box \Diamond p_b$[,] or $\Box p_a \wedge \Box \Diamond p_b$, where $a > b$ (D.2.1)

(r>s)>(((#p&s)+(#%p&s))+((#p&r)&(#p&s))) ;
`FFFF` `TTTT CTCT CTCT` (D.2.2)

Eqs. D.1.2 and D2.2 are *not* tautologous. This means the equivalences of the surveillance objective forms are *not* tautologous. Hence the reduction of the multi-agent surveillance synthesis problem to solving single-sensor surveillance subgames is refuted.

Denial of Suzko's problem

Abstract: We examine a sentential logic description, as based on set theory, in support of Suzko's theorem that only two truth values are required as a universal logic. Under syntactic notions, we evaluate three definitions (monotonicity, transivity, permeability) out of six definitions (trivial are substitution-invariance, reflexivity, combined consequence relation). Monotonicity and transivity are *not* tautologous. Right-to-left permeability is *not* tautologous. What follows is that a Malinowski extension of *mixed-consequence* by relaxation of the two values for three logical values is spurious, especially due to the fact that Suzko's theorem is a conjecture based on the *assumption* of set theory. What also follows is that compositionality as based on Suzko-Scott reductions are *not* bivalent and exact, but rather a vector space and probabilistic. Our results point further to the equations analyzed as being *non* tautologous fragments of the universal logic VL4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables.
(See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊔ ;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Chemla, E.; Egrvé, P. (2019).
Suszko's problem: mixed consequence and compositionality.
arxiv.org/pdf/1707.08017.pdf paul.egre@ens.fr

Definition 2.5 (Monotonicity). A consequence relation ⊢ is *monotonic* if:

$\forall\ \Gamma_1 \subseteq \Gamma_2, \Delta_1 \subseteq \Delta_2 : \Gamma_1 \vdash \Delta_1$ implies $\Gamma_2 \vdash \Delta_2$. (2.5.1)

LET p, q, r, s, t: Γ_1 or Γ, Γ_2 or Γ', Δ_1 or Δ, Δ_2 or Δ' or Σ, L.

(~(#q<#p)&~(#s<#r))>((#r>#p)>(#s>#q)) ;
TTTT TTTT TTTT TTCT (2.5.2)

Definition 2.7 (Transitivity). A consequence relation ⊢ is *transitive* iff:

if Γ⊬ Δ, then there are Γ'⊇ Γ , Δ'⊇ Δ such that Γ'⊬Δ' and Γ'∪ Δ'=L. (2.7.1)

((q>p)>(~(p>q)&~(r<s)))>((~(s>q)&(q+s))=t) ;
TTTT TTTT T**F**TT T**F**TT,

T**FF**T TT**F**T TT**F**T TT**F**T (2.7.2)

We introduce here a formal property that a consequence relation should *not* have:

Definition 2.9 (Permeability)**.** A consequence relation is *permeable* if it is *left-to-right* or *right-to-left permeable*, in the following sense: ...

Right-to-left permeability: $\forall$ Γ, Δ, Σ : Γ⊢Σ, Δ ⇒ Γ, Σ⊢Δ (2.9.2.1)

((#s&#q)>#p)>(#q>(#p&#s)) ; TTCC TTCC TTTT TTTT (2.9.2.2)

By extension, a logic is called *permeable* if its consequence relation is permeable. If a logic is not permeable, then its consequence relation is neither universal nor trivial ...

Eqs. 2.5.2, 2.7.2, and 2.9.2 are *not* tautologous. This means those three of six equations refute the goal of mixed-consequence before subsequent machinations including entertainment of 3- or 4-values and the Appendix A compositionality of Suzko-Scott reductions which are *not* bivalent but a vector space.

Refutation of symmetry breaking, Boolean skeletons, ensemble technique, and SMT solver

Abstract: The SMT problem as stated is not a theorem, and the derivation of Boolean skeletons, while equivalent, are not the SMT problem. This refutes the symmetry breaking technique and also the attendant ensemble technique. What follows is the SMT solver is refuted, forming a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Dingliwal, S.; Agarwal, R.; Mittal, H.; Singla, P. (2019). CVC4-SymBreak: Derived SMT solver at SMT Competition 2019. arxiv.org/pdf/1908.00860.pdf

1 Introduction

Satisfiability Modulo Theories (SMT) is a decision problem for logical first order formulas combined with operations defined over additional constructs such as integers, reals, arrays, and uninterpreted functions.. . Symmetry breaking.. has been an effective technique for improving the efficiency of propositional logic solvers for a long time. It involves identifying variable permutations (known as symmetry permutations) applying which does not alter the theory, and then using them to add constraints to the problem without changing it's satisfiability and thereby reducing the search space.

3.1 Symmetry Breaking Technique

SMT problem Ω (x<8)∧(y<8)∧((x+y<10)∨(x+y>3))) (3.1.1.1)

LET p, q, r, s: T, Q, R, S.

((x<q)&(y<q))&((x+(y<r))+(x+(y>p))) ;

FFFF FFFF FFFF FFFF(48)
TT**FF** TT**FF** TT**FF** TT**FF**(16) (3.1.1.2)

Boolean skeleton Ψ Q∧R∧(S∨T) (3.1.2.1)

(q&r)&(s+p) ; **FFFF FFF**T **FFFF FF**TT (3.1.2.2)

Constraints set ...
Symmetry permut. ...
SBP added …

New skeleton Ψ′ Q∧R∧(S∨T)∧(¬Q∨R) (3.1.6.1)

(q&r)&((s+p)&(~q+r)) ; **FFFF FFF**T **FFFF FF**TT (3.1.6.2)

Eqs. 3.1.1.2, 3.1.2.2, and 3.1.6.2 are *not* tautologous. Eqs. 3.1.2.2 and 3.16.2 are the same. The SMT problem is not a theorem, and the Boolean skeletons, while equivalent, are not the SMT problem. This refutes the symmetry breaking technique and also the attendant ensemble technique. What follows is the SMT solver is also refuted.

Refutation of symplectic vector space for physics of biology

Abstract: The symplectic vector space is refuted as the basis for the Borsuk-Ulam theorem (BUT) and the ham sandwich theorem, demoting those to conjecture status. Consequently, arguments derived therefrom cannot be proved for use in physics of biology.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: *ω* lc_omega; u; v; V;
~ Not; + Or; & And; > Imply, greater than; < Not imply, lesser than, ∈;
= Equivalent; @ Not Equivalent; (s@s) zero.

From: Tozzi, Arturo; Peters, James F.; Torday, John S. (2018). An operational definition of life, evolution and their primeval occurrence. vixra.org/pdf/1810.0132v1.pdf; en.wikipedia.org/wiki/Symplectic_vector_space:

alternating: ω(v,v)=0 holds for all v∈V (2.1.1)

(#r<s)&((p&(q&r))=(s@s)) ; **FFFF** NNN**F** **FFFF** **FFFF** (2.1.2)

nondegenerate: ω(u,v)=0 for all v∈V implies that u is zero. (3.1.1)

((#r<s)&((p&(q&r))=(s@s)))>(q=(s@s)) ; TTTT TT<u>C</u>T TTTT TTTT (3.1.2)

Eqs. 2.1.2 and 3.1.2 as rendered are *not* tautologous, with Eq. 3.1.2 diverging by one contingency value of C as falsity. This refutes the symplectic vector space.

The basis of the Borsuk-Ulam theorem (BUT) and all cases of the ham sandwich theorem is symplectic vector space. Therefore, those theorems are refuted at their inception and are demoted to conjectures for use in physics of biology. This results in unprovable results, such as definition of life, evolution, and primeval instance of the authors, as based on such conjectures.

Refutation of Tarski's geometric axioms and betweenness

Abstract: Of 13 equations evaluated, five are tautologous and eight are *non* tautologous. This refutes Tarski's geometric axioms and betweenness which form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Tarski%27s_axioms

[Alfred] Tarski's … axiom set for the substantial fragment of Euclidean geometry ... is formulable in first-order logic with identity, and requiring no set theory ([from] 1959) (i.e., that part of Euclidean geometry that is formulable as an elementary theory).

Congruence axioms
Reflexivity of congruence xy≡yx (1.1)

Remark 1.1: xy is not read point x is less than y as equivalent to point y is greater than x but rather x&y is equivalent to y&x.

LET p, q, r, s, t, u, v, w, x, y, z: a, b, d, e, u, v, w, x, y, z.

(x&y)=(y&x) ; TTTT TTTT TTTT TTTT (1.2)

Identity of congruence xy≡zz→x=y (2.1)

((x&y)=(z&z))>(x=y) ; TTTT TTTT TTTT TTTT(16)
FFFF FFFF FFFF FFFF(32)
TTTT TTTT TTTT TTTT(80) (2.2)

Transivity of congruence (xy≡zu∧xy≡vw)→zu≡vw (3.1)

(((x&y)=(z&u))&((x&y)=(v&w)))>((z&u)=(v&w)) ;
TTTT TTTT TTTT TTTT (3.2)

Betweenness axioms
Identity of betweenness Bxyx→x=y (4.1)

((y>x)&(x>y))>(x=y) ; TTTT TTTT TTTT TTTT (4.2)

Axiom of Pasch (Bxuz∧Byvz)→∃a(Buay∧Bvax) (5.1)

(((u>x)&(z>u))&((v>y)&(z>v)))>(((%p>u)&(y>%p))&((%p>v)&(x>%p))) ;

N**F**N**F** N**F**N**F** N**F**N**F** N**F**N**F**(2)}x2
TTTT TTTT TTTT TTTT(6)}
FFFF FFFF FFFF FFFF(4)}x2
TTTT TTTT TTTT TTTT(4)}
FFFF FFFF FFFF FFFF(2)}x4
TTTT TTTT TTTT TTTT(2)}
FFFF FFFF FFFF FFFF(6)}x2
CTCT CTCT CTCT CTCT(2)}
TTTT TTTT TTTT TTTT(48)
TTTT TTTT TTTT TTTT(6)}x2
CTCT CTCT CTCT CTCT(2)} (5.2)

Axiom schema of continuity
∃a∀x∀y[(φ (x)∧ψ(y))→Baxy] → ∃b∀x∀y[(φ (x)∧ψ(y))→Bxby] (11.1)

LET u, v: φ, ψ.

(((u&#x)&(v&#y))>((#x>%p)&(#y>#x)))>(((u&#x)&(v&#y))>((%q>#x)&(#y>%q))) ;

TTTT TTTT TTTT TTTT(54)
TCTT TCTT TCTT TCTT(2)
TTTT TTTT TTTT TTTT(6)
TCTT TCTT TCTT TCTT(2) (11.2)

Lower dimension ∃a∃b∃c[¬Babc∧¬Bbca∧¬Bcab] (6.1)

(~((%q>%p)&(%r>%q))&~((%r>%q)&(%p>%r)))&~((%p>%r)&(%q>%p)) ;

FFFF FFFF FFFF FFFF (6.2)

Congruence and betweenness
Upper dimension (xu≡xy∧yu≡yv∧zu≡zv∧u≠v) → (Bxyz∨Byzx∨Bzxy) (7.1)

((((x&u)=(x&v))&((y&u)=(y&v)))&(((z&u)=(z&v))&(u@v)))>
((((y>x)&(z>y))&((z>y)&(x>z)))&((x>z)&(y>x))) ;

TTTT TTTT TTTT TTTT (7.2)

Axiom of Euclid
A: (Bxyw∧xy≡yw)∧(Bxuv∧xu≡uv)∧(Byuz∧yu≡zu))→yz≡vw (8.1.1)

((((((y>x)&(w>y))&(x&y))=(y&w))&((((u>x)&(v>u))&(x&u))=(u&v)))&
((((u>y)&(z>u))&(y&u))=(z&u)))>((y&z)=(v&w)) ;

TTTT TTTT TTTT TTTT(12)
FFFF FFFF FFFF FFFF(2)

TTTT TTTT TTTT TTTT(14)
FFFF FFFF FFFF FFFF(4)
TTTT TTTT TTTT TTTT(28)
FFFF FFFF FFFF FFFF(2)
TTTT TTTT TTTT TTTT(14)
FFFF FFFF FFFF FFFF(2)
TTTT TTTT TTTT TTTT(14)
FFFF FFFF FFFF FFFF(2)
TTTT TTTT TTTT TTTT(2)
FFFF FFFF FFFF FFFF(6)
TTTT TTTT TTTT TTTT(18)
FFFF FFFF FFFF FFFF(2)
TTTT TTTT TTTT TTTT(6) (8.1.2)

B: Bxyz∨Byzx∨Bxzy∨∃a(xa≡ya∧xa≡za) (8.2.1)

((((y>x)&(z>y))&((z>y)&(x>z)))&((x>z)&(y>x)))&(((x&%p)=(y&%p))&((x&%p)=(z&%p))) ;

TTTT TTTT TTTT TTTT(16)
FFFF FFFF FFFF FFFF(96)
TTTT TTTT TTTT TTTT(16) (8.2.2)

C: (Bxuv∧Byuz∧x≠u)→∃a∃b(Bxya≠Bxzb∧Bavb) (8.3.1)

((((u>x)&(v>u))&((u>y)&(z>u)))&(x@u))>
((((y>x)&(%p>y))&((z>x)&(%q>z)))&((v>%p)&(%q>v))) ;

TTTT TTTT TTTT TTTT(10)
TTTT TTTT TTTT TTTT(6) }x2
NN**FF** NN**FF** NN**FF** NN**FF**(2) }
TTTT TTTT TTTT TTTT(6) }
NN**FF** NN**FF** NN**FF** NN**FF**(2) }
TTTT TTTT TTTT TTTT(16) }
TTTT TTTT TTTT TTTT(54) (8.3.2)

Five segment (x≠y∧Bxyz∧Bx'y'z'∧xy≡x'y'∧yz≡y'z'∧xu≡x'u'∧yu≡y'u')→zu≡z'u'

(9.1)

LET p, q, r, t, u, x, y, z:
x', y', z', u', u, x, y, z

(((x@y)&(((y>x)&(z>y))&((q>p)&(r>q))))&
((((x&y)=(p&q))&((y&z)=(q&r)))&(((x&u)=(p&t))&((y&u)=(q&t)))))>((z&u)=(r&t)) ;

TTTT TTTT TTTT TTTT (9.2)

Segment construction ∃z[Bxyz∧yz≡ab](10.1)

LET p, q: a, b

((y>x)&(#z>y))&((y&#z)=(p&q)) ;

TTT**F** TTT**F** TTT**F** TTT**F**(32)
FFFF FFFF FFFF FFFF(16)
TTT**F** TTT**F** TTT**F** TTT**F**(16)

```
CCCF CCCF CCCF CCCF(32)
FFFF FFFF FFFF FFFF(16)
CCCN CCCN CCCN CCCN(16)        (10.2)
```

Of 13 eqs. evaluated, five are tautologous and eight are *non* tautologous. This refutes Tarski's geometric axioms and betweenness.

Refutation of Tarski's undefinability of truth theorem

From: Salehji, S. Theorems of Tarski's undefinability and Gödel's second incompleteness--computationally". 2017. arxiv.org/pdf/1509.00164.pdf

> "Gödel's first incompleteness theorem is usually stated as "*no sound and R*[ecursively] *E*[numerable] *extension of P*[eano's] *A*[rithmetic] *can be complete*"; in notation $\mathrm{PA} \subseteq T \ \& \ T \in \Sigma_1 \ \& \ T \subseteq \mathrm{Th}(N) \Rightarrow T \neq \mathrm{Th}(N)$." (2.2.1)

> "So, Tarski's theorem states that for any n, $\mathrm{Th}(N) \notin \Sigma_n$. For the sake of unifying it with Gödel's theorem let us present this theorem as $(*)_n$ $\mathrm{PA} \subseteq T \ \& \ T \in \Sigma_n \ \& \ T \subseteq \mathrm{Th}(N) \Rightarrow \mathrm{Th}(N) \not\subseteq T$ stating that "no definable and sound extension of PA can be complete"." (2.2.2)

We rewrite Eq. 2.2.2 because "for any n, $\mathrm{Th}(N) \notin \Sigma_n$" is not expressed correctly.

$\mathrm{PA} \subseteq T \ \& \ \mathrm{T} \in \Sigma_n \ \& \ T \subseteq \mathrm{Th}(N) \Rightarrow \mathrm{Th}(N) \not\subseteq \forall n \Sigma_n$. (2.2.3)

We assume the apparatus and method of Meth8/VŁ4 to evaluate Eq. 2.2.3.

The designated proof value is `T` for tautology; `F` is for contradiction.
The 16-valued truth table is presented row-major and horizontally.

LET: ~ Not; & And; + Or; > Imply, greater than; < Not Imply, less than;
necessity, all; % possibility, some;
pqrs: "PA"; T; n; N
r Σ_n; #r $\forall n \Sigma_n$; (%s>#s) non-contingency truth value for Th(N); ~(q<p) ($p \subseteq q$).

(~(q<p) & ((q<r)&~((%s>#s)<q))) > (#r<(%s>#s)) ; `TTTF TTTT TTTF TTTT` (2.2.4)

Eq. 2.2.4 as rendered for Eq. 2.2.3 is *not* tautologous.

Refutation of Tarski-Grothendieck set theory

From: en.wikipedia.org/wiki/Tarski–Grothendieck_set_theory

Axiom: $\forall x \exists y\, [x \in y \wedge \forall z \in y\, (P(z) \subseteq y \wedge P(z) \in y) \wedge \forall z \in P(y)\, ((\neg z \approx y) \rightarrow z \in y)]$ (1.1)

We assume the Meth8/VŁ4 apparatus and method.

LET p q r s: x, y, z, P.
necessity, for all; % possibility, for one or some;
\> Imply, greater than; < Not Imply, less than, ∈; = Equivalent to, ≈

```
(#p&%q) & ((p<q)&((#r<q)&((~((s&r)>q)&((s&r)>q))&((#r<(s&q))&((~r=q)>(r<q)))))) ;
FFFF FFFF FFFF FFFF (1.2)
```

Remark: The consequent in Eq. 1.2 has the same table result as Eq. 1.2. Therefore the quantifier as the antecedent has no affect on the result.

Eq. 1.2 as rendered is *not* tautologous. This means the axiom of Tarski-Grothendieck set theory in Eq. 1.1 is refuted.

Refutation of Tarski-Grothendieck theorem by Metamath theorem prover

Abstract: We evaluate six conjectures and one theorem, as proffered by Metamath staff. The conjectures are *not* tautologous. The Tarski-Grothendieck theorem is also *not* tautologous. Metamath fails our analysis.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

[C]onsider the theorem

-. A. y E. z x = y (1.1)

which is provable in FOL.

LET p, q, r: x, y, z;
~ Not, -.; & And, /\; Or; = Equivalent, <->; @ Not Equivalent;
> Imply, ->, greater than; < Not Imply, lesser than;
necessity, for all or every, A.; % possibility, for one or some, E.;
(p=p) Tautology.

(~#q&(%r&p))=q ; TN**F**C T**FF**C TN**F**C T**FF**C (1.2)

Remark 1.1: Eq. 1.2 as rendered is *not* tautologous, hence not provable in FOL per Meth8/VŁ4.

The Exists.z part is trivial because z is not in the statement, so it says that not every y is equal to some fixed variable x.

We consider Eqs. 1.0, 1.1 without the z as r:

(~#q&p)=q ; T**FF**C T**FF**C T**FF**C T**FF**C (1.2.2)

Remark 1.2.2: Table results for Eq. 1.2.2 differ from Eq. 1.2 by the lines TN**F**C, so the exists z part is not so trivial (even though z isn't in the statement).

(If the free x is uncomfortable, [one] can also bind it [Eq. 1.1] as

A. x -. A. y E. z x = y.) (2.1)

((#p&~#q)&(%r&p))=q ; TT**FF** TC**FF** TT**FF** TC**FF** (2.2)

Remark 2.2: Table results for Eqs. 1.2, 1.2.2, 2.2 are different, so the binding in 2.1 is not equivalent to 1.1.

... [one] would translate this [Eq. 2.2] to:

~#%(p=q) (3.2)

~(#(%(p=q)=(p=p))) = (p=p) ; CTTC CTTC CTTC CTTC (3.2.2)

Remark 3.2.2: Eq. 3.2.2 does not produce the same table result as Eq. 2.2, meaning this translation of equivalence is mistaken.

and

%(p=q) evaluates to T, (4.1)

%(p=q) = (p=p) ; TCCT TCCT TCCT TCCT (4.2)

Remark 4.2: Eq. 4.1 is *not* tautologous, hence mistaken as an evaluation.

so

~#T evaluates to F. (5.1)

~(#(p=p)=(p=p)) = (p@p) ; NNNN NNNN NNNN NNNN (5.2)

Remark 5.2: The table result for Eq. 5.2 is truthity, and hence at a single value stage closest to tautology.

The Tarski–Grothendieck [*ax-groth*] is rendered as:

((-. (x -> E. y) /\ -. (A. z -> E. y)) /\
(((z -> A. w) -> -. (A. w -> E. y)) /\
(-. (E. w -> E. y) /\ (-. (-. z -> A. v) -> -. (A. v -> w))))) /\
(-. (-. y -> A. z) -> ((A. z <-> y) /\ -. (A. z -> y))) (6.1)

LET w, x, y, z: w, x, y, z.

((~(x>%y)&~(#z>%y)) &
(((z>#w)>~(#w>%y)) &
(~(%w>%y)&(~(~z>#v)>~(#v>w))))) &
(~(~y>#z)>((#z=y)&~(#z>y))) ;
TTFF TCFF TTFF TCFF (6.2)

Remark 6.2: Eq. 6.2 is *not* tautologous, meaning that *ax-groth* is refuted.

Refutation of the planar Euclidean R-geometry of Tarski

Abstract: We evaluate the axioms of the title. The axiom of identity of betweenness and axiom of Euclid are tautologous, but the others are not. The commonplace expression of the axiom of Euclid does not match its other two variations which is troubling. This effectively refutes the planar R-geometry.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r, s, u, v, w, x, y, z:
a, b, r, B, u, v, w, x, y, z;
~ Not, ¬ ; + Or, ∨ ; & And, ∧; > Imply, →;
= Equivalent, ≡; @ Not Equivalent, ≠; % possibility, for one or some, ∃.

From: en.wikipedia.org/wiki/Tarski's_axioms

Congruence axioms

Identity of congruence

x y ≡ z z → x = y (2.1)

((x&y)=(z&z))>(x=y) ; `TTTT TTTT TTTT TTTT` (x96),
`FFFF FFFF FFFF FFFF` (x32) (2.2)

Transivity of congruence

(x y ≡ z u ∧ x y ≡ v w) → z u ≡ v w (3.1)

((x&y)=(((z&u)&(x&y))=(v&w)))>((z&u)=(u&w)) ;
`TTTT TTTT TTTT TTTT` (x124),
`FFFF FFFF FFFF FFFF` (x 4) (3.2)

Betweenness axioms

Identity of betweenness

B x y x → x = y (4.1)

((s&x)&(y&x))>(x=y) ; `TTTT TTTT TTTT TTTT` (x128) (4.2)

Axiom of Pasch

B x u z ∧ B y v z) → ∃ a (B u a y ∧ B v a x) (5.1)

(((s&x)&(u&z))&((s&y)&(v&z))) >
(((s&u)&(%p&y))&((s&v)&(%p&x))) ;
TTTT TTTT TTTT TTTT (x128),
TTTT TTTT CTCT CTCT (x 4) (5.2)

Axiom schema of continuity

LET u, v: ϕ, ψ.

Let $\varphi(x)$ and $\psi(y)$ be first-order formulae containing no free instances of either *a* or *b*.t there also be no free instances of *x* in $\psi(y)$ or of *y* in $\varphi(x)$. Then all instances of the following schema are axioms:

$$\exists a \forall x \forall y [(\phi (x) \wedge \psi (y)) \rightarrow B a x y] \rightarrow$$
$$\exists b \forall x \forall y [(\phi (x) \wedge \psi (y)) \rightarrow B x b y]$$
(6.1)

(((u&#x)&(v&#y)) > (s&(%p&(#x&#y)))) >
(((u&#x)&(v&#y)) > (s&((#x&%q)&#y))) ;
TTTT TTTT TTTT TTTT (x120),
TTTT TTTT CTCT CTCT (x 8) (6.2)

Lower dimension

LET p, q, r, s: a, b, c, B.

$$\exists a \exists b \exists c [\neg B a b c \wedge \neg B b c a \wedge \neg B c a b] \quad (7.1)$$

(((~s&%p)&(%q&%r)) & ((~s&%q)&(%r&%p))) &
((~s&%r)&(%p&%q)) ; CCCC CCCT TTTT TTTT (x128) (7.2)

Congruence and betweenness

Upper dimension

$$(x u \equiv x v \wedge y u \equiv y v \wedge z u \equiv z v \wedge u \neq v) \rightarrow (B x y z \vee B y z x \vee B z x y)$$
(8.1)

(((x&u)=((x&v)&(y&u)))=(((y&v)&(z&u))=((z&v)&(u@v)))) >
((((s&x)&(y&z))&((x&y)&(z&z)))&((s&z)&(x&y))) ;
FFFF FFFF TTTT TTTT, TTTT TTTT TTTT TTTT,
FFFF FFFF FFFF FFFF (8.2)

Axiom of Euclid

Each of the three variants of this axiom, all equivalent over the remaining Tarski's axioms to Euclid's parallel postulate, has an advantage over the others:

A dispenses with existential quantifiers;
B has the fewest variables and atomic sentences;
C requires but one primitive notion, betweenness. This variant is the usual one given in the literature.

A: $((B\, x\, y\, w \wedge x\, y \equiv y\, w) \wedge (B\, x\, u\, v \wedge x\, u \equiv u\, v) \wedge (B\, y\, u\, z \wedge y\, u \equiv z\, u)) \rightarrow y\, z \equiv v\, w$ (9.1)

((((s&(x&y))&(w&(x&y)))=(y&w))&((((s&x)&((u&v)&(x&u)))=(u&v))&(((s&y)&((u&z)&(y&u)))=(z&u))))>((y&z)=(v&w)) ;
TTTT TTTT TTTT TTTT, **FFFF FFFF FFFF FFFF**,
TTTT TTTT **FFFF FFFF** (9.2)

B: $B\, x\, y\, z \vee B\, y\, z\, x \vee B\, z\, x\, y \vee \exists\, a (x\, a \equiv y\, a \wedge x\, a \equiv z\, a)$ (10.1)

((((s&x)&(y&z))+((s&y)&(z&x)))+((s&z)&(x&y))) +
(((x&%p)=(y&%p))&((x&%p)=(z@%p))) ;
N**F**N**F** N**F**N**F** N**F**N**F** N**F**N**F**, TTTT TTTT TTTT TTTT,
CTCT CTCT CTCT CTCT, **FFFF FFFF FFFF FFFF**,
FFFF FFFF TTTT TTTT (10.2)

C: $(B\, x\, u\, v \wedge B\, y\, u\, z \wedge x \neq u) \rightarrow \exists\, a\, \exists\, b (B\, x\, y\, a \wedge B\, x\, z\, b \wedge B\, a\, v\, b)$ (11.1)

((s&(x&(u&v)))&((s&(y&(u&z)))&(x@u))) >
(((s&(x&y))&(%p&(s&(x&z))))&(%q&(s&((%p&v)&%q)))) ;
TTTT TTTT TTTT TTTT (x128) (11.2)

Eqs. 4.2 and 11.2 as rendered are tautologous, but the others are not. The commonplace expression of the axiom of Euclid is tautologous, but oddly the other two such expressions are not. This effectively refutes the planar Euclidean R-geometry of Tarski.

Refutation of temporal logic via instant- and interval/period-based models of time

Abstract: The basic properties in seven equations are *not* tautologous. This refutes temporal logic via instant- and interval/period-based models of time and forms it as a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Galton, A.; Valentin Goranko, V. Temporal logic. (2015).
plato.stanford.edu/entries/logic-temporal/ valentin.goranko@philosophy.su.se

2.1 Instant-based models of the flow of time

Some further basic properties … can be expressed with first-order sentences as follows:

reflexivity: ∀*x*(*x*≺*x*) (2.1.1.1)

LET p, q, r: x, y, z

#p<#p ; **FFFF FFFF FFFF FFFF** (2.1.1.2)

density (between every two precedence-related instants there is an instant):
∀x∀y(x≺y→∃z(x≺z∧z≺y)) (2.1.2.1)

(#p<#q)>((#p<%r)&(%r<#q)) ;
`TCTT TCTT TCTT TCTT` (2.1.2.2)

no beginning: ∀x∃y(y≺x);∀x∃y(y≺x) (2.1.3.1)

%q<#p ; `CTCC CTCC CTCC CTCC` (2.1.3.2)

no end: ∀x∃y(x≺y) (2.1.4.1)

#p<%q ; **F**`N`**FF F**`N`**FF F**`N`**FF F**`N`**FF** (2.1.4.2)

every instant has an immediate successor:
$\forall x\exists y(x\prec y\wedge\forall z(x\prec z\rightarrow y\leq z))\forall x\exists y(x\prec y\wedge\forall z(x\prec z\rightarrow y\leq z))$ (2.1.5.1)

(#p<%q)&((#p<#r)>~(#r<%q)) ;
FN**FF** **F**N**FF** **F**N**FF** **F**N**FF** (2.1.5.2)

every instant has an immediate predecessor:
$\forall x\exists y(y\prec x\wedge\forall z(z\prec x\rightarrow z\leq y))$ (2.1.6.1)

(%q<#p)&((#r<#p)>~(%q<#r)) ;
CCTC CCTC CCTC CCTC 2.1.6.2)

2.2 Interval/period based models of time

Some natural basic properties of such interval-based relations and models include:

atomicity of $\sqsubseteq$ (for discrete time): $\forall x\exists y(y\sqsubseteq x\wedge\forall z(z\sqsubseteq y\rightarrow z=y))$ (2.2.1.1)

~(#p<%q)&(~(%q<#r)>(#r=%q)) ;
TCTT CCTT TCTT CCTT (2.2.1.2)

Basic properties in these seven equations are *not* tautologous. This refutes temporal logic via instant- and interval/period-based models of time.

Refutation of completeness of temporal logics over infinite intervals

Abstract: We evaluate three pairs of equations for interval temporal logics [ITL]. None is tautologous, refuting completeness of temporal logics for finite *and* infinite intervals. These form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Wang, H.; Xu, Q. (2004). Completeness of temporal logics over infinite intervals. fst.um.edu.mo/en/staff/documents/fstqx/CompletenessInfinite.pdf

Abstract Interval temporal logics over infinite intervals are studied. First, the ordinary possible worlds models are extended to infinite possible world models. Accordingly, an axiomatic system is proposed and it has been proved complete. Secondly, infinite intervals are included in a logic over abstract intervals. A corresponding axiomatic system is given and proven to be complete also.

2 Interval temporal logic [ITL] **over finite intervals**
2.4 System S'
Axiomatic system S' concentrates on reasoning about intervals rather than just possible worlds.
Temporal and duration domain A duration domain D is a non-empty set equipped with a binary operation + and at least one element 0 which satisfy the conditions D1–D5 below

D5 $(\exists z)(x + z = y \vee y + z = x)$, (2.4.5.1.1)
$(\exists z)(z + x = y \vee z + y = x)$. (2.4.5.2.1)

LET p, q, r: x, y, z.

((%r+p)=q)+((%r+q)=p) ; NCCT **F**TTT NCCT **F**TTT (2.4.5.1.2)
((p+%r)=q)+((q+%r)=p) ; NCCT **F**TTT NCCT **F**TTT (2.4.5.1.2)

4.1 Infinite interval models
Duration domain Let D be an algebra with a binary operation + and two distinct constants 0 and ∞. D is called a duration domain if the algebra satisfies the following conditions

(6) There exists z such that x+z=y or y+z=x, and (4.1.6.1.1)
there exists z such that z+x=y or z+y=x. (4.1.6.2.1)

((%r+p)=q)+((%r+q)=p) ; `NCCT` **F**`TTT NCCT` **F**`TTT` (4.1.6.1.2)
((p+%r)=q)+((q+%r)=p) ; `NCCT` **F**`TTT NCCT` **F**`TTT` (4.1.6.1.2)

4.2 System S'$_\infty$
System S'$_\infty$ for infinite interval models is obtained by adding the following new axioms to S$_\infty$ [about Duration domain]:

D6 $(\exists x)(x + z = y \lor y + z = x)$, (4.2.6.1.1)
$(\exists x)(z + x = y \lor z + y = x)$; (4.2.6.2.1)

((%p+r)=q)+((q+r)=%p) ; `N`**F**`CT CTTT N`**F**`CT CTTT` (4.2.6.1.2)
((r+%p)=q)+((q+r)=%p) ; `N`**F**`CT CTTT N`**F**`CT CTTT` (4.2.6.2.2)

Eqs. 2.4.5, 4.1.6, and 4.2.6 are *not* tautologous. This refutes the conjectures of completeness of temporal logics over finite *and* infinite intervals.

Refutation of the term rewriting approach to automated theorem proving

Abstract: Using three group axioms, two examples for an original and rewritten proofs are *not* tautologous. This refutes the term rewriting approach for automated theorem proving to form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ←, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Hsiang, J.; Kirchner, H.; Lescanne, P.; Rusinowitch, M. (1992). The term rewriting approach to automated theorem proving. J. Logic Programming 1992:14:71-99.
core.ac.uk/download/pdf/82371298.pdf

Example 2.1. … Consider the following set, called Group of axioms for groups:
x*e=x; x*i(x) =e; (x*y)*z=x*(y*z). ... An equational proof of e * x =x in groups is given below:

e*x=e*(x*e) =e*(x*(i(x)*i(i(x))))=e*((x*i(x))*i(i(x)))
=e*(e*i(i(x))) =(e*e)*i(i(x)) =e*i(i(x))
=(x*i(x))*i(i(x)) =x*(i(x)*i(i(x))) =x*e=x. (2.1.1)

LET p, q, r: e, I, x; & for *.

(((((p&r)=(p&(r&p)))=((p&(r&(q&r)))&(q&(q&r))))=(((p&((r&q)&r))&(q&(q&r)))=
(p&(p&(q&(q&r))))))=(((p&p)&(q&(q&r)))=(p&(q&(q&r))))) =
(((((r&q)&r)&(q&(q&r)))=((r&(q&r))&(q&(q&r))))=((r&p)=r)) ;
FFFF T**F**TT **FFFF** T**F**TT (2.1.2)

Example 2.2. For instance, x*e→x; x*i(x)→e; (x*y)*z→x*(y*z) is a rewrite system. Rewriting a term with a rewrite system R consists in replacing a subterm which matches a left-hand side of a rewrite rule by the right-hand side whose variables are bound to values computed by the matching algorithm. This relation is denoted by $\rightarrow_R$. Iterating this process is called reducing. If two terms can be reduced to a same one, a special equational proof is obtained, called a rewrite proof. A term which cannot be rewritten is said to be in normal form.

Example 2.3. The rewrite rules of Example 2.2 are used in the equational proof of Example 2.1 as follows:

e*x←e*(x*e)←e*(x*(i(x)*i(i(x))))←e*((x*i(x))*i(i(x)))
→e*(e*i(i(x)))←(e*e)*i(i(x))→e*i(i(x))
←(x*i(x))*i(i(x))→x*(i(x)*i(i(x)))→x*e→x. (2.3.1)

(((((p&r)<(p&(r&p)))<((p&(r&(q&r)))&(q&(q&r))))<(((p&((r&q)&r))&(q&(q&r)))>
(p&(p&(q&(q&r))))))<(((p&p)&(q&(q&r)))>(p&(q&(q&r))))) <
(((((r&q)&r)&(q&(q&r)))>((r&(q&r))&(q&(q&r))))>((r&p)>r)) ;
FFFF FFFF FFFF FFFF (2.3.2)

Obviously it is not a rewrite proof. There are peaks, i.e., terms from which issue two sequences of rewritings, and valleys, i.e., terms where rewriting is not applied any more. Such terms, for instance e * x, e *(e * i(i(x))), e * i(i(x) and x are in normal form for the rewrite system.

Eqs. 2.1.2 and 2.3.2 as rendered are *not* tautologous. This refutes the term rewriting approach for automated theorem proving.

Refutation of logical theory based on compatible consequence in set theory

Abstract: We evaluate canonically logical compatibility relations (CM) and complements (CR_S), each in three sets of definitions. None is tautologous, so we avoid the subsequent ten relations. This refutes the "the possibility of a notion of compatibility that allows either for glutty or gappy reasoning". (By extension, paraconsistent logic is rendered untenable.) Therefore the bivalent standard notion of formal theory in logic is confirmed as allowing both assertion and denial as equally valid. In fact, this refutation further disallows injection of a bilateralist approach on many dimensions. This also indirectly reiterates that set theory is *not* bivalent, and hence derivations therefrom, such as the instant relations, are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And; > Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊔;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Blasio, C.; Caleiro, C.; Marcos, J. (2019).
What is logical theory? On theories containing assertions and denials.
arxiv.org/pdf/1903.02338.pdf jmarcos@dimap.ufrn.br

Remark 0: The reproduction below captures use of italic and single quote within the expository narrative only.

1 Capturing the notion of logical consequence

Let L be a non-empty set of *sentences*. We will assume that the judgments of *assertion* and *denial* are primitive in our metalanguage, and in what follows we will intuitively think of the *consecution* (Δ_1, Δ_0) as a meta-logical expression concerning the 'compatibility' of certain judgments, namely, the assertion of all sentences in $\Delta_1 \subseteq$ L and the denial of all sentences in $\Delta_0 \subseteq$ L. Building on that idea, a (*canonical logical*) *compatibility relation* (*on* L) will be here defined as any relation ► on ℘ (L) × ℘ (L) satisfying, for every Π, Π', Σ, Σ', Δ ⊆ L:

LET p, q, r, s, t, u, v: Π, Π', Σ, Σ', L, Δ, Δ';
► Imply; ▷ Not Imply (the complement of Imply).

(CM0) if Π' ∪ Π ► Σ ∪ Σ', then Π ► Σ

```
~(#t<#u)>(((#q+#p)>(#r+#s))>(#q>#r)) ;
                        TTTT TTTT TTCC TTTT(1),
                        TTTT TTTT TTTT TTTT(1),
                        TTTT TTTT TTCC TTTT(3),
                        TTTT TTTT TTTT TTTT(1),
                        TTTT TTTT TTCC TTTT(2)
```

(CM0.2)

(CM1) if Π ▸ Σ, then Π ∩ Σ = Ø

```
~(#t<#u)>(((#p>#r)>(#p&#r))=(z@z)) ;
                        TCTC TCTC TCTC TCTC(1)
                        TTTT TTTT TTTT TTTT(1),
                        TCTC TCTC TCTC TCTC(3)
                        TTTT TTTT TTTT TTTT(1),
                        TCTC TCTC TCTC TCTC(2)
```

(CM1.2)

(CM2) if Π ▸ Σ, then there is some Δ' ⊆ Δ such that Δ' ∪ Π ▸ Σ ∪ (Δ \ Δ')

```
~(#t<#u)>
(((#p>#r)>%(~(#u<#v)=(z=z)))>((#v+#p)>(#r+(#u\#v)))) ;
                        TTTT TTTT TTTT TTTT(6),
                        CCCC TTTT CCCC TTTT(2)
```

(CM2.2)

The reading of (CM0) is immediate: in any state of affairs in which a certain set of sentences $\Delta_1 = \Pi' \cup \Pi$ is asserted while a certain set of sentences $\Delta_0 = \Sigma \cup \Sigma'$ is denied, one may in particular say that all subsets of Δ_1 are asserted and that all subsets of Δ_0 are denied. Furthermore, on the one hand, taking Π = Σ = {A}, property (CM1) says that the sentence A may not be simultaneously asserted and denied; on the other hand, taking Δ = {A}, property (CM2) says that the sentence A must be either asserted or denied (in a context where the sentences in Π are asserted and those in Σ are denied). One might say thus that (CM1) provides a meta-logical formulation of the 'Principle of Non-Contradiction', and disallows for *glutty* states of affairs in which a sentence is simultaneously asserted and denied: In any given (consistent) state of affairs, asserting a given sentence A should not be compatible with denying it. Dually, one might say that (CM2) provides a meta-logical formulation of the 'Principle of Excluded Middle', and disallows for *gappy* states of affairs in which a sentence is neither asserted nor denied: In no state of affairs can a sentence A fail to be either asserted or denied.

The complement ▹ of a compatibility relation ▸ on ℘ (L) × ℘ (L) will here be called an S-*consequence relation* (*on* L). It should be clear that it satisfies the following properties, for every Π, Π', Σ, Σ', Δ ⊆ L:

(CR$_S$0) if Π ▹ Σ, then Π' ∪ Π ▹ Σ ∪ Σ'

```
~(#t<#u)>((#p<#r)>((#q&#p)<(#r+#s))) ;
                        TCTT TTTT TCTC TTTT(1),
                        TTTT TTTT TTTT TTTT(1),
                        TCTT TTTT TCTC TTTT(3),
                        TTTT TTTT TTTT TTTT(1),
                        TCTT TTTT TCTC TTTT(2)
```

(CR$_S$0.2)

(CR$_S$1) if $\Pi \cap \Sigma \neq \emptyset$, then $\Pi \triangleright \Sigma$

```
~(#t<#u)>(((#p&#r)=(z@z))>(#p<#r)) ;
                    FNFN FNFN FNFN FNFN(1),
                    NNNN NNNN NNNN NNNN(1),
                    FNFN FNFN FNFN FNFN(3),
                    NNNN NNNN NNNN NNNN(1),
                    FNFN FNFN FNFN FNFN(2)          (CRS1.2)
```

(CR$_S$2) if $\Delta' \cup \Pi \triangleright \Sigma \cup (\Delta \setminus \Delta')$ for every $\Delta' \subseteq \Delta$, then $\Pi \triangleright \Sigma$

```
~(#t<#u)>
(#(~(#t<#v)=(z=z))>(((#v+#p)<(#r+(#u\#v)))>(#p<#r))) ;
                    TTTT TTTT TTTT TTTT(6),
                    CTCT TTTT CTCT TTTT(2)          (CRS2.2)
```

None of the equations is tautologous, with conclusions in the abstract.

The new bivalent, three-valued logic VŁ3

Abstract: We build and test a new bivalent three-valued logic named VŁ3. Recent advances are support of the classical tautologies, modal definitions, the law of excluded fourth, and extended contradiction principle.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q : A, B;
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⊢; < Not Imply, less than, ∈;
= Equivalent, ≡, ⊨; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(%p<#p) C as contingency; (p=p) T as tautology; (p@p) **F** as contradiction;
~(y < x) (x ≤ y), (x ⊆ y).

From: en.wikipedia.org/wiki/Three-valued_logic

We build bivalent truth tables for the connectives Or, And, Imply, and Equivalent using the 2-tuple in bits as: 00 False (contradiction); 01,10 (unknown); and 11 Tautology (designated truth value).

+	**00**	**01,10**	**11**	
00	00	01,10	11	
01,10	01,10	01,10,11	11	
11	11	11	11	(1.1)

&	**00**	**01,10**	**11**	
00	00	00,00	00	
01,10	00,00	00,01,10	01,10	
11	00	01,10	11	(1.2)

>	**00**	**01,10**	**11**	
00	11	11,11	11	
01,10	10,01	11,10,01	11	
11	00	01,10	11	(1.3)

=	**00**	**01,10**	**11**	
00	11	00,00	00	
01,10	00,00	00,11	00,00	
11	00	00,00	11	(1.4)

We rewrite Eqs. 1 removing: 01,10 for U.

+	**F**	**U**	**T**	
F	F	U	T	
U	U	U,T	T	
T	T	T	T	(2.1)

&	**F**	**U**	**T**	
F	F	F,F	F	
U	F,F	F,U	U	
T	F	U	T	(2.2)

>	**F**	**U**	**T**	
F	T	T,T	T	
U	U	T,U	T	
T	F	U	T	(2.3)

=	**F**	**U**	**T**	
F	T	F,F	F	
U	F,F	F,T	F,F	
T	F	F,F	T	(2.4)

We rewrite Eqs. 2 by removing: x, U for U; T,T for T; F,F for F; F,T for U.

+	**F**	**U**	**T**	
F	F	U	T	
U	U	U	T	
T	T	T	T	(3.1)

&	**F**	**U**	**T**	
F	F	F	F	
U	F	U	U	
T	F	U	T	(3.2)

>	**F**	**U**	**T**	
F	T	T	T	
U	U	U	T	
T	F	U	T	(3.3)

=	**F**	**U**	**T**	
F	T	F	F	
U	F	U	F	
T	F	F	T	(3.4)

~		
F	T	
U	U	
T	F	(3.5)

We evaluate two classical tautologies usually falsified by common three-valued logic systems (L3):

A+~A (4.1)

p+~p ; TTTT TTTT TTTT TTTT (4.2)

$\sim(A\&\sim A)$ (5.1)

~(p&~p) = (p=p) ; TTTT TTTT TTTT TTTT (5.2)

We evaluate three classical tautologies:

$A \vee B = (A \to B) \to B$ (6.1)

(p+q)=((p>q)>q) ; TTTT TTTT TTTT TTTT (6.2)

$A \wedge B = \neg(\neg A \vee \neg B)$ (7.1)

(p&q)=~(~p+~q) ; TTTT TTTT TTTT TTTT (7.2)

$A \leftrightarrow B = (A \to B) \wedge (B \to A)$ (8.1)

(p=q)=((p>q)&(q>p)) ; TTTT TTTT TTTT TTTT (8.2)

We evaluate three modal definitions:

$\mathbf{M}A = \neg A \to A$, corrected as: (9.1)

%p=(#~p>#p) ; TTTT TTTT TTTT TTTT (9.2)

$\mathbf{L}A = \neg\mathbf{M}\neg A$ (10.1)

#p=~%~p ; TTTT TTTT TTTT TTTT (10.2)

$\mathbf{I}A = \mathbf{M}A \wedge \neg\mathbf{L}A$,
with **I**A meaning "it is contingent that" (11.1)

(%p<#p)=(%p&~#p) ; TTTT TTTT TTTT TTTT (11.2)

We also evaluate:

$A \vee \mathbf{I}A \vee \neg A$ (law of excluded fourth) (12.1)

(p+((%p>#p)=(#p&~%p)))+~p ; TTTT TTTT TTTT TTTT (12.2)

$\neg(A \wedge \neg\mathbf{I}A \wedge \neg A)$ (extended contradiction principle) (13.1)

~((p+((%p>#p)=(#p&~%p)))+~p) = (p@p) ;
TTTT TTTT TTTT TTTT (13.2)

Eqs. 1-13 are tautologous, confirming VŁ3 as a bivalent, three-valued logic in support of five classical tautologies, three modal definitions, the law of excluded fourth, and extended contradiction principle.

Evaluation of t_B, t_N, t_A in a new algorithm for time

Abstract: Two main renditions of the algorithm are evaluated as *not* tautologous and are contradictory. An attempt to resuscitate the algorithm by the conjectured standard is *not* tautologous and a falsity. These results form a contradictory and *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
\> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Jarvis, S.H. (2017). Golden ratio axioms for time and space. vixra.org/pdf/1704.0169v6.pdf

"Now consider the following as a standard for time's flow:

$t_N = 1$ (4.1)

LET p, r, s:
φ, 5^0.5 [irrational part of φ], s [with ordinal 1 as (%s>#s), and 2 as (%<#s)]

t_N = (%s>#s) (4.2)

... Let us also consider a standard: $t_N = t_A - t_B$ (5.0)

Remark 5.0: We rewrite Eq. 5.0 as

$t_B = t_A - t_N$ (5.1)

$t_B = t_A$ − (%s>#s) (5.2)

Simply, t_B when applied to space (as 1, t_N) leads to t_A, as a proposed equation for "time". Thus:

$(t_A + t_B) / t_A = (t_A / t_B)$ (6.1.1)

This equation is significant, for it represents the "golden ratio", φ, which is solved as a quadratic equation for t_B as -0.61803... or 1.61803... In using both quadratic results together for t_A (6.2.1)

t_A = p = ((((%s>#s)+r)\(%s<#s))+(((%s>#s)-r)\(%s<#s))) ; (6.2.2)

Remark 6.1.2: By substitution from Eq. 6.2.2, Eq. 5.2 becomes:

t_B = (((((%s>#s)+r)\(%s<#s))+(((%s>#s)-r)\(%s<#s)))-(%s>#s)) ; (5.3)

By substitution from Eq. 5.3, Eq. 6.1.1 becomes:

((((((%s>#s)+r)\(%s<#s))+(((%s>#s)-r)\(%s<#s)))+(((((%s>#s)+r)\(%s<#s))+
(((%s>#s)-r)\(%s<#s)))-(%s>#s)))\((((%s>#s)+r)\(%s<#s))+(((%s>#s)-r)\(%s<#s))))=
(((((%s>#s)+r)\(%s<#s))+(((%s>#s)-r)\(%s<#s)))\(((((%s>#s)+r)\(%s<#s))+(((%s>#s)-
r)\(%s<#s)))-(%s>#s))) ; **FFFF FFFF FFFF FFFF** (6.1.2)

(which technically breaks equation 6[.1.1], yet is nonetheless how time is proposed to operate as symmetry-breaking):

$t_B{}^2 = \varphi \cdot -1/\varphi = -1$ (7.1)

(((((%s>#s)+r)\(%s<#s))+(((%s>#s)-r)\(%s<#s)))&~((%s>#s)\((((%s>#s)+r)\
(%s<#s))+(((%s>#s)-r)\(%s<#s)))))=~(%s>#s) ;
FFFF FFFF FFFF FFFF (7.2)

Thus, t_N as "1" is the opposite of a future event “-1”, hence t_N sending itself to t_A as a negative inverse flip (in much the same way as t_B regarding t_A), thus a type of continual process of this equation as a “now” event. Yet according to the result here, the following is effected: $t_B = i$ …”

Remark 7.2: To resuscitate the conjectured standard we evaluate Eq. 5.0 as is by substitutions from Eqs. 4.2, 6.2.2, and 5.3 as 7.3:

(%s>#s)=(((((%s>#s)+r)\(%s<#s))+(((%s>#s)-r)\(%s<#s)))
-(((((%s>#s)+r)\(%s<#s))+(((%s>#s)-r)\(%s<#s)))-(%s>#s))) ;
CCCC CCCC CCCC CCCC (7.3)

Eqs. 6.1.2 and 7.2 are *not* tautologous and in fact contradictions. Eq. 7.3, a statement of the standard, is *not* tautologous and produces a truth table value of C for falsity. These results deny the conjectured new algorithm for time.

Time as God conjecture

If God knows that past, present, and future are tautologous [and that past implies present, implies future], then:

God as past implies God as present, implies past as present;
or
God as past implies God as future, implies past as future;

or

God as present implies God as future, implies present as future
{ or past as present implies pas as future, implies present as future }

Proof for time as God in Meth8 script.

LET p God, q past, r present, s future, [also t time = q & r & s]

```
(p & (((q=q)&(s=s))&(r=r)))
>
(   ((((p=q)>(p=r))>(q=r))
    +
    (((p=q)>(p=s))>(q=s)))
+
(((p=r)>(p=s))>(r=s))  ) ; tautologous
```

For the additional bracketed and braced expressions:

```
((p&(((q=q)&(s=s))&(r=r)))&(((q=q)>(s=s))>(r=r)))
>
(   ((((p=q)>(p=r))>(q=r))
    +
     (((p=q)>(p=s))>(q=s)))
+
((((p=r)>(p=s))>(r=s))+(((q=r)&(q=s))&(r=s)))  ) ; tautologous
```

Refutation of time as the coexistence of past, present, future

As attributed to physicists Einstein, Feynman, and Hawking:

The past, present, future as equivalents coexist to define time. (1.1)

We rewrite Eq. 1.1 as:

If past, present, future are equivalents, then past, present, and future imply time. (2.1)

We attempt to strengthen Eq. 2.1 with the modal operator of necessity as the universal quantifier.

If past, present, future are necessarily equivalents, then past, present, and future imply the necessity of time. (3.1)

LET p q r s: past, present, future, time; # necessity, for all.

```
((p=q)=r)>s ;                    TFFT FTTF FTTF TFFT      (1.2)
((p=q)=r)>((p&(q&r))>s) ;        TTTT TTTF TTTT TTTT      (2.2)
#((p=q)=r)>((p&(q&r))>#s) ;      TTTT TTTC TTTT TTTT      (3.2)
```

Eq. 3.2 strengthens Eq. 2.2 marginally in the proof table by replacing the contradiction value with the falsity value of contingency.

Eqs. 1.2, 2.2, and 3.2 are *not* tautologous. This refutes Eqs. 1.1, 2.1, and 3.1.

The converse implication operator EQT as a tense connective

Abstract

The converse implication operator named EQT arises to study the inequality in the tense of time for Past > Present > Future. EQT is symmetrically bivalent with the 8-bit pattern {1101 1101} as decimal 187. It is shown that: Past in terms of Present is a falsity; Present in terms of Present is a tautology; and Future in terms of Present is a tautology. Derivations are by Peirce NOR and a 2-tuple truth table.

Background

The four commonly used logical connectives as operators at the bit level are AND, IMP, OR, and XOR. The two most commonly used negations are NAND for NOT(AND) and NOR for NOT(OR). The four commonly used operators were developed by Charles S Peirce (1880) using NOR and by Henry M Sheffer (1913) using NAND.

The value and definitions of the 2-tuple as four valued bit code (4VBC) are the bit pairs of 00 F for contradiction F, 01 N for truthity, 10 C for falsity, and 00 T for tautology (the designated *proof* value). The bit pairs have a left, sinistro, and falsity side, and also a right, dextro, and truthity side in Table 1.

Bit pair	**Left side, sinistro**	**Right side, dextro**	Meaning	
01	0 NOT(Falsity)	1 Truthity	N	Truthity, Non contingency
10	1 Falsity	0 NOT(Truthity)	C	Falsity, Contingency
00	0 NOT(Falsity)	0 NOT(Truthity)	F	Contradiction (truthity AND falsity)
11	1 Falsity	1 Truthity	T	Tautology (truthity OR falsity)

Table 1. Bit pair meanings

Mapping tense with 4VBC

The states of the time continuum are evaluated as relation with the tense of the time segments assigned to bit pairs in Table 2.

10	>	01	>	11
Past	>	Present	>	Future

Table 2. Bit pairs for tenses in time

Past is defined as falsity because it has transpired and is no longer truthity as Present. Future is defined as truthity or falsity, a tautology, because it is undetermined. No tense is assigned to both truthity and falsity at the same time, a contradiction. There is also no number line associated here because the bits {00} as the absence of a variable or lack of a proposition do not exist as a tense, excluded in Table 2.

Three relations are deduced around the fraction of tense / Present as a unity in Table 3.

Past / Present > Present / Present (unity) > Future / Present

Table 3. Inequality of tense based on Present

The fraction of Present / Present is taken as the only fiducial point of unity.

The groups in relation Table 3 are replaced by letters, the division symbol is removed, and the tenses are substituted with bit pairs from 4VBC in Table 2 to make Table 4, where D is Unity.

B	>	D	>	F
10		01		11
01		01		01

Table 4. 4VBC for tense from Table 3

We translate the relations of Table 6 relations into words to obtain an outcome.

B: Past in terms of Present is a falsity because the Past is transpired and is no longer a truthity as is Present.
D: Present in terms of itself is a tautology as being either truthity or a falsity.
F: Future in terms of Present is either truthity or a falsity as a tautology.

We rewrite Table 4 with proposed outcomes in Table 5.

	B	>	D	>	F
p	10		01		11
q	01		01		01
(p?q)	10		11		11

Table 5. Tense relations with results

We search for a symmetrical bivalent operator to produce the results of Table 7. We find the converse implication operator and name it "equate"or EQT (negation NEQT). The bit-wise operation is in Table 8. The decimal numbers are the binary equivalents from our 8-bit canon of 256-operators.

```
  p  0000 1111           p  0000 1111
  q  0101 0101           q  0101 0101
EQT  1010 1111 187    NEQT  0101 0000   68
```

Table 6. Bit-wise operation of the converse implication operator

Therefore using the converse implication operator as EQT, Table 5 can be rewritten in Table 7.

	B	>	D	>	F
p	10		01		11
q	01		01		01
EQT	10		11		11

Table 7. 4VBC for tense defined by EQT

Alternate definition of the converse implication (EQT)

We begin with the AND operator and the equivalence EQV operator for p 0011 and q 0101 in Table 8.

	0011	p		0011
	0101	q		0101
AND	0001		EQV	1001

Table 8. The logical bit operators AND and EQV

The converse implication (EQT) decomposed as (p AND q) EQV (q) equal to (p EQT q) in Table 9.

	0011	p		0011
	0101	q		0101
AND	0001			:
	0101			:
EQV	1011		EQT	1011

Table 9. (p EQT q) shown identical to (p AND q) EQV (q)

Derivation of converse implication (EQT) by NOR

The definition of the operator in Table 9 may be reduced to NOR operations as follows.

NOT(p) = p NOR p; NOT(q) = q NOR q;
p EQT q ≡ ((p AND q) OR (q))
≡ ((((NOT(p)) NOR (NOT(q)))) NOR ((NOT(p)) NOR (NOT(q)))))
NOR (p NOR q))

Lookup table for converse implication (EQT)

For the converse implication (EQT) operator, the results of the operators on the propositions of p and q are tabulated as row major with p as the index to the rows and q as the index to the columns. The headings for the rows and columns arrange the bits of a 2-tuple in the order of <00, 01, 10, 11> for <contradiction, truthity, falsity, tautology> because the bits as binary numbers, with least significant bit to the right, increase in value as <0, 1, 2, 3>, in Table 10 below.

EQT	00	01	10	11
00	**11**	**11**	**11**	**11**
01	**10**	**11**	**10**	**11**
10	**01**	**01**	**11**	**11**
11	**00**	**01**	**10**	**11**

Table 10. Lookup table for the EQT operator

Pattern of converse implication (EQT) in Meth8/VŁ4

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET p, q: p; q; & And; = Equivalent;
% possibility, possibly, for one or some; # necessity, necessarily, for all or every.

(p EQT q): (p AND q) EQV (q) (1.1)
(p&q) = q ; TFTF TFTF TFTF TFTF (1.2)

Eq. 1.2 is *not* tautologous. We attempt to strengthen Eq.1.2 by injecting the existential quantifier in the first antecedent.

(%p AND q) EQV (q) (2.1)
(%p&q) = q ; TTCT TTCT TTCT TTCT (2.2)

Eq. 1.2.2 is *not* tautologous, but approaches tautology more closely than Eq. 1.2.

Evaluation of tense relations with results in Meth8/VŁ4

We evaluate Table 9 as a relational expression, and rewrite it using Table 11.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	(p=p)	T	tautology	proof	11	3
2	(p@p)	F	contradiction	absurdum	00	0
3	(%p>#p)	N	non-contingency	truthity	01	1
4	(%p<#p)	C	contingency	falsity	10	2

Table 11. Axioms of Meth8/VŁ4

From Table 7:

	B.1	>	D.1	>	F.1
p	10		01		11
q	01		01		01
EQT	10		11		11

((%p<#p)&(%p>#p))=(%p<#p) ; NNNN NNNN NNNN NNNN (B.2)
((%p>#p)&(%p>#p))=(p=p) ; NNNN NNNN NNNN NNNN (D.2)
((p=p)&(%p>#p))=(p=p) ; NNNN NNNN NNNN NNNN (F.2)

We rewrite the expression as: B.1 > D.1 > F.1 (3.1)
(((((%p<#p)&(%p>#p))=(%p<#p)) > (((%p>#p)&(%p>#p))=(p=p)))=(p=p))
> (((p=p)&(%p>#p))=(p=p)) ; NNNN NNNN NNNN NNNN (3.2)

Eqs. B.2, D.2, and F.2 as rendered are *not* tautologous, but are respectively truthity as non-contingent.

This means that the converse implication (EQT) as derived implies that tense, and hence time, is not a tautology. Therefore time can only be assumed as non-contingent and truthity, and not as a theorem.

Proof that transition function of a topological manifold is not tautologous (but close to being so)

From public source: en.wikipedia.org/wiki/Topological_manifold, under Coordinate Charts

> "Given two charts φ and ψ with overlapping domains *U* and *V* there is a transition function
>
> $$\psi\varphi^{-1}: \varphi(U \cap V) \to \psi(U \cap V). \qquad (1)$$
>
> Such a map is a homeomorphism between open subsets of $\mathbf{R}^n$. That is, coordinate charts agree on overlaps up to homeomorphism. Different types of manifolds can be defined by placing restrictions on types of transition maps allowed. For example, for differential manifolds the transition maps are required to be diffeomorphism."

We map Eq 1 into Meth8 script to validate it:

> LET: p q r s, ψ φ *U* *V*, nvt not tautologous, Tautologous (Evaluated) Designated truth values
> & ∩ And, \ Not And, → ":" > Imply, ($\psi\varphi^{-1}$) (ψ \ φ)
>
> (p\q)>((q&(r&s))>(p&(r&s))) ; nvt (2)

The truth table of Eq 2 (each model is the concatenation of four table rows of four values):

Model 1 .Model 2.1 .Model 2.2 .Model 2.3.1 .Model 2.3.2
TTTTTTTTTTTTTT**F**T.EEEEEEEEEEEEEE**U**E.EEEEEEEEEEEEEE**U**E.EEEEEEEEEEEEEE**U**E.EEEEEEEEEEEEEE**U**E
(p\q)>((q&(r&s))>(p&(r&s))) Step: 15

The non truth values contradictory (Unevaluated) are in bold above to show how closely Eq 2 diverges.

(If in Eq 2 the main connective > Imply is changed to & And or to < Not Imply, or the order of main terms are juxtaposed around those connectives, or the order of p,q is changed in combinations, then those expressions are also nvt.)

We ask what does this mean regarding the transition function of the topological manifold?

If it is not tautologous, then the notion of manifolds is suspicious for:

> Discrete spaces (0-manifold); Curves (1-manifold); Surfaces (2-manifolds);
> Volumes (3-manifolds); and General (n-manifolds).

This is troubling because Volumes (3-manifolds) resulting from Thurston's geometrization conjecture was proved by Grigori Perelman, but the prize was not accepted.

If the transition function of the topological manifold is not validated, then the set theory of Volumes (3-manifolds) apparently fails.

What follows is that branes, as predicated on manifolds, are also suspicious.

Refutation of modal logic with the difference modality of topological T_0-spaces

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s;
~ Not, ¬; + Or, ∨; & And, ∧; = Equivalent; > Imply;
# necessity, for all or every, □, ∀; % possibility, for one or some, ◊;
[≠] ~#; ⟨≠⟩ ~%; ◊A = ¬□¬A; ⟨≠⟩A = ¬[≠]¬A; [≠]A∧A = [∀]A.

From: Aghamov, R. (2018). Modal logic with the difference modality of topological T_0-spaces. arxiv.org/pdf/1810.02150.pdf agamov@phystech.edu

In this paper we will use the following axioms:

($D_□$) [∀]p → □p, (2.4.1)

(~#p&p)>#p ; TNTN TNTN TNTN TNTN (2.4.2)

(B_D) p → [≠]⟨≠⟩p, (2.5.1)

p>~(#(~#~p=(p=p))=(p=p)); TCTC TCTC TCTC TCTC (2.5.2)

(AT_0) (p ∧ [≠]¬p ∧ ⟨≠⟩(q ∧ [≠]¬q)) → (□¬q ∨ ⟨≠⟩(q ∧ □¬p)) (2.7.1)

((p&~#~p)&~(#(~(q&~#~q)=(p=p))=(p=p)))>
(#~q+~(~(#(~(q&#~p)=(p=p))=(p=p))=(p=p))) ;
TNTN TNTN TNTN TNTN (2.7.2)

We introduce the notation for the following logics:

S4D = $\mathbf{K_2}$ + $T_□$ + $4_□$ + $D_□$ + B_D + 4_D
$\mathbf{S4DT_0}$ = **S4D** + AT_0

Eqs. 2.4.2, 2.5.2, and 2.7.2 as rendered are *not* tautologous. This means logics **S4D** and $\mathbf{S4DT_0}$ are also *not* tautologous. Hence, modal logic with the difference modality of topological T_0-spaces is refuted.

Refutation of the totherian set definition

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: x, y, E;
~ Not; + Or; - Not Or; & And; = Equivalent; > Imply; < Not Imply;
% possibility, for any one or some, ∃; # necessity, for every or all, ∀.
(s=s) T tautology, good; (s@s) **F** contradiction, bad.

From: Bado, I.O. (2018). From the totherian analysis to the hypothesis of Riemann.
vixra.org/pdf/1809.0554v1.pdf (olivier.bado@ensea.edu.ci)

Let E be a nonempty set, E is totherian if and only if

∀(x,y) ∈ E^2, (x+y) ∈ E, (x-y) ∈ E (2.1.1)

We decompose the clauses.

∀(x,y) ∈ E^2 (2.1.1.1)
(#p&#q)<(r&r) ; FFFN FFFF FFFN FFFF (2.1.1.2)

(x+y) ∈ E (2.1.2.1)
(p+q)<r ; FTTT FFFF FTTT FFFF (2.1.2.2)

(x-y) ∈ E (2.1.3.1)
(p-q)<r ; TFFF FFFF TFFF FFFF (2.1.3.2)

The argument from Eq. 2.1.1 expands to:

∀(x,y) ∈ E^2 & (x+y) ∈ E & (x-y) ∈ E (2.1.4.1)
((#p&#q)<(r&r))&(((p+q)<r)&((p-q)<r));
FFFF FFFF FFFF FFFF (2.1.4.2)

Eqs. 2.1.1.1, 2.1.2.2, 2.1.3.2, and 2.1.4.2 as rendered are *not* tautologous. This refutes the definition of totherian sets.

Remark: If the And connectives in Eq. 2.1.4.1 are replaced by the Imply connective to the strengthen the argument toward tautology, the result remains *not* tautologous.

((#p&#q)<(r&r))>(((p+q)<r)&((p-q)<r)) ; TTTC TTTT TTTC TTTT (2.1.5.2)

Refutation of translation invariance in superposition calculus

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

> LET p, q, r, s: A, B, X, Y;
> ~ Not; + Or; & And; > Imply, greater than, →; < Not Imply, less than;
> # necessity, ∀, for all or every; % possibility, ∃, for one or some.

From: en.wikipedia.org/wiki/Knuth–Bendix_completion_algorithm

> [I]f a rewriting system is used to calculate minimal representatives then the order < should also have the property ... called translation invariance.

A < B → XAY < XBY for all words A,B,X,Y (1.1)

((p<q) > (((r&p)&s) < ((r&q)&s))) & #((p&q)&(r&s)) ;
`FFFF FFFF FFFF FFF``N` (1.2)

If we distribute the universal quantifier over each word, and remove the quantified consequent in Eq. 1.2, then:

(#p<#q) > (((#r&p)&#s) < ((#r&#q)&#s)) ; `TCTT TCTT TCTT TTTT` (1.3)

Eqs. 1.2 and 1.3 are *not* tautologous. Eq. 1.2 is nearly contradictory, excepting one non-contingency value `N`. Eq. 1.3 is closest to tautology, excepting the three contingency values `C`. Hence the property of translation invariance is refuted.

Solution to the traveling salesman problem as a theorem, unrelated to P, NP

Abstract: We evaluate the traveling salesman problem (TSP) to confirm it as a theorem but with *multiple* solutions for n = 4 cities. The number of solutions here is also given by n = 4. Our results do not relate to P, NP, or NP-hard. Hence the salesman problem as an outstanding mathematical problem of optimization is refuted, and as such becomes a non tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Travelling_salesman_problem

Conjecture: If distances between cities are unique and not zero, then the relations of unique cities imply the order of respective distances to be traversed as least to greatest. (1.0)

Remark 1.0: We write the conjecture as: If n numbered locations are unique and not zero and the respective distances are unique and not zero, then relations of locations imply the respective distances as assigned sequentially from least to greatest. (1.1)

LET p, q, r, s, t, u, v, w, x, y: city_1, city_2, city_3, city_4, (p-q), (p-r), (p-s), (q-r), (q-s), (r-s).

Remark 1.1: Distance assignments can be mapped separately in one dimension, overlaid from a fiducial point as least to greatest, with no two distances as equal. In vectors, the distances are ordered from t to y as least to greatest:

(~(((p&q)&(r&s))=(s@s))&
(~((((t=(p-q))&(u=(p-r)))&(v=(p-s)))&(((w=(q-r))&(x=(q-s)))&(y=(r-s))))=(s@s))) >
((((((p>q)>r)<s)+(((p>q)<r)<s))+((((p<q)>r)<s)+(((p<q)<r)<s)))>(((((t<u)<v)<w)<x)<y)) ;
TTTT TTTT TTTT TTTT (1.2)

Remark 1.2: The solutions by city number relation paths are: (((p>q)>r)<s) ; (((p>q)<r)<s)); (((((p<q)>r)<s); or (((p<q)<r)<s). In this rendition, there are four cities n = 4 and four solutions n = 4.

Eq. 1.2 is tautologous, meaning the conjecture is confirmed. This appears at first glance to support P=NP, but the theorem is logically unrelated to P, NP, or NP-Hard. Hence, the conjecture does not quality as an outstanding mathematical problem of optimization. (No prize incentives exist for removing a problem.)

Confirmation of the triangle inequality

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: point p, point q, point r; ~ Not; & And; + Or; - Not Or;
> Imply, greater than; < Not Imply, less than; = Equivalent;
((p-q)=(q-p)) The absolute value of the distance p to q is equivalent to that of q to p;
((p-r)=(r-p)) The absolute value of the distance p to r is equivalent to that of r to p;
((q-r)=(r-q)) The absolute value of the distance q to r is equivalent to that of r to q.

From: en.wikipedia.org/wiki/Triangle_inequality

"[T]he triangle inequality states that for any triangle, the sum of the lengths of any two sides must be greater than or equal to the length of the remaining side." (1.0)

We rewrite Eq. 1.0 as contradiction F implies tautology T.

"If qr is not greater than pq with pr, then both qr is greater than pq and qr is greater than pr." (1.1)

~(((q-r)=(r-q))>(((p-q)=(q-p))+((p-r)=(r-p)))) >
((((q-r)=(r-q))>((p-q)=(q-p)))&(((q-r)=(r-q))>((p-r)=(r-p)))) ;
TTTT TTTT TTTT TTTT (1.2)

Eq. 1.2 as rendered is tautologous, hence confirming the triangle inequality.

Remark: This exercise indirectly speaks to the fact that the vector space is *not* bivalent.

Trivial proofs for a troll

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET: + Or; > Imply; = Equivalent; # necessity, for all or every; % possibility, for one or some.

For all p, p+p=p.		(1.1)
#p>((p+p)=p) ;	`TTTT TTTT TTTT TTTT`	(1.2)
For one p, p+p=p.		(2.1)
%p>((p+p)=p) ;	`TTTT TTTT TTTT TTTT`	(2.2)
Axiom of associativity		(3.1)
((p+q)+r)=(p+(q+r)) ;	`TTTT TTTT TTTT TTTT`	(3.2)

Remark: As expected, replacing the Or connective with the And connective produces the same results.

Resolution of the ethical trolley problem

Abstract: We evaluate the trolley problem. While neither outcome is tautologous, the lesser of two evils is chosen as the ethical resolution because resulting values from one logic table are closer to the ideal state of tautology. In other words, while both outcomes are *non* tautologous fragments of the universal logic VŁ4, the *relative* value of the results implies the ethical choice.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊔;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1;
(%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Trolley_problem

The trolley problem is a thought experiment in ethics. The general form of the problem is this: You see a runaway trolley moving toward five tied-up (or otherwise incapacitated) people lying on the tracks. You are standing next to a lever that controls a switch. If you pull the lever, the trolley will be redirected onto a side track, and the five people on the main track will be saved. However, there is a single person l[a]ying on the side track. You have two options:

1. Do nothing and allow the trolley to kill the five people on the main track.
2. Pull the lever, diverting the trolley onto the side track where it will kill one person.

Which is the more ethical option?

Remark 1.: The modal attributes of variables for the thought experiment are the necessity of main and side rails, the possibility of people, and the possibility of death.

We map the problem to avoid the case of no people on rails as:

If the total number of possible people on the necessary main and side rails is not zero, then the total number of possible deaths from the necessary rails is not zero. (1.1)

LET p, q, r, s: person, train of death, main rail line, side rail line

(((%p&#r)+(%p&#s))@(p@p))>(((%q&(%p&#r))+(%q&(%p&#s)))@(p@p)) ;
TTTT TCTT TCTC TCTC (1.2)

Remark 2.: We pose the ethical question as two states of affairs.

If Eq.1.1, then if more possible people are on the necessary main rail than possible people on the necessary side rail, then what is the relative truth table result of the necessary side rail implying the necessary main rail; and (2.1.1)

((((%p&#r)+(%p&#s))@(p@p))>(((%q&(%p&#r))+
(%q&(%p&#s)))@(p@p)))>(((p&r)>(p&s))>(#s>#r)) ;
TTTT TTTT CTCC TTTT (2.1.2)

If Eq.1.1, then if more possible people are on the necessary main rail than possible people on the necessary side rail, then what is the relative truth table result of the necessary main rail implying the necessary side rail; (2.2.1)

((((%p&#r)+(%p&#s))@(p@p))>(((%q&(%p&#r))+
(%q&(%p&#s)))@(p@p)))>(((p&r)>(p&s))>(#r>#s)) ;
TTTT CTCT TTTT TTTT (2.2.2)

Eq. 2.1.2 and 2.2.2 are *not* tautologous. For Eqs. 2.1.2 and 2.2.2 respectively, the unique row results are CTCC and CTCT. Because CTCT is closer to a tautologous row of TTTT than is CTCC, we choose Eq. 2.2.2 as the preferred outcome to resolve the trolley problem. This means the ethical choice is made to throw the switch toward the side rail, thereby minimizing death. We note that this thought experiment resolution cannot be generalized properly to alphabetical issues of morality such as abortion, capital punishment, euthanasia, and gender choice.

Refutation of Turing's halting problem: not a problem

Taken from: en.wikipedia.org/wiki/Halting_problem

Given:

There is at least one n such that N(n) is equal to the statement H(a, i) meaning a halts on input i.

What follows is that:

Either there is an n such that N(n) = H(a,i), (1.1)
or there is an n' such that N(n') =~H(a,i). (2.1)

This means that this gives an algorithm to decide the halting problem [as Eq. 1.1 or Eq. 2.1 is a proof].

[There is an n such that N(n)=H(a,i)] Or [There is an n' such that N(n')=~H(a,i)] = proof (3.1)

"Since we know that there cannot be such an algorithm, it follows that the assumption that there is a consistent and complete axiomatization of all true first-order logic statements about natural numbers must be false." (4.1)

We assume the apparatus and method of Meth8 implementing variant logic system VŁ4.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	p=p	T	Tautology	proof	11	3
2	p@p	F	Contradiction	absurdum	00	0
3	%p>#p	N	Non-contingency	truth	01	1
4	%p<#p	C	Contingency	falsity	10	2

LET: p q r s N n n' H(a,i); % possibility, for some (one); # necessity, for all;
~ Not; & And; + Or; > Imply; = Equivalent; @ Not equivalent
(p=p) 11, Tautology; (p@p) 00, Contradiction

The designated *proof* value is T.
The 16-valued truth tables are presented horizontally as row-major.

Eq. 1.1 is mapped as

(%q>((p&q)>s)) (1.2)

Eq. 2.1 is mapped as

(%r>((p&r)>~s)) (2.2)

Eq. 3.1 is mapped as

((%q>((p&q)>s)) + (%r>((p&r)>~s))) = (p=p) ; `TTTT TTTT TTTT TTTT` (3.2)

Because the truth table of Eq. 3.2 is tautologous (all T), this means the halting problem is in fact a theorem and not a problem. In other words:

> The assumption that there is a consistent and complete axiomatization of all true first-order logic statements about natural numbers must be *tautologous*. (4.2)

However, if Eq. 3.1 is written to replace the ">s" (implies s) in the antecedent parts with "=s" (equivalent to s), then Eq. 3.1 maps as

((%q>((p&q)=s)) + (%r>((p&r)=~s))) = (p=p) ; `TTNT TTTT TTTT TNTT` (3.3)

Because the truth table of Eq. 3.3 is not tautologous (not all T, but with some N as the non-contingent value of truth), this means the halting problem is not a problem of contradiction but rather an expression with values close to but not quite tautologous.

If the universal quantifier is applied to Eq. 3.3 on both main segments of the antecedent and consequent, then Eq. 3.3 maps as

#((%q>((p&q)=s)) + (%r>((p&r)=~s))) = #(p=p) ; `TTTT TTTT TTTT TTTT` (3.3)

and the halting problem becomes tautologous with the same status of theorem and result as in Eq. 3.2.

We conclude that Alan Turing's difficulty was in expressing the halting problem in the format of a two-valued logic which was not as expressive as in a four-valued logic to show nuances of what exactly the equation stated.

In comparison to Gödel's incompleteness theorems, Turing's halting problem has no superficial similarities other than being refuted as not a problem. Hence in contrast, both expressions are disparate and ultimately unrelated as to content meaning.

Refutation of Turing's halting problem as logically unsolvable

Abstract: We confirm the halting conjecture as tautologous and hence refute the halting problem as unsolvable. What follows is that first order logic is decidable. This proof was made possible by the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

We cast Turing's halting problem as follows.

If stop is contradictory and a program step is less than the program executing it, then:
if the predictor indicates stop, then
if the program is no longer executing then the program step is stopped
or
if the predictor indicates not to stop, then
if the program is executing then the program step is not stopped. (1.1)

LET p, q, r, s, t:
P prediction; Q step within the program loop; loop; stop switch; instant experiment.

(((s=(s@s))&(q<(r>r)))>(((p>s)>((r<r)>(q>s)))+((p>~s)>((r>r)>(q>~s)))))=(s=s) ;
TTTT TTTT TTTT TTTT (1.2)

The iteration aspect of the problem is handled as follows in the form of (T=T)>(T>T).

If the instant experiment is equivalent to the halting problem in Eq. 1.1, then
if the halting problem in Eq. 1.1, then
the instant experiment. (2.1)

(t=(((s=(s@s))&(q<(r>r)))>(((p>s)>((r<r)>(q>s)))+((p>~s)>((r>r)>(q>~s))))))>
((((s=(s@s))&(q<(r>r)))>(((p>s)>((r<r)>(q>s)))+((p>~s)>((r>r)>(q>~s)))))>t);
TTTT TTTT TTTT TTTT(128) in 75 steps (2.2)

Eq.1.2 is tautologous, refuting the halting problem as unsolvable. What follows is that first order logic is decidable.

The twin paradox is not a paradox by mathematical logic

We define the twin paradox without resort to stopping because we assume that instant velocity commences and terminates at an instant state of rest.

> Twins occupy the same fiducial point from which one twin obtains
> an instant velocity to a non-fiducial point, then obtains another instant velocity
> back to the fiducial point. The question is are the twins the same
> at the fiducial point before and after the separation and travel of the one twin. (0.0)

We test this in words as:

> If the fiducial point implies the twins are equivalent, then
> if a twin implies a velocity to a non-fiducial point,
> then if that same twin implies a reverse velocity to the fiducial point,
> then the fiducial point point implies the twins are equivalent. (1.1)

We assume the apparatus and method of Meth8/VŁ4: ~ Not; > Imply; = Equivalent to.

LET: pq twins; r the fiducial point; ~r not the fiducial point;
s velocity to a non-fiducial point; ~s velocity from a non-fiducial point.

The designated *proof* value is `T` for tautology; `F` is the designated *contradiction* value.
The 16-valued truth table is presented row-major and horizontally.

((r>(p=q))>(p>(s>~r))) > ((p>(~s>r))>(r>(p=q))) ; `TTTT TFFT TTTT TFFT` (1.2)

This describes the state of affairs *without* special relativity. Eq. 1.2 as rendered is *not* tautologous.

We test the counter example in words as:

> If the fiducial point implies the twins are equivalent, then
> if a twin implies a velocity to a non-fiducial point,
> then if that same twin implies a reverse velocity to the fiducial point,
> then the fiducial point point implies the twins are *not* equivalent. (2.1)

((r>(p=q))>(p>(s>~r))) > ((p>(~s>r))>(r>~(p=q))) ; `TTTT FTTF TTTT FTTT` (2.2)

The describes the state of affairs *with* special relativity. Eq. 2.2 as rendered is *not* tautologous.

The paradox is supposed to arise by numerical calculation of special relativity. This would mean that the respective states of affairs are both tautologous (or both contradictory) at the same time.

However Eqs. 1.2 and 2.2 are not both tautologous (or both contradictory), and *not* inversive.

This means the twin paradox is not a paradox, but rather something else, namely, a state of affairs that is *not* tautologous and *not* contradictory. What follows is that special relativity is suspicious.

Refutation of the two-sided page paradox

Abstract: We evaluate the two-sided page conjecture that: if either the front page implies the back page is false or the back page implies the front page is true is a paradox (contradiction). The conjecture is a theorem and hence refuted as a paradox.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency).
The 16-valued truth table is row-major and horizontal.

LET: ~ Not; + Or; = Equivalent; @ Not Equivalent;
% possibility, for one or some; # necessity, for all or every;
p, ~p: page front side, page back side (not page front side);
(p=p) Tautology; (pp) **F** as contradiction;
(%p>#p) N as truthity (non-contingency); (%p<#p) C as falsity (contingency).

From: scigod.com/index.php/sgj/article/download/162/193; en.wikipedia.org/wiki/Card_paradox

> [C]onsider the double contradiction represented by a single sheet of paper with contradictory signs on each face:
>
> The statement on the other side is false. (1.0)
> The statement on the other side is true. (2.0)
>
> If we accept either in its entirety, we are in a double-bind, for each leads us into a state of global contradiction, when the other is taken into account. (3.0)

We write Eqs. 1.0 and 2.0 as:

Front page implies back page is false. (1.1)

(p>~p)>(p@p) ; **F**T**F**T **F**T**F**T **F**T**F**T **F**T**F**T (1.2)

Back page implies front page is true (2.1)

(~p>p)>(p=p) ; TTTT TTTT TTTT TTTT (2.2)

If either front page implies back page is false or
back page implies front page is true implies contradiction. (3.1)

(((p>~p)>(pp))+((~p>p)>(p=p)))>(p@p) ;
FFFF FFFF FFFF FFFF (3.2)

Remark 3.2: Eq. 3.2 is *not* tautologous as asserted in Eq. 3.1, is contradictory, and hence is refuted.

We rewrite Eqs. 1.0, 2.0, and 3.0 to replace false and true respectively with falsity and truthity so as to weaken the assertions.

Front page implies back page is falsity. (4.1)

(p>~p)>(%p<#p) ; `CTCT CTCT CTCT CTCT` (4.2)

Back page implies front page is truthity. (5.1)

(~p>p)>(%p>#p) ; `TNTN TNTN TNTN TNTN` (5.2)

If either front page implies back page is falsity or
back page implies front page is truthity implies contradiction. (6.1)

(((p>~p)>(%p<#p))+((~p>p)>(%p>#p)))>(p@p) ;
`FFFF FFFF FFFF FFFF` (6.2)

Remark 6.2: Eq. 6.2 is *not* tautologous as asserted in Eq. 6.1, is contradictory, and hence is refuted.

From Eqs. 3.2 and 6.2 as rendered, no paradox (contradiction) exists, and in fact the conjectures in Eqs. 3.1 and 6.1 are theorems.

Refutation of type theory

Abstract: We evaluate the subset equations for the basis of type theory. They are *not* tautologous. Therefore the basis of type theory is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1;
(%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Coquand, T. (2018). Type theory.
plato.stanford.edu/entries/type-theory/ coquand@chalmers.se

Consider the following subset of X: A={x∈X|x∉F(x)}. (1.1)

LET p, q, r, s: a, F, X, x.

[A]=(~(s<(q&s))>(s<r)) ; **`FFFF FFFF`** `TTTT TT`**`FF`** (1.2)

This subset cannot be in the range of F.
For if A=F(a), for some a, then a∈F(a) iff a∈A, iff a∉F(a) which is a contradiction. (2.1)

((~(s<(q&s))>(s<r))=(q&%p))>
(((p<(~(s<(q&s))>(s<r)))+~(p<~(q&p)))>(p<(q&p))) ;
`F``TCT` **`F`**`TCT TTNT TTCT` (2.2)

Remark 2.2: Eq. 2.2 as rendered is *not* tautologous, but it is also *not* contradictory. This means the basis of type theory is a *non* tautologous fragment of the universal logic VŁ4.

Refutation of projective determinancy via the ultrapower

Abstract: We evaluate the definition of ultrapower as a convention. The two states equated to 1 as designated *proof* value and as ordinal value are *not* tautologous. This refutes the ultrapower and hence colors the subsequent exposition to deny projective determinancy. Therefore the ultrapower and projective determinancy are *non* tautologous fragments of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
\> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ↞, ⋦, ↾ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Martin, D.A.; Steele, J.R. (1989). A proof of projective indeterminacy.
Journal of the American Mathematical Society. 2:1. 01.1989. 71/125.
ams.org/journals/jams/1989-02-01/
S0894-0347-1989-0955605-X/S0894-0347-1989-0955605-X.pdf
dam@math.ucla.edu steel@math.berkeley.edu

I. Extenders (pg 75), **Convention.**

Let D be a directed nonempty set of sets: if a, b ∈ D then there is a c ∈ D such that a ∪ b ⊆ c.

Suppose that Z is a set and $\langle \mu_a \mid a \in D \rangle$ is such that (75.1.1)

(1) each μ_a is a countably additive measure on ${}^aZ = \{f \mid f: a \to Z\}$;
(2) the μ_a are *compatible*: if a ⊆ b and $\mu_a(X) = 1$, then μ_b $(\{f \mid f \restriction a \in X\}) = 1$. (75.2.1)

We wish to define the *ultrapower* (of the universe V) *by* $\langle \mu_a \mid a \in D \rangle$. (This will really be a direct limit of ultrapowers rather than an ultrapower proper, but calling it an "ultrapower" is by now standard.)

Remark 75: We build the conjecture for ultrapower as Eqs. 75.1 implies 75.2. (75.3.0)
There are two states with 1 as T as tautology (the designated *proof* value) (75.3.1.1)

LET p, q, r, s, u, X:
a, b, D, f, μ, X;

(((p<r)>(u&p))>(~(q<p)&(((u&p)&x)=(p=p))))>(((u&q)&((s<(p<x))>s))=(p=p)) ;
T**F**T**F** TTTT T**F**T**F** TTTT(2), TTTT TTTT TTTT TTTT(2),
T**F**TT TTTT T**F**TT TTTT(2), T**F**T**F** T**F**T**F** T**F**TT T**F**TT(2) x 4 (75.3.1.2)

or 1 as ordinal one. (75.3.2.1)

(((p<r)>(u&p))>(~(q<p)&(((u&p)&x)=(%p>#p))))>(((u&q)&((s<(p<x))>s))=
(%p>#p)) ;
TCTC TTTT TCTC TTTT(2), TTTT TTTT TTTN TTTN(2),
TCTC TTTT TCTC TTTT(2), TCTC TCTC TCTT TCTT(2) x 4 (75.3.2.2)

Eqs. 75.3.1.2 and 3.2.2 as rendered are *not* tautologous. This refutes Eq. 75.1.1 ⟨μ_a | a ∈ D⟩ as defining the ultrapower. What follows is the coloring and denial of the subsequent exposition.

No unanswered qustion

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal.

Is it true that any question implies at least one answer? (1.1)

LET: p, q: answer, question;
> Imply;
% possibility, for one or some;
# necessity, for every or all .

#q>%p ; TTCT TTCT TTCT TTCT (1.2)

Eq. 1.2 as rendered is *not* tautologous, therefore the answer to Eq. 1.1 is no.

The reciprocal reads as:

Is it true that at least one question implies any answer? (2.1)

%q>#p ; NN**F**N NN**F**N NN**F**N NN**F**N (2.2)

Eq. 2.2 rendered is *not* tautologous, therefore the answer to Eq.2.1 is no.

However, we combine the Eqs. to read as:

Is it true that if at least one question implies any answer,
then any question implies at least one answer? (3.1)

(%q>#p) > (#q>%p) ; TTTT TTTT TTTT TTTT (3.2)

Eq. 3.2 *is* tautologous, therefore the answer to Eq. 3.1 is yes.

Refutation of unfalsifiable conjectures in mathematics and science

Abstract: In bivalent mathematical logic, the unfalsifiable conjecture is not contradictory, and hence tautologous to be a theorem. The theorem by definition is not contradictory, tautologous, and hence unfalsifiable. There is no distinction between the states of unfalsifiable or confirmable as opposed to falsifiable or refutable.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p: p; ~ Not; + Or; & And; > Imply; = Equivalent;
(p=p) T tautology; (p@p) **F** contradiction.

From: Feinstein, C.A. (2018). Unfalsifiable conjectures in mathematics. *Progress in physics*. 14:4. vixra.org/pdf/1809.0454v1.pdf

Let us assume that the ZFC axioms are consistent ... . Then what are the implications of proving that a mathematical conjecture is unfalsifiable? (1.1.0)

The answer is that even though an unfalsifiable conjecture might not be true, there is still no harm in assuming that it is true, since there is no chance that one could derive any provably false statements from it; (1.2.0)

if one could derive any provably false statements from an unfalsifiable conjecture, this would imply that the conjecture is falsifiable, which is a contradiction. (1.3.0)

Remark: In Eq. 1.1.0, The words "provably" or "proving" are redundant. We take "false" and "falsifiable" as equivalent to avoid semantic confusion and as equivalent to contradictory, and "unfalsifiable" to mean not contradictory. To assume ZFC as consistent (which we show elsewhere is *not* the case) is the equivalent to stating it is tautologous.

We write Eq. 1.1.0 as:

"If ZFC axioms are tautologous, then if a conjecture is not contradictory, [then subsequent implications follow]." (1.1.1)

(p=p)>~(p@p) ; TTTT TTTT TTTT TTTT (1.1.2)

Remark: In Eq. 1.2.0, the inexact use of the words "might", "no chance", and "any" are ignored to avoid injection of the modal states of possibility, not necessarily, and necessity. The words "no harm" are a metaphysical term.

We rearrange the verbiage order in Eq. 1.2.0 to read as:

"If a conjecture is not contradictory, then if it is tautologous, then it is not contradictory." (1.2.1)

~(p@p)>((p=p)>~(p@p)) ; TTTT TTTT TTTT TTTT (1.2.2)

We rearrange the verbiage order in the second sentence fragment to read:

"The sentence ((If a conjecture is not contradictory, then it is contradictory), then it is contradictory) is a contradiction." (1.3.1)

((~(p@p)>(p@p))>(p@p))=(p@p) ; FFFF FFFF FFFF FFFF (1.3.2)

We assume the semicolon between Eqs. 1.2.1 and 1.3.1 to mean an "and" pause to what follows and to serve as the operator And.

This produces the sentence of Eqs. 1.1.1 implies 1.2.1 or 1.3.1. (1.4.1)

((p=p)>~(p@p))>((~(p@p)>((p=p)>~(p@p)))&(((~(p@p)>(p@p))>(p@p))
=(p@p))) ; FFFF FFFF FFFF FFFF (1.4.2)

Eqs. 1.3.2 and 1.4.2 are *not* tautologous, hence refuting the proposition of unfalsifiable conjectures in mathematics and science.

Remark: In bivalent mathematical logic, the unfalsifiable conjecture is not contradictory, and hence tautologous and a theorem. The theorem by definition is not contradictory, tautologous, and hence unfalsifiable. There is no distinction between the states of unfalsifiable or confirmable as opposed to falsifiable or refutable. This is in the spirit of Popper's *Conjecture and Refutation*.

The advantage of Meth8/VŁ4 is that a conjecture can be not contradictory *and* not tautologous at the same time, meaning it has some proof table result state *between* contradiction and tautology, but neither. This means a conjecture can be effectively falsified if it is *not* unfalsifiable. For example, a proof table with all values for truthity or for falsity, or with mixed values of truthity and falsity, is not contradictory *and* not tautologous.

Refutation of unification nets (canonical proof net quantifiers)

Abstract: Using the drinker's paradox, as rendered in two equations, we evaluate the unification net, in two equations, as *not* tautologous. To extend the unification net to additives is similarly defective, forming a *non* tautologous fragment of the universal logic VŁ4. We also supply analysis of Smullyan's drinking principle.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Hughes, D.J.D. (2018). Unification nets: canonical proof net quantifiers. arxiv.org/pdf/1802.03224.pdf

1.3 Towards combinatorial proofs for classical first-order logic
A first-order combinatorial proof of Smullyan's drinker paradox [is shown]

∃x(Px ⇒ ∀yPy) . (1.3.0.1)

LET p, q, r, s: P, D, x, y.

%r&((p&r)>(#s&(p&s))) ; CCCC T**F**T**F** CCCC TNTN (1.3.0.2)

Remark 1.3.0.2: We rewrite Eq. 1.3.0.1 for clarity by distributing the respective quantifiers. (1.3.0.1.1)

(p&%r)>(p&#s) ; TNTN T**F**T**F** TNTN TNTN (1.3.0.1.2)

Eqs. 1.3.0.2 and 1.3.0.1.2 are *not* tautologous and *not* equivalent; also, Smullyan's drinker paradox is stated differently elsewhere*.

By using a semi-combinatorial presentation style ... the unification net becomes more apparent.

$\exists x(\overline{P}x \vee \forall y\ Py)$ (1.3.4.1)

%r&((~p&r)+(#s&(p&s))) ; **FFFF** T**F**T**F** **FFFF** T**F**TN (1.3.4.2)

Remark 1.3.4.2: We rewrite Eq. 1.3.4.1 for clarity by distributing the respective quantifiers. (1.3.4.1.1)

(~p&%r)+(p&#s) ; C**F**C**F** T**F**T**F** CNCN TNTN (1.3.4.1.2)

Eqs. 1.3.4.2 and 1.3.4.1.2 are *not* tautologous and *not* equivalent, refuting unification nets. To extend the unification net to additives is similarly defective.

*From: en.wikipedia.org/wiki/Drinnker_paradox [sic]

[Raymond Smullyan's drinking principle is known as the drinker's paradox.]

"There is someone in the pub such that, if he is drinking, then everyone in the pub is drinking." (2.0.1)

The formal statement of the theorem is, where D is an arbitrary predicate and P is an arbitrary nonempty set

$\exists x \in P.[(D(x) \Rightarrow \forall y \in P.D(y)].$ (2.0.1.1)

Remark 2.0.1: We disagree that Eq. 2.0.1 maps to 2.0.1.1 (it is *no*t tautologous). Instead we map 2.0.1 in words as:

"If one is in the bar, then if that one in the bar is drinking, then all in the bar are drinking." (2.0.2.1)

LET p, q, r, s: P pub, D drinking, x one, y all.

(%r<p)>(((%r<p)&q)>((#s<p)&q)) ;
TTNT TT**F**T TTNT TTNT (2.0.2.2)

Remark 2.0.2.2: Eq. 2.0.2.2 is also equivalent to (%r<p)>((%r&q)>((#s<p)&q)), excluding the repetitive second "%r<p" for "%r".

Eq. 2.0.2.2 as rendered is not tautologous, refuting the drinker's paradox as a paradox, and forming another *non* tautologous fragment of the universal logic VŁ4.

Refutation of the unification type of simple symmetric modal logics

Abstract: We evaluate the definitions of two new modal connectives as box-plus and box-minus. Neither is tautologous and both are equivalent. This refutes the unification type of simple symmetric modal logics and implies it is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⇔, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) C non-contingency, ∇, ordinal 2; (%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Balbiani, P.; Gencer, Ç. (2019). About the unification type of simple symmetric modal logics. arxiv.org/pdf/1902.03770.pdf philippe.balbiani@irit.fr

2 Syntax.
For all parameters p, we write "p^0" to mean "¬p" and we write "p^1" to mean "p".
Let ⊞ and ⊟ be the modal connectives defined as follows: (2.0)

Remark 2.0: We name the respective symbols ⊞ and ⊟ as box-plus and box-minus.

$⊞\phi ::= (p^0 \wedge q^0 \rightarrow \Box(p^1 \wedge q^0 \rightarrow \Box(p^0 \wedge q^1 \rightarrow \Box(p^0 \wedge q^0 \rightarrow \phi))))$,
(2.1.1)

LET p, q, r: p^1=p, q^1=q, p^0=~p, q^0=~q, ϕ.

((~p&~q)>#((p&~q)>#((~p&q)>#((~p&~q)>t)))) = (p=p) ;
NTTT NTTT NTTT NTTT (2.1.2)

$⊟\phi ::= (p^0 \wedge q^0 \rightarrow \Box(p^0 \wedge q^1 \rightarrow \Box(p^1 \wedge q^0 \rightarrow \Box(p^0 \wedge q^0 \rightarrow \phi))))$. (2.2.1)

((~p&~q)>#((~p&q)>#((p&~q)>#((~p&~q)>t)))) = (p=p) ;
NTTT NTTT NTTT NTTT (2.2.2)

Eqs. 2.1.2 and 2.2.2 are *not* tautologous and logically equivalent because the respective conjunctive clauses are identical. This refutes the unification type of simple symmetric modal logics.

Refutation of the universal finite set

From:

Hamkins, J.D.; Woodin, W.H. (2017). The universal finite set.
arxiv.org/pdf/1711.07952.pdf.jdh.hamkins.org/the-universal-finite-set/

We evaluated two parts of the proof of Lemma 2 (Folklore).

We assume the apparatus of the Meth8 modal logic model checker implementing variant system VŁ4. Meth8 allows to mix four logical values with four analytical values. The designated *proof* value is T.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	p=p	T	Tautology	proof	11	3
2	p@p	F	Contradiction	absurdum	00	0
3	%p>#p	N	Non-contingency	truth	01	1
4	%p<#p	C	Contingency	falsity	10	2

LET: ~ Not; + Or; & And; \ Not and; > Imply; < Not imply; = Equivalent to; @ Not equivalent to; # all; % some; (p@p) 00; (p=p) 11

Results are the proof table of 16-values in row major horizontally.

(3 -> 2):

Not evaluated (3.2)

(2->1):

LET pqrs ψ lc_psi θ lc_theta uc_V x

((%q&r)&(q>p)) = ((%q&%s) & (((s=(r&q))&s)>p));
`NNNN TTNC NNFF TTTT` (2.1)

(1-> 3:

LET pqrs φ lc_phi x y H; k is uncountable, so k=(p>(p=p)).

((%q&#r)&((p&(p@p))&(q&r))) = ((%(p>(p=p))&(s&(p>(p=p))))
>((%q&#r)&((p&(p@p))&(q&r)))) ; `FFFF FFFF TTTT TTTT` (1.3)

Eqs. 2.1 and 1.3 as rendered are *not* tautologous.

Refutation of a universal operator for interpretable deep convolution networks

Abstract: We evaluate a universal operator then apply the specified parameters to form operators for AND, OR, XOR, and MP (modus ponens). None are tautologous. This refutes the universal operator as proposed for interpretable deep convolution networks.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r, s, t, u, v, w, x, y, z:
P, ϕ, A, B, α, β, γ, b, x, y, z;
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⊢, ↦; < Not Imply, less than, ∈;
= Equivalent, ≡, ⊨, :=, ⇔, ↔; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology; (z@z) **F** as contradiction, Ø, Null;
(%z<#z) C non-contingency, ∇, ordinal 2;
(%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y).

From: Chan, C.S.; Fan, L.; Ng, K.W. (2019).
A universal logic operator for interpretable deep convolution networks.
arxiv.org/ftp/arxiv/papers/1901/1901.08551.pdf
cs.chan@um.edu.my, lixin.fan@jd.com, kamwoh@siswa.um.edu.my, zqs1022@sjtu.edu.cn

Table 1: Comparison between our proposed universal logical operator (ULO) and four classical probabilistic logical inference rules (AND, OR, XOR, MP) under independence assumption.

Note that x = P(ϕx), y = P(ϕy); MP stands for modus ponens, for which P(ϕx) = P(A); P(ϕy) = P(B|A) [ie, conditional probability meaning P(A And B)/P(A)]; and P(ϕc) = P(B).

Inference rule: U(ϕx, ϕy)	Output: P(ϕc) = P(U(ϕx, ϕy))	Logical operator parameters:	
ULO (ϕx, ϕy)	αxy + βy + γx + b	α, β, γ, b to be optimized	(1.0)

P(B)=αP(A)P(B)+βP(B|A)+b (1.1)

(p&s)=((t&((p&r)&(p&s)))+(((u&(p&(r&s)))\(p&r))+w)) ;

```
FFFF FFFF FTFT FTFT( 2),
FFFF FFFF FTFT FFFF( 1),
FFFF FFFF FTFT FTFT( 3),
FFFF FFFF FTFT FFFF( 1),
FFFF FFFF FTFT FTFT( 9)
```
(1.2)

Remark 1.2: Eq. 1.2 serves as the antecedent along with the specified parameters to imply the consequent of the designated operator.

AND (ϕx, ϕy) xy α=1, β=0, γ=0, b=0 (2.0)

Eq. 1.1 and specific parameters imply P(B)=P(A)P(B|A). (2.1)

```
(((p&s)=((t&((p&r)&(p&s)))+(((u&(p&(r&s)))\(p&r))+w)))&
(((t=(%z>#z))&(u=(z@z)))&((v=(z@z))&(w=(z@z)))))>
((p&s)=((p&r)&((u&(p&(r&s)))\(p&r)))) ;
                              TTTT TTTT TCTC TTTT( 1),
                              TTTT TTTT TNTN TTTT( 1),
                              TTTT TTTT TTTT TTTT(14)
```

(2.2)

OR (ϕx, ϕy) x + y − xy α=−1, β=1, γ=1, b=0 (3.0)

Eq. 1.1 and specific parameters imply
P(B)=P(A)+P(B|A)-P(A)P(B). (3.1)

```
((((p&s)=((t&((p&r)&(p&s)))+(((u&(p&(r&s)))\(p&r))+w)))&
(((t=(%z>#z))&(u=(z@z)))&((v=(z@z))&(w=(z@z)))))>
((p&s)=((p&r)+(((u&(p&(r&s)))\(p&r))-((p&r)&(p&s))))) ;
                              TTTT TTTT TNTN TTTT( 1),
                              TTTT TTTT TCTC TTTT( 1),
                              TTTT TTTT TTTT TTTT(14)
```

(3.2)

XOR (ϕx, ϕy) x + y − 2xy α=−2, β=1, γ=1, b=0 (4.0)

Eq. 1.1 and specific parameters imply
P(B)=P(A)+P(B)-2P(A)P(B|A). (4.1)

```
((((p&s)=((t&((p&r)&(p&s)))+(((u&(p&(r&s)))\(p&r))+w)))&
(((t=(%z>#z))&(u=(z@z)))&((v=(z@z))&(w=(z@z)))))>
((p&s)=(((p&r)+(p&s))-
((%z<#z)&((p&r)&((u&(p&(r&s)))\(p&r)))))) ;
                              TTTT TTTT TNTN TNTN( 1),
                              TTTT TTTT TCTC TCTC( 1),
                              TTTT TTTT TTTT TTTT(14)
```

(4.2)

MP (ϕx, ϕy) xy + (1 − x)/2 α=1, β=0, γ=−0.5, b=0.5 (5.0)

Eq. 1.1 and specific parameters imply
P(B)=P(A)P(B|A)+(1-P(A))/2. (5.1)

```
(((p&s)=((t&((p&r)&(p&s)))+(((u&(p&(r&s)))\(p&r))+w)))&
(((t=(%z>#z))&(u=(z@z)))&((v=(z@z))&(w=(z@z)))))>
((p&s)=(((p&r)&((u&(p&(r&s)))\(p&r)))+(((%z>#z)-(p&r))\
(%z<#z)))) ;
```

```
TTTT TTTT TNTN TTTT( 1),
TTTT TTTT TTTT TTTT(15)
```
(5.2)

Eqs. 1.2-5.2 as rendered are *not* tautologous. This refutes a proposed universal operator for interpretable deep convolution networks.

Universal logic VŁ4

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1. Abstract

This paper demonstrates why logic system VŁ4 is a universal logic composed of any refutation as a non-tautologous fragment. Recent advances are a definitive answer to criticism of logic Ł4, modal equations for lines and angles of the Square of Opposition, confirmation of the 24-syllogisms by updating Modus Cesare and Camestros, and proving that respective quantified and modal operators are equivalent. The parser Meth8 implements VŁ4 as the modal logic model checker Meth8/VŁ4. About 575 artifacts are tested in 3060 assertions with a refutation rate of 82.45%.

2. Introduction

2.1. Outline

This paper proves that an exact, bivalent, quaternary logic is *not* a probabilistic, vector space. From the four-values of the 2-tuple, logical assignments are derived for two models in logic system B4. Modal values are further ascribed for system Ł4 with truth tables for connectives. A criticism of Ł4 is answered by trivial proof. The Square of Opposition is corrected with modal equations for vertices and edges. Corrections are made to two of the 24-syllogisms as confirmed. The quantifiers are shown equivalent to the respective modal operators as a distinguishing feature for system VŁ4. The Meth8 parser hosts and implements VŁ4 as a modal logic model checker. Meth8/VŁ4 tested 575 artifacts in 3060 assertions for a rate of 17.55% confirmation and 82.45% refutation. The seven examples given are for refutations.

2.2. Overview of literature

Universal logic owns a public domain corpus published at encyclopedia web sites with lists of marginal, secondary references. A few primary sources describe non-standard and paraconsistent logic as appropriated by three writers, but traceable to earlier concepts as minimized or suppressed. Until now, there is *no* literature on bivalent, modal, quaternary, universal logic.

3. B4 as a group, ring, module

In (James, 2010), the 2-tuple of logic B4 was described as:

> Four value bit code (4vbc) consists of four dibits that have the semantic meanings of True {01} and False {10} and the syntactic meanings of Bivalent {11} and Not Bivalent {00}. The respective left- and right-bits are further variables for false and true. Two dibits (4-bits) form the basis of PMDL, a universal logic for propositional, modal, and deontic logics. PMDL has three levels of tabular proofs as negation, rotation, and reflection. This paper proves that 4vbc constitutes its own mathematical category as a group, ring, and module.

The outline of the proof was:

> 4vbc contains unique 8-bit operators that are tabulated into 256 look up tables (LUTs). The additive table for the small finite field $\mathbf{F}_4$ is isomorphic in 4vbc to the LUT of the logical operator "XOR". 4vbc is not isomorphic to the $\mathbf{F}_4$ multiplicative table which is *bit-inconsistent*. Hence 4vbc is not a

vector space. The modulo 2 additive table of the elementary Abelian group $(Z/2Z)^2$ is isomorphic in 4vbc to the LUT of the logical operator "Necessarily XOR". $(Z/2Z)^2$ is the finite group C_2xC_2 that is a distinct group of order 4 and is not cyclic. A Cayley table as the representation of a multiplicative table of C_2xC_2 table is isomorphic in 4vbc to the LUT of the logical operator "10 EQV(XOR)". (Another distinct group of order 4 is the C_4 group that is cyclic; in 4vbc that multiplicative table is *bit-inconsistent*.) 4vbc is further isomorphic to multiplicative table of the abstract Vierergruppe or Klein V_4 Group. 4vbc meets the five axioms required for an Abelian group under addition. 4vbc meets the three axioms required for a monoid group under multiplication. 4vbc meets the six axioms required for a ring to include left- and right-distributivity through 12 brute force combinations. 4vbc meets the four axioms required for a left R-module. Because the right and left R-modules are commutative, 4vbc is also an R-module.

The term above *bit-inconsistent* describes non-bivalent, vector spaces.

LET s = sinister (left-handed); d = dexter (right-handed). The x below is connective AND.

Table 4. **F**4 multiplicative table in 4vbc

	s|d	s|d	s|d	s|d	
x	0|0	0|1	1|0	1|1	
0|0	0|0	0|0	0|0	0|0	Line 1
0|1	0|0	0|1	1|0	1|1	Line 2
1|0	0|0	1|0	1|1	0|1	Line 3
1|1	0|0	1|1	0|1	1|0	Line 4
	s|d	s|d	s|d	s|d	

Table 4 is bit inconsistent in the left and right bits, respectively. We show right (d) bits only in Table 5.

0&0=0 Lines 1, 3;	0&0=1 Line 3;
0&1=0 Lines 1, 3;	0&1=1 Line 3;
1&0=0 Lines 2, 4;	1&0=1 Line 4; and
1&1=1 Lines 2, 4;	1&1=0 Line 4.

Each clausex above is a contradiction, meaning **F**4 is not bivalent, but a vector space.

4. Two model types on B4 with 2-tuple values

The two model types on B4 are named Model 1 (M1) and Model 2 (M2). The logical values of the 2-tuple {00, 10, 01, 11} are described respectively as:

{**F**alse for contradiction; `C`ontingent for falsity; `N`on contingent for truthity; `T`autology for proof} (M1)

and

{**U**nevaluated; `I`mproper; `P`roper; `E`valuated}. (M2)

The respective values of { F, C, N, T} in M1 are equivalent to { U, I, P, E} in M2. The designated *proof* value is `T` for tautology and `E` for evaluated.

5. Modal values on Ł4

Model 2.1 (M2.1) is equivalent to Model 1 (M1) but with { U, I, P, E} instead of { F, C, N, T}. M2.1 is included for completeness. M2 also contains the sub-models of M2.2 and M2.3. These are required for combinations of logical values in B4 to produce modal values in Ł4. The derivation is based on the up-and down-functors of Łukasiewicz below. (The symbols are & for AND and v for OR, and [] for necessity and <> for possibility.)

Łukasiewicz' Up-functor [p]

```
M1     []: { F,C,N,T} & C = { F,C,F,C};  M1     <>: { F,C,N,T} v N = { N,T,N,T}
M2.1   []: { U,I,P,E} & E = { U,I,P,E};  M2.1   <>: { U,I,P,E} v U = { U,I,P,E}
M2.2   []: { U,I,P,E} & U = { U,U,U,U};  M2.2   <>: { U,I,P,E} v E = { E,E,E,E}
M2.3.1 []: { U,I,P,E} & P = { U,U,P,P};  M2.3.1 <>: { U,I,P,E} v I = { I,I,E,E}
M2.3.2 []: { U,I,P,E} & I = { U,I,U,I};  M2.3.2 <>: { U,I,P,E} v P = { P,E,P,E}
```

Łukasiewicz' Down-functor [~p]

```
M1     []: { T,N,C,F} & C = { C,F,C,F};  M1     <>: { T,N,C,F} v N = { T,N,T,N}
M2.1   []: { E,P,I,U} & E = { E,P,I,U};  M2.1   <>: { E,P,I,U} v U = { E,P,I,U}
M2.2   []: { E,P,I,U} & U = { E,E,E,E};  M2.2   <>: { E,P,I,U} v E = { U,U,U,U}
M2.3.1 []: { E,P,I,U} & P = { E,E,I,I};  M2.3.1 <>: { E,P,I,U} v I = { E,E,I,I}
M2.3.2 []: { E,P,I,U} & I = { I,U,I,U};  M2.3.2 <>: { E,P,I,U} v P = { E,P,E,P}
```

The look up tables (LUTs) are stored in binary and decimal as

{ 00, 10, 01, 11} and with substitution LUTs for: { F, C, N, T} and
{ 0, 3, 2, 1} { U, I, P, E}.

Rule 1 states that for any expression falling within the scope of a modal operator, only M2.1 applies for all truth constructs of the expression.

Symbols are: & for AND; + for OR; # for □ necessity; and % for ◊ possibility. The modal results for #p, %p, #~p, and %~p of each model are below:

Row index	Column index Model	0 **p**	1 **#p**	2 **%p**	3 **#~p**	4 **%~p**
0	**B4** # * 3, % + 2	0 3 2 1	0 3 0 3	2 1 2 1	3 0 3 0	1 2 1 2
1	**B4** # &1 1, % + 01	00 10 01 11	00 10 00 10	01 11 01 11	10 00 10 00	11 01 11 01
2	**M1** # & C, % + N	F C N T	F C F C	N T N T	C F C F	T N T N
3	**M2.1** # & E, % + U	U I P E	U I U I	P E P E	E P E P	I U I U
4	**M2.2** # & U, % + E	U I P E	U U U U	E E E E	E E E E	U U U U
5	**M2.3.1** # & P, % + I	U I P E	U U P P	I I E E	E E I I	P P U U
6	**M2.3.2** # & I, % + P	U I P E	U I U I	P E P E	E P E P	I U I U

More compact LUTs are described as:

```
VŁ4:     M1  M2      ~VŁ4: ~M1  ~M2
         F   U             T    E
         C   I             N    P
         N   P             C    I
         T   E             F    U
```

	1			2.1		2.2		2.31		2.32	
	#	%		#	%	#	%	#	%	#	%
F.	F	C	U.	U	U	U	E	U	P	U	I
C.	F	C	I.	I	I	U	E	I	E	U	I
N.	N	T	P.	P	P	U	E	U	P	P	E
T.	N	T	E.	E	E	U	E	I	E	P	E

< Definitions of the five models.
Necessity, All or every;
% Possibility, One or some
(The equivalence of modal and quantified operators is derived in Section 9 below.)

The connectives are from standard logic and in one character as

{and, or, imply, equivalent} for {&, +, >, =};

and with the negated connectives as

{nand; nor; not imply; exclusive-or} for {\, -, <, @}.

The 16-valued look up truth tables are by four rows-major and presented horizontally.

1	&	.	F,F,F,F .	F,C,F,C .	F,F,N,N .	F,C,N,T
1	\	.	T,T,T,T .	T,N,T,N .	T,T,C,C .	T,N,C,F
1	+	.	F,C,N,T .	C,C,T,T .	N,T,N,T .	T,T,T,T
1	-	.	T,N,C,F .	N,N,F,F .	C,F,C,F .	F,F,F,F
1	<	.	F,F,F,F .	C,F,C,F .	N,N,F,F .	T,N,C,F
1	=	.	T,N,C,F .	N,T,F,C .	C,F,T,N .	F,C,N,T
1	>	.	T,T,T,T .	N,T,N,T .	C,C,T,T .	F,C,N,T
1	@	.	F,C,N,T .	C,F,T,N .	N,T,F,C .	T,N,C,F
2	&	.	U,U,U,U .	U,I,U,I .	U,U,P,P .	U,I,P,E
2	\	.	E,E,E,E .	E,P,E,P .	E,E,I,I .	E,P,I,U
2	+	.	U,I,P,E .	I,I,E,E .	P,E,P,E .	E,E,E,E
2	-	.	E,P,I,U .	P,P,U,U .	I,U,I,U .	U,U,U,U
2	<	.	U,U,U,U .	I,U,I,U .	P,P,U,U .	E,P,I,U
2	=	.	E,P,I,U .	P,E,U,I .	I,U,E,P .	U,I,P,E
2	>	.	E,E,E,E .	P,E,P,E .	I,I,E,E .	U,I,P,E
2	@	.	U,I,P,E .	I,U,E,P .	P,E,U,I .	E,P,I,U

6. Answer to an Ł4 objection

This proposition is supposed to be egregious to logic system Ł$_4$:

$$(\Diamond p \& \Diamond q) \to \Diamond (p \& q). \qquad (6.1.1)$$

If possibly the cat is alive and possibly the cat is dead, then possibly both the cat is alive and the cat is dead. (6.1.0)

LET p, q: Schrödinger's cat is alive; Schrödinger's cat is dead

(%p&%q)>%(p&q) ; `TTTT TTTT TTTT TTTT` (6.1.2)

Assumptions: ((exists(p) & exists(q))).
Goals (exists(p&q)). Exhausted. (6.1.3)

Prover9 invalidates Eq. 6.1.3 to show Ł$_4$ is untenable as an alethic logic.

If we preload p=~q as the antecedent to Eq. 6.1.0, then:

If possibly the cat is alive is equivalent to Not (the cat is dead), then
if possibly the cat is alive and possibly the cat is dead, then
possibly both the cat is alive and the cat is dead. (6.2.0)

%(p=~q)>(%(p&q)>(%p&%q)) ;
TTTT TTTT TTTT TTTT (6.2.2)

Assumptions: (exists(p<->-q)).
Goals: (exists(p)&exists(q))->(exists(p&q)).
Exhausted. (6.2.3)

Prover9 invalidates Eq. 6.2.3 to show Ł$_4$ is untenable as an alethic logic.

Remark 6.2.3: Eq. 6.2.3 shows Prover9 also does not distribute the existential quantifier.

We rewrite Eq. 6.2.1 using one variable and its negation as respectively *alive* and *not alive*:

$(\Diamond p\&\Diamond\sim p)\rightarrow\Diamond(p\&\sim p)$. (6.3.1)

If possibly the cat is alive and possibly the cat is not alive, then
possibly both the cat is alive and the cat is not alive. (6.3.0)

(%p&%~p)>%(p&~p) ; TTTT TTTT TTTT TTTT (6.3.2)

Assumptions: (exists(p)&-exists(p)).
Goals: (exists(p&-p). Theorem. (6.3.3)

Prover9 validates Eq. 6.3.3 to show Ł4 is tenable as an alethic logic.

We explain Eqs. 6.1.2, 6.2.2, and 6.3.2 as rendered as tautologous in Meth8/VŁ4, but 6.1.3 as exhausted in Prover9 in this way. For more than one variable, the vector space for arity with Prover9 diverges from the bivalance inherent in VŁ$_4$, in which modal operators and quantifiers are distributive. This speaks to Meth8/VŁ4, based on the *corrected* modern Square of Opposition for an exact bivalent system, as opposed to Prover9, based on the uncorrected modern Square of Opposition for an inexact probabilistic vector space.

Remark 6.3.2: Meth8/VŁ4 also distinguishes between Eqs. 2.2 and 3.2 by protasis and apodosis as:

%p&%q ; CCCT CCCT CCCT CCCT (6.1.2.1.2)
%(p&q)=(p=p) ; CCCT CCCT CCCT CCCT (6.1.2.2.2)

and

%p&%~p ; CCCC CCCC CCCC CCCC (6.3.2.1.2)
%(p&~p)=(p=p) ; CCCC CCCC CCCC CCCC (6.3.2.2.2)

7. Corrected Square of Opposition

We include the Square of Opposition as corrected by Meth8 and confirmation of the Łukasiewicz Square of Opposition via logic VŁ4, including the Seuren Cube of Opposition which vindicates its mistaken criticism (although still *not* tautologous).

7.1. Square of Opposition Meth8 corrected

The modern revision of the square of opposition is not validated as tautologous by the Meth8 logic model checker, as based on system variant VŁ4. Consequently we redefine the square so that it is validated as tautologous my Meth8. Instead of definientia using implication for universal terms or conjunction for existential terms, we adopt the equivalent connective for all terms. The modal modifiers necessity and possibility map quantifiers as applying to the entire terms rather than to the antecedent within the terms.

The Meth8 symbols here are: ~ Negation ; \ Nand ; > Imply ; + Or ; # modal necessity for universal quantifier ; % modal possibility for existential quantifier ; ? unspecified connective.

Sources Type	Definientia	* Modern Revision Script	Valid as	** Meth8 Correction Script	Valid as
Corner	A	#s> p		#(s= p)	
	E	#s>~p		#(s=~p)	
	I	%s&p		%(s= p)	
	O	%s&~p		%(s=~p)	
Contraries	AE	(#s>p) + (#s>~p)	A + E	#(s= p) \ #(s=~p)	A \ E
Subalterns	AI	(#s>p) **?** (%s&p)		#(s= p) > %(s= p)	A > I
Contradictories	AO	(#s>p) + (%s&~p)	A + O	#(s= p) \ %(s=~p)	A \ O
Contradictories	EI	(#s>~p) + (%s&p)	E + I	#(s=~p) \ %(s= p)	E \ I
Subalterns	EO	(#s>~p) **?** (%s&~p)		#(s=~p) > %(s=~p)	E > O
Subcontraries	IO	(%s&p) \ (%s&~p)	I \ O	%(s= p) + %(s=~p)	I + O

* The quantifier may refer to the entire term as #(p=q) or to the antecedent of the term as (#p=q). In Meth8 there is a difference. We adopt the latter because it returns more validated connectives. For example from the traditional square: #(A?E), #(I?O) versus (A+E), (I\O).

The modern revision of the square of opposition is not validated as tautologous by the Meth8 logic checker in five models for all expressions. This leads us to consider that any logic system based on the square of opposition is spurious. What follows then is that a first order predicate logic based on the square of opposition is now suspicious.

** The Meth8 validated square of opposition redefines A, E, I, O to match the words more clearly. For example on A, "All S is P" is mapped as "#(s=p)", not as in the note above with "#s=p" because the connective of equivalence is stricter than that of implication and consistent for all definiens. By changing the connective in the term from implication or conjunction to equivalence makes the Meth8 validated square of opposition suitable as a basis for other logics such as first order predicate logic.

We note the validating connectives for the edges on the square are: \ Nand for the Contraries and Contradictories; > Imply for the Subalterns; and + Or for the Subcontraries.

7.2. Confirmation of the Łukasiewicz Square of Opposition via logic VŁ4

We evaluate the existential import of the Revised Modern Square of Opposition. We confirm that the Łukasiewicz syllogistic was intended to apply to *all* terms. What follows is that Aristotle was mistaken in his mapping of vertices, which we correct and show fidelity to Aristotle's intentions. We also evaluate the Cube of Opposition of Seuren. Two final claims are not tautologous, hence refuting the Cube, which also contradict criticism of Seuren that was not based on those claims.

See: Read, S. (2015). Aristotle and Łukasiewicz on Existential Import.
st-andrews.ac.uk/~slr/Existential_import.pdf

We map vertices of the first Square of Opposition on page 4 with its words below.

(A)	Every S is P.	#(s= p)=(p=p) ;	N**F**N**F** N**F**N**F** **F**N**F**N **F**N**F**N	(7.2.0.1.2)
(E)	No S is P.	#(s=~p)=(p=p) ;	**F**N**F**N **F**N**F**N N**F**N**F** N**F**N**F**	(7.2.0.3.2)
(I)	Some S is P.	%(s= p)=(p=p) ;	TCTC TCTC CTCT CTCT	(7.2.0.5.2)
(O)	Not every S is P.	%(~s=p)=(p=p) ;	CTCT CTCT TCTC TCTC	(7.2.0.7.2)

Remark 7.2.0: The above is from our *revised* Modern Square of Opposition as in Section 7.1.

We map the relations which Aristotle accepts as preserved here.

A- and E-propositions are contrary (cannot both be true) [(A)=T & (E)=T] (7.2.1.1.1)

(#(s= p)=(p=p))&(#(s=~p)=(p=p)) ; **FFFF FFFF FFFF FFFF** (7.2.1.1.2)

and I- and O-propositions are subcontrary (cannot both be false)
[(I)=**F** & (O)=**F**] (7.2.1.2.1)

(%(s= p)=(p@p))&(%(s=~p)=(p@p)) ; **FFFF FFFF FFFF FFFF** (7.2.1.2.2)

A- and O-propositions are contradictories, [(A)&(O)] (7.2.2.1.1)

#(s= p)&%(s=~p) ; **FFFF FFFF FFFF FFFF** (7.2.2.1.2)

as are I- and E-propositions [(I) & (E)] (7.2.2.2.1)

%(s= p)&#(s=~p) ; **FFFF FFFF FFFF FFFF** (7.2.2.2.2)

A-propositions imply their subaltern I-proposition, [(A) > (I)] (7.2.3.1.1)

#(s= p)>%(s= p) ; TTTT TTTT TTTT TTTT (7.2.3.1.2)

and E-propositions their subaltern O-proposition [(E) > (O)] (7.2.3.2.1)

#(s=~p)>%(s=~p) ; TTTT TTTT TTTT TTTT (7.2.3.2.2)

I- propositions convert simply ‘Some *S* is *P* ’ implies ‘Some *P* is *S*’, (7.2.4.1.1)

%(s= p)>%(p= s) ; TTTT TTTT TTTT TTTT (7.2.4.1.2)

and E-propositions ‘No *S* is *P* ’ implies ‘No *P* is *S*’ (7.2.4.2.1)

#(~s=p)>#(~p=s) ; TTTT TTTT TTTT TTTT (7.2.4.2.2)

A-propositions convert accidentally (‘Every *S* is *P* ’ implies ‘Some *P* is *S*’) (7.2.5.1.1)

#(s= p)>%(p= s) ; TTTT TTTT TTTT TTTT (7.2.5.1.2)

and O-propositions don’t convert at all.
[Some S is not P implies Every P is not S.] (7.2.5.2.1)

%(s=~p)>#(p=~s) ; NNNN NNNN NNNN NNNN (7.2.5.2.2)

We present these six equations for the six directed rays in the Square, as in Section 7.1.

(A\E)	#(s= p) \ #(s=~p) ;	TTTT TTTT TTTT TTTT	(7.2.6.1.2)
(A>I)	#(s= p) > %(s= p) ;	TTTT TTTT TTTT TTTT	(7.2.6.2.2)
(A\O)	#(s= p) \ %(s=~p) ;	TTTT TTTT TTTT TTTT	(7.2.6.3.2)
(E\I)	#(s=~p) \ %(s= p) ;	TTTT TTTT TTTT TTTT	(7.2.6.4.2)
(E>O)	#(s=~p) > %(s=~p) ;	TTTT TTTT TTTT TTTT	(7.2.6.5.2)
(I+O)	%(s= p) + %(s=~p) ;	TTTT TTTT TTTT TTTT	(7.2.6.6.2)

Remark 7.2.6: The new connective distribution is as follows with count. The mappings above allow for replication and confirmation of the 24-syllogisms and with our claim of a minor correction each to Modus Camestros and Modus Cesare.

(1)	Contraries	Not And (\);
(1)	Subcontraries	Or (+);
(2)	Subalterns	Imply (>); and
(2)	Contradictories	Not And (\)

We conclude that Łukasiewicz was not mistaken in his rendition of the Square of Opposition.

We now turn to the criticism of the Cube of Opposition of Seuren to map and interleave the additional vertices from the diagram on page 8. While * marks predicate negation with the term "-P", we use $ to mark copula negation with the term "not P", and mark the negation of $ using !.

(A) Every S is P. #(s= p) =(p=p) ;
N**F**N**F** N**F**N**F** **F**N**F**N **F**N**F**N (7.2.7.1.1)

(A*) Every S is not-P. ~(#(s=p)=(p=p))=(p=p) ;
as Not (Every S is P.)
CTCT CTCT TCTC TCTC (7.2.7.1.2)

(A$) Every S is not P. #(s=~p)=(p=p) ;

FN**F**N **F**N**F**N N**F**N**F** N**F**N**F** (7.2.7.1.3)

(A!) Not (Every S is not P.) ~(#(s=~p)=(p=p))=(p=p) ;

TCTC TCTC CTCT CTCT (7.2.7.1.4)

(E) No S is P. #(s=~p)=(p=p) ;

FN**F**N **F**N**F**N N**F**N**F** N**F**N**F** (7.2.7.2.1)

(E*) No S is not-P. ~(#(s=~p)=(p=p))=(p=p) ;

as Not (No S is P.) TCTC TCTC CTCT CTCT (7.2.7.2.2)

(E$) No S is not P. #(~s=~p)=(p=p) ;

N**F**N**F** N**F**N**F** **F**N**F**N **F**N**F**N (7.2.7.2.3)

(E!) Not (No S is not P.) ~(#(~s=~p)=(p=p))=(p=p) ;

CTCT CTCT TCTC TCTC (7.2.7.2.4)

(I) Some S is P. %(s= p)=(p=p) ;

TCTC TCTC CTCT CTCT (7.2.7.3.1)

(I*) Some S is not-P. ~(%(s= p)=(p=p))=(p=p) ;

as Not (Some S is P). **F**N**F**N **F**N**F**N N**F**N**F** N**F**N**F** (7.2.7.3.2)

(I$) Some S is not P. %(s=~p)=(p=p) ;

CTCT CTCT TCTC TCTC (7.2.7.3.3)

(I!) Not (Some S is not P.)~(%(s=~p)=(p=p))=(p=p) ;

N**F**N**F** N**F**N**F** **F**N**F**N **F**N**F**N (7.2.7.3.4)

(O) Not every S is P. %(~s=p)=(p=p) ;

CTCT CTCT TCTC TCTC (7.2.7.4.1)

(O*) Not every S is not-P. ~(%(~s=p)=(p=p))=(p=p) ;

as Not(Not every S is P.) N**F**N**F** N**F**N**F** **F**N**F**N **F**N**F**N (7.2.7.4.2)

(O$) Not every S is not P. %(~s=~p)=(p=p) ;

TCTC TCTC CTCT CTCT (7.2.7.4.3)

(O!) Not (Not every S is not P.) ~(%(~s=~p)=(p=p))=(p=p) ;

FN**F**N **F**N**F**N N**F**N**F** N**F**N**F** (7.2.7.4.4)

The following are supposed to hold:

~I* = *E: ~(~(%(s= p)=(p=p))=(p=p)) = (~(#(s=~p)=(p=p))=(p=p)) ;

TTTT TTTT TTTT TTTT (7.2.8.1.1)

~A* = O*: ~(~(#(s=p)=(p=p))=(p=p)) = (~(%(~s=p)=(p=p))=(p=p)) ;

TTTT TTTT TTTT TTTT (7.2.8.1.2)

A* > E: (~(#(s=p)=(p=p))=(p=p)) > (#(s=~p)=(p=p)) ;

NNNN NNNN NNNN NNNN (7.2.9.1.1)

A > E*: (#(s= p) =(p=p)) > (~(#(s=~p)=(p=p))=(p=p)) ;

TTTT TTTT TTTT TTTT (7.2.9.1.2)

I > O*: (%(s= p)=(p=p)) > (~(%(~s=p)=(p=p))=(p=p)) ;

NNNN NNNN NNNN NNNN 7.2.9.1.3)

I* > O: (~(%(s= p)=(p=p))=(p=p)) > (%(~s=p)=(p=p)) ;

`TTTT TTTT TTTT TTTT` (7.2.9.1.4)

Eqs. 7.2.9.1.1 (A* > E) and 7.2.9.1.3 (I > O*) are *not* tautologous, albeit truthities. This means that the final claims of Seuren's Cube of Opposition are mistaken, but also that the criticism of Seuren as based not on those claims is also mistaken.

8. Corrected syllogisms

The original Square of Opposition produced four combinations for each corner A, I, E, O for 4 ^ 4 = 256 syllogisms. Medieval scholars determined 24 of the 256 syllogisms were tautologous deductions. Of those, 9 were made tautologous but only after additional *known* assumptions were applied as fix ups. Meth8/VŁ4 found tautologous none of the 24 syllogisms *before* fix ups. Meth8 also *discovered* correct additional assumptions to render the other 15 syllogisms found as tautologous. The fix ups in bold were verified independently by Prover9 (2007).

From: en.wikipedia.org/wiki/Syllogism

LET q, r, s: M, P, S.

Original syllogisms in Meth8 script:

Code	Name	Assumptions: 1, 2,	3	Conclusion	Test	Comments
AAA-1	Modus Barbara	((#q&r)&(#s&q))		>(#s&r)	tautologous	
AAI-1	Modus Barbari	(((#q&r)&(#s&q))	&%s)	>(%s&r)		* not needed
		((#q&r)&(#s&q))		>(%s&r)	tautologous	
AAI-4	Modus Bamalip	(((#r&q)&(#q&s))	&%r	>(%s&r)		* not needed
		((#r&q)&(#q&s))		>(%s&r)	tautologous	
EAE-1	Modus Celarent	((~q&r)&(#s&q))		>(~s&r)	tautologous	
EAE-2	**Modus Cesare**	((~r&q)&(#s&q))		>(~s&r)	~ tautologous	* *Mistake*
		(((~r&q)&(#s&q))	**&%r)**	>(~s&r)	tautologous	* **Meth8 fix**
AEE-2	Modus Camestres	((#r&q)&(~s&q))		>(~s&r)	tautologous	
AEE-4	Modus Calemes	((#r&q)&(~q&s))		>(~s&r)	tautologous	
EAO-1	Modus Celaront	(((~q&r)&(#s&q))	&%s)	>(~s&r)		* not needed
		((~q&r)&(#s&q))		>(~s&r)	tautologous	
EAO-2	Modus Cesaro	(((~r&q)&(#s&q))	&%s)	>(%s&~r)		* not needed
		((~r&q)&(#s&q))		>(%s&~r)	tautologous	
AEO-2	**Modus Camestros**	(((#r&q)&(~s&q))	**&%s)**	>(%s&~r)	tautologous	* **needed**
		((#r&q)&(~s&q))		>(%s&~r)	~ tautologous	* *Mistake*
AEO-4	Modus Calemos	(((#r&q)&(~q&s))	&%s)	>(%s&~r)		* not needed
		((#r&q)&(~q&s))		>(%s&~r)	tautologous	
AII-1	Modus Darii	((#q&r)&(%s&q))		>(%s&r)	tautologous	

Code	Name	Assumptions: 1, 2,	3	Conclusion	Test	Comments
AII-3	Modus Datisi	((#q&r)&(%q&s))		>(%s&r)	tautologous	
IAI-3	Modus Disamis	((%q&r)&(#q&s))		>(%s&r)	tautologous	
IAI-4	Modus Diamatis	((%r&q)&(#q&s))		>(%s&r)	tautologous	
EIO-1	Modus Ferio	((~q&r)&(%s&q))		>(%s&~r)	tautologous	
EIO-2	Modus Festino	((~r&q)&(%s&q))		>(%s&~r)	tautologous	
EIO-3	Modus Ferison	((~q&r)&(%q&s))		>(%s&r)	tautologous	
EIO-4	Modus Fresison	((~r&q)&(%q&s))		>(%q&~r)	tautologous	
AOO-2	Modus Baroco	((#r&q)&(%s&~q))		>(%s&~r)	tautologous	
OAO-3	Modus Bocardo	((%q&~r)&(#q&s))		>(%s&~r)	tautologous	
AAI-3	Modus Darapti	(((#q&r)&(#q&s))	&%q)	>(%s&r)		* not needed
		((#q&r)&(#q&s))		>(%s&r)	tautologous	
EAO-3	Modus Felapton	(((~q&r)&(#q&s))	&%q)	>(%s&~r)		* not needed
		((~q&r)&(#q&s))		>(%s&~r)	tautologous	
EAO-4	Modus Fesapo	(((~r&q)&(#q&s))	&%q)	>(%s&~r)		* not needed
		((~r&q)&(#q&s))		>(%s&~r)	tautologous	

9. Quantifiers equivalent to modal operators

The rationale for rendering quantifiers as modal operators in Meth8/VŁ4 has arguments from reproducability of formulas for vertices and edges in modal logic for the Square of Opposition in Section 7, reproducability of evaluating syllogisms as tautologous (with two corrections) in Section 8, and from satisfiability (contra Kuhn) below.

From: Kuhn, S.T. (1979). "Quantifiers as modal operators". Studia Logica. 39.2-3/80: 147. faculty.georgetown.edu/kuhns/supp_files/quantifiers.pdf

"Either [with Montague's approach as first order models or with Prior's approach as "sequences of individuals"], there is a problem. The atomic formulas of predicate logic cannot all be treated as atoms in the modal language. If we regard *Pxy* and *Pyx*, for example, as distinct sentence letters of the modal language then

$$\exists x \, \exists y \, Pxy \, \& \, -\exists \mathbf{x} \, \exists \mathbf{y} \, Pyx \tag{9.1.1}$$

LET p, q, r: p, x, y

(p&(%q&%r))&~(p&(%r&%q)) ;

`FFFF FFFF FFFF FFFF` (9.1.2)

will be satisfiable.

Remark 9.1.2: Eq. 9.1.2 is *not* tautologous and a contradiction.

If we regard them as identical sentence letters then [this] will be unsatisfiable."

$\exists x \exists y\ (Pxy\ \&\ -Pyx)$ (9.2.1)

((p&(%q&%r))&~(p&(%r&%q))) = (p=p) ;
FFFF FFFF FFFF FFFF (9.2.2)

> **Remark 9.2.2:** Eq. 9.2.2 is *not* tautologous, is a contradiction, and is identical to Eq. 9.1.2.

Because Eqs. 9.1.2 and 9.2.2 are identical as contradictions, so that rendition of the satisfiability for quantifiers to modal operators is contradictory. For Meth8/VŁ4 to show that the contradictions are equivalent implies Meth8/VŁ4 is consistent in finding those definitions as equivalent.

What follows is that there is no reason to rely on

> "the variable-free formulations of logic by Tarski, Bernays, Halmos, Nolin and Quine ... [for] the effect of arbitrary permutations and identifications of the variables occurring in a formula."

We further show that Eq. 9.1.1 (or 9.2.1) is *not* a fragment contained within the universally quantified variables of p&(#q&#r): (9.3.1)

((p&(%q&%r))&~(p&(%r&%q)))<(#q&#r) ;
FFFF FFFF FFFF FFFF (9.3.2)

10. Meth8/VŁ4 implementation

The Meth8 script uses literals and connectives in one-character. Propositions are p-z, and theorems are A-B. The connectives for {and, or, imply, equivalent} are {&, +, >, =}. The negated connectives for {nand; nor; not imply; exclusive-or} are {\, -, <, @}. The operators for {not; possibility ◇ ∃ ; necessity □ ∀ } are {~, %, #}. Expressions are adopted for clarity as: (p=p) for tautologous; (p@p) for contradiction; and (x<y) for x∈y. The expression x≤y as "x less than or equal to y" is rendered in the negative as ~(y<x) or as (~x>~y). Variables are defined as:

Definition	Axiom	Symbol	Name	Meaning	Binary	Decimal
1	p=p	T	tautology	proof	11	3
2	p@p	F	contradiction	absurdum	00	0
3	%p>#p	N	non-contingency	truthity	01	1
4	%p<#p	C	contingency	falsity	10	2

Note the meaning of (%p>#p): a possibility of p implies the necessity of p; and some p implies all p. In other words, if a possibility of p then the necessity of p; and if some p then all p.

This shows equivalence of respective modal operators and quantified operators as in Section 9 above.

Meth8 contains recent advances in parsing technology named sliding window. It is written in 7,100 lines of industrial grade code in True BASIC, the educator's language, and ANSI standard. The novel installation wrapper is for one user per seat per CPU, and licensed by number of logical LUT accesses at run time. The is no internet access, and no asymmetric key encryption. Hence Meth8 is ITAR compliant and exportable.

Meth8 use variables for 4 propositions, 4 theorems, and 11 propositions. The size of truth tables are respectively for 16-, 256-, and 2048- truth values, using recent advances in look up table indexing. In RAM look up tables (LUTs) are for 4 theorems (16 result tables), 4 propositional variables (1 result table), 11 propositional variables (128 result tables). Larger numbers of variables scale via LUTs on external media.

11. Notable refutations

We evaluate 575 artifacts in 3060 assertions to confirm 537 as tautology and 2523 as *not* (82.45%). We use Meth8, a modal logic checker in five models. The mapping of formulas in Meth8 script was performed by hand, checked, and tested for accuracy of intent.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⇐⇒, ↔, ≜, ≈; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) `C` as contingency, Δ, ordinal 1; (%z>#z) `N` as non-contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).

Note: For clarity we usually distribute quantifiers on each variable as designated.

Seven refutations are discovered as non-tautologous fragments of VŁ4.

11.1. Refutation of Bell's inequality

From: Maccone, L. (2013). "A simple proof of Bell’s inequality". arxiv.org/pdf/1212.5214.pdf

The summation of the respective probabilities for q equivalent to r, r equivalent to s, and q equivalent to s is equal to or greater than one, and hence is equivalent to a theorem. (11.1.1.1)

~((((p&q)=(p&r)) + (((p&r)=(p&s)) + ((p&q)=(p&s)))) < (%p>#p))= (p=p) ;
`NNNN NNNN NNNN NNNN` 11.1.1.2)

Remark 11.1.1.1: For further qualification to strengthen Eq. 11.1.1.1, we rewrite it as:

If the respective probabilities for q, r, s are equivalent to and equal to one, then the summation of the respective probabilities for q equivalent to r, r equivalent to s, and q equivalent to s is equal to or greater than one. (11.1.2.1)

(((p&q)=((p&r)=(p&s)))=(%p>#p)) > ~((((p&q)=(p&r)) + (((p&r)=(p&s)) +
((p&q)=(p&s)))) < (%p>#p)) ; `NNNT TTNN TTNN NNTT` 11.1.2.2)

Eqs. 11.1.1.2 and 11.1.2.2 as rendered are *not* tautologous. Hence, Bell's inequality as Eqs. 11.1.1.1 or 11.1.2.1 is refuted.

11.2. Refutation of the Gödel-Löb axiom

This example replicates the proof for provability logic of the Gödel-Löb axiom GL as

$$\Box(\Box p \to p) \to \Box p. \qquad (11.2.1.1)$$

If p is "*choice*", this transcribes in words to:
"The necessity of *choice*, as always implying *a choice*, implies always *a choice*." (11.2.1.0)

#(#p>p)>#p ; `CTCT CTCT CTCT CTCT` (11.2.1.2)

To coerce the GL axiom to be a tautology, the expression is rewritten as

$\Box(\Box p \to p) \leftrightarrow (p \lor \neg p)$, `TTTT TTTT TTTT TTTT` (11.2.2.1)

in words: "The necessity of *choice*, as always implying *a choice*, is equivalent to always *a choice* or *no choice*." (11.2.2.0)

A simpler rendition of a tautologous GL-type axiom is either

$\Box(\Box\neg p \to p) \leftrightarrow \Box p$, or (11.2.3.1)

$\Box(\Box p \to \neg p) \leftrightarrow \Box\neg p$ (11.2.4.1)

as respectively in words: "The necessity of *no choice*, as always implying *a choice*, is equivalent to always *a choice*."; or (11.2.3.0)

"The necessity of *choice*, as always implying *no choice*, is equivalent to always *no choice*." (11.2.4.0)

Remark 11.2: If GL fails, then so also does Zermelo-Fraenkel set theory and the axiom of choice (ZFC) as the basis of modern mathematics.

11.3. Refutation of the Löb theorem and Gödel incompleteness by substitution of contradiction

From: Gross, J. et al. (2016). Löb’s Theorem. jasongross.github.io/lob-paper/nightly/lob.pdf
jgross@mit.edu, jack@gallabytes.com, benya@intelligence.org

This, in a nutshell, is Löb’s theorem: to prove *X*, it suffices to prove that *X* is true whenever *X* is provable. If we let $\Box X$ denote the assertion “*X* is provable,” then, symbolically, Löb’s theorem becomes: $\Box(\Box X \to X) \to \Box X$. (11.3.1.1)

LET p, q: *X*.

#(#p>p)>#p ; `CTCT CTCT CTCT` (11.3.1.2)

Remark 11.3.1.2: Eq 11.3.1.2 as rendered is *not* tautologous, thus refuting Löb's theorem.

Note that Gödel's incompleteness theorem follows trivially from Löb's theorem: by instantiating *X* with a contradiction [⊥], we can see that it's impossible for provability to imply truth for propositions which are not already true. (11.3.2.1)

#(#(p@p)>(p@p))>#(p@p) ; CCCC CCCC CCCC CCCC (11.3.2.2)

Remark 11.3.2.2: Eq. 11.3.2.2, rendered as Eq. 11.3.1.2 with p substituted by (p@p), is *not* tautologous but consistently falsity as C for contingency. Hence Gödel's incompleteness theorem, as following trivially, is also refuted.

This means that the type of Löb's theorem becomes either □(□X→X)→□X [Eq. 11.3.1.1], which is not strictly positive, or □(X→X)→□X, (11.3.3.1)

#(p>p)>#p ; CTCT CTCT CTCT CTCT (11.3.3.2)

which, on interpretation, must be filled with a general fixpoint operator. Such an operator is well-known to be inconsistent.

Remark on Fn. 2: Eq. 11.3.3.2 as rendered produces the same truth table result as Eq. 11.3.1.2 and as another trivial refutation.

11.4. Refutation of the Löwenheim–Skolem theorem

From: en.wikipedia.org/wiki/Löwenheim–Skolem_theorem

In its general form, the Löwenheim–Skolem theorem states that for every signature σ, every infinite σ-structure *M,* and every infinite cardinal number κ ≥ |σ|, (11.4.1.1)

LET p, q, r, s: κ, M, N, σ ; (p@p) 0, zero;
(s>(p@p)) |σ|; (q>(p@p)) |*M*|; (r>(p@p)) |*N*|; ~(p<q) (p≥q).

#(s&((s&q)&(~(p<(s>(p@p)))))) ; **FFFF FFFF FF**N**F FF**N**F** (11.4.1.2)

there is a σ-structure *N* (11.4.2.1)

%(s&r) ; CCCC CCCC CCCC TTTT (11.4.2.2)

such that |*N*| = κ and
if κ < |*M*| then *N* is an elementary substructure of *M*; [and/or]
if κ > |*M*| then *N* is an elementary extension of *M*. (11.4.3.1)

(((r>(p@p))=p)&(((p<(q>(p@p)))>(q<r)) [&,+] ((p>(q>(p@p)))>(q>r)))) ;
FT**F**T T**F**T**F** **F**T**F**T T**F**T**F** (11.4.3.2)

Eq. 11.4.1.1 implies 11.4.2.1. (11.4.4.1)

#(s&((s&q)&~(p<(s>(p@p)))))>%(s&r) ;
TTTT TTTT TTCT TTTT (11.4.4.2)

Eq. (11.4.4.1 = 11.4.1.1 implies 11.4.2.1) implies 11.4.3.1. (11.4.5.1)

(#(s&((s&q)&~(p<(s>(p@p)))))>%(s&r)) >
(((r>(p@p))=p)&(((p<(q>(p@p)))>(q<r))+((p>(q>(p@p)))>(q>r)))) ;
FT**F**T T**F**T**F** **F**T**F**T T**F**T**F** (11.4.5.2)

Eq. 11.4.1.2 as rendered is *not* tautologous, and not contradictory. Eq. 11.4.11.4.4.1 is *not* tautologous due to one C falsity value. Eq. 11.4.4.2 is *not* tautologous, and the same result table as Eq. 11.4.5.2. This means the Löwenheim–Skolem theorem is refuted.

11.5. Refutation of Peirce's abduction and induction, and confirmation of deduction

From: iep.utm.edu/peir-log/

C.S. Peirce originally defined the three forms of inference in logic as:

Abduction: (Q is S) and (Q is P) imply (S is P) (11.5.1.1.1)

LET p, q, s: P, Q, S.

((q=s)&(q=p))>(s=p) ; TTTT TTTT TTTT TTTT 11.5.1.1.2)

Induction: (S is Q) and (P is Q) imply (S is P) (11.5.2.1.1)

((s=q)&(p=q))>(s=p) ; TTTT TTTT TTTT TTTT (11.5.2.1.2)

Deduction: (S is Q) and (Q is P) imply (S is P) (11.5.3.1.1)

((s=q)&(q=p))>(s=p) ; TTTT TTTT TTTT TTTT (11.5.3.1.2)

Peirce described Eqs. 11.5.1 - 11.5.3 as inversions of the same.

Remark 11.5.1.1.1: If the word "is" is taken to mean the word "implies" then the connective = is replaced with the connective > below.

Abduction: (Q implies S) and (Q implies P) imply (S implies P) (11.5.1.2.1)

((q>s)&(q>p))>(s>p) ; TTTT TTTT **F**TTT **F**TTT (11.5.1.2.2)

Induction: (S implies Q) and (P implies Q) imply (S implies P) (11.5.2.2.1)

((s>q)&(p>q))>(s>p) ; TTTT TTTT TT**F**T TT**F**T (11.5.2.2.2)

Deduction: (S implies Q) and (Q implies P) imply (S implies P) (11.5.3.2.1)

((s>q)&(q>p))>(s>p) ; TTTT TTTT TTTT TTTT (11.5.3.2.2)

Eqs. 11.5.1.2.2 - 11.5.2.2.2 as rendered for abduction and induction are *not* tautologous, but Eq. 11.5.3.2.2 is tautologous. This means that abduction and induction are not inversions of deduction, leaving deduction as the only form of tautologous inference in logic.

11.6. Erwin Schrödinger's cat thought-experiment

From: en.wikipedia.org/wiki/Schrödinger's_cat

If the monitor is tautologous, that is not activated, along with the box, cat, and poison apparatus in place, then there is no death. (11.6.H_0.1)

LET p, q, r, s t: box, cat, poison, monitor, death

((s=s)&((p&q)&r)) > ~t ; **FF**N**F** **FF**N**F** **FF**N**F** **FF**N**F** (11.6.H_0.2)

If the monitor is contradictory, that is activated, along with the box, cat, and poison apparatus in place, then there is death. (11.6.H_1.1)

((s@s)&((p&q)&r)) > t ; TTTT TTTT TTTT TTTT (11.6.H_1.2)

Hence when opening the box at any time, the cat is either still alive or dead, but not "entangled" as both dead and alive (a contradiction). Therefore the experiment is *not* a paradox from Eq. 11.6.H_1.2 but a contradiction.

11.7. Refutation of the ZF axiom of the empty set

From: en.wikipedia.org/wiki/Axiom_of_empty_set

In the formal language of the Zermelo–Fraenkel axioms, the axiom reads ... in words:

There is a set such that no element is a member of it: $\exists x \forall y \neg (y \in x)$ (11.7.1.0)

We distribute the quantifiers to the respective variables as:

Not(necessarily y as a member of possibly x). (11.7.1.1)

(#q>%p) = (p=p) ; TTCT TTCT TTCT TTCT (11.7.1.2)

From: plato.stanford.edu/entries/set-theory/ZF.html by Joan Bagaria (joan.bagaria@icrea.cat)

The null set, equivalent to the empty set, is defined as: $\exists x \neg \exists y (y \in x)$ (11.7.2.0)

We distribute the quantifiers to the respective variables as:
Not(possibly y as a member of possibly x). (11.7.2.1)

(%q>%p) = (p=p) ; TTCT TTCT TTCT TTCT (11.7.2.2)

Eqs. 11.7.1.2 and 11.7.2.2, with the same truth table result, are *not* tautologous. This refutes the ZF axiom of the empty set.

12. Conclusion

This paper:

1. Introduces the bivalent logic B4;
2. Adopts a four-valued system based on the 2-tuple in two models M1 and M2;
3. Derives modal values in Ł4;
4. Answers an objection by trivial proof;
5. Corrects the Square of Opposition with modal equations for lines and angles;
6. Confirms the 24-syllogisms by modifying two;
7. Shows respective quantified and modal operators are equivalent;
8. Describes the Meth8 software implementation of VŁ4;
9. Tests 2200 assertions for a refutation rate of 80%;
10. Provides seven worked examples of refutation;
11. Classifies refutations as non tautologous fragments of VŁ4; and
12. Concludes that VŁ4 is a universal logic.

13. Future research

Continued testing of artifacts burgeons the table of contents of results, with details usually as one or two paged papers. The mapping of sentences into script for Meth8/VŁ4 could be automated for repetitive testing, however there is no substitute for hand-coding as best-by-test for catching most errors of symbolic assignments. The parsing component of Meth8 is mature enough to rapidly detect incorrect grammatical for the input script. For Meth8 an immediate further application is mapping sentences of natural language into logical formulas, so a semi-automation of that linguistic process is proceeding.

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Validation of Veblen's Axiom for Euclidean geometry

From en.wikipedia.org/wiki/List_of_first-order_theories# , Veblen's axiom is published as:

if *ab* and *cd* lie on intersecting lines, then so do *ac* and *bd*. (1)
as $\forall a \forall b \forall c \forall d \forall e \forall G \forall H aH \wedge bH \wedge eH \wedge cG \wedge dG \wedge eG \rightarrow \exists\ f \exists I \exists J aI \wedge cI \wedge fI \wedge bJ \wedge dJ \wedge fJ$ (2)

LET p,q, r,s, t,u,v,w, x,y = a,b,c,d,e,f,G,H,I,J; #, % as universal, existential quantifier; > as Imply.
This maps (2) into Meth8 script of (3):

```
((#p&(#q&(#r&(#s&(#t&(#v&#w))))))&((p&w)&((q&w)&((t&w)&((r&v)&((s&v)&(t&v)))))))
>
((%u&(%x&%y))&((p&x)&((r&x)&((u&x)&((q&y)&((s&y)&(u&y))))))) ; not Validated
```
(3)

These 16 rows are repeated 128 times in the truth tables Note that the last truth value is not designated as T or E.

```
Model 1              Model 2.1            Model 2.2            Model 2.3.1          Model 2.3.2
TTTT TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTC* EEEE EEEE EEEE EEEU* EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEP* EEEE EEEE EEEE EEEI*
TTTT TTTT TTTT TTTT  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE
TTTT TTTT TTTT TTTC* EEEE EEEE EEEE EEEU* EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEP* EEEE EEEE EEEE EEEI*
. . . . . . . . . ^  . . . . . . . . . ^                       . . . . . . . . . ^  . . . . . . . . . ^
Model 1              Model 2.1            Model 2.2            Model 2.3.1          Model 2.3.2
```

Inspection of this schema shows a case which does not hold when the points labeled *ac* and *bd* fall on parallel lines. To accommodate this anomaly, the following code is included on the main antecedent to produce:

"and *ac* plus a constant *z* is not equivalent to *bd*"(3) to mean " (4)

```
(((p&r)+z)@(q&s))
```
(5)

```
(((#p&(#q&(#r&(#s&(#t&(#v&#w))))))&((p&w)&((q&w)&((t&w)&((r&v)&((s&v)&(t&v)))))))
&
(((p&r)+z)@(q&s))) >
((%u&(%x&%y))&((p&x)&((r&x)&((u&x)&((q&y)&((s&y)&(u&y))))))); tautologous
```
(6)

This is an example of correcting a logical expression to its intended meaning from the result as rendered by a Meth8 script.

Confirmation of a trivial vector conjecture

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r: point p, point q, point s;
~ Not; & And; + Or; - Not Or; > Imply, greater than; = Equivalent;
# necessity, for all or every; % possibility, for one or some;
((p-q)=(q-p)) The absolute value of the distance p to q is equivalent to that of q to p;
((p-r)=(r-p)) The absolute value of the distance p to r is equivalent to that of r to p;
((q-r)=(r-q)) The absolute value of the distance q to r is equivalent to that of r to q.

"From the distance between two points, a third point always has the same distance to the other points." (1.0)

We rewrite Eq. 1.0 as:

"If two points imply the necessity of a third point, then the respective distances are possibly the same." (1.1)

((p&q)>#r)>%(((p-q)=(q-p))=(((p-r)=(r-p))=((q-r)=(r-q)))) ;
TTTT TTTT TTTT TTTT (1.2)

Eq. 1.2 is tautologous, hence confirming the conjecture.

Remark: This exercise indirectly speaks to the fact that the vector space is *not* bivalent.

Bivalent correction of IEEE Std 1800-2017 (Verilog) and Std 1164-1993 (VHDL)

Abstract: We evaluate the multivalued logic SystemVerilog in IEEE Std 1800-2017. The classical logic proof tables for the connectives And, Or, Xor, and negations are based on the bivalency of 1, 0, X, Z as 0=~1 and Z=~X. This refutes and corrects the standard. We also retrofit and correct IEEE Std 1164-1993 (SynopsysVHDL) for the same.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET: ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⊢; < Not Imply, less than, ∈;
= Equivalent, ≡, ⊨; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊; # necessity, for every or all, ∀, □;
~(y < x) (x ≤ y), (x ⊆ y); (p=p) `T`autology.

See: ieeexplore.ieee.org/document/1560791;

Table 28-3—Truth tables for multiple input logic gates

(28.3.1)

and	**0**	**1**	**x**	**z**
0	0	0	0	0
1	0	1	x	x
x	0	x	x	x
z	0	x	x	x

or	**0**	**1**	**x**	**z**
0	0	1	x	x
1	1	1	1	1
x	x	1	x	x
z	x	1	x	x

xor	**0**	**1**	**x**	**z**
0	0	1	x	x
1	1	0	x	x
x	x	x	x	x
z	x	x	x	x

(28.3.2)

and	**0**	**1**	**x**	**z**
0	0	0	0	0
1	0	1	x	z
x	0	x	x	0
z	0	z	0	z

or	**0**	**1**	**x**	**z**
0	0	1	x	z
1	1	1	1	1
x	x	1	x	1
z	z	1	1	z

xor	**0**	**1**	**x**	**z**
0	0	1	x	z
1	1	0	z	x
x	x	z	0	1
z	z	x	1	0

See: perso.telecom-paristech.fr/guilley/ENS/20161206/TP/tp_syn/doc/IEEE_VHDL_1164-1993.pdf

`VHDL MODEL INTEROPERABILITY (Std_logic_1164_1993)` (1164-1993.1)

```
-- truth table for "not" function
CONSTANT not_table: stdlogic_1d :=
-- -------------------------------------------------
-- U    X    0    1    Z    W    L    H    -
-- -------------------------------------------------
( 'U', 'X', '1', '0', 'X', 'X', '1', '0', 'X' );

-- truth table for "and" function
CONSTANT and_table : stdlogic_table := (
```

```
-- ---------------------------------------------------
-- U    X    0    1    Z    W    L    H    -
-- ---------------------------------------------------
( 'U', 'U', '0', 'U', 'U', 'U', '0', 'U', 'U' ), -- | U |
( 'U', 'X', '0', 'X', 'X', 'X', '0', 'X', 'X' ), -- | X |
( '0', '0', '0', '0', '0', '0', '0', '0', '0' ), -- | 0 |
( 'U', 'X', '0', '1', 'X', 'X', '0', '1', 'X' ), -- | 1 |
( 'U', 'X', '0', 'X', 'X', 'X', '0', 'X', 'X' ), -- | Z |
( 'U', 'X', '0', 'X', 'X', 'X', '0', 'X', 'X' ), -- | W |
( '0', '0', '0', '0', '0', '0', '0', '0', '0' ), -- | L |
( 'U', 'X', '0', '1', 'X', 'X', '0', '1', 'X' ), -- | H |
( 'U', 'X', '0', 'X', 'X', 'X', '0', 'X', 'X' )  -- | - |);

-- truth table for "or" function
CONSTANT or_table : stdlogic_table := (
-- ---------------------------------------------------
-- U    X    0    1    Z    W    L    H    -
-- ---------------------------------------------------
( 'U', 'U', 'U', '1', 'U', 'U', 'U', '1', 'U' ), -- | U |
( 'U', 'X', 'X', '1', 'X', 'X', 'X', '1', 'X' ), -- | X |
( 'U', 'X', '0', '1', 'X', 'X', '0', '1', 'X' ), -- | 0 |
( '1', '1', '1', '1', '1', '1', '1', '1', '1' ), -- | 1 |
( 'U', 'X', 'X', '1', 'X', 'X', 'X', '1', 'X' ), -- | Z |
( 'U', 'X', 'X', '1', 'X', 'X', 'X', '1', 'X' ), -- | W |
( 'U', 'X', '0', '1', 'X', 'X', '0', '1', 'X' ), -- | L |
( '1', '1', '1', '1', '1', '1', '1', '1', '1' ), -- | H |
( 'U', 'X', 'X', '1', 'X', 'X', 'X', '1', 'X' )  -- | - |);

-- truth table for "xor" function
CONSTANT xor_table : stdlogic_table := (
-- ---------------------------------------------------
-- U    X    0    1    Z    W    L    H    -
-- ---------------------------------------------------
( 'U', 'U', 'U', 'U', 'U', 'U', 'U', 'U', 'U' ), -- | U |
( 'U', 'X', 'X', 'X', 'X', 'X', 'X', 'X', 'X' ), -- | X |
( 'U', 'X', '0', '1', 'X', 'X', '0', '1', 'X' ), -- | 0 |
( 'U', 'X', '1', '0', 'X', 'X', '1', '0', 'X' ), -- | 1 |
( 'U', 'X', 'X', 'X', 'X', 'X', 'X', 'X', 'X' ), -- | Z |
( 'U', 'X', 'X', 'X', 'X', 'X', 'X', 'X', 'X' ), -- | W |
( 'U', 'X', '0', '1', 'X', 'X', '0', '1', 'X' ), -- | L |
( 'U', 'X', '1', '0', 'X', 'X', '1', '0', 'X' ), -- | H |
( 'U', 'X', 'X', 'X', 'X', 'X', 'X', 'X', 'X' )  -- | - |);
```

Remark 1164-1993.2: Areas amended are shaded in gray with modifications underlined.

(1164-1993.2)

```
-- truth table for "not" function
CONSTANT not_table: stdlogic_1d :=
-- -------------------------------------------------
--   U    X    0    1    Z    W    L    H    -
-- -------------------------------------------------
```

```
(   'U', 'X', '1', '0', 'X', 'X', '1', '0', 'X' ); =      not[...]
(   'U', 'X', '1', '0', 'X', 'X', '0', '1', 'X' ); = Not( not[...])

-- truth table for "and" function
CONSTANT and_table : stdlogic_table := (
-- ------------------------------------------------------
-- U    X    0    1    Z    W    L    H    -
-- ------------------------------------------------------
( 'U', 'U', '0', 'U', 'U', 'U', '0', 'U', 'U' ), -- | U |
( 'U', 'X', '0', '0', 'X', 'X', '0', 'X', 'X' ), -- | X |
( '0', '0', '0', '0', '0', '0', '0', '0', '0' ), -- | 0 |
( 'U', 'X', '0', 'Z', 'X', 'X', '0', '1', 'X' ), -- | 1 |
( 'U', '0', '0', 'Z', 'Z', 'X', '0', 'X', 'X' ), -- | Z |
( 'U', 'X', '0', 'X', 'X', 'X', '0', 'X', 'X' ), -- | W |
( '0', '0', '0', '0', '0', '0', '0', '0', '0' ), -- | L |
( 'U', 'X', '0', '1', 'X', 'X', '0', '1', 'X' ), -- | H |
( 'U', 'X', '0', 'X', 'X', 'X', '0', 'X', 'X' )  -- | - |);

-- truth table for "or" function
CONSTANT or_table : stdlogic_table := (
-- ------------------------------------------------------
-- U    X    0    1    Z    W    L    H    -
- --- ------------------------------------------------
( 'U', 'U', 'U', '1', 'U', 'U', 'U', '1', 'U' ), -- | U |
( 'U', 'X', 'X', '1', '1', 'X', 'X', '1', 'X' ), -- | X |
( 'U', 'X', '0', '1', 'Z', 'X', '0', '1', 'X' ), -- | 0 |
( '1', '1', '1', '1', '1', '1', '1', '1', '1' ), -- | 1 |
( 'U', '1', 'Z', '1', 'Z', 'X', 'X', '1', 'X' ), -- | Z |
( 'U', 'X', 'X', '1', 'X', 'X', 'X', '1', 'X' ), -- | W |
( 'U', 'X', '0', '1', 'X', 'X', '0', '1', 'X' ), -- | L |
( '1', '1', '1', '1', '1', '1', '1', '1', '1' ), -- | H |
( 'U', 'X', 'X', '1', 'X', 'X', 'X', '1', 'X' )  -- | - |);

-- truth table for "xor" function
CONSTANT xor_table : stdlogic_table := (
-- ------------------------------------------------------
-- U    X    0    1    Z    W    L    H    -
-- ------------------------------------------------------
( 'U', 'U', 'U', 'U', 'U', 'U', 'U', 'U', 'U' ), -- | U |
( 'U', '0', 'X', 'Z', '1', 'X', 'X', 'X', 'X' ), -- | X |
( 'U', 'X', '0', '1', 'Z', 'X', '0', '1', 'X' ), -- | 0 |
( 'U', 'Z', '1', '0', 'X', 'X', '1', '0', 'X' ), -- | 1 |
( 'U', '1', 'Z', 'X', '0', 'X', 'X', 'X', 'X' ), -- | Z |
( 'U', 'X', 'X', 'X', 'X', 'X', 'X', 'X', 'X' ), -- | W |
( 'U', 'X', '0', '1', 'X', 'X', '0', '1', 'X' ), -- | L |
( 'U', 'X', '1', '0', 'X', 'X', '1', '0', 'X' ), -- | H |
( 'U', 'X', 'X', 'X', 'X', 'X', 'X', 'X', 'X' )  -- | - |);
```

On difficulties with definitions of the Veronoï region

LET: E a plane, x all points, s a generating point, q another generating point

$\| x - s \| \le \| x - q \|$: point x is necessarily nearer generating point s than any other possible generating point q. (1)

We rewrite Eq 1 in a single operator with negation as $\sim(\| x - s \| > \| x - q \|)$. (2)

To avoid absolute value arithmetic, we specify that x, s, q are not less than zero:

~((x+(s+q))<((x+(s+q))-(x+(s+q)))) ; (3)

We rewrite Eq 2 with Eq 3:

(~((x-s)>(x-q))&~((x+(s+q))-(x+(s+q)))) ; (4)

The generating points (s, q) are a part of necessarily all points in a plane (E):

((s&q)<(#x<E)). (5)

A Veronoï region for generating point (s) is then the combination of Eq 5 and 4:

((s&q)<(#x<E)) & (~((x-s)>(x-q))&~((x+(s+q))-(x+(s+q)))) ; (6)

In Meth8 model checker, in Eq 6 we redefine (x, E) as (p, r):

((s&q)<(#p<r)) & (~((p-s)>(p-q))&~((p+(s+q))-(p+(s+q)))) ; nvt; contradictory (7)

We simplify Eq 7 by removing the plane r (as E):

((s&q)<#p) & (~((p-s)>(p-q))&~((p+(s+q))<((%p<%#p)))) ; nvt; contradictory (8)

We further simply Eq 8 by removing the expressions to avoid absolute value arithmetic:

((s&q)<#p) & ~((p-s)>(p-q)) ; nvt; contradictory (9)

We turn to another definition from en.wikipedia.org/wiki/Voronoi_diagram:

"Let X be a metric space with distance function d. Let K be a set of indices and let $(P_k)_{k \in K}$ be a tuple (ordered collection) of nonempty subsets (the sites) in the space X. The Voronoi cell, or Voronoi region, R_k, associated with the site P_k is the set of all points in X whose distance to P_k is not greater than their distance to the other sites P_j, where j is any index different from k. In other words, if $d(x , A) = \inf \{ d(x , a) \mid a \in A \}$ denotes the distance between the point x and the subset A, then

$$R_k = \{ x \in X \mid d(x , P_k) \le d(x , P_j) \text{ for all } j \neq k \} \quad (10)$$

The Voronoi diagram is simply the tuple of cells $(R_k)_{k \in K}$."

We map this into Meth8 assuming a metric space to be all space for this region:

~((x-k)>(x-j)) & ~(#j=#k) (11)

We rewrite Eq 11 by redefining (x, j, k) as (p, q, r):

~((p-r)>(p-q))&~(#q=#r) ; nvt (12)

```
FFNF FFFF FFNF FFFF   UUEU UUUU UUEU UUUU   UUUU UUUU UUUU UUUU   UUIU UUUU UUIU UUUU   UUPU UUUU UUPU UUUU
Model 1               Model 2.1             Model 2.2             Model 2.3.1           Model 2.3.2
```

What concerns us in Eq 10 is the phrase "of nonempty subsets", and later on "subset A", because that assumes such a thing as an empty set, which we do not validate tautologous.

This leads us to believe that Eq 10 is mistaken, which reminds one again that Wikipedia can be a source of misinformation.

Our conclusion is that the Veronoï region is not tautologous.

Confirmation of the Vickrey auction theorem

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal.

> LET p, q, r, s: value, bidder, i, j; & And; + Or; - Not Or; > Imply, greater than; < Not Imply, lesser than; = Equivalent; @ Not Equivalent; (p@p) ordinal zero, 0; (p=p) `T`, *proof*.

From: en.wikipedia.org/wiki/Vickrey_auction

> <u>Proof of dominance of truthful bidding</u>: The dominant strategy in a Vickrey auction with a single, indivisible item is for each bidder to bid their true value of the item.

Let v_i be bidder i's value for the item. Let b_i be bidder i's bid for the item. (10.1)

p&r ; `FFFF FTFT FFFF FTFT` (10.2)

Let v_i be bidder i's value for the item. Let b_i be bidder i's bid for the item. (11.1)

q&r ; `FFFF FFTT FFFF FFTT` (11.2)

The payoff for bidder i is v_i - $\max_{j \neq i} b_j$ if $b_i > \max_{j \neq i} b_j$, or 0 otherwise. (12.0)

We rewrite Eq. 12.0 because it does not take into account the third state of payoff as a negative amount, also amounting as a net no payoff:

The payoff for bidder i is v_i - $\max_{j \neq i} b_j$ if $b_i > \max_{j \neq i} b_j$, or [≤] 0 otherwise. (12.1)

payoff for (q&r): ((((q&r)>((r@s)&(q&s)))>((p&r)-((r@s)>(q&s))))>(p@p))
+~((~((q&r)>((r@s)&(q&s)))>((p&r)-((r@s)>(q&s))))>(p@p)) ;
`TTTT TTTF TTTT TTFF` (12.2)

The strategy of overbidding is dominated by bidding truthfully.

Assume that bidder i bids $b_i > v_i$. (20.1)

(q&r)>(p&r) ; `TTTT TTFT TTTT TTFT` (20.2)

If $\max_{j \neq i} b_j < v_i$ then the bidder would win the item with a truthful bid as well as an overbid. (21.1)

(((q&r)>(p&r))&(((r@s)>(q&s))<(p&r))) > (((((q&r)>((r@s)&(q&s)))>((p&r)-
((r@s)>(q&s))))>(p@p))+~((~((q&r)>((r@s)&(q&s)))>((p&r)-((r@s)>(q&s))))
>(p@p))) ; `TTTT TTTT TTTT TTTT` (21.2)

The bid's amount does not change the payoff so the two strategies have equal payoffs in this case.

If $\max_{j \neq i} b_j > b_i$ then the bidder would lose the item either way so the strategies have equal payoffs in this case. (22.1)

(((q&r)>(p&r))>(((r@s)&(q&s))>(q&r))) > (((((q&r)>((r@s)&(q&s)))>((p&r)-((r@s)>(q&s))))>(p@p))+~((~((q&r)>((r@s)&(q&s)))>((p&r)-((r@s)>(q&s))))>(p@p))) ; TTTT TTTF TTTT TTFF (22.2)

If $v_i < \max_{j \neq i} b_j < b_i$ then only the strategy of overbidding would win the auction. (23.1)

(((q&r)>(p&r))>((p&r)<(((r@s)&(q&s))<(q&r)))) > (((((q&r)>((r@s)&(q&s)))>((p&r)-((r@s)>(q&s))))>(p@p))+~((~((q&r)>((r@s)&(q&s)))>((p&r)-((r@s)>(q&s))))>(p@p))) ; TTTT TTTF TTTT TTFF (23.2)

The payoff would be negative for the strategy of overbidding because they paid more than their value of the item, while the payoff for a truthful bid would be zero. Thus the strategy of bidding higher than one's true valuation is dominated by the strategy of truthfully bidding. The strategy of underbidding is dominated by bidding truthfully.

Assume that bidder i bids $b_i < v_i$. (30.1)

(q&r)<(p&r) ; FFFF FFTF FFFF FFTF (30.2)

If $\max_{j \neq i} b_j > v_i$ then the bidder would lose the item with a truthful bid as well as an underbid, so the strategies have equal payoffs for this case. (31.1)

(((q&r)<(p&r))>(((r@s)>(q&s))>(p&r))) > (((((q&r)>((r@s)&(q&s)))>((p&r)-((r@s)>(q&s))))>(p@p))+~((~((q&r)>((r@s)&(q&s)))>((p&r)-((r@s)>(q&s))))>(p@p))) ; TTTT TTTF TTTT TTTF (31.2)

If $\max_{j \neq i} b_j < b_i$ then then the bidder would win the item either way so the strategies have equal payoffs in this case. (32.1)

(((q&r)<(p&r))>(((r@s)>(q&s))<(q&r))) > (((((q&r)>((r@s)&(q&s)))>((p&r)-((r@s)>(q&s))))>(p@p))+~((~((q&r)>((r@s)&(q&s)))>((p&r)-((r@s)>(q&s))))>(p@p))) ; TTTT TTTF TTTT TTTF (32.2)

If $b_i < \max_{j \neq i} b_j < v_i$ then only the strategy of truthfully bidding would win the auction. (33.1)

(((q&r)>(p&r))>((q&r)<(((r@s)&(q&s))<(p&r)))) > (((((q&r)>((r@s)&(q&s)))>((p&r)-((r@s)>(q&s))))>(p@p))+~((~((q&r)>((r@s)&(q&s)))>((p&r)-((r@s)>(q&s))))>(p@p))) ; TTTT TTTF TTTT TTFF (33.2)

The payoff for the truthful strategy would be positive as they paid less than their value of the item, while the payoff for an underbid bid would be zero. Thus the strategy of underbidding is dominated by the strategy of truthfully bidding. Truthful bidding
dominates the other possible strategies (underbidding and overbidding) so it is an

optimal strategy. (40.0)

We write as: Eqs. (((21.1 and 22.1) and 23.1) or ((31.1 and 32.1) and 33.1)) imply 12.1. (40.1)

(((((((q&r)>(p&r))&(((r@s)>(q&s))<(p&r))) > (((((q&r)>((r@s)&(q&s)))>
((p&r)-((r@s)>(q&s))))>(p@p))+~((~((q&r)>((r@s)&(q&s)))>((p&r)-
((r@s)>(q&s))))>(p@p))))&((((q&r)>(p&r))>(((r@s)&(q&s))>(q&r))) >
(((((q&r)>((r@s)&(q&s)))>((p&r)-((r@s)>(q&s))))>(p@p))
+~((~((q&r)>((r@s)&(q&s)))>((p&r)-
((r@s)>(q&s))))>(p@p)))))&((((q&r)>(p&r))>((p&r)<(((r@s)&(q&s))<
(q&r)))) > (((((q&r)>((r@s)&(q&s)))>((p&r)-((r@s)>(q&s))))>(p@p))
+~((~((q&r)>((r@s)&(q&s)))>((p&r)-((r@s)>(q&s))))>(p@p)))))
+
((((((q&r)<(p&r))>(((r@s)>(q&s))>(p&r))) > (((((q&r)>((r@s)&(q&s)))>
((p&r)-((r@s)>(q&s))))>(p@p))+~((~((q&r)>((r@s)&(q&s)))>((p&r)-
((r@s)>(q&s))))>(p@p))))&((((q&r)<(p&r))>(((r@s)>(q&s))<(q&r))) >
(((((q&r)>((r@s)&(q&s)))>((p&r)-((r@s)>(q&s))))>(p@p))
+~((~((q&r)>((r@s)&(q&s)))>((p&r)-
((r@s)>(q&s))))>(p@p)))))&((((q&r)>(p&r))>((q&r)<(((r@s)&(q&s))<
(p&r)))) > (((((q&r)>((r@s)&(q&s)))>((p&r)-((r@s)>(q&s))))>(p@p))
+~((~((q&r)>((r@s)&(q&s)))>((p&r)-((r@s)>(q&s))))>(p@p))))))
>
(((((q&r)>((r@s)&(q&s)))>((p&r)-((r@s)>(q&s))))>(p@p))
+~((~((q&r)>((r@s)&(q&s)))>((p&r)-((r@s)>(q&s))))>(p@p))) ;
TTTT TTTT TTTT TTTT (40.2)

Eq. 40.2 as rendered is tautologous. This means the Vickrey auction theorem is confirmed.

Remark: Processing Eq. 40.2 required 519 steps.

Refutation of modal forms on Vietoris space

Abstract: We use modal logic to evaluate the defined modal forms on Vietoris space to find them *not* tautologous.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

LET p, q, r, s: C, U, V, X;
~ Not; & And, ∩; > Imply; < Not Imply, less than, ∈ ;
= Equivalent; @ Not Equivalent, ≠;
% possibility, for one or some, ◊; # necessity, for all or every, □;
(q@q) null, Ø ; ≤ ⊆; ~(y < x) (x ≤ y); (y > x) (x | y).

From: Borlido, C.; Gehrke, M. (2018). A note on powers of Boolean spaces with internal semigroups. arxiv.org/pdf/1811.12339.pdf cborlido@unice.fr

The power construction on compact Hausdorff spaces is the so-called Vietoris space. Vietoris is a covariant endofunctor on compact Hausdorff spaces which restricts to the category of Boolean spaces. At the level of objects it assigns to a space X the set of all its closed subsets, denoted V(X) and called the Vietoris space of X, equipped with the topology generated by the sets of the form [where U ⊆ X ranges over all open subsets of X]

◊U = {C ∈ V(X) | C ∩ U ≠ Ø} and (3.1.1)

~(s<q)>(%q=(((p&q)@(q@q))>(p<(r&s)))) ;
CCTT CCTT TTTT TTT**F** (3.1.2)

□U = {C ∈ V(X) | C ⊆ U}, (3.2.1)

~(s<q)>(#q=(~(q<p)>(p<(r&s)))) ; T**F**NN T**F**NN T**F**NN TTNC (3.2.2)

Eqs. 3.1.2 and 3.2.2 as rendered are *not* tautologous. This means the defined modal forms on Vietoris space are refuted.

Refutation of a fast algorithm for network forecasting time series by visibility graph definition

Abstract: We evaluate a definition of the visibility graph as *not* tautologous to deny a fast algorithm for forecasting time series. Hence the conjecture of a forecasting algorithm is denied. This forms a *non* tautologous fragment of the universal logic VŁ4. However, we resuscitate the conjecture using the Kanban cell neuron network (KCNN), a linear step-wise function, for the desired conjecture without injected data.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Liu, F.; Deng, Y. (2019). A fast algorithm for network forecasting time series. vixra.org/pdf/1905.0080v1.pdf liufanuestc@gmail.com, dengentropy@uestc.edu.cn

Definition 2 Connectivity in time series is defined as follows [27].

Remark Def. 2: The definition as the basis of the titled conjecture for a fast algorithm is derived from the footnote source below.

[27] Lacasa, L. et al. (2008). From time series to complex networks: the visibility graph. Proceedings of the National Academy of Sciences of the United States of America. 105. 13:4972–5. pnas.org/content/pnas/105/13/4972.full.pdf lucas@dmae.upm.es

More formally, we can establish the following visibility criteria: two arbitrary data values (t_a, y_a) and (t_b, y_b) will have visibility, and consequently will become two connected nodes of the associated graph, if any other data (t_c, y_c) placed between them fulfills: $y_c < y_b + (y_a - y_b)((t_b - t_c)/(t_b - t_a))$. (1.1)

LET p, q, r, t, y: *a* , *b* , *c* , *t* , *y* .

((((t&p)&(y&p))<((t&r)&(y&r)))<((t&q)&(y&q))) >
((t&r)<((t&q)+(((y&p)-(y&q))&(((t&q)-(t&r))\((t&q)-(t&p)))))) ;
TTTT TTTT TTTT TTTT(1),
T**F**TT TTTT T**F**TT TTTT(1) (1.2)

Eq. 1.2 as rendered is *not* tautologous. This means the original definition in Eq. 1.1, from which Definition 2 is derived, is refuted. Hence the conjecture of a forecasting algorithm is denied. However, we resuscitate Eq. 1.2 using the Kanban cell neuron network (KCNN), a linear step-wise function, for the desired conjecture without resorting to any other injected data between extrema: (((t&p)&(y&p))<((t&q)&(y&q)))> ((y&q)+(((y&p)-(y&q))\((t&q)-(t&p)))). TTTT TTTT TTTT TTTT (1.3)

Confirmation of VŁ4 as complete

Abstract: Logic VŁ4 is defined as a bivalent classical logic that maps quantifiers to modalities as a tautology making VŁ4 complete. Paraconsistent, non bivalent, vector logics are defined as *non* tautologous fragments of VŁ4 as a universal logic.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , ·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

Logic VŁ4, for variant MŁ$_4$, is a bivalent classical logic that maps quantifiers to modalities: the existential quantifier is equivalent to the modal operator of possibility; and the universal quantifier is equivalent to the modal operator of necessity. This definition is expressed in words as:

The possibility of p implying the necessity of p implies
the possibility of q implying the necessity of q. (1.1)

((%p>#p)>((%q>#q); `TTTT TTTT TTTT TTTT` (1.2)

Eq. 1.2 as rendered invokes the equivalence of the quantifiers to modal operators to map the logical value of non contingency `N` or truthity to imply the logical value of non contingency `N` or truthity. Eq. 1.2 results in `T` or tautology as self proving and complete.

Paraconsistent, non-bivalent, vector logics are expressed in words as:

The possibility of p implying the necessity of p implies
the possibility of q *not* implying the necessity of q. (1.1)

(%p>#p)>(%q<#q) ; `CCCC CCCC CCCC CCCC` (2.2)

Eq. 2.2 invokes the logical value of non contingency `N` or truthity to imply the logical value of contingency `C` or falsity. Eq. 2.2 results in `C` or falsity as *not* tautologous.

VŁ4 classifies conjectures as a tautologous or *not* tautologous result, with the latter to include the contradictory result. This qualifies VŁ4 as a universal logic because it maps known logics, some of which as *non* tautologous fragments of VŁ4, another indication that VŁ4 is complete.

Confirmation of VŁ4 as sound

Abstract: Logic VŁ4 is defined as a bivalent classical logic that maps tautology correctly and hence is sound. Because paraconsistent, non bivalent, vector logics cannot map *non* tautology correctly, they are defined as *non* tautologous fragments of VŁ4 as a universal logic.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊔ ;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1;
(%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

Löb's theorem also named the Gödel-Löb (GL) axiom is:

□(□p→p)→□p (1.1)

LET p, q, r, s: bivalent logic, multi-valued logic, modal logic, s.

#(#p>p)>#p ; CTCT CTCT CTCT CTCT (1.2)

Remark 1.2: Eq. 1.2 as rendered is *not* tautologous.

We define a bivalent logic, implying multivalues and modalities, as *not* implied by the GL axiom. (2.1)

((#(#p>p)>#p)=(s@s))>(p>(q&r)) ; TTTT TTTT TTTT TTTT (2.2)

We define a *non* bivalent logic, such as paraconsistent logic, implying multivalues and modalities, as implied by the GL axiom. (3.1)

(#p>p)>#p)=(s=s))>(p>(q&r)) ; T**F**T**F** T**F**TT T**F**T**F** T**F**TT (3.2)

Two VŁ4 theorems not in S5

Abstract: We evaluate two equations not found as theorems in S5; both are theorems in VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⇔, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) `C` non-contingency, ∇, ordinal 2; (%z>#z) `N` as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).

From: Font, J.M.; Hájek, P. (2000). On Łukasiewicz's four-valued modal logic.
citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.25.8024&rep=rep1&type=pdf

Remark 0: The modal truth table of the authors, while mistaken and corrected below, does not affect the logical results which follow:

	given as		should be		
	◊	□	◊	□	
11	11	10	11	01	11
10	11	10	10	00	10
01	01	00	11	01	01
00	01	00	10	00	00

Theorem 2 proof: It is straightforward to show that

◊φ≡(◊⊤& φ) (2.1.1)

%p=(%(p=p)&p) ; `NTNT NTNT NTNT NTNT` (2.1.2)

and □φ≡(◊⊤→φ) (2.2.1)

#p=(%(p=p)>p) ; `TNTN TNTN TNTN TNTN` (2.2.2)

hold in *L*.

Remark 2: Eqs. 2.1.2 and 2.2.2 are *not* tautologous, meaning Theorem 2 is refuted.

Certainly *L* is a rather unusual class of Kripke models; e.g.
[unnumbered equations on page 6]

◊φ→φ (6.1.1)

%p>p ; `NTNT NTNT NTNT NTNT` (6.1.2)

and φ→□φ (6.2.1)

p>#p ; `TNTN TNTN TNTN TNTN` (6.2.2)

hold in *L,*

while φ→◊φ (6.3.1)

p>%p ; `TTTT TTTT TTTT TTTT` (6.3.2)

and □φ→φ (6.4.1)

#p>p ; `TTTT TTTT TTTT TTTT` (6.4.2)

don't.

> **Remark 6:** Eqs. 6.1.2 and 6.2.2 are *not* tautologous and do not hold in *L* while6.3.2 and 6.4.2 are tautologous and hold in *L*. These results are the opposite of what the authors claim.

[I]t is clear that the resulting formulas [unnumbered equations on page 16]

LET p, q, r, s: x, a, b, c

(□(∀x)(b(x)→a(x))& (∀x)(c(x)→b(x)))→□(∀x)(c(x)→a(x)) (16.1.1)

(#((r&#p)>(q&#p))& ((s&#p)>(r&#p)))>
#((s&#p)>(q&#p)) ; `TTTT TTTT TTTT TTTT` (16.1.2)

((∀x)(b(x)→a(x))&□(∀x)(c(x)→b(x)))→□(∀x)(c(x)→a(x)) (16.2.1)

(((r&#p)>(q&#p))&#((s&#p)>(r&#p)))>
#((s&#p)>(q&#p)) ; `TTTT TTTT TTTT TTTT` (16.2.2)

are not theorems of any of the normal modal logics, as they are not theorems of predicate S5.

> **Remark 16:** Eqs. 16.1.2 and 16.2.2 are theorems of the normal modal logic VŁ4, the opposite result of what the authors claim.

Results for eight Eqs. are the opposite of what the authors claim, thereby refuting their assertions.

Refutation of Vongehr's paradigm shift rendering QM "natural"

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal.

> LET: p, q: state of affairs, observation; ~ Not; & And; + Or; > Imply, greater than; @ Not Equivalent; % possibility, for one or some; # necessity, for all or every.

From: Vongehr, S. (ca. 2011). Realism escaping Wittgenstein's silence: the paradigm shift that renders quantum mechanics natural. fqxi.org/data/essay-contest-files/Vongehr_Vongehr_1.pdf; science20.com/alpha_meme/wheeler's_utterly_simple_idea_demands_quantum-93600 .

> (Page 3 of 10, with emphasis as quoted):
>
> **1**) **Totality** encompasses the total of all possibilities. Something *im*possible is, for example, the square of a real number being negative. The impossible is always unobservable, but the observable/unobservable distinction should differ somehow from the possible/impossible one, in order to be significant language. Thus, we separate "possible" from "observable": Some **unobservable is possible** (0.0)
>
> Totality encompasses the total of all possibilities. (1.1)
>
> #(p&q)>#(((%p&q)+(%p&~q))+((~%p&q)+(~%p&~q))) ; `TTTT TTTT TTTT TTTT` (1.2)
>
> The impossible is always unobservable [impossible state is always unobservable state] (2.1)
>
> ~%p>#(p&~q) ; `CTCT CTCT CTCT CTCT` (2.2)
>
> observable/unobservable distinction should differ ... from the possible/impossible one (3.1)
>
> ((%p&q)@(%p&~q))@((~%p&q)@(~%p&~q)) ; `TTTT TTTT TTTT TTTT` (3.2)
>
> Some unobservable is possible [some unobservable state is possible state] (4.1)
>
> (%p&~q)>%p ; `TTTT TTTT TTTT TTTT` (4.2)
>
> We test the argument: If (necessarily all possibilities), then ((impossible is always unobservable) and (distinct combinations differ) and (some unobservable is possible)). (5.0)
>
> If 1.2, then ((2.2 and 3.2) and 4.2). (5.1)
>
> (#(p&q)>#(%p&%q))>(((~%p>#(p&~q))&(((%p&q)@(%p&~q))@((~%p&q)@(~%p&~q))))&((%p&~q)>%p)) ; `CTCT CTCT CTCT CTCT` (5.2)

Eq. 5.2 as rendered is *not* tautologous. This refutes Vongehr's paradigm shift rendering QM "natural".

W in system K4W

From (1971) Sergerberg, p. 68.

#(#p>p)>#p ; not tautologous

Refutation of Wadge order and application to the Scott domain

Abstract: We evaluate the definition of the Wadge order. It is *no*t tautologous. Hence application of the Wadge hierarchy to and also the Scott domain are similarly suspicious.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

> LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And; > Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
> = Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
> % possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
> (z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
> (%z<#z) `C` non-contingency, ∇, ordinal 2; (%z>#z) `N` as non-contingency, Δ, ordinal 1;
> ~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
> Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Duparc, J.; Vuilleumier, L. (2019).
The Wadge order on the Scott domain is not a well-quasi-order. arxiv.org/pdf/1902.09419.pdf
jacques.duparc@unil.ch, louis.vuilleumier@etu.univ-paris-diderot.fr

> The Wadge order $\leq_w$ ... on the subsets of a topological space X is the quasi-order induced by reductions via continuous functions. More precisely, if A,B⊆X, then A$\leq_w$B if there exists a continuous function f:X→X such that $f^{-1}[B]=A$, i.e., x∈A⇔f(x)∈ B for all x∈X. (1.0)

Remark 1.0: We map Eq. 1.0, with $\leq_w$ as ≤ for purposes of testing here, as:

If (if for all x∈X, then (x∈A⇔ f(x)∈ B)), then if A,B⊆X, then A≤B. (1.1)

LET p, q, r, x, u: A, B, f, x, X

```
(#(x<u)>((x<p)=((r&x)<q)))>(%r&u)>u)>(~(u<(p&q))>~(q<p))) ;
                         TTCT TTCT TTCT TTCT( 8),
                         TTTT TTTT TTTT TTTT(24)        (1.2)
```

Remark 1.2: Distributing the respective quantifiers on x or from f as r does not change the table result output.

Eq. 1.2 as rendered is *not* tautologous. This means the Wadge order is refuted. What follows is that the Wadge hierarchy and the incidental Scott domain are similarly suspicious.

Refutation of a weak set theory H that proves its own consistency

Abstract: For theory H, the axioms of extensionality and separation are *not* tautologous. The theorem to prove any instances of the scheme of ε-induction is also *not* tautologous. These conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Pakhomov, F. (2019). A weak set theory that proves its own consistency. arxiv.org/pdf/1907.00877.pdf

1. Introduction
We define a theory $H_{<\omega}$ and show that it proves its own Hilbert-style consistency. Unlike Willard's theories, our theory isn't arithmetical but rather a system in the language of set theory with additional unary function V. The axioms of H are

1. x = y ↔ ∀z(z ∈ x ↔ z ∈ y) (Extensionality); (1.1.1)

LET p, q, r, s: x, y, z, ϕ.

(p=q)=((#r<p)=(#r<q)) ; T**FF**T TNNT T**FF**T TNNT (1.1.2)

2. ∃y∀z(z ∈ y ↔ z ∈ x∧ϕ(z)), where ϕ(z) ranges over first-order formulas without free occurrences of y (Separation); (1.2.1)

(#r<%q)=(#r<(p&(s&#r))) ; TTTT TTCC TTTT TCCT (1.2.2)

A. Theory H is non-Gödelian
Lemma 10. Theory H prove any instances of the scheme of ε-induction:

∀x((∀y ∈ x)ϕ(y) → ϕ(x)) → ∀x ϕ(x). (10.1)

(((#r<#q)&(p& r))>(p&#q))>(p&#q) ;
FFFN **F**N**F**N **FFF**N **F**N**F**N (10.2)

For theory H, the axioms of extensionality and separation in Eqs. 1.1.2 and 1.2.2 as rendered and the theorem to prove any instances of the scheme of ε-induction in 10.2 are *not* tautologous.

Well ordering property

From plato.stanford.edu/entries/logic-higher-order/

$$\exists x\ Px \rightarrow \exists x(Px\ \&\ \forall y(Py \rightarrow (y = x \text{ v } x < y))) \quad (1)$$

$$\forall P[\exists x\ Px \rightarrow \exists x(Px\ \&\ \forall y(Py \rightarrow (y = x \text{ v } x < y)))] \quad \text{(formalization)} \quad (2.1)$$

(#p&(%x&(p&x))) > (#p&(%y&((p&x)&(#y&((p&y)>((y=x)+(x<y))))))) ; nvt (2.2)

```
Model 1               Model 2.1             Model 2.2             Model 2.3.1           Model 2.3.2
TTTT TTTT TTTT TTTT   EEEE EEEE EEEE EEEE   EEEE EEEE EEEE EEEE   EEEE EEEE EEEE EEEE   EEEE EEEE EEEE EEEE
TCTC TCTC TCTC TCTC   EUEU EUEU EUEU EUEU   EEEE EEEE EEEE EEEE   EPEP EPEP EPEP EPEP   EIEI EIEI EIEI EIEI
```

Refutation of the wellfoundedness of the multiset order

Abstract: The two equations for an inductive proof of the wellfoundedness of the multiset order are *not* tautologous, and form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Nipkow, T.; Buchholz, W. (1998). An inductive proof of the wellfoundedness of the multiset order. www21.in.tum.de/~nipkow/misc/multiset.ps [partial file]

Given a binary relation < on a set S, the subset of S called the *well-founded par*t of S w.r.t < is defined inductively as follows ..:

$$\frac{\forall y<x.y\in W}{x\in W} \tag{1.1.1}$$

LET p, q, r, s: P, w, x, y.

(#s<(r&(s<q)))>(r<q) ; TTTT TTTT CCCC TTCC (1.1.2)

The corresponding induction principle easily yields the principle of *well-founded part induction*:

$$\frac{\forall x\in W.(\forall y<x.P(y))\Rightarrow P(x)}{\forall x\in W.P(x)} \tag{1.2.1}$$

(((#r<q)&(#s<(r&(p&s))))>(p&r))>(#r<(q&(p&r))) ;
FFFF NNN**F** **FFFF** NNN**F** (1.2.2)

Eqs. 1.1.2 and 1.2.2 as rendered are *not* tautologous. This refutes the conjecture of an inductive proof of the wellfoundedness of the multiset order.

Refutation of White's model for creation

Abstract: From the introduction, we evaluate a system of four postulates (P1, P2, P3, P4). P1 implies P2; P4 implies P3; but (P1 implies P2) does not imply (P4 implies P3). Hence the system is *not* tautologous. Two subsequent postulates (P5, P6) are not examined.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And; > Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⊢, ⊨, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: White, P.B. (2019). A model for creation: part 1.
vixra.org/pdf/1903.0084v1.pdf pbwx@att.net

LET p, q, r, s: P1, P1, P3, P4.

1. For creation of the physical universe, the basic information element is a type of projection --- more specifically, a projection from a prior level. (1.1)

p=((q>r)>s) ; TFFT TFTF FTFT FTFT (1.2)

2. The basic information structure is a sequence of such projections.
With respect to the first postulate, we may refer to both projections and levels as "elements" (or basic elements) of the system, but will reserve the term "basic information element" for the projections alone. (2.1)

p>((q>r)>s) ; TFTT TFTF TTTT TTTT (2.2)

We now add two more postulates:
3. Each such projection is a one-dimensional vector, constituting a different, but related, one-dimensional space. (The basic relations between these projections/vectors are stated in the next postulate.) (3.1)

(p@q)@(r@s) ; FTTF TFFT TFFT FTTF (3.2)

4. Prior things (e.g., projections, levels, and constructions from them) are independent of subsequent things; and, conversely, subsequent things are dependent on prior things. (The terms prior, subsequent, dependent, and independent denote here logical/ontological relations. See e.g. [4].) (4.1)

~((p>q)>(r>s)) = (p=p) ; **FFFF** T**F**TT **FFFF** **FFFF** (4.2)

Using these four postulates (and two more that will be stated later), we develop a model for the basic construction of the physical universe ... (5.1, 6.1)

Remark 1.-4.: The postulates are related in pairs, then we relate the pairs.

P1 implies P2: P1>P2 (10.1)

(p=((q>r)>s))>(p>((q>r)>s)) ; TTTT TTTT TTTT TTTT (10.2)

P4 implies P3: P4>P3 (11.1)

~((p>q)>(r>s))>((p@q)@(r@s)) ; TTTT TT**F**T TTTT TTTT (11.2)

Remark 11.2: For P3 implies P4: P3>P4 (11.2.1)

((p@q)@(r@s))>~((p>q)>(r>s)) ; T**FF**T TTTT **F**TT**F** T**F**FT (11.2.2)

The truth table of Eq. 11.2.2 is relatively farther from tautology than that of 11.2; hence we choose use 11.1 for P4>P3.

(P1 implies P) implies (P4 implies P3): (P1>P2)>(P4>P3) (12.1)

((p=((q>r)>s))>(p>((q>r)>s)))>(~((p>q)>(r>s))>((p@q)@(r@s))) ;
TTTT TT**F**T TTTT TTTT (12.2)

Eq. 12.2 is not tautologous. Therefore the model of creation based on four postulates so far is refuted. We did not examine the subsequent two postulates.

Wittgenstein's *ab*-notation (and **Quine-McCloskey algorithm**)

Lampert, Timm. "Wittgenstein's *ab*-notation: an iconic proof procedure". *History and Philosophy of Logic.* 2017. dx.doi.org/10.1080/01445340.2017.1312222.

From:
researchgate.net/publication/316898292_Wittgenstein%27s_ab_-Notation_An_Iconic_Proof_Procedure

We use the apparatus of Meth8 modal logic model checker (system Ł4 as resuscitated in variant VŁ4).

The designated proof value is T (tautology); other logic values are: Contingent (contradictory); Non-contingent (tautologous); and F (contradiction). The 2-tuple is respectively { 11, 10, 01, 00 }.

Truth tables are presented in16-values with four row major horizontally.

We evaluate two of Wittgenstein's expressions.

LET: p q r s x y z F; ~ Not; & And;
universal quantifier; % existential quantifier

(~#p&(q&p)) = (%p&(~q&p)) ; TFTN TFTN TFTN TFTN ; (*9.01, pg 16)

Eq *9.01 contains the expression "&p" in both the antecedent and consequent. If that is removed, then the following equation is tested with a different, more negative result.

(~#p&q)=(%p&~q) ; NFFN NFFN NFFN NFFN ; (*9.01.1)

((#r&(%q&(#p&(s&(p&(q&r))))))&(#p&(%q&(s&(p&(q&p)))))) &
((#p&(#q&(s&(p&(p&q))))))&(%p&(s&(p&(p&p))))) ;
FFFF FFFF FFFF FFF*N* ; (7), pg 28

We note that Eq 7 results in a truth which is one logical value off (*N*) from being a proof of contradiction (F).

We surmise that the *ab*-notation of Ludwig Wittgenstein is not bivalent.

In the process of evaluation above, we validated the equations given for the Quine-McCloskey algorithm to minimize reductive disjunctive normal forms (RDNFs) as follows.

LET p q r P Q R; + Or

((p&~q)+(~p&q)) + ((p&r)+(q&r))	;	FTTF FTTT	(4) pg 14
((p&~q)+(~p&q)) + (p&r)	;	FTTF FTTT	(5)
((p&~q)+(~p&q)) + (q&r)	;	FTTF FTTT	(6)

Refutation of X-homology over manifolds in topology

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: α, d, X, s; + Or; & And, ∧; = Equivalent; @ Not Equivalent; (s@s) zero, 0.

From: Balan, A. (2018). The X-cohomology. vixra.org/pdf/1810.0075v1.pdf [no email proffered]

Demonstration 1 *Indeed*: d(dα+X∧α)+X∧(dα+X∧α)=0 (1.1)

((q&((q&p)+(r&q)))+(r&((q&p)+(r&p))))=(s@s) ; TTT**F** T**FFF** TTT**F** T**FFF** (1.2)

Eq.1.2 as rendered is not tautologous, hence refuting the X-homology over manifolds in topology.

Yalcin log

Holliday, W.H.; Icard, III, T.F. "Indicative conditionals and dynamic epistemic logic". July 2017. DOI: 10.4204/EPTCS.251.24. From: researchgate.net/publication/318709156

Figure 1. Axioms of the Yalcin logic, page 339:

LET p lc_phi, q lc_psi, r lc_alpha, s lc_beta

(p>r) > (p>#r) ; `TNTN TTTT TNTN TTTT` ; (14)

((p>(r+%s))&(p>~s)) > (p>r) ; `TNTN TTTT TTTT TTTT` ; (16)

Yalcin logic has two axioms not validated as tautology by Meth8.

Correction to the yinyang logic schema

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

LET + truthity, 01; - falsity, 10; 0 contradiction, 00; ± tautology, 11;
$\hat{+}$ Or, And with And as Not Or, as in Not(01 Or 10) **= 00**.

From: From: Xu, W. (2015). On the origin of physical states. vixra.org/pdf/1811.0010v1.pdf

The yinyang interactions operates with the addition rule $\hat{+}$ for yinyang signs:

$\hat{+}$(+,+)=+; $\hat{+}$(-,-)=-; $\hat{+}$(+,-)=o; $\hat{+}$(o,±)=±. (1.1), (2.1), (3.1), (4.1)

Remark: The operator $\hat{+}$ described as an addition rule is mistaken as such because it is not bivalent and exact, but rather a vector space and probabilistic. The intention of the captioned paper is to be exact. The operator $\hat{+}$ is re-named as "the operator rules of classical logic" because of the following schema:

01 Or 01 = 01; 10 Or 10 = 10; 01 And 10 **= 00**; 00 Or 11 = 11.
(1.2), (2.2), (3.2), (4.2)

This advance gives the yinyang interactions a basis in classical mathematical logic.

One is reminded that Fenyman, who needed to make a living as an academic, was always *privately* suspicious of quantum mechanics as a specialized artifact far removed from the actual state of affairs in the universe.

First Zadeh's logical operators on fuzzy logic

Said Broumi, Said; Majumdar, Pinaki; Smarandache, Florentin. 2014. "New operations on intuitionistic fuzzy soft sets based on First Zadeh's logical operators". doi: 10.5281/zenodo.30235.

From: researchgate.net/publication/281103677_New_Operations_on_Intuitionistic_Fuzzy_Soft_Sets_Based_on_First_Zadeh%27s_Logical_Operators

We assume the Meth8 apparatus of VŁ4 (variant of our resuscitated Łukasiewicz modal B_4). Table fragments of 16-values are from the full proof of 128-valued truth tables. The designated proof value is T.

LET: p q r s t u v A B C F G H; ~ Not; & And; \ Nand; + Or; - Nor;
= Equivalent to; @ Not Equivalent to; > Imply; < Not Imply;
~(A>B) A<=B; T tautology; F contradiction

pg 279. Def. 2.7:

(r=(p+q))>(((s&p)+(t&q))=(u&r)) ; TTTT TTFF TTTT TFFF

pg. 281. Prop. 3.2.2:

(r=(p&q))>(((s&p)\(t&q))=(u&r)) ; FFFT TTTF FFFT TTTF

pg 282. Prop. 3.2.3:

((s&p)\(t&q))>~((u&r)<(((s&p)>(t&q))\((t&q)>(u&r)))) ; TTTT FFFF TTTT FTFT

pg 283. Ex. 3.3.2:

(r=(p&q))>(((s&p)-(t&q))=(u&r)) ; FFFT TTTT FTFT TTTF

pg 284. Prop. 3.3.4:

((s&p)-(t&q))>~((u&r)<(((s&p)>(t&q))-((t&q)>(u&r)))) ; TTTT FFTT TTTT FTTT

Evaluation:

Because Meth8 does not validate the above definition, example, and propositions as tautology, we deem Zadeh fuzzy logic as not bivalent, a probabilistic vector space, and hence suspicious.

Refutation in one variable of the historical basis for fuzzy logic

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

LET p; ~ Not, ¬; & And, ∧; Imply >; = Equivalent; @ Not Equivalent;
% possibility, for one or some; # necessity, for all or every; (p=p) T.

From: Dubois, D.; et al. (2007). Fuzzy-set based logics: an history-oriented presentation of their main developments. *Handbook of the history of logic*. Volume 8. Dov M. Gabbay, John Woods (Editors).

However the proposition “possible p” is not the same as p, and “possible ¬p” is not the negation of “possible p”. Hence the fact that the proposition

“possible p” ∧ “possible ¬p”

may be true does not question the law of non-contradiction since “possible p” and “possible ¬p” are not mutually exclusive. This situation leads to interpretation problems for a fully truth-functional calculus of possibility, since even if p is “possible” and ¬p is “possible”, still p ∧ ¬p is ever false.

“possible p” is not the same as p (1.1)

%p@p ; C**F**C**F** C**F**C**F** C**F**C**F** C**F**C**F** (1.2)

“possible ¬p” is not the negation of “possible p” (2.1)

%~p=~%p ; NNNN NNNN NNNN NNNN (2.2)

Hence the fact that the proposition “possible p” ∧ “possible ¬p” may be true (3.1)

(%p&%~p)=%(p=p) ; CCCC CCCC CCCC CCCC (3.2)

“possible p” and “possible ¬p” are not mutually exclusive (4.1)

~(%p@~p)=(p=p) ; C**F**C**F** C**F**C**F** C**F**C**F** C**F**C**F** (4.2)

even if p is “possible” and ¬p is “possible”, still p ∧ ¬p is ever false (5.1)

((p=%(p=p))&(~p=%(p=p)))>#((p&~p)=(p@p)) ;
TTTT TTTT TTTT TTTT (5.2)

Remark: Eqs. 1.2-4.2 are *not* tautologous, and 5.2 is (as expected with a contradictory antecedent). Hence an historical basis for fuzzy logic is refuted, and in one variable.

Refutation of perfect and strong functions in fuzzy logic

Abstract: For fuzzy logic, we evaluate the definition of the residuated lattice operator as *not* tautologous. We evaluate the perfect fuzzy function and strong (surjective) fuzzy function as *not* tautologous, but logical equivalents. This refutes the fuzzy functions and distinctions. These form a *non* tautologous fragment of the universal logic VL4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Perfilieva, I. (2011). Fuzzy function: theoretical and practical point of view. EUSFLAT-LFA 2011. 480/6. pdfs.semanticscholar.org/ef1c/b4b8045fd36591daffdd27a396856e18e0ba.pdf

Abstract The aim of this investigation is to reconsider two notions of fuzzy function, namely: a fuzzy function as a special fuzzy relation and a fuzzy function as a mapping between fuzzy spaces. We propose to combine both notions in such a way that a fuzzy function as a relation determines a fuzzy function as a mapping. We investigate conditions which guarantee that dependent values of the related fuzzy functions coincide. Moreover, we investigate properties and relationship of the related fuzzy functions in the case when both of them are "fuzzified" versions of the same ordinary function.

1. Introduction The notion of fuzzy function has at least two different meanings in fuzzy literature. From theoretical point of view (see e.g., ..), a fuzzy function is a special fuzzy relation with a generalized property of uniqueness. According to this approach, each element from a range of a fuzzy function can be assigned with a certain degree to each element from its domain. Thus, instead of working with direct functional values we have to work with degrees. Another, practical point of view on a fuzzy function originates from the early work of L. Zadeh .. where he proposed the well known extension principle. According to this principle, every function (in an ordinary sense) can be "fuzzified", i.e. extended to arguments given by fuzzy sets. Thus, any ordinary function determines a mapping from a set of fuzzy subsets of its domain to a set of fuzzy subsets of its range. In .. , we have used this approach and defined a fuzzy function as an ordinary mapping between two universes of fuzzy sets. Similar definition appeared in .. and implicitly, in many other papers devoted to fuzzy IF-THEN rules models where these models are used as partially given fuzzy functions. The aim of this investigation is to reconsider both notions and combine them in such a way that a fuzzy function as a relation determines a fuzzy function as a mapping (we will say that they are related). We will investigate conditions which guarantee that dependent values of related fuzzy functions coincide. Moreover, we will investigate properties and relationship of related fuzzy functions in the case when they are "fuzzified" versions of the same ordinary function.

2.2. Residuated lattice ...
Definition 1 A residuated lattice is an algebra … such that … the operation → is a residuation with respect to ∗, i.e. a ∗ b ≤ c iff a ≤ b → c. (2.2.1.1)

LET p, q, r: a, b, c.

(~(q<p)>r)>~(r<(p&q)) ; TTTT **FFF**T TTTT **FFF**T (2.2.1.2)

Remark 2: Eq. 2.2.1.2 is *not* tautologous. Hence the definition for the operation → as a residuation is refuted.

3.1. (E−F)-fuzzy function
...
Definition 3 Let E, F be respective fuzzy equivalences on X and Y . An (E−F)-fuzzy function is a binary fuzzy relation ρ on X × Y such that ... the following axioms hold true: … An (E−F)-fuzzy function is called perfect ... **F.4** for all x∈X, there exists y∈Y, such that ρ(x,y) = 1. (3.1.3.4.1)

LET r, x, y, u, v: ρ, x, y, X, Y.

((#x<u)&(%y<v))>((r&(#x&%y))=(%z>#z)) ;

TTTT TTTT TTTT TTTT } x48
TTTT CCCC TTTT CCCC } x 2 *} x 2*
TTTT TTTT TTTT TTTT } x 6 *}* (3.1.3.4.2)

An (E−F)-fuzzy function is called (strong) surjective ... if **F.5** for all y∈Y, there exists x∈X, such that ρ(x, y) = 1. (3.1.3.5.1)

((#y<v)&(%x<u))>((r&(%x&#y))=(%z>#z)) ;

TTTT TTTT TTTT TTTT } x48
TTTT CCCC TTTT CCCC } x 2 *} x 2*
TTTT TTTT TTTT TTTT } x 6 *}* (3.1.3.5.2)

Remark 3: Eqs. 3.3.4.2 and 3.1.3.5.2 as rendered are *not* tautologous, but produce sames truth table values. Hence the axioms for the (E−Z)-functions as perfect or strong (surjective) are refuted and equivalents.

Refutation of measures for resolution and symmetry in fuzzy logic of Zadeh Z-numbers

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal.

LET p, q, r, s: A, B, H, Z;
~ Not; > Imply, greater than; < Not Imply, less than; = Equivalent;
% possibility, for one or some, sharpened; # necessity, for every or all;
(%p>#p) ordinal 1, truthity. Note: ~(x<y) = (x≥y).

From: Deng, Y.; Lia, Y. (2018). Measuring fuzziness of Z-numbers and its application in sensor data fusion. vixra.org/pdf/1807.0245v1.pdf dengentropy@uestc.edu.cn

Proof. Assume the fuzziness measure, H, ...

For G3 [resolution], denoted A∗ = (A∗, B∗), where A∗, B∗ are [a] sharpened version of A and B, respectively. So H(A)≥H(A∗) and H(B)≥H(B∗), therefore
H(A)+H(B)≥H(A∗)+H(B∗)) > H(Z)≥H(Z∗). (3.1)

(~((r&p)<(r&#p))&~((r&q)<(r&#q))) > (~(((r&p)+(r&q))<((r&#p)+(r&#q)))>
~((r&s)<(r&#s))) ; TTTT TTTT TTTT NTTT (3.2)

For G4, [symmetry] H(A)=H(1−A) and H(B)=H(1−B), so H(A)+H(B)=
(H(1−A))+(H(1−B))) > HZ(Z)=HZ(Z(1−A,1−B)). (4.1)

(((r&p)=(r&((%p>#p)-p)))&((r&q)=(r&((%p>#p)-q)))) >
((((r&p)+(r&q))=((r&((%p>#p)-p))+(r&((%p>#p)-q))))>
(((r&s)&s)=((r&s)&(s&(((%p>#p)-p)&((%p>#p)-q)))))) ; TTTT TTTT TTTT CTTT (4.2)

Eqs. 3.2 and 4.2 as rendered are *not* tautologous. This means the commonly accepted measures G3 (resolution) and G4 (symmetry) for the Zadeh (Z-numbers) fuzzy logic are refuted.

Refutation of Zadeh's Swedes and Italians challenge as a logic problem

Abstract: For Zadeh's Swedes and Italians challenge problem, we evaluate the linguistic weighted average (LWA) for the conjecture that Swedes are on average taller than Italians. None is tautologous, refuting the challenge as a logic problem. These form a *non* tautologous fragment of the universal logic VL4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Rajati, M.R.; Mendel, J.M. (circa 2011). Solving Zadeh's Swedes and Italians challenge problem. cs.utep.edu/vladik/nafips12special.sessions/104.pdf

Abstract—In this paper, we present a solution to Zadeh's Swedes and Italians challenge problem which involves linguistic quantifiers and linguistic attributes. First, we argue that Zadeh's solution to this problem via the Generalized Extension Principle is very difficult to implement. Then, we use a syllogism based on the entailment principle to interpret the problem so that it can be solved via Linguistic Weighted Averages. ...

III. OUR SOLUTION TO THE SWEDES AND ITALIANS PROBLEM
This section presents our solution to the Swedes and Italians problem using Linguistic Weighted Averages (LWAs). …

In such a framework, we can calculate the following LWA to obtain the average value that Swedes are taller than most Italians, $\tilde{AH}_1$: $\tilde{AH}_1 \equiv \mathbb{E}\{\tilde{B}_1\}$ =

[*Most* x *Much taller* + *Few* x *not Much taller*] / [*Most* + *Few*] (18.1.1)

Remark 18.1.1: We reduce the number of variables where Most means Not Few, and Few means Not Most. [A separate variable for Few produces the *same* truth table values below.]

LET p, q, r, s: Most, Italians, Much taller, Swedes.

((p&r)+(q&~r))\(p+~p) ; `TT`**FF** `T`**F**`T`**F** `TT`**FF** `T`**F**`T`**F** (18.1.2)

Remark 18.1.2: Eq. 18.1.2 as rendered is *not* tautologous, in other words, the average value is *not* a theorem.

This implies that on average, Swedes are *AH*˜1 taller than most Italians. (18.2.1)

Remark 18.2.1: We write *AH*˜1 to mean the average value from Eq. 18.1.1.

(((p&r)+(q&~r))\(p+~p))>(s>q) ;
TTTT TTTT **FF**TT **F**TTT (18.2.2)

Eqs. 18.1.2 and 18.2.2 are *not* tautologous, hence refuting the Swedes and Italians challenge as a (fuzzy) logic problem.

Refutation of affine varieties in Zariski topology and denial of Grothendieck's scheme theory

Abstract: From the affine varieties of Zariski topology, we evaluate two definitions. Neither is tautologous. In fact, the two definitions are equivalents. This refutes the conjecture of affine varieties in Zariski topology. Therefore the affine varieties of Zariski topology are *non* tautologous fragments of the universal logic VŁ4. What follows is that the scheme theory of Grothendieck is *non* tautologous.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩, · ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ;
< Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊔ ;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1;
(%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B); (B>A) (A⊢B); (B>A) (A⊨B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From : en.wikipedia.org/wiki/Zariski_topology

Zariski topology of varieties

In classical algebraic geometry (that is, the part of algebraic geometry in which one does not use schemes, which were introduced by Grothendieck around 1960), the Zariski topology is defined on algebraic varieties. The Zariski topology, defined on the points of the variety, is the topology such that the closed sets are the algebraic subsets of the variety. As the most elementary algebraic varieties are affine and projective varieties, it is useful to make this definition more explicit in both cases. We assume that we are working over a fixed, algebraically closed field *k* (in classical geometry *k* is almost always the complex numbers).

Affine varieties

First we define the topology on affine spaces A n , which as sets are just *n*-dimensional vector spaces over *k*. The topology is defined by specifying its closed sets, rather than its open sets, and these are taken simply to be all the algebraic sets in A n . That is, the closed sets are those of the form $V(S) = \{ x \in A^n \mid f(x) = 0, \forall f \in S \}$ where *S* is any set of polynomials in *n* variables over *k*. It is a straightforward verification to show that: V(S) = V((S)), where (S) is the ideal generated by the elements of S; For any two ideals of polynomials I, J, we have

$V(I) \cup V(J) = V(IJ)$; (1.1)

LET p, q, r, s: I, J, V

((r&p)+(r&q))=(r&(p&q)) ; TTTT T**FF**T TTTT T**FF**T (1.2)

$V(I) \cap V(J) = V(I + J)$. (2.1)

((r&p)&(r&q))=(r&(p+q)) ; TTTT T**FF**T TTTT T**FF**T (2.2)

Remark 1.-2.: Eqs. 1.2 and 2.2 as rendered are *not* tautologous, but are equivalent.

This refutes the conjecture of Zariski topology of affine varieties, thereby denying Grothendieck's scheme theory.

Refutation: constructive Zermelo-Fraenkel set theory (CZF) and extended Church's thesis (ECT)

Abstract: We evaluate constructive Zermelo-Fraenkel set theory (CZF) of intuitionistic logic. Of the eight CZF axioms, only the induction scheme is tautologous. From CZF axioms for infinity, set induction, and extensionality the deduction of one set denoted ω is *not* tautologous. An equation for extended Church's thesis (ECT) is *not* tautologous. This supports previous work that intuitionistic logic is *not* tautologous.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) `C` non-contingency, ∇, ordinal 2; (%z>#z) `N` as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

From: Rathjen, M. (2005). Constructive Zermelo-Fraenkel Set Theory CZF.
jucs.org/jucs_11_12/constructive_set_theory_and/jucs_11_12_2008_2033_rathjen.pdf
m.rathjen@leeds.ac.uk

Definition: 2.1 (Axioms of CZF) The language of CZF is the first order language of Zermelo-Fraenkel set theory, LST, with the non logical primitive symbol ∈. CZF is based on intuitionistic predicate logic with equality. The set theoretic axioms of axioms of CZF are the following:

1. Extensionality: $\forall a\, \forall b\, (\forall y\, (y \in a \leftrightarrow y \in b) \rightarrow a = b)$ (2.1.1)

LET a, b, y, x: p, q, r, s

((#r<#p)=(#r<#q))>(#p=#q) ; `TCCT TTTT TCCT TTTT` (2.1.2)

2. Pair: $\forall a\, \forall b\, \exists x\, \forall y\, (y \in x \leftrightarrow y = a \vee y = b)$ (2.2.1)

LET p, q, r, s: a, x, y, z

((#r<%s)=#r)=((#p+%s)=q) ; `N`**F**`CT N`**F**`CT` **FF**`TT NNCC` (2.2.2)

3. Union: $\forall a\, \exists x\, \forall y\, (y \in x \leftrightarrow \exists z \in a\, y \in z)$ (2.3.1)

(#r<%q)=(%s<((#p&#r)<%s)) ; `NNNN` **FF**`NN` **FFFF** `NN`**FF** (2.3.2)

4. Restricted separation scheme: $\forall a\ \exists x\ \forall y\ (y \in x \leftrightarrow y \in a \wedge \phi(y))$, for every restricted formula $\phi(y)$, where a formula $\phi(x)$ is *restricted*, or $\Delta 0$, if all the quantifiers occurring in it are restricted, i.e. of the form $\forall\ x \in b$ or $\exists\ x \in b$. (2.4.1)

LET p, q, r, s: ϕ, a, x, y

```
((s<r)=(x<q))&(p&s) ;          FFFF FFFF FFFF FTFT(16),
                               FFFF FFFF FTFF FFFT(16)
```
(2.4.2)

5. Subset collection scheme: $\forall\ a\ \forall\ b\ \exists\ c\ \forall\ u\ \forall\ x \in a\ \exists\ y \in b\ \phi(x,y,u) \rightarrow$
$\exists\ d \in c\ (\forall x \in a\ \exists\ y \in d\ \phi(x,y,u) \wedge \forall\ y \in d\ \exists\ x \in a\ \phi(x,y,u))$
for every formula $\phi(x,y,u)$ (2.5.1)

```
((#x<#q)&(%y<(#r&#(#p&((x&y)&u)))))>((%z<%s)&(((#x<#q)&((%y<
%z)&#(p&(x&(y&#u)))))&((#y<%z)&((%x<#q)&#(p&(x&(y&#u)))))) ;
                               TTTT TTTT TTTT TTTT(48),
                               CCTT CCTT CCTT CCTT(16)
```
(2.5.2)

6. Strong collection scheme: $\forall\ a\ \forall\ x \in a\ \ \exists\ y\ \phi(x,y) \rightarrow$
$\exists\ b\ (\forall\ x \in a\ \exists\ y \in b\ \phi(x,y) \wedge \forall y \in b\ \exists\ x \in a\ \phi(x,y))$ for every formula $\phi(x,y)$ (2.6.1)

```
((#x<#q)&(%y&#(p&(x&y))))>(((#x<#q)&((%y<%r)&#(p&(x&y))))&
((#y<%r)&((%x<#q)&#(p&(x&y))))) ;
                               TTTT TTTT TTTT TTTT(48),
                               TTTT TCTT TTTT TCTT(16)
```
(2.6.2)

7. Infinity: $\exists\ x \forall u\ [\ u \in x \leftrightarrow 0 = u \vee \exists\ v \in x\ (u = v \cup \{v\})$ where y+1 is $y \cup \{y\}]$, and 0 is the empty set, defined in the obvious way

```
((y+(%y>#y))=(y&y))>((#u<%x)=((y@y)=(#u&(%v<(#x&(#u=(#u&#u))))))) ;
                               NNNN NNNN NNNN NNNN( 4),
                               FFFF FFFF FFFF FFFF(28)
```
(2.7.2)

8. Set induction scheme: (IND $\in$) $\forall a\ (\forall x \in a\ \phi(x) \rightarrow \phi(a)) \rightarrow \forall a\ \phi(a)$, for every formula $\phi(a)$ (2.8.1)

```
#(p&q)>(((#x<(#q&#(p&q)))>#(p&q))>(p&#q)) ;
                               TTTT TTTT TTTT TTTT
```
(2.8.2)

From Infinity, Set induction, and Extensionality one can deduce that there exists exactly one set x such that $\forall u[u \in x \leftrightarrow 0 = u \vee \exists v \in x(u = v \cup \{v\})]$; this set will be denoted by ω. (2.9.1)

```
((((#x<#q)&(%y&#(p&(x&y))))>(((#x<#q)&((%y<%r)&#(p&(x&y))))&
((#y<%r)&((%x<#q)&#(p&(x&y))))))&((#(p&q)>(((#x<(#q&#(p&q)))>
#(p&q))>(p&#q)))&(((#r<#p)=(#r<#q))>(#p=#q))))>(((#u<x)=(((y@y)=
(#u&%v))<(x&(#u=(%v&%v)))))>%x) ;
                               CCCC CCCC CCCC CCCC( 4),
                               TTTT TTTT TTTT TTTT(28)
```
(2.9.2)

Definition: 3.3 Extended Church's Thesis, ECT, asserts that

$$\forall n \in N\ \psi(n) \rightarrow \exists\ m \in N \phi(n,m) \text{ implies}$$
$$\exists\ e \in N \forall n \in N\ \psi(n) \rightarrow \exists\ m,p \in N\ [T\ (e,n,p) \wedge U\ (p,m) \wedge \phi(n,m)]$$
(3.3.1)

LET p, q, r, s, t, u, v, w, x:
p, ϕ, ψ, e, T, U, m, n, N.

```
((#w<(x&(r&w)))>(%v<((x&y)&(w&v)))) >
(((%s<x)&(#w<(x&(r&w))))>
(((%v&%p)<x)&(((t&s)&(w&p))&((u&(p&v))&(q&(w&v)))))) ;
                    TTTT TTTT CCCT CCCT( 1),
                    TTTT TTTT CCCC CCCC( 3),
                    TTTT TTTT TTTT TTTT(28)
```
(3.3.2)

Based on intuitionistic logic, CZF axioms as rendered in Eqs. 2.1-2.7 are *not* tautologous. Eq. 2.8 as the induction scheme is tautologous. Eq. 2.9 to derive one set named ω is *no*t tautologous. The definition for extended Church's thesis in Eq. 3 is *not* tautologous. These results support previous work that intuitionistic logic is *not* tautologous.

Shortest refutations of the Zermelo-Fraenkel (ZF) axioms

Abstract: The Zermelo-Fraenkel (ZF) axioms evaluated are below.

1. Null Set: ∃x¬∃y(y∈x)[=Ø].
2. Extensionality: ∀x∀y[∀z(z∈x↔z∈y)→x=y].
3. Pairs: ∀x∀y∃z∀w(w∈z↔w=x∨w=y).
4. Power Set: ∀x∃y∀z[z∈y↔∀w(w∈z→w∈x)] (1).
4. Power Set: ∀x∃y∀z(z∈y↔z⊆x) (2).
5. Unions: ∀x∃y∀z[z∈y↔∃w(w∈x∧z∈w)].
6. Infinity: ∃x[Ø∈x∧∀y(y∈x→(y∪(y))∈x)].
7. Separation Schema: ∀uk[∀w∃v∀r(r∈v↔r∈w∧ψx,u[r,u])].
8. Replacement Schema: ∀uk[∀x∃!yϕ(x,y,u)→∀w∃v∀r(r∈v↔∃s(s∈w∧ϕx,y,u[s,r,u]))].
9. Regularity: ∀x[x≠Ø→∃y(y∈x∧∀z(z∈x→¬(z∈y)))].

None are tautologous.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s, t, u, v, w, x, y, z: ϕ, ψ, r, s, k, u, v, w, x, y, z;
~ Not, !; + Or, ∪ ; & And; > Imply; < Not Imply, less than;
= Equivalent; @ Not Equivalent;
necessity, ∀, for all or every; % possibility, ∃, for one or some;
~%p=#p; (q>p) ~(q∈p), not lt.eq.; ~(q>p) (q∈p) lt.eq., ⊆.

From: plato.stanford.edu/entries/set-theory/ZF.html by Joan Bagaria (joan dot bagaria@icrea dot cat)

1. **Null Set**:This axiom asserts the existence of the empty set. Since it is provable from this axiom and the next axiom that there is a unique such set, we may introduce the notation 'Ø' to denote it

∃x¬∃y(y∈x) [=Ø] (1.1)

(%x&~%y)&~(y>x) ; `FFFF FFFF FFFF FFFF, FFFF FFFF FFFF FFFF` (1.2)

2. **Extensionality**: This axiom asserts that when sets x and y have the same members, they are the same set.

∀x∀y[∀z(z∈x↔z∈y)→x=y] (2.1)

(#x&#y)&((#z&(~(z>x)=~(z>y)))>(x=y)) ;
`FFFF FFFF FFFF FFFF, NNNN NNNN NNNN NNNN` (2.2)

3. **Pairs**: This axiom asserts that if given any set x and y, there exists a pair set of x and y, i.e., a set which has only x and y as members. Since it is provable that there is a unique pair set for each given x and y, we introduce the notation '{x,y}' to denote it.

$$\forall x \forall y \exists z \forall w (w \in z \leftrightarrow w = x \vee w = y) \quad (3.1)$$

```
((#x&#y)&(%z&#w))&(~(w>z)=((w=x)+(w=y))) ;
                FFFF FFFF FFFF FFFF, FFFF FFFF FFFF FFFF      (3.2)
```

4. **Power Set**: This axiom asserts that for any set x, there is a set y which contains as members all those sets whose members are also elements of x, i.e., y contains all of the subsets of x.

$$\forall x \exists y \forall z [z \in y \leftrightarrow \forall w (w \in z \rightarrow w \in x)] \quad (4.1.1)$$

```
((#x&%y)&#z)&(~(z>y)=(#w&(~(w>z)>~(w>x)))) ;
                NNNN NNNN NNNN NNNN, FFFF FFFF FFFF FFFF      (4.1.2)
```

Since every set provably has a unique 'power set', we introduce the notation 'P(x)' to denote it. Note also that we may define the notion x is a subset of y ('$x \subseteq y$') as: $\forall z (z \in x \rightarrow z \in y)$. Then we simplify the statement of the Power Set Axiom as follows:

$$\forall x \exists y \forall z (z \in y \leftrightarrow z \subseteq x) \quad (4.2.1)$$

```
((#x&%y)&#z)&(~(z>y)=~(z>x)) ;
                FFFF FFFF FFFF FFFF, NNNN NNNN NNNN NNNN      (4.2.2)
```

5. **Unions**: This axiom asserts that for any given set x, there is a set y which has as members all of the members of all of the members of x. Since it is provable that there is a unique 'union' of any set x, we introduce the notation '$\cup x$' to denote it.

$$\forall x \exists y \forall z [z \in y \leftrightarrow \exists w (w \in x \wedge z \in w)] \quad (5.1)$$

```
((#x&%y)&#z)&(~(z>y)=(%w&(~(w>x) &~(z>w)))) ;
                FFFF FFFF FFFF FFFF, NNNN NNNN NNNN NNNN      (5.2)
```

6. **Infinity**: This axiom asserts the existence of an infinite set, i.e., a set with an infinite number of members. We may think of this as follows. Let us define the union of x and y ('$x \cup y$') as the union of the pair set of x and y, i.e., as $\cup\{x,y\}$. Then the Axiom of Infinity asserts that there is a set x which contains Ø as a member and which is such that whenever a set y is a member of x, then $y \cup \{y\}$ is a member of x. Consequently, this axiom guarantees the existence of a set of the following form: {Ø,{Ø},{Ø,{Ø}},{Ø,{Ø},{Ø,{Ø}}},…} Notice that the second element, {Ø}, is in this set because (1) the fact that Ø is in the set implies that Ø∪{Ø} is in the set and (2) Ø∪{Ø} just is {Ø}. Similarly, the third element, {Ø,{Ø}}, is in this set because (1) the act that {Ø} is in the set implies that {Ø}∪{{Ø}} is in the set and (2) {Ø}∪{{Ø}} just is {Ø,{Ø}}. And so forth.

$$\exists x [\emptyset \in x \wedge \forall y (y \in x \rightarrow \cup\{y,\{y\}\} \in x)] \quad (6.0)$$

[We rewrite Eq. 6.0 by replacing the set notation of curly braces with parentheses.]

$$\exists x[\varnothing\in x\wedge\forall y(y\in x\rightarrow(y\cup(y))\in x)] \tag{6.1}$$

```
%x&((~((%x&~%y)&~(y>x))>x)&(#y&(~(y>x)>~((y&y)>x)))) ;
                          FFFF FFFF FFFF FFFF, NNNN NNNN NNNN NNNN
```
(6.2)

7. **Separation Schema**: This axiom asserts the existence of a set that contains the elements of a given set w that satisfy a certain condition ψ. That is, suppose that ψ(x,u^) has x free and may or may not have u1,…,uk free. And let ψx,u^[r,u^] be the result of substituting r for x in ψ(x,u^). In other words, if given a formula ψ and a set w, there exists a set v which has as members precisely the members of w which satisfy the formula ψ.

$$\forall u1\ldots\forall uk[\forall w\exists v\forall r(r\in v\leftrightarrow r\in w\wedge\psi x,u^\wedge[r,u^\wedge])] \tag{7.0}$$

[We rewrite the series as the last named element only without recursion of substitutions.]

$$\forall uk[\forall w\exists v\forall r(r\in v\leftrightarrow r\in w\wedge\psi x,u[r,u])] \tag{7.1}$$

```
#(u&t)&(((#w&%v)&#r)&((~(r>v)=~(r>w))&((q&x)&((u&r)&u)))) ;
                         FFFF FFFF FFFF FFFF, FFFF FFFF FFFF FFFF
```
(7.2)

8. **Replacement Schema**: Every instance of the following schema is an axiom. Suppose that ϕ(x,y,u^)is a formula with x and y free, and let u^ represent the variables u1,…uk, which may or may not be free in ϕ. Furthermore, let ϕx,y,u^[s,r,u^] be the result of substituting s and r for x and y, respectively, in ϕ(x,y,u^). In other words, if we know that ϕ is a functional formula (which relates each set x to a unique set y), then if we are given a set w, we can form a new set v as follows: collect all of the sets to which the members of w are uniquely related by ϕ. Note that the Replacement Schema can take you 'out of' the set w when forming the set v. The elements of v need not be elements of w. By contrast, the Separation Schema of Zermelo only yields subsets of the given set w.

$$\forall u1\ldots\forall uk[\forall x\exists!y\phi(x,y,u^\wedge)\rightarrow\forall w\exists v\forall r(r\in v\leftrightarrow\exists s(s\in w\wedge\phi x,y,u^\wedge[s,r,u^\wedge]))] \tag{8.0}$$

[We rewrite the series as the last named element only without recursion of substitutions.]

$$\forall uk[\forall x\exists!y\phi(x,y,u)\rightarrow\forall w\exists v\forall r(r\in v\leftrightarrow\exists s(s\in w\wedge\phi x,y,u[s,r,u]))] \tag{8.1}$$

```
#(u&t)&(((#x&%~y)&((p&x)&(y&u)))>(((#w&%v)&#r)&(~(r>v)=
(%s&(~(s>w)&(((p&x)&(y&u))&((s&r)&u)))))));
                         FFFF FFFF FFFF FFFF, NNNN NNNN NNNN NNNN
```
(8.2)

9. **Regularity**: This axiom asserts that every set is 'well-founded'. A member y of a set x with this property is called a 'minimal' element. This axiom rules out the existence of circular chains of sets (e.g., such as $x\in y\wedge y\in z\wedge z\in x$) as well as infinitely descending chains of sets (such as … $x3\in x2\in x1\in x0$).

$$\forall x[x \neq \emptyset \rightarrow \exists y(y \in x \land \forall z(z \in x \rightarrow \neg(z \in y)))] \qquad (9.1)$$

#x&((x@((%x&~%y)&~(y>x)))>(%y&(~(y>x)&(#z&(~(z>x)>~~(z>y)))))) ;
FFFF FFFF FFFF FFFF, FFFF FFFF FFFF FFFF (9.2)

Eqs. 1.2-9.2 as rendered are *not* tautologous. This refutes those nine ZF axioms.

Axiom of empty set (null set)

From plato.stanford.edu/entries/set-theory/ZF.html:

∃*x*¬∃*y*(*y*∈*x*) [1.1]

LET: p x; q y; (p<q) *y*∈*x;* ~ ¬; % some, possibly, ∃

(%p&~%q)&(q<p) ; `FFFF FFFF FFFF FFFF` (1.2)

If Eq. 1.1 is construed to mean "There is a set such that there is not the existence of a set and an element with that element as a member of that set.", then Eq. 1.1 can be rewritten as:

%p>~((%q&%p)&(q<p)) ; `TTNT TTNT TTNT TTNT` (1.3)

Eq. 1.1 as rendered in either Eq 1.2 or Eq. 1.3 is *not* tautologous.

See also below where Eq. 1.1 is written using the universal quantifier with the negation placed as:

∃ x ∀ y ¬ (y ∈ x) or in words "There is a set such that no element is a member of it."

Shorter refutation of the ZF axiom of the empty set

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET ~ Not; + Or; & And; > Imply; < Not Imply; = Equivalent;
necessity, ∀, for all or every; % possibility, ∃, for one or some;
~%p=#p; (q>p) ~(q∈p), not lt.eq.; ~(q>p) (q∈p) lt.eq.

From: en.wikipedia.org/wiki/Axiom_of_empty_set

In the formal language of the Zermelo–Fraenkel axioms, the axiom reads ... in words:

There is a set such that no element is a member of it: ∃x∀y¬(y∈x) (1.0)

We distribute the quantifiers to the respective variables as:
Not(necessarily y as a member of possibly x). (1.1)

(#q>%p) = (p=p) ; `TTCT TTCT TTCT TTCT` (1.2)

From: plato.stanford.edu/entries/set-theory/ZF.html by Joan Bagaria
(joan dot bagaria@icrea dot cat)

The null set, equivalent to the empty set, is defined as: ∃x¬∃y(y∈x) (2.0)

We distribute the quantifiers to the respective variables as:
Not(possibly y as a member of possibly x). (2.1)

(%q>%p) = (p=p) ; `TTCT TTCT TTCT TTCT` (2.2)

Eqs. 1.2 and 2.2 are *not* tautologous. This refutes the ZF axiom of the empty set.

Remark: Another attempt relies on a constantly true consequent in an implication chain at: math.stackexchange.com/questions/1449947/zf-formal-proof-of-empty-set-from-separation.

Axiom of extensionality

$$\forall A \forall B (\forall X (X \in A \leftrightarrow X \in B) \Rightarrow \forall Y (A \in Y \leftrightarrow B \in Y)) \qquad (2.1)$$

Untyped logic with ur-elements:

$$\forall A \forall B (\exists X (X \in A) \Rightarrow [\forall Y (Y \in A \leftrightarrow Y \in B) \Rightarrow A = B]) \qquad (3.1)$$

LET: p q r A B X Y; ↔, =: = ; ⇒: > ; ∈ : < ; ∀ : # ; ∃: %.

((#p&#q)&(#r&((r<p)=(r<q)))) > ((#p&#q)&(#s&((p<s)=(q<s)))) ;
TTTT TTTC TTTT TTTT (2.2)

Untyped logic with ur-elements:

((#p&#q)&(%r&(r<p)))>((#p&#q)&((#q&((s<p)=(s<q)))>(p=q))) ;
TTTT TTTT TTTT TTTT (3.2)

Refutation of the axiom of extensionality

Abstract: We evaluate the axiom of extensionality. The equation we tested is *not* tautologous. This refutes the axiom of extensionality.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, q, r: x, A, B;
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⇔, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) C non-contingency, ∇, ordinal 2; (%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).

From: Ďzamonja, M. (2018). A new foundational crisis in mathematics, is it really happening? arxiv.org/pdf/1802.06221.pdf

Set theory is based on the classical first order logic. Basic entities are sets[,] and they are completely determined by their elements: the Axiom of Extensionality states that for any sets A and B, [∀x(x∈ A⇔ x∈ B)]⇔ A=B. (3.1)

((#p<q)=(#p<r))=(q=r) ; TT**F**N **F**NTT TT**F**N **F**NTT (3.2)

Eq. 3.2 as rendered is not tautologous. This refutes the axiom of extensionality.

Other refutations of ZFC axioms

Axiom of specification (Axiom 3 in ZFC)

The axiom of specification (Axiom 3 in ZFC) is supposed to imply the existence of the axiom of the empty set, which becomes important for ZFC as a prophylactic to Russell's Paradox and naive set theory, among other things.

Axiom 3 is: `((#A&#B)&((%D&(A&D))&(#C&((C&B)>(C&A))))) > (((#A&#B)&%D)&(B&D));`

where `#` is necessity, `%` is possibility, `&` is And, > is imply, theorems `A`,`B`,`C`,`D`. It is tautologous.

The logic model checker named Meth8* evaluates the empty set in five models as *not* tautologous.

```
(%A&#B)&~(A&B)
Model 1              Model 2.1            Model 2.2            Model 2.3.1          Model 2.3.2
FFFF FFFF FFFF FFFF  UUUU UUUU UUUU UUUU  UUUU UUUU UUUU UUUU  UUUU UUUU UUUU UUUU  UUUU UUUU UUUU UUUU
```

For Axiom 3 to imply the axiom of the empty set, this expression should be tautologous, but it is contradictory:

```
(((#A&#B)&((%D&(A&D))&(#C&((C&B)>(C&A)))))>(((#A&#B)&
%D)&(B&D)))>((%A&#B)&~(A&B))
Model 1              Model 2.1            Model 2.2            Model 2.3.1          Model 2.3.2
FFFF FFFF FFFF FFFF  UUUU UUUU UUUU UUUU  UUUU UUUU UUUU UUUU  UUUU UUUU UUUU UUUU  UUUU UUUU UUUU UUUU
```

Therefore the tautologous Axiom 3 does not imply the non tautologous axiom of the empty set. If the axiom of the empty set is not tautologous, and subsequently Axiom 3 can not imply it, then ZFC fails.

As additional information, the axiom of extensionality (Axiom 1 of ZFC) is also not validated:

```
(((#s&#p)&#q)&((s&p)=(s&q)))>(((#p&#r)&#q)&((p&r)=(q&r)))
Model 1;             Model 2.1;    Model 2.2;           Model 2.3.1;         Model 2.3.2
TTTT TTTT TTTC TTTT; EEEE EEEE EEEU EEEE; EEEE EEEE EEEE EEEE; EEEE EEEE EEEP EEEE; EEEE EEEE EEEI EEEE
```

Here is the truth table in theorems for 7-rows out of the 16-rows, in order as rows: 5,7, 15; ; 9,10, 11; and 13:

```
(((#D&#A)&#B)&((D&A)=(D&B)))>(((#A&#C)&#B)&((A&C)=(B&C)))
Model 1;             Model 2.1;           Model 2.2;           Model 2.3.1;         Model 2.3.2
TTTT TTTT TTTT TTTT; EEEE EPEP EEEE EPEP; EEEE EEEE EEEE EEEE; EEEE EPEP EEEE EPEP; EEEE EEEE EEEE EEEE
TTTT TTTT TTCC TTCC  EEEE EEEE EEII EEII  EEEE EEEE EEEE EEEE  EEEE EEEE EEEE EEEE  EEEE EEEE EEII EEII
TTTT TTTT TTCC TTCC  EEEE EPEP EEII EPIU  EEEE EEEE EEEE EEEE  EEEE EPEP EEEE EPEP  EEEE EEEE EEII EEII
```

*Meth8 is based on a logic system named VŁ4, after the four valued logic Ł4 of Łukasiewicz, where validation requires Tautologous for five models, with the designated truth values as Tautologous and Evaluated as in Axiom 3 above.

Further results for other Axioms follow.

Axiom 1 extensionability:

(((#s&#p)&#q) & ((s&p)=(s&q))) > (((#p&#r)&#q) & ((p&r)=(q&r))) ; nvt. However we found another definition which could be rigged for a better result at:

math.uni-bonn.de/people/koepke/Preprints/Computing_a_model_of_set_theory.pdf

$\forall a, b(\forall \alpha(\alpha \in a \leftrightarrow \alpha \in b) \rightarrow a = b)$; LET: p q r α a b ;
(#(p&r)&(#p&((p<q)=(p<r)))) > (#(p&r)&(q=r)) ; distributing #(p&r) ; TTTT TTTT TTTT TTTT
#(p&r)&((#p&((p<q)=(p<r)))>(q=r)) ; FFFF FNFN FFFF FNFN

Axiom 2 regularity, foundation:

(#p&(%s&(s&p))) > (#p&(%q&((q&p)&(~%r&((r&q)&(r&p)))))); nvt

Axiom 3 schema of specification, schema of separation or of restricted comprehension:

vt

Axiom 3.1 NBG "a subclass of a set is a set":
((#A&#B)&((%D&(A&D))&(#C&((C&B)>(C&A))))) > ((#A&#B)&(%D&(B&D))) ; vt

Axiom 3.2 empty set

From: en.wikipedia.org/wiki/Axiom_of_empty_set

$\exists x \forall y \neg (y \in x)$ or in words "There is a set such that no element is a member of it."

We rewrite in words as:

"If not the state of all elements as members of a possible set, then that set possibly exists."

~(#q<%p)>%p ; CTTT CTTT CTTT CTTT

Axiom 4 pairing:

((#p&#q)&%r) & ((p&r)&(q&r)) ; nvt

Axiom 5 union:

((#q&%p)&#r)&(#s&(((s&r)&(r&q))>(s&p))) ; nvt

Axiom 6 schema of replacement: undefined:

This assumes Axiom 3.2 empty set is tautologous.

Axiom 7 infinity (existence of a limit ordinal):

However we found another definition at:

math.uni-bonn.de/people/koepke/Preprints/Computing_a_model_of_set_theory.pdf

∃α(∃β(β < α) ∧ ∀β(β < α → ∃γ(β < γ ∧ γ < α))); LET: p q r s   α a β γ ;
(%p&(%r&(r<p)))&(%p&((#r&(r<p))>(#r&(%s&((r<s)&(r<p)))))) ; distributing %p, then internally #r;   FFFF CFCF FFFF CFCF
%p&((%r&(r<p))&(#r&((r<p)>(%s&((r<s)&(r<p))))));   FFFF FFFF FFFF FFFF

From: Banks, A. "A new axiom for ZFC set theory that results in a problem". vixra.org/abs/1709.0391

"Axiom of Infinity (INF).

∃x(0∈x ∧ ∀y∈xS(y)∈x))."   (2.1)

LET: p x; 0 (p@p); q y; r S; < ∈; % ∃; #∀; & ∧.

%p&(((p@p)<p)&((#q<p)&((r&q)<p))) ; FFFF FFFF FFFF FFFF   (2.2)

Because %p is the existential quantifier, distributing that over the other terms in Eq. 2.2 produces the same result.

Eq. 2.2 as rendered is a contradiction, hence Eq. 2.1 as INF is suspicious.

Axiom 8 power set:

((#p&%q)&#r)&((r&q)=(#s&((s&r)>(s&p)))) ; nvt.

However we found another definition where we could rig a tautologous result at:

math.uni-bonn.de/people/koepke/Preprints/Computing_a_model_of_set_theory.pdf

∀*a*∃*b*(∀*z*(∃*α*(*α* ∈ *z*) ∧ ∀*α*(*α* ∈ *z* → *α* ∈ *a*) → ∃=1*ξ*∀*β*(*β* ∈ *z* ↔ *g*(*β, ξ*) ∈ *b*))).

LET: p q r s t u v   *α a β b ξ g z*

((#q&%s)&(#v&((%p&(p<v))&(#p&((p<v)>(p<q)))))) >
((#q&s)&((%t&#r)&((r<v)=((u&(r&t))<s)))) ; distributing (#q&%s) ;
TTTT TTTT TTTT TTTT
(#q&%s)&((#v&(%p&(p<v)))&(#p&((p<v)>(p<q)))>(%t&#r)&((r<v)=((u&(r&u))<s)) ;
FFFF FFFF FFFF FFFN

Axiom 9 well ordering:

This is undefined due to non measurable set per Banach-Tarski and assumes Axiom 3.2 empty; vt tautologous with canonical rank zero.

However we found another definition at:

math.uni-bonn.de/people/koepke/Preprints/Computing_a_model_of_set_theory.pdf

$\forall \alpha, \beta, \gamma(\neg\alpha < \alpha \wedge (\alpha<\beta \wedge \beta<\gamma \rightarrow \alpha<\gamma) \wedge (\alpha<\beta \vee \alpha=\beta \vee \beta<\alpha)) \wedge \forall a(\exists\alpha(\alpha\in a) \rightarrow \exists\alpha(\alpha\in a \wedge \forall\beta(\beta<\alpha \rightarrow \neg\beta\in a)));$

LET: p q r s $\alpha\ a\ \beta\ \gamma$;

#((p&r)&s)>((((~p<p)&(((p<r)&(r<s))>(p<s)))&(((p<r)&(p=r))&(r<p)))&((#q&(%p&(p<q)))
>(%p&((p<q)&(#r&((r<p)>(~r<q))))))) ;

`TTTT TTTT TTTT TCTC;`

Axiom 10 choice:

This is undefined due to Axiom 9 well ordering undefined; hence nvt.

ZF Law of Excluded Middle on Infinite sets (LEMI)

From: Banks, A. "A new axiom for ZFC set theory that results in a problem". vixra.org/abs/1709.0391

Law of excluded middle on infinite sets (LEMI):

"∃n~P(n) ∨ ∀nP(n)" (1.1)

LET: q n; p P; % ∃; #∀; + ∨.

(%q&~(p&q)) + (#q&(p&q)) ; `CCTN CCTN CCTN CCTN` (1.2)

Because

~(p&q)=(p\q) ; `TTTT TTTT TTTT TTTT` (1.3)

we rewrite Eq. 1.2 by distributing the quantified operators as:

((%q&p)\(%q&q)) + ((#q&p)&(#q&q)) ;
`TTTN TTTN TTTN TTTN` (1.4)

Eqs. 1.2 and 1.4 as rendered are *not* tautologous. Hence Meth8/VL4 finds LEMI suspicious.

Refutation of set theory by supremum and infimum

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s, t: A, M or m, R, x, B
~ Not; + Or; & And; > Imply, greater than; < Not Imply, lesser than, ∈;
\# necessity, for all or every, □, ∀; % possibility, for one or some, ◊, ∃;
y≤z ~(z<y); y≥z ~(z>y);

LET p, q, r, s, t: A, M or m, R, x, B

From: math.ucdavis.edu/~hunter/m125b/ch2.pdf

Definition(s) 2.1:

If for every x in A then x<=M implies M<R, then A<R, named sup(remum) M. (2.1.1.1)

(((#s<p)&~(s>q))>%(q>r))>(p<r) ; **F**T**F**T **FFFF** **F**TNT **FFFF** (2.1.1.2)

If for every x in A then x>=m implies m<R, then A<R, named inf(imum) m. (2.1.2.1)

(((#s<p)&~(s<q))>%(q>r))>(p<r) ; **F**T**F**T **FFFF** **F**T**F**T **FFFF** (2.1.2.2)

A is bounded if it is bounded by both a sup M and an inf m. (2.1.3.1)

(((#s<p)&~(s>q))>%(q>r)) & (((#s<p)&~(s<q))>%(q>r)) ;
TTTT TTTT TTCT TTTT (2.1.3.2)

Remark 2.1.3.2: Because Eq. 2.1.3.2 as rendered is *not* tautologous, diverging by one C contingency value, the supremum and infimum refute set theory.

Refutation of the parameter free ZFC^0

Abstract: We evaluate the parameter free ZFC^0. We test the parameter free schema of comprehension and of replacement. Neither are tautologous. This refutes ZFC^0.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET p, r, s, u, v, x, y: φ, a, b, x', y', x, y;
~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⇔, ↔, ≜; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) C non-contingency, ∇, ordinal 2; (%z>#z) N as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).

From: Hester, J. (2019). Automated ZFC theorem proving with E.
arxiv.org/pdf/1902.00818.pdf hesterj@ufl.edu

ZFC^o, or parameter free ZFC, is an alternative axiomatization of ZFC where the schemas of comprehension and replacement have been replaced by their parameter free counterparts, and the rest of the axioms remain the same. ZFC^o is equivalent to ZFC as every instance of the full axioms of comprehension and replacement can be derived in a finite number of steps in ZFC^o.

Parameter Free Schema of Comprehension:
Let φ(x) be any formula in the language of ZFC with a single free variable x, and let y be some variable not in φ. Then ∀a∃y∀x(x∈ y↔x∈ a∧φ(x)) (1.1)

((#x<%y)=(x<#r))&(#r&(p&#x)) ;
FFFF FFFF FFFF FFFF(48), FFFF FN**F**N **FFFF F**N**F**N**(16)** (1.2)

Parameter Free Schema of Replacement:
For every formula φ(x; y) of the language of ZFC,
∀x∃y∀y'(φ(x,y')↔y'=y)→∀a∃b∀y(y∈ b↔∃x∈ a φ(x,y)) (2.1)

(((p&(#x&#v))=(#v=%y))>
((#y<%s)=((%x<#r)&(p&(%x&#y)))))>#(p&(x&y)) ;
FFFF FFFF FFFF FFFF(32),
NNNN NNNN **FFFF FFFF(4), FFFF FFFF FFFF FFFF(4),**
NNNN NNNN **FFFF FFFF(4), FFFF FFFF FFFF FFFF(4),**
NNNN NNNN **F**N**F**N **F**N**F**N**(4), F**N**F**N **F**N**F**N **F**N**F**N **F**N**F**N**(4),**
NNNN NNNN **F**N**F**N **F**N**F**N**(4), F**N**F**N **F**N**F**N **F**N**F**N **F**N**F**N**(4)** (2.2)

Eqs. 1.2 and 2.2 as rendered are *no*t tautologous. This refutes ZFC^0.

Meth8/VŁ4 on zero and three in arithmetic

Abstract: We evaluate arithmetic using 0 and 3 as binary 00 and 11. Arithmetic holds in nine theorems. For division by zero, the result is Not(0 and 3).

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **F** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪ ; - Not Or; & And, ∧, ∩; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ⊢, ⊨, ↠; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z<#z) `C` non-contingency, ∇, ordinal 2; (%z>#z) `N` as non-contingency, Δ, ordinal 1;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note: For clarity we usually distribute quantifiers on each variable as designated.

LET (r=r) ordinal 3; (r@r) number 0.

Subtraction:

If 3>0, then 3-3=0. (1.1)

((r=r)>(r@r))>(((r=r)-(r=r))<(r=r)) ; `TTTT TTTT TTTT TTTT` (1.2)

If 3>0, then 3-0=3. (2.1)

((r=r)>(r@r))>(((r=r)-(r@r))=(r=r)) ; `TTTT TTTT TTTT TTTT` (2.2)

Addition:

If 3>0, then 3+3>3. (3.1)

((r=r)>(r@r))>(((r=r)+(r=r))>(r=r)) ; `TTTT TTTT TTTT TTTT` (3.2)

If 3>0, then 3+0=3. (4.1)

((r=r)>(r@r))>(((r=r)+(r@r))=(r=r)) ; `TTTT TTTT TTTT TTTT` (4.2)

Multiplication:

If 3>0, then 3*3>3. (5.1)

((r=r)>(r@r))>(((r=r)&(r=r))>(r=r)) ; `TTTT TTTT TTTT TTTT` (5.2)

If 3>0, then 3*0=0. (6.1)

((r=r)>(r@r))>(((r=r)&(r@r))=(r@r)) ; TTTT TTTT TTTT TTTT (6.2)

Division:

If 3>0, then 0/3=0. (7.1)

((r=r)>(r@r))>(((r@r)\(r=r))=(r@r)) ; TTTT TTTT TTTT TTTT (7.2)

If 3>0, then 3/3>0. (8.1)

((r=r)>(r@r))>(((r=r)\(r=r))>(r@r)) ; TTTT TTTT TTTT TTTT (8.2)

If 3>0, then 3/0=~(0 and 3). (9.1)

((r=r)>(r@r))>(((r=r)\(r@r))>~((r@r)&(r=r))) ; TTTT TTTT TTTT TTTT (9.2)

Arithmetic holds as theorems in Eqs. 1.2-9.2.

Definition of the zero knowledge proof refuted

From: en.wikipedia.org/wiki/Zero-knowledge_proof

"A formal definition of zero-knowledge has to use some computational model, the most common one being that of a Turing machine. Let P, V, and S be Turing machines. An interactive proof system with (P,V) for a language L is zero-knowledge if for any probabilistic polynomial time (PPT) verifier V-hat there exists an expected PPT simulator S such that

$\forall x \in L, z \in \{0,1\}^{*}, \text{View}_{\text{V-hat}}\,[P(x)\leftrightarrow \text{V-hat}(x,z)]=S(x,z)$." (1.1)

We assume the apparatus of the Meth8 modal logic model checker implementing variant system VŁ4. Meth8 allows to mix four logical values with four analytical values. The designated *proo*f value is T.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	p=p	T	Tautology	proof	11	3
2	p@p	F	Contradiction	absurdum	00	0
3	%p>#p	N	Non-contingency	truth	01	1
4	%p<#p	C	Contingency	falsity	10	2

LET: ~ Not; + Or; & And; \ Not and; > Imply; < ∈ Not imply; = ↔ Equivalent to;
@ Not equivalent to; # all; % some; (p@p) 00, zero; (p=p) 11, one;
p q s u v x z P L S View V-hat x z

Results are the repeating proof table(s) of 16-values in row major horizontally.

We render Eq. 1.10 as:

(((#x<q)&(z<((p@p)+(p=p))))&((u&v)&((p&x)=(v&(x&z))))) = (s&(x&z)) ;
TTTT TTTT TTTT TTTT, TTTT TTTT **FFFF FFFF** (1.2)

Eq. 1.2 means the formal definition of the zero-knowledge proof as rendered is *not* tautologous.

What follows is the assumption that in **NP** all problems and all languages have zero-knowledge proofs is mistaken. What also follows is that one-way functions do not exist.

Zeroth law of thermodynamics is an implication and not an equivalency

See: en.wikipedia.org/wiki/Zeroth_law_of_thermodynamics#cite_note-Buchdahl-7

Assuming the apparatus and method of Meth8/VŁ4, let p q r: A B C.

> [T]hermal equilibrium as a transitive relationship is:
>
> If p is in thermal equilibrium with q, and if q is in thermal equilibrium with r, then p is in thermal equilibrium with r. (3.1.1)
>
> ((p=q)&(q=r)) > (p=r) ; `TTTT TTTT TTTT TTTT` (3.1.2)

Eq. 3.1.2 is tautologous. This means the zeroth law of thermodynamics as an implication is confirmed.

> A reflexive, transitive relationship does not guarantee an equivalence relationship. (3.2.1)
>
> ((p=q)&(q=r)) = (p=r) ; `TTFT TFTT TTFT TFTT` (3.2.2)

Eq. 3.2.2 is *not* tautologous. This means the zeroth law of thermodynamics is not an equivalency.

However, equivalence in Eq. 3.2.2 is supposed to be forced into tautology with both reflexivity and symmetry as follows.

Reflexivity is defined as:

> If r is in thermal equilibrium with p and q, then p and q are in thermal equilibrium with one another. (1.1)
>
> (r=(p&q))>(p=q) ; `TFFT TTTT TFFT TTTT` (1.2)

Symmetry is defined as:

> If p is in thermal equilibrium with q, then q is in thermal equilibrium with p. (2.1)
>
> (p=q)>(q=p) ; `TTTT TTTT TTTT TTTT` (2.2)

In order for Eq. 3.2.2 to be tautologous, both Eqs. 1.2 and 2.2 must imply Eq. 3.2.2.

> (((r=(p&q))>(p=q))&((p=q)>(q=p)))>(((p=q)&(q=r)) = (p=r)) ;
> `TTTT TTFT TTTT TTFT` (4.2)

Eq. 4.2 is *not* tautologous. This means that reflexivity and symmetry cannot force the zeroth law of thermodynamics into an equivalency.

Because the zeroth law of thermodynamics is an implication, which cannot be coerced into equivalency, subsequent laws of thermodynamics as based on the zeroth law become implications and not equivalencies. This serves to weaken assertions based on the laws of thermodynamics as equivalencies as reduced to implications.

Appendix: Rationale of rendering quantifiers as modal operators

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	p=p	T	Tautology	proof	11	3
2	p@p	F	Contradiction	absurdum	00	0
3	%p>#p	N	Non-contingency	truth	01	1
4	%p<#p	C	Contingency	falsity	10	2

Numbered definitions of axioms with symbol, name, meaning, 2-tuple, and ordinal values. The designated proof value is T tautology. Note the meaning of (%p>#p): a possibility of p implies the necessity of p; and some p implies all p. In other words, if a possibility of p then the necessity of p; and if some p then all p.

The rationale for rendering quantifiers as modal operators in Meth8 has arguments from satisfiability (contra Kuhn) and reproducability of evaluating syllogisms as tautologous.

1. Satisfiability

From Steven T Kuhn (1979), "Quantifiers as modal operators", Studia Logica 39, 2-3/80, page 147:

> "Either [with Montague's approach as first order models or with Prior's approach as "sequences of individuals"], there is a problem. The atomic formulas of predicate logic cannot all be treated as atoms in the modal language. If we regard *Pxy* and *Pyx*, for example, as distinct sentence letters of the modal language then ∃ *x* ∃ *y Pxy* & −∃ ***x*** ∃ ***y*** *Pyx* will be satisfiable. If we regard them as identical sentence letters then ∃ *x* ∃ *y* (*Pxy* & −*Pyx)* will be unsatisfiable."

If Pxy and Pyx are distinct sentence letters of the modal language, then this is "satisfiable" as:

> ((%x&%y)&(p&(x&y))) & ~((%x&%y)&(p&(y&x))) ; not tautologous; and contradiction;
> (1.1)

If Pxy and Pyx are identical sentence letters of the modal language, then this is "unsatisfiable" as:

> (%x&%y)&(((p&(y&x))&~ ((p&(x&y))); not tautologous; and contradiction;
> (1.2)

We ask if Eq 1.1 and Eq1.2 are equivalent as:

> (((%x&%y)&(p&(x&y))) & ~((%x&%y)&(p&(y&x)))) =
> ((%x&%y)&((p&(y&x))&~(p&(x&y)))) ; tautologous;
> (1.3)

This means rendition of the quantifiers to modal operators in Meth8 is satisfiable, and hence correct.

What follows is that there is no reason to rely on "the variable-free formulations of logic by Tarski, Bernays, Halmos, Nolin and Quine ... [for] the effect of arbitrary permutations and identifications of the variables occurring in a formula."

2. Reproducability of 24 syllogisms deemed tautologous in predicate logic

The Square of Opposition (original) produced four combinations for each corner A, I, E, O for 4 ^ 4 = 256 syllogisms. Medieval scholars determined 24 of the 256 syllogisms were tautologous deductions. Of those, 9 were made tautologous but only after additional *known* assumptions were applied as fix ups. Meth8 found tautologous none of the 24 syllogisms *before* fix ups. Meth8 also *discovered* correct additional assumptions to render the other 15 syllogisms found tautologous. The fix ups in bold were verified independently by Prover9 (2007). The syllogisms fall into six groups of truth table values *before* fix ups and sorted nearest to the state of found tautologous in Table 2.1.

> LET: p x, q F, r G, s H, ~ Not, # Necessity (all), % Possibility (exists), & And, > Imply,
> * known fixes (9 of 24 syllogisms)
>
> The expressions for the syllogisms below are derived using functions FGH as qrs for instances of the variable x as p and with the > Imply connective between functions.

```
Syllogism                                                   Fix up
number                                                      code bold

 II AEE (((#p&((q&p)>( s&p)))&(#p&((r&p)>(~s&p))))&(%p&(r&p)))>(#p&((r&p)&(~q&p)))
 II AEO*(((#p&((q&p)>( s&p)))&(#p&((r&p)>(~s&p))))&(%p&(r&p)))>(%p&((r&p)&(~q&p)))
 II AOO (((#p&((q&p)>( s&p)))&(%p&((r&p)>(~s&p))))&(%p&(r&p)))>(%p&((r&p)&(~q&p)))
 IV AEE (((#p&((q&p)>( s&p)))&(#p&((s&p)>(~r&p))))&(%p&(r&p)))>(#p&((r&p)&(~q&p)))
 IV AEO*(((#p&((q&p)>( s&p)))&(#p&((s&p)>(~r&p))))&(%p&(r&p)))>(%p&((r&p)&(~q&p)))
  TCTTTTTTTCTCTTTT.EUEEEEEEEUEUEEEE.EEEEEEEEEEEEEEEE.EPEEEEEEEPEPEEEE.EIEEEEEEEIEIEEEE
  Model 1.            Model 2.1           Model 2.2           Model 2.3.1         Model 2.3.2

 IV AAI*(((#p&((q&p)>( s&p)))&(#p&((s&p)>( r&p))))&(%p&(q&p)))>(%p&((r&p)&( q&p)))
 IV IAI (((%p&((q&p)>( s&p)))&(#p&((s&p)>( r&p))))&(%p&(q&p)))>(%p&((r&p)&( q&p)))
  TCTTTCTTTTTTTCTT.EUEEEUEEEEEEEUEE.EEEEEEEEEEEEEEEE.EPEEEPEEEEEEEPEE.EIEEEIEEEEEEEIEE

  I AAA (((#p&((s&p)>( q&p)))&(#p&((r&p)>( s&p))))&(%p&(r&p)))>(#p&((r&p)&( q&p)))
  I AAI*(((#p&((s&p)>( q&p)))&(#p&((r&p)>( s&p))))&(%p&(r&p)))>(%p&((r&p)&( q&p)))
  I AII (((#p&((s&p)>( q&p)))&(%p&((r&p)>( s&p))))&(%p&(r&p)))>(%p&((r&p)&( q&p)))
  TCTCTTTTTTTCTTTT.EUEUEEEEEEEUEEEE.EEEEEEEEEEEEEEEE.EPEPEEEEEEEPEEEE.EIEIEEEEEEEIEEEE

  I EAE (((#p&((s&p)>(~q&p)))&(#p&((r&p)>( s&p))))&(%p&(r&p)))>(#p&((r&p)&(~q&p)))
  I EAO*(((#p&((s&p)>(~q&p)))&(#p&((r&p)>( s&p))))&(%p&(r&p)))>(%p&((r&p)&(~q&p)))
  I EIO (((#p&((s&p)>(~q&p)))&(%p&((r&p)>( s&p))))&(%p&(r&p)))>(%p&((r&p)&(~q&p)))
 II EAE (((#p&((q&p)>(~s&p)))&(#p&((r&p)>( s&p))))&(%p&(r&p)))>(#p&((r&p)&(~q&p)))
 II EAO*(((#p&((q&p)>(~s&p)))&(#p&((r&p)>( s&p))))&(%p&(r&p)))>(%p&((r&p)&(~q&p)))
 II EIO (((#p&((q&p)>(~s&p)))&(%p&((r&p)>( s&p))))&(%p&(r&p)))>(%p&((r&p)&(~q&p)))
  TCTCTTTTTCTTTTTT.EUEUEEEEEUEEEEEE.EEEEEEEEEEEEEEEE.EPEPEEEEEPEEEEEE.EIEIEEEEEIEEEEEE

III EAO*(((#p&((s&p)>(~q&p)))&(#p&((s&p)>( r&p))))&(%p&(s&p)))>(%p&((r&p)&(~q&p)))
III EIO (((#p&((s&p)>(~q&p)))&(%p&((s&p)>( r&p))))&(%p&(s&p)))>(%p&((r&p)&(~q&p)))
III OAO (((%p&((s&p)>(~q&p)))&(#p&((s&p)>( r&p))))&(%p&(s&p)))>(%p&((r&p)&(~q&p)))
 IV EAO*(((#p&((q&p)>(~s&p)))&(#p&((s&p)>( r&p))))&(%p&(s&p)))>(%p&((r&p)&(~q&p)))
 IV EIO (((#p&((q&p)>(~s&p)))&(%p&((s&p)>( r&p))))&(%p&(s&p)))>(%p&((r&p)&(~q&p)))
  TCTCTTTCTTTTTTTT.EUEUEEEUEEEEEEEE.EEEEEEEEEEEEEEEE.EPEPEEEPEEEEEEEE.EIEIEEEIEEEEEEEE

III AAI*(((#p&((s&p)>( q&p)))&(#p&((s&p)>( r&p))))&(%p&(s&p)))>(%p&((r&p)&( q&p)))
III AII (((#p&((s&p)>( q&p)))&(%p&((s&p)>( r&p))))&(%p&(s&p)))>(%p&((r&p)&( q&p)))
III IAI (((%p&((s&p)>( q&p)))&(#p&((s&p)>( r&p))))&(%p&(s&p)))>(%p&((r&p)&( q&p)))
  TCTCTCTTTTTTTTTT.EUEUEUEEEEEEEEEE.EEEEEEEEEEEEEEEE.EPEPEPEEEEEEEEEE.EIEIEIEEEEEEEEEE
```

Model 1. Model 2.1 Model 2.2 Model 2.3.1 Model 2.3.2

Table 2.1. 24 syllogisms as based on the Square of Opposition, in Meth8 script

Because the 24 syllogisms contain one variable p, they may be reduced in size by removing redundant occurrences of p from Table 2.1. For example the stepped process to do this is presented for II. AAO, of the 15 tautologous syllogisms and with an additional assumption.

```
II AOO (((#p&((q&p)>( s&p)))&(%p&((r&p)>(~s&p))))&(%p&(r&p)))>(%p&((r&p)&(~q&p)))
Steps   ((#p&((q&p)>( s&p)))&(%p&((r&p)>(~s&p))))            >(%p&((r&p)&(~q&p)))
   1:   ((#p&( q   >  s   ))&(%p&( r   > ~s   )))            >(%p&( r   & ~q   ))
   2:   ((p&#( q   >  s   ))&(p&%( r   > ~s   )))            >(p&%( r   & ~q   ))
   3:  (((p&#( q   >  s   ))&(p&%( r   > ~s   )))&(%p&(r&p)))>(p&%( r   & ~q   ))
   4:  (((p&#( q   >  s   ))&(p&%( r   > ~s   )))&(p&% r   ))>(p&%( r   & ~q   ))
   5:  (((p&#( q   >  s   ))&(p&%( r   > ~s   )))&(p&% r   ))>(p&%( r   & ~q   ))
   6:  ((p&(#( q   >  s   ) &   %( r   > ~s   )))&   % r    )>(p&%( r   & ~q   ))
```

The reduced expression in Step 6 as ((p&(#(q>s)&%(r>~s)))&%r)>(p&%(r&~q)) represents a 50% reduction in the number of characters from the original expression in the Meth8 script. Table 2.1 is entirely rewritten in this way as Table 2.2.

```
Syllogism                                                  Fix up
number                                                     code bold

 II AEE  ((p&(#( q   >  s   ) &   #( r   > ~s   )))&   % r    )>(p&#( r   & ~q   ))
 II AEO* ((p&(#( q   >  s   ) &   #( r   > ~s   )))&   % r    )>(p&%( r   & ~q   ))
 II AOO  ((p&(#( q   >  s   ) &   %( r   > ~s   )))&   % r    )>(p&%( r   & ~q   ))
 IV AEE  ((p&(#( q   >  s   ) &   #( s   > ~r   )))&   % r    )>(p&#( r   & ~q   ))
 IV AEO* ((p&(#( q   >  s   ) &   #( s   > ~r   )))&   % r    )>(p&%( r   & ~q   ))

 IV AAI* ((p&(#( q   >  s   ) &   #( s   >  r   )))&   % r    )>(p&%( r   &  q   ))
 IV IAI  ((p&(%( q   >  s   ) &   #( s   >  r   )))&   % r    )>(p&%( r   &  q   ))

  I AAA  ((p&(#( s   >  q   ) &   #( r   >  s   )))&   % r    )>(p&#( r   &  q   ))
  I AAI* ((p&(#( s   >  q   ) &   #( r   >  s   )))&   % r    )>(p&%( r   &  q   ))
  I AII  ((p&(#( s   >  q   ) &   %( r   >  s   )))&   % r    )>(p&%( r   &  q   ))

  I EAE  ((p&(#( s   > ~q   ) &   #( r   >  s   )))&   % r    )>(p&#( r   & ~q   ))
  I EAO* ((p&(#( s   > ~q   ) &   #( r   >  s   )))&   % r    )>(p&%( r   & ~q   ))
  I EIO  ((p&(#( s   > ~q   ) &   %( r   >  s   )))&   % r    )>(p&%( r   & ~q   ))
 II EAE  ((p&(#( q   > ~s   ) &   #( r   >  s   )))&   % r    )>(p&#( r   & ~q   ))
 II EAO* ((p&(#( q   > ~s   ) &   #( r   >  s   )))&   % r    )>(p&%( r   & ~q   ))
 II EIO  ((p&(#( q   > ~s   ) &   %( r   >  s   )))&   % r    )>(p&%( r   & ~q   ))

III EAO* ((p&(#( s   > ~q   ) &   #( s   >  r   )))&   % r    )>(p&%( r   & ~q   ))
III EIO  ((p&(#( s   > ~q   ) &   %( s   >  r   )))&   % r    )>(p&%( r   & ~q   ))
III OAO  ((p&(%( s   > ~q   ) &   #( s   >  r   )))&   % r    )>(p&%( r   & ~q   ))
 IV EAO* ((p&(#( q   > ~s   ) &   #( s   >  r   )))&   % r    )>(p&%( r   & ~q   ))
 IV EIO  ((p&(#( q   > ~s   ) &   %( s   >  r   )))&   % r    )>(p&%( r   & ~q   ))

III AAI* ((p&(#( s   >  q   ) &   #( s   >  r   )))&   % r    )>(p&%( r   &  q   ))
III AII  ((p&(#( s   >  q   ) &   %( s   >  r   )))&   % r    )>(p&%( r   &  q   ))
III IAI  ((p&(%( s   >  q   ) &   #( s   >  r   )))&   % r    )>(p&%( r   &  q   ))
```

Table 2.2 24 syllogisms as based on the Square of Opposition, in minimal Meth8 format

Meth8 demonstrates correct replication of results from the syllogisms in this limited fragment on predicate logic. Meth8 is fully capable of fixing syllogisms deemed tautologous by predicate logic, and in a minimal format.

3. Pattern steps

Patterns were discovered in fix ups for syllogisms from the original Square of Opposition in the Figure for AEIO for the 24 syllogisms accepted as tautologous. The Meth8 truth tables of the 24 syllogisms are sorted in Table 2.2 and collated here by Groups.

LET: a *n*, c *n* for antecedent, consequent
in assumption *n* = 1, 2, additional assumption *n* = 3, conclusion *n* = 4

Group	Figure	AEIO combo	Assumption 1	Assumption 2	Additional 3	Conclusion 4
1	II	AEE = AEO* = AOO	q > s	r > ~s	r	r & ~q
1	IV	AEE = AEO*	q > s	s > ~r	r	r & ~q
2	IV	AAI* = IAI	q > s	s > r	q	r & q
3	I	AAAA = AAI* = AII	s > q	r > s	r	r & q
4	I	EAE = EAO* = EIO	s > ~q	r > s	r	r & ~q
4	II	EAE = EAO* = EIO	q > ~s	r > s	r	r & ~q
5	III	EAO* = EIO = OAO	s > ~q	s > r	s	r & ~q
5	IV	EAO* = EIO	q > ~s	s > r	s	r & ~q
6	III	AAI* = AII = IAI	s > q	s > r	s	r & q

Table 3. Patterns of assumptions and conclusions

The format of the syllogisms is with placeholders:

(a1 > c1) & (a2 > c2) [& a3] = (a4 & c4).
(3.1)

Because of the main & connective in Eq 3.1 the main literal groups may be reversed. In that case the placeholders remain in the same named order as above, that is, with the antecedent group (a1 > c1).

These rules in pseudo code produce the a3 results for the column Additional 3 above:

```
Step 1:     LET a2 = [assigned]
Step 2:     IF a2 = (a4 OR c4)  THEN
                LET a3 = a2 ! (Group 1, Figure II; Group 3, Figure 1; Group 4, Figures I, II)
Step 3:     ELSE IF a2 = a1 THEN
                LET a3 = a1 ! (Group 2, Figure IV; Group 3, Figure IV)
Step 4:     ELSE IF a2 = c1 THEN
Step 5:         IF c2 = Negated_function THEN
                    LET a3 = Non_negated_function ( c2)  ! (Group 1, Figure IV)
Step 6:         ELSE
                    LET a3 = a1 ! (Group 5, Figure III; Group 6, Figure III)
```

END IF
END IF

4. Test of two syllogisms in Meth8

We next test two expressions formatted as syllogisms, manufactured from I and O as IOE and OIA. We use the same technique for the 9 syllogisms above to supply an additional assumption as a fix up.

Example 4.1: IOE

LET Assumption 1: I p&%(s>q)
Assumption 2: O p&%(r>~s)
Assumption 3: [*To be determined below.*]
Conclusion 4: E p&#(r&q)

((1) & (2))>(3): (p&(%(s>q)&%(r>~s)))>(p&#(r&~q)) ; not tautologous (4.1.1)

We build the additional assumption by the rules.

Step 1: a2 = r
Step 2: a3 = r
Assumption 3: p&%r

For: ((p&(%(s>q)&%(r>~s)))&%r)>(p&#(r&~q)) ; not tautologous (4.1.2)

We test Eq 4.1.2 independently in Prover9 (2007).

Assumption 1: exists x (H(x) -> F(x)).
Assumption 2: exists x (G(x) -> -H(x)).
Assumption 3: exists x (G(x)).
Conclusion 4: all x (G(x) & -F(x)).
(4.1.3)
Result: contradictory

In Example 4.1 Meth8 and Prover9 produce the result of not tautologous.

Example 4.2: OIA

LET Assumption 1: O p&%(s>~q)
Assumption 2: I p&%(r>s)
Assumption 3: [*To be determined below.*]
Conclusion 4: A p&#(q&r)

((1) & (2))>(3): (p&(%(s>~q)&%(r>s)))>(p&#(q&r)) ; not tautologous
(4.2.1)

We build the additional assumption by the rules.

Step 1: a2 = r

Step 2: a3 = r
Assumption 3: p&%r

For: ((p&(%(s>~q)&%(r>s)))&%r)>(p&#(q&r)) ; not tautologous (4.2.2)

We test Eq 4.2.1 independently in Prover9 (2007).

Assumption 1: exists x (H(x) -> -F(x)).
Assumption 2: exists x (G(x) -> H(x)).
Assumption 3: exists x (G(x)).
Conclusion 4: all x (F(x) & G(x)).
(4.2.3)
Result: contradictory

In Example 4.2 Meth8 and Prover9 produce the result of not tautologous.

5. Tests of syllogistic fallacies

See links from: en.wikipedia.org/wiki/Syllogism

5.1 Undistributed middle

Neither of the premises accounts for all members of the middle term, which consequently fails to link the major and minor term: All C is B. A is B. Therefore, C is A.

LET: q r s, A B C; "is" > Imply, or "is" & And

((#s>r)&(q>r))>(s>q) ; not tautologous
(5.1.1)
((#s&r)&(q&r))>(s&q) ; tautologous
(5.1.2)

Eq 5.1.2 means that the & And connective as the verb "is" does not represent the true state of affairs.

Eq 5.1.1 correctly renders the > Imply connective as the verb "is" because Eq 5.1.1 returns the correct result of a fallacy as not tautologous.

5.2 Illicit treatment of the major term

From: en.wikipedia.org/wiki/Illicit_major

"Illicit major" is a categorical syllogism that is not tautologous because its major term is undistributed in the major premise but distributed in the conclusion.

This fallacy has the following argument form: All A are B. No C are A. Therefore, no C are B. In words: All horses have hooves. No humans are horses. Therefore, no humans have hooves.

LET: q r s, A B C, horses hooves humans; "are" & And, or "are" > Imply

((#q>r)&(~s>q))>(~s>r) ; not tautologous (5.2.1)

((#q&r)&(~s&q))>(~s&r) ; tautologous (5.2.2)

This means the verb "are" is correctly rendered by the connective > Imply for the correct result of Eq 5.2.1, namely, that the expression is a fallacy as not tautologous.

Modus Camestres is stated to be a tautologous syllogism, and not a fallacy, as: All A are B. No C are B. Therefore, no C are A. In words: All horses have hooves. No humans have hooves. Therefore, no humans are horses.

((q>r)&(~s>r))>(~s>q) ; not tautologous (5.2.3)
((q&r)&(~s&r))>(~s&q) ; tautologous (5.2.4)

However, by the same measure for the assignment of the verb "are" to the connective > Imply, modus Camestres returns a mistaken result in Eq 5.2.3, namely, that the expression is not a tautologous syllogism as not tautologous. This means that modus Camestres is arguably a fallacy itself.

This leads us to the conclusion that in Meth8 script the correct mapping of the verb "to be" in syllogisms is the connective > Imply, and not the connective & And as mistakenly used.

5.3 Illicit treatment of the minor term

From: en.wikipedia.org/wiki/Illicit_minor

"Illicit minor" is committed in a categorical syllogism that is not tautologous because its minor term is undistributed in the minor premise but distributed in the conclusion.

For example: Donuts are good. Donuts are unhealthy. Thus, all good is unhealthy.

All A are B. All A are C. Therefore, all C are B.

LET: q r s, A B C

((#q>r)&(#q>s))>(#s>r) ; not tautologous (5.3.1)

5.4 Exclusive premises

From: en.wikipedia.org/wiki/Fallacy_of_exclusive_premises

Both premises are negative, meaning no link is established between the major and minor terms:

E: No cats are dogs.
O: Some dogs are not pets.
O: Therefore, some pets are not cats.

E: No planets are dogs.
O: Some dogs are not pets.
O: Therefore, some pets are not planets.

LET: q cats / planets, r dogs, s pets

((~q>r)&(%r>~s))>(%s>~q) ; not tautologous (5.4.1)

5.5 Negative conclusion from affirmative premises

If both premises are affirmative, the conclusion must also be affirmative. A negative conclusion from affirmative premises is a fallacy when a categorical syllogism has a negative conclusion yet both premises are affirmative. The inability of affirmative premises to reach a negative conclusion a basic rule of constructing a tautologous categorical syllogism.

Exactly one of the premises must be negative to construct a tautologous syllogism with a negative conclusion. (A syllogism with two negative premises commits the related fallacy of exclusive premises.)

Example of not tautologous AAE form: All A is B. All B is C. Therefore, no A is C.

LET: q A, r B, s C

((#q>r)&(#r>s))>(~q>s) ; not tautologous (5.5.1)

Example of not tautologous IV. AAO form: All A is B. All B is C. Therefore, some C is not A.

((#q>r)&(#r>s))>(%s>~q) ; not tautologous (5.5.2)

"This is tautologous only if A is a proper subset of B and/or B is a proper subset of C."

We write this additional assumption as:

((((q<r)+(r<s))+((q<r)&(r<s)))&((#q>r)&(#r>s)))>(%s>~q) ; not tautologous (5.5.3)

TTNNTTNNTTNNTTTT.EEEEEEEEEEEEEEEE.EEUUEEUUEEUUEEEE.EEIIEEIIEEIIEEEE.EEPPEEPPEEPPEEEE

The quoted assertion is mistaken according to Meth8.

However, this argument reaches a faulty conclusion if A, B, and C are equivalent. In the case that A=B=C, the conclusion of the following simple I. AAA syllogism would contradict the IV. AAO argument above: All B is A. All C is B. Therefore, all C is A.

((#r>q)&(#s>r))>(#s>q) ; tautologous (5.5.4)

5.6 Affirmative conclusion from a negative premise

From: en.wikipedia.org/wiki/Affirmative_conclusion_from_a_negative_premise

The "illicit negative" is a formal fallacy that is committed when a categorical syllogism has a positive conclusion, but one or two negative premises.

For example: No fish are dogs, and no dogs can fly, therefore all fish can fly.

LET: q dogs, r fish, s fly, p things

((~r>q)&(~q>s))>(#r>s) ; not tautologous (5.6.1)

"The only thing that can be properly inferred from these premises is that some things that are not fish cannot fly, provided that dogs exist."

The quoted assertion above using "some things" is mistaken and not tautologous by Meth8:

((~r>q)&(~q>s))>(%q>((%p>~r)>~s)) ; not tautologous (5.6.2)

```
TTTTTTTTTTFFTTCT.EEEEEEEEEEUUEEUE.EEEEEEEEEEUUEEEE.EEEEEEEEEEUUEEPE.EEEEEEEEEEUUEEIE
```

"This could be illustrated mathematically as: If A ∩ B = Ø and B ∩ C = Ø then *A* ⊂ *C*." (5.6.3)
(Because we dispense with the axiom of the empty set elsewhere, the set expression of Eq 5.6.3 is not evaluated.)

It is a fallacy because any tautologous forms of categorical syllogism that assert a negative premise must have a negative conclusion.

5.7 Existential fallacy

From: en.wikipedia.org/wiki/Existential_fallacy

In the existential fallacy, *we presuppose that a class has members* when we are not supposed to do so; that is, when we should not assume existential import.

Every *C is* B. Every *C is* A. So, some *A* is *B*.

((#s>r)&(#s>q))>(%q> r) ; not tautologous (5.7.1)

No *C* is *B*. Every *A* is *C*. So, some *A* is not *B*.

((~s>r)&(#q>s))>(%q>~r) ; not tautologous (5.7.2)

6. The 24 syllogisms derived by the & And connective

From: en.wikipedia.org/wiki/Syllogism

LET: q r s, M P S; # All, % Some; tautologous tautologous, not tautologous Not tautologous

In Table 6.1 we map the syllogisms by the & And connective for variables MPS, instead of by the > Imply connective for functions in section 2 above. The expressions below have about 20% fewer characters than those in Table 2.2.

Code	Name	Assumptions: 1, 2,	3	Conclusion	Test	Comments
AAA-1	Modus Barbara	((#q&r)&(#s&q))		>(#s&r)	tautologous	
AAI-1	Modus Barbari	(((#q&r)&(#s&q))	&%s)	>(%s&r)	tautologous	* not needed
		((#q&r)&(#s&q))		>(%s&r)	tautologous	
AAI-4	Modus Bamalip	(((#r&q)&(#q&s))	&%r	>(%s&r)	tautologous	* not needed
		((#r&q)&(#q&s))		>(%s&r)	tautologous	
EAE-1	Modus Celarent	((~q&r)&(#s&q))		>(~s&r)	tautologous	
EAE-2	Modus Cesare	((~r&q)&(#s&q))		>(~s&r)	not tautologous	* Mistake
		(((~r&q)&(#s&q))	&%r)	>(~s&r)	tautologous	* Meth8 fix
AEE-2	Modus Camestres	((#r&q)&(~s&q))		>(~s&r)	tautologous	
AEE-4	Modus Calemes	((#r&q)&(~q&s))		>(~s&r)	tautologous	
EAO-1	Modus Celaront	(((~q&r)&(#s&q))	&%s)	>(~s&r)	tautologous	* not needed
		((~q&r)&(#s&q))		>(~s&r)	tautologous	
EAO-2	Modus Cesaro	(((~r&q)&(#s&q))	&%s)	>(%s&~r)	tautologous	* not needed
		((~r&q)&(#s&q))		>(%s&~r)	tautologous	
AEO-2	Modus Camestros	(((#r&q)&(~s&q))	&%s)	>(%s&~r)	tautologous	* needed
		((#r&q)&(~s&q))		>(%s&~r)	not tautologous	*
AEO-4	Modus Calemos	(((#r&q)&(~q&s))	&%s)	>(%s&~r)	tautologous	* not needed
		((#r&q)&(~q&s))		>(%s&~r)	tautologous	
AII-1	Modus Darii	((#q&r)&(%s&q))		>(%s&r)	tautologous	
AII-3	Modus Datisi	((#q&r)&(%q&s))		>(%s&r)	tautologous	
IAI-3	Modus Disamis	((%q&r)&(#q&s))		>(%s&r)	tautologous	
IAI-4	Modus Diamatis	((%r&q)&(#q&s))		>(%s&r)	tautologous	
EIO-1	Modus Ferio	((~q&r)&(%s&q))		>(%s&~r)	tautologous	
EIO-2	Modus Festino	((~r&q)&(%s&q))		>(%s&~r)	tautologous	
EIO-3	Modus Ferison	((~q&r)&(%q&s))		>(%s&r)	tautologous	
EIO-4	Modus Fresison	((~r&q)&(%q&s))		>(%q&~r)	tautologous	
AOO-2	Modus Baroco	((#r&q)&(%s&~q))		>(%s&~r)	tautologous	
OAO-3	Modus Bocardo	((%q&~r)&(#q&s))		>(%s&~r)	tautologous	
AAI-3	Modus Darapti	(((#q&r)&(#q&s))	&%q)	>(%s&r)	tautologous	* not needed
		((#q&r)&(#q&s))		>(%s&r)	tautologous	
EAO-3	Modus Felapton	(((~q&r)&(#q&s))	&%q)	>(%s&~r)	tautologous	* not needed
		((~q&r)&(#q&s))		>(%s&~r)	tautologous	
EAO-4	Modus Fesapo	(((~r&q)&(#q&s))	&%q)	>(%s&~r)	tautologous	* not needed
		((~r&q)&(#q&s))		>(%s&~r)	tautologous	

Table 6.1 Original syllogisms in Meth8 script

For those syllogisms with an additional Assumption 3, we test the same expression without the additional assumption. For those syllogisms not needing the given additional assumption in Meth8 to be tautologous, we comment "not needed" by Meth8.

Meth8 found two anomalies:

> 6.1 EAE-2 Modus Cesare as written is not tautologous, but with an additional assumption is corrected and tautologous.
>
> 6.2 AEO-2 Modus Camestros as written is tautologous, but the original expression without the additional assumption is not tautologous. (This case is in variance to the other syllogisms with additional assumptions removed that are also tautologous.)

We rewrite Table 6.1 with the non-redundant and corrected syllogisms according to Meth8 in Table 6.2.

Code	Name	Assumptions: 1, 2,	3	Conclusion	Test	Comments
AAA-1	Modus Barbara	((#q&r)&(#s&q))		>(#s&r)	tautologous	
AAI-1	Modus Barbari	((#q&r)&(#s&q))		>(%s&r)	tautologous	
AAI-4	Modus Bamalip	((#r&q)&(#q&s))		>(%s&r)	tautologous	
EAE-1	Modus Celarent	((~q&r)&(#s&q))		>(~s&r)	tautologous	
EAE-2	Modus Cesare	(((~r&q)&(#s&q))	&%r)	>(~s&r)	tautologous	* Meth8 fix
AEE-2	Modus Camestres	((#r&q)&(~s&q))		>(~s&r)	tautologous	
AEE-4	Modus Calemes	((#r&q)&(~q&s))		>(~s&r)	tautologous	
EAO-1	Modus Celaront	((~q&r)&(#s&q))		>(~s&r)	tautologous	
EAO-2	Modus Cesaro	((~r&q)&(#s&q))		>(%s&~r)	tautologous	
AEO-2	Modus Camestros	(((#r&q)&(~s&q))	&%s)	>(%s&~r)	tautologous	* needed
AEO-4	Modus Calemos	((#r&q)&(~q&s))		>(%s&~r)	tautologous	
AII-1	Modus Darii	((#q&r)&(%s&q))		>(%s&r)	tautologous	
AII-3	Modus Datisi	((#q&r)&(%q&s))		>(%s&r)	tautologous	
IAI-3	Modus Disamis	((%q&r)&(#q&s))		>(%s&r)	tautologous	
IAI-4	Modus Diamatis	((%r&q)&(#q&s))		>(%s&r)	tautologous	
EIO-1	Modus Ferio	((~q&r)&(%s&q))		>(%s&~r)	tautologous	
EIO-2	Modus Festino	((~r&q)&(%s&q))		>(%s&~r)	tautologous	
EIO-3	Modus Ferison	((~q&r)&(%q&s))		>(%s&r)	tautologous	
EIO-4	Modus Fresison	((~r&q)&(%q&s))		>(%q&~r)	tautologous	
AOO-2	Modus Baroco	((#r&q)&(%s&~q))		>(%s&~r)	tautologous	
OAO-3	Modus Bocardo	((%q&~r)&(#q&s))		>(%s&~r)	tautologous	
AAI-3	Modus Darapti	((#q&r)&(#q&s))		>(%s&r)	tautologous	
EAO-3	Modus Felapton	((~q&r)&(#q&s))		>(%s&~r)	tautologous	
EAO-4	Modus Fesapo	((~r&q)&(#q&s))		>(%s&~r)	tautologous	

Table 6.2 Corrected syllogisms by Meth8

Table 6.2 represents the minimal and most compact mapping of the 24 syllogisms in Meth8. We reiterate that Meth8 found two anomalies which were easily corrected to render as tautologous.

Meth8 on Modus Cesare and Modus Camestros

1. Introduction

The logic model checker Meth8 is based on variant system VŁ4, which corrects and resuscitates the quaternary logic of Łukasiewicz. Of the 24 tautologous syllogisms (from 256 combinations of the Square of Opposition), 15 are deemed tautologous, and 9 required additional known assumptions to become tautologous.

We use Meth8 to replicate the 24 tautologous syllogisms derived from the original Square of Opposition. In the process we make three recent advances.

2. A third assumption is needed to fix up Modus Cesare EAE-2

3. The third assumption cannot be removed from Modus Camestros AEO-2
(as in other syllogisms with known third assumptions); and

4. No third assumptions are required for the other 22 syllogisms.

In our discussion we also present:

5. Analysis of Modus Cesare EAE-2 with Modus Camestros AEO-2

We use public domain definitions from en.wikipedia.org/wiki/Syllogism as mapped to Meth8 script.

LET: # All, % Exists; tautologous tautologous, not tautologous Not tautologous;
T E, True Evaluated as designated values

2. Additional assumptions required for Modus Cesare EAE-2

LET: q r s, MPS; reptiles fur snakes

The original definition for Modus Cesare EAE-2 is:

No fur is on reptiles.	(PeM)	(~r&q) &
All snakes are reptiles.	(SaM)	(#s&q) >
∴ No snakes have fur.	(SeP)	(~s&r) ; not tautologous

Here are truth tables in the five models:

```
TTTTTTTTTTCCTTTT.EEEEEEEEEEUUEEEE.EEEEEEEEEEEEEEEE.EEEEEEEEEEPPEEEE.EEEEEEEEEEIIEEEE
Model 1         .Model 2.1       .Model 2.2       .Model 2.3.1     .Model 2.3.2
((~r&q)&(#s&q))>(~s&r)   Step: 11
```

The original definition is not tautologous by Meth8.

We test an additional existential assumption for "some fur exists".

No fur is on reptiles.	(PeM)	(~r&q) &
All snakes are reptiles.	(SaM)	(#s&q) &
Some fur exists.		(%r) >

∴ No snakes have fur. (SeP) (~s&r) ; tautologous

Here are truth tables in the five models:

```
TTTTTTTTTTTTTTTT.EEEEEEEEEEEEEEEE.EEEEEEEEEEEEEEEE.EEEEEEEEEEEEEEEE.EEEEEEEEEEEEEEEE
Model 1         .Model 2.1       .Model 2.2       .Model 2.3.1     .Model 2.3.2
(((~r&q)&(#s&q))&%r)>(~s&r)   Step: 13
```

The modified definition is tautologous by Meth8.

3. Additional known assumption required for Modus Camestros AEO-2

LET: q r s, MPS; reptiles snakes fur

The original definition is:

All snakes are reptiles. (PaM) (#r&q) &
No fur is on reptiles. (SeM) (~s&q) &
Some fur exists. (%s) >
∴ Some fur is not on snakes. (SoP) (%s&~r) ; tautologous

The original definition is tautologous by by Meth8.

In all syllogisms with a known additional assumption, it can be removed with the syllogism still being tautologous by Meth8. We test Modus Camestros AEO-2 for this condition.

All snakes are reptiles. (PaM) (#r&q) &
No fur is on reptiles. (SeM) (~s&q) >
∴ Some fur is not on snakes. (SoP) (%s&~r) ; not tautologous

The original definition without the known additional assumption is not tautologous by Meth8.

Here are truth tables in the five models:

```
TTTTTTCCTTTTTTTT.EEEEEEUUEEEEEEEE.EEEEEEEEEEEEEEEE.EEEEEEPPEEEEEEEE.EEEEEEIIEEEEEEEE
Model 1         .Model 2.1       .Model 2.2       .Model 2.3.1     .Model 2.3.2
((#r&q)&(~s&q))>(%s&~r)   Step: 11
```

This means that Modus Camestros AEO-2 does not behave as all other tautologous syllogisms with known additional assumptions when those assumptions are removed.

4. No additional assumptions required for other 22 syllogisms

We show that the other tautologous syllogisms with known additional assumptions removed are tautologous by Meth8 in Table 6.2 above.

5. Analysis of Modus Cesare EAE-2 with Modus Camestros AEO-2

We set Modus Cesare EAE-2 and Modus Camestros AEO-2 in opposite columns for comments about shaded variables.

	q r s MPS reptiles fur snakes			q r s MPS reptiles snakes fur	
EAE-2	**Modus Cesare**			**Modus Camestros**	**AEO-2**
(PeM)	No fur is on reptiles.	(~r&q) &	(#r&q) &	All snakes are reptiles.	(PaM)
(SaM)	All snakes are reptiles	(#s&q) &	(~s&q) &	No fur is on reptiles.	(SeM)
	Some fur exists.	(%r) >	(%s) >	Some fur exists.	
(SeP)	∴ No snakes have fur.	(~s&r) ; tautologous	(%s&~r) ; tautologous	∴ Some fur is not on snakes.	(SoP)

5.1 While syllogism models differentiate between first and second premises as antecedent and consequent around the & And connective, Meth8 does not. Therefore if the variable r is replaced by s or vice versa, in one column, then expressions are the same in both columns.

5.2 If the order of the premises is interchanged, then: Modus Cesare EAE-2 becomes Modus Camestres AEE-2 (without or with an additional assumption of %r); and Modus Camestros AEO-2 becomes EAO-3 or EAO-4 (without or with an additional assumption of %r) if the assignments change to q r s MPS fur reptiles snakes.

5.3 The respective conclusions are identical by variable replacement.

5.4 If the modal operators are removed then both syllogisms with the additional assumptions are still tautologous. This speaks to what we name the *core voracity* of the syllogisms.

6. Conclusion

Variant system VŁ4 as implemented in the Meth8 modal logic checker in five models:

6.1 Corrects Modus Cesare EAE-2 by an additional assumption;

6.2 Shows Modus Camestros AEO-2 must retain its known additional assumption (unlike the other syllogisms that are tautologous also without it); and

6.3 Presents the table of correct syllogisms in compact Meth8 scripts.

We further demonstrate that:

6.4 The modal operators of necessity and possibility are useful to map exactly the quantifiers of all and exists; and

6.5 The Meth8 tool is qualified to map, evaluate, analyze, and correct this limited fragment of predicate logic.

Availability of Meth8/VŁ4

Manifest of files distributed in current release 2019 June 4 v.10 (2019.06.04.10):

1. Install programs:

   M8_installer.exe
   machine.exe

2. Logic system parameter file:

   meth8_parameter_file.txt
   meth8_parameter_file.txt.bak (backup)

3. $ 99.97 program of 4 propositional variables (p...s), 4 theorems (A...D):

   M8_04.exe

4. $149.97 program of 11 propositional variables (p...z), no theorems:

   M8_00.exe (companion program to M8_04.exe)

5. Meth8 input file for equations:

   METH8_INPUT_FILE.txt
   METH8_INPUT_FILE.txt.bak (backup)

6. Parsing anomalies:

   M8_parsing_anomalies.pdf

Standard discounts apply to qualified resellers and university labs.

ersatz-systems.com info@cec-services.com (719) 210-9534

Availability of Meth8/VŁ4 demo for 2-variables (p,q) with unlimited sequents

This is free on request from info@ersatz-systems.com.

Please state name and organization to receive:

Unrestricted m8_executable.exe;

Instructions.txt with known anomalies; and

Editable sample meth8_input_file.txt.

The input file contains the shortest confirmation of McCune's proof of Huntington's equation.

From: en.wikipedia.org/wiki/Robbins_algebra

LET p, q: a, b.

(~(~p+q)+~(~p+~q))=p ; TTTT TTTT TTTT TTTT

The input file contains the shortest refutation of paraconsistent logic.

From: en.wikipedia.org/wiki/Paraconsistent_logic#An_ideal_three-valued_paraconsistent_logic

(4) To establish that a formula Γ is equivalent to Δ in the sense that either can be substituted for the other wherever they appear as a subformula, one must show

$$((\Gamma\rightarrow\Delta)\wedge(\Delta\rightarrow\Gamma))\wedge((\neg\Gamma\rightarrow\neg\Delta)\wedge(\neg\Delta\rightarrow\neg\Gamma)).$$

LET p, q: Γ, Δ.

((p>q)&(q>p))&((~p>~q)&(~q>~p)) ; T**FF**T T**FF**T T**FF**T T**FF**T

The input file also contains the refutation for provability logic of the Gödel-Löb axiom GL as, "The necessity of *choice*, as always implying *a choice*, implies always *a choice*."

$$\Box(\Box p\rightarrow p)\rightarrow\Box p.$$

LET p: *choice*.

#(#p>p)>#p ; CTCT CTCT CTCT CTCT

Scalability of Meth8/VŁ4

Abstract: We offer two more packages in Meth8 to process increasing numbers of propositional variables.

To derive scalability statistics of Meth8, the test platform was

HPE-519c, AMD II X6 1065T 2.96 GHz, 16.0 GB RAM, 64-bit OS with ordinary load.

We tabulate: the number of propositional variables; time in seconds (or minutes, hours, days) for building the look up tables (LUTs) in real time; the size of the LUTs built; the number of rows output for the resulting truth table (with each row being a 16-byte table by five models); and the output in bytes for the resulting truth table. The designations in powers of 1024 bytes are prefixed as "...bibytes", where **K**,**M**,**G** are kibibytes, mebibytes, gibibytes; and **s**, **m**, **h**, **d** are second, minute, hour, day.

Variables	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
LUT build **s**	0	0	0	0	0	0	0.1	0.3	0.6	1.6	4.1	12.2	55.2	3.7**m**	14.5	58.8	3.9**h**	15.7	2.6**d**
LUT size	1**K**	3	8	18	41	92	205	451	983	2.1*M*	4.6	9.8	21	44.6	94.4	19.	421	888	1.9**G**
Result rows	1	2	4	8	16	32	64	128	256	512	1024	2048	8192	16**K**	32	64	128	256	512
Result bytes	80	160	320	640	1**K**	2.6	5.1	10.2	20.5	41	81.9	20.5	655	1.3**M**	2.62	5.24	10	21	41

The literature rarely invokes more than 11 variables except in the cases of inductive, brute-force proofs. Building the logic LUTs on the fly for 11 or less variables takes less than one second. Therefore for 12 or more variables we recommend using external media LUTs that are pre-computed. For example, the build of LUTs for 7 theorems takes 15m. Testing conjectures with variables only is an effective way to avoid the enormous overhead of theorems and even larger output table results.

The maximum number of alphabetic variables allowed in one character is 24 because "i" and "o" are not allowed for clarity if capitalized. The maximum RAM footprint of the Meth8 engine as implemented is about 2^31 bytes or 2.1 GB. However on our test platform, LUTs of 12 or more variables take more time to execute than one second in real time to build in RAM. Therefore, we build external storage files of LUTs for 12 or more variables. The size of the external drive limits the number of such LUTs by variable number. A DVD capacity of 4.70 GB stores LUTs of 4 to 20 variables or 4 to 22 variables. We offer two options for externalized storage with pricing:

$ 97	4 propositional variables, and 4 theorem variables (included in $147 below)
$147	4 to 11 propositional variables
$497	4 to 20 propositional variables (DVD 4.70 GB)
$797	4 to 22 propositional variables (DVD 4.70 GB)

The engine software is licensed according to the package. LUT performance is linear as a function of table size. Output performance is based on the number of rows of LUTs printed to the screen and hard disk.