

Abstract: We evaluate these topics using the Meth8/VŁ4 modal logic model checker:

Role of logic for probabilistic computation in artificial intelligence (AI)

New approach: Most of the papers we evaluate are found as titles at a private preprint system of Cornell Library. Since 2017 we found so many mistakes that now we opt to streamline our approach. For the instant refutation digest title above, we list artifacts in the abstract section above. The respective from-reference, text block, and remarks then follow beginning on separate pages. Conjectures refuted form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, **N** as truthity (non-contingency), and **C** as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET \sim Not, \neg ; + Or, \vee , \cup , \sqcup ; - Not Or; & And, \wedge , \cap , \prod , \cdot , \circ , \otimes ; \ Not And;
 $>$ Imply, greater than, \rightarrow , \Rightarrow , \mapsto , $>$, \supset , \Rightarrow ; $<$ Not Imply, less than, \in , $<$, \subset , \neq , \neq , \leftarrow , \lesssim ;
 $=$ Equivalent, \equiv , $:=$, \Leftrightarrow , \leftrightarrow , \triangleq , \approx , \cong ; @ Not Equivalent, \neq , \oplus ;
 $\%$ possibility, for one or some, \exists , $\exists!$, \diamond , M ; # necessity, for every or all, \forall , \square , L ;
 $(z=z)$ **T** as tautology, \top , ordinal 3; $(z@z)$ **F** as contradiction, \emptyset , Null, \perp , zero;
 $(\%z>\#z)$ **N** as non-contingency, Δ , ordinal 1; $(\%z<\#z)$ **C** as contingency, ∇ , ordinal 2;
 $\sim(y < x)$ $(x \leq y)$, $(x \subseteq y)$, $(x \sqsubseteq y)$; $(A=B)$ $(A\sim B)$.
 Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Darwiche, A. (2020). Three modern roles for logic in AI. arxiv.org/pdf/2004.08599.pdf

Abstract. We consider three modern roles for logic in artificial intelligence, which are based on the theory of tractable Boolean circuits: (1) logic as a basis for computation, (2) logic for learning from a combination of data and knowledge, and (3) logic for reasoning about the behavior of machine learning systems.

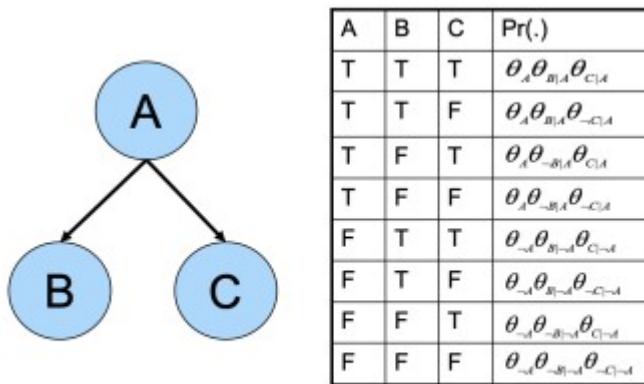


Figure 4: A Bayesian network and its distribution.

$$\text{LET } p, q, r: A, B, C. \tag{4.1}$$

$$p > (q+r); \quad \mathbf{TFTT} \quad \mathbf{TTTT} \quad \mathbf{TFTT} \quad \mathbf{TTTT} \tag{4.2}$$

Remark 4.2: Eq. 4.2 is *not* tautologous. Note that the result consists of one 16-valued truth tables presented horizontally, row-major, to save space. Hence there are a total number of 16 logical values above for 2 **F** and 1984 **T**.

Our approach to map Fig. 4 into respective probabilities is arbitrarily assigning the **FCNT** logic values as based on a scale of 4, presuming contradiction and tautology are respectively the least and most desirable statistical states.

4-valued logic: F {00} contradiction	C {10} falsity	N {01} truthity	T {11} tautology
Probability: 1/4	2/4	3/4	4/4

Therefore the P of Eq. 4.2 is calculated as $14/16 = 0.875$. According to the same method on the table in Fig. 4, there are a total of 24 logical values for 12 **F** and 12 **T** for $12/24 = 0.5$ as P.

The 24 logical values in Fig. 4 lead to marginal probabilities in ten parameters for eight models below.

Consider the Bayesian network in Figure 4, which has three binary variables A, B and C . Variable A has one distribution $(\theta_A, \theta_{\neg A})$. Variable B has two distributions, which are conditioned on the state of its parent A : $(\theta_{B|A}, \theta_{\neg B|A})$ and $(\theta_{B|\neg A}, \theta_{\neg B|\neg A})$. Variable C also has two similar distributions: $(\theta_{C|A}, \theta_{\neg C|A})$ and $(\theta_{C|\neg A}, \theta_{\neg C|\neg A})$. We will refer to the probabilities θ as *network parameters*. The Bayesian network in Figure 4 has ten parameters.

This Bayesian network induces the distribution depicted in Figure 4, where the probability of each variable instantiation is simply the product of network parameters that are compatible with that instantiation; see [27, Chapter 3] for a discussion of the syntax and semantics of Bayesian networks. We will next show how one can efficiently construct a Boolean formula Δ from a Bayesian network, allowing one to compute marginal probabilities on the Bayesian network by performing weighted model counting on formula Δ .

The main insight is to introduce a Boolean variable P for each network parameter θ , which is meant to capture the presence or absence of parameter θ given an instantiation of the network variables (i.e., a row of the table in Figure 4). For the network in Figure 4, this leads to introducing ten Boolean variables: $P_A, P_{\neg A}, P_{B|A}, \dots, P_{\neg C|\neg A}$. In the second row of Figure 4, which corresponds to variable instantiation $A, B, \neg C$, parameters $\theta_A, \theta_{B|A}$ and $\theta_{\neg C|A}$ are present and the other seven parameters are absent.

We can capture such presence/absence by adding one expression to the Boolean formula Δ for each network parameter. For example, the parameters associated with variable A introduce the following expressions: $A \iff P_A$ and $\neg A \iff \neg P_{\neg A}$. Similarly, the parameters of variable B introduce the expressions $A \wedge B \iff P_{B|A}$, $A \wedge \neg B \iff \neg P_{\neg B|A}$, $\neg A \wedge B \iff P_{B|\neg A}$ and $\neg A \wedge \neg B \iff \neg P_{\neg B|\neg A}$. The parameters of variable C introduce similar expressions.

The resulting Boolean formula Δ will have exactly eight models, which correspond to the network instantiations. The following is one of these models which correspond to instantiation $A, B, \neg C$:

$$\begin{aligned}
 & A \ B \ \neg C \ P_A \ P_{B|A} \ P_{\neg C|A} \\
 & \neg P_{\neg A} \ \neg P_{\neg B|A} \ \neg P_{B|\neg A} \ \neg P_{\neg B|\neg A} \ \neg P_{C|A} \ \neg P_{C|\neg A} \ \neg P_{\neg C|\neg A}. \quad (1)
 \end{aligned}$$

In this model, all parameters associated with instantiation $A, B, \neg C$ appear positively (present) while others appear negatively (absent).

We do not evaluate here the schemata above.