Table of Integrals

Basic Forms

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \ n \neq -1 \tag{1}$$

$$\int \frac{1}{x} dx = \ln |x| \tag{2}$$

$$\int u dv = uv - \int v du \tag{3}$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln |ax+b| \tag{4}$$

Integrals of Rational Functions

$$\int \frac{1}{(x+a)^2} dx = -\frac{1}{x+a} \tag{5}$$

$$\int (x+a)^n dx = \frac{(x+a)^{n+1}}{n+1}, n \neq -1 \tag{6}$$

$$\int x(x+a)^n dx = \frac{(x+a)^{n+1}((n+1)x-a)}{(n+1)(n+2)} \tag{7}$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x \tag{8}$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} \tag{9}$$

$$\int \frac{x}{a^2+x^2}dx = \frac{1}{2}\ln|a^2+x^2| \tag{10}$$

$$\int \frac{x^2}{a^2+x^2}dx = x - a\tan^{-1}\frac{x}{a} \tag{11}$$

$$\int \frac{x^3}{a^2+x^2}dx = \frac{1}{2}x^2 - \frac{1}{2}a^2\ln|a^2+x^2| \tag{12}$$

$$\int \frac{1}{ax^2+bx+c}dx = \frac{2}{\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} \tag{13}$$

$$\int \frac{1}{(x+a)(x+b)}dx = \frac{1}{b-a}\ln\frac{a+x}{b+x},\ a \neq b \tag{14}$$

$$\int \frac{x}{(x+a)^2}dx = \frac{a}{a+x} + \ln|a+x| \tag{15}$$

$$\int \frac{x}{ax^2+bx+c}dx = \frac{1}{2a}\ln|ax^2+bx+c| - \frac{b}{a\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} \tag{16}$$

Integrals with Roots

$$\int \sqrt{x-a}\ dx = \frac{2}{3}(x-a)^{3/2} \tag{17}$$

$$\int \frac{1}{\sqrt{x\pm a}}\ dx = 2\sqrt{x\pm a} \tag{18}$$

$$\int \frac{1}{\sqrt{a-x}}\ dx = -2\sqrt{a-x} \tag{19}$$

(20) $$\int x\sqrt{x-a}\,dx = \begin{cases} \frac{2a}{3}(x-a)^{3/2} + \frac{2}{5}(x-a)^{5/2}, & \text{or} \\ \frac{2}{3}x(x-a)^{3/2} - \frac{4}{15}(x-a)^{5/2}, & \text{or} \\ \frac{2}{15}(2a+3x)(x-a)^{3/2} \end{cases}$$

(21) $$\int \sqrt{ax+b}\,dx = \left(\frac{2b}{3a} + \frac{2x}{3}\right)\sqrt{ax+b}$$

(22) $$\int (ax+b)^{3/2}\,dx = \frac{2}{5a}(ax+b)^{5/2}$$

(23) $$\int \frac{x}{\sqrt{x\pm a}}\,dx = \frac{2}{3}(x\mp 2a)\sqrt{x\pm a}$$

(24) $$\int \sqrt{\frac{x}{a-x}}\,dx = -\sqrt{x(a-x)} - a\tan^{-1}\frac{\sqrt{x(a-x)}}{x-a}$$

(25) $$\int \sqrt{\frac{x}{a+x}}\,dx = \sqrt{x(a+x)} - a\ln\left[\sqrt{x}+\sqrt{x+a}\right]$$

(26) $$\int x\sqrt{ax+b}\,dx = \frac{2}{15a^2}(-2b^2+abx+3a^2x^2)\sqrt{ax+b}$$

(27)
$$\int \sqrt{x(ax+b)}\,dx = \frac{1}{4a^{3/2}}\left[(2ax+b)\sqrt{ax(ax+b)} - b^2\ln\left|a\sqrt{x}+\sqrt{a(ax+b)}\right|\right]$$

(28)
$$\int \sqrt{x^3(ax+b)}\,dx = \left[\frac{b}{12a} - \frac{b^2}{8a^2x} + \frac{x}{3}\right]\sqrt{x^3(ax+b)} + \frac{b^3}{8a^{5/2}}\ln\left|a\sqrt{x}+\sqrt{a(ax+b)}\right|$$

(29) $$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}x\sqrt{x^2\pm a^2} \pm \frac{1}{2}a^2\ln\left|x+\sqrt{x^2\pm a^2}\right|$$

(30) $$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}x\sqrt{a^2 - x^2} + \frac{1}{2}a^2 \tan^{-1} \frac{x}{\sqrt{a^2 - x^2}}$$

(31) $$\int x\sqrt{x^2 \pm a^2}\, dx = \frac{1}{3}\left(x^2 \pm a^2\right)^{3/2}$$

(32) $$\int \frac{1}{\sqrt{x^2 \pm a^2}}\, dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right|$$

(33) $$\int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \sin^{-1}\frac{x}{a}$$

(34) $$\int \frac{x}{\sqrt{x^2 \pm a^2}}\, dx = \sqrt{x^2 \pm a^2}$$

(35) $$\int \frac{x}{\sqrt{a^2 - x^2}}\, dx = -\sqrt{a^2 - x^2}$$

(36) $$\int \frac{x^2}{\sqrt{x^2 \pm a^2}}\, dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \mp \frac{1}{2}a^2 \ln\left|x + \sqrt{x^2 \pm a^2}\right|$$

(37)
$$\int \sqrt{ax^2 + bx + c}\, dx = \frac{b + 2ax}{4a}\sqrt{ax^2 + bx + c} + \frac{4ac - b^2}{8a^{3/2}} \ln\left|2ax + b + 2\sqrt{a(ax^2 + bx^+ c)}\right|$$

(38)
$$\int x\sqrt{ax^2 + bx + c}\, dx = \frac{1}{48a^{5/2}}\left(2\sqrt{a}\sqrt{ax^2 + bx + c}\left(-3b^2 + 2abx + 8a(c + ax^2)\right)\right.$$
$$\left. +3(b^3 - 4abc)\ln\left|b + 2ax + 2\sqrt{a}\sqrt{ax^2 + bx + c}\right|\right)$$

(39) $$\int \frac{1}{\sqrt{ax^2+bx+c}}\,dx = \frac{1}{\sqrt{a}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx+c)}\right|$$

(40) $$\int \frac{x}{\sqrt{ax^2+bx+c}}\,dx = \frac{1}{a}\sqrt{ax^2+bx+c} - \frac{b}{2a^{3/2}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx+c)}\right|$$

(41) $$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}}$$

Integrals with Logarithms

(42) $$\int \ln ax\;dx = x\ln ax - x$$

(43) $$\int x\ln x\;dx = \frac{1}{2}x^2\ln x - \frac{x^2}{4}$$

(44) $$\int x^2\ln x\;dx = \frac{1}{3}x^3\ln x - \frac{x^3}{9}$$

(45) $$\int x^n\ln x\;dx = x^{n+1}\left(\frac{\ln x}{n+1} - \frac{1}{(n+1)^2}\right), \quad n\neq -1$$

(46) $$\int \frac{\ln ax}{x}\;dx = \frac{1}{2}(\ln ax)^2$$

(47) $$\int \frac{\ln x}{x^2}\;dx = -\frac{1}{x} - \frac{\ln x}{x}$$

(48) $$\int \ln(ax+b)\;dx = \left(x+\frac{b}{a}\right)\ln(ax+b) - x, a\neq 0$$

(49) $$\int \ln(x^2+a^2)\,dx = x\ln(x^2+a^2) + 2a\tan^{-1}\frac{x}{a} - 2x$$

(50) $$\int \ln(x^2-a^2)\,dx = x\ln(x^2-a^2) + a\ln\frac{x+a}{x-a} - 2x$$

(51) $$\int \ln\left(ax^2+bx+c\right)\,dx = \frac{1}{a}\sqrt{4ac-b^2}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} - 2x + \left(\frac{b}{2a}+x\right)\ln\left(ax^2+bx+c\right)$$

(52) $$\int x\ln(ax+b)\,dx = \frac{bx}{2a} - \frac{1}{4}x^2 + \frac{1}{2}\left(x^2 - \frac{b^2}{a^2}\right)\ln(ax+b)$$

(53) $$\int x\ln\left(a^2-b^2x^2\right)\,dx = -\frac{1}{2}x^2 + \frac{1}{2}\left(x^2 - \frac{a^2}{b^2}\right)\ln\left(a^2-b^2x^2\right)$$

(54) $$\int (\ln x)^2\,dx = 2x - 2x\ln x + x(\ln x)^2$$

(55) $$\int (\ln x)^3\,dx = -6x + x(\ln x)^3 - 3x(\ln x)^2 + 6x\ln x$$

(56) $$\int x(\ln x)^2\,dx = \frac{x^2}{4} + \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2\ln x$$

(57) $$\int x^2(\ln x)^2\,dx = \frac{2x^3}{27} + \frac{1}{3}x^3(\ln x)^2 - \frac{2}{9}x^3\ln x$$

Integrals with Exponentials

$$\int e^{ax}\ dx = \frac{1}{a}e^{ax} \tag{58}$$

$$\int \sqrt{x}e^{ax}\ dx = \frac{1}{a}\sqrt{x}e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}}\mathrm{erf}\left(i\sqrt{ax}\right),\ \text{where } \mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt \tag{59}$$

$$\int xe^{x}\ dx = (x-1)e^{x} \tag{60}$$

$$\int xe^{ax}\ dx = \left(\frac{x}{a} - \frac{1}{a^2}\right)e^{ax} \tag{61}$$

$$\int x^2e^{x}\ dx = \left(x^2 - 2x + 2\right)e^{x} \tag{62}$$

$$\int x^2e^{ax}\ dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right)e^{ax} \tag{63}$$

$$\int x^3e^{x}\ dx = \left(x^3 - 3x^2 + 6x - 6\right)e^{x} \tag{64}$$

$$\int x^ne^{ax}\ dx = \frac{x^ne^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,\mathrm{d}x \tag{65}$$

$$\int x^ne^{ax}\ dx = \frac{(-1)^n}{a^{n+1}}\Gamma[1+n, -ax],\ \text{where } \Gamma(a,x) = \int_x^\infty t^{a-1}e^{-t}\,\mathrm{d}t \tag{66}$$

$$\int e^{ax^2}\ dx = -\frac{i\sqrt{\pi}}{2\sqrt{a}}\mathrm{erf}\left(ix\sqrt{a}\right) \tag{67}$$

(68) $$\int e^{-ax^2}\ dx = \frac{\sqrt{\pi}}{2\sqrt{a}}\mathrm{erf}\left(x\sqrt{a}\right)$$

(69) $$\int xe^{-ax^2}\ dx = -\frac{1}{2a}e^{-ax^2}$$

(70) $$\int x^2e^{-ax^2}\ dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}\mathrm{erf}(x\sqrt{a}) - \frac{x}{2a}e^{-ax^2}$$

Integrals with Trigonometric Functions

(71) $$\int \sin ax\ dx = -\frac{1}{a}\cos ax$$

(72) $$\int \sin^2 ax\ dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

(73) $$\int \sin^3 ax\ dx = -\frac{3\cos ax}{4a} + \frac{\cos 3ax}{12a}$$

(74) $$\int \sin^n ax\ dx = -\frac{1}{a}\cos ax\ {}_2F_1\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3}{2}, \cos^2 ax\right]$$

(75) $$\int \cos ax\ dx = \frac{1}{a}\sin ax$$

(76) $$\int \cos^2 ax\ dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

(77) $$\int \cos^3 axdx = \frac{3\sin ax}{4a} + \frac{\sin 3ax}{12a}$$

(78) $$\int \cos^p ax dx = -\frac{1}{a(1+p)} \cos^{1+p} ax \times {}_2F_1 \left[\frac{1+p}{2}, \frac{1}{2}, \frac{3+p}{2}, \cos^2 ax \right]$$

(79) $$\int \cos x \sin x \ dx = \frac{1}{2} \sin^2 x + c_1 = -\frac{1}{2} \cos^2 x + c_2 = -\frac{1}{4} \cos 2x + c_3$$

(80) $$\int \cos ax \sin bx \ dx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)}, a \neq b$$

(81) $$\int \sin^2 ax \cos bx \ dx = -\frac{\sin[(2a-b)x]}{4(2a-b)} + \frac{\sin bx}{2b} - \frac{\sin[(2a+b)x]}{4(2a+b)}$$

(82) $$\int \sin^2 x \cos x \ dx = \frac{1}{3} \sin^3 x$$

(83) $$\int \cos^2 ax \sin bx \ dx = \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b} - \frac{\cos[(2a+b)x]}{4(2a+b)}$$

(84) $$\int \cos^2 ax \sin ax \ dx = -\frac{1}{3a} \cos^3 ax$$

(85)
$$\int \sin^2 ax \cos^2 bx dx = \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)} + \frac{\sin 2bx}{8b} - \frac{\sin[2(a+b)x]}{16(a+b)}$$

(86) $$\int \sin^2 ax \cos^2 ax \ dx = \frac{x}{8} - \frac{\sin 4ax}{32a}$$

(87) $$\int \tan ax \ dx = -\frac{1}{a} \ln \cos ax$$

$$\int \tan^2 ax \; dx = -x + \frac{1}{a} \tan ax \tag{88}$$

$$\int \tan^n ax \; dx = \frac{\tan^{n+1} ax}{a(1+n)} \times {}_2F_1 \left(\frac{n+1}{2}, 1, \frac{n+3}{2}, -\tan^2 ax \right) \tag{89}$$

$$\int \tan^3 ax dx = \frac{1}{a} \ln \cos ax + \frac{1}{2a} \sec^2 ax \tag{90}$$

$$\int \sec x \; dx = \ln |\sec x + \tan x| = 2 \tanh^{-1} \left(\tan \frac{x}{2} \right) \tag{91}$$

$$\int \sec^2 ax \; dx = \frac{1}{a} \tan ax \tag{92}$$

$$\int \sec^3 x \; dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x| \tag{93}$$

$$\int \sec x \tan x \; dx = \sec x \tag{94}$$

$$\int \sec^2 x \tan x \; dx = \frac{1}{2} \sec^2 x \tag{95}$$

$$\int \sec^n x \tan x \; dx = \frac{1}{n} \sec^n x, n \neq 0 \tag{96}$$

$$\int \csc x \; dx = \ln \left| \tan \frac{x}{2} \right| = \ln |\csc x - \cot x| + C \tag{97}$$

(98) $$\int \csc^2 ax \; dx = -\frac{1}{a} \cot ax$$

(99) $$\int \csc^3 x \; dx = -\frac{1}{2} \cot x \csc x + \frac{1}{2} \ln |\csc x - \cot x|$$

(100) $$\int \csc^n x \cot x \; dx = -\frac{1}{n} \csc^n x, n \neq 0$$

(101) $$\int \sec x \csc x \; dx = \ln |\tan x|$$

Products of Trigonometric Functions and Monomials

(102) $$\int x \cos x \; dx = \cos x + x \sin x$$

(103) $$\int x \cos ax \; dx = \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax$$

(104) $$\int x^2 \cos x \; dx = 2x \cos x + \left(x^2 - 2\right) \sin x$$

(105) $$\int x^2 \cos ax \; dx = \frac{2x \cos ax}{a^2} + \frac{a^2 x^2 - 2}{a^3} \sin ax$$

(106) $$\int x^n \cos x dx = -\frac{1}{2}(i)^{n+1} \left[\Gamma(n+1, -ix) + (-1)^n \Gamma(n+1, ix)\right]$$

(107) $$\int x^n \cos ax \; dx = \frac{1}{2}(ia)^{1-n} \left[(-1)^n \Gamma(n+1, -iax) - \Gamma(n+1, ixa)\right]$$

$$\int x \sin x \; dx = -x \cos x + \sin x \tag{108}$$

$$\int x \sin ax \; dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a^2} \tag{109}$$

$$\int x^2 \sin x \; dx = \left(2 - x^2\right) \cos x + 2x \sin x \tag{110}$$

$$\int x^2 \sin ax \; dx = \frac{2 - a^2 x^2}{a^3} \cos ax + \frac{2x \sin ax}{a^2} \tag{111}$$

$$\int x^n \sin x \; dx = -\frac{1}{2}(i)^n \left[\Gamma(n+1, -ix) - (-1)^n \Gamma(n+1, -ix)\right] \tag{112}$$

$$\int x \cos^2 x \; dx = \frac{x^2}{4} + \frac{1}{8} \cos 2x + \frac{1}{4} x \sin 2x \tag{113}$$

$$\int x \sin^2 x \; dx = \frac{x^2}{4} - \frac{1}{8} \cos 2x - \frac{1}{4} x \sin 2x \tag{114}$$

$$\int x \tan^2 x \; dx = -\frac{x^2}{2} + \ln \cos x + x \tan x \tag{115}$$

$$\int x \sec^2 x \; dx = \ln \cos x + x \tan x \tag{116}$$

Products of Trigonometric Functions and Exponentials

$$\int e^x \sin x \; dx = \frac{1}{2} e^x (\sin x - \cos x) \tag{117}$$

$$\int e^{bx} \sin ax \; dx = \frac{1}{a^2 + b^2} e^{bx} (b \sin ax - a \cos ax) \tag{118}$$

$$\int e^x \cos x \; dx = \frac{1}{2} e^x (\sin x + \cos x) \tag{119}$$

$$\int e^{bx} \cos ax \; dx = \frac{1}{a^2 + b^2} e^{bx} (a \sin ax + b \cos ax) \tag{120}$$

$$\int x e^x \sin x \; dx = \frac{1}{2} e^x (\cos x - x \cos x + x \sin x) \tag{121}$$

$$\int x e^x \cos x \; dx = \frac{1}{2} e^x (x \cos x - \sin x + x \sin x) \tag{122}$$

Integrals of Hyperbolic Functions

$$\int \cosh ax \; dx = \frac{1}{a} \sinh ax \tag{123}$$

$$\int e^{ax} \cosh bx \; dx = \begin{cases} \dfrac{e^{ax}}{a^2 - b^2} [a \cosh bx - b \sinh bx] & a \neq b \\ \dfrac{e^{2ax}}{4a} + \dfrac{x}{2} & a = b \end{cases} \tag{124}$$

$$\int \sinh ax \; dx = \frac{1}{a} \cosh ax \tag{125}$$

$$\int e^{ax} \sinh bx \; dx = \begin{cases} \frac{e^{ax}}{a^2 - b^2}[-b \cosh bx + a \sinh bx] & a \neq b \\ \frac{e^{2ax}}{4a} - \frac{x}{2} & a = b \end{cases} \tag{126}$$

$$\int \tanh ax \, dx = \frac{1}{a} \ln \cosh ax \tag{127}$$

$$\int e^{ax} \tanh bx \; dx = \begin{cases} \frac{e^{(a+2b)x}}{(a+2b)} {}_2F_1\left[1 + \frac{a}{2b}, 1, 2 + \frac{a}{2b}, -e^{2bx}\right] \\ \qquad -\frac{1}{a} e^{ax} {}_2F_1\left[1, \frac{a}{2b}, 1 + \frac{a}{2b}, -e^{2bx}\right] & a \neq b \\ \frac{e^{ax} - 2 \tan^{-1}[e^{ax}]}{a} & a = b \end{cases} \tag{128}$$

$$\int \cos ax \cosh bx \; dx = \frac{1}{a^2 + b^2} [a \sin ax \cosh bx + b \cos ax \sinh bx] \tag{129}$$

$$\int \cos ax \sinh bx \; dx = \frac{1}{a^2 + b^2} [b \cos ax \cosh bx + a \sin ax \sinh bx] \tag{130}$$

$$\int \sin ax \cosh bx \; dx = \frac{1}{a^2 + b^2} [-a \cos ax \cosh bx + b \sin ax \sinh bx] \tag{131}$$

$$\int \sin ax \sinh bx \; dx = \frac{1}{a^2 + b^2} [b \cosh bx \sin ax - a \cos ax \sinh bx] \tag{132}$$

$$\int \sinh ax \cosh ax dx = \frac{1}{4a} [-2ax + \sinh 2ax] \tag{133}$$

$$\int \sinh ax \cosh bx \; dx = \frac{1}{b^2 - a^2} [b \cosh bx \sinh ax - a \cosh ax \sinh bx] \tag{134}$$