THE GRAND UNIFIED THEORY OF CLASSICAL QUANTUM MECHANICS

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A theory of classical quantum mechanics (CQM) is derived from first principles that successfully applies physical laws on all scales. Using Maxwell's equations, the classical wave equation is solved with the constraint that a bound electron cannot radiate energy. By further application of Maxwell's equations to electromagnetic and gravitational fields at particle production, the Schwarzschild metric (SM) is derived from the classical wave equation which modifies general relativity to include conservation of spacetime in addition to momentum and matter/energy. The result gives a natural relationship between Maxwell's equations, special relativity, and general relativity. It gives gravitation from the atom to the cosmos.

INTRODUCTION

A theory of classical quantum mechanics (CQM), derived from first principles, successfully applies physical laws on all scales [1]. The classical wave equation is solved with the constraint that a bound electron cannot radiate energy. The mathematical formulation for zero radiation based on Maxwell's equations follows from a derivation by Haus [2]. The function that describes the motion of the electron must not possess spacetime Fourier components that are synchronous with waves traveling at the speed of light. CQM gives closed form solutions for the atom including the stability of the $n = 1$ state and the instability of the excited states, the equation of the photon and electron in excited states, the equation of the free electron, and photon which predict the wave particle duality behavior of particles and light. The current and charge density functions of the electron may be directly physically interpreted. For example, spin angular momentum results from the motion of negatively charged mass moving systematically, and the equation for angular momentum, $\mathbf{r} \times \mathbf{p}$, can be applied directly to the wave function (a current density function) that describes the electron. The magnetic moment of a Bohr magneton, Stern Gerlach experiment, g factor, Lamb shift, resonant line width and shape, selection rules, correspondence principle, wave particle duality, excited states, reduced mass, rotational energies, and momenta, orbital and spin splitting, spin-orbital coupling, Knight shift, and spin-nuclear coupling, ionization of two electron atoms, inelastic electron scattering from helium atoms, and the nature of the chemical bond are derived in closed form equations based on Maxwell's equations. The calculations agree with experimental observations.

For any kind of wave advancing with limiting velocity and capable of transmitting signals, the equation of front propagation is the same as the equation for the front of a light wave. By applying this condition to electromagnetic and gravitational fields at particle production, the Schwarzschild metric (SM) is derived from the classical wave equation which modifies general relativity to include conservation of spacetime in addition to momentum and matter/energy. The result gives a natural relationship between Maxwell's equations, special relativity, and general relativity. It gives gravitation from the atom to the cosmos. The Universe is time harmonically oscillatory in matter energy and spacetime expansion and contraction with a minimum radius that is the gravitational radius. In closed form equations with fundamental constants only, CQM gives the deflection of light by stars, the precession of the perihelion of Mercury, the particle masses, the Hubble constant, the age of the Universe, the observed acceleration of the expansion, the power of the Universe, the power spectrum of the Universe, the microwave background temperature, the uniformity of the microwave background radiation, the microkelvin spatial variation of the microwave background radiation, the observed violation of the GZK cutoff, the mass density, the large scale structure of the Universe, and the identity of dark matter which matches the criteria for the structure of galaxies. In a special case wherein the gravitational potential energy density of a blackhole equals that of the Plank mass, matter converts to energy and spacetime expands with the release of a gamma ray burst. The singularity in the SM is eliminated.

CLASSICAL QUANTUM THEORY OF THE ATOM BASED ON MAXWELL'S EQUATIONS

One-electron atoms include the hydrogen atom, He^{+}, Li^{2+}, Be^{3+}, and so on. The mass-energy and angular momentum of the electron are constant; this requires that the equation of motion of the electron be temporally and spatially harmonic. Thus, the classical wave equation applies and

$$\left[\nabla^2 - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right]\rho(r,\theta,\phi,t) = 0 \tag{1}$$

where $\rho(r,\theta,\phi,t)$ is the charge density function of the electron in time and space. In general, the wave equation has an infinite number of solutions. To arrive at the solution which represents the electron, a suitable boundary condition must be imposed. It is well known from experiments that each single atomic electron of a given isotope radiates to the same stable state. Thus, the physical boundary condition of nonradiation of the bound electron was imposed on the solution of the wave equation for the charge density function of the electron [1]. The condition for radiation by a moving point charge given by Haus [2] is that its spacetime Fourier transform does possess components that are synchronous with waves traveling at the speed of light. Conversely, it is proposed that the condition for nonradiation by an ensemble of moving point charges that comprises a current density function is

For non-radiative states, the current-density function must NOT possess spacetime Fourier components that are synchronous with waves traveling at the speed of light.

The time, radial, and angular solutions of the wave equation are separable. The motion is time harmonic with frequency ω_n. A constant angular function is a solution to the wave equation. The solution for the radial function which satisfies the boundary condition is a delta function

$$f(r) = \frac{1}{r^2}\delta(r - r_n) \tag{2}$$

which defines a constant charge function on a spherical shell where $r_n = nr_1$. Given time harmonic motion and a radial delta function, the relationship between an allowed radius and the electron wavelength is given by

$$2\pi r_n = \lambda_n \tag{3}$$

Using the de Broglie relationship for the electron mass where the coordinates are spherical,

$$\lambda_n = \frac{h}{p_n} = \frac{h}{m_e v_n} \tag{4}$$

and the magnitude of the velocity for *every* point on the orbitsphere is

$$v_n = \frac{\hbar}{m_e r_n} \tag{5}$$

The sum of the $\mathbf{L}_i$, the magnitude of the angular momentum of each infinitesimal point of the orbitsphere of mass m_i, must be constant. The constant is $\hbar$.

$$\sum |\mathbf{L}_i| = \sum \left|\mathbf{r} \times m_i \mathbf{v}\right| = m_e r_n \frac{\hbar}{m_e r_n} = \hbar \tag{6}$$

Thus, an electron is a spinning, two-dimensional spherical surface, called an *electron orbitsphere*, that can exist in a bound state at only specified distances from the nucleus as shown in Figure 1. The corresponding current function shown in Figure 2 which gives rise to the phenomenon of *spin* is derived in the "Spin Function" section.

Nonconstant functions are also solutions for the angular functions. To be a harmonic solution of the wave equation in spherical coordinates, these angular functions must be spherical harmonic functions. A zero of the spacetime Fourier transform of the product function of two spherical harmonic angular functions, a time harmonic function, and an unknown radial function is sought. The solution for the radial function which satisfies the boundary condition is also a delta function given by Eq. (2). Thus, bound electrons are described by a charge-density (mass-density) function which is the product of a radial delta function, two angular functions (spherical harmonic functions), and a time harmonic function.

$$\rho(r,\theta,\phi,t) = f(r)A(\theta,\phi,t) = \frac{1}{r^2}\delta(r - r_n)A(\theta,\phi,t); \qquad A(\theta,\phi,t) = Y(\theta,\phi)k(t) \tag{7}$$

In these cases, the spherical harmonic functions correspond to a traveling charge density wave confined to the spherical shell which gives rise to the phenomenon of orbital angular momentum. The orbital functions which modulate the constant "spin" function shown graphically in Figure 3 are given in the "Angular Functions" section.

SPIN FUNCTION

The orbitsphere spin function comprises a constant charge density function with moving charge confined to a two-dimensional spherical shell. The current pattern of the orbitsphere spin function comprises an infinite series of correlated orthogonal great circle current loops wherein each point moves time harmonically with angular velocity

$$\omega_n = \frac{\hbar}{m_e r_n^2} \tag{8}$$

The current pattern is generated over the surface by a series of nested rotations of two orthogonal great circle current loops where the coordinate axes rotate with the two orthogonal great circles. Half of the pattern is generated as the z-axis rotates to the negative z-axis during a 1st set of nested rotations. The mirror image, second half of the pattern is generated as the z-axis rotates back to its original direction during a 2nd set of nested rotations.

Points on Great Circle Current Loop One:

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} cos(\Delta\alpha) & -sin^2(\Delta\alpha) & -sin(\Delta\alpha)cos(\Delta\alpha) \\ 0 & cos(\Delta\alpha) & -sin(\Delta\alpha) \\ sin(\Delta\alpha) & cos(\Delta\alpha)sin(\Delta\alpha) & cos^2(\Delta\alpha) \end{bmatrix} \begin{bmatrix} x_1' \\ y_1' \\ z_1' \end{bmatrix} \tag{9}$$

and $\Delta\alpha' = -\Delta\alpha$ replaces $\Delta\alpha$ for $\sum_{n=1}^{\frac{\sqrt{2}\pi}{\Delta\alpha}} \Delta\alpha = \sqrt{2}\pi$; $\sum_{n=1}^{\frac{\sqrt{2}\pi}{|\Delta\alpha'|}} |\Delta\alpha'| = \sqrt{2}\pi$

Points on Great Circle Current Loop Two:

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} cos(\Delta\alpha) & -sin^2(\Delta\alpha) & -sin(\Delta\alpha)cos(\Delta\alpha) \\ 0 & cos(\Delta\alpha) & -sin(\Delta\alpha) \\ sin(\Delta\alpha) & cos(\Delta\alpha)sin(\Delta\alpha) & cos^2(\Delta\alpha) \end{bmatrix} \begin{bmatrix} x_2' \\ y_2' \\ z_2' \end{bmatrix} \tag{10}$$

and $\Delta\alpha' = -\Delta\alpha$ replaces $\Delta\alpha$ for $\sum_{n=1}^{\frac{\sqrt{2}\pi}{\Delta\alpha}} \Delta\alpha = \sqrt{2}\pi$; $\sum_{n=1}^{\frac{\sqrt{2}\pi}{|\Delta\alpha'|}} |\Delta\alpha'| = \sqrt{2}\pi$

The orbitsphere is given by reiterations of Eqs. (9) and (10). The output given by the non primed coordinates is the input of the next iteration corresponding to each successive nested rotation by the infinitesimal angle where the summation of the rotation about each of the x-axis and the y-axis is $\sum_{n=1}^{\frac{\sqrt{2}\pi}{\Delta\alpha}} \Delta\alpha = \sqrt{2}\pi$ (1st set) and $\sum_{n=1}^{\frac{\sqrt{2}\pi}{|\Delta\alpha'|}} |\Delta\alpha'| = \sqrt{2}\pi$ (2nd set). The current pattern corresponding to great circle current loop one and two shown with 8.49 degree increments of the infinitesimal angular variable $\Delta\alpha(\Delta\alpha')$ of Eqs. (9) and (10) is shown from the perspective of looking along the z-axis in Figure 2. The true orbitsphere current pattern is given as $\Delta\alpha(\Delta\alpha')$ approaches zero. This current pattern gives rise to the phenomenon corresponding to the spin quantum number of the electron.

MAGNETIC FIELD EQUATIONS OF THE ELECTRON

The orbitsphere is a shell of negative charge current comprising correlated charge motion along great circles. For ℓ = 0, the orbitsphere gives rise to a magnetic moment of 1 Bohr magneton [3].

$$\mu_B = \frac{e\hbar}{2m_e} = 9.274\,X\,10^{-24}\,JT^{-1}, \tag{11}$$

The magnetic field of the electron shown in Figure 4 is given by

$$\mathbf{H} = \frac{e\hbar}{m_e r_n^3}(\mathbf{i}_r \cos\theta - \mathbf{i}_\theta \sin\theta) \qquad \text{for } r < r_n \tag{12}$$

$$\mathbf{H} = \frac{e\hbar}{2m_e r^3}(\mathbf{i}_r 2\cos\theta - \mathbf{i}_\theta \sin\theta) \qquad \text{for } r > r_n \tag{13}$$

The energy stored in the magnetic field of the electron is

$$E_{mag} = \frac{1}{2}\mu_o \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} H^2 r^2 \sin\theta dr d\theta d\Phi \tag{14}$$

$$E_{mag\,\text{total}} = \frac{\pi\mu_o e^2\hbar^2}{m_e^2 r_1^3} \tag{15}$$

STERN-GERLACH EXPERIMENT

The Stern-Gerlach experiment implies a magnetic moment of one Bohr magneton and an associated angular momentum quantum number of 1/2. Historically, this quantum number is called the spin quantum number, s ($s = \frac{1}{2}$; $m_s = \pm\frac{1}{2}$). The superposition of the vector projection of the orbitsphere angular momentum on to an axis S that precesses about the z-axis called the spin axis at an angle of $\theta = \frac{\pi}{3}$ and an angle of $\phi = \pi$ with respect to $\langle \mathbf{L}_{xy} \rangle_{\Sigma\Delta\alpha}$ is

$$\mathbf{S} = \pm\sqrt{\frac{3}{4}}\hbar \tag{16}$$

$\mathbf{S}$ rotates about the z-axis at the Larmor frequency. $\langle \mathbf{S}_z \rangle$, the time averaged projection of the orbitsphere angular momentum onto the axis of the applied magnetic field is

$$\langle \mathbf{L}_z \rangle_{\Sigma\Delta\alpha} \pm \frac{\hbar}{2}. \tag{17}$$

ELECTRON g FACTOR

Conservation of angular momentum of the orbitsphere permits a discrete change of its "kinetic angular momentum" $(\mathbf{r} \times m\mathbf{v})$ by the applied magnetic field of $\frac{\hbar}{2}$, and concomitantly the "potential angular momentum" $(\mathbf{r} \times e\mathbf{A})$ must change by $-\frac{\hbar}{2}$.

$$\Delta \mathbf{L} = \frac{\hbar}{2} - \mathbf{r} \times e\mathbf{A} \tag{18}$$

$$= \left[\frac{\hbar}{2} - \frac{e\phi}{2\pi} \right] \hat{z} \tag{19}$$

In order that the change of angular momentum, $\Delta \mathbf{L}$, equals zero, ϕ must be $\mathbf{\Phi}_0 = \frac{h}{2e}$, the magnetic flux quantum. The magnetic moment of the electron is parallel or antiparallel to the applied field only. During the spin-flip transition, power must be conserved. Power flow is governed by the Poynting power theorem,

$$\nabla \bullet (\mathbf{E} \times \mathbf{H}) = -\frac{\partial}{\partial t}\left[\frac{1}{2} \mu_o \mathbf{H} \bullet \mathbf{H} \right] - \frac{\partial}{\partial t}\left[\frac{1}{2} \varepsilon_o \mathbf{E} \bullet \mathbf{E} \right] - \mathbf{J} \bullet \mathbf{E} \tag{20}$$

Eq. (21) gives the total energy of the flip transition which is the sum of the energy of reorientation of the magnetic moment (1st term), the magnetic energy (2nd term), the electric energy (3rd term), and the dissipated energy of a fluxon treading the orbitsphere (4th term), respectively.

$$\Delta E_{mag}^{spin} = 2\left(1 + \frac{\alpha}{2\pi} + \frac{2}{3}\alpha^2 \left(\frac{\alpha}{2\pi} \right) - \frac{4}{3}\left(\frac{\alpha}{2\pi} \right)^2 \right) \mu_B B \tag{21}$$

$$\Delta E_{mag}^{spin} = g\mu_B B \tag{22}$$

where the stored magnetic energy corresponding to the $\frac{\partial}{\partial t}\left[\frac{1}{2} \mu_o \mathbf{H} \bullet \mathbf{H} \right]$ term increases, the stored electric energy corresponding to the $\frac{\partial}{\partial t}\left[\frac{1}{2} \varepsilon_o \mathbf{E} \bullet \mathbf{E} \right]$ term increases, and the $\mathbf{J} \bullet \mathbf{E}$ term is dissipative. The spin-flip transition can be considered as involving a magnetic moment of g times that of a Bohr magneton. The g factor is redesignated the fluxon g factor as opposed to the anomalous g factor. The calculated value of $\frac{g}{2}$ is $1.001\ 159\ 652\ 137$. The experimental value [4] of $\frac{g}{2}$ is $1.001\ 159\ 652\ 188(4)$.

ANGULAR FUNCTIONS

The time, radial, and angular solutions of the wave equation are separable. Also based on the radial solution, the angular charge and current-density functions of the electron, $A(\theta, \phi, t)$, must be a solution of the wave equation in two dimensions (plus time),

$$\left[\nabla^2 - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right] A(\theta, \phi, t) = 0 \tag{23}$$

where $\rho(r, \theta, \phi, t) = f(r)A(\theta, \phi, t) = \frac{1}{r^2}\delta(r - r_n)A(\theta, \phi, t)$ and $A(\theta, \phi, t) = Y(\theta, \phi)k(t)$

$$\left[\frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)_{r,\phi} + \frac{1}{r^2 \sin^2\theta}\left(\frac{\partial^2}{\partial\phi^2}\right)_{r,\theta} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right] A(\theta, \phi, t) = 0 \tag{24}$$

where v is the linear velocity of the electron. The charge-density functions including the time-function factor are

$\ell = 0$

$$\rho(r, \theta, \phi, t) = \frac{e}{8\pi r^2}[\delta(r - r_n)]\left[Y_\ell^m(\theta, \phi) + Y_0^0(\theta, \phi)\right] \tag{25}$$

$\ell \neq 0$

$$\rho(r, \theta, \phi, t) = \frac{e}{4\pi r^2}[\delta(r - r_n)]\left[Y_0^0(\theta, \phi) + Re\left\{Y_\ell^m(\theta, \phi)\left[1 + e^{i\omega_n t}\right]\right\}\right] \tag{26}$$

$$Re\left\{Y_\ell^m(\theta, \phi)\left[1 + e^{i\omega_n t}\right]\right\} = Re\left[Y_\ell^m(\theta, \phi) + Y_\ell^m(\theta, \phi)e^{i\omega_n t}\right] = P_\ell^m(\cos\theta)\cos m\phi + P_\ell^m(\cos\theta)\cos(m\phi + \omega_n t)$$

and $\omega_n = 0$ for $m = 0$.

SPIN AND ORBITAL PARAMETERS

The total function that describes the spinning motion of each electron orbitsphere is composed of two functions. One function, the spin function, is spatially uniform over the orbitsphere, spins with a quantized angular velocity, and gives rise to spin angular momentum. The other function, the modulation function, can be spatially uniform—in which case there is no orbital angular momentum and the magnetic moment of the electron orbitsphere is one Bohr magneton—or not spatially uniform—in which case there is orbital angular momentum. The modulation function also rotates with a quantized angular velocity.

The spin function of the electron corresponds to the nonradiative $n = 1$, $\ell = 0$ state of atomic hydrogen which is well known as an s state or orbital. (See Figure 1 for the charge function and Figure 2 for the current function.) For orbitals with the ℓ quantum number not equal to zero, the constant spin function is modulated by a time and spherical harmonic function as given by Eq. (26) and shown in Figure 3. The modulation or traveling charge density wave corresponds to an orbital angular momentum in addition to a spin angular momentum. These states are typically referred to as p, d, f, etc. orbitals. Application of Haus's [2] condition also predicts nonradiation for a constant spin function modulated by a time and spherically harmonic orbital function. There is acceleration without radiation. (Also see Abbott and Griffiths and Goedecke [5-6]). However, in the case that such a state arises as an excited state by photon absorption, it is radiative due to a radial dipole term in its current density function since it possesses spacetime Fourier Transform components synchronous with waves traveling at the speed of light [2]. (See "Instability of Excited States" section.)

Moment of Inertia and Spin and Rotational Energies

$\boldsymbol{\ell = 0}$

$$I_z = I_{spin} = \frac{m_e r_n^2}{2} \tag{27}$$

$$L_z = I\omega \mathbf{i}_z = \pm\frac{\hbar}{2} \tag{28}$$

$$E_{rotational} = E_{rotational,\,spin} = \frac{1}{2}\left[I_{spin}\left(\frac{\hbar}{m_e r_n^2}\right)^2\right] = \frac{1}{2}\left[\frac{m_e r_n^2}{2}\left(\frac{\hbar}{m_e r_n^2}\right)^2\right] = \frac{1}{4}\left[\frac{\hbar^2}{2I_{spin}}\right] \tag{29}$$

$\boldsymbol{\ell \neq 0}$

$$I_{orbital} = m_e r_n^2\left[\frac{\ell(\ell+1)}{\ell^2+\ell+1}\right]^{\frac{1}{2}} \tag{30}$$

$$L_z = m\hbar \tag{31}$$

$$L_{z\ total} = L_{z\ spin} + L_{z\ orbital} \tag{32}$$

$$E_{rotational,\ orbital} = \frac{\hbar^2}{2I}\left[\frac{\ell(\ell+1)}{\ell^2+2\ell+1}\right] \tag{33}$$

$$T = \frac{\hbar^2}{2m_e r_n^2} \tag{34}$$

$$\left\langle E_{rotational,\ orbital}\right\rangle = 0 \tag{35}$$

From Eq. (35), the time average rotational energy is zero; thus, the principal levels are degenerate except when a magnetic field is applied.

NONRADIATION CONDITION (Acceleration Without Radiation)

The Fourier transform of the electron charge density function is a solution of the four-dimensional wave equation in frequency space ($\mathbf{k},\omega\, space$). Then the corresponding Fourier transform of the current density function $K(s,\Theta,\Phi,\omega)$ is given by multiplying by the constant angular frequency.

$$K(s,\Theta,\Phi,\omega) = 4\pi\omega_n \frac{\sin(2s_n r_n)}{2s_n r_n} \otimes 2\pi\sum_{\upsilon=1}^{\infty} \frac{(-1)^{\upsilon-1}(\pi\sin\Theta)^{2(\upsilon-1)}}{(\upsilon-1)!(\upsilon-1)!} \frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\upsilon+\frac{1}{2}\right)}{(\pi\cos\Theta)^{2\upsilon+1}2^{\upsilon+1}} \frac{2\upsilon!}{(\upsilon-1)!} s^{-2\upsilon}$$
$$\otimes 2\pi\sum_{\upsilon=1}^{\infty} \frac{(-1)^{\upsilon-1}(\pi\sin\Phi)^{2(\upsilon-1)}}{(\upsilon-1)!(\upsilon-1)!} \frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\upsilon+\frac{1}{2}\right)}{(\pi\cos\Phi)^{2\upsilon+1}2^{\upsilon+1}} \frac{2\upsilon!}{(\upsilon-1)!} s^{-2\upsilon} \frac{1}{4\pi}[\delta(\omega-\omega_n)+\delta(\omega+\omega_n)] \quad (36)$$

$\mathbf{s}_n \bullet \mathbf{v}_n = \mathbf{s}_n \bullet \mathbf{c} = \omega_n$ implies $r_n = \lambda_n$ Spacetime harmonics of $\frac{\omega_n}{c} = k$ or $\frac{\omega_n}{c}\sqrt{\frac{\varepsilon}{\varepsilon_o}} = k$ for which the Fourier transform of the current-density function is nonzero do not exist. Radiation due to charge motion does not occur in any medium when this boundary condition is met.

FORCE BALANCE EQUATION

The radius of the nonradiative ($n = 1$) state is solved using the electromagnetic force equations of Maxwell relating the charge and mass density functions wherein the angular momentum of the electron is given by Planck's constant bar. The reduced mass arises naturally from an electrodynamic interaction between the electron and the proton.

$$\frac{m_e}{4\pi r_1^2}\frac{v_1^2}{r_1} = \frac{e}{4\pi r_1^2}\frac{Ze}{4\pi\varepsilon_o r_1^2} - \frac{1}{4\pi r_1^2}\frac{\hbar^2}{m r_n^3} \quad (37)$$

$$r_1 = \frac{a_H}{Z} \quad (38)$$

ENERGY CALCULATIONS

From Maxwell's equations, the potential energy V, kinetic energy T, electric energy or binding energy E_{ele} are

$$V = \frac{-Ze^2}{4\pi\varepsilon_o r_1} = \frac{-Z^2e^2}{4\pi\varepsilon_o a_H} = -Z^2\, X\ 4.3675\, X\, 10^{-18}\, J = -Z^2\, X\, 27.2\ eV \quad (39)$$

$$T = \frac{Z^2e^2}{8\pi\varepsilon_o a_H} = Z^2\, X\, 13.59\ eV \quad (40)$$

$$T = E_{ele} = -\frac{1}{2}\varepsilon_o\int_{\infty}^{r_1}\mathbf{E}^2 dv \quad \text{where } \mathbf{E} = -\frac{Ze}{4\pi\varepsilon_o r^2}. \quad (41)$$

$$E_{ele} = -\frac{Z^2e^2}{8\pi\varepsilon_o a_H} = -Z^2\, X\, 2.1786\, X\, 10^{-18}\ J = -Z^2\, X\, 13.598\ eV \quad (42)$$

The calculated Rydberg constant is $109{,}677.58\ cm^{-1}$; the experimental Rydberg constant is $109{,}677.58\ cm^{-1}$.

EXCITED STATES

CQM gives closed form solutions for the resonant photons and excited state electron functions. The angular momentum of the photon given by

$$\mathbf{m} = \frac{1}{8\pi}\operatorname{Re}[\mathbf{r} \times (\mathbf{E} \times \mathbf{B}^*)] = \hbar \tag{43}$$

is conserved [7]. The change in angular velocity of the electron is equal to the angular frequency of the resonant photon. The energy is given by Planck's equation. The predicted energies, Lamb shift, hyperfine structure, resonant line shape, line width, selection rules, etc. are in agreement with observation.

The orbitsphere is a dynamic spherical resonator cavity which traps photons of discrete frequencies. The relationship between an allowed radius and the "photon standing wave" wavelength is

$$2\pi r = n\lambda \tag{44}$$

where n is an integer. The relationship between an allowed radius and the electron wavelength is

$$2\pi(nr_1) = 2\pi r_n = n\lambda_1 = \lambda_n \tag{45}$$

where $n = 1,2,3,4,\ldots$ The radius of an orbitsphere increases with the absorption of electromagnetic energy. The radii of excited states are solved using the electromagnetic force equations of Maxwell relating the field from the charge of the proton, the electric field the photon, and charge and mass density functions of the electron wherein the angular momentum of the electron is given by Planck's constant bar (Eq. (37)). The solutions to Maxwell's equations for modes that can be excited in the orbitsphere resonator cavity give rise to four quantum numbers, and the energies of the modes are the experimentally known hydrogen spectrum. The relationship between the electric field equation and the "trapped photon" source charge-density function is given by Maxwell's equation in two dimensions.

$$\mathbf{n} \bullet (\mathbf{E}_1 - \mathbf{E}_2) = \frac{\sigma}{\varepsilon_0} \tag{46}$$

The photon standing electromagnetic wave is phase matched with the electron

$$\mathbf{E}_{r\,photon\ n,l,m} = \frac{e(na_H)^{\ell}}{4\pi\varepsilon_o}\frac{1}{r^{(\ell+2)}}\left[-Y_0^0(\theta,\phi) + \frac{1}{n}\left[Y_0^0(\theta,\phi) + Re\left\{Y_\ell^m(\theta,\phi)\left[1 + e^{i\omega_n t}\right]\right\}\right]\right] \tag{47}$$

$$\omega_n = 0 \ \ for\ m = 0$$

$$\ell = 1,2,\ldots,n-1$$

$$m = -\ell, -\ell + 1, \ldots, 0, \ldots, +\ell$$

$$\mathbf{E}_{r\,total} = \frac{e}{4\pi\varepsilon_o r^2} + \frac{e(na_H)^\ell}{4\pi\varepsilon_o}\frac{1}{r^{(\ell+2)}}\left[-Y_0^0(\theta,\phi) + \frac{1}{n}\left[Y_0^0(\theta,\phi) + Re\left\{Y_\ell^m(\theta,\phi)\left[1+e^{i\omega_n t}\right]\right\}\right]\right] \quad (48)$$

$\omega_n = 0$ for $m = 0$

For $r = na_H$ and $m = 0$, the total radial electric field is

$$\mathbf{E}_{rtotal} = \frac{1}{n}\frac{e}{4\pi\varepsilon_o (na_H)^2} \quad (49)$$

The energy of the photon which excites a mode in the electron spherical resonator cavity from radius a_H to radius na_H is

$$E_{photon} = \frac{e^2}{8\pi\varepsilon_o a_H}\left[1 - \frac{1}{n^2}\right] = h\nu = \hbar\omega \quad (50)$$

The change in angular velocity of the orbitsphere for an excitation from $n = 1$ to $n = n$ is

$$\Delta\omega = \frac{\hbar}{m_e (a_H)^2} - \frac{\hbar}{m_e (na_H)^2} = \frac{\hbar}{m_e (a_H)^2}\left[1 - \frac{1}{n^2}\right] \quad (51)$$

The kinetic energy change of the transition is

$$\frac{1}{2} m_e (\Delta v)^2 = \frac{e^2}{8\pi\varepsilon_0 a_H}\left[1 - \frac{1}{n^2}\right] = \hbar\omega \quad (52)$$

The change in angular velocity of the electron orbitsphere is identical to the angular velocity of the photon necessary for the excitation, ω_{photon}. *The correspondence principle holds*. It can be demonstrated that the resonance condition between these frequencies is to be satisfied in order to have a net change of the energy field [8].

ORBITAL AND SPIN SPLITTING

The ratio of the square of the angular momentum, M^2, to the square of the energy, U^2, for a pure (ℓ, m) multipole is [9]

$$\frac{M^2}{U^2} = \frac{m^2}{\omega^2} \quad (53)$$

The magnetic moment is defined as

$$\mu = \frac{\text{charge x angular momentum}}{\text{2 x mass}} \quad (54)$$

The radiation of a multipole of order (ℓ, m) carries m$\hbar$ units of the z component of angular momentum per photon of energy $\hbar\omega$. Thus, the z component of the angular momentum of the corresponding excited state electron orbitsphere is

$$L_z = m\hbar \quad (55)$$

Therefore,

$$\mu_z = \frac{em\hbar}{2m_e} = m\mu_B \quad (56)$$

where μ_B is the Bohr magneton. The orbital splitting energy is

$$E_{mag}^{orb} = m\mu_B B \tag{57}$$

The spin and orbital splitting energies superimpose; thus, the principal excited state energy levels of the hydrogen atom are split by the energy $E_{mag}^{spin/orb}$.

$$E_{mag}^{spin/orb} = m\frac{e\hbar}{2m_e}B + m_s g\frac{e\hbar}{m_e}B \text{ where} \tag{58}$$

$$n = 2,3,4,\ldots$$

$$\ell = 1,2,\ldots,n-1$$

$$m = -\ell,-\ell+1,\ldots,0,\ldots,+\ell$$

$$m_s = \pm\frac{1}{2}$$

For the electric dipole transition, the selection rules are

$$\begin{aligned}\Delta m &= 0,\pm 1\\ \Delta m_s &= 0\end{aligned} \tag{59}$$

RESONANT LINE SHAPE AND LAMB SHIFT

The spectroscopic linewidth shown in Figure 5 arises from the classical rise-time band-width relationship, and the Lamb Shift is due to conservation of energy and linear momentum and arises from the radiation reaction force between the electron and the photon. It follows from the Poynting power theorem with spherical radiation that the transition probabilities are given by the ratio of power and the energy of the transition [10]. The transition probability in the case of the electric multipole moment is

$$\frac{1}{\tau} = \frac{power}{energy} \tag{60}$$

$$\frac{1}{\tau} = \frac{\left[\frac{2\pi c}{[(2l+1)!!]^2}\left(\frac{l+1}{l}\right)k^{2l+1}\left|Q_{lm}+Q_{lm}^{'}\right|^2\right]}{[\hbar\omega]} = 2\pi\left(\frac{e^2}{h}\right)\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{2\pi}{[(2l+1)!!]^2}\left(\frac{l+1}{l}\right)\left(\frac{3}{l+3}\right)^2(kr_n)^{2l}\omega \tag{61}$$

$$\mathbf{E}(\omega) \propto \int_0^\infty e^{-\alpha t}e^{-i\omega t}dt = \frac{1}{\alpha - i\omega} \tag{62}$$

The relationship between the rise-time and the band-width for exponential decay is

$$\tau\Gamma = \frac{1}{\pi} \tag{63}$$

The energy radiated per unit frequency interval is

$$\frac{dI(\omega)}{d\omega} = I_0\frac{\Gamma}{2\pi}\frac{1}{(\omega-\omega_0-\Delta\omega)^2+(\Gamma/2)^2} \tag{64}$$

LAMB SHIFT

The Lamb Shift of the ${}^2P_{1/2}$ state of the hydrogen atom is due to conservation of linear momentum of the electron, atom, and photon. The electron component is

$$\Delta f = \frac{\Delta\omega}{2\pi} = \frac{E_{h\upsilon}}{h} = 3\frac{(E_{h\upsilon})^2}{h2m_ec^2} = 1052\ MHz \tag{65}$$

where $E_{h\upsilon}$ is

$$E_{h\nu} = 13.6\left(1 - \frac{1}{n^2}\right)\frac{1}{\left|X_{lm}\right|^2_{\ell=1}} - h\Delta f \tag{66}$$

$$E_{h\nu} = 13.6\left(1 - \frac{1}{n^2}\right)\frac{3}{8\pi} - h\Delta f \tag{67}$$

$$h\Delta f <<< 1 \tag{68}$$

Therefore,

$$E_{h\nu} = 13.6\left(1 - \frac{1}{n^2}\right)\frac{3}{8\pi} \tag{69}$$

The atom component is

$$\Delta f = \frac{\Delta\omega}{2\pi} = \frac{E_{h\upsilon}}{h} = \frac{1}{2}\frac{(E_{h\upsilon})^2}{2m_Hc^2} = 6.5\ MHz \tag{70}$$

The sum of the components is

$$\Delta f = 1052\ MHz + 6.5\ MHz = 1058.5\ MHz \tag{71}$$

The experimental Lamb Shift is 1058 MHz.

INSTABILITY OF EXCITED STATES

For the excited energy states of the hydrogen atom, σ_{photon}, the two dimensional surface charge due to the "trapped photons" at the electron orbitsphere, given by Eq. (46) and Eq. (47) is

$$\sigma_{photon} = \frac{e}{4\pi(r_n)^2}\left[Y_0^0(\theta,\phi) - \frac{1}{n}\left[Y_0^0(\theta,\phi) + Re\left\{Y_\ell^m(\theta,\phi)\left[1 + e^{i\omega_n t}\right]\right\}\right]\right]\delta(r - r_n) \tag{72}$$

where $n = 2,3,4,\ldots,$. Whereas, $\sigma_{electron}$, the two dimensional surface charge of the electron orbitsphere given by Eq. (26) is

$$\sigma_{electron} = \frac{-e}{4\pi(r_n)^2}\left[Y_0^0(\theta,\phi) + Re\left\{Y_\ell^m(\theta,\phi)\left[1 + e^{i\omega_n t}\right]\right\}\right]\delta(r - r_n) \tag{73}$$

The superposition of σ_{photon} (Eq. (72)) and $\sigma_{electron}$ is equivalent to the sum of a radial electric dipole represented by a doublet function and a radial electric monopole represented by a delta function.

$$\sigma_{photon} + \sigma_{electron} = \frac{e}{4\pi (r_n)^2}$$

$$\left[Y_0^0(\theta,\phi)\dot{\delta}(r-r_n) - \frac{1}{n}Y_0^0(\theta,\phi)\delta(r-r_n) - \left(1+\frac{1}{n}\right)\left[Re\left\{Y_\ell^m(\theta,\phi)\left[1+e^{i\omega_n t}\right]\right\}\right]\delta(r-r_n)\right] \quad 74)$$

where $n = 2,3,4,...,$. Due to the radial doublet, excited states are radiative since spacetime harmonics of $\frac{\omega_n}{c} = k$ or $\frac{\omega_n}{c}\sqrt{\frac{\varepsilon}{\varepsilon_o}} = k$ do exist for which the spacetime Fourier transform of the current density function is nonzero.

PHOTON EQUATIONS

The time-averaged angular-momentum density, $\mathbf{m}$, of an emitted photon is

$$\mathbf{m} = \frac{1}{8\pi}\mathrm{Re}[\mathbf{r}\times(\mathbf{E}\times\mathbf{B}^*)] = \hbar \quad (75)$$

A linearly polarized photon orbitsphere is generated from two orthogonal great circle field lines shown in Figure 6 rather than two great circle current loops as in the case of the electron spin function. The right-handed circularly polarized photon orbitsphere shown in Figure 7 corresponds to the case wherein the summation of the rotation about each of the x-axis and the y-axis is $\sum_{n=1}^{\frac{\sqrt{2}\pi}{\Delta\alpha}}\Delta\alpha = \sqrt{2}\pi$, and the mirror image left-handed circularly polarized photon orbitsphere corresponds to the case wherein the summation of the rotation about each of the x-axis and the y-axis is $\sum_{n=1}^{\frac{\sqrt{2}\pi}{|\Delta\alpha'|}}|\Delta\alpha'| = \sqrt{2}\pi$.

Nested Set of Great Circle Field Lines Generates the Photon Function

H Field:

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} cos(\Delta\alpha) & -sin^2(\Delta\alpha) & -sin(\Delta\alpha)cos(\Delta\alpha) \\ 0 & cos(\Delta\alpha) & -sin(\Delta\alpha) \\ sin(\Delta\alpha) & cos(\Delta\alpha)sin(\Delta\alpha) & cos^2(\Delta\alpha) \end{bmatrix} \begin{bmatrix} x_1' \\ y_1' \\ z_1' \end{bmatrix} \quad (76)$$

and $\Delta\alpha' = -\Delta\alpha$ replaces $\Delta\alpha$ for $\sum_{n=1}^{\frac{\sqrt{2}\pi}{\Delta\alpha}}\Delta\alpha = \sqrt{2}\pi$; $\sum_{n=1}^{\frac{\sqrt{2}\pi}{|\Delta\alpha'|}}|\Delta\alpha'| = \sqrt{2}\pi$

E Field:

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} cos(\Delta\alpha) & -sin^2(\Delta\alpha) & -sin(\Delta\alpha)cos(\Delta\alpha) \\ 0 & cos(\Delta\alpha) & -sin(\Delta\alpha) \\ sin(\Delta\alpha) & cos(\Delta\alpha)sin(\Delta\alpha) & cos^2(\Delta\alpha) \end{bmatrix} \begin{bmatrix} x'_2 \\ y'_2 \\ z'_2 \end{bmatrix} \tag{77}$$

and $\Delta\alpha' = -\Delta\alpha$ replaces $\Delta\alpha$ for $\sum_{n=1}^{\frac{\sqrt{2}\pi}{\Delta\alpha}} \Delta\alpha = \sqrt{2}\pi$; $\sum_{n=1}^{\frac{\sqrt{2}\pi}{|\Delta\alpha'|}} |\Delta\alpha'| = \sqrt{2}\pi$

The field lines in the lab frame follow from the relativistic invariance of charge as given by Purcell [11]. The relationship between the relativistic velocity and the electric field of a moving charge shown schematically in Figure 8. From Eqs. (76-77) with $\sum_{n=1}^{\frac{\sqrt{2}\pi}{\Delta\alpha}} \Delta\alpha = \sqrt{2}\pi$, the photon equation in the lab frame of a right-handed circularly polarized photon orbitsphere is

$$\mathbf{E} = \mathbf{E}_0 [\mathbf{x} + i\mathbf{y}] e^{-jk_z z} e^{-j\omega t} \tag{78}$$

$$\mathbf{H} = \left(\frac{\mathbf{E}_0}{\eta}\right)[\mathbf{y} - i\mathbf{x}] e^{-jk_z z} e^{-j\omega t} = \mathbf{E}_0 \sqrt{\frac{\varepsilon}{\mu}} [\mathbf{y} - i\mathbf{x}] e^{-jk_z z} e^{-j\omega t} \tag{79}$$

with a wavelength of

$$\lambda = 2\pi \frac{c}{\omega} \tag{80}$$

The relationship between the photon orbitsphere radius and wavelength is

$$2\pi r_0 = \lambda_0 \tag{81}$$

The electric field lines of a right-handed circularly polarized photon orbitsphere as seen along the axis of propagation in the lab inertial reference frame as it passes a fixed point is shown in Figure 9.

Spherical Wave

Photons superimpose, and the amplitude due to N photons is

$$\mathbf{E}_{total} = \sum_{n=1}^{N} \frac{e^{-ik_r|\mathrm{r}-\mathrm{r}'|}}{4\pi|\mathbf{r}-\mathbf{r'}|} f(\theta,\phi) \tag{82}$$

In the far field, the emitted wave is a spherical wave

$$\mathbf{E}_{total} = E_o \frac{e^{-ikr}}{r} \tag{83}$$

The Green Function is given as the solution of the wave equation. Thus, the superposition of photons gives the classical result. As r goes to infinity, the spherical wave becomes a plane wave. The double slit interference pattern is predicted. From the equation of a photon, the wave-particle duality arises naturally. The energy is always given by Planck's equation; yet, an interference pattern is observed when photons add over time or space.

EQUATIONS OF THE FREE ELECTRON

Charge Density Function

The radius of an electron orbitsphere increases with the absorption of electromagnetic energy [12]. With the absorption of a photon of energy exactly equal to the ionization energy, the electron becomes ionized and is a plane wave (spherical wave in the limit) with the de Broglie wavelength. The ionized electron traveling at constant velocity is nonradiative and is a two dimensional surface having a total charge of e and a total mass of m_e. The solution of the boundary value problem of the free electron is given by the projection of the orbitsphere into a plane that linearly propagates along an axis perpendicular to the plane where the velocity of the plane and the orbitsphere is given by

$$v = \frac{\hbar}{m_e \rho_0} \tag{84}$$

and the radius of the orbitsphere in spherical coordinates is equal to the radius of the free electron in cylindrical coordinates ($\rho_0 = r_0$). The mass density function of a free electron shown in Figure 10 is a two dimensional disk having the mass density distribution in the xy(ρ)-plane

$$\rho_m(\rho, \phi, z) = \frac{m_e}{\frac{2}{3}\pi \rho_0^{\,3}} \pi\left(\frac{\rho}{2\rho_o}\right) \sqrt{\rho_0^{\,2} - \rho^2}\, \delta(z) \tag{85}$$

and charge-density distribution, $\rho_e(\rho, \phi, z)$, in the xy-plane given by replacing m_e with e. The charge density distribution of the free electron has recently been confirmed experimentally [13-14]. Researchers working at the Japanese National Laboratory for High Energy Physics (KEK) demonstrated that the charge of the free electron increases toward the particle's core and is symmetrical as a function of ϕ. In addition, the wave-particle duality arises naturally, and the result is consistent with scattering experiments from helium and the double split experiment [1].

Current Density Function

Consider an electron initially bound as an orbitsphere of radius $r = r_n = r_o$ ionized from a hydrogen atom with the magnitude of the angular velocity of the orbitsphere is given by

$$\omega = \frac{\hbar}{m_e r^2} \tag{86}$$

The current-density function of the free electron propagating with velocity v_z along the z-axis in the inertial frame of the proton is given by the vector projection of the current into xy-plane as the radius increases from $r = r_o$ to $r = \infty$. The current-density function of the free electron, is

$$\mathbf{J}(\rho,\phi,z,t) = \left[\pi\left(\frac{\rho}{2\rho_o}\right) \frac{e}{\frac{4}{3}\pi\rho_0^{\,3}} \frac{\hbar}{m_e\sqrt{\rho_0^{\,2}-\rho^2}} \mathrm{i}_\phi \right] \tag{87}$$

where $\rho_o = r_o$. The angular momentum, $\mathbf{L}$, is given by

$$\mathbf{L}\mathbf{i}_z = m_e r^2 \omega \tag{88}$$

Substitution of m_e for e in Eq. (87) followed by substitution into Eq. (88) gives the angular momentum density function, $\mathbf{L}$

$$\mathbf{L}\mathbf{i}_z = \pi\left(\frac{\rho}{2\rho_o}\right) \frac{m_e}{\frac{4}{3}\pi\rho_0^{\,3}} \frac{\hbar}{m_e\sqrt{\rho_0^{\,2}-\rho^2}} \rho^2 \tag{89}$$

The total angular momentum of the free electron is given by integration over the two dimensional disk having the angular momentum density given by Eq. (89).

$$\mathbf{L}\mathbf{i}_z = \int_0^{2\pi}\int_0^{\rho_0} \pi\left(\frac{\rho}{2\rho_o}\right) \frac{m_e}{\frac{4}{3}\pi\rho_0^{\,3}} \frac{\hbar}{m_e\sqrt{\rho_0^{\,2}-\rho^2}} \rho^2 \rho\, d\rho\, d\phi = \hbar \tag{90}$$

The four dimensional spacetime current-density function of the free electron that propagates along the z-axis with velocity given by Eq. (84) corresponding to $r = r_o = \rho_0$ is given by substitution of Eq. (84) into Eq. (88).

$$\mathbf{J}(\rho,\phi,z,t) = \left[\pi\left(\frac{\rho}{2\rho_o}\right) \frac{e}{\frac{4}{3}\pi\rho_0^{\,3}} \frac{\hbar}{m_e\sqrt{\rho_0^{\,2}-\rho^2}} \mathbf{i}_\phi \right] + \frac{e\hbar}{m_e\rho_0}\delta\left(z - \frac{\hbar}{m_e\rho_0}t\right)\mathbf{i}_z \tag{91}$$

The spacetime Fourier Transform of is

$$\frac{e}{\frac{4}{3}\pi\rho_0^{\,3}} \frac{\hbar}{m_e} \mathrm{sinc}(2\pi s\rho_o) + 2\pi e \frac{\hbar}{m_e\rho_0} \delta(\omega - \mathbf{k}_z \bullet \mathbf{v}_z) \tag{92}$$

Spacetime harmonics of $\frac{\omega_n}{c} = k$ or $\frac{\omega_n}{c}\sqrt{\frac{\varepsilon}{\varepsilon_o}} = k$ do not exist. Radiation due to charge motion does not occur in any medium when this boundary condition is met. Thus, no Fourier components that are synchronous with light velocity with the propagation constant $|\mathbf{k}_z| = \frac{\omega}{c}$ exist. Radiation due to charge motion does not occur when this boundary condition is met. It follows from Eq. (84) and the relationship $2\pi\rho_o = \lambda_o$ that the wavelength of the free electron is the de Broglie wavelength.

$$\lambda_o = \frac{h}{m_e v_z} = 2\pi\rho_o \tag{93}$$

In the presence of a z-axis applied magnetic field, the free electron precesses. The time average vector projection of the total angular momentum of the fee electron onto an axis **S** that rotates about the z-axis is $\pm\sqrt{\frac{3}{4}}\hbar$, and the time averaged projection of the angular momentum onto the axis of the applied magnetic field is $\pm\frac{\hbar}{2}$. Magnetic flux is linked by the electron in units of the magnetic flux quantum with conservation of angular momentum as in the case of the orbitsphere as the projection of the angular momentum along the magnetic field axis of $\frac{\hbar}{2}$ reverses direction. The energy, ΔE_{mag}^{spin}, of the spin flip transition corresponding to the $m_S = \frac{1}{2}$ quantum number is given by Eq. (22).

$$\Delta E_{mag}^{spin} = g\mu_B B \tag{94}$$

The Stern-Gerlach experiment implies a magnetic moment of one Bohr magneton and an associated angular momentum quantum number of 1/2. Historically, this quantum number is called the spin quantum number, m_S, and that designation is maintained.

TWO ELECTRON ATOMS

Two electron atoms may be solved from a central force balance equation with the nonradiation condition. The force balance equation is

$$\frac{m_e}{4\pi r_2^2}\frac{v_2^2}{r_2} = \frac{e}{4\pi r_2^2}\frac{(Z-1)e}{4\pi\varepsilon_o r_2^2} + \frac{1}{4\pi r_2^2}\frac{\hbar^2}{Zm_e r_2^3}\sqrt{s(s+1)} \tag{95}$$

which gives the radius of both electrons as

$$r_2 = r_1 = a_0\left(\frac{1}{Z-1} - \frac{\sqrt{s(s+1)}}{Z(Z-1)}\right); s = \frac{1}{2} \tag{96}$$

Ionization Energies Calculated using the Poynting Power Theorem

For helium, which has no electric field beyond r_1

$$Ionization\ Energy(He) = -E(electric) + E(magnetic) \tag{97}$$

where,

$$E(electric) = -\frac{(Z-1)e^2}{8\pi\varepsilon_o r_1} \tag{98}$$

$$E(magnetic) = \frac{2\pi\mu_0 e^2\hbar^2}{m_e^2 r_1^3} \tag{99}$$

For $3 \le Z$

$$Ionization\ Energy = -Electric\ Energy - \frac{1}{Z} Magnetic\ Energy \tag{100}$$

The energies of several two-electron atoms are given in Table 1.

ELASTIC ELECTRON SCATTERING FROM HELIUM ATOMS

The aperture distribution function, $a(\rho,\phi,z)$, for the elastic scattering of an incident electron plane wave represented by $\pi(z)$ by a helium atom represented by

$$\frac{2}{4\pi(0.567a_o)^2}[\delta(r-0.567a_o)] \tag{101}$$

is given by the convolution of the plane wave with the helium atom function:

$$a(\rho,\phi,z) = \pi(z)\otimes\frac{2}{4\pi(0.567a_o)^2}[\delta(r-0.567a_o)] \tag{102}$$

The aperture function is

$$a(\rho,\phi,z) = \frac{2}{4\pi(0.567a_o)^2}\sqrt{(0.567a_o)^2 - z^2}\,\delta\left(r-\sqrt{(0.567a_o)^2 - z^2}\right) \tag{103}$$

Far Field Scattering (circular symmetry)

Applying Huygens' principle to a disturbance caused by the plane wave electron over the helium atom as an aperture gives the amplitude of the far field or Fraunhofer diffraction pattern $F(s)$ as the Fourier Transform of the aperture distribution. The intensity I_1^{ed} is the square of the amplitude.

$$F(s) = \frac{2}{4\pi(0.567a_o)^2}2\pi\int_0^\infty\int_{-\infty}^\infty\sqrt{(0.567a_o)^2 - z^2}\,\delta\left(\rho-\sqrt{(0.567a_o)^2 - z^2}\right)J_o(s\rho)e^{-iwz}\rho\, d\rho\, dz \tag{104}$$

$$I_1^{ed} = F(s)^2$$

$$= I_e\left\{\begin{bmatrix}\left[\frac{2\pi}{(z_ow)^2+(z_os)^2}\right]^{\frac{1}{2}}\\ \left\{2\left[\frac{z_os}{(z_ow)^2+(z_os)^2}\right]J_{3/2}\left[((z_ow)^2+(z_os)^2)^{1/2}\right]\right.\\ \left.-\left[\frac{z_os}{(z_ow)^2+(z_os)^2}\right]^2 J_{5/2}\left[((z_ow)^2+(z_os)^2)^{1/2}\right]\right\}\end{bmatrix}\right\}^2 \tag{105}$$

$$s = \frac{4\pi}{\lambda}\sin\frac{\theta}{2};\ w = 0\ (units\ of\ \mathring{A}^{-1}) \qquad (106)$$

The experimental results of Bromberg [17], the extrapolated experimental data of Hughes [17], the small angle data of Geiger [18] and the semiexperimental results of Lassettre [17] for the elastic differential cross section for the elastic scattering of electrons by helium atoms is shown graphically in Figure 11. The elastic differential cross section as a function of angle numerically calculated by Khare [17] using the first Born approximation and first-order exchange approximation also appear in Figure 11. These results which are based on a quantum mechanical model are compared with experimentation [17-18]. The closed form function (Eqs. (105) and (106)) for the elastic differential cross section for the elastic scattering of electrons by helium atoms is shown graphically in Figure 12. The scattering amplitude function, $F(s)$ (Eq. (104), is shown as an insert. It is apparent from Figure 11 that the quantum mechanical calculations fail completely at predicting the experimental results at small scattering angles; whereas, there is good agreement between Eq. (105) and the experimental results.

THE NATURE OF THE CHEMICAL BOND OF HYDROGEN

The hydrogen molecule charge and current density functions, bond distance and energies are solved from the Laplacian in ellipsoidal coordinates with the constraint of nonradiation.

$$(\eta - \zeta)R_\xi \frac{\partial}{\partial \xi}(R_\xi \frac{\partial \phi}{\partial \xi}) + (\zeta - \xi)R_\eta \frac{\partial}{\partial \eta}(R_\eta \frac{\partial \phi}{\partial \eta}) + (\xi - \eta)R_\zeta \frac{\partial}{\partial \zeta}(R_\zeta \frac{\partial \phi}{\partial \zeta}) = 0 \qquad (107)$$

The force balance equation for the hydrogen molecule is

$$\frac{\hbar^2}{m_e a^2 b^2} 2ab^2 X = \frac{e^2}{4\pi\varepsilon_o} X + \frac{\hbar^2}{2m_e a^2 b^2} 2ab^2 X \qquad (108)$$

where

$$X = \frac{1}{\sqrt{\xi + a^2}} \frac{1}{\xi + b^2} \frac{1}{c} \sqrt{\frac{\xi^2 - 1}{\xi^2 - \eta}} \qquad (109)$$

Eq. (108) has the parametric solution

$$r(t) = \mathbf{i}a\cos\omega t + \mathbf{j}b\sin\omega t \qquad (110)$$

when the semimajor axis, a, is

$$a = a_o \qquad (111)$$

The internuclear distance, $2c'$, which is the distance between the foci is

$$2c' = \sqrt{2}a_o \qquad (112)$$

The experimental internuclear distance is $\sqrt{2}a_o$. The semiminor axis is

$$b = \frac{1}{\sqrt{2}} a_o \tag{113}$$

The eccentricity, e, is

$$e = \frac{1}{\sqrt{2}} \tag{114}$$

The Energies of the Hydrogen Molecule

The potential energy of the two electrons in the central field of the protons at the foci is

$$V_e = \frac{-2e^2}{8\pi\varepsilon_o \sqrt{a^2 - b^2}} ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = -67.813\, eV \tag{115}$$

The potential energy of the two protons is

$$V_p = \frac{e^2}{8\pi\varepsilon_o \sqrt{a^2 - b^2}} = 19.23\, eV \tag{116}$$

The kinetic energy of the electrons is

$$T = \frac{\hbar^2}{2m_e a\sqrt{a^2 - b^2}} ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = 33.906\, eV \tag{117}$$

The energy, V_m, of the magnetic force between the electrons is

$$V_m = \frac{-\hbar^2}{4m_e a\sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = -16.9533\ eV \tag{118}$$

The total energy is

$$E_T = V_e + T + V_m + V_p \tag{119}$$

$$E_T = -13.6\, eV \left[\left(2\sqrt{2} - \sqrt{2} + \frac{\sqrt{2}}{2} \right) ln \frac{\sqrt{2}+1}{\sqrt{2}-1} - \sqrt{2} \right] = -31.63\, eV \tag{120}$$

The energy of two hydrogen atoms is

$$E(2H[a_H]) = -27.21\, eV \tag{121}$$

The bond dissociation energy, E_D, is the difference between the total energy of the corresponding hydrogen atoms (Eq. (121)) and E_T (Eq. (120)).

$$E_D = E(2H[a_H]) - E_T = 4.43\, eV \tag{122}$$

The experimental energy determined by calorimetry is

$$E_D = 4.45\ eV \tag{123}$$

COSMOLOGICAL THEORY BASED ON MAXWELL'S EQUATIONS

Maxwell's equations and special relativity are based on the law of propagation of a electromagnetic wave front in the form

$$\frac{1}{c^2}\left(\frac{\partial\omega}{\partial t}\right)^2-\left[\left(\frac{\partial\omega}{\partial x}\right)^2+\left(\frac{\partial\omega}{\partial y}\right)^2+\left(\frac{\partial\omega}{\partial z}\right)^2\right]=0 \tag{124}$$

For any kind of wave advancing with limiting velocity and capable of transmitting signals, the equation of front propagation is the same as the equation for the front of a light wave. Thus, the equation

$$\frac{1}{c^2}\left(\frac{\partial\omega}{\partial t}\right)^2-(grad\,\omega)^2=0 \tag{125}$$

acquires a general character; it is more general than Maxwell's equations from which Maxwell originally derived it.

A discovery of the present work is that the classical wave equation governs: (1) the motion of bound electrons, (2) the propagation of any form of energy, (3) measurements between inertial frames of reference such as time, mass, momentum, and length (Minkowski tensor), (4) fundamental particle production and the conversion of matter to energy, (5) a relativistic correction of spacetime due to particle production or annihilation (Schwarzschild metric), (6) the expansion and contraction of the Universe, (7) the basis of the relationship between Maxwell's equations, Planck's equation, the de Broglie equation, Newton's laws, and special, and general relativity.

The relationship between the time interval between ticks t of a clock in motion with velocity v relative to an observer and the time interval t_0 between ticks on a clock at rest relative to an observer is [19]

$$(ct)^2=(ct_0)^2+(vt)^2 \tag{126}$$

Thus, the time dilation relationship based on the constant maximum speed of light c in any inertial frame is

$$t=\frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}} \tag{127}$$

The metric $g_{\mu\nu}$ for Euclidean space is the Minkowski tensor $\eta_{\mu\nu}$. In this case, the separation of proper time between two events x^μ and $x^\mu+dx^\mu$ is $d\tau^2=-\eta_{\mu\nu}dx^\mu dx^\nu$.

THE EQUIVALENCE OF THE GRAVITATIONAL MASS AND THE INERTIAL MASS

The equivalence of the gravitational mass and the inertial mass, $m_g/m_i=\text{universal constant}$, which is predicted by Newton's law of mechanics and gravitation is experimentally confirmed to less $1\,X\,10^{-11}$ [20]. In physics, the discovery of a universal constant often leads to the development of an entirely new theory. From the universal constancy of the velocity of light, c the special theory of relativity was derived; and from Planck's constant h, the quantum theory was deduced. Therefore, the universal constant m_g / m_i should be the key to the gravitational problem. The energy equation of Newtonian gravitation is

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r} = \frac{1}{2}mv_0^2 - \frac{GMm}{r_0} = \text{constant} \tag{128}$$

Since h, the angular momentum per unit mass, is

$$h = L/m = |\mathbf{r} \times \mathbf{v}| = r_0 v_0 \sin\phi$$

the eccentricity e may be written as

$$e = [1 + \left(v_0^2 - \frac{2GM}{r_0}\right)\frac{r_0^2 v_0^2 \sin^2\phi}{G^2 M^2}]^{1/2} \tag{129}$$

where m is the inertial mass of a particle, v_0 is the speed of the particle, r_0 is the distance of the particle from a massive object, ϕ is the angle between the direction of motion of the particle and the radius vector from the object, and M is the total mass of the object (including a particle). The eccentricity e given by Newton's differential equations of motion in the case of the central field permits the classification of the orbits according to the total energy E [21] (column 1) and the orbital velocity squared, v_0^2, relative to the gravitational velocity squared, $\frac{2GM}{r_0}$ [21] (column 2):

$E < 0$	$v_0^2 < \frac{2GM}{r_0}$	$e < 1$	ellipse
$E < 0$	$v_0^2 < \frac{2GM}{r_0}$	$e = 0$	circle (special case of ellipse)
$E = 0$	$v_0^2 = \frac{2GM}{r_0}$	$e = 1$	parabolic orbit
$E > 0$	$v_0^2 > \frac{2GM}{r_0}$	$e > 1$	hyperbolic orbit

(130)

CONTINUITY CONDITIONS FOR THE PRODUCTION OF A PARTICLE FROM A PHOTON TRAVELING AT LIGHT SPEED

A photon traveling at the speed of light gives rise to a particle with an initial radius equal to its Compton wavelength bar.

$$r = \lambda\!\!\!^{-}_C = \frac{\hbar}{mc} = r_\alpha^* \tag{131}$$

The particle must have an orbital velocity equal to Newtonian gravitational escape velocity v_g of the antiparticle.

$$v_g = \sqrt{\frac{2Gm}{r}} = \sqrt{\frac{2Gm_0}{\lambda\!\!\!^{-}_C}} \tag{132}$$

The eccentricity is one. The orbital energy is zero. The particle production trajectory is a parabola relative to the center of mass of the antiparticle.

A Gravitational Field as a Front Equivalent to Light Wave Front

The particle with a finite gravitational mass gives rise to a gravitational field that travels out as a front equivalent to a light wave front. The form of the outgoing gravitational field front traveling at the speed of light is $f\left(t-\frac{r}{c}\right)$, and $d\tau^2$ is given by

$$d\tau^2 = f(r)dt^2 - \frac{1}{c^2}\left[f(r)^{-1}dr^2 + r^2 d\theta^2 + r^2 sin^2\,\theta d\phi^2\right] \tag{133}$$

The speed of light as a constant maximum as well as phase matching and continuity conditions of the electromagnetic and gravitational waves require the following form of the squared displacements:

$$(c\tau)^2 + (v_g t)^2 = (ct)^2 \tag{134}$$

$$f(r) = \left(1-\left(\frac{v_g}{c}\right)^2\right) \tag{135}$$

In order that the wave front velocity does not exceed c in any frame, spacetime must undergo time dilation and length contraction due to the particle production event. *The derivation and result of spacetime time dilation is analogous to the derivation and result of special relativistic time dilation* wherein the relative velocity of two inertial frames replaces the gravitational velocity.

The general form of the metric due to the relativistic effect on spacetime due to mass m_0 with v_g given by Eq. (132) is

$$d\tau^2 = \left(1-\left(\frac{v_g}{c}\right)^2\right)dt^2 - \frac{1}{c^2}\left[\left(1-\left(\frac{v_g}{c}\right)^2\right)^{-1}dr^2 + r^2 d\theta^2 + r^2 sin^2\,\theta d\phi^2\right] \tag{136}$$

The gravitational radius, r_g, of each orbitsphere of the particle production event, each of mass m_0, and the corresponding general form of the metric are respectively

$$r_g = \frac{2Gm}{c^2}, \tag{137}$$

$$d\tau^2 = \left(1-\frac{r_g}{r}\right)dt^2 - \frac{1}{c^2}\left[\left(1-\frac{r_g}{r}\right)^{-1}dr^2 + r^2 d\theta^2 + r^2 sin^2\,\theta d\phi^2\right] \tag{138}$$

The metric $g_{\mu\nu}$ for non-Euclidean space due to the relativistic effect on spacetime due to mass m_0 is

$$g_{\mu\nu} = \begin{pmatrix} -\left(1-\frac{2Gm_0}{c^2 r}\right) & 0 & 0 & 0 \\ 0 & \frac{1}{c^2}\left(1-\frac{2Gm_0}{c^2 r}\right)^{-1} & 0 & 0 \\ 0 & 0 & \frac{1}{c^2} r^2 & 0 \\ 0 & 0 & 0 & \frac{1}{c^2} r^2 \sin^2\theta \end{pmatrix} \tag{139}$$

Masses and their effects on spacetime *superimpose*. The separation of proper time between two events x^μ and $x^\mu + dx^\mu$ is

$$d\tau^2 = \left(1-\frac{2GM}{c^2 r}\right)dt^2 - \frac{1}{c^2}\left[\left(1-\frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right] \tag{140}$$

The *Schwarzschild metric (Eq. (140)) gives the relationship whereby matter causes relativistic corrections to spacetime that determines the curvature of spacetime and is the origin of gravity.*

Particle Production Continuity Conditions from Maxwell's Equations, and the Schwarzschild Metric

The photon to particle event requires a transition state that is continuous wherein the velocity of a transition state orbitsphere is the speed of light. The radius, r, is the Compton wavelength bar, $\lambda\!\!\!^{-}_C$, given by Eq. (131). At production, the Planck equation energy, the electric potential energy, and the magnetic energy are equal to $m_0 c^2$.

The Schwarzschild metric gives the relationship whereby matter causes relativistic corrections to spacetime that determines the masses of fundamental particles. Substitution of $r = \lambda\!\!\!^{-}_C$; $dr = 0$; $d\theta = 0$; $\sin^2\theta = 1$ into the Schwarzschild metric gives

$$d\tau = dt\left(1-\frac{2Gm_0}{c^2 r^*_\alpha} - \frac{v^2}{c^2}\right)^{\frac{1}{2}} \tag{141}$$

with $v^2 = c^2$, the relationship between the proper time and the coordinate time is

$$\tau = ti\sqrt{\frac{2GM}{c^2 r^*_\alpha}} = ti\sqrt{\frac{2GM}{c^2 \lambda\!\!\!^{-}_c}} = ti\frac{v_g}{c} \tag{142}$$

When the orbitsphere velocity is the speed of light, continuity conditions based on the constant maximum speed of light given by Maxwell's equations are mass energy = Planck equation energy = electric potential energy = magnetic energy = mass/spacetime metric energy. Therefore,

$$m_o c^2 = \hbar\omega^* = V = E_{mag} = E_{spacetime} \tag{143}$$

$$m_0 c^2 = \hbar\omega^* = \frac{\hbar^2}{m_0 \lambda\!\!\!^{-2}_C} = \alpha^{-1}\frac{e^2}{4\pi\varepsilon_0 \lambda\!\!\!^{-}_C} = \alpha^{-1}\frac{\pi\mu_0 e^2\hbar^2}{(2\pi m_0)^2 \lambda\!\!\!^{-3}_C} = \frac{\alpha h}{1\,sec}\sqrt{\frac{\lambda\!\!\!^{-}_C c^2}{2Gm}} \tag{144}$$

The continuity conditions based on the constant maximum speed of light given by the Schwarzschild metric are:

$$\frac{\text{proper time}}{\text{coordinate time}} = \frac{\text{gravitational wave condition}}{\text{electromagnetic wave condition}} = \frac{\text{gravitational mass phase matching}}{\text{charge/inertial mass phase matching}} \tag{145}$$

$$\frac{\text{proper time}}{\text{coordinate time}} = i\frac{\sqrt{\frac{2Gm}{c^2\lambda\!\!\!^{-}_C}}}{\alpha} = i\frac{v_g}{\alpha c}$$

MASSES OF FUNDAMENTAL PARTICLES

Each of the Planck equation energy, electric energy, and magnetic energy corresponds to a particle given by the relationship between the proper time and the coordinate time. The electron and down-down-up neutron correspond to the Planck equation energy. The muon and strange-strange-charmed neutron correspond to the electric energy. The tau and bottom-bottom-top neutron correspond to the magnetic energy. The particle must possess the escape velocity v_g relative to the antiparticle where $v_g < c$. According to Newton's law of gravitation, the eccentricity is one and the particle production trajectory is a parabola relative to the center of mass of the antiparticle.

The Electron-Antielectron Lepton Pair

A clock is defined in terms of a self consistent system of units used to measure the particle mass. The proper time of the particle is equated with the coordinate time according to the Schwarzschild metric corresponding to light speed. The special relativistic condition corresponding to the Planck energy gives the mass of the electron.

$$2\pi\frac{\hbar}{mc^2} = sec\sqrt{\frac{2Gm^2}{c\alpha^2\hbar}} \tag{146}$$

$$m_e = \left(\frac{h\alpha}{\sec c^2}\right)^{\frac{1}{2}}\left(\frac{c\hbar}{2G}\right)^{\frac{1}{4}} = 9.1097\,X\,10^{-31}\ kg \tag{147}$$

$$m_e = 9.1097\,X\,10^{-31}\ kg - 18\ eV(\nu_e) = 9.1094\,X\,10^{-31}\ kg \tag{148}$$

$$m_{e\ \text{experimental}} = 9.1095\,X\,10^{-31}\ kg \tag{149}$$

Down-Down-Up Neutron (DDU)

The corresponding equation for production of the neutron is

$$2\pi \frac{2\pi\hbar}{\frac{m_N}{3}\left[\frac{1}{2\pi}-\frac{\alpha}{2\pi}\right]c^2} = sec\sqrt{\frac{2G\left[\frac{m_N}{3}\left[\frac{1}{2\pi}-\frac{\alpha}{2\pi}\right]\right]^2}{3c(2\pi)^2\hbar}} \tag{150}$$

$$m_{N\ calculated} = (3)(2\pi)\left(\frac{1}{1-\alpha}\right)\left(\frac{2\pi h}{\sec c^2}\right)^{\frac{1}{2}}\left(\frac{2\pi(3)ch}{2G}\right)^{\frac{1}{4}} = 1.6744X10^{-27}\ kg \tag{151}$$

$$m_{N\mathrm{experimental}} = 1.6749X10^{-27}\ kg \tag{152}$$

GRAVITATIONAL POTENTIAL ENERGY

The gravitational radius, α_G or r_G, of an orbitsphere of mass m_0 is defined as

$$\alpha_G = r_G = \frac{Gm_0}{c^2} \tag{153}$$

When $r_G = r_\alpha^* = \lambda\!\!\!^{-}_C$, the gravitational potential energy equals m_0c^2

$$r_G = \frac{Gm_0}{c^2} = \lambda\!\!\!^{-}_c = \frac{\hbar}{m_0c}, \tag{154}$$

$$E_{grav} = \frac{Gm_0^2}{r} = \frac{Gm_0^2}{\lambda\!\!\!^{-}_C} = \frac{Gm_0^2}{r_\alpha^*} = \hbar\omega^* = m_0c^2 \tag{155}$$

The mass m_0 is the Planck mass, m_u,

$$m_u = m_0 = \sqrt{\frac{\hbar c}{G}} \tag{156}$$

The corresponding gravitational velocity, v_G, is defined as

$$v_G = \sqrt{\frac{Gm_0}{r}} = \sqrt{\frac{Gm_0}{\lambda\!\!\!^{-}_C}} = \sqrt{\frac{Gm_u}{\lambda\!\!\!^{-}_C}} \tag{157}$$

Relationship of the Equivalent Planck Mass Particle Production Energies

For the Plank mass particle, the relationships corresponding to Eq. (144) are: (mass energy = Planck equation energy = electric potential energy = magnetic energy = gravitational potential energy = mass/spacetime metric energy)

$$m_oc^2 = \hbar\omega^* = V = E_{mag} = E_{grav} = E_{spacetime} \tag{158}$$

$$m_0c^2 = \hbar\omega^* = \frac{\hbar^2}{m_0\lambda\!\!\!^{-}_C{}^2} = \alpha^{-1}\frac{e^2}{4\pi\varepsilon_0\lambda\!\!\!^{-}_C} = \alpha^{-1}\frac{\pi\mu_0e^2\hbar^2}{(2\pi m_0)^2\lambda\!\!\!^{-}_C{}^3} = \alpha^{-1}\frac{\mu_0e^2c^2}{2h}\sqrt{\frac{Gm_0}{\lambda\!\!\!^{-}_C}}\sqrt{\frac{\hbar c}{G}} = \frac{\alpha h}{1\sec}\sqrt{\frac{\lambda\!\!\!^{-}_Cc^2}{2Gm}} \tag{159}$$

These equivalent energies give the particle masses in terms of the gravitational velocity, v_G and the Planck mass, m_u

$$m_0 = \alpha^{-1} \frac{\mu_0 e^2 c}{2h} \frac{\sqrt{\frac{Gm_0}{\bar{\lambda}_C}}}{c} m_u = \alpha^{-1} \frac{\mu_0 e^2 c}{2h} \sqrt{\frac{Gm_0}{c^2 \bar{\lambda}_C}} m_u = \alpha^{-1} \frac{\mu_0 e^2 c}{2h} \frac{v_G}{c} m_u = \frac{v_G}{c} m_u \qquad (160)$$

Planck Mass Particles

A pair of particles each of the Planck mass corresponding to the gravitational potential energy is not observed since the velocity of each transition state orbitsphere is the gravitational velocity v_G that in this case is the speed of light; whereas, the Newtonian gravitational escape velocity v_g is $\sqrt{2}$ the speed of light. In this case, an electromagnetic wave of mass energy equivalent to the Planck mass travels in a circular orbit about the center of mass of another electromagnetic wave of mass energy equivalent to the Planck mass wherein the eccentricity is equal to zero and the escape velocity can never be reached. The Planck mass is a "measuring stick." The extraordinarily high Planck mass ($\sqrt{\frac{\hbar c}{G}} = 2.18\ X\ 10^{-8}\ kg$) is the unobtainable mass bound imposed by the angular momentum and speed of the photon relative to the gravitational constant. It is analogous to the unattainable bound of the speed of light for a particle possessing finite rest mass imposed by the Minkowski tensor.

Astrophysical Implications of Planck Mass Particles

The limiting speed of light eliminates the singularity problem of Einstein's equation that arises as the radius of a blackhole equals the Schwarzschild radius. General relativity with the singularity eliminated resolves the paradox of the infinite propagation velocity required for the gravitational force in order to explain why the angular momentum of objects orbiting a gravitating body does not increase due to the finite propagation delay of the gravitational force according to special relativity [22]. When the gravitational potential energy density of a massive body such as a blackhole equals that of a particle having the Planck mass, the matter may transition to photons of the Planck mass. Even light from a blackhole will escape when the decay rate of the trapped matter with the concomitant spacetime expansion is greater than the effects of gravity which oppose this expansion. Gamma-ray bursts are the most energetic phenomenon known that can release an explosion of gamma rays packing 100 times more energy than a supernova explosion [23]. The annihilation of a blackhole may be the source of γ*-ray bursts*. The source may be due to conversion of matter to photons of the Planck mass/energy which may also give rise to cosmic rays which are the most energetic particles known, and their origin is also a mystery [24]. According to the GZK cutoff, the cosmic spectrum cannot extend beyond $5\,X\,10^{19}\ eV$, but AGASA, the world's largest air shower array, has shown that the spectrum is extending beyond $10^{20}\ eV$ without any clear sign of cutoff [25]. Photons each of the Planck mass may be the source of these inexplicably energetic cosmic rays.

RELATIONSHIP OF MATTER TO ENERGY AND SPACETIME EXPANSION

The Schwarzschild metric gives the relationship whereby matter causes relativistic corrections to spacetime. The limiting velocity c results in the contraction of spacetime due to particle production, which is given by $2\pi r_g$ where r_g is the gravitational radius of the particle. This has implications for the expansion of spacetime when matter converts to energy. Q the mass/energy to expansion/contraction quotient of spacetime is given by the ratio of the mass of a particle at production divided by T, the period of production.

$$Q = \frac{m_0}{T} = \frac{m_0}{\frac{2\pi r_g}{c}} = \frac{m_0}{\frac{2\pi \frac{2Gm_0}{c^2}}{c}} = \frac{c^3}{4\pi G} = 3.22\,X\,10^{34}\ \frac{kg}{\sec} \qquad (161)$$

The gravitational equations with the equivalence of the particle production energies (Eq. (144)) permit the conservation of mass/energy ($E = mc^2$) and spacetime ($\frac{c^3}{4\pi G} = 3.22\ X\,10^{34}\ \frac{kg}{\sec}$). With the conversion of $3.22\,X\,10^{34}\ kg$ of matter to energy, spacetime expands by 1 sec. The photon has inertial mass and angular momentum, but due to Maxwell's equations and the implicit special relativity it does not have a gravitational mass.

Cosmological Consequences

The Universe is closed (it is *finite but with no boundary*). It is a 3-sphere Universe-Riemannian three dimensional hyperspace plus time of constant positive curvature at each r-sphere. The *Universe is oscillatory in matter/energy and spacetime* with a finite minimum radius, the gravitational radius. Spacetime expands as mass is released as energy which provides the basis of the atomic, thermodynamic, and cosmological arrows of time. Different regions of space are isothermal even though they are separated by greater distances than that over which light could travel during the time of the expansion of the Universe [26]. Presently, stars and large scale structures exist which are older than the elapsed time of the present expansion as stellar, galaxy, and supercluster evolution occurred during the contraction phase [27–33]. The maximum power radiated by the Universe which occurs at the beginning of the expansion phase is $P_U = \frac{c^5}{4\pi G} = 2.89\,X\,10^{51}\,W$. Observations beyond the beginning of the expansion phase are not possible since the Universe was entirely matter filled.

The Period of Oscillation of the Universe Based on Closed Propagation of Light

Mass/energy is conserved during harmonic expansion and contraction. The gravitational potential energy E_{grav} given by Eq. (155) with $m_0 = m_U$ is equal to $m_U c^2$ when the radius of the Universe r is the gravitational radius r_G. The gravitational velocity v_G (Eq. (157) with $r = r_G$ and $m_0 = m_U$) is the speed of light in a circular orbit wherein the eccentricity is equal to zero and the escape velocity from the Universe can never be reached. The period of the oscillation of the Universe and the period for light to transverse the Universe corresponding to the gravitational radius r_G must be equal. The harmonic oscillation period, T, is

$$T = \frac{2\pi r_G}{c} = \frac{2\pi G m_U}{c^3} = \frac{2\pi G\left(2\ X\ 10^{54}\ kg\right)}{c^3} = 3.10\ X\ 10^{19}\ sec = 9.83\ X\ 10^{11}\ years \tag{162}$$

where the mass of the Universe, m_U, is approximately $2\ X\,10^{54}\ kg$. (The initial mass of the Universe of $2\ X\,10^{54}\ kg$ is based on internal consistency with the size, age, Hubble constant, temperature, density of matter, and power spectrum.) Thus, the observed Universe will expand as mass is released as photons for $4.92\ X\,10^{11}\ years$. At this point in its world line, the Universe will obtain its maximum size and begin to contract.

THE DIFFERENTIAL EQUATION OF THE RADIUS OF THE UNIVERSE

Based on conservation of mass/energy ($E = mc^2$) and spacetime ($\frac{c^3}{4\pi G} = 3.22\ X\,10^{34}\ \frac{kg}{\sec}$). The Universe behaves as a simple harmonic oscillator having a restoring force, $\mathbf{F}$, which is proportional to the radius. The proportionality constant, k, is given in terms of the potential energy, E, gained as the radius decreases from the maximum expansion to the minimum contraction.

$$\frac{E}{\aleph^2} = k \tag{163}$$

Since the gravitational potential energy E_{grav} is equal to $m_U c^2$ when the radius of the Universe r is the gravitational radius r_G

$$F = -k\aleph = -\frac{m_U c^2}{r_G^2}\aleph = -\frac{m_U c^2}{\left(\frac{G m_U}{c^2}\right)^2}\aleph \tag{164}$$

And, the differential equation of the radius of the Universe, is

$$m_U \ddot{\aleph} + \frac{m_U c^2}{r_G^2}\aleph = m_U \ddot{\aleph} + \frac{m_U c^2}{\left(\frac{G m_U}{c^2}\right)^2}\aleph = 0 \tag{165}$$

The *maximum radius of the Universe*, the amplitude, r_o, of the time harmonic variation in the radius of the Universe, is given by the quotient of the total mass of the Universe and Q ((Eq. (161)), the mass/energy to expansion/contraction quotient.

$$r_0 = \frac{m_U}{Q} = \frac{m_U}{\frac{c^3}{4\pi G}} = \frac{2\ X\,10^{54}\ kg}{\frac{c^3}{4\pi G}} = 1.97\ X\,10^{12}\ light\ years \tag{166}$$

The *minimum radius* which corresponds to the gravitational radius, r_g, given by Eq. (137) with $m_0 = m_U$ is

$$r_g = \frac{2Gm_U}{c^2} = 2.96\ X\,10^{27}\ m = 3.12\ X\,10^{11}\ light\ years \tag{167}$$

When the radius of the Universe is the gravitational radius, r_g, the proper time is equal to the coordinate time by Eq. (142), and the gravitational escape velocity v_g of the Universe is the speed of light. The radius of the Universe as a function of time as shown in Figure 13 is

$$\aleph = \left(r_g + \frac{cm_U}{Q}\right) - \frac{cm_U}{Q} cos\left(\frac{2\pi t}{\frac{2\pi r_G}{c}}\right) = \left(\frac{2Gm_U}{c^2} + \frac{cm_U}{\frac{c^3}{4\pi G}}\right) - \frac{cm_U}{\frac{c^3}{4\pi G}} cos\left(\frac{2\pi t}{\frac{2\pi G m_U}{c^3}}\right) \tag{168}$$

The expansion/contraction rate, $\dot{\aleph}$, as shown in Figure 14 is given by time derivative of Eq. (168)

$$\dot{\aleph} = 4\pi c\ X\,10^{-3}\ sin\left(\frac{2\pi t}{\frac{2\pi G m_U}{c^3}}\right)\frac{km}{sec} \tag{169}$$

THE HUBBLE CONSTANT

The *Hubble constant* is given by the ratio of the expansion rate given in units of $\frac{km}{sec}$ divided by the radius of the expansion in Mpc. The radius of expansion is equivalent to the radius of the light sphere with an origin at the time point when the Universe stopped contracting and started to expand.

$$H = \frac{\dot{\aleph}}{t\ Mpc} = \frac{4\pi c\ X\,10^{-3}\ sin\left(\frac{2\pi t}{\frac{2\pi G m_U}{c^3}}\right)\frac{km}{sec}}{t\ Mpc} \tag{170}$$

For $t = 10^{10}\ light\ years = 3.069\ X\,10^{3}\,Mpc$, the Hubble constant, H_0, is

$$H_0 = 78.6\,\frac{km}{sec\cdot Mpc} \tag{171}$$

The experimental value [34] as shown in Figure 15 is

$$H_0 = 80 \pm 17 \frac{km}{sec \cdot Mpc} \tag{172}$$

THE DENSITY OF THE UNIVERSE AS A FUNCTION OF TIME

The density of the Universe as a function of time $\rho_U(t)$ given by the ratio of the mass as a function of time and the volume as a function of time as shown in Figure 16 is

$$\rho_U(t) = \frac{m_U(t)}{V(t)} = \frac{m_U(t)}{\frac{4}{3}\pi \aleph(t)^3} = \frac{\frac{m_U}{2}\left(1 + cos\left(\frac{2\pi t}{\frac{2\pi G m_U}{c^3}}\right)\right)}{\frac{4}{3}\pi\left(\left(\frac{2Gm_U}{c^2} + \frac{cm_U}{\frac{c^3}{4\pi G}}\right) - \frac{cm_U}{\frac{c^3}{4\pi G}} cos\left(\frac{2\pi t}{\frac{2\pi G m_U}{c^3}}\right)\right)^3}, \tag{173}$$

For $t = 10^{10}$ *light years*, $\rho_U = 1.7\ X\ 10^{-32}\ g/cm^3$. The density of luminous matter of stars and gas of galaxies is about $\rho_U = 2\ X\ 10^{-31}\ g/cm^3$ [35–36].

THE POWER OF THE UNIVERSE AS A FUNCTION OF TIME, $P_U(t)$

From $E = mc^2$ and Eq. (161),

$$P_U(t) = \frac{c^5}{8\pi G}\left(1 + cos\left(\frac{2\pi t}{\frac{2\pi r_G}{c}}\right)\right) \tag{174}$$

For $t = 10^{10}$ *light years*, $P_U(t) = 2.88\ X\ 10^{51}\ W$. The observed power is consistent with that predicted. The power of the Universe as a function of time is shown in Figure 17.

THE TEMPERATURE OF THE UNIVERSE AS A FUNCTION OF TIME

The temperature of the Universe as a function of time, $T_U(t)$, as shown in Figure 18, follows from the Stefan-Boltzmann law.

$$T_U(t) = \left(\frac{1}{1 + \frac{Gm_U(t)}{c^2 \aleph(t)}}\right)\left[\frac{R_U(t)}{e\sigma}\right]^{\frac{1}{4}} = \left(\frac{1}{1 + \frac{Gm_U(t)}{c^2 \aleph(t)}}\right)\left[\frac{\frac{P_U(t)}{4\pi \aleph(t)^2}}{e\sigma}\right]^{\frac{1}{4}} \tag{175}$$

The calculated uniform temperature is about 2.7 K which is in agreement with the observed microwave background temperature [34].

POWER SPECTRUM OF THE COSMOS

The power spectrum of the cosmos, as measured by the Las Campanas survey, generally follows the prediction of cold dark matter on the scales of 200 million to 600 million light-years. However, the power increases dramatically on scales of 600 million to 900 million light-years [33]. This discrepancy means that the Universe is much more structured on those scales than current theories can explain.

The Universe is oscillatory in matter/energy and spacetime with a finite minimum radius. The *minimum radius* which corresponds to the gravitational radius, r_g, given by Eq. (167) is $3.12\, X\, 10^{11}$ *light years*. The minimum radius is larger than that provided by the current expansion, approximately 10 billion light years [34]. The Universe is a four dimensional hyperspace of constant positive curvature at each r-sphere. The coordinates are spherical, and the space can be described as a series of spheres each of constant radius r whose centers coincide at the origin. The existence of the mass m_U causes the area of the spheres to be less than $4\pi r^2$ and causes the clock of each r-sphere to run so that it is no longer observed from other r-spheres to be at the same rate. The Schwarzschild metric given by Eq. (140) is the general form of the metric which allows for these effects. Consider the present observable Universe that has undergone expansion for 10 billion years. The radius of the Universe as a function of time from the coordinate r-sphere is of the same form as Eq. (168). The average size of the Universe, r_U, is given as the sum of the gravitational radius, r_g, and the observed radius, 10 billion light years.

$$r_U = r_g + 10^{10}\ \textit{light yrs} = 3.12\, X\, 10^{11}\ \textit{light yrs} + 10^{10}\ \textit{light years} = 3.22\, X\, 10^{11}\ \textit{light yrs} \quad (176)$$

The frequency of Eq. (168) is one half the amplitude of spacetime expansion from the conversion of the mass of Universe into energy according to Eq. (161). Thus, keeping the same relationships, the frequency of the current expansion function is the reciprocal of one half the current age. Substitution of the average size of the Universe, the frequency of expansion, and the amplitude of expansion, 10 billion light years, into Eq. (168) gives *the radius of the Universe as a function of time for the coordinate r-sphere*.

$$\aleph = 3.22\, X\, 10^{11} - 1\, X\, 10^{10} \cos\left(\frac{2\pi t}{5\, X\, 10^{9}\ \textit{light years}}\right) \textit{light years} \quad (177)$$

The Schwarzschild metric gives the relationship between the proper time and the coordinate time. The infinitesimal temporal displacement, $d\tau^2$, is given by Eq. (140). In the case that $dr^2 = d\theta^2 = d\phi^2 = 0$, the relationship between the proper time and the coordinate time is

$$d\tau^2 = \left(1 - \frac{2Gm_U}{c^2 r}\right) dt^2 \tag{178}$$

$$\tau = t\sqrt{1 - \frac{r_g}{r}} \tag{179}$$

The maximum power radiated by the Universe is given by Eqs. (174) which occurs when the proper radius, the coordinate radius, and the gravitational radius r_g are equal. For the present Universe, the coordinate radius is given by Eq. (176). The gravitational radius is given by Eq. (167). The maximum of the power spectrum of a trigonometric function occurs at its frequency [37]. Thus, the coordinate maximum power according to Eq. (177) occurs at $5\,X\,10^9$ *light years*. The maximum power corresponding to the proper time is given by the substitution of the coordinate radius, the gravitational radius r_g, and the coordinate power maximum into Eq. (179). The power maximum in the proper frame occurs at

$$\tau = 5\,X\,10^9\ \textit{light years}\sqrt{1 - \frac{3.12\,X\,10^{11}\ \textit{light years}}{3.22\,X\,10^{11}\ \textit{light years}}} \tag{180}$$

$$\tau = 880\,X\,10^6\ \textit{light years}$$

The power maximum of the current observable Universe is predicted to occur on the scale of $880\,X\,10^6$ *light years*. There is excellent agreement between the predicted value and the experimental value of $600 - 900\,X\,10^6$ *light years* [33].

THE EXPANSION/CONTRACTION ACCELERATION, $\ddot{\aleph}$

The expansion/contraction acceleration rate, $\ddot{\aleph}$, as shown in Figure 19, is given by the time derivative of Eq. (169).

$$\ddot{\aleph} = 2\pi \frac{c^4}{Gm_U} cos\left(\frac{2\pi t}{\frac{2\pi G m_U}{c^3}\ sec}\right) \ddot{\aleph} = H_o = 78.7\, cos\left(\frac{2\pi t}{3.01\,X\,10^5\ Mpc}\right) \frac{km}{sec \cdot Mpc} \tag{181}$$

The differential in the radius of the Universe, $\Delta\aleph$, due to its acceleration is given by $\Delta\aleph = 1/2\ddot{\aleph}t^2$. The differential in expanded radius for the elapsed time of expansion, $t = 10^{10}$ *light years* corresponds to a decease in brightness of a supernovae standard candle of about an order of magnitude of that expected where the distance is taken as $\Delta\aleph$. This result based on the predicted rate of acceleration of the expansion is consistent with the experimental observation [38–40].

Furthermore, the microwave background radiation image obtained by the Boomerang telescope [41] is consistent with a Universe of nearly flat geometry since the commencement of its expansion. The data is consistent with a large offset radius of the Universe with a fractional increase in size since the commencement of expansion about 10 billion years ago.

THE PERIODS OF SPACETIME EXPANSION/CONTRACTION AND PARTICLE DECAY/PRODUCTION FOR THE UNIVERSE ARE EQUAL

The period of the expansion/contraction cycle of the radius of the Universe, T, is given by Eq. (162). It follows from the Poynting power theorem with spherical radiation that the transition lifetimes are given by the ratio of energy and the power of the transition (Eqs. (60-61)). Exponential decay applies to electromagnetic energy decay

$$h(t) = e^{-\frac{1}{T}t}u(t) \tag{182}$$

The coordinate time is imaginary because energy transitions are spacelike due spacetime expansion from matter to energy conversion. For example, the mass of the electron (a fundamental particle) is given by

$$\frac{2\pi\bar{\lambda}_C}{\sqrt{\frac{2Gm_e}{\bar{\lambda}_C}}} = \frac{2\pi\bar{\lambda}_C}{v_g} = i\alpha^{-1}\ sec \tag{183}$$

where v_g is Newtonian gravitational velocity (Eq. (132)). When the gravitational radius r_g is the radius of the Universe, the proper time is equal to the coordinate time by Eq. (142), and the gravitational escape velocity v_g of the Universe is the speed of light. Replacement of the coordinate time, t, by the spacelike time, it, gives

$$h(t) = Re\left[e^{-i\frac{1}{T}t}\right] = cos\frac{2\pi}{T}t \tag{184}$$

where the period is T (Eq. (162)). The continuity conditions based on the constant maximum speed of light (Maxwell's equations) are given by Eqs. (143-144). The continuity conditions based on the constant maximum speed of light (Schwarzschild metric) are given by Eq. (145). The periods of spacetime expansion/contraction and particle decay/production for the Universe are equal because only the particles which satisfy Maxwell's equations and the relationship between proper time and coordinate time imposed by the Schwarzschild metric may exist.

WAVE EQUATION

The general form of the light front wave equation is given by Eq. (124). The equation of the radius of the Universe, $\aleph$, may be written as

$$\aleph = \left(\frac{2Gm_U}{c^2} + \frac{cm_U}{\frac{c^3}{4\pi G}}\right) - \frac{cm_U}{\frac{c^3}{4\pi G}} cos\left(\frac{2\pi}{\frac{2\pi Gm_U}{c^3}sec}\left(t - \frac{\aleph}{c}\right)\right)m, \tag{185}$$

which is a solution of the wave equation for a light wave front.

CONCLUSION

Maxwell’s equations, Planck’s equation, the de Broglie equation, Newton’s laws, and special, and general relativity are unified.

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Table 1. The calculated electric (per electron), magnetic (per electron), and ionization energies for some two-electron atoms.

Atom	( )[a]	Electric Energy[b] (eV)	Magnetic Energy[c] (eV)	Calculated Ionization Energy[d] (eV)	Experimental[e] Ionization [14-15] Energy (eV)
	0.567	-23.96	0.63	24.59	24.59
	0.356	-76.41	2.54	75.56	75.64
	0.261	-156.08	6.42	154.48	153.89
	0.207	-262.94	12.96	260.35	259.37
	0.171	-396.98	22.83	393.18	392.08
	0.146	-558.20	36.74	552.95	552.06
	0.127	-746.59	55.35	739.67	739.32
	0.113	-962.17	79.37	953.35	953.89

[a] from Equation (96)
[b] from Equation (98)
[c] from Equation (99)
[d] from Equations (97) and (100)

Figure 1. The orbitsphere is a two dimensional spherical shell with the Bohr radius of the hydrogen atom.

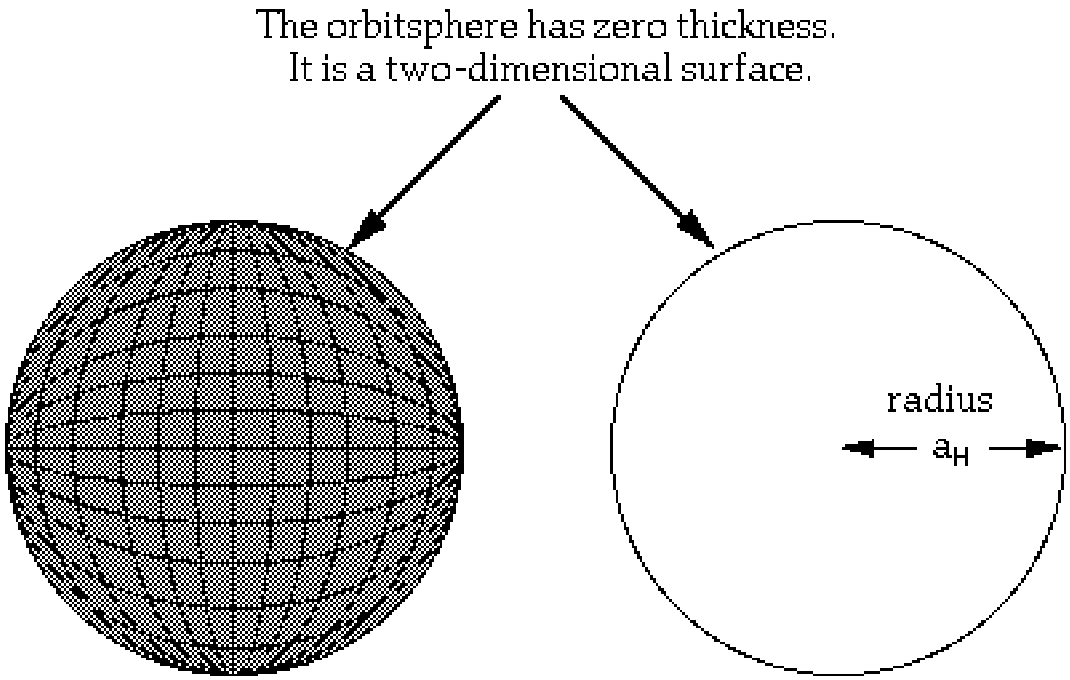
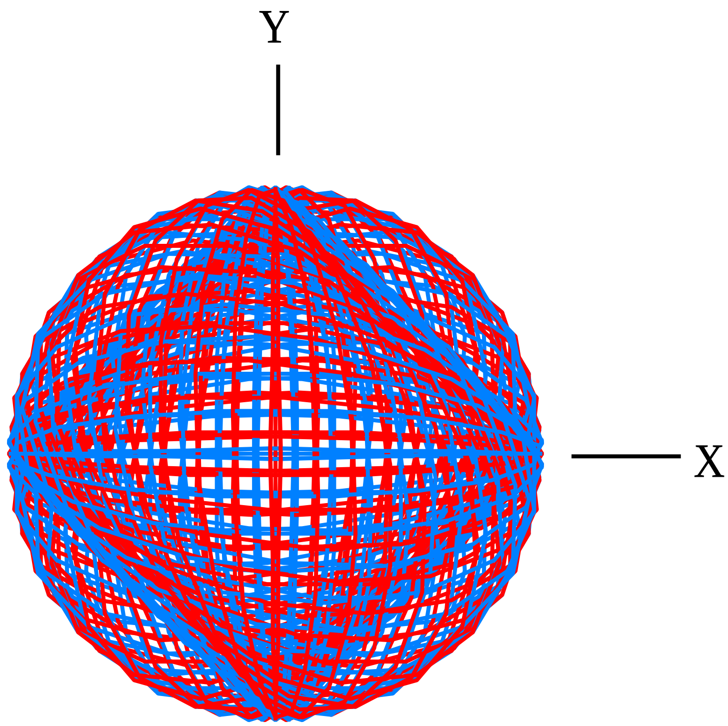


Figure 2. The current pattern of the orbitsphere from the perspective of looking along the z-axis. The current and charge density are confined to two dimensions at $r_n = nr_1$. The corresponding charge density function is uniform.



VIEW ALONG THE Z AXIS

Figure 3. The orbital function modulates the constant (spin) function (shown for t = 0; cross-sectional view).

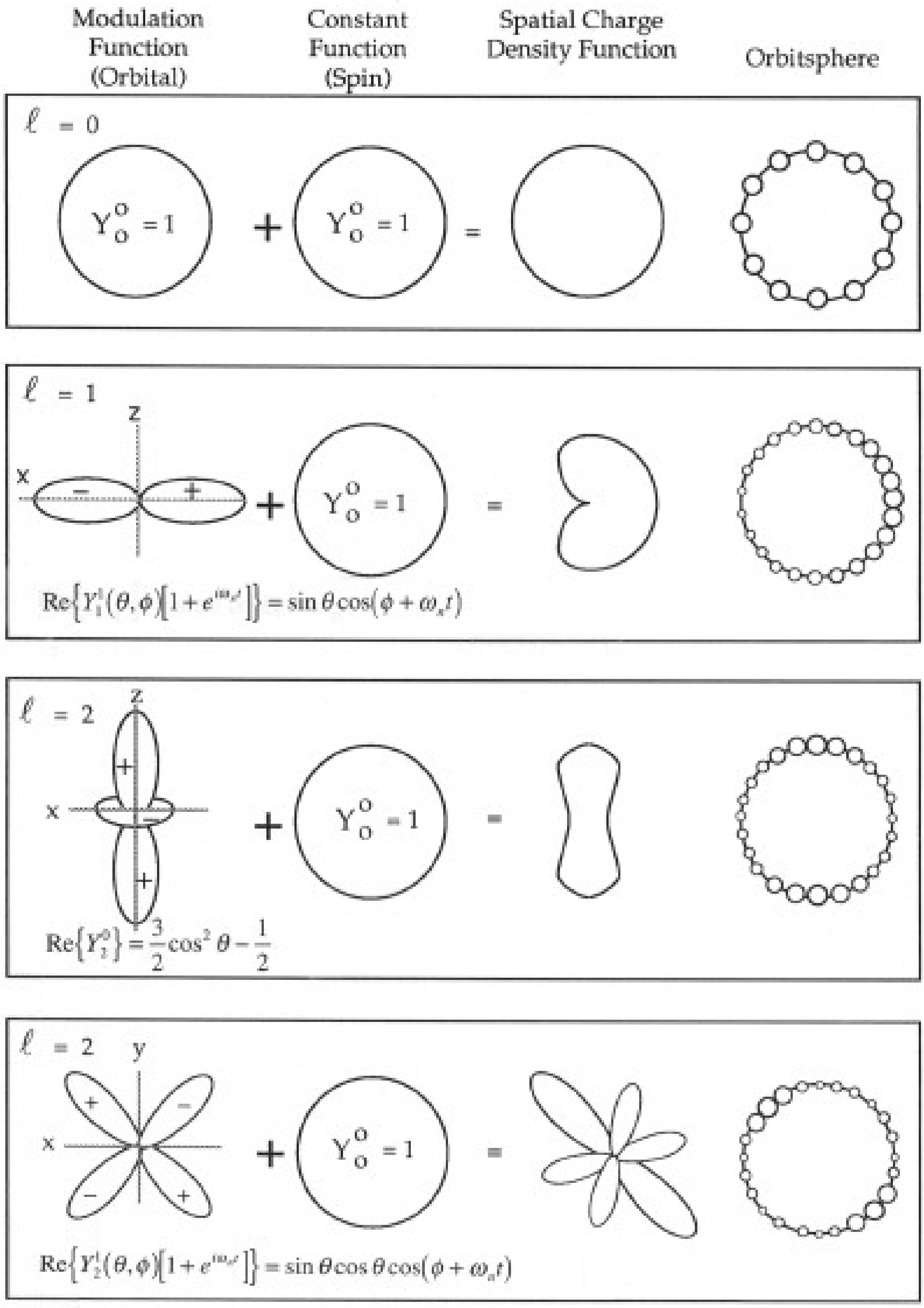


Figure 4. The magnetic field of an electron orbitsphere.

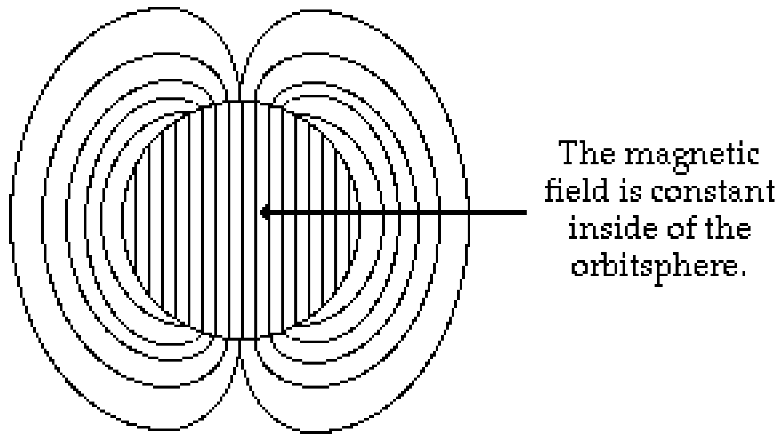


Figure 5. Broadening of the spectral line due to the rise-time and shifting of the spectral line due to the radiative reaction. The resonant line shape has width Γ. The level shift is $\Delta\omega$.

Figure 6. The Cartesian coordinate system wherein the first great circle magnetic field line lies in the yz-plane, and the second great circle electric field line lies in the xz-plane is designated the photon orbitsphere reference frame of a photon orbitsphere.

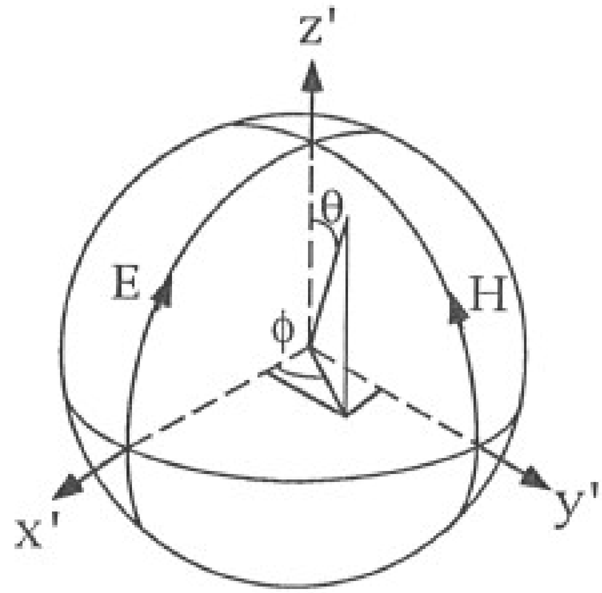


Figure 7. The field line pattern from the perspective of looking along the z-axis of a right-handed circularly polarized photon.

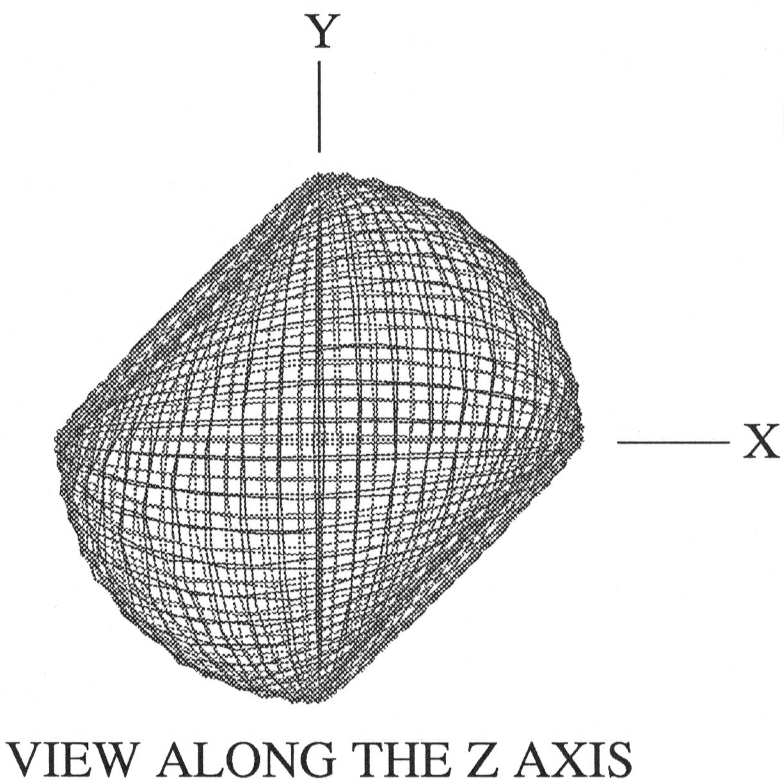


Figure 8. The electric field of a moving point charge ($v = \frac{4}{5}c$).

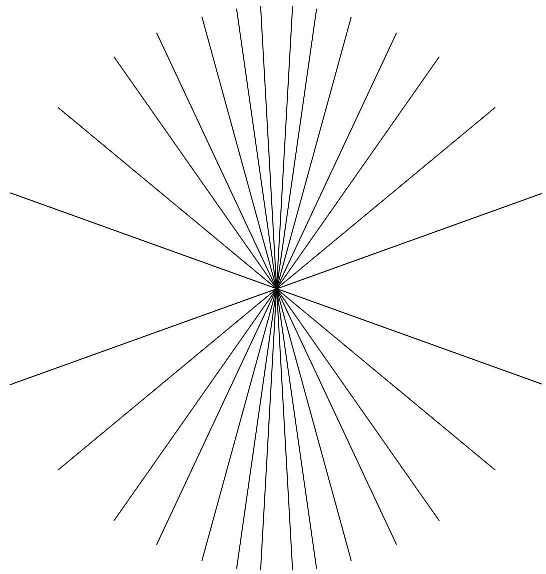


Figure 9. The electric field lines of a right-handed circularly polarized photon orbitsphere as seen along the axis of propagation in the lab inertial reference frame as it passes a fixed point.

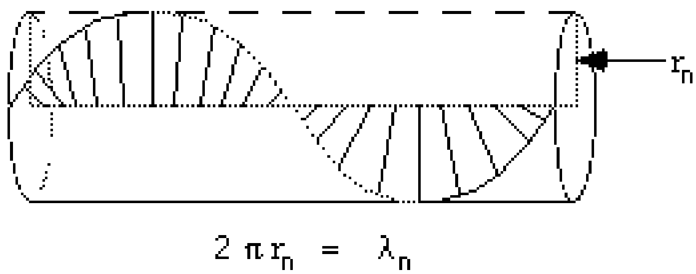
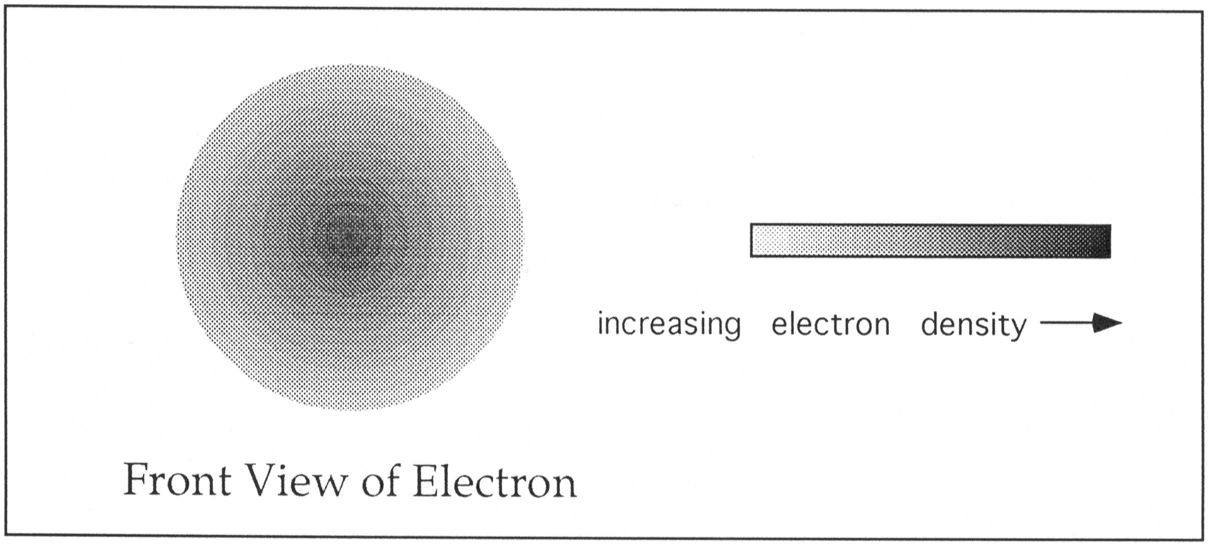


Figure 10. The front view of the magnitude of the mass (charge) density function in the xy-plane of a free electron; side view of a free electron along the axis of propagation--z-axis.



$$\rho_0 = \frac{\hbar}{mv_z}$$

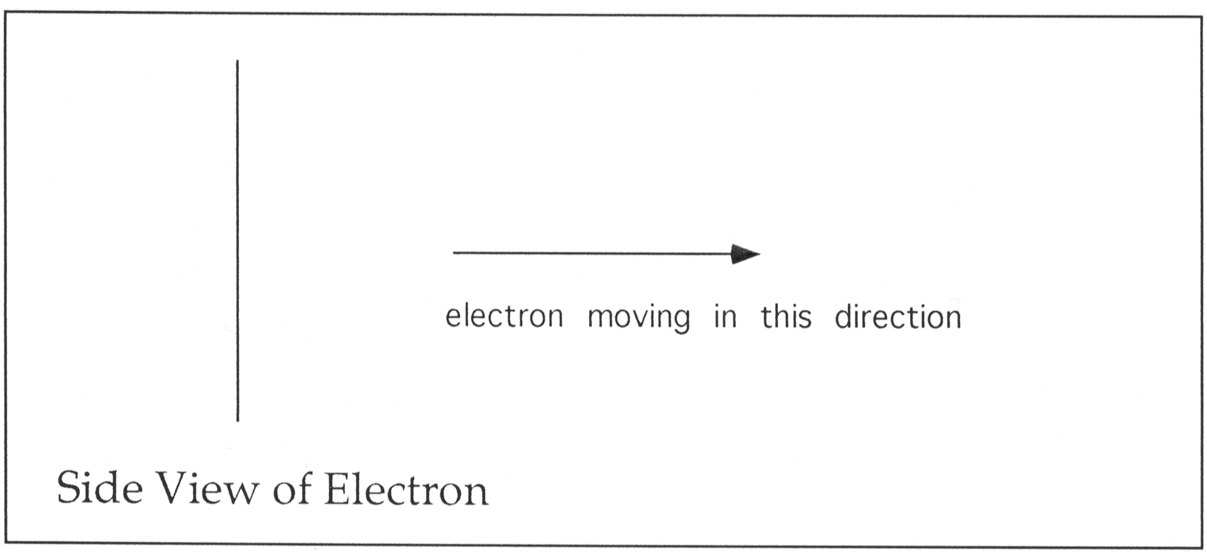


Figure. 11. The experimental results for the elastic differential cross section for the elastic scattering of electrons by helium atoms and a Born approximation prediction.

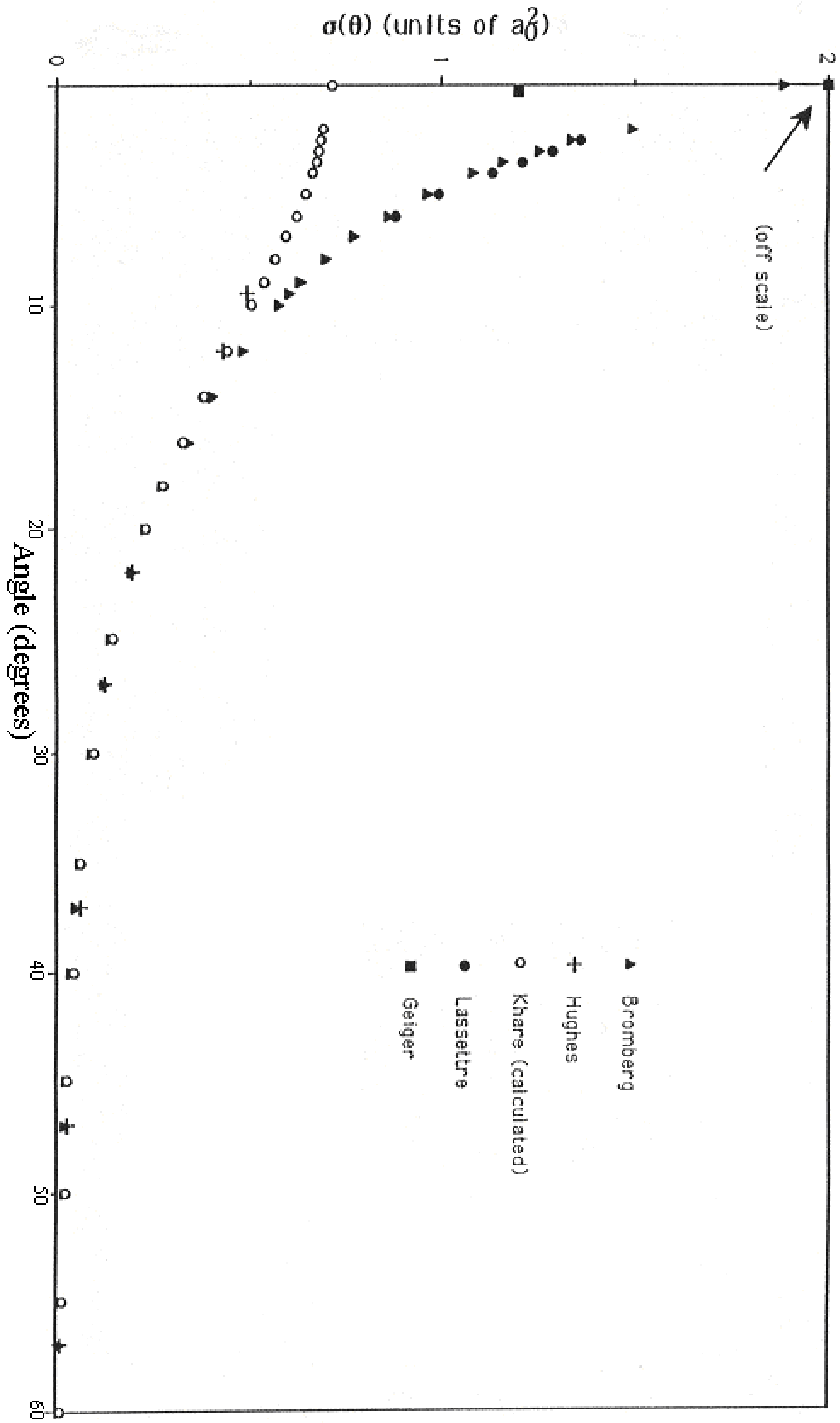


Figure. 12. The closed form function [Eqs. (105) and (106)] for the elastic differential cross section for the elastic scattering of electrons by helium atoms. The scattering amplitude function, $F(s)$ (Eq. (104), is shown as an insert.

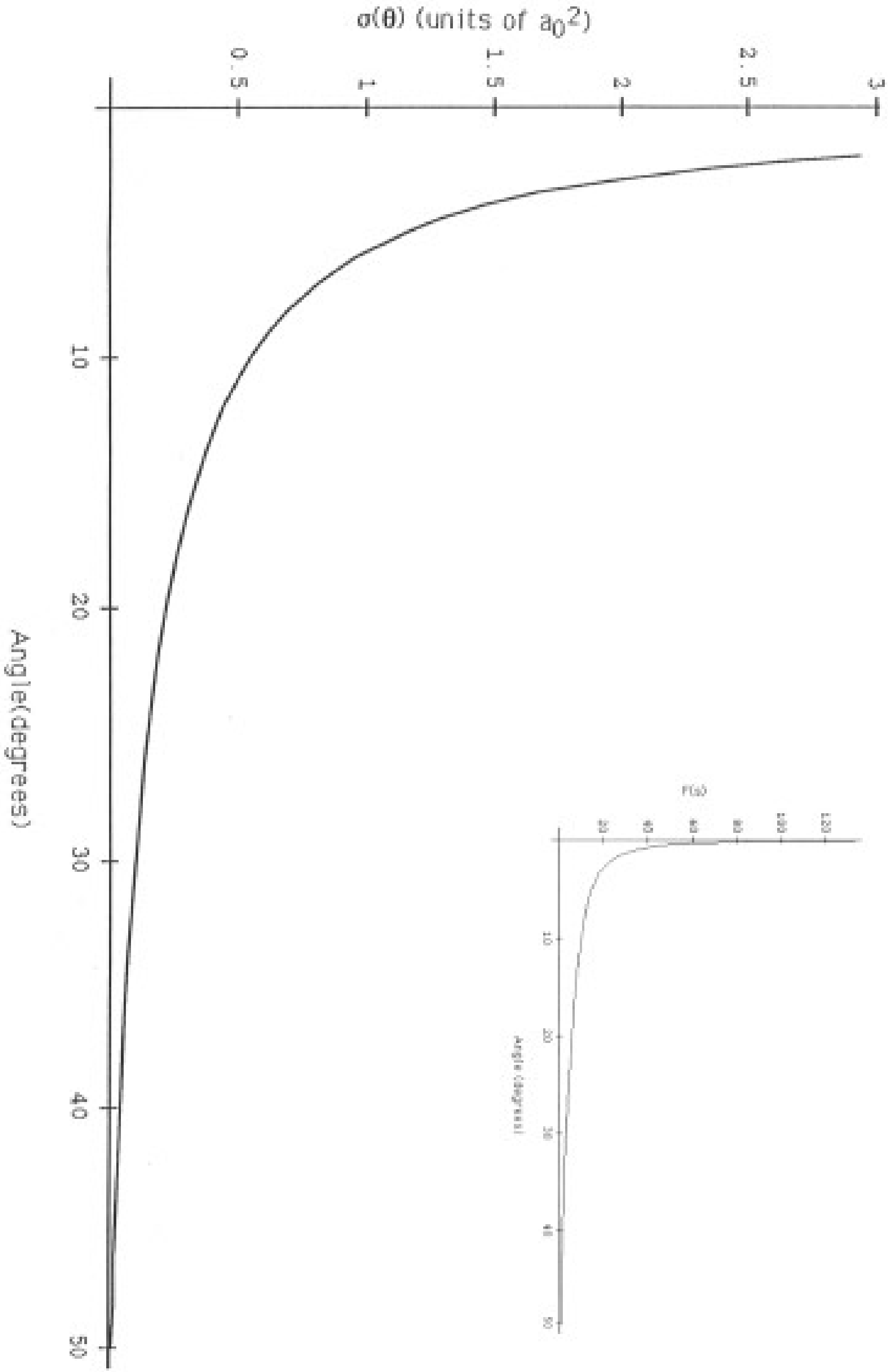


Figure 13. The radius of the universe as a function of time.

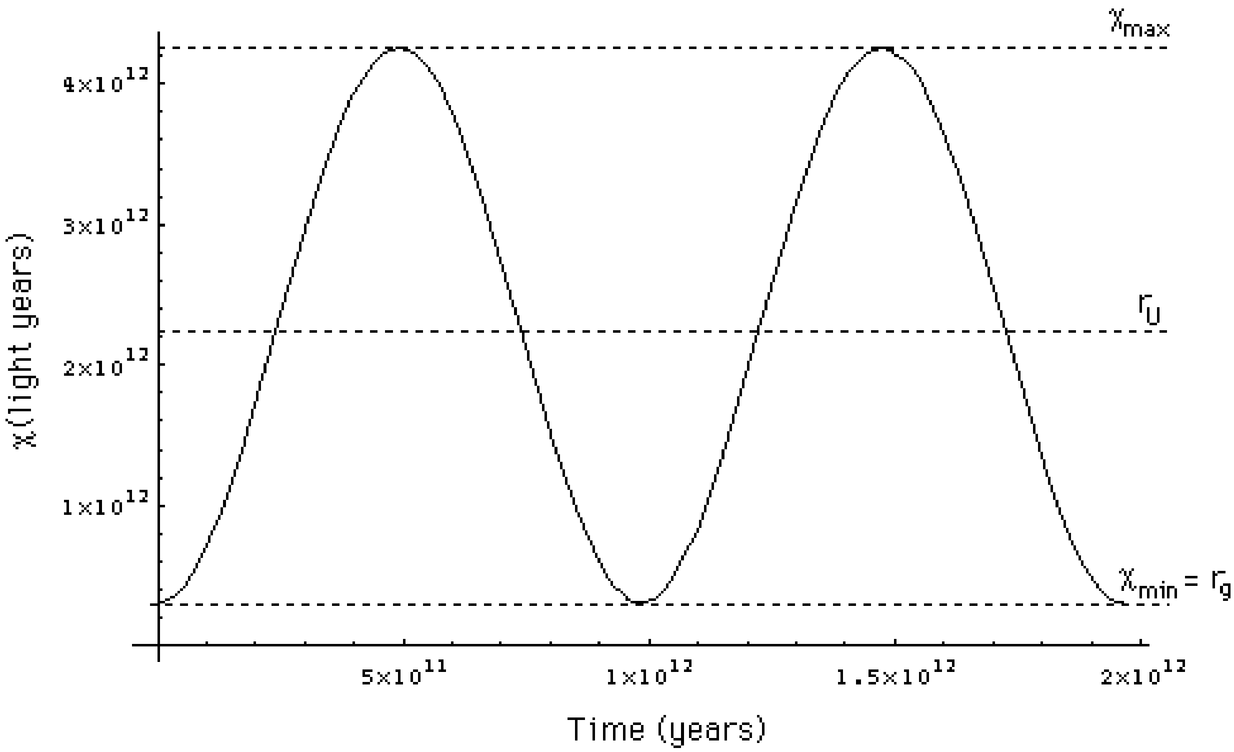


Figure 14. The expansion/contraction rate of the universe as a function of time.

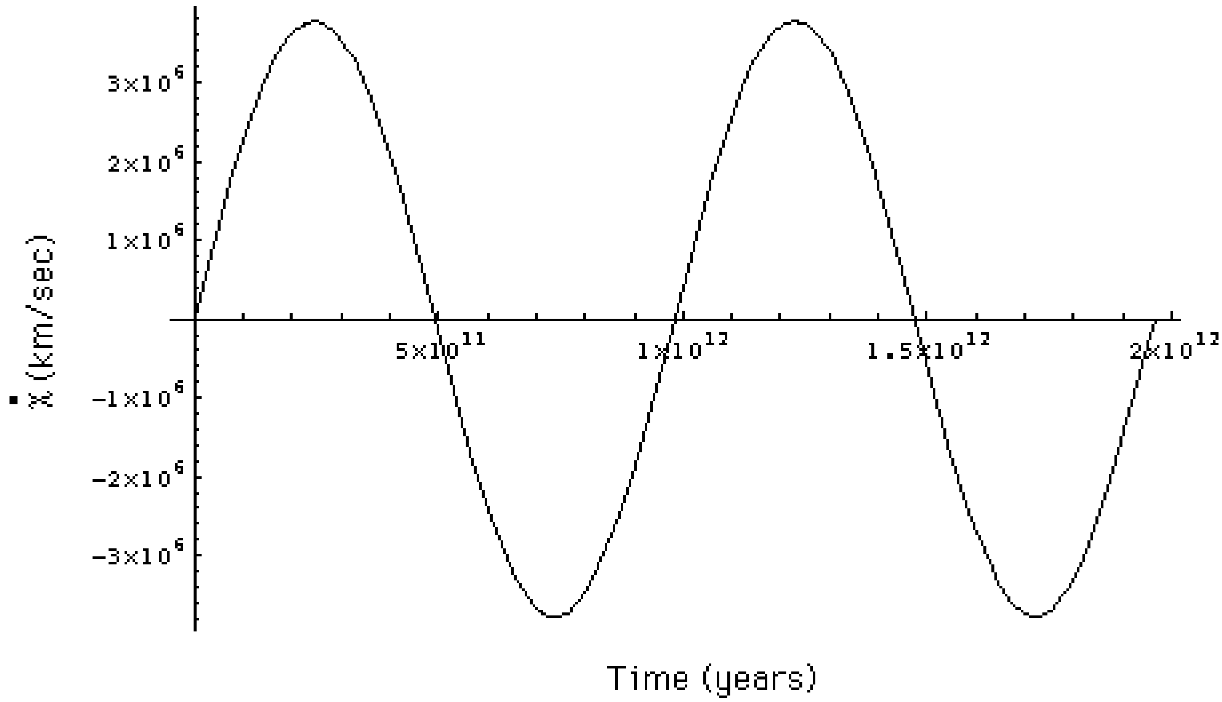


Figure 15. The Hubble constant of the universe as a function of time.

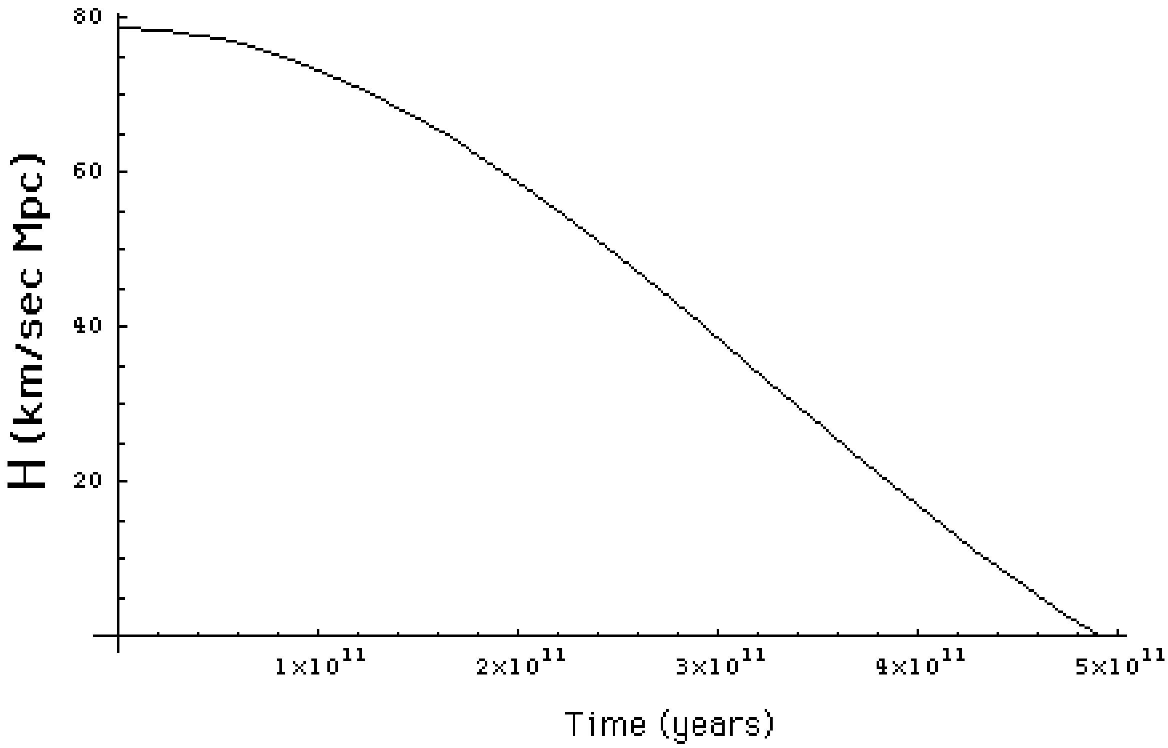


Figure 16. The density of the universe as a function of time.

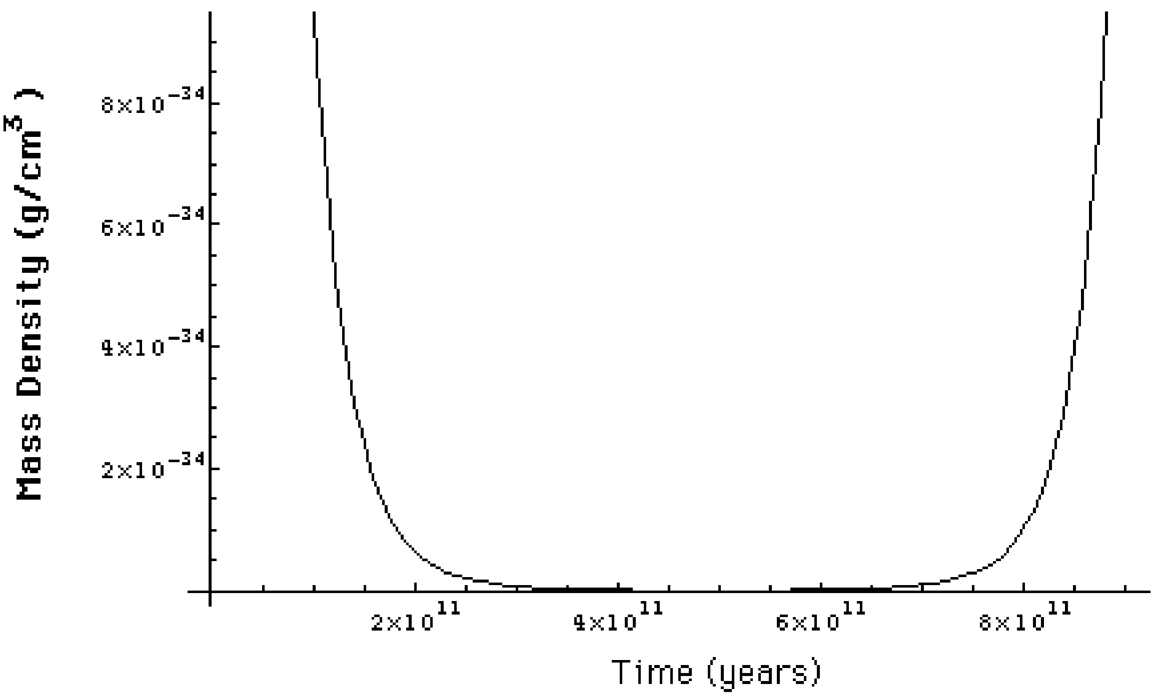


Figure 17. The power of the universe as a function of time.

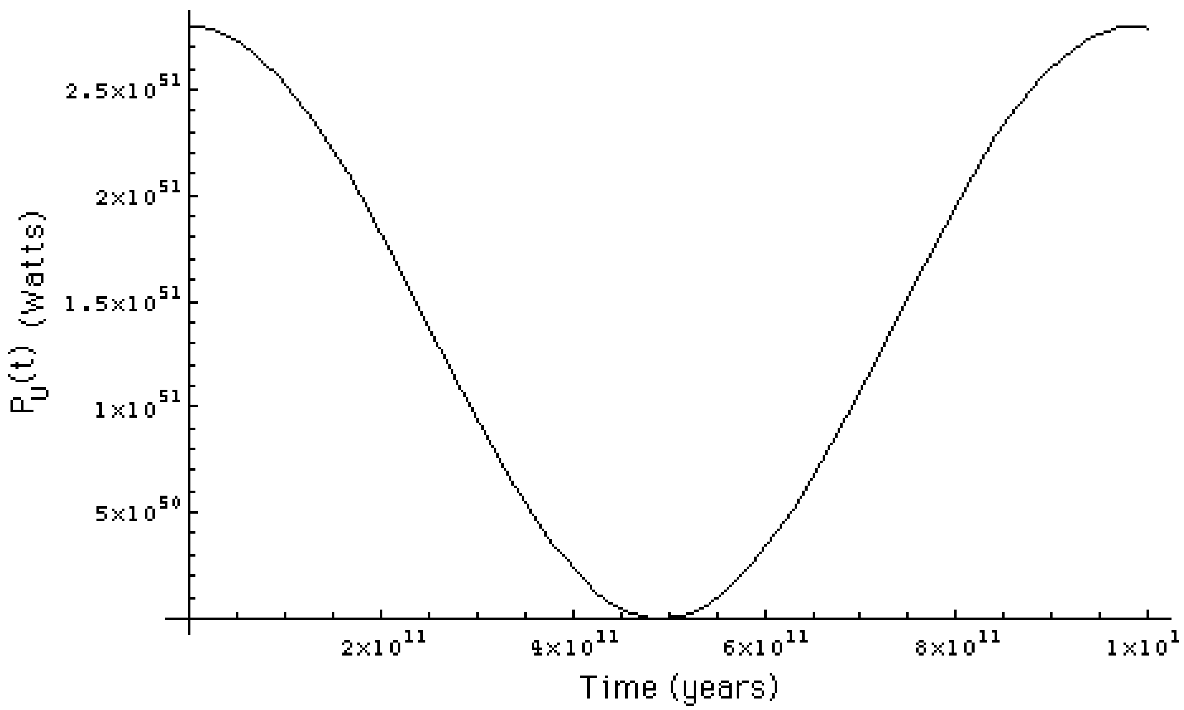


Figure 18. The temperature of the universe as a function of time during the expansion phase.

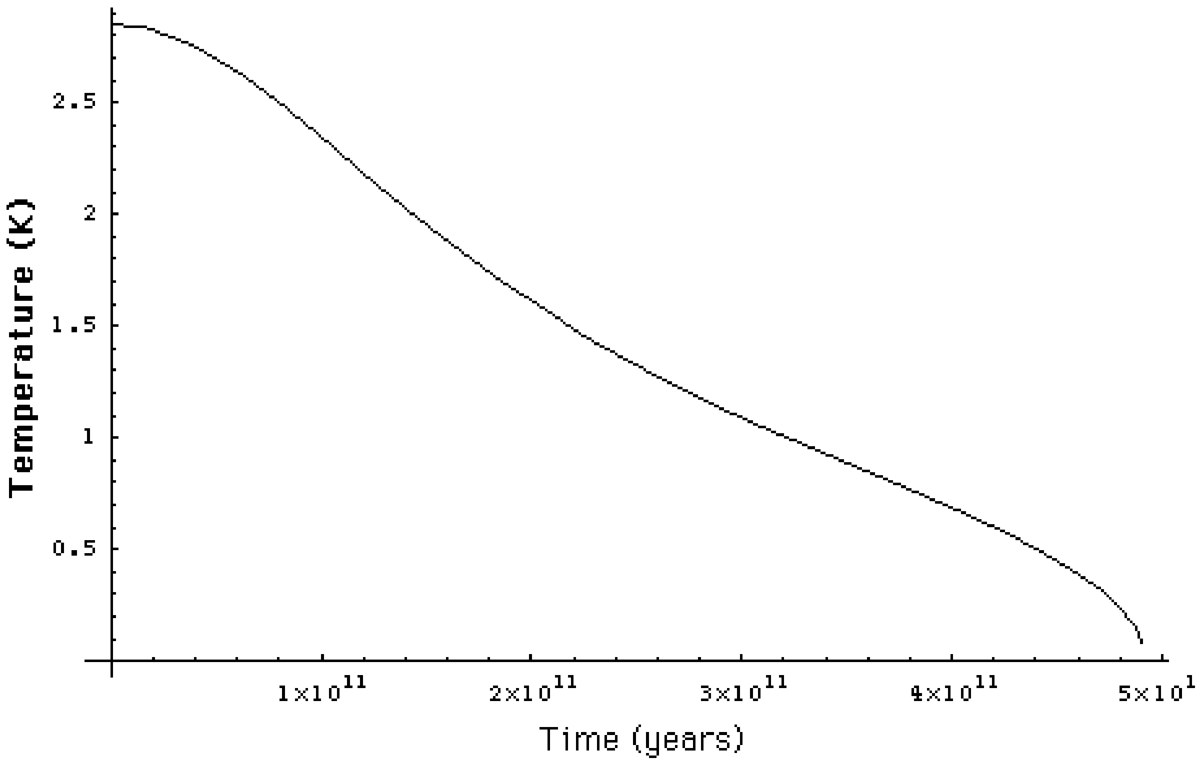


Figure 19. The differential expansion of the light sphere due to the acceleration of the expansion of the cosmos as a function of time.