The Spekken toy model and CHSH / Bell inequalities

From: en.wikipedia.org/wiki/Spekkens_Toy_Model (edited)

> The *knowledge balance principle* of the Spekken toy model ensures that any measurement of a system from within itself yields incomplete knowledge of itself. This implies that observable states of a system are epistemic, that is, only relate to the study of knowledge.
>
> The Spekken toy model implicitly assumes that there is an ontic state of a system at any instant, but which is unobserved.
>
> The model can not derive quantum mechanics due to a disparity of model and quantum theory.
>
> The model contains local and noncontextual variables, so based on Bell's theorem[*] the model can not replicate predictions made by quantum mechanics.
>
> The toy model produces strange quantum effects, interpreted in support the epistemic view.

For an elementary system, the four ontic states are p,q,r,s.

LET { 1, 2, 3, 4, |0>, |1>, |+>, |->, |i>, |-i>, *I*/2 }, where *I* is not defined at the link (Eq 1)
= { p, q, r, s, t, u, v, w, x, y, , z }

For an elementary system, the four ontic states are p, q, r, s.

These map into 6 qubit states, with + And, = Equivalent, @ Not Equivalent, > Imply, < Not Imply:

LET p+q = t; r+s = u; p+r = v; q+s = w; p+s = x; q+r = y; p+q+r+s = z; (Eqs 2a)
Derived for: r = (((u-s)+(v-p))+(y-q)); s = (((u-r)+(w-q))+(x-p)); (Eqs 2b)
All states: (((((p+q)=t)&((r+s)=u))&(((p+r)=v)&((q+s)=w)))&
((((p+s)=x)&((q+r)=y))&(((p+q)+(r+s))=z))) ; (Eq 2c)

The knowledge balance principle [**] is satisfied by transformations on the ontic state of the system in permutations of the four ontic states. For example:

(((p&q)&(r&s))&(p+q)) > (p+q) ; (Eq 3)
(((p&q)&(r&s))&(p+r)) > (q+s) ; (Eq 4)
(((p&q)&((((u-s)+(v-p))+(y-q))&(((u-r)+(w-q))+(x-p))))&(p+r)) > (q+r) ; (Eq 5)

The example given of an antiunitary map on Hilbert space is the antecedent of Eq 5:

(((p&q)&((((u-s)+(v-p))+(y-q))&(((u-r)+(w-q))+(x-p))))&(p+r)) ; (Eq 6)

For the permutations of the six states below, no single transformation as the antecedent serves as a universal state inverter to imply the properties of these consequents:

(p+q)<(r+s) ; (p+r)<(q+s) ; (p+s)<(q+r) ; (Eqs 7a)
(r+s)<(p+q) ; (q+s)<(p+r) ; (q+r)<(p+s) ;

We rewrite Eqs 7a by substitution of Eqs 2a as:

(t<u) ; (v<w) ; (x<y) ; (Eqs 7b)
(u<t) ; (w<v) ; (y<x) ;

We ask if any or all of Eqs 7b are validated as True, that is, are *not* allowed as implied transformations. This means we test Eqs 7b for each equation as separate and also for all of the equations as combined.

To test Eqs 7b for any equation, we use the Or connective (+) as sum of equations below in Eq 7c:

((((t<u)+(v<w))+((x<y)+(u<t)))+((w<v)+(y<x))) ; (Eq 7c)

To test Eqs 7b for all equations, we use the And connective (&) as product of equations below in Eq 7d.

((((t<u)&(v<w))&((x<y)&(u<t)))&((w<v)&(y<x))) ; (Eq 7d)

We also note that for Eq 7c, 7d to be complete, we must account for the definitions of variables in Eq 2c. We therefore rewrite Eq 7c, 7d in Eq 7e, 7f below:

((((((p+q)=t)&((r+s)=u))&(((p+r)=v)&((q+s)=w)))&((((p+s)=x)&((q+r)=y))&(((p+q)+
(r+s))=z))) & ((((t<u)+(v<w))+((x<y)+(u<t)))+((w<v)+(y<x)))) ; (Eq 7e)

((((((p+q)=t)&((r+s)=u))&(((p+r)=v)&((q+s)=w)))&((((p+s)=x)&((q+r)=y))&(((p+q)+
(r+s))=z))) & ((((t<u)&(v<w))&((x<y)&(u<t)))&((w<v)&(y<x)))) ; (Eq 7f)

Our experiment tests Eqs 7e, 7f for the Truth value of (z=z) in Eqs 8.1,8.2 and Eqs 9.1,9.2.

(z=z) = ((((((p+q)=t)&((r+s)=u))&(((p+r)=v)&((q+s)=w)))&((((p+s)=x)&((q+r)=y))
&(((p+q)+(r+s))=z))) & ((((t<u)+(v<w))+((x<y)+(u<t)))+((w<v)+(y<x)))) ; (Eq 8.1)
not validated as true, and False ; 73 steps

(z=z) > ((((((p+q)=t)&((r+s)=u))&(((p+r)=v)&((q+s)=w)))&((((p+s)=x)&((q+r)=y))
&(((p+q)+(r+s))=z))) & ((((t<u)+(v<w))+((x<y)+(u<t)))+((w<v)+(y<x)))) ; (Eq 8.2)
not validated as true, and False ; 73 steps

(z=z) = ((((((p+q)=t)&((r+s)=u))&(((p+r)=v)&((q+s)=w)))&((((p+s)=x)&((q+r)=y))
&(((p+q)+(r+s))=z))) & ((((t<u)&(v<w))&((x<y)&(u<t)))&((w<v)&(y<x)))) ; (Eq 9.1)
not validated as true, and False ; 73 steps

(z=z) > ((((((p+q)=t)&((r+s)=u))&(((p+r)=v)&((q+s)=w)))&((((p+s)=x)&((q+r)=y))
&(((p+q)+(r+s))=z))) & ((((t<u)&(v<w))&((x<y)&(u<t)))&((w<v)&(y<x)))) ; (Eq 9.2)
not validated as true, and False ; 73 steps

To our question if any or all of Eqs 7b are validated as True, our answer is no, meaning some or all of Eqs 7b are allowed as transformations. This means that the knowledge based principle, as applied to elementary ontic values and transformations therefrom, is not validated as true.

What follows is that according to the VŁ4 modal propositional logic of Meth8, the Spekken toy model as an epistemic foundation of the quantum model is suspicious.

[*] The CHSH inequality and Bell inequality

1. The CHSI inequality is an acronym for John Clauser, Michael Horne, Abner Shimony, and Richard Holt, and is described at en.wikipedia.org/wiki/CHSH_inequality :.

|S| ≤ 2 where (Eq 10)
E = (w-x-y+z)\(w+x+y+z) (Eq 11)
S = E(p,q) - E(p,s) +E(r, q) + E(r,s), (Eq 12)

LET |s| ≤ 2 be: ((s<((s\s)-(s\s))) > ((s&((s\s)-((s\s)-(s\s))))<((s\s)+((s\s)+(s\s))))) ; (Eq 13)
LET E be: u = (((w-x)-(y+z))\((w+x)+(y+z))) ; (Eq 14)
LET S be: s = u&(((p&q)-(p&s))+((r&q)+(r&s))) ; (Eq 15)

((u=(((w-x)-(y+z))\((w+x)+(y+z))))&(s=(u&(((p&q)-(p&s))+((r&q)+(r&s)))))) >
((s<((s\s)-(s\s)))>((s&((s\s)-((s\s)-(s\s))))<((s\s)+((s\s)+(s\s))))) ; validated as true (Eq 16)

The CHSH inequality is validated as true.

2. The original Bell inequality named after John Stewart Bell is described at en.wikipedia.org/wiki/Bell%27s_theorem :

Ch(a,b) = E(A(a,z),B(b,z)), where z is lower case lambda (Eq17)
[Ch(a,c) - Ch(b,a) - Ch(b,c)] ≤ 1 (Eq18)

LET Ch(a,b) be: (y&(p&z)) = (u&((w&(p&z))&(x&(q&z)))) ; (Eq 19)
LET Ch(a,c) be: (y&(p&r)) = (u&((w&(p&z))&(x&(r&z)))) ; (Eq 20)
LET Ch(b,c) be: (y&(q&r)) = (u&((w&(q&z))&(x&(r&z)))) ; (Eq 21)
LET [Ch(a,c) - Ch(b,a) - Ch(b,c)] be: (Eq 22)
(((y&(p&z))=(u&((w&(p&z))&(x&(q&z))))) - (((y&(p&r))=(u&((w&(p&z))&(x&(r&z))))) -
((y&(q&r))=(u&((w&(q&z))&(x&(r&z))))))) ; (Eq 23)
LET ≤ 1 be: > ((z\z)+(z\z)) ; (Eq 24)

(((y&(p&z))=(u&((w&(p&z))&(x&(q&z))))) - (((y&(p&r))=(u&((w&(p&z))&(x&(r&z))))) -
((y&(q&r))=(u&((w&(q&z))&(x&(r&z))))))) > ((z\z)+(z\z)) ; not validated (Eq 25)

Bell's inequality as written here in Section 2 should be validated as true because the CHSH inequality in Section 1, validated as true, is supposed to be an abstraction.

3. We then test the truth relationship between the CHSH inequality and Bell's inequality.

3.1 We ask if Bell's inequality implies the CHSH inequality as it's a more general form.

(((u=(((w-x)-(y+z))\((w+x)+(y+z))))&(s=(u&(((p&q)-(p&s))+((r&q)+(r&s)))))) >
((s<((s\s)-(s\s)))>((s&((s\s)-((s\s)-(s\s))))<((s\s)+((s\s)+(s\s)))))) >
((((y&(p&z))=(u&((w&(p&z))&(x&(q&z))))) - (((y&(p&r))=(u&((w&(p&z))&(x&(r&z))))) -
((y&(q&r))=(u&((w&(q&z))&(x&(r&z))))))) > ((z\z)+(z\z))) ; not validated as true (141 steps)
(Eq 26)

Here are unique fragments of the truth table result:

```
Model 1               Model 2.1             Model 2.2             Model 2.3.1           Model 2.3.2
TTTT TTTT TTTT TTTT   EEEE EEEE EEEE EEEE   EEEE EEEE EEEE EEEE   EEEE EEEE EEEE EEEE   EEEE EEEE EEEE EEEE
TTTF TTTT TTTF TTTT   EEEU EEEE EEEU EEEE   EEEU EEEE EEEU EEEE   EEEU EEEE EEEU EEEE   EEEU EEEE EEEU EEEE
TFTF TFTT TFTF TFTT   EUEU EUEE EUEU EUEE   EUEU EUEE EUEU EUEE   EUEU EUEE EUEU EUEE   EUEU EUEE EUEU EUEE
```

3.2 We also ask if the CHSH inequality implies Bell's inequality as a more specific axiom. This is accomplished by changing the main connective from > Imply to < Not Imply.

(((u=(((w-x)-(y+z))\((w+x)+(y+z))))&(s=(u&(((p&q)-(p&s))+((r&q)+(r&s)))))) >
((s<((s\s)-(s\s)))>((s&((s\s)-((s\s)-(s\s))))<((s\s)+((s\s)+(s\s)))))) <
((((y&(p&z))=(u&((w&(p&z))&(x&(q&z))))) - (((y&(p&r))=(u&((w&(p&z))&(x&(r&z))))) -
((y&(q&r))=(u&((w&(q&z))&(x&(r&z))))))) > ((z\z)+(z\z))) ; not validated as true (141 steps)
(Eq 27)

Here are unique fragments of the truth table result:

```
Model 1               Model 2.1             Model 2.2             Model 2.3.1           Model 2.3.2
FFFF FFFF FFFF FFFF   UUUU UUUU UUUU UUUU   UUUU UUUU UUUU UUUU   UUUU UUUU UUUU UUUU   UUUU UUUU UUUU UUUU
FFFT FFFF FFFT FFFF   UUUE UUUU UUUE UUUU   UUUE UUUU UUUE UUUU   UUUE UUUU UUUE UUUU   UUUE UUUU UUUE UUUU
FTFT FTFF FTFT FTFF   UEUE UEUU UEUE UEUU   UEUE UEUU UEUE UEUU   UEUE UEUU UEUE UEUU   UEUE UEUU UEUE UEUU
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As expected, Eq 27 is the negation of Eq 26.

What follows is that the CHSH inequality and Bell's inequality are not logically related. This means that both are now suspicious as proofs of Bell's theorem, raising a more general doubt that the foundation of quantum mechanics is questionable from the standpoint of system VŁ4.

[**] We note that the term "knowledge balance principle", as defined above at the instant wiki site, was no where else found in the extant quantum literature.