



The thermonuclear instability of the solar core

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I present here a list of the most important astrophysical problems related to the energy and neutrino producing solar core. Attempting to find a physical connection between the solar activity and the nuclear processes of the solar core, I present the relation between the LOCAL thermal instability and the conditions in the solar core. The result show that if a mechanism would be able to produce a local heating above 10^8 K, this perturbation would push the local region of the core into the instability regime. Tidal-electromagnetic inductive coupling may elicit such a large local heating. Some consequences of the dynamic solar model are discussed.

1. Astrophysical observations showing the dynamical nature of the Sun

There exists a global phenomenon showing the dynamical nature of the Sun: the solar activity. The origin of solar activity points to the solar core [1]. Moreover, the magnetism of the Solar System's planets is linearly proportional to the tidal effects of their moons [2]. This phenomenon may indicate that the solar magnetism also is linearly proportional to the tidal effects of the planets. A remarkable list of unsolved, tantalising problems surfaced in the last decades in relation to the activity of the solar core, like:

- the existence of rigid rotation of the "activity centres", "hot spots", "sunspot nests" ([3,4]) with a rotation rate of the rigidly rotating deep solar core. Are there rigid funnels from the core?

- the global appearance of surface explosions, with 180 degree symmetry of active longitudes [5]. Does the core participate in the surface activity?

- the slow rotation of the core, instead of 4-15 times of the surface rate it is only 0.7-1.3 of the surface rate. Does a coupling exist between the core and the surface?

- the rotation rate of the inner core seems to correlate with the surface activity cycle [6];

- the nitrogen-enigma: N^{15}/N^{14} enhancement with 50% of the solar wind as measure in the Moon's surface in the last 3×10^9 years (instead of decrease, [7]). N^{15} production: in hot CNO cycle, above 10^8 K. Are there hot and rigid funnels

from the core?

- the solar wind, and especially the active regions, are enhanced in all the heavy elements with $Z > 2$, relative to the general photosphere. Fe: > 20 enhancements, Ca: twofold, $He^3/He^4 < 10$ 000-fold enhancements. Fe production is possible above 10^9 K. Are there hot and rigid volcanic funnels?

- sudden changes of the giant cells set up simultaneously with the rotation rate change of the background field, accompanied by the violent eruption of the solar activity and neutrino production [8];

- the more than 300 years correlation of the planetary tides with the sunspot number [9]. Do planetary tides trigger hot and rigid funnels from the solar core?

- planetary co-alignments of the inner planets have periods around 11.2 years, coinciding with the average of the solar activity cycle [10];

- planetary co-alignments of the outer planets trigger a large amplitude motion of the mass centre of the Solar System relative to the centre of the Sun with a period of 11.2 years [11];

- the dynamic solar model offers a natural solution to the solar neutrino problems and to the tantalising astrophysical problems of solar activity and the solar core [1,10,12];

- Basu [13] confirmed the earlier results of Basu [14] and McNutt [15], finding a statistically significant ($> 98\%$) correlation between the solar neutrino flux of Homestake and the mass-flux of

the solar wind. Basu noted that "The logical conclusion is that the two may have a common cause of origin in the solar interior which needs further investigation".

These tantalising problems indicate that the solar activity is ultimately related to the solar core. In this paper I attempt to reveal how such a connection may arise between the solar activity on the surface and the solar neutrino production rates.

2. The thermonuclear stability of the Sun

The nuclear timescale of the Sun as a whole is indicated by

$$\tau_K \approx 3P/2\rho\epsilon \approx 10^{15} s. \quad (1)$$

But in the presence of energy fluctuations [16] a convective instability arises above $10^8 K$. If T is large, $\epsilon > 10^{12} \text{ ergs/g/s}$, $\tau_K < 10^3 s$. The time-scale of volume expansion is

$$t_{ff} \approx (2R/g)^{1/2} \approx (2G\rho)^{-1/2} \approx 10^3 - 10^4 s. \quad (2)$$

The time-scale of energy production is: $\tau_e \approx C_v T/\epsilon\nu$. The energy production rate with the CNO-cycle is

$$\epsilon_{CNO} \approx \epsilon_0 \rho X_H X_{CNO} (T/T_0)^\nu, \quad (3)$$

therefore, if T grows, τ_e decreases, and instability may arise. With $\rho \approx 0.041 T^3$, see [17]:

$$T_{crit} = [C_v (2G/0.041^{1/2} \nu / \epsilon_0)]^{\nu+n/2-1}. \quad (4)$$

Now with $T = 1.7 \times 10^8 K$, $\epsilon_{CNO} = 8 \times 10^{13} \text{ ergs/g/s}$, $\Delta T \approx 7 \times 10^5 K$, and so on, the times-scale of the thermal expansion $\Delta t \approx 10^3 s$ the resulting heating will reach $\Delta T \approx 7 \times 10^8 K$. Therefore, the condition of the local thermonuclear instability is $\Delta T > 10^8 K$, and a heating mechanism is needed.

Considering the electro-magnetohydrodynamic heating: $E = 1/c(v \times B)$, v from tides, B from global magnetic field $v = 2\pi R/T_{rot} \approx 10^5 \text{ cm/s}$, $\rho \approx 10^2 \text{ g/cm}^3$, $B_{eq} \approx 4 \times 10^6 \text{ G}$, $E \approx 10^4 - 10^5 \text{ V/cm}$, $U = l_{acc} e E$, $\Delta T \approx U/k$. If $l_{acc} \approx l_{mfp} \approx 10^{-6} \text{ cm}$, $\Delta T_{max} \approx 0.1 \text{ eV} \approx 10^3 K$. Therefore, a simple MHD heating does not work, since five orders of magnitude are missing. A catalysing effect is needed:

1. tidal heating from time to time may be fast and localised, since the tidal waves may concentrate the dissipation of the kinetic energy at the front of the magnetic flux bundles;

2. if E is parallel to B , $l_{acc} \gg l_{mfp}$, so $\Delta T_{max} \gg 10^3 K$. In the outer solar atmosphere, E is parallel to B at most solar flares, which is a well-known phenomenon of the "magnetic shear" present at flare onsets;

3. when electron and proton currents are separated, and concentrated into filamentary currents by the pinch effect, $l_{acc} \gg l_{mfp}$ and so $l_{acc} \approx 1 \text{ cm}$ is available, $U \approx 10-100 \text{ keV}$, $\Delta T \approx 2 \times 10^8 K$. Reconnection can heat to large temperatures in the solar core. Flares are indicated to be present in the energy producing solar core.

Astrophysical investigations suggest that three empirical phenomenon may be of interest in studying the thermonuclear instability of the Sun:

- the time development of the tidal heating and the conditions of 'dissipation events' occurring in the solar core

- flares in the solar atmosphere, the generation of the magnetic shear, and the origin of the filamentary current structure of the solar flares

- how the hot spots, the deep volcanic funnels of the Earth are triggered in the core, and how the hot bubbles are able to travel from the deep core through the mantle to the surface, and similarly in the Sun.

2.1. Chemical anomalies of the active regions as indicators of hot bubbles

In the solar active regions the high first-ionization-potential (FIP) elements (Fe, Mg, etc) are enhanced by a factor 6 to 31, and the high FIP elements (Ne, O, etc.) are enhanced by a factor > 1.75 , relatively to hydrogen. Apparently, there is a continuous enhancement in the He^3/He^4 ratio in the solar wind during the last billions of years. In the C1 meteorites this ratio is 1.5×10^{-4} , in lunar regolith samples 3.3×10^{-4} , and in the solar wind at present it is 4×10^{-4} . In the meteorites, $Fe/H \approx 3.2 \times 10^{-5}$, in the solar wind it is 10^{-4} . In the non-active photospheric regions the iron abundance is around 4×10^{-5} , in the coronal active regions it is 3×10^{-4} . The He^3/H in the protosolar nebula was around 1.1×10^{-5} , while

in solar flares it is around 3×10^{-5} [1]. Regarding these data, I point out that the heavy element enhancements of the solar wind, corona and chromosphere, especially in the active regions calls the attention to a connection between the outermost and the innermost solar regions.

2.2. The physical characteristics of the hot bubbles

Let us see some relations determining the conditions of the bubbles on their way towards the surface. Following Gorbacky [18], if the hot bubble's surplus energy is dominated by radiation, and their initial energy surplus when they start to rise is Q_0 , then $Q_0 = 4/3\pi R^3 a T_2^4$ where T_2 is the temperature in the bubble and R is the radius of the bubble, $p_1 = a/3 \times T_2^4$, p_1 is the pressure outside the bubble:

$$R_b = 1/(4\pi)^{1/3} Q_0/p_1^{1/3} \approx 0.43 Q_0/p_1^{1/3}. \quad (5)$$

Using a value for $Q_0 = 10^{33} - 10^{35}$ ergs, and $p_1 \approx 10^{17}$ dyn, one can get for $T_2 \approx 8 \times 10^7$ K that $R \approx 8.6 \times 10^4$ cm – 4×10^5 cm. The mass of the bubble may be estimated from $m_b \approx L_b/\epsilon_b \approx 10^{16} - 10^{19}$ g.

On its travel the bubble meets with resistance, therefore it is necessary to put a term in the equation of motion of the bubble describing it. Since the molecular viscosity is extremely low, it is enough to take into account only the turbulent drag here:

$$(4/3\pi R^3 \rho_2 + 2/3\pi R^3 \rho_1) d^2 r/dt^2 = -4/3\pi R^3 (dp_1/dr + \rho_2 GM_r/r^2) - c_x \rho_1 v^2/2\pi r^2 \quad (6)$$

where c_x is the coefficient of the turbulent drag, v is the velocity of the bubble. The quantity $2/3(\pi R^3 \rho_1)$ represents the so-called "induced-mass" term occurring for a body moving in a hydrodynamic medium (see [19]) in case of a spherical bubble. Using this equation, the resulting velocity of the steady-state will be reached within 1.4 s with a value given by the following equation:

$$v = [8R/3c_x 1/\rho_1 (-dp_1/dr)(1 - T_1/T_2)]^{1/2} \approx 10^5 - 10^6 \text{ cm/s}. \quad (7)$$

In this way, the bubbles formed in the core will reach the surface activity centres within a day, i.e. almost immediately.

It has to be noted here that recent experimental and computational results suggest that for extremely high Rayleigh numbers $Ra > 10^7$ the turbulent convection turns to a thready flow. Actually, in the solar core $Ra_{act} > 10^{16}$ even when the characteristic length is only $d = 10^5$ cm. "The flow is driven entirely (in the limit of infinite Ra) by these threads. The heat flux is carried by flows that maintain their identity...and can cross a convecting layer with little mixing between them. The width of the threads, in spite of entrainment, decreases with Rayleigh number instead of increasing as one might have expected on the basis of the simple 'higher Ra means more turbulence means more mixing' line of argument" ([20]).

The time-scale of radiative diffusion is very slow in the solar core. The specific radiant energy content of the Sun is $Q(rad) = 7.5 \times 10^{-15} T^4 \text{ erg/cm}^3$ so that $Q(rad, total) = 3 \times 10^{46}$ erg. Therefore, the time for radiation to travel from the centre to the surface $t(rad) \approx Q(rad, total)/L \approx 7 \times 10^{12}$ s (see e.g. [21]). So the average propagation speed of the photons in the solar core $v_{ph} < 10^{-2}$ cm/s. Now the expansion velocity of the relatively sharp "edge" of the bubble may be estimated as $v_b(edge) \approx \Delta R_b/\Delta t \approx [\rho(surface)/\rho(core)]^{1/3} \times R_b(core)/t_{risetime} \approx 10^3$ cm/s. Therefore, the slowness makes radiative dissipation ineffective and negligible.

What amount of temperature surplus is necessary for a bubble to reach the solar envelope from the core? The adiabatic temperature gradient is given as $(dT/dr)_{ad} = -g/c_p$, here g is the local gravitational acceleration, and c_p is the specific heat at constant pressure. In the solar core, the adiabatic temperature gradient $(dT/dr)_{ad}$ is lower than 10^{-3} K/cm. This means that the temperature difference between an adiabatic gradient and the actual one in the solar core, starting from the bottom of the convective zone to the centre, will be $\Delta[T_{ad} - T_{act}] < 5 \times 10^7$ K. The estimation shows that with an initial temperature surplus above 10^8 K all the bubbles may reach the subphotospheric regions if the dissipative effects are ignorable. When dissipation is included as turbulent drag, it is easy to determine how far may travel a bubble with a certain initial heat surplus, when solving equation (34) of Gor-

backy [18], who determined that in a dM star like Castor C ($M = 0.65M_{Sun}$), the bubbles with an initial heat surplus $Q_0 \approx 10^{33}$ ergs cannot reach the bottom of convective zone if they start from $r < 0.2R_{CastorC}$, but with $Q_0 \approx 10^{35}$ ergs they may reach until $0.98 R_{CastorC}$.

What kind of nuclear reactions occur in a hot bubble travelling outwards from the solar core? In a bubble the density is relatively low, therefore the helium-burning does not play a dominant role over the CNO cycle even above $T_b > 5 \times 10^8 K$, and so for the energy production the most important reaction in the bubble will be the CNO cycle. Nevertheless, this CNO cycle is different from the one occurring in the quiet solar core, since in the bubble there is no time enough for the nuclear equilibrium to set up between the reaction products. For example, if the bubble rises from a certain depth, it will expand and cool down, the reaction rates continuously change and deviate from the respective values present at their site of origin. During the characteristic time-scale of the $N^{14}(p, \gamma)O^{15}$ reaction at $T_b > 10^8 K$, the bubble rises and cools down, therefore the time-scale of the $N^{15}(p, \alpha)C^{12}$ reaction (\gg days) will be lower than in the regions which did not rise with the bubble. Therefore, relatively less $N^{15}(p, \alpha)C^{12}$ occurs in the bubble when compared to the region of the Sun from where the bubble have arisen. In other words, it is inevitable that a relative enhancement of N^{15} will set up in the bubble. Moreover, above $T = 2 \times 10^8 K$, the CNO cycle becomes explosive, which leads to a large enhancement of N^{15} [22]. Flare-related He^3/He^4 enhancements, observed to reach a rate 10^4 , may find their explanation partly in the fact that their travel time 10^5 s is less than the time-scales of reactions $He^3(He^3, 2p)He^4$ and $He^3(He^4, \gamma)Be^7$ (which are \gg years), therefore most of the deuterons produced in two-proton collisions will produce He^3 as the end product.

One may ask, whether the hot bubbles shall produce a significant neutrino surplus or not, since in them the CNO cycle burns the hydrogen, and it is well known, that the enhancement of the CNO cycle would produce much more events at the neutrino detectors, thus worsening the solar

neutrino problems. Since the hydrogen-burning reactions (pp and CNO-cycle) produce neutrons in an equal number with protons, it is not necessary for the nuclear reactions to convert additional protons to neutrons besides that of the hydrogen-burning, therefore in a fixed mass element these reactions do not produce neutrinos at a rate larger than hydrogen burning. On the other hand, the CNO-neutrinos have larger energy than the pp-neutrinos, therefore they elicit more significant rates in the neutrino detectors. The question may be answered by a quantitative estimation of the CNO neutrino flux of the hot bubbles. The CNO neutrino flux of the bubble as measured by the neutrino detectors $\Phi_b(CNO)$ will be proportional to the number of reactions producing the neutrinos, therefore, with the mass of the bubble and with the energy production of the bubble. Moreover, $\Phi_b(CNO)$ will be proportional with the ratio of the calculated standard CNO and pp neutrino fluxes:

$$\Phi_b(CNO) < L_b/L_{Sun} \Phi_{SSM}(CNO) \quad (8)$$

and so with $L_b \approx 10^{-1}L_{Sun}$, and accepting that $\Phi_{SSM}(CNO)/\Phi_{SSM} \approx 3$ to 5 [23], calculated for the case when CNO cycle dominates the energy production, one can get $\Phi_b(CNO) < 3 \times 10^{-1}\Phi_{SSM}$. With larger bubble luminosity one can get even larger CNO neutrino flux than the one predicted by the SSM. Occasionally, the run-aways in the solar core may reach such a high temperature, $T > 10^9 K$, that they may contribute to a measurable flare effect, i.e. a correlation of outstandingly large flares with the solar neutrino flux. A prediction of the dynamic solar model (DSM) besides the above, is the presence of a small flux of extremely high-energy neutrinos arising from three proton collisions. High-energy neutrino excess is already indicated by the SuperKamiokande measurements (see e.g. in [24]). The dynamical solar model indicates that the excess is lower at solar activity minimum since there are less (or zero) bubbles then [12].

The different helioseismic measurements until now may present only contradicting results in the most central region $r < 0.2R_{Sun}$ [25]. At the same time, the model presented here may be easily consistent with the present helioseismic results

as well, when $\Delta T_c < 1\%$.

3. The conjectures of the dynamical solar model to the solar neutrino problems and the mass of the neutrino

The following results are reached: (1) an energy source outside the SSM pp, CNO energy source is present in the solar core, and the Sun is not in a thermodynamic equilibrium, (2) this non-SSM source distorts the standard neutrino energy spectrum, and perhaps the MSW effect also contributes to the spectrum distortion (3) The Homestake, Gallex and SuperKamiokande results contain a term arising from the non-pp, CNO source, which has the largest contribution to SuperKamiokande, less to Homestake, and the smallest to Gallex.

The results presented here suggest that (in the case of a static core) the neutrino-temperature seen by the gallium-detector is $T_{Ga} \approx 0.922T(SSM)$ (actually, this is a lower limit, for the case when ignoring the MSW effect). This leads to a pp luminosity of the Sun around $L_{pp} \approx 72\%L(SSM)$. The remaining part of the solar luminosity should be produced by the hot bubbles, $L_b \approx 28\%L(SSM)$. The runaway nuclear reactions proceeding in the bubbles (and possibly in the microinstabilities) should also produce neutrinos, and this additional neutrino-production, Φ_b should generate the surplus terms in the chlorine and water Cherenkov detectors as well. The estimated upper limit for the bubble luminosity ($\approx 28\%$) may be easily consistent with the calculated non-SSM neutrino fluxes $\Delta S_{Cl} = S_C(T_{Cl} = 0.949) - S_C(T_{Ga} = 0.922) \approx 1.04$, therefore the relative rate of the bubble neutrinos at the Homestake detector is $\Delta S_{Cl}/S_{Cl} \approx 41\%$. The same quantity for the SuperKamiokande detector is larger, $\Delta S_{SK} = S_{SK}(T_{SK} = 0.970) - S_{SK}(T_{Ga} = 0.922) \approx 1.74$, $\Delta S_{SK}/S_{SK} \approx 71\%$. These values arise in a dynamic solar model without taking into account the MSW effect, therefore since the neutrinos seem to oscillate, the real relative bubble luminosities and neutrino fluxes have to be smaller (detailed calculations are in preparation).

The dynamic solar model predicts a beryl-

lium neutrino flux $< 43\%$ of the SSM value, corresponding to a temperature of $T(DSM) < 93\%$. This estimation offers a prediction for the Borexino neutrino detector $\Phi(Borexino) = \Phi_{Be}(SSM) \times T(DSM)^{11.5} + \Phi(bubbles) < 0.43\Phi_{Be}(SSM) + \Phi(bubbles)$. Regarding the SNO detector, I can assume that the electron neutrinos of the SSM-like core plus all kinds of neutrinos produced by the hot bubbles produce the neutral currents. Therefore, the prediction of the DSM is $\Phi(SNO) = T(DSM)^{24.5}\Phi(SSM) + \Phi(bubbles) \approx 0.17\Phi(SSM) + \Phi(bubbles)$. These predictions differ significantly from the SSM-values calculated with the MSW effect. Therefore, the future observations may definitively decide which model describes better the actual Sun, the SSM-based MSW effect or the dynamic solar model. In the interpretation if the future measurements it will be important also to take into account the possible dependence of the neutrino fluxes on the solar cycle.

4. Discussion and Conclusions

The calculated solutions of the neutrino flux equations are consistent with the data of the neutrino detectors even for standard neutrinos with zero mass. I have shown that introducing the runaway energy source, it is possible to resolve the apparent contradiction between the different neutrino detectors even assuming standard neutrinos. Moreover, the results presented here suggest that the physical neutrino problems of the atmospheric neutrinos may be consistent with the solution of the solar neutrino problems even without introducing sterile neutrinos.

The dynamic solar model has a definite suggestion that below 0.10 solar radius the standard solar model is to be replaced by a significantly (1 - 7 %) cooler and possibly varying core. These predictions can be checked with future helioseismic observations. Helioseismology is not able to tell us the temperature in this deepermost central region below $0.2R_{Sun}$, see e.g. [25]. On the other hand, the presence of the thermonuclear micro-instabilities causes a significant departure from the thermal equilibrium and changes the Maxwell-Boltzmann distribution

of the plasma particles. It is shown that such a modification leads to increase the temperature of the solar core, which can compensate the non-standard cooling [26] and so the simple dynamic solar model can be easily consistent with the helioseismic results as well.

The indicated presence of a runaway energy source in the solar core - if it will be confirmed - will have a huge significance in our understanding of the Sun, the stars, and the neutrinos. This subtle and compact phenomena turns the Sun from a simple gaseous mass being in hydrostatic balance to a complex and dynamic system being far from the thermodynamic equilibrium. This complex, dynamic Sun ceases to be a closed system, because its energy production is partly regulated by tiny outer influences like planetary tides. This subtle dynamics is possibly related to stellar activity and variability. Modifying the participation of the MSW effect in the solar neutrino problem, the dynamic energy source has a role in the physics of neutrino mass and oscillation. An achievement of the suggested dynamic solar model is that it may help to solve the physical and astrophysical neutrino problems without the introduction of sterile neutrinos, and, possibly, it may improve the bad fit (acceptance lower than 7 %) of the MSW effect [24].

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