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Magnetic and Electric Fields Produced in the Sea During Geomagnetic Disturbances

D. H. BOTELER[1] and R. J. PIRJOLA[1,2]

Abstract — To understand geomagnetic effects on systems with long conductors it is necessary to know the electric field those systems experience. For surface conductors such as power systems and pipelines this can easily be calculated from the magnetic field variations at the surface using the surface impedance of the earth. However, for calculating the electric fields in pipelines and submarine cables at the seafloor it is necessary to take account of the attenuating effect of the conducting seawater. Assuming that the fields are vertically propagating plane waves, we derive the transfer functions between the electric and magnetic fields at the seafloor and the magnetic field variations at the sea surface. These transfer functions are then used, with surface magnetic field data, to determine the power spectra of the seafloor magnetic and electric fields in a shallow sea (depth 100 m) and in the deep ocean (depth 5 km) for different values of the Kp magnetic activity index. For the period range considered (2 min to 3 hrs) the spectral characteristics of the seafloor magnetic and electric fields for a 100 m deep sea are very similar to those of the surface fields. For the deep ocean the seafloor spectra show a faster decrease in spectral density with increasing frequency compared to the surface fields. The results obtained are shown to be consistent with seafloor observations. Assessment of the seafloor electric fields produced by different levels of geomagnetic activity can be useful in the design of the power feed equipment for submarine cables and cathodic protection for undersea pipelines.

Key words: Space weather, electromagnetic induction, seafloor electric fields, submarine cables.

Introduction

Geomagnetic effects on technology have been a problem since the first long conductors were stretched over the earth's surface for use in the early telegraph system (see review by BOTELER *et al.*, 1998). During geomagnetic disturbances, the changing magnetic field induces an electric field within the earth and in electrical conductors such as power transmission lines, pipelines, and phone cables (e.g. LANZEROTTI and GREGORI, 1986). The currents driven by this electric field can damage equipment and cause problems with system operation. On pipelines, the fluctuations in pipe-to-soil potentials produced during geomagnetic disturbances interfere with pipeline potential surveys and possibly contribute to corrosion

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[1] Geomagnetic Laboratory, Geological Survey of Canada, 7 Observatory Crescent, Ottawa, Ontario K1A 0Y3, Canada. E-mail: DBoteler@nrcan.gc.ca

[2] *Permanent address*: Geophysical Research Division, Finnish Meteorological Institute, P.O. Box 503, FIN-00101 Helsinki, Finland.

(SHAPKA, 1993; GUMMOW, 2002). Geomagnetically induced currents in power systems have caused transformers to burn out, misoperation of relays, and system blackout (KAPPENMAN, 1996; BOZOKI *et al.*, 1996). On submarine phone cables, communication has been disrupted during geomagnetic disturbances, with voices transmitted alternately as shrieks and whispers as the induced voltage added to, or subtracted from, the driving voltage for the cable (ANDERSON, 1978; MELONI *et al.*, 1983). For modern submarine cables, including optical-fibre cables, geomagnetically induced voltages have to be considered in the design of the equipment feeding power to the amplifiers ("repeaters") along the cable.

The first step in assessing the possible impact of geomagnetic disturbances on a technological system is to determine the electric fields that the system will experience. The electric fields can then be used with a system model to determine the electric current flow across the system and identify where problems may occur. Several studies have been made to assess the geomagnetic hazard to surface systems, such as power systems (COLES and BOTELER, 1992; PULKINNEN *et al.*, 2000; BOTELER, 2001) and pipelines (PULKINNEN *et al.*, 2001; RIX and BOTELER, 2001). Studies of geomagnetic effects on submarine cables and subsea pipelines, however, have been hindered by an absence of calculations of the electric fields produced at the seafloor.

Electric fields experienced by conductors at the surface of the earth can be calculated from the magnetic field variations and a surface impedance function dependent on the conductivity structure of the earth. The surface impedance for a layered earth is easily calculated using a recursion relation for the impedance at the top and bottom of successive layers from the basement up to the surface (eg. WEAVER, 1994). The relation between the magnetic and electric fields at the seafloor can be calculated in a similar fashion by omitting the top layer (ocean) in the recursion calculations. However, magnetic field observations at the seafloor are limited. To study geomagnetic effects on submarine cables and subsea pipelines, it is more useful to be able to calculate the electric fields at the seafloor from magnetic field observations made at the surface.

In this paper we examine the attenuation of the electric and magnetic fields in the sea and derive the relations between the electric and magnetic fields at the seafloor and at the surface. Surface magnetic field spectra are then used with the seafloor/surface relations to calculate the spectra of the electric and magnetic fields on the seafloor. These calculations are made for different levels of magnetic activity, as given by Kp, and for water depths of 100 m and 5 km corresponding to the cases of a shallow sea and a deep ocean.

Theory

For a horizontally layered conductivity structure the surface impedance associated with plane waves propagating vertically can easily be calculated using

Maxwell's equations and boundary conditions. These techniques are well-established (see, for example, WAIT, 1981; WARD and HOHMANN, 1988; WEAVER, 1994) and are widely used in magnetotelluric studies using measurements of the electric and magnetic fields at the surface of the earth or at the seafloor for determining the underlying conductivity structure (e.g., SCHMUCKER, 1970; WANNAMAKER *et al.*, 1989; FERGUSON *et al.*, 1990). In all these studies the focus is on the relationship between electric and magnetic fields at the same depth. However, for examining geomagnetic effects on submarine cables and undersea pipelines we need to understand the attenuation of the fields in the sea and know the relation between the electric field at the seafloor and the magnetic field at the sea surface. We will show that the relations needed are easily obtained from the existing theory.

We consider large-scale geomagnetic disturbances at a mid-ocean position away from the coast. This is represented as a plane wave vertically incident on a planar ocean of depth d, above a layered half-space. We use the geomagnetic coordinate system where x is northward, y is eastward, and z is vertically down, and assume that the fields have a time dependence $e^{i\omega t}$. The electric and magnetic fields in the ocean can be represented as a superposition of a wave travelling downwards and a reflected wave travelling back up:

$$E_x = Se^{-kz} + Re^{kz} \tag{1}$$

$$H_y = \frac{1}{Z}\left(Se^{-kz} - Re^{kz}\right) \ , \tag{2}$$

where S and R are the amplitude of the downward and upward waves. The propagation constant, k, and the characteristic impedance, Z, of the ocean are given by

$$k = (i\omega\mu_0\sigma)^{1/2} \tag{3}$$

and

$$Z = \left(\frac{i\omega\mu_0}{\sigma}\right)^{1/2} , \tag{4}$$

where σ is the conductivity of the ocean. The magnetic permeability is assumed to always have its free space value, $\mu_0 = 4\pi.10^{-7}$ H/m. Displacement currents are neglected in equations (3) and (4), which is acceptable due to the low frequencies considered in connection with geomagnetic effects.

Within an ocean of depth d, the ratio of the electric field at depth z $(0 \le z \le d)$ to the surface magnetic field is obtained from (1) and (2) and is given by

$$\frac{E(z)}{H_0} = Z\frac{e^{-kz} + \alpha e^{kz}}{1 - \alpha} \ . \tag{5}$$

Similarly, the ratio between the magnetic fields at a depth z and at the sea surface is

$$\frac{H(z)}{H_0} = \frac{e^{-kz} - \alpha e^{kz}}{1 - \alpha} \tag{6}$$

and the ratio between the electric fields at a depth z and at the sea surface is

$$\frac{E(z)}{E_0} = \frac{e^{-kz} + \alpha e^{kz}}{1 + \alpha} \ , \tag{7}$$

where $\alpha = R/S$ is given by

$$\alpha = \left(\frac{Z_d - Z}{Z_d + Z}\right) e^{-2kd} \tag{8}$$

in terms of Z_d $(= E_d/H_d)$, the impedance at the seafloor representing the underlying layers.

Note, because the medium is the same at depth z and at the surface, equation (7) also gives the ratio of the electric current densities at depth z and at the surface. If the seafloor conductivity is assumed to be negligible compared to the conductivity of the seawater, $Z/Z_d \approx 0$ and $\alpha \approx e^{-2kd}$. Equations (6) and (7) are then equivalent to equations (11) and (9) respectively of PRICE (1965) with his source wavenumber term, v, set to zero for the plane-wave source considered here.

The fields at the seafloor can be obtained from equations (5) to (7) by setting $z = d$. Substituting α from equation (8) into equation (5) and re-arranging gives

$$\frac{E_d}{H_0} = Z \frac{(1 + Z/Z_d)e^{-kd} + (1 - Z/Z_d)e^{-kd}}{(1 + Z/Z_d) - (1 - Z/Z_d)e^{-2kd}} \tag{9}$$

which reduces to

$$\frac{E_d}{H_0} = Z \frac{2}{(1 + Z/Z_d)e^{kd} - (1 - Z/Z_d)e^{-kd}} \tag{10}$$

which can be written

$$\frac{E_d}{H_0} = Z_d \frac{1}{\cosh(kd) + Z_d/Z \sinh(kd)} \ . \tag{11}$$

Similarly we obtain the relations between the magnetic fields at the seafloor and the sea surface

$$\frac{H_d}{H_0} = \frac{Z}{Z_d} \frac{2}{(1 + Z/Z_d)e^{kd} - (1 - Z/Z_d)e^{-kd}} \tag{12}$$

$$\frac{H_d}{H_0} = \frac{1}{\cosh(kd) + Z_d/Z \sinh(kd)} \tag{13}$$

and the relations between the electric fields at the seafloor and the sea surface

$$\frac{E_d}{E_0} = \frac{2}{(1 + Z/Z_d)e^{kd} + (1 - Z/Z_d)e^{-kd}} \tag{14}$$

$$\frac{E_d}{E_0} = \frac{1}{\cosh(kd) + Z/Z_d \sinh(kd)} \ . \tag{15}$$

Equations (10) to (15) now give us the means to calculate the electric field E_d and the magnetic field H_d at the seafloor from the magnetic field H_0 observed at the sea surface.

In practical applications we are concerned with the magnetic flux density B that is actually measured rather than the magnetic field strength H. Using the relation $B = \mu_0 H$ the field ratios are easily rewritten in terms of B.

Fields in the Sea

To illustrate the attenuation of the electric field and the magnetic field in the sea, Figure 1 shows the amplitudes of E/E_0 and of B/B_0 versus depth for oceans with depths of 100 m, 1 km, and 5 km. Calculations are made for a period of 5 min and a uniform resistivity beneath the seafloor, $\rho_e = 10\,\Omega$-m. The seawater resistivity is $\rho_s = 0.25\,\Omega$-m. The skin depth in seawater for a variation with a 5-min period is 4.36 km therefore the three plots in Figure 1 represent situations where the ocean depth is (i) much less, (ii) less, and (iii) greater than the seawater skin depth. These results show that, in cases (i) and (ii) the electric field is nearly constant with depth, while in case (iii) the electric field decays approximately exponentially with depth. In all cases the attenuation of the magnetic field is much greater than that of the electric field.

Fields at the Seafloor

Equations (10) to (15) show that the fields at the seafloor are dependent on kd, the product of the propagation constant in seawater and the ocean depth. The propagation constant is related to the skin depth, δ

$$k = \frac{1+i}{\delta} \quad \text{where} \quad \delta = \sqrt{\frac{2\rho_s}{\omega\mu_0}} \ . \tag{16}$$

Thus

$$kd = (1+i)\frac{d}{\delta} \tag{17}$$

showing that the value of kd depends on the relative size of the ocean depth and the skin depth in seawater. Note that $\mathrm{Re}[kd] = \mathrm{Imag}[kd] = \frac{|kd|}{\sqrt{2}} = \frac{d}{\delta}\cdot$

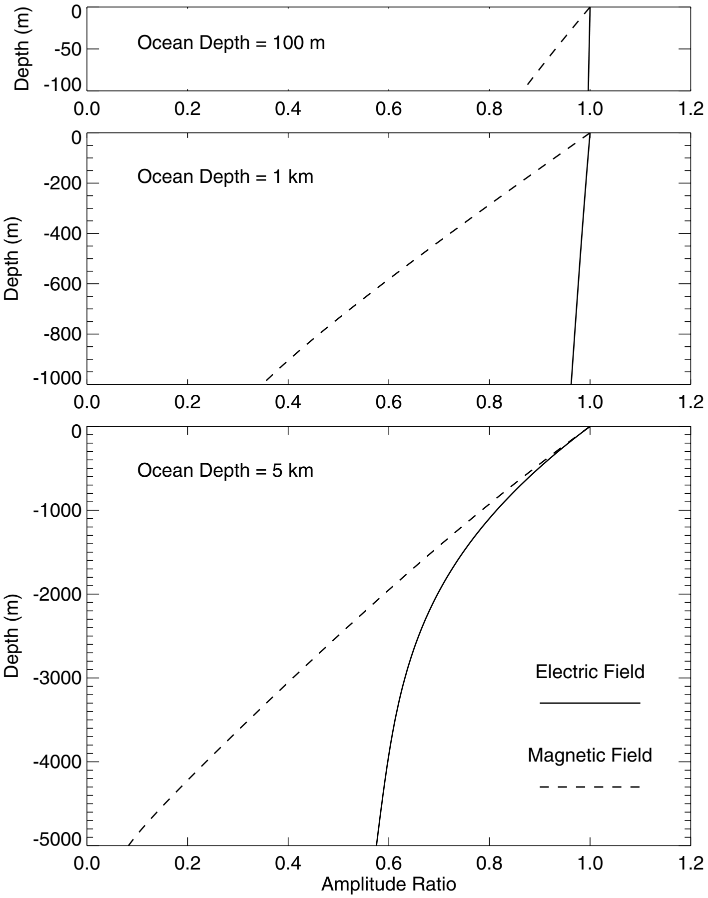


Figure 1

Attenuation of the electric field and the magnetic field within the seawater for a period of 5 minutes for oceans of different depth. The seawater resistivity is 0.25 ohm-m and the earth below the seafloor is taken to be uniform with $\rho_e = 10\,\Omega$-m.

Two limiting cases can be considered. When the ocean depth is considerably less than the skin depth in seawater (the "thin sheet" case) then $|kd| \ll 1$ and equations (10), (12) and (14) simplify to

$$\frac{E_d}{H_0} = Z_d\left(\frac{1}{1+\frac{Z_d}{Z}kd}\right) \tag{18}$$

$$\frac{H_d}{H_0} = \frac{1}{1+\frac{Z_d}{Z}kd} \tag{19}$$

$$\frac{E_d}{E_0} = \frac{1}{1+\frac{Z}{Z_d}kd}\ . \tag{20}$$

Usually $Z_d/Z \gg 1$, so in (18) and (19) the $\frac{Z_d}{Z}kd$ term is not necessarily insignificant and there is a change in the magnetic field across the thin sheet (the ocean). However, in (20) $\frac{Z}{Z_d}kd$ is negligible, showing that there is no significant attenuation of the electric field in the seawater.

In the case when the ocean depth is more than twice the skin depth, $|e^{kd}|$ is considerably greater than $|e^{-kd}|$, so the terms involving the latter exponential can be dropped. Then equations (10), (12) and (14) simplify to

$$\frac{E_d}{H_0} = Z\frac{2}{(1 + Z/Z_d)}e^{-kd} \tag{21}$$

$$\frac{H_d}{H_0} = \frac{Z}{Z_d(1 + Z/Z_d)}\frac{2}{}e^{-kd} \tag{22}$$

$$\frac{E_d}{E_0} = \frac{2}{(1 + Z/Z_d)}e^{-kd} \tag{23}$$

showing that the electric and magnetic fields decrease exponentially with an increase in ocean depth.

Also, noting that Z/Z_d is usually much less than 1, these can be further simplified to

$$\frac{E_d}{H_0} == 2Ze^{-kd} \tag{24}$$

$$\frac{H_d}{H_0} = 2\frac{Z}{Z_d}e^{-kd} \tag{25}$$

$$\frac{E_d}{E_0} = 2e^{-kd} \quad . \tag{26}$$

These formulas can be compared to the fields that would be observed if the depth d represented a mid-ocean position in an infinitely deep ocean, instead of a position at the seafloor.

$$\frac{E(z = d)}{H_0} = Ze^{-kd} \tag{27}$$

$$\frac{H(z = d)}{H_0} = e^{-kd} \tag{28}$$

$$\frac{E(z = d)}{E_0} = e^{-kd} \quad . \tag{29}$$

These equations show the expected exponential decay of the fields within a uniform half-space. Comparing equations (24) and (26) to the corresponding infinite-ocean expressions shows an extra factor of 2 because of the doubling of the electric field due to the reflection of the electromagnetic field at the resistive seafloor. In contrast, reflection at the seafloor reduces the magnetic field (equation 25) by a factor $2Z/Z_d$ compared to what it would be at the same depth in an infinitely-deep ocean.

Calculations of the seafloor fields can be made for any layered-earth model of the structure below the seafloor. However, some general results can be presented if the earth beneath the seafloor is assumed to have uniform resistivity ρ_e. Then, for seawater resistivity ρ_s, the ratio of the impedances can be written

$$\frac{Z}{Z_d} = \sqrt{\frac{\rho_s}{\rho_e}} \tag{30}$$

and is independent of frequency. The frequency dependence of H_d/H_0 and E_d/E_0 is now only in kd.

Figures 2 and 3 show the the variation of B_d/B_0 and E_d/E_0 as a function of Re[kd] for uniform resistivities, ρ_e, of 10, 100, and 1000 ohm-m. There are many combinations of frequency and depth that will give the kd values shown; however for geomagnetic induction we are mainly concerned with periods from days to seconds. These periods give Re[kd] values from 0.001 to 0.1 for a shallow sea with $d = 100$ m, up to Re[kd] values of 0.1 to 10.0. for a deep ocean with $d = 5$ km.

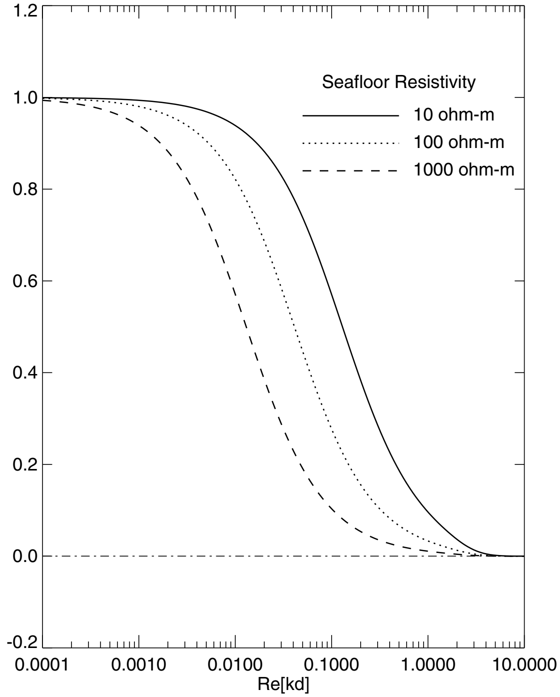
Figures 2 and 3 show the limiting values of B_d/B_0 and E_d/E_0 for high and low values of kd, described above. Between those limiting cases, Figure 2 shows that the resistivity below the seafloor influences the size of the magnetic field at the seafloor for values of Re[kd] from 0.001 to 1.0. (This is the basis of the vertical gradient method for sounding seafloor resistivity structure, see, for example, POEHLS and VON HERZEN, 1976; LAW and GREENHOUSE, 1981.) Figure 3 shows the variation of E_d/E_0 with Re[kd] for the same resistivity values, and shows that the resistivity below the seafloor has little influence on the ratio of the seafloor and surface electric fields. The reason for this different behaviour can be seen by looking at equations (13) and (15) and noting that cosh(kd) is always larger than sinh(kd). In equation (15) Z/Z_d is less than 1 so the sinh term is negligible and the actual Z/Z_d value has little effect on the final E_d/E_0 ratio. In equation (13) Z_d/Z is greater than 1, consequently the sinh term is significant and the value of Z/Z_d influences the H_d/H_0 ratio.

Figures 2b and 3b show dramatic changes in the phase of E_d/E_0 and H_d/H_0 for higher values of Re[kd]. For a specified value for the ocean depth d, an increase in Re[kd] corresponds to an increase in frequency and associated reduction in skin depth. Conversely, if the frequency (and skin depth) are constant, an increase in Re[kd] corresponds to an increase in the depth of the ocean. In either case, an increase in Re[kd] corresponds to an increase in ocean depth as a function of the

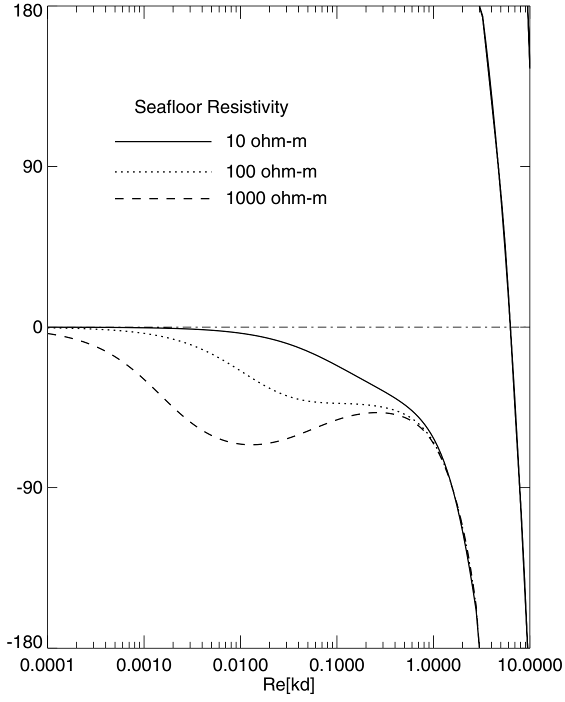
▶

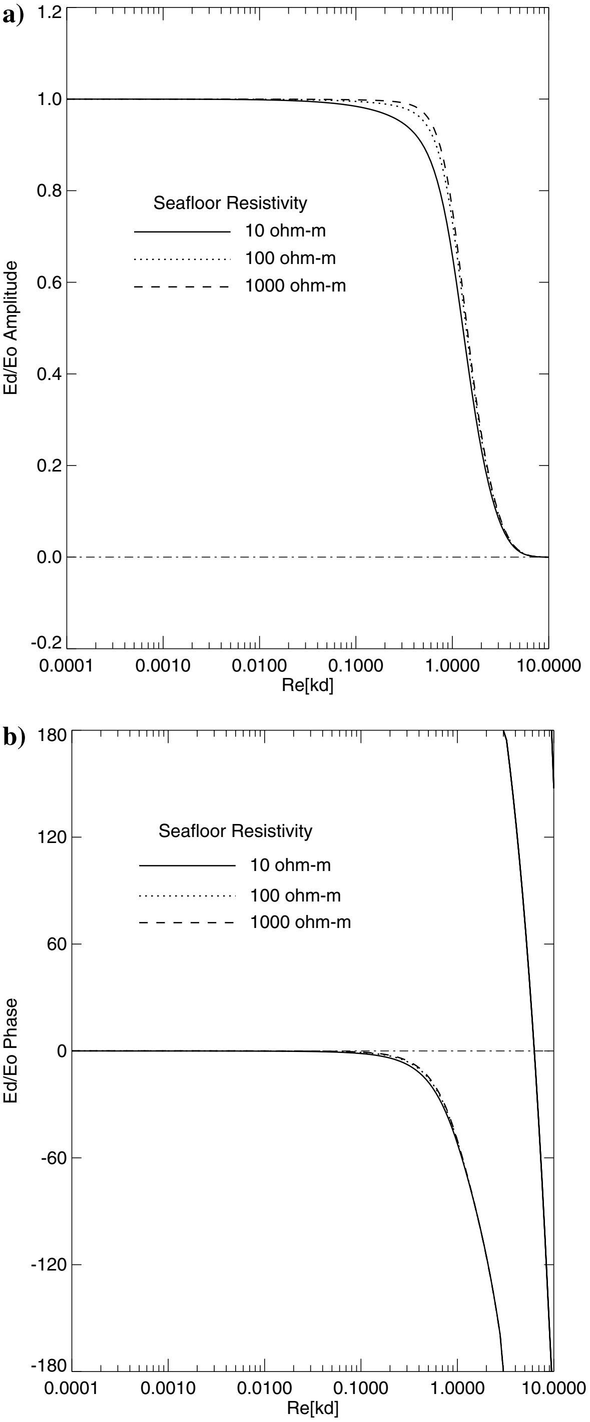
Figure 2
Ratio of seafloor and surface magnetic fields, B_d/B_0 as a function of the product of the propagation constant and depth of the seawater, kd, for sea with resistivity of 0.25 ohm-m above an earth with resistivities of 10, 100, and 1000 ohm-m. (a) Amplitude of B_d/B_0. (b) Phase of B_d/B_0. The x axis is Re[kd]. This is equal to Imag[kd] $= |kd|/\sqrt{2}$. Phase $[B_d/B_0]$ goes through zero at Re[kd] $= 2\pi, 4\pi, \ldots$

a)


Seafloor Resistivity
10 ohm-m
100 ohm-m
1000 ohm-m
Bd/Bo Amplitude
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
0.0001
0.0010
0.0100
0.1000
1.0000
10.0000
Re[kd]

b)


Seafloor Resistivity
10 ohm-m
100 ohm-m
1000 ohm-m
Bd/Bo Phase
180
90
0
-90
-180
0.0001
0.0010
0.0100
0.1000
1.0000
10.0000
Re[kd]


a)
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
Ed/Eo Amplitude
Seafloor Resistivity
10 ohm-m
100 ohm-m
1000 ohm-m
0.0001
0.0010
0.0100
0.1000
1.0000
10.0000
Re[kd]
b)
180
120
60
0
-60
-120
-180
Ed/Eo Phase
Seafloor Resistivity
10 ohm-m
100 ohm-m
1000 ohm-m
0.0001
0.0010
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0.1000
1.0000
10.0000
Re[kd]

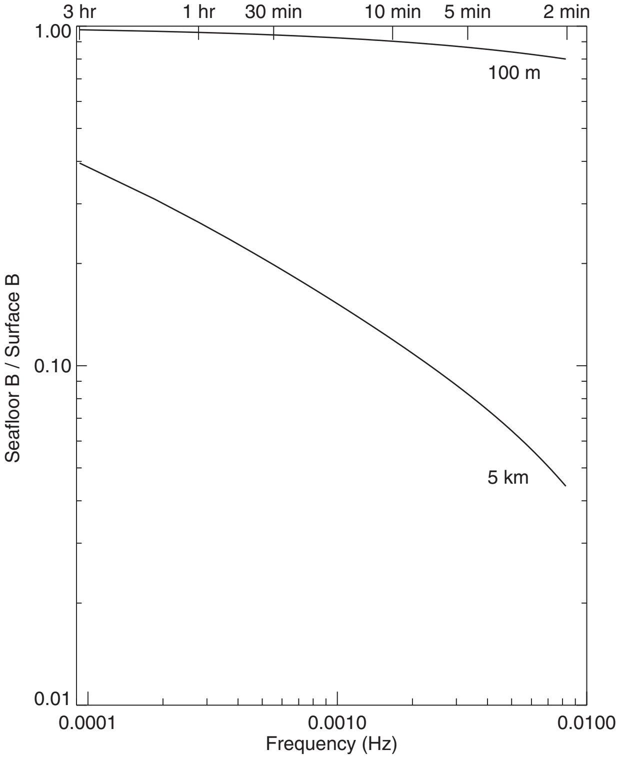

Figure 4
Relation between magnetic fields at the seafloor and magnetic fields at the sea surface as a function of frequency, for sea depths of 100 m and 5 km, for sea with resistivity of 0.25 ohm-m above an earth with resistivity of 10 ohm-m.

wavelength, $2\pi\delta$, in seawater. Thus Figures 2b and 3b show a change in phase which goes through zero when $\mathrm{Re}[kd] = 2\pi, 4\pi, \ldots$, i.e., every time the ocean depth is an integral number of wavelengths.

Seafloor/Surface Transfer Functions

As examples of the transfer functions between seafloor electric and magnetic fields and surface magnetic fields, calculations have been made, using equations (10) and (12) for periods from 2 min to 3 hrs. Calculations are made for a seafloor depth

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Figure 3
Ratio of seafloor and surface electric fields, E_d/E_0 as a function of the product of the propagation constant and depth of the seawater, *kd*, for sea with resistivity of 0.25 ohm-m above an earth with resistivities of 10, 100, and 1000 ohm-m. (a) Amplitude of E_d/E_0. (b) Phase of E_d/E_0. The x axis is Re[*kd*]. This is equal to $\mathrm{Imag}[kd] = |kd|/\sqrt{2}$. Phase $[E_d/E_0]$ goes through zero at $\mathrm{Re}[kd] = 2\pi, 4\pi, \ldots$

of 100 m, representing a typical depth for seas on continental shelves, and a seawater depth of 5 km representing the deep oceans. The conductivity of seawater is taken as 4 S/m ($\rho_s = 0.25$ ohm-m), and the earth below the seafloor is represented by a uniform half-space with a conductivity of 0.1 S/m ($\rho_e = 10$ ohm-m).

The amplitude ratios between the magnetic fields on the seafloor and the magnetic field at the surface are shown in Figure 4. This shows that for a 100 m deep sea the seafloor magnetic fields are little different than those at the surface, while for a 5 km depth the seafloor magnetic field is significantly attenuated compared to the surface fields. Figure 5 shows the amplitude ratios between the electric field on the seafloor and the magnetic field at the surface for ocean depths of 100 m and 5 km. Also shown, for comparison, is the relation between the electric field and magnetic field for an ocean of zero depth. This corresponds to the relation between the electric and magnetic fields on the seafloor and represents the response of the underlying

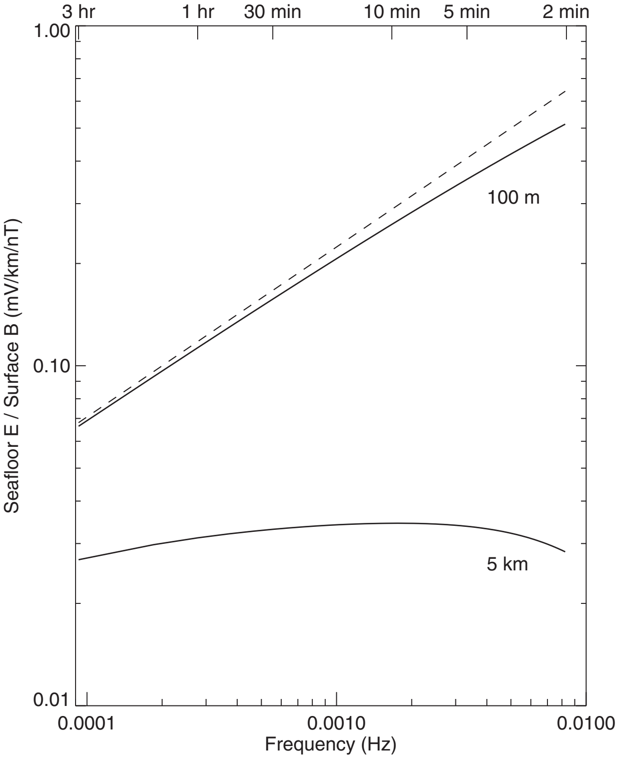

Figure 5

Relation between electric fields at the seafloor and magnetic fields at the sea surface as a function of frequency, for sea depths of 100 m and 5 km, for sea with resistivity of 0.25 ohm-m above an earth with resistivity of 10 ohm-m. The dashed line shows the relation between the electric and magnetic fields at the seafloor.

conductivity structure (in this case a half-space with resistivity 10 ohm-m). For a 100 m deep sea the seafloor electric field is little different from the electric field that would occur without the sea; while, for a 5-km deep ocean the seafloor electric field is considerably reduced.

Seafloor Power Spectra

The transfer functions shown in Figures 4 and 5 can be used with surface magnetic field data to determine the spectra of the electric fields and magnetic fields on the seafloor. Magnetic field data from the sea surface are generally not available (exceptions to this are observations made on the sea ice of the Arctic Ocean, e.g., NIBLETT *et al.*, 1987). Consequently we use magnetic data from a land-based observatory. This assumes that the magnetic field variations at the surface of the ocean are the same as at a site on land. It can be shown (see Appendix) that, for a plane wave source, the reflected magnetic field produced by induced currents is the same as the incident magnetic field. This relation is independent of the choice of layered conductivity structure for the earth. Thus the total magnetic field at the surface of the earth is twice the incident field whether the site is at the surface of the ocean or on land. Coastal observatories provide the closest measure of the magnetic field variations at the ocean surface. However, the magnetic field near the coast can be distorted by concentrations of induced currents flowing parallel to the coast, and an observatory further inland may provide a better record of the magnetic field variations. Here we use average magnetic field spectra (Fig. 6) obtained from the Ottawa Magnetic Observatory at geomagnetic latitude 57.0°N and longitude 351.5°E.

To examine the spectra of the electric field and magnetic field variations at the seafloor we present calculations for a shallow sea (depth 100 m) and for a deep ocean (depth 5 km). By using surface spectra from Ottawa we obtain spectra for the seafloor fields that would occur at a similar geomagnetic latitude. The surface magnetic field spectra (Fig 6) are multiplied by the square of the transfer functions shown in Figures 4 and 5 to give the electric field and magnetic field power spectra at the seafloor for different levels of magnetic activity. For a shallow sea the magnetic field spectra at the seafloor is nearly identical to the surface magnetic field spectra (Fig. 6) and shows the same decrease in power with increasing frequency. In contrast, the seafloor electric field power decreases more slowly with increasing frequency as shown in Figure 7. For a deep ocean, the seafloor magnetic field power (Fig. 8) has a steeper fall-off with increasing frequency compared to the surface spectra. The deep-ocean seafloor electric field power (Fig. 9) has a slower decrease with increasing frequency compared to the seafloor magnetic field. However, the electric field spectra have a much steeper decrease at the deep ocean seafloor compared to the shallow sea

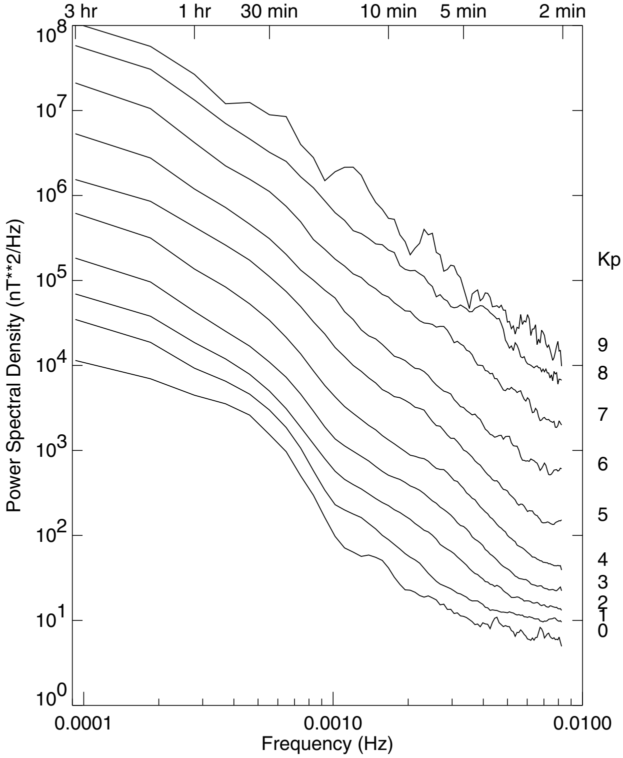

Figure 6
Average magnetic field spectra for different levels of magnetic activity derived from 1-minute data recorded at Ottawa Magnetic Observatory.

seafloor electric field spectra. Thus the spectral shape for the seafloor magnetic and electric fields changes considerably with the depth of the ocean.

Discussion

Seafloor measurements of magnetic fields and electric fields have been made for over 20 years (Cox *et al.*, 1971; Filloux, 1987) and are used for magnetotelluric studies of the seafloor and studies of tidal water movements. Power spectra of seafloor magnetic and electric fields have been reported by Ferguson (1988) and Lilley *et al.* (1989) using measurements from the Tasman Sea, at an approximate geomagnetic latitude of 44°S. Power spectra from one of these sites are shown in Figure 10. These show lower spectral levels compared to the calculations for 57°N (Figs. 8 and 9) as would be expected for a lower latitude site. However, the same differences in spectral slopes between the electric field spectra and magnetic field spectra can be seen in the North Atlantic calculations and the Tasman observations.

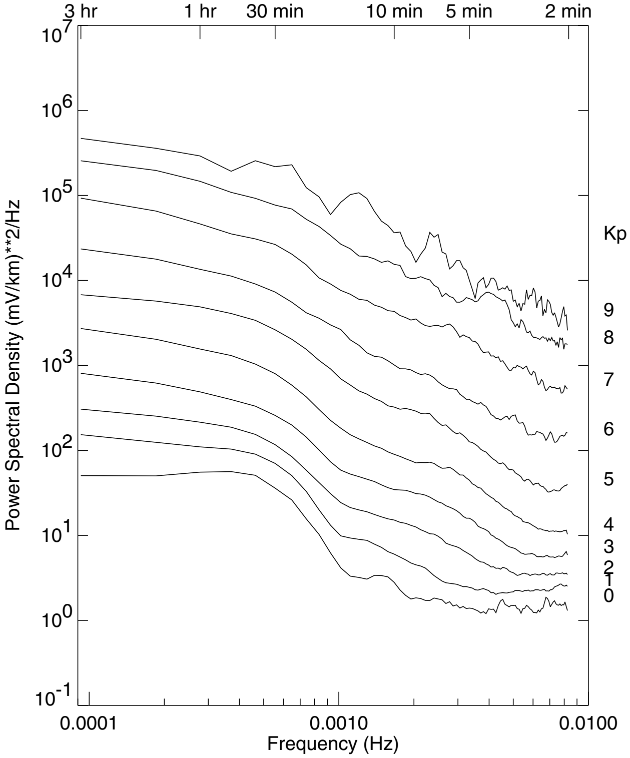

Figure 7
Spectra of the seafloor electric fields for a 100 m deep sea for different levels of magnetic activity, derived by multiplying the surface spectra of Figure 6 by the square of the transfer function in Figure 5.

Measurements of power spectra of seafloor electric fields have been made by Fujii *et al.* (1995) using two decommissioned and unpowered cables in the Pacific, HAW-1 cable (4050 km) from California to Hawaii, and the TPC-1 cable from Guam to the Philippines (2716 km). These show power spectral levels that increase with greater magnetic activity and all show a decrease in power with increasing frequency as seen in the calculated seafloor spectra (Figs. 7 and 9). The HAW-1 cable has also been used to measure the power spectra at longer periods (Chave *et al.*, 1992). Chave *et al.* found that the signal up to periods of 20 days was correlated with magnetic field variations and beyond this was due to motional effects.

The relations derived in this paper use the assumptions that the ocean and earth structure are laterally uniform and that the incident field is also laterally uniform. These assumptions are probably reasonable for a mid-ocean site at mid-latitudes. However, near the coast the situation is complicated, not only by the changing depth of the seafloor, but by the distortion to the fields produced at a coastline (Fischer, 1979). This distortion of the fields also influences the conductivity structure derived from seafloor MT observations (Heinson and Constable, 1992; Constable and

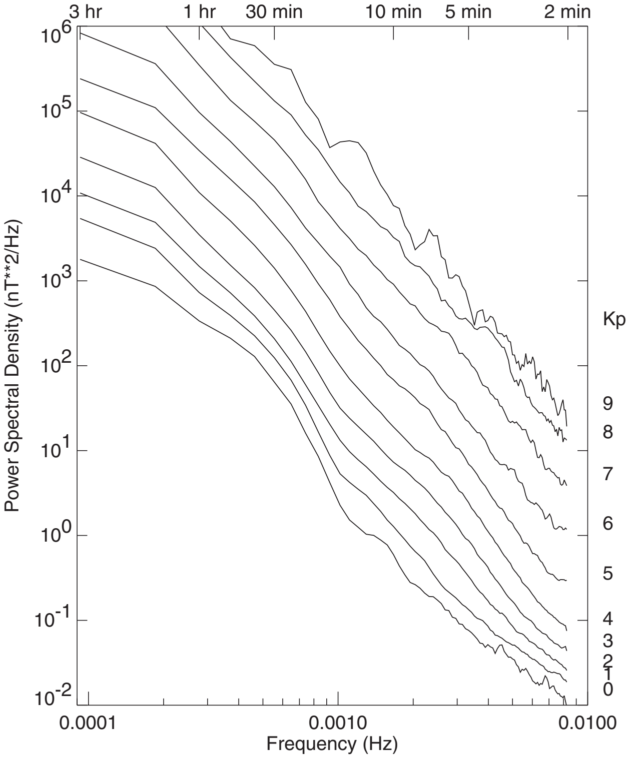

Figure 8
Spectra of the seafloor magnetic fields for a 5 km deep ocean for different levels of magnetic activity, derived by multiplying the surface spectra of Figure 6 by the square of the transfer function in Figure 4.

Heinson, 1993; Heinson and Lilley, 1993). At auroral latitudes (or near the equator) the magnetic disturbances are produced by the auroral (equatorial) electrojet systems and the non-uniformity of the source field also needs to be included in the calculation (see Pirjola *et al.*, 2000).

The electric fields at the seafloor are significant because of the effect they can have on the tranmsission of signals on submarine cables. The repeater amplifiers in a submarine cable are connected in series and supplied with a current driven by power-feed equipment at the ends of the cable. To preserve full operation of modern cables during magnetic disturbances the power feed equipment must be able to regulate the current fed to the repeaters in spite of the rapidly varying induced voltages (Axe, 1968; Root, 1979). In recent years optical-fibre cables have been used; however, a conductor along the cable is needed to carry power to the repeaters, therefore these cables are still subject to geomagnetically induced currents and voltages. For example, during the March 13, 1989 magnetic disturbance fluctuations of 700 V were observed on the TAT-8 fibre-optic trans-atlantic cable (Medford *et al.*, 1989).

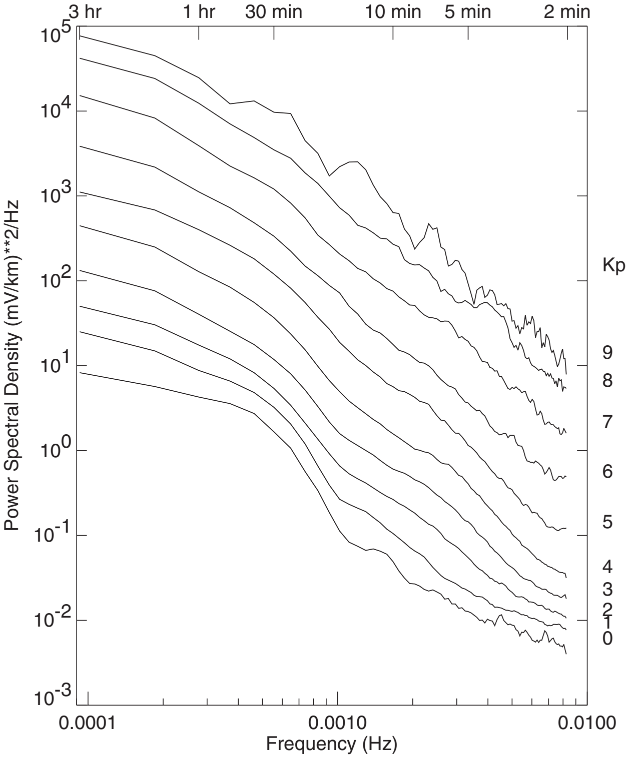

Figure 9
Spectra of the seafloor electric fields for a 5-km deep ocean for different levels of magnetic activity, derived by multiplying the surface spectra of Figure 6 by the square of the transfer function in Figure 5.

To assess the likelihood of extreme induced voltages that could cause problems on cable systems, statistical studies can be made using observations on an existing cable (LANZEROTTI *et al.*, 1993). However, the data sets available for this type of study are limited, and this approach is obviously not possible when trying to assess the geomagnetic influence expected on a new cable. With the calculations presented here it is now possible to calculate the electric fields expected on a submarine cable during different levels of magnetic activity. These calculations can be combined with statistics of magnetic activity levels to determine the frequency at which high induced voltages can be expected on the cable. Such occurrence information will be a useful parameter in the design of the power-feed equipment for new cables. Even for existing cables these calculations have advantages, as they provide a more reliable estimate of the occurrence of extreme values based on the long records of geomagnetic activity instead of the limited recordings possible using the cable itself.

The present analysis has concentrated on geomagnetic fluctuations with periods greater than 2 minutes, using magnetic field spectra and the Kp index which are both

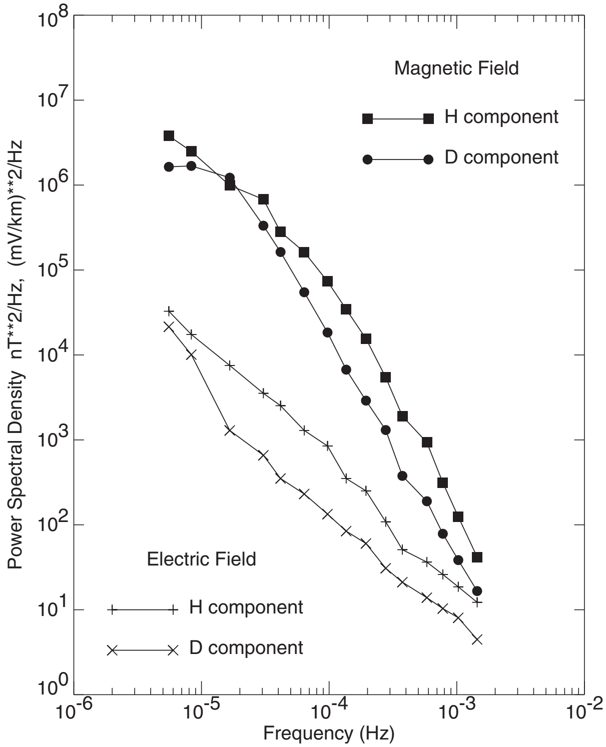

Figure 10
Power spectra of the horizontal magnetic and electric fields observed at seafloor site TP5 in the Tasman Sea at geographic coordinates 36°43′S, 153°35′E, at depth 4660 m. (Figure redrawn from Figure 4 of LILLEY *et al.* (1989).

derived from 1-minute sampled data. A system designer will also be concerned about the electric fields produced by magnetic disturbances with periods less than 2 minutes. Figure 5 shows that at 100-metres depth the transfer function between the seafloor electric field and surface magnetic field is trending upward with increasing frequency so large electric fields can be expected at periods less than 2 minutes. For a depth of 5 km there is considerable attenuation of the higher frequencies which leads to the downward turn in the ‘5 km’ transfer function seen in Figure 5. Thus electric fields with periods less than 2 minutes will not be less significant in the deep ocean. A more thorough analysis of higher frequency seafloor fields is hoped for in future work.

Conclusions

The electric field at the floor of an ocean of depth *d* is related to the magnetic field at the ocean surface by the expression

$$\frac{E_d}{B_0} = \frac{Z}{\mu_0} \frac{2}{(1 + Z/Z_d)e^{kd} - (1 - Z/Z_d)e^{-kd}} , \tag{31}$$

where k and Z are the propagation constant and characteristic impedance of the seawater, and Z_d is the surface impedance at the seafloor. Similarly the ratio of the seafloor and surface magnetic fields can be written

$$\frac{B_d}{B_0} = \frac{Z}{Z_d} \frac{2}{(1 + Z/Z_d)e^{kd} - (1 - Z/Z_d)e^{-kd}} \tag{32}$$

and the ratio of the seafloor and surface electric fields is given by

$$\frac{E_d}{E_0} = \frac{2}{(1 + Z/Z_d)e^{kd} + (1 - Z/Z_d)e^{-kd}} . \tag{33}$$

These equations can be used, with surface magnetic field data, to calculate the electric and magnetic fields on the seafloor.

Using the above relations, seafloor electric and magnetic field power spectra have been calculated for a shallow sea (depth = 100 m) and for a deep ocean (depth = 5 km). For a shallow sea the spectral slopes are similar to those for surface fields, while for a deep ocean the electric and magnetic fields show a much steeper fall off in power with increasing frequency. Power spectra are provided for different levels of geomagnetic activity, as given by Kp, and can be used to assess the electric fields induced in submarine cables and undersea pipelines during geomagnetic disturbances.

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Appendix

For a plane wave source, the magnetic field at the earth's surface is twice the field due to the source and is independent of the choice of layered conductivity structure for the earth.

To show this, consider the plane wave electric and magnetic fields in the air as given by

$$E = S_0 e^{-k_0 z} + R_0 e^{k_0 z} \tag{A.1}$$

and the magnetic field is then given by

$$H = \frac{S_0 e^{-k_0 z}}{Z_0} - \frac{R_0 e^{k_0 z}}{Z_0} \ . \tag{A.2}$$

where S_0 and R_0 are the amplitude of the incident and reflected waves, $k_0 = i\omega\sqrt{\mu_0 \varepsilon_0}$, and $Z_0 = \sqrt{\mu_0/\varepsilon_0} = 377\,\Omega$ are the propagation constant and impedance in the air.

The ratio of the electric and magnetic fields at the earth's surface can be found by combining equations (A.1) and (A.2) with $z = 0$, and this equals the surface impedance of the earth, Z_e.

$$Z_e = \left(\frac{S_0 + R_0}{S_0 - R_0}\right) Z_0 \ . \tag{A.3}$$

Rearranging equation (A.3) gives the ratio of the reflected and incident waves in the air

$$\frac{R_0}{S_0} = \frac{Z_e - Z_0}{Z_e + Z_0} \ . \tag{A.4}$$

For all reasonable conductivity values of the earth layers. $Z_0 \gg |Z_e|$ making R_0 equal to $-S_0$. Consequently, the incident and reflected electric fields practically cancel each other at the earth's surface. Substituting for R_0 in equation (A.2) shows that the total horizontal magnetic field is twice the incident magnetic field.

Kaufman and Keller (1981, pp. 49–50) derive the same result regarding the factor of 2 between the incident and total horizontal magnetic fields differently. They apply the fact that infinite ionospheric current sheets create an incident magnetic field below the ionosphere that is independent of the vertical coordinate z . Considering the earth as a half-space with a horizontally-layered conductivity distribution, induced currents will flow in infinite horizontal sheets and create a secondary (or reflected) magnetic field outside the earth that also is independent of z. Since the total field has to be zero at infinity, the secondary field due to sheet currents flowing in earth layers exactly cancels the incident field at $z = \infty$. The secondary field is equal in magnitude but opposite in sign at $z = \infty$ and at the earth's surface. Consequently, the incident horizontal magnetic field and the reflected magnetic field are the same at the earth's surface resulting in the factor of 2 for the total field.

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