Journal of Atmospheric and Solar-Terrestrial Physics 71 (2009) 239–245

Contents lists available at ScienceDirect

Journal of
Atmospheric and Solar-Terrestrial Physics

journal homepage: www.elsevier.com/locate/jastp

Forecasting the parameters of sunspot cycle 24 and beyond

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ARTICLE INFO

Article history:
Received 4 August 2008
Received in revised form
19 October 2008
Accepted 3 November 2008
Available online 13 December 2008

Keywords:
Solar variability
Solar dynamo
Sunspots
Geomagnetic index

ABSTRACT

Solar variability is controlled by the internal dynamo which is a non-linear system. We develop a physical–statistical method for forecasting solar activity that takes into account the non-linear character of the solar dynamo. The method is based on the generally accepted mechanisms of the dynamo and on recently found systematic properties of the long-term solar variability. The amplitude modulation of the Schwabe cycle in dynamo's magnetic field components can be decomposed in an invariant transition level and three types of oscillations around it. The regularities that we observe in the behaviour of these oscillations during the last millennium enable us to forecast solar activity. We find that the system is presently undergoing a transition from the recent Grand Maximum to another regime. This transition started in 2000 and it is expected to end around the maximum of cycle 24, foreseen for 2014, with a maximum sunspot number $R_{max} = 68 \pm 17$. At that time a period of lower solar activity will start. That period will be one of regular oscillations, as occurred between 1730 and 1923. The first of these oscillations may even turn out to be as strongly negative as around 1810, in which case a short Grand Minimum similar to the Dalton one might develop. This moderate-to-low-activity episode is expected to last for at least one Gleissberg cycle (60–100 years).

1. Introduction

Presently existing methods for forecasting sunspot activity may be subdivided into three types (cf. Usoskin and Mursula, 2003): (a) statistical methods that consider inherent statistical properties of solar activity (e.g. Kane, 1999; Ogurtsov, 2005a, b), (b) physical methods, based on an assumed mechanism for the solar dynamo that yields links between the poloidal and the toroidal magnetic field components and that are further based on some precursor type parameters of solar activity (e.g. Schatten, 2005; Svalgaard et al., 2005), and (c) physical–statistical methods which are combinations of the approaches (a) and (b) (e.g. Hathaway and Wilson, 2006; Duhau, 2003).

All the three methods have been applied for forecasting solar activity. In spite of the fact that some of these forecasts are fairly sophisticated, published predictions of the maximum sunspot number (R_{max}) for the coming cycle are disappointingly divergent (see, e.g. Usoskin and Mursula, 2003; Li et al., 2001). They range from very high, as in the last 50 years (Hathaway and Wilson, 2006; Dikpati et al., 2006; Charvàtovà, 2008), over intermediate R_{max} values (Schatten, 2002; Duhau, 2003; Le and Wang, 2003; de Meyer, 2003; Sello, 2003; Ogurtsov, 2004, 2005a, b; Svalgaard et al., 2005; Schatten, 2005; Kane, 2007; Aguirre et al., 2008) to very small R_{max} values (Badalyan et al., 2001; Komitov and Kaftan, 2003; Callebaut et al., 2003). These latter forecasts might lead to another Grand Minimum episode. These conflicting results of predictions may be due to the fact that most of the methods used to forecast solar activity (for reviews see, e.g. Hathaway et al., 1999; Schatten, 1998) assume the relation between the involved variables to be linear. However, the solar dynamo is a non-linear system with deterministic chaotic elements (Weiss, 1987; Feynman and Gabriel, 1990; Ostryakov and Usoskin, 1990; Kremliovsky, 1995; Usoskin and Mursula, 2003; Duhau, 2003; Weiss and Tobias, 2004; De Jager, 2005; Aguirre et al., 2008). Hence, the solar dynamo behaviour, as manifested in its temporal evolution, differs fundamentally from that assumed in most predictions in which the non-linearity was not considered. The divergence between presently existing methods to forecast sunspot activity calls for a further development of the prognosis technique.

The dynamo system in an axial-symmetric model has 4 degrees of freedom, i.e. the toroidal and poloidal magnetic field components and the meridional and azimuthal components of the velocity field (see, e.g. Knobloch et al., 1998; Durney, 2000; Dikpati et al., 2004). The geomagnetic index aa at minima, aa_{min} (Mayaud, 1972), and the sunspot number at maxima, R_{max}, are measures of the amplitudes of the poloidal and toroidal magnetic components of the solar cycle, respectively (cf. Duhau, 2003 and references therein). The non-linear evolution of the dynamo system from 1844 to 2000 was shown by means of an R_{max} vs.

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1364-6826/$ - see front matter
doi:10.1016/j.jastp.2008.11.006

aa_{min} phase diagram introduced by Duhau and Chen (2002). Their method was improved by Duhau and De Jager (2008) and applied by them to study the non-linear evolution of the solar dynamo during the last millennium, from proxy data of the sunspot numbers (Nagovitsyn, 1997, 2005, 2007) and the geomagnetic index aa (Nagovitsyn, 2006) time series. It was found that the dynamo system is characterized by an invariant, sharply defined, transition state around which the system states oscillate. The oscillations were decomposed in multidecadal and Gleissberg oscillations. The first (henceforth called 'decadal' for simplicity) is defined for the present purpose as the superposition of all wavelet components with periods in the 15–72 year band, where the lower limit is chosen such that the influence of the Schwabe cycle is eliminated from the resulting data. The Gleissberg oscillations are defined here as the superposition of all wavelet components with periods above 72 years, to which the linear trend is added and from which the transition level's coordinate is subtracted. This decomposition was made to facilitate solar activity predictions, since we (Duhau and De Jager, 2008) found that the decadal oscillation includes the odd–even rule and that the Gleissberg cycle is a succession of harmonic oscillations with a period in the Suess band (Great Episodes) and in the Gleissberg band (Regular Episodes). Moreover, we found evidence that solar variability is mainly time-correlated in the long-term (Gleissberg) time scale, hardly so in the decadal one. The method, developed here, belongs essentially to the above-described method (c) but is substantially improved by including the non-linear character of the solar dynamo.

The system appears to move sequentially towards the three types of quasi-periodic behaviours in brief phase transitions. The character of these transitions appears to depend on the distance of the path of the dynamo components in phase space to the transition point.

On the basis of this new information we study in this paper the forthcoming solar variability and in particular the nature of the next solar dynamo episode and the characteristics of solar cycle 24. This is done in Sections 2–4. In Section 5 we discuss the causes of the large dispersion in presently existing predictions of sunspot maximum #24. A summary of the results and our main conclusions are given in Section 6.

2. The Gleissberg cycle in solar variability and the forecast of the next dynamo episode

Outline: In this section we study the phase diagram of the Gleissberg cycle in the plane of the two magnetic field components for the forthcoming half century in order to forecast the behaviour of the solar dynamo for that period.

As mentioned in Section 1, the dynamo is characterized by a sharply defined transition state (coordinates: $R_{max} = 93.38 \pm 0.69$ spot number units and $aa_{min} = 10.34 \pm 0.08$ nT). There are three types of dynamo behaviour around the transition point: the Grand Minima (M), the Grand Maxima (H), and the Regular Oscillations (R). The first two last for half a strong oscillation, negative and positive respectively. These are periods of time in the upper part of Gleissberg band of periods. An example of a Grand Maximum is the 1923–2008 large loop; it is also shown in the second quadrant of the phase diagram of Fig. 1. The R-type oscillations are rather weaker and can last longer than the H and M episodes, viz. for time periods of 60–200 years, equivalent to one to a few Gleissberg cycles (cf. the loops from 1730 to 1923 in Fig. 1).

Between the various types of episodes there are brief phase transitions with different durations, roughly of the order of a Schwabe cycle. There are two types of such phase transitions. We called them C- and G-types. The G-type transitions occur when

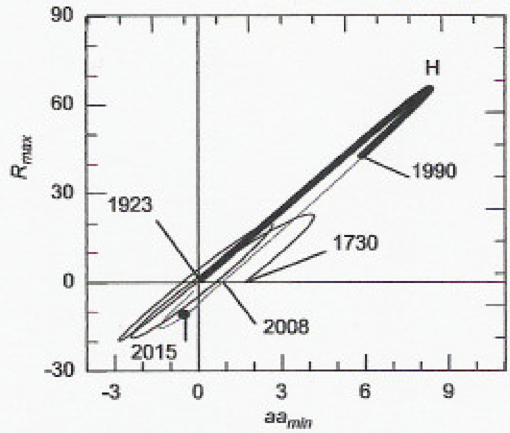

Fig. 1. The phase diagram (R_{max}, aa_{min}) for the Gleissberg cycle for the period 1730–1990 (cf. Fig. 6d in Duhau and De Jager, 2008) (full line) and its extrapolation to 2015 (dashed line). The two coordinates are proxies for the toroidal (R_{max}) and the poloidal (aa_{min}) components of the solar dynamo. Light lines in the diagram refer to counterclockwise motion and dark and dashed lines to clockwise motion.

the values of the Gleissberg oscillations around the transition state, both in the poloidal and in the toroidal magnetic field components, differ simultaneously from zero by less than 0.1% of the transition point's coordinates. A G-type transition leads invariably to a Grand episode. If, however, one of the two components does not differ by less than 0.1% from the corresponding transition state coordinate a C-type transition occurs. Such a transition is always followed by an R-type episode.

The Gleissberg oscillation, as determined for the interval 1923–1990 (cf. Fig. 6 in Duhau and De Jager, 2008), is well represented by a sine function (cross-correlation coefficient 0.99). It appears that in 2008 (see Fig. 1) its value in R_{max} is zero to within a high degree of accuracy (0.0004 sunspot numbers) while aa_{min} was still deviating from zero by 0.84 nT, which is 8% of the relevant transition state coordinate. This fact implies that the current transition is of a C-type. Hence it will be followed by an R-type episode, during which the sense of motion of the path in the phase diagram is always equal to that of the previous Grand Episode.

A similar situation occurred at the end of the episode in the 12th century that we called H_{-2} (cf. Fig. 6a in Duhau and De Jager, 2008). In analogy with, and according to the rules that appear to govern the solar dynamo, we therefore expect that the forthcoming episode will be of the R-type and that its track in the phase diagram, after the past H-type episode, should be clockwise. Following our earlier designations we label the forthcoming episode R_{+1}.

To estimate the error in the predicted path beyond the year 2008 we extrapolate the forthcoming R_{+1} Gleissberg cycle in R_{max} by assuming it successively equal to the two extreme cases that occurred during the last millennium. One of these was the R_{-2} episode (from 1165 to 1230). It was the weakest episode of regular oscillations of the last millennium. The other is the R episode (1730–1923), which had the largest amplitude. We find that the differences between the two cases are not discernible before 2015 in the scale of Fig. 1. This is due to the fact that, as follows from an analysis of the proxies for the last millennium, the length of a Gleissberg cycle depends directly on its amplitude, where lengths of 60 and 95 years correspond with amplitudes of 15 and 25 sunspot number units, respectively. Hence, the strongest cycle is varying slower than the weakest (cf. also Fig. 4).

Any polar (poloidal) cycle starts at the maximum of the preceding cycle and it ends one maximum later (Svalgaard et al., 2005; Callebaut, personal comm.). Therefore, the polar cycle at which the current C transition occurs started in the year 2000, which was the year of sunspot maximum #23. It is expected to end at the time of sunspot maximum #24. Hence, the last sunspot (toroidal) cycle of the 20th century Grand Maximum is cycle #23 and the first one of the forthcoming Regular Episode, R_{+1}, will be cycle #24. In analogy with earlier episodes, the R_{+1} episode may last for one or more Gleissberg cycles. If it would be like the R_{-2} episode (1165–1230) it would last for about 60 years. Moreover, if the cyclical repetition of the three types of episodes of the last millennium would continue, one might expect that after the forthcoming R_{+1} episode a G-type transition will occur. That transition will then be followed by an M episode, which we label M_{+1}. That episode will not start before about 2070.

We summarize this section: The 20th century Grand Maximum terminated around 2000. Thereafter a C-type transition period started. After that we expect an episode of regular oscillations. As such earlier episodes, it may last for one or more Gleissberg cycles. If that episode would be like the R_{-1} episode (1165–1230) it may last as short as about 60 years. It is expected to be followed by a Grand Minimum.

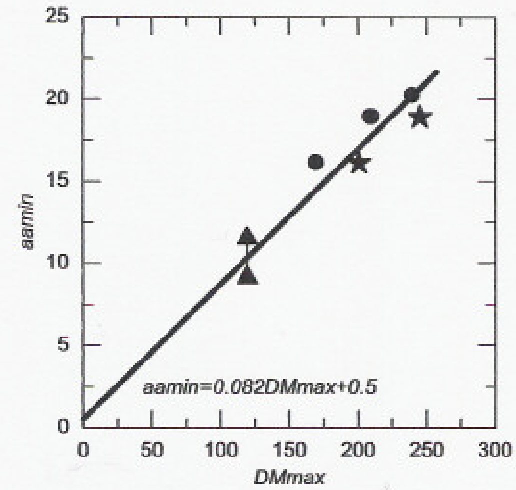

Fig. 2. Values of aa_{min} as a function of DM_{max} as obtained by Svalgaard et al. (2005) from Mount Wilson Solar Observatory data (stars) and from Wilcox Observatory data (dots). The line is the regression line that results from including the origin in the computation. The triangles are the prediction from the value of *DM* during sunspot cycle 23.

3. The amplitude and length of the ongoing poloidal magnetic cycle and of the next sunspot cycle

Outline: As a first step towards predicting sunspot maximum #24, we derive in this section an empirical relation between the geomagnetic index *aa* at sunspot minima and the polar component of the solar magnetic field as observed in the Mount Wilson and Wilcox solar observatories during a few years before and around the sunspot minima.

Svalgaard et al. (2005) observed that the polar magnetic field strength (*DM*) as measured in the Mount Wilson Solar observatory for sunspot cycles 20–22 reached its maximum yearly value 2 or 3 years before sunspot minimum. From their Fig. 4 we observe that 2 or 3 years before sunspot minimum until a year after that *DM* oscillated around its average level by less than 10% of its absolute value. From the 12-month smoothed sunspot number and Mayaud's *aa* index (Mayaud, 1972) time series for the last 160 years we find that the geomagnetic and sunspot minima are almost synchronous. Actually, the geomagnetic minima are lagging behind sunspot minima by less than 0.4 year. This enables us to establish an empirical relation between the corresponding values of aa_{min} and DM_{max}. It is based on data from the last three cycles and is shown in Fig. 2. The cross-correlation coefficient increases from 0.83 to 0.97 when we include the origin in the computations. This result is based on three recent cycles only and evidently calls for confirmation by future observations, but it allows one to assume that

$$aa_{min}\ (\mathrm{nT}) = (0.082 \pm 0.010) DM_{max}(\mu\mathrm{T}), \tag{1}$$

where the error in the slope gives the 90% confidence level. The linearity of the relation between aa_{min} and DM_{max} gives further confidence in the common claim that aa_{min} is a proxy for the poloidal cycle's amplitude.

Eq. (1) has two important consequences: (a) it allows one to forecast aa_{min} by a few years in advance of a given sunspot minimum; and (b) conversely, it quantifies aa_{min} in terms of the amplitude of the polar cycle, thus providing accurate proxy data for this amplitude.

The average DM_{max} value measured by Svalgaard, Cliver, and Kamide for 2004 is 119.3 μT, this being quite similar to the value derived from Wilcox observatory data. With data from http://wso.stanford.edu/gifs/Polar.gif we verified that since 2004 the *DM* values have oscillated at most by 10% from its average value. By introducing the observed value in Eq. (1) we predict for the next minimum: $aa_{min} = 9.8 \pm 1.2$ (the filled triangles in Fig. 2). This is well below all values observed during the Grand Maximum of the 20th century. It is comparable to those at the beginning of the 20th century. This is consistent with our finding in Section 2 that the forthcoming dynamo episode will be of an R-type.

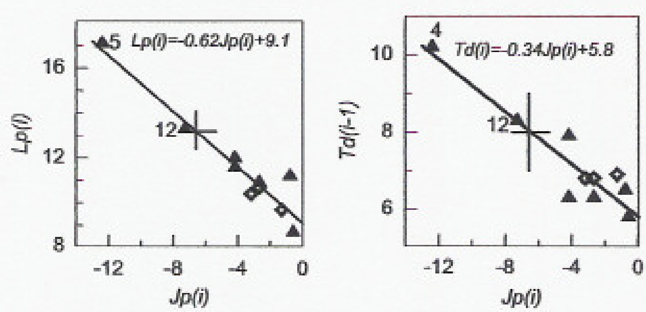

Fig. 3. The length $Lp(i)$ of polar cycle *i* (left) and the time $Td(i-1)$ elapsed between sunspot maximum #$(i-1)$ and the next minimum (right) since 1730. In both diagrams the abscissa $Jp(i)$ is the difference between the amplitudes, as measured by aa_{min}, of polar cycles #*i* and #$(i-1)$. The triangles and the open diamonds refer to cycles that belong to the R and the H episodes, respectively. Some cycles are labelled by their number; cycle 5 is the first of the Dalton minimum. The plusses are the predicted *Lp* and *Td* values for sunspot cycles 24 and 23, and their errors.

So far we have predicted the aa_{min} value for the forthcoming polar cycle (#24). We now look for a relation that will allow us to predict its length. We recall that a given poloidal cycle starts at the sunspot maximum before the cycle under investigation ends at the subsequent maximum. For the present purpose we therefore define the length $Lp(i)$ of the polar cycle #*i* as the difference between the time of occurrence of sunspot maximum for cycle $(i-1)$ and that of the subsequent one, cycle #*i*. We next define the 'decay time', *Td*, of a sunspot cycle as the time between its maximum and the subsequent minimum. Finally we define $Jp(i)$ as the difference between values of $aa_{min}(i)$ and $aa_{min}(i-1)$. The best linear fit appears to be found when $Lp(i)$ and $Td(i-1)$ are plotted vs. $Jp(i)$ (see Fig. 3). The cross-correlation coefficients are

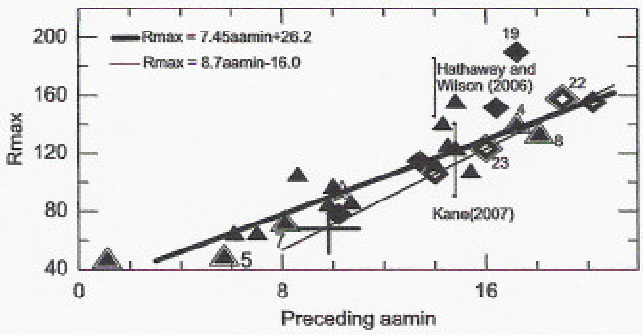

Fig. 5. Empirical determination of the relation between R_{max} and aa_{min} and comparison with other investigations. The filled diamonds correspond to the first four cycles of the H episode, and the other symbols are as in Fig. 3. The regression lines for the R-type episode (heavy line) and for the last four cycles of the H episode (light line) are shown. Also shown are the prediction by Hathaway and Wilson (2006) and Kane (2007) (vertical thin lines). The heavy cross is our prediction with its error bars and the open star represents the transition point.

As to the calculation of the value of the decadal oscillation in R_{max} we note the following. We showed above that the decadal variation in polar cycle 24 is expected to be negative, a prediction based on the measured *DM* value. Another look at Fig. 4 shows that negative values of the decadal oscillation in aa_{min} correspond always with negative values in R_{max} for the same oscillation. Hence, the decadal oscillation in R_{max} will be negative during sunspot minimum #24. This oscillation was positive during sunspot cycle maxima #21 through #23. This is consistent with the observation that never in the past millennium more than three consecutive decadal oscillations had the same sign (see, e.g. Fig. 4). We showed earlier (Duhau and De Jager, 2008, with more references, cf. also our comments to Figs. 4 and 5 of this paper) that the relationship between the decadal variation in aa_{min} and R_{max} for the same magnetic cycle is by no means linear. However, the length of the decadal oscillation has never exceeded 45 years, and therefore, no more that three successive cycles can have the same sign. Therefore, as the decadal oscillation has been positive for sunspot cycles 21–23, we may forecast that it will be zero or negative at sunspot maximum #24. On the other hand, the largest amplitude of the negative oscillations in R_{max} that was ever observed during an R-type episode (this being −31), which we expect to start with cycle 24, occurred during cycle 12. For that cycle the decadal oscillation in aa_{min} was −3.52, which is stronger that the strongest value predicted for polar cycle 24, viz. −2.72 (see Table 1). Briefly, a comparison of R_{max} and aa_{min} for the R-type episodes during the past millennium, also considering the forecasted aa_{min} values, shows the bounds between which R_{max} can maximally vary. Putting all evidence together we may safely assume that the decadal oscillation in R_{max} at sunspot maximum #24 will be between −31 and 0. Therefore we forecast (Table 1) the value of -15.5 ± 15.5. Thus, by adding this last to the Gleissberg cycle's and transition point's values we find for R_{max} in cycle 24: 68 ± 17. The error is the largest conceivable deviation. The larger part of the error in our prediction comes from the decadal variation. The smaller predicted value is in the range of the Dalton minimum values, but a Dalton-type minimum will only develop when the future decadal oscillation would have a length of about twice the Hale length. This is so because only in this case the low values would maintain for longer than a sunspot cycle. In the contrary, if the decadal oscillation would have a length in the Hale band, it would be positive at sunspot cycle 25, and so sunspot maximum #25 will be moderate.

Summarizing: A consistent picture is the following. After the past Grand Maximum that lasted from 1923 to 2000, a C-type transition started. It will end around the time of maximum of the forthcoming cycle, which will occur in 2014.0 ± 0.5. That cycle will be the first of the coming R-type episode. For it, R_{max} will assume a value of 68 ± 17, where the error is the largest conceivable one.

5. Comparison with some other forecasts

A variety of methods is being used to forecast solar activity (for reviews, see Kremliovsky, 1995; Hathaway et al., 1999; Usoskin and Mursula, 2003). Several of these are based on the assumption that the solar dynamo has basic periodicities. Others, such as Ohl's (1966) method, are based on some kind of precursor of the sunspot cycle that has proved to bear some degree of statistical significance, but without a clear physical explanation. The first method that is physically founded on a dynamo theory is the one introduced by Schatten et al. (1978) (see also Layden et al., 1991; Schatten, 2002, 2003), which is based on a Babcock-type dynamo model.

In the past few years many studies have been published on forecasts of solar activity for cycle 24. An overview is given by Pesnell. His review (http://www.sec.noaa.gov/SolarCycle/SC24/May_24_2007_table.pdf) summarizes 45 predictions. Predicted years of maximum sunspot numbers range between 2009 and 2014, while the forecasted maximum sunspot numbers vary between 42 and 197. A NOAA-NASA panel of 13 members and 8 consultants (http://www.sec.noaa.gov/SolarCycle/SC24/index.html) presented a prediction for cycle 24. The panel did not reach consensus; its split opinion was that cycle 24 might either peak in October 2011 (with $R_{max} = 140 \pm 20$) or in August 2013 ($R_{max} = 90 \pm 10$). Approaches that are not listed in Pesnell are by Makarov et al. (2003) and Callebaut et al. (2003). Their study considers the 'rest latitudes', i.e. the lower heliographic latitudes of the solar residual global unipolar magnetic field areas. They found that these latitudes gradually decreased during the past four solar cycles and by extrapolation it is concluded that for cycle 24 the value of R_{max} will be 'less than 80'. This conclusion agrees fairly well with the values predicted from precursor methods, including the present one.

We next discuss some recent predictions in more detail. Some of these are based, like ours, on the relationship between the poloidal field strength at the end of a given cycle and the toroidal one at the maximum of the next cycle. The difference with the present investigation is that we have based our work on the non-linear relationship between these two variables. This is illustrated by Fig. 5, which gives the values of *aa* at a geomagnetic minimum in comparison with R_{max} of the next cycle. As discussed in Section 4 the large dispersion in the data points occurs for the larger part in the decadal scale. This dispersion was even larger around and during the Dalton minimum (framed triangles in Fig. 5) and during the H episode (diamonds). In particular, the large deviation for the first four cycles of the H episode (black diamonds in the figure) is due to a sharp increase in the amplitude of the decadal amplitude oscillation of the toroidal field, while that in the poloidal field is close to zero (see Fig. 4).

In seemingly contrast with the above statement is that Svalgaard et al. (2005), using the measured DM_{max} values from the Wilcox observatory, found $R_{max} = 75 \pm 10$, which is not too far from our prediction, in spite of the fact that these authors assumed a linear relationship between DM_{max} (for which aa_{min} is a proxy) and R_{max}. The explanation is that for the three cycles (open diamonds in Fig. 5), which are those used by Svalgaard et al., the relationship between these variables is indeed linear and does not differ significantly from the light line in Fig. 5. We foresee, though, a departure from this linear relationship in the forthcoming years because these authors have made their

0.96 and 0.92 for the left-hand and right-hand diagrams of Fig. 3, respectively.

These diagrams enable us to forecast the decay time of sunspot cycle 23 and the length of the polar cycle 24: $Td(23) = 8.0 \pm 0.7$ years and $Lp(24) = 13.5 \pm 0.5$ years (crosses in Fig. 3). From these values and the observed time of sunspot maximum #23: the year 2000.5, we obtain the time of the next sunspot minimum, 2008.5 ± 0.7 year. The year of sunspot maximum for cycle #24 is 2014 ± 0.5.

We briefly discuss some qualitative comparisons with other cycles. The pairs of cycles (11, 12) and (23, 24) have very similar Jp values. The same applies to their values of aa_{min}: (14.2, 7.2) and (16.1, 9.8), respectively. Hence we claim that the poloidal cycle 24 will in these respects be similar to poloidal cycle 12 (marked in Fig. 3). This enables us to make a comparison with observational evidence: the first spot of the new cycle was observed January 4, 2008 (NOAA 0981) and the first magnetogram that indicated a weak evidence of a magnetically new region was observed January 2, 2008, at 14:28 UT. So the first activity of the new cycle was observed just at the beginning of 2008. For sunspots cycles such as cycle 12, which is comparable to the forthcoming one, the time difference between the appearance of the first high-latitude spot and the subsequent minimum is 1–2 years (Waldmeier, 1935). The thus roughly estimated time of the next minimum is in agreement with our determination.

Summarizing this section: We derived a relation between aa_{min} and DM_{max} (Fig. 2) and two relationships between parameters describing the shapes of the toroidal (sunspot) and poloidal (polar) cycles (Fig. 3). From these we predict $aa_{min}(24) = 9.8 \pm 1.2$ and from this value we infer that sunspot minimum and maximum of cycle 24 will occur in 2008.5 ± 0.7 and in 2014.0 ± 0.5, respectively.

4. The prediction of the amplitude of sunspot maximum 24

Outline: Based on the result of Sections 2 and 3, we predict the shape and amplitude of the Gleissberg cycle for the toroidal and poloidal components of the solar magnetic cycle at the time of occurrence of sunspot maximum #24. We also predict the value of the decadal oscillation in the geomagnetic aa index at the preceding minimum. From these data and from the non-linear relationship that appears to exist between the decadal oscillations in the aa geomagnetic index at each sunspot minimum with those in the sunspot number during the next maximum, we predict the decadal oscillation in R_{max} for sunspot maximum #24. By adding the so-found Gleissberg and decadal oscillations to the respective transition state coordinate, sunspot maximum #24 is predicted. We also briefly discuss the characteristics of the decadal oscillations that are expected to occur during the forthcoming episode of solar activity.

Essential for this section is Fig. 4, which shows the time variation since 1724 of the decadal oscillations in aa_{min} and R_{max} and of the Gleissberg oscillation in R_{max}. In this figure the variations of R_{max} and aa_{min} are plotted relative to the transition point's coordinates. We describe some aspects. The amplitude of the decadal oscillation in R_{max} (the bold line with dots in Fig. 4) steadily decreased since sunspot cycle #3. It had its minimum during cycle 15, just prior to the 1923 G-type transition. Thereafter it increased until a maximum was reached during sunspot cycle 19, while it again decreased after that. A similar behaviour was followed by aa_{min} (thin line with stars in Fig. 4), which was practically zero between cycles 14 and 19. It was negative after that (i.e. after the beginning of the 20th century Grand Maximum) to reach a value of about −40 during polar cycle 20. After that both components went in pace again. This indicates

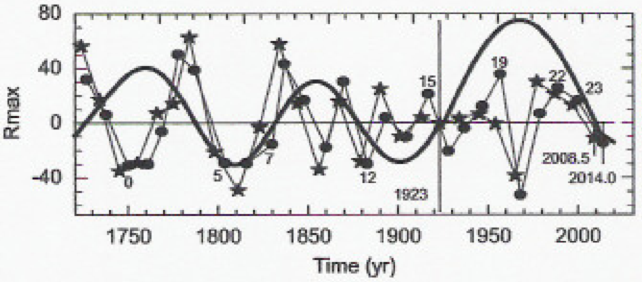

Fig. 4. The decadal oscillations in aa_{min} and in R_{max} since 1724 and their dates of occurrence (stars and points connected by thin lines). The heavy line represents the Gleissberg cycle in R_{max}. The star and dot at 2008.5 and 2014.0 are the predicted values for aa_{min} and R_{max}, respectively, and their errors (the thin lines).

Table 1
The calculation of aa_{min} and R_{max} for the poloidal and toroidal cycle 24

	aa_{min}	R_{max}
Transition coordinate	10.34 ± 0.08	93.38 ± 0.7
Gleissberg cycle	0.70 ± 0.20	-10.00 ± 0.4
Decadal oscillation	-1.24 ± 1.48	-15.5 ± 15.5
Predicted	9.8 ± 1.2	68 ± 17

that another variable, in addition to those of the two magnetic field components, determines dynamo activity (cf. our comments in Section 5).

The Gleissberg cycle (heavy line in Fig. 4) shows its non-linear nature in the fact that its length increases with its amplitudes. A similar behaviour is observed in the decadal oscillations. For a cycle with moderate amplitude, as cycles 9 through 15, the length lies in the Hale time scale, below 25 years, and so the decadal oscillations alternately change sign at each sunspot cycle, which is a behaviour that leads to the well-known odd–even rule. For the moderate cycles 9 through 15 this changeover period lies in the Hale time scale. Contrary to that, the strong decadal oscillations that started after the C-type transitions of 1730 and 1923 had lengths of about twice the Hale cycle length. As a consequence the odd–even rule was violated twice around the Dalton minimum and once during the H episode.

We next present the values of aa_{min} and R_{max} for cycle 24. The values result from the addition of the Gleissberg cycle and the decadal oscillation value to the transition point value. This is done for the years 2008.8 (for aa_{min}) and 2014.0 (for R_{max}). The relevant numbers are given in Table 1, and their calculation is explained here: In Section 3 we have predicted that the next sunspot minimum will occur between 2007.8 and 2009.2. From the extrapolation in Fig. 1 we find that the Gleissberg cycle ordinate varies for these years from 0.9 to 0.5. Hence, we estimate that at the time of the next sunspot minimum (poloidal maximum) the value of the Gleissberg cycle will be 0.7 ± 0.2 (see Table 1). By subtracting this last and the transition point ordinate from the aa_{min} value as predicted in the previous section the decadal variation in aa_{min} at the next sunspot minimum is found. Hence, the error in the decadal oscillation in aa_{min} is the addition of the error in the predicted value and in the Gleissberg and transition values. The result is represented in Fig. 4 by the star at 2008.7. The dot at 2014.0 is the predicted value of the decadal oscillation in R_{max} at sunspot cycle #24. The way in which this latter is found is described below.

Analogously, taking into account that sunspot maximum #24 is predicted to occur between 2013.5 and 2014.5, we derive from the extrapolation of Fig. 1 the value of the Gleissberg cycle at sunspot maximum #24.

prediction assuming that R_{max} depends linearly on *DM*, and we have shown that the relationship is significantly non-linear. This suggests again the existence, next to the poloidal and toroidal field strengths, of an additional variable that determines the time evolution of the toroidal field and hence of R_{max}. This is not surprising since even in axial-symmetric solar dynamo models the dynamo states are described by the associated two-dimensional velocity field next to the two magnetic field components. These additional variables might be identified with the differential rotation and the large-scale meridional circulation (the 'conveyor belt') in the convection zone. This is a matter that asks for further investigation.

The neglect of the non-linear nature of the dynamo and of the fact that solar dynamo is at present undergoing a phase transition may also explain the difference between our result and that of Dikpati et al. (2006). They predicted an R_{max} value that is 30–50% higher than that of cycle 23, which leads to $R_{max} = 155–180$ (Pesnell). Their work is based on a sophisticated elaboration of the physical theory of the solar dynamo, but it neglects the non-linear character of the dynamo and the consequent possibility of phase transitions between the three types of episodes.

In other forecasts the polar field strength is estimated from the geomagnetic *aa* index, as in the present paper and in those by Hathaway and Wilson (2004, 2006) and Kane (2007) (see the corresponding vertical lines in Fig. 6). The most recent prediction by Hathaway and Wilson (2006) yields $R_{max} = 160 \pm 25$, which is one of the largest predicted R_{max} values. It is even close to that of cycle 19 (cf. the labelled diamond in Fig. 5). The cause of the strong difference with our results is that the polar field's contribution to the *aa* geomagnetic index was estimated by splitting the geomagnetic index in a toroidal and a poloidal component by a procedure proposed by Feynman (1982). This leads to a contribution that is well above the one derived here from the observed values of aa_{min}. Kane (2007) based his prediction on Ohl's method (Ohl, 1966) and consequently used a regression line that is very similar to that of Fig. 5. However he took for aa_{min} the running mean value over the 12 months prior to March 2007. The resulting value is still fairly high (14.7 nT) as compared to the one that we derived (9.8 ± 1.2 nT). We must add that if the minimum in *aa* would occur as late as 2009–2010, the aa_{min} value may show to be very low, as is already suggested by the measured value of *DM* during the current cycle.

Our result may be compared with a prediction by Ogurtsov (2004, 2005a, b, see also references therein) for the next 50 years. He found an average value of solar activity of about $\langle R \rangle = 45$. His work was based on a study of the evolution of the system's trajectory in a multi-dimensional pseudo-phase space, on the basis of proxy data for sunspot numbers for the last 10,000 years. Similarly, a recent study by Aguirre et al. (2008), also based on a non-linear dynamo model, predicts values of $R_{max} = 65 \pm 16$ and 87 ± 13, respectively. Their first value is in excellent agreement with ours.

6. Summarizing conclusions

We have used information on the temporal evolution of the non-linear relationship between the poloidal and the toroidal solar magnetic cycles as found by Duhau and De Jager (2008) for the last millennium, to predict future solar activity in so far as such is possible.

As suggested before, we confirm that, while dynamo's behaviour is predictable in the long-term time scale, it is less so in the decadal. This is consistent with results found from a study of the Lyapunov exponents (e.g. Ostryakov and Usoskin, 1990; Sello, 2001, 2003 and references therein) indicating that solar activity can be predicted only for a few years in advance. This is so because in the short-term time scales (of the same order or smaller than the 11-year cycle) the effects of the long-term variations are small or non-existent. Also, the effect of the presence of chaotic and stochastically relevant components at decadal and smaller time scales on the behaviour of the solar dynamo is a reason for limited predictability. Therefore, because of its local nature (Sello, 2001, 2003), the Lyapunov exponent only detects lack of a regular behaviour in the short-term variation, which is in the 'decadal time scales' as defined by Duhau and De Jager (2008).

Along this line we predicted the long-term behaviour of solar activity and we found that presently a C-type transition is going on. It started in 2000 and it will end in 2014 ± 0.5, when sunspot maximum #24 is predicted to occur. This transition period will be followed by an R-type episode. It is foreseen that it will be an episode, characterized by a Gleissberg cycle of moderate amplitude (not larger than 25 sunspot numbers) and a length in the lower side of the Gleissberg band (60–100 years). This episode is starting in 2008 with a negative oscillation. The path in the phase diagram of the magnetic field coordinates (cf. Fig. 1) will be counterclockwise.

For an R-type episode, such as the one that is expected to start after 2008, we (Duhau and De Jager, 2008) found that the decadal oscillations may be of moderate or of strong amplitude. In the first case the length of the oscillations is in the band of Hale periods and then the odd–even rule will be fulfilled during that whole episode. If the amplitude would be large, the length of the oscillation is expected to be in the semi-secular time scale as was the case around the Dalton-type minima and during the H episodes. This would lead to a violation of the odd–even rule.

We found that polar cycle 24 is expected to be similar to polar cycle 12. Both have in their positive phase strong decadal oscillations. These led to the Dalton minimum. We stressed that the maxima of sunspot cycles 23 and 24 will be quite similar to those of the cycle pair 11 and 12. However, this does not mean that the length and amplitude of sunspot cycle 24 would be like those of sunspot cycle 13, since according to our results in Section 3, it depends on the amplitude of polar cycle 25. This latter might differ from that of polar cycle 13. This is a consequence of the short horizon of predictability that exists in the decadal time scale.

We have predicted the next geomagnetic minimum to occur at 2008.5 ± 0.7 when aa_{min} should have a value of 9.8 ± 1.2. From cycle 24 the maximum sunspot number R_{max} is predicted to be 68 ± 17, where the largest likely error was assumed. However, a strong decadal oscillation is presently going on; it is expected to continue during the forthcoming R episode. Since the Gleissberg cycle and these strong decadal oscillations are predicted to be both in their negative phases during sunspot maximum #24, the lower values, comparable to those during the Dalton minimum, are more likely to occur at maximum of cycle 24. As to the long-term decomposition, the forthcoming episode is forecasted to be of the R-type. We conclude that, even in the case that sunspot cycle 24 would appear to have the lowest predicted value (52) it would not cause an M-type episode, like the Maunder Minimum. Instead, a short Dalton-type minimum would rather occur. It may last for at most three sunspot cycles, viz. cycles 24 through 26. In that case the odd–even rule would be violated for the pair of cycles 25, 26. This period of low solar activity may be followed by a Grand Minimum.

Acknowledgments

We are greatly obliged to the anonymous reviewers of this paper for their well-considered comments and constructive suggestions.

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