

Hilbert #10: Does there exist a universal algorithm for solving Diophantine equations?

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We examine **linear Diophantine equations** of the form $ax + by = c$, rewritten in Meth8 scripts below.

LET $rp + sq = t$:

$((r\&p)+(s\&q)) < t$; not validated; false;	not < t
$((r\&p)+(s\&q)) > t$; validated;	> t
$((r\&p)+(s\&q)) = t$; not validated;	not = t

LET $rp + sq = 1$:

$((r\&p)+(s\&q)) < (t\ t)$; not validated;	not < 1
$((r\&p)+(s\&q)) > (t\ t)$; not validated;	not > 1
$((r\&p)+(s\&q)) = (t\ t)$; not validated;	not = 1

LET $rp + sq = 0$:

$((r\&p)+(s\&q)) < ((t\ t)-(t\ t))$; not validated; false ;	not < 0
$((r\&p)+(s\&q)) > ((t\ t)-(t\ t))$; validated;	> 0
$((r\&p)+(s\&q)) = ((t\ t)-(t\ t))$; not validated;	not = 0

LET $rp + sq = -1$:

$((r\&p)+(s\&q)) < (((t\ t)-(t\ t))-(t\ t))$; not validated;	not < -1
$((r\&p)+(s\&q)) > (((t\ t)-(t\ t))-(t\ t))$; not validated;	not > -1
$((r\&p)+(s\&q)) = (((t\ t)-(t\ t))-(t\ t))$; not validated;	not = -1

LET $rp + sq = 1/2$: [not Diophantine]

$((r\&p)+(s\&q)) < ((t\ t)\ ((t\ t)+(t\ t)))$; not validated; false;	not < 1/2
$((r\&p)+(s\&q)) > ((t\ t)\ ((t\ t)+(t\ t)))$; validated;	> 1/2
$((r\&p)+(s\&q)) = ((t\ t)\ ((t\ t)+(t\ t)))$; not validated;	not = 1/2

Results are validated only for $ax + by = t$ as $t > 0$, but not for $t = 0$, $t < 1$, $t = 1$, or $t > 1$.

The proof is that there exist no integers greater than zero and less than 1 as integer solutions to linear Diophantine equations. Consequently, no universal solution exists for *all* Diophantine equations.