Nonlinear Dyn (2019) 95:143–150
https://doi.org/10.1007/s11071-018-4556-8

ORIGINAL PAPER

Generation and control of multiple solitons under the influence of parameters

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Received: 7 June 2018 / Accepted: 1 September 2018 / Published online: 12 September 2018

Abstract In this paper, the analytic three-soliton solution for a high-order nonlinear Schrödinger equation is obtained by the Hirota's bilinear method. The transmission characteristics of three solitons are discussed. By selecting relevant parameters, soliton interactions are presented, and the method of generating new solitons is suggested. The influences of corresponding parameters on soliton transmission and interactions are analyzed. Results of this paper are helpful for enriching the soliton theory and studying the signal routing system.

Keywords Optical solitons · Soliton interactions · Hirota's bilinear method · Soliton transmission

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1 Introduction

Soliton, which is one of three branches of nonlinear science, has been developing vigorously since the discovery of solitons [1–12]. And so far, some nonlinear evolution equations have been studied to obtain soliton solutions [13–17], and some soliton phenomena have been observed in such fields as nonlinear optics and optical communications [18–22]. For the optical soliton, it was first predicted theoretically in 1973 [23]. In 1980, it was successfully generated by a color-mode-locked soliton laser experimentally [24]. Since then, optical solitons have rapidly became a rising star and attracted some researchers to conduct in-depth exploration [25–38].

Optical soliton is a pulse-modulated wave with the coherent optical carrier frequency. The optical soliton in an ideal lossless single-mode fiber satisfies the nonlinear Schrödinger (NLS) equation:

$$iu_z \pm \frac{1}{2}u_{tt} + |u|^2u = 0. \tag{1}$$

However, in reality, this ideal state cannot be easy to implement. Therefore, some other equations are investigated when considering the effects of loss, higher-order disturbances, etc. An integrable NLS hierarchy can be used to describe the soliton transmission in the reality optical fibers as follows [39–42],

$$\begin{aligned} & iu_x + \alpha_2(u_{tt} + 2u|u|^2) - i\alpha_3(u_{ttt} + 6u_t|u|^2) \\ & \quad + \alpha_4(u_{tttt} + 6u^* u_t^2 + 4u|u_t|^2 \\ & \quad + 8|u|^2 u_{tt} + 2u^2 u_{tt}^* + 6|u|^4 u) - i\alpha_5(u_{ttttt} \\ & \quad + 10|u|^2 u_{ttt} + 30|u|^4 u_t + 10uu_t u_{tt}^* \\ & \quad + 10uu_t^* u_{tt} + 10u_t^2 u_t^* + 20u^* u_t u_{tt}) \\ & \quad + \alpha_6\{u_{tttttt} + u^2[60|u_t|^2 u^* + 50u_{tt}(u^*)^2 \\ & \quad + 2u_{tttt}^*] + u[12u_{tttt}u^* + 8u_t u_{ttt}^* \\ & \quad + 22|u_{tt}|^2 + 18u_{ttt}u_t^* + 70u_t^2(u^*)^2] \\ & \quad + 20u_t^2 u_{tt}^* + 10u_t(5u_{tt}u_t^* + 3u_{ttt}u^*) \\ & \quad + 20u_{tt}^2 u^* + 10u^3[(u_t^*)^2 + 2u^* u_{tt}^*] \\ & \quad + 20u|u|^6\} + \cdots = 0. \end{aligned} \tag{2}$$

Here, $u(x,t)$ denotes the normalized complex amplitude of the optical pulse envelope, and $*$ represents the conjugation. $\alpha_l (l = 2, 3, 4\ldots)$ are all real constant parameters. x and t are the propagation variable and transverse variable, respectively. As far as we know, the first-order and second-oder solutions of Eq. (2) have been obtained by the Darboux transformation (DT) [39]. In this paper, we set $\alpha_m = 0$ $(m = 4, 5, 6\ldots)$ and consider the real situation of soliton transmission in optical fibers. The variable coefficients third-order NLS equations investigated here can be presented as [43]

$$iu_x + \alpha_2(x)(u_{tt} + 2u|u|^2) - i\alpha_3(x)(u_{ttt} + 6u_t|u|^2) = 0. \tag{3}$$

In Eq. (3), u_{tt}, u_{ttt}, $|u|^2 u$ and $|u|^2 u_t$ represent the group velocity dispersion (GVD), third-order dispersion (TOD), self-phase modulation and self-steepening effects, respectively. $\alpha_2(x)$ and $\alpha_3(x)$ are both real functions and represent GVD and TOD coefficients [44, 45]. Equation (3) can be used to describe the optical soliton propagation in inhomogeneous optical fibers [43]. In this paper, based on Hirota's bilinear method [46], "Mathematica 9.0" is used to solve Eq. (3) and plot the corresponding figures. Third-soliton solutions of Eq. (3) will be derived. Optical soliton interactions based on Eq. (3) will be studied to generate new solitons.

The structure of this paper is as follows. In Sect. 2, analytic three-soliton solutions for Eq. (3) will be derived by the Hirota's bilinear method. In Sect. 3, soliton interaction will be discussed, and the phenomenon of new soliton generation is analyzed. In Sect. 4, conclusions will be given.

2 Analytic three-soliton solutions for Eq. (3)

Next, we will use Hirota's bilinear method to obtain the analytical three-soliton solution of Eq. (3). At first, through introducing the rational dependent variable transformation [46]

$$u = \frac{G(x,t)}{F(x,t)}, \tag{4}$$

where $F(x,t)$ is a real differentiable function and $G(x,t)$ is a complex one, we can obtain the following bilinear forms of Eq. (3) after some operations and simplify,

$$(iD_x + \alpha_2(x)D_t^2 - i\alpha_3(x)D_t^3)G \cdot F = 0, \tag{5}$$

$$D_t^2 F \cdot F - 2|G|^2 = 0. \tag{6}$$

Here, the D operator is defined as [47,48],

$$\begin{aligned} & D_x^k D_t^l G(x,t) \cdot F(x,t) \\ & \quad = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^k \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^l G(x,t)F(x',t')\Big|_{x'=x,t'=t} \end{aligned} \tag{7}$$

with k and l as any integer.

In order to obtain three-soliton solutions for Eq. (3), $G(x,t)$ and $F(x,t)$ are assumed to be the series form of the formal parameters,

$$G(x,t) = \varepsilon G_1(x,t) + \varepsilon^3 G_3(x,t) + \varepsilon^5 G_5(x,t) + \cdots, \tag{8}$$

$$F(x,t) = 1 + \varepsilon^2 F_2(x,t) + \varepsilon^4 F_4(x,t) + \varepsilon^6 F_6(x,t) + \cdots. \tag{9}$$

Substituting expressions (8) and (9) into Eqs. (5) and (6) and setting $\varepsilon = 1$, we can obtain the analytic three-soliton solutions as

$$u(x,t) = \frac{G_1(x,t) + G_3(x,t) + G_5(x,t)}{1 + F_2(x,t) + F_4(x,t) + F_6(x,t)}, \tag{10}$$

where

$$G_1(x,t) = e^{\theta_1} + e^{\theta_2} + e^{\theta_3},$$
$$\begin{aligned} F_2(x,t) = {} & A_{21}e^{\theta_1+\theta_1^*} + A_{22}e^{\theta_1+\theta_2^*} \\ & + A_{23}e^{\theta_1+\theta_3^*} + A_{24}e^{\theta_2+\theta_1^*} \\ & + A_{25}e^{\theta_2+\theta_2^*} + A_{26}e^{\theta_2+\theta_3^*} \\ & + A_{27}e^{\theta_3+\theta_1^*} + A_{28}e^{\theta_3+\theta_2^*} \\ & + A_{29}e^{\theta_3+\theta_3^*}, \end{aligned}$$
$$\begin{aligned} G_3(x,t) = {} & B_{31}e^{\theta_1+\theta_2+\theta_1^*} + B_{32}e^{\theta_1+\theta_2+\theta_2^*} \\ & + B_{33}e^{\theta_1+\theta_2+\theta_3^*} \\ & + B_{34}e^{\theta_1+\theta_3+\theta_1^*} + B_{35}e^{\theta_1+\theta_3+\theta_2^*} \\ & + B_{36}e^{\theta_1+\theta_3+\theta_3^*} \\ & + B_{37}e^{\theta_2+\theta_3+\theta_1^*} + B_{38}e^{\theta_2+\theta_3+\theta_2^*} \\ & + B_{39}e^{\theta_2+\theta_3+\theta_3^*}, \end{aligned}$$
$$\begin{aligned} F_4(x,t) = {} & C_{41}e^{\theta_1+\theta_2+\theta_1^*+\theta_2^*} + C_{42}e^{\theta_1+\theta_2+\theta_1^*+\theta_3^*} \\ & + C_{43}e^{\theta_1+\theta_2+\theta_2^*+\theta_3^*} + C_{44}e^{\theta_1+\theta_3+\theta_1^*+\theta_2^*} \\ & + C_{45}e^{\theta_1+\theta_3+\theta_1^*+\theta_3^*} + C_{46}e^{\theta_1+\theta_3+\theta_2^*+\theta_3^*} \\ & + C_{47}e^{\theta_2+\theta_3+\theta_1^*+\theta_2^*} + C_{48}e^{\theta_2+\theta_3+\theta_1^*+\theta_3^*} \\ & + C_{49}e^{\theta_2+\theta_3+\theta_2^*+\theta_3^*}, \end{aligned}$$
$$\begin{aligned} G_5(x,t) = {} & D_{51}e^{\theta_1+\theta_2+\theta_3+\theta_1^*+\theta_2^*} \\ & + D_{52}e^{\theta_1+\theta_2+\theta_3+\theta_1^*+\theta_3^*} \\ & + D_{53}e^{\theta_1+\theta_2+\theta_3+\theta_2^*+\theta_3^*}, \end{aligned}$$
$$F_6(x,t) = E_{61}e^{\theta_1+\theta_2+\theta_3+\theta_1^*+\theta_2^*+\theta_3^*}.$$

Here, $\theta_j(x,t) = \sigma_j(x) + \eta_j t + \varphi_j$. We assume $\sigma_j = \sigma_{j1}(x) + i\sigma_{j2}(x)$, $\eta_j = \eta_{j1} + i\eta_{j2}$, and $\varphi_j = \varphi_{j1} + i\varphi_{j2}$ $(j = 1, 2, 3)$. $\sigma_{j1}(x)$ and $\sigma_{j2}(x)$ are real differentiable functions. The values of $\eta_{j1}, \eta_{j2}, \varphi_{j1}$ and φ_{j2} are real constants. After some calculation, $\sigma_{j1}(x)$ and $\sigma_{j2}(x)$ can be derived as

$$\sigma_{j1}(x) = \int \Big(-2\eta_{j1}\eta_{j2}\alpha_2(x) + (\eta_{j1}^3 - 3\eta_{j1}\eta_{j2}^2)\alpha_3(x)\Big)\mathrm{d}x,$$
$$\sigma_{j2}(x) = \int \Big((\eta_{j1}^2 - \eta_{j2}^2)\alpha_2(x) + (3\eta_{j1}^2\eta_{j2} - \eta_{j2}^3)\alpha_3(x)\Big)\mathrm{d}x.$$

The other corresponding parameters can also be calculated as

$$A_{21} = \frac{1}{4\eta_{11}^2}, \quad B_{31} = \zeta_{31}^2 A_{21} A_{24},$$
$$C_{41} = \frac{\kappa_{11}^2}{16\eta_{11}^2\eta_{21}^2\kappa_{12}^2},$$
$$A_{22} = \frac{1}{\zeta_{21}^2}, \quad B_{32} = \zeta_{31}^2 A_{22} A_{25},$$
$$C_{42} = \frac{\zeta_{31}^2\zeta_{42}^2}{4\eta_{11}^2\zeta_{11}^2\zeta_{22}^2\zeta_{23}^2},$$
$$A_{23} = \frac{1}{\zeta_{22}^2}, \quad B_{33} = \zeta_{31}^2 A_{23} A_{26},$$
$$C_{43} = \frac{\zeta_{31}^2\zeta_{43}^2}{4\eta_{21}^2\zeta_{21}^2\zeta_{23}^2\zeta_{23}^2},$$
$$A_{24} = \frac{1}{\zeta_{11}^2}, \quad B_{34} = \zeta_{32}^2 A_{21} A_{27},$$
$$C_{44} = \frac{\zeta_{32}^2\zeta_{41}^2}{4\eta_{11}^2\zeta_{21}^2\zeta_{12}^2\zeta_{13}^2},$$
$$A_{25} = \frac{1}{4\eta_{21}^2}, \quad B_{35} = \zeta_{32}^2 A_{22} A_{28},$$
$$C_{45} = \frac{\kappa_{21}^2}{16\eta_{11}^2\eta_{31}^2\kappa_{22}^2},$$
$$A_{26} = \frac{1}{\zeta_{23}^2}, \quad B_{36} = \zeta_{32}^2 A_{23} A_{29},$$
$$C_{46} = \frac{\zeta_{32}^2\zeta_{43}^2}{4\eta_{31}^2\zeta_{21}^2\zeta_{22}^2\zeta_{13}^2},$$
$$A_{27} = \frac{1}{\zeta_{12}^2}, \quad B_{37} = \zeta_{33}^2 A_{24} A_{27},$$
$$C_{47} = \frac{\zeta_{33}^2\zeta_{41}^2}{4\eta_{21}^2\zeta_{11}^2\zeta_{12}^2\zeta_{13}^2},$$
$$A_{28} = \frac{1}{\zeta_{13}^2}, \quad B_{38} = \zeta_{33}^2 A_{25} A_{28},$$
$$C_{48} = \frac{\zeta_{33}^2\zeta_{42}^2}{4\eta_{31}^2\zeta_{11}^2\zeta_{23}^2\zeta_{12}^2},$$
$$A_{29} = \frac{1}{4\eta_{31}^2}, \quad B_{39} = \zeta_{33}^2 A_{26} A_{29},$$
$$C_{49} = \frac{\kappa_{31}^2}{16\eta_{21}^2\eta_{31}^2\kappa_{32}^2},$$
$$D_{51} = \frac{\kappa_{11}^2\zeta_{32}^2\zeta_{33}^2}{16\eta_{11}^2\eta_{21}^2\zeta_{11}^2\zeta_{21}^2\zeta_{12}^2\zeta_{13}^2},$$
$$D_{52} = \frac{\kappa_{21}^2\zeta_{31}^2\zeta_{33}^2}{16\eta_{11}^2\eta_{31}^2\zeta_{11}^2\zeta_{22}^2\zeta_{23}^2\zeta_{12}^2},$$
$$D_{53} = \frac{\kappa_{31}^2\zeta_{31}^2\zeta_{32}^2}{16\eta_{21}^2\eta_{31}^2\zeta_{21}^2\zeta_{22}^2\zeta_{23}^2\zeta_{13}^2},$$
$$E_{61} = \frac{\kappa_{11}^2\kappa_{21}^2\kappa_{31}^2}{64\eta_{21}^2\eta_{11}^2\eta_{21}^2\eta_{31}^2\zeta_{21}^2\zeta_{22}^2\zeta_{11}^2\zeta_{32}^2\zeta_{12}^2\zeta_{13}^2}.$$

with

$$\zeta_{11} = \eta_1^* + \eta_2,\ \zeta_{21} = \eta_1 + \eta_2^*,$$
$$\zeta_{31} = \eta_1 - \eta_2,\ \zeta_{41} = \eta_1^* - \eta_2^*,$$
$$\zeta_{12} = \eta_1^* + \eta_3,\ \zeta_{22} = \eta_1 + \eta_3^*,$$
$$\zeta_{32} = \eta_1 - \eta_3,\ \zeta_{42} = \eta_1^* - \eta_3^*,$$

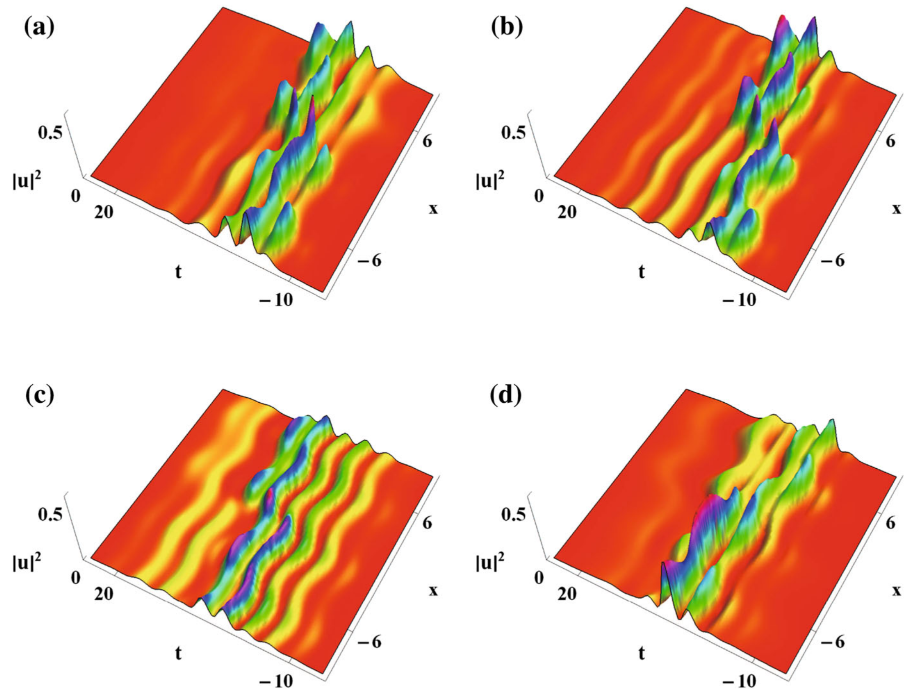

Fig. 1 Three-soliton transmission process. The corresponding parameters are choosen as $\varphi_1 = 0.27 - 0.17i$, $\varphi_2 = -0.98 - 0.41i$, $\varphi_3 = 0.31 + 0.42i$. **a** $\eta_1 = -0.5 + 0.83i$, $\eta_2 = -0.26 - 0.75i$, $\eta_3 = -0.13 + 0.42i$. **b** $\eta_1 = -0.5 + 0.83i$, $\eta_2 = -0.15 - 0.75i$, $\eta_3 = -0.13 + 0.42i$. **c** $\eta_1 = -0.22 + 0.83i$, $\eta_2 = -0.15 - 0.75i$, $\eta_3 = -0.13 + 0.42i$. **d** $\eta_1 = -0.22 + 0.83i$, $\eta_2 = 0.4 - 0.75i$, $\eta_3 = -0.13 + 0.42i$

$$\zeta_{13} = \eta_2^* + \eta_3,\ \zeta_{23} = \eta_2 + \eta_3^*,$$
$$\zeta_{33} = \eta_2 - \eta_3,\ \zeta_{43} = \eta_2^* - \eta_3^*,$$
$$\kappa_{11} = (\eta_{11} - \eta_{21})^2 + (\eta_{12} - \eta_{22})^2,$$
$$\kappa_{21} = (\eta_{11} - \eta_{31})^2 + (\eta_{12} - \eta_{32})^2,$$
$$\kappa_{31} = (\eta_{21} - \eta_{31})^2 + (\eta_{22} - \eta_{32})^2.$$
$$\kappa_{12} = (\eta_{11} + \eta_{21})^2 + (\eta_{12} - \eta_{22})^2,$$
$$\kappa_{22} = (\eta_{11} + \eta_{31})^2 + (\eta_{12} - \eta_{32})^2,$$
$$\kappa_{32} = (\eta_{21} + \eta_{31})^2 + (\eta_{22} - \eta_{32})^2.$$

3 Discussion

Before analyzing soliton interactions, it may be desirable to assign $\alpha_2(x)$ and $\alpha_3(x)$ to the determined values. Here, we assign $\alpha_2(x) = sech(x), \alpha_3(x) = cos(x)$ and discuss the effect of η_{j1} and η_{j2} on soliton interactions. Firstly, keeping the value of φ_j as constant, we can see from Fig. 1a, b, and when η_{21} changes from -0.26 to -0.15, some new visible waveforms appear on the left side of Fig. 1b. And in Fig. 1c, when the value of η_{11} increases from -0.50 to -0.22, the number of multiple solitons has been increased on the basis of Fig. 1b. However, when we adjust the value of η_{21} again and take $\eta_{21} = 0.4$, the number of solitons is obviously reduced in Fig. 1d. Therefore, we can see from Fig. 1 that the value of η_{j1} has an effect on the number of multiple solitons. Also, when η_{j1} are numerically close to each other, the number of multiple solitons will be more.

Next, we change η_{22} to observe the effect of parameter η_{j2} on the number of multiple solitons. In turn, the value of η_{22} is taken as -1.3, -0.88, 0.063 and 1.5 in Fig. 2. In the process of taking different values of η_{22}, the number of solitons decreases first and then increases. Therefore, the imaginary part of η_j is also an important reason for adjusting the number of multiple solitons. Not only that, we observe that the lateral vibrations of Fig. 2b, c are relatively weak, while Fig. 2a, d are more intense. So the larger the value of $|\eta_{22}|$ is, the wider the lateral amplitude of the pulse formed. It makes the interaction between solitons be more severe and distorts the information transferred in optical fibers. Therefore, when considering more soliton numbers, we must also think over that increasing the lateral amplitude is equivalent to increase the width of the pulse, which is more likely to cause interactions and lead to pulse deformation. Thus, the value of η_{j2} should be controlled within a reasonable range.

In one-soliton solutions, φ_j just control the position of the pulse and have no effect on peaks and waveforms. Similarly, in the three solitons, φ_j dominate the initial phase of the corresponding pulses. And the relative

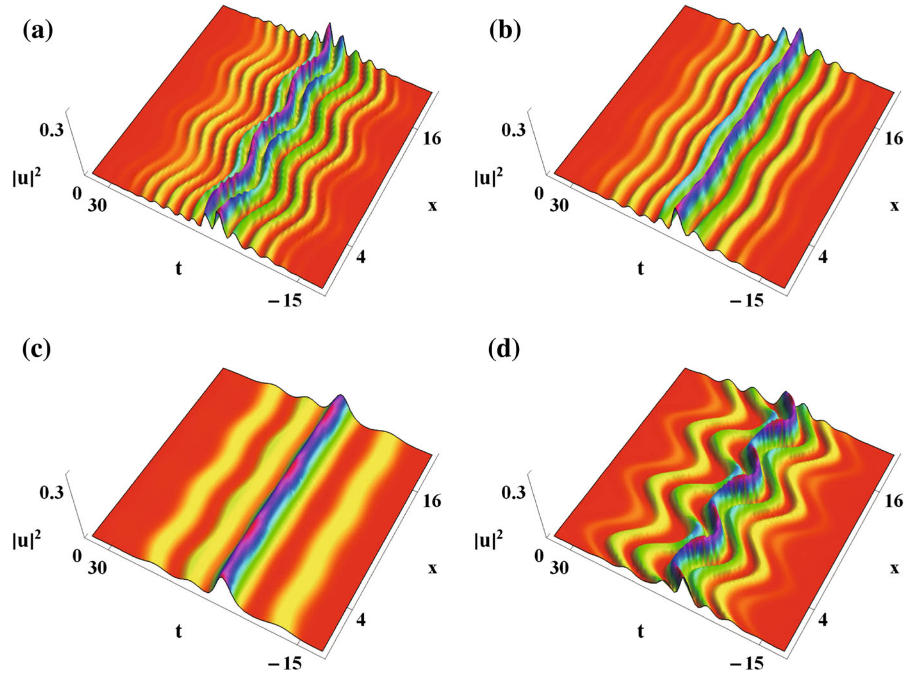

Fig. 2 Three-soliton transmission process. The corresponding parameters are choosen as $\varphi_1 = 0.27 - 0.17i$, $\varphi_2 = -0.98 - 0.41i$, $\varphi_3 = 0.31 + 0.42i$, $\eta_1 = -0.22 + 0.41i$, $\eta_3 = -0.13 + 0.66i$. **a** $\eta_2 = -0.11 - 1.3i$. **b** $\eta_2 = -0.11 - 0.88i$. **c** $\eta_2 = -0.11 + 0.063i$. **d** $\eta_2 = -0.11 + 1.5i$

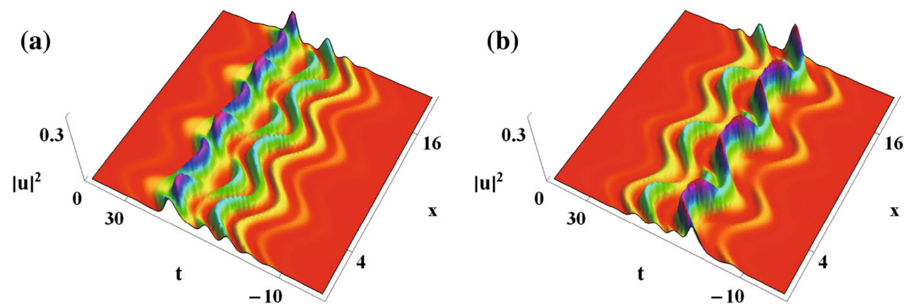

Fig. 3 Three-soliton transmission process. The corresponding parameters are choosen as $\varphi_2 = -0.98$, $\varphi_3 = 0.31$, $\eta_1 = -0.22 + 0.83i$, $\eta_2 = -0.11 + 1.5i$, $\eta_3 = -0.13 + 0.42i$. **a** $\varphi_1 = 2.8$. **b** $\varphi_1 = 0.56$

distance between solitons directly affects the generation of interactions, so φ_j can command the strength of the interaction by adjusting the transmission position of solitons. In Figs. 3a, b, when the value of φ_j changes from 2.8 to 0.56, the pulse with the higher peak obviously moves to the negative direction of t, and the number of multiple solitons decreases. Therefore, φ_j are also an important parameter affecting the generation of multiple solitons. Moreover, when the values of φ_j are close to each other, the interval between the pulses is reduced, and the number of multiple solitons generated by the interaction is increased. Therefore, η_j and φ_j determine the soliton phase, which is an important reason for determining the generation of multiple solitons. The more the phases tend to be consistent, the more number of solitons are produced.

The influence of $\alpha_2(x)$ and $\alpha_3(x)$ on solitons will be discussed in the next. As shown in Fig. 4a, b, when $\alpha_2(x)$ takes different values, the period of interaction between solitons is adjusted. Therefore, the region where the soliton interaction occurs can be achieved by changing the function of $\alpha_2(x)$ as needed. As shown in Fig. 4c, d, $\alpha_3(x)$ can determine the propagation path of solitons; when $\alpha_3(x) = cos(x)$, the propagation path of the soliton is a periodic vibration wave in Fig. 4c. When $\alpha_3(x) = x$, it is a parabolic soliton in Fig. 4d.

4 Conclusion

In this paper, Eq. (3) has been solved by the Hirota's bilinear method, and analytic three-soliton solution (10) has been obtained. Three solitons have been split into

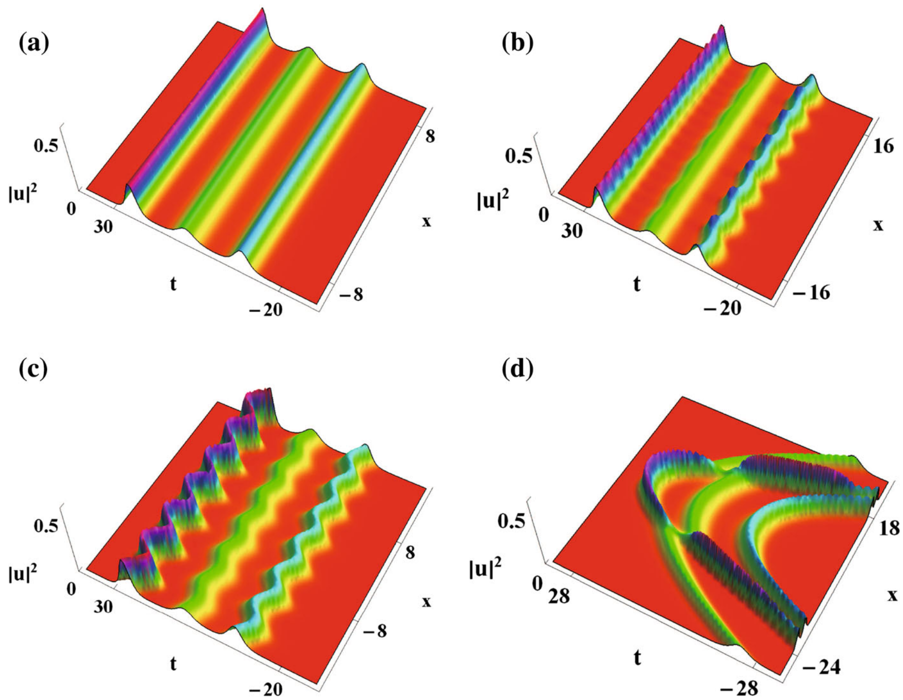

Fig. 4 Three-soliton transmission process. The corresponding parameters are choosen as $\varphi_1 = 1$, $\varphi_2 = 3 - 0.41i$, $\varphi_3 = -5$, $\eta_1 = 0.3$, $\eta_2 = 0.4$, $\eta_3 = 0.5$. **a** $\alpha_2(x) = 1$, $\alpha_3(x) = 0.1$. **b** $\alpha_2(x) = 15$, $\alpha_3(x) = 0.1$. **c** $\alpha_2(x) = 1$, $\alpha_3(x) = 10cos(x)$. **d** $\alpha_2(x) = 1$, $\alpha_3(x) = x$

multiple solitons under certain conditions. Moreover, the number of solitons in multiple solitons has been affected by the relevant parameters. During the process of three-soliton transmission, the smaller the difference in η_{j1} $(j = 1, 2, 3)$ to each other, the larger the number of multiple solitons is. η_{j2} are important parameters that determine the direction of soliton transmission. The lateral amplitude of the soliton has been decided by η_{j2}, and the pulse width has been reduced. Besides, η_{j2} have the significant effects on the number of multiple solitons. φ_j have been used to adjust the transmission position of solitons and alter the intensity of interactions between solitons. When the distance between solitons is considerable, the interaction between solitons has decreased, and the number of the generation of solitons has been reduced. In summary, η_j and φ_j are decisive parameters for controlling the number of multiple solitons, which together determine the phase of solitons. When the soliton phases are approximated, multiple solitons can be generated. $\alpha_2(x)$ plays the role of adjusting the interaction period, and $\alpha_3(x)$ can control the transmission path of solitons.

Acknowledgements This work of Wenjun Liu was supported by the National Natural Science Foundation of China (Grant Nos. 11674036 and 11875008), by the Beijing Youth Top-notch Talent Support Program (Grant No. 2017000026833ZK08) and by the Fund of State Key Laboratory of Information Photonics and Optical Communications (Beijing University of Posts and Telecommunications, Grant Nos. IPOC2016ZT04 and IPOC2017ZZ05). This work of Qin Zhou was supported by the National Natural Science Foundation of China (Grant Nos. 11705130 and 1157149), and this author was also sponsored by the Chutian Scholar Program of Hubei Government in China. The research work of Milivoj Belic was supported by Qatar National Research Fund (QNRF) under the Grant Number NPRP 8-028-1-001.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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