Refutation of fuzzy logic

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***Abstract*—We evaluate eight seminal conjectures of fuzzy logic for sets, logic, operators, axioms, Z-numbers, intuitionistic logic, paraconsistent logic, and neutrosophic logic. These are *not* tautologous, to form a fragment of the universal logic VŁ4.**

Keywords—fuzzy axiom, fuzzy logic, fuzzy operator, fuzzy set, intuitionistic logic, Meth8/VŁ4, modal operator, modern square of opposition, Modus Cesare, Modus Camestros, neutrosophic logic, paraconsistent logic, syllogism, quantifier

I. Introduction

This paper evaluates five seminal aspects of fuzzy logic. These include sets, logic, operators, axioms, and Z-numbers. Fuzzy sets are pseudo-triangular bases. Fuzzy logic is in *one* variable from a "historical" context. Fuzzy operators are from intuitionistic soft sets. Fuzzy axioms are from preference relations. Fuzzy Z-numbers are measured for resolution and symmetry. Fuzzy logics by three extensions apply to intuitionistic, paraconsistent, and neurtrosophic logics, with the last claimed as a generaliztion of the previous.

We use our resuscitation of the four-valued modal logic of Łukasiewicz[1] The modal logic model checker named Meth8/VŁ4 implents the universal logic VŁ4[2]. A student demo for two variables and unlimited sequents is free by request.

Symbolic values in VŁ4 are presented to replicate results.

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~ Not; + Or; - Not Or; & And; \ Not And;
> Imply, greater than; < Not Imply, less than;
= Equivalent; @ Not Equivalent;
% possibility, for one; # necessity, for all;
(z=z)   T as tautology, ordinal 3, binary 11;
(z@z)   F as contradiction, zero,  binary 00;
(%z>#z) N as truthity, ordinal 1,  binary 01;
(%z<#z) C as falsity, ordinal 2,   binary 10;
~( y < x) as ( x ≤ y), ( x ⊆ y).
Quantifiers are distributed onto variables.
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```
Model 1    Models 2
- - - -    - - - - - - - - - -
  M1       M21  M22  M231 M232
  # %      # %  # %  # %  # %
F.F C   U. U U  U E  U P  U I
C.F C   I. I I  U E  I E  U I
N.N T   P. P P  U E  U P  P E
T.N T   E. E E  U E  I E  P E
```

```
Model 1 connectives as table rows 1-4 from left.
1 & . F F F F . F C F C . F F N N . F C N T
1 \ . T T T T . T N T N . T T C C . T N C F
1 + . F C N T . C C T T . N T N T . T T T T
1 - . T N C F . N N F F . C F C F . F F F F
1 < . F F F F . C F C F . N N F F . T N C F
1 = . T N C F . N T F C . C F T N . F C N T
1 > . T T T T . N T N T . C C T T . F C N T
1 @ . F C N T . C F T N . N T F C . T N C F
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II. Fuzzy components

We test fuzzy set, logic, operator, axiom, and Z-number.

A. Fuzzy sets

Pseudo triangular bases of fuzzy sets[2]

2. Properties of fuzzy sets, Lemma 2.1. A fuzzy set f: [0,1]→[0,1] is min-convex if, and only if, for any 0≤x<z<y≤ 1 we have that if f(z)<f(x) then f(y)≤f(z). Moreover, it is strictly min-convex if, and only if, for any 0≤x<z<y≤1 we have that if f(z)≤f(x) then f(y)<f(z). Proof. This is a straightforward verification. (2.1.2.1)

LET p, q, r, s: f, x, y, z.
z.#((~(q<(s@s))<s)<~((%s>#s)<r))>(~((p&q)<(p&s))>((p&q)<(p&s))) ; CCTT TTTT TTTT TTTT (2.1.2.2)

Remark 2.1.2.2: Distributing the universal quantifier on variables in the antecedent produces the same truth table result.

Eq. 2.1.2.2 as rendered is *not* tautologous, hence refuting strictly min-convex and subsequent conjectures, constituting the briefest refutation of fuzzy logic.

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1 A trivial objection to Łukasiewicz M_4 is (◊p&◊q)→◊(p&q). For example if Schrödinger's cat is p for alive or q for dead, the sentence reads: If possibly the cat is alive and possibly the cat is dead, then possibly the cat is dead and alive. This is tautologous in Meth8/VŁ4, but *hard-wired* as not tautologous in assistants as Molle and Prover9. The easy answer is casting the dual of ◊p,◊q to a reduced, single variable dual of ◊p,~◊p for (◊p&~◊p)→◊(p&~p) to read: If possibly the cat is alive and not possibly the cat is alive, then possibly the cat is alive and not alive. This is tautologous in the provers listed.

2 After proof of modal operators as respective quantifiers, two recent advances followed. The Modern Square of Opposition adopted new formulas for vertices and edges. These in turn validated the 24-syllogisms, to make minor corrections to Modus Cesare and Camestros. Further proofs cascaded to refute the Löb axiom □(□p>p)>□p, disallowing Gödel logic as a quantum basis, and the axiom of the empty set, disqualifying ZFC as a mathematical foundation. The model version of $MŁ_4$ became the universal logic system named variant VŁ4.

B. *Fuzzy logic*

Refutation in one variable of the historical basis for fuzzy logic[4]

However the proposition "*possible* p" is not the same as p (1.1), and "*possible* ¬p" is not the negation of "*possible* p" (2.1). Hence the fact that the proposition "*possible p*" ∧ "*possible* ¬p" may be true (3.1) does not question the law of non-contradiction since "*possible* p" and "*possible* ¬p" are not mutually exclusive (4.1). This situation leads to interpretation problems for a fully truth-functional calculus of possibility, since even if p is "possible" and ¬p is "possible", still p ∧ ¬p is ever false (5.1).

%p@p ; C**F**C**F** C**F**C**F** C**F**C**F** C**F**C**F** (1.2)

%~p=~%p ; NNNN NNNN NNNN NNNN (2.2)

(%p&%~p)=%(p=p) ;
CCCC CCCC CCCC CCCC (3.2)

~(%p@~p)=(p=p) ;
C**F**C**F** C**F**C**F** C**F**C**F** C**F**C**F** (4.2)

(%p&%~p)>(p&~p) ;
NNNN NNNN NNNN NNNN (5.2)

Remark: Eqs. 1.2-5.2 are *not* tautologous. Hence an historical basis for fuzzy logic is refuted, and in one variable.

C. *Fuzzy operators*

First Zadeh's logical operators on intuitionistic fuzzy soft set[1]

Definition 2.7. … [T]he union of (F,A) and (G,B) is denoted by '(F,A)∪(G,B)' and is defined by (F,A) ∪ (G,B)=(H,C), where C=A∪B … (2.7.1)

LET p, q, r, s, t, u: A, B, C, F, G, H.
(r=(p+q))>(((s&p)+(t&q))=(u&r)) ;
TTTT TT**FF** TTTT T**FFF** . . . (2.7.2)

3.2. First Zadeh's intuitionistic fuzzy conjunction of intuitionistic fuzzy soft set

Example 3.2.2. (F,A)$\tilde{\wedge}_z$(G,B)=(H,C), where C=A∩B … (3.2.2.1)

(r=(p&q))>(((s&p)\(t&q))=(u&r)) ;
FFFT TTT**F** **FFF**T TTT**F** . . . (3.2.2.2)

Proposition 3.2. 3. (F,A)$\tilde{\wedge}_{z,1}$(G,B)$_{z,1}$→(H,C)⊇ [(F,A)$_{z,1}$→(H,C)]$\tilde{\wedge}_{z,1}$ [(G,B)$_{z,1}$→(H,C)] (3.2.3.1)

((s&p)\(t&q))>~((((s&p)>(t&q))\((t&q)>(u&r)))<(u&r)) ;
TTTT TTTT T**F**T**F** T**F**T**F** . . . (3.2.3.2)

Example 3.3.2. (F,A)$\tilde{\vee}_{z,1}$ (G,B)=(H,C), where C=A∩B … (3.3.2.1)

(r=(p&q))>(((s&p)-(t&q))=(u&r)) ;
FFFT TTTT **F**T**F**T TTT**F** . . . (3.3.2.2)

Example 3.3.6. It is obviously that
(F,A)$\tilde{\wedge}_{z,1}$(G,B)≠(G,B)$\tilde{\wedge}_{z,1}$(F,A) (3.3.6.1)

((s&p)\(t&q))@((t&q)\(s&p)) ;
FFFF FFFF FFFF FFFF (3.3.6.2)

Because the above definition and example as rendered are not tautologous, First Zadeh's logical operators on intuitionistic logic fuzzy soft set is refuted.

D. *Fuzzy axioms*

Axiomatizing logics of fuzzy preferences[8]

2. Preliminaries on fuzzy preference relations … [W]e will assume that a weak A-valued preference relation on a set U will be now a fuzzy ∧-preorder P : U ×U → A, where P(a, b) is interpreted as the degree in which v is at least as preferred as u, that is, satisfying … ∧-transitivity: P(u,v)∧P(v,w)≤P(u,w) for each u,v,w∈U (2.5.1)

Remark 2.5.1: We ignore the subset clause for evaluation of the assumed ∧-transitivity theorem.

LET p, q, r, s: P, u, v, w.
~((p&(q&s))<((p&(q&r))&(p&(r&s)))) = (p=p) ;
TTTT TTTT TTT**F** TTTT (2.5.2)

Remark 2.5.2: Eq. 2.5.2 as rendered is *not* tautologous. This also refutes subsequent conjectures in the text, notably, the minimal modal logics of a finite residuated lattice and the Bulldozed method.

E. *Z-numbers*

Refutation of measures for resolution and symmetry in fuzzy logic of Zadeh Z-numbers[3]

Proof. Assume the fuzziness measure, H, … For G3 [resolution], denoted A∗ = (A∗, B∗), where A∗, B∗ are [a] sharpened version of A and B, respectively. So H(A)≥H(A∗) and H(B)≥H(B∗), therefore H(A)+H(B)≥H(A∗)+H(B∗)) > H(Z)≥H(Z∗). (3.1)

LET p, q, r, s: A, B, H, Z;
(~((r&p)<(r&#p))&~((r&q)<(r&#q))) > (~(((r&p)+(r&q))<((r&#p)+(r&#q)))>~((r&s)<(r&#s))) ;
TTTT TTTT TTTT <u>N</u>TTT (3.2)

For G4, [symmetry] H(A)=H(1−A) and H(B)=H(1−B), so H(A)+H(B)=(H(1−A))+(H(1−B))) > Z(Z)=HZ(Z(1−A,1−B)). (4.1)

(((r&p)=(r&((%p>#p)-p)))&((r&q)=(r&((%p>#p)-q)))) > ((((r&p)+(r&q))=((r&((%p>#p)-p))+(r&((%p>#p)-q))))> (((r&s)&s)=((r&s)&(s&(((%p>#p)-p)&((%p>#p)-q)))))) ;
TTTT TTTT TTTT <u>C</u>TTT (4.2)

Eqs. 3.2 and 4.2 as rendered are *not* tautologous. This means the commonly accepted measures G3 (resolution) and G4 (symmetry) for the Zadeh (Z-numbers) fuzzy logic are refuted.

III. Related logics

We test fuzzy logic as often related to intuitionistic, parconsistent, and neutrosophic logics.

A. *Intuitionistic logic*

Contra intuitionistic logic[6]

Intuitionistic logic is not based on the *a priori* existence of truth values (although it is possible to give a truth values semantics for it, for example, via Heyting algebras or Kripke frames). (1.1)

In intuitionistic logic the meaning of a connective is given by describing how a proof of the compound formula can be obtained from proofs of the constituents. (2.1)

Remark 1.1: Eq. 1.1 means a universal, designated proof value does not exist, hence renderng intuitionistic logic without an exact bivalent solution and forcing it into a probabilisitic vector space, equivalent to an inexact guess.

Remark 2.1: Eq. 2.1 means a connective cannot be consistent between proofs and further implies a connective has no truth table. Therefore coupled with Eq. 1.1, this represents the briefest refutation of intuitionistic logic known.

B. Paraconsistent logic

Refutation of paraconsistent logic on one conjecture[5]

[To prove the seminal equivalence and replacement formula of paraconsistent logic is]
(4) To establish that a formula Γ is equivalent to Δ in the sense that either can be substituted for the other wherever they appear as a subformula, one must show

((Γ→Δ)∧(Δ→Γ))∧((¬Γ→¬Δ)∧(¬Δ→¬Γ)). (4.1)

LET p, q: Γ, Δ.
((p>q)&(q>p))&((~p>~q)&(~q>~p)) ;
T**FF**T T**FF**T T**FF**T T**FF**T (4.2)

Remark 4.2: Eq. 4.2 as rendered is *not* tautologous. This refutes the seminal theorem of replacement and serves as the briefest refutation of paraconsistent logic known

C. Neutrosophic logic

Refutation of neutrosophic logic as generalization of intuitionistic, fuzzy logic[7]

For neutrosophic logic (N), we map the respective values of truth, falsity, and indeterminacy as:

Nt (%p>#p); Nf (%p<#p);
Ni (((%p>#p)+(%p<#p))+~((%p>#p)+(%p<#p))). (1.1)

We simplify our evaluation by ignoring the numeric scaling factor of *ε*. That serves to push a single numeric value of the combined, summed state of Nt+Ni+Nf outside an interval definition of q on "]0,1[" and into "]0,3[", or ultimately to natural numbers, including zero.

#(((q>(p-p))&(q<(p\p)))+((q=(p-p))+(q=(p\p)))) >
%(q=(((%p>#p)+(%p<#p))+~((%p>#p)+(%p<#p)))) ;
TCTT TCTT TCTT TCTT (1.2)

In Eq. 1.2: the antecedent establishes the necessity of 0≤q≤1; the consequent establishes the possibility that q is the summation of Nt+Ni+Nf; and the result of the sentence is *not* tautologous, meaning neutrosophic logic is refuted and hence its use as a generalization of intuitionistic, fuzzy logic is likewise unworkable.

We expand our evaluation by including more neutrosophic values for absolute truth +1, absolute falsity -0, and absolute indeterminacy on the interval written "]-0,1+[", as respectively:

N+t (#p>#p); N+f (#p<#p);
N+i (((#p>#p)+(#p<#p))+~((#p>#p)+(#p<#p))). (2.1)

We substitute values of Eq. 2.1 into Eq. 1.2.

#(((q<(p-p))&(q>(p\p)))+((q=(p-p))+(q=(p\p)))) > %
(q=(((#p>#p)+(#p<#p))+~((#p>#p)+(#p<#p)))) ;
TCTT TCTT TCTT TCTT (2.2)

In Eq. 2.2: the antecedent establishes the necessity of 1≤q≤0; the consequent establishes the possibility that q is the summation of (N+t)+(N+i)+(N+f); and the result of the sentence is *not* tautologous, with the same table result as in Eq. 1.2. Therefore neutrosophic logic as a generalization to include intutionistic and paraconsistent logics is unworkable.

IV. Concluding Comments

We tested equations for 19 conjectures which are *not* tautologous. This refutes eight aspects of fuzzy logic and its derivatives. These results form a non tautologous fragment of the universal logic VŁ4.

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